

Title: Quantum Theory - Lecture 220921

Speakers: Dan Wohns

Collection: Quantum Theory (2022-2023)

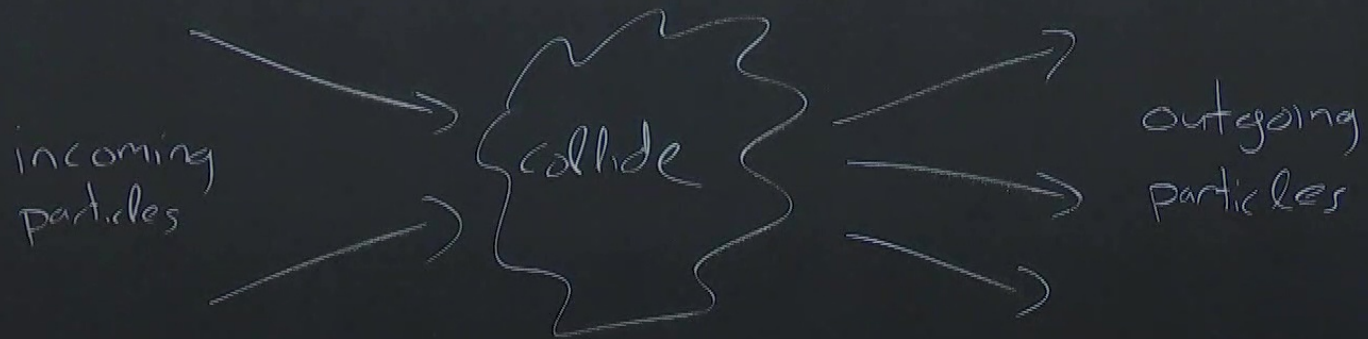
Date: September 21, 2022 - 10:45 AM

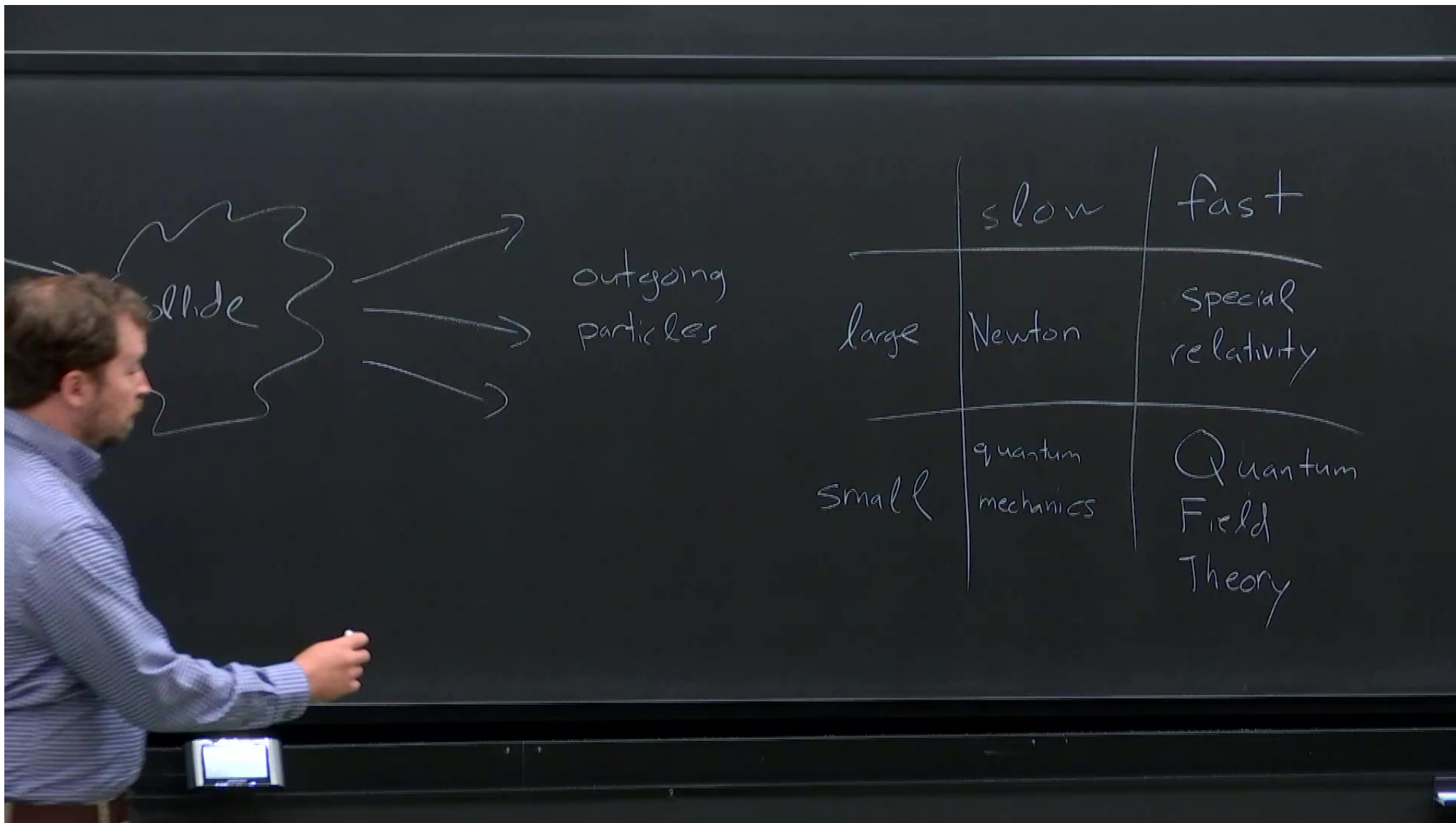
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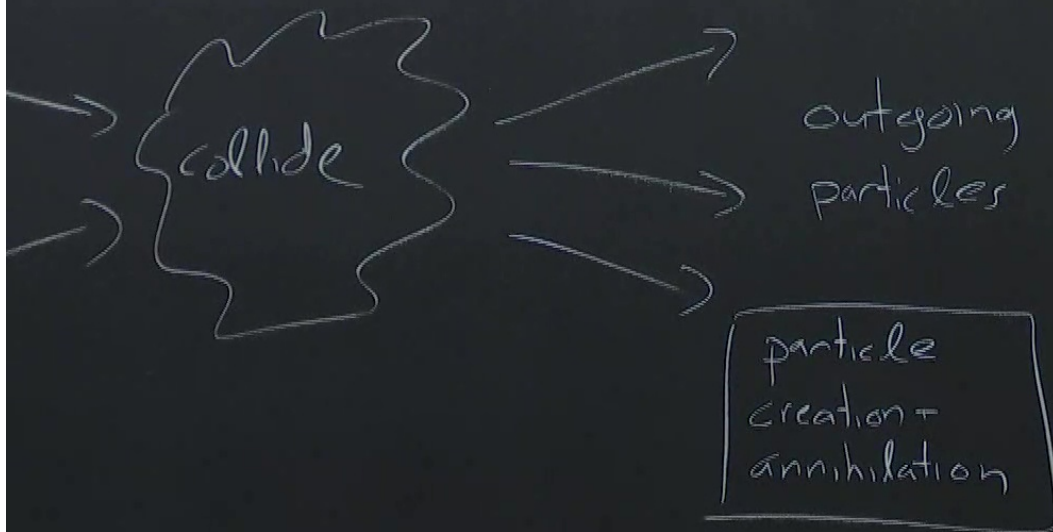
ativistic)

m)

ory







	slow	fast
large	Newton	special relativity
small	quantum mechanics	Quantum Field Theory

Natural units

$$\hbar = c = 1$$

$$[m] = 1 \quad (m = m^1)$$

↑
mass dimension

$$[E] = 1 = [p]$$

$E = mc^2$ $E = \frac{p^2}{2m}$

$$[x] = [t]$$

$x = ct$

Natural units

$$\hbar = c = 1$$

$$[m] = 1 \quad (m = m^1)$$

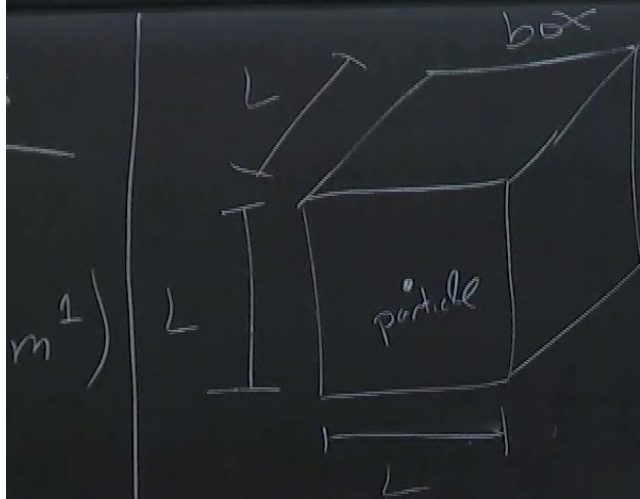
↑
mass dimension

$$[E] = 1 = [p]$$

$E = mc^2$ $E = \frac{p^2}{2m}$

$$[x] = [t] = -1$$

$x = ct$ $[x, p] = \hbar$



$$\Delta p \approx \frac{1}{L} \quad (\text{QM})$$

$$E \approx p \quad (\text{SR } p \gg m)$$

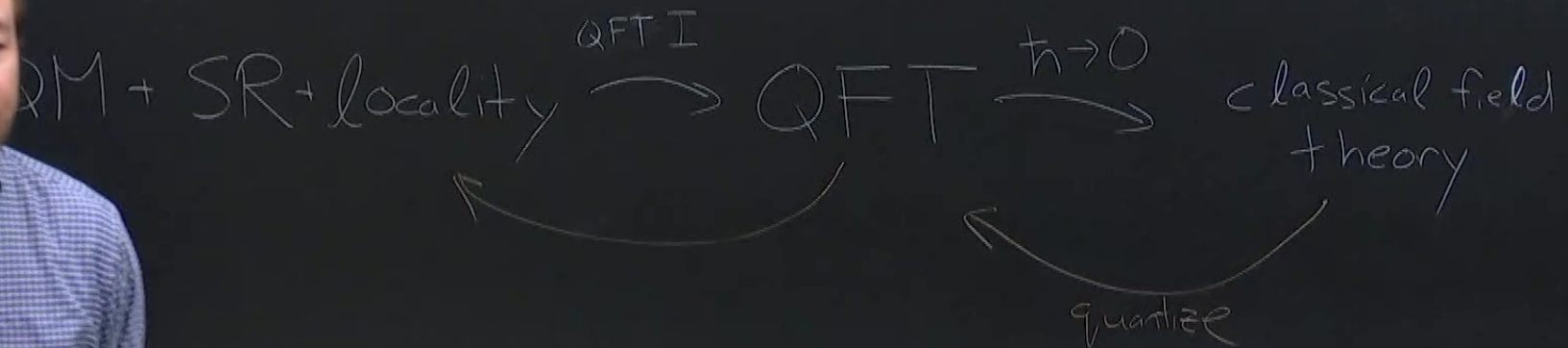
$$E = m \quad (\text{SR})$$

$$\Delta E \approx 2m \quad (\text{one or two particles?})$$



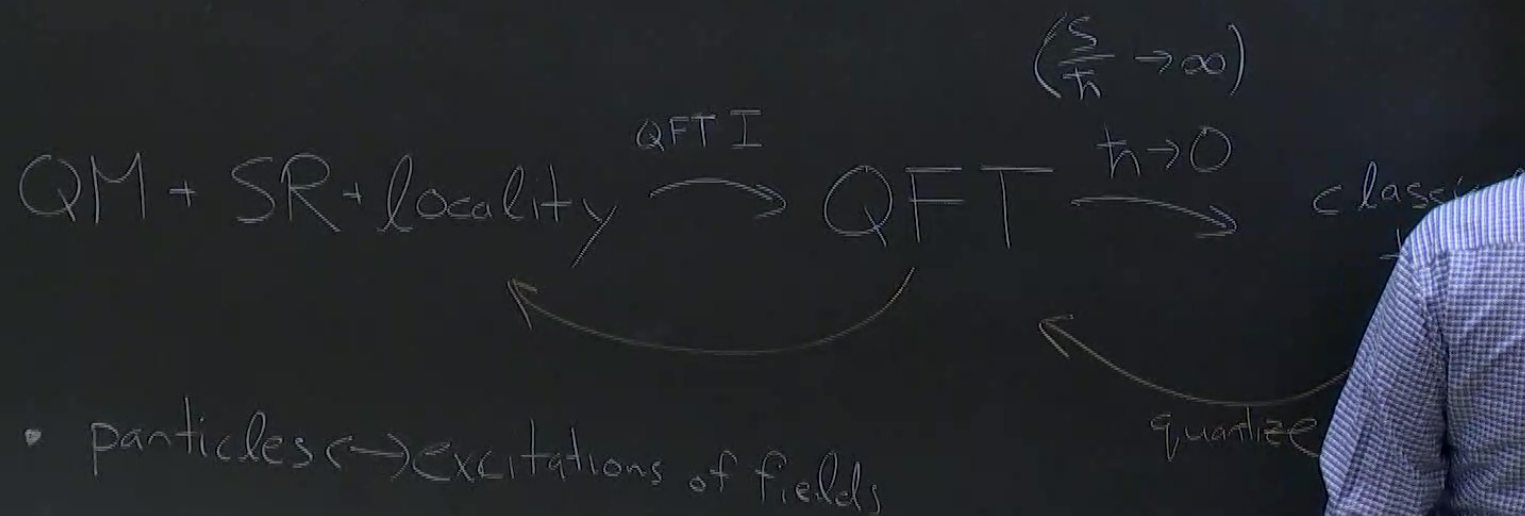
Why fields?

- locality



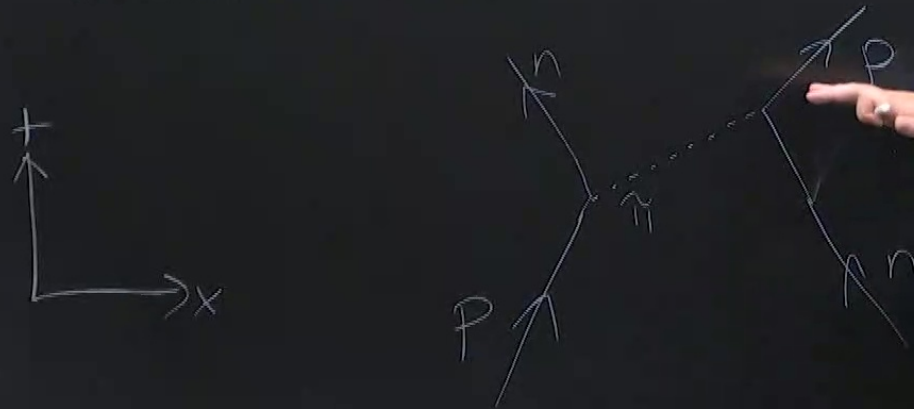
Why fields?

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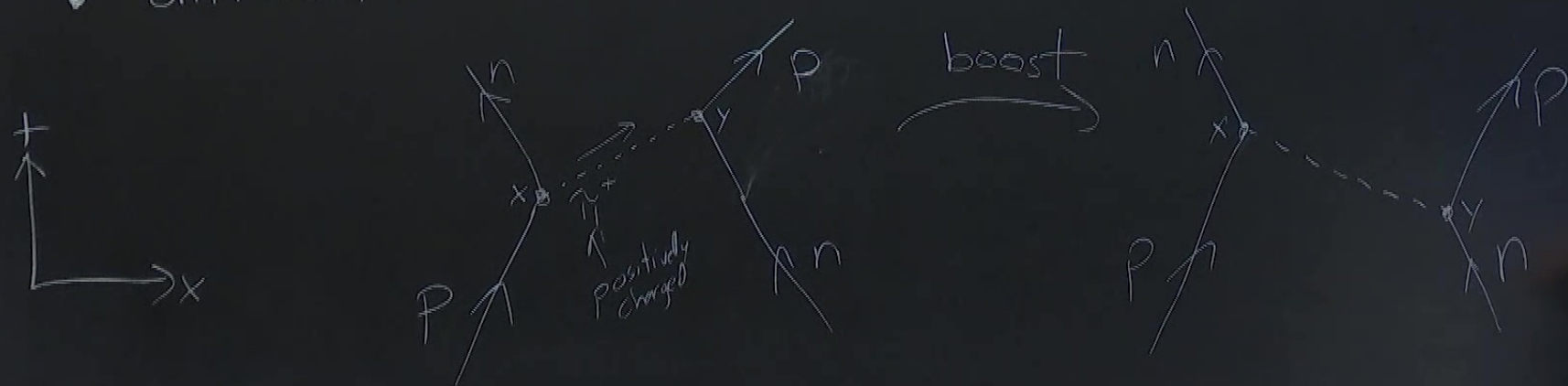
Mysteries Solved

- identical particles
- antimatter



Mysteries Solved

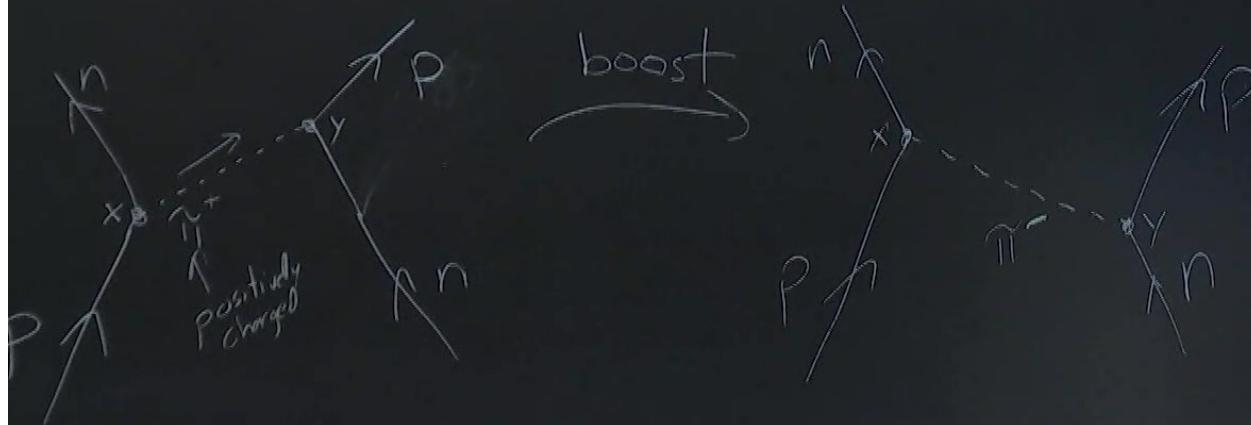
- identical particles
- antimatter



Solved

particles

er ($\pi^+ + \pi^-$ are antiparticles)



• spin-statistics
 $s=0,1,2,\dots \leftrightarrow [,]$

• spin-statistics
 $s=0,1,2,\dots \leftrightarrow [,]$
 $s=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots \leftrightarrow \{ , \}$

Quantization

• $\varphi \rightarrow \hat{\varphi}$

canonical quantization - QT, QFT I

$\varphi \rightarrow \int \mathcal{D}\varphi$

path or functional integral - QFT II

RT, QFT I

- QFT II

Position + Time

$$\hat{X} + \xrightarrow[\text{promote}]{+} \hat{X} + \hat{f}(\tau)$$

$$\xrightarrow[\text{add parameter}]{} \hat{X}^M(\tau, \sigma)$$

spatial parameter
string theory

Position + Time

$$\hat{X}, + \xrightarrow{\text{promote}} \hat{X}, + \xrightarrow{\text{add parameter}} \hat{X}^\mu(z, \sigma)$$

$f(z)$

spatial parameter

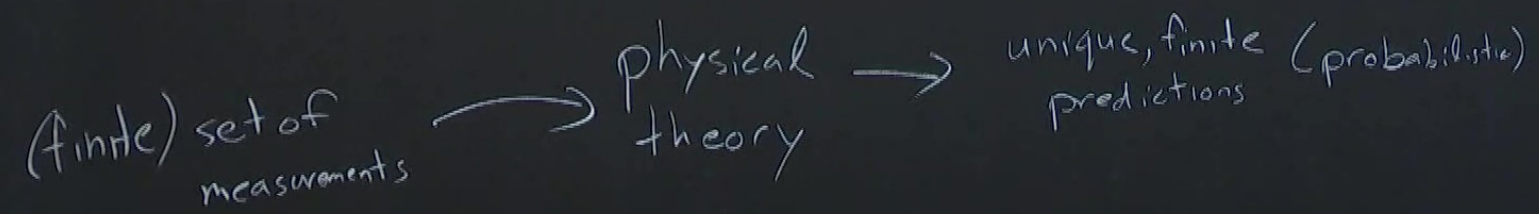
string theory

(denote
 $\hat{X}$)

$X, +$

will not discuss
position operator

What can we measure?



infinities in unobservable quantities

What can we measure?

set of measurements $\rightarrow$ physical theory $\rightarrow$ unique, finite (probabilistic) predictions

infinities in unobservable quantities
renormalization

Classical fields

$$\phi(\vec{x}, t) = \phi(x)$$

Scalar field $\nearrow$ $\nearrow$ 3-vector (bold) d^3x $\nwarrow$ 4-vector (italic) d^4x

$$\eta^{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$x^2 = t^2 - \vec{x}^2$$



Classical fields

$$\phi(\vec{x}, t) = \phi(x)$$

↑
scalar
field

↑
3-vector (bold)
 d^3x

↑ 4-vector (italic)

d^4x

↑ generalized
coordinates

$$\phi(\vec{x}, t) \leftrightarrow q_a(t)$$

$$\eta^{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$x^2 = t^2 - \vec{x}^2$$

c)

Lagrangian

L

$$S = i\hbar \int d^3x \mathcal{L}(\varphi, \partial_\mu \varphi)$$

↑
Lagrangian
(density)

(probabilistic)

$$\varphi(\vec{x}, t) = \varphi(x)$$

Scalar
field

3-vector (bold)

d^3x

4-vector (italic)

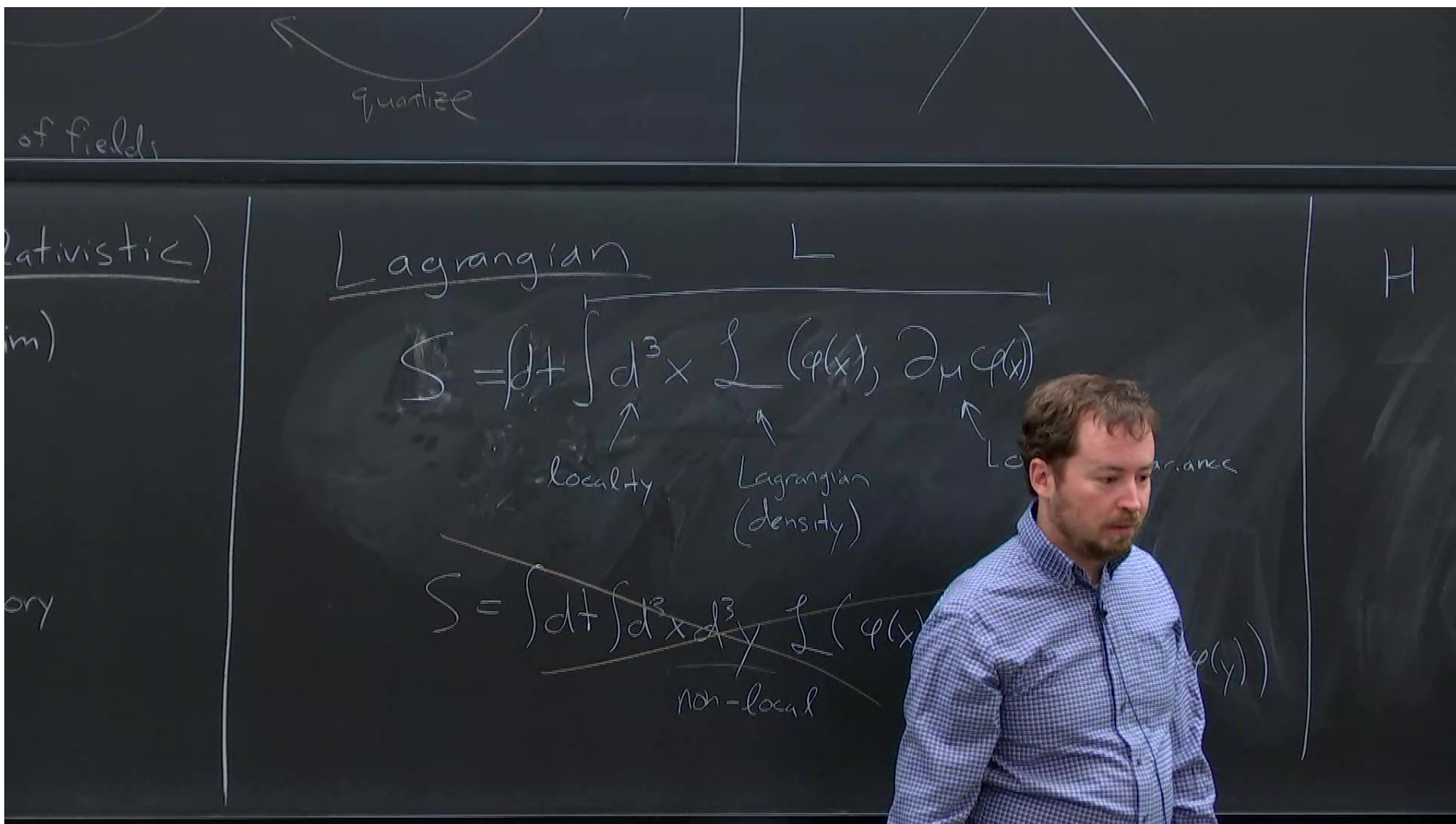
d^4x

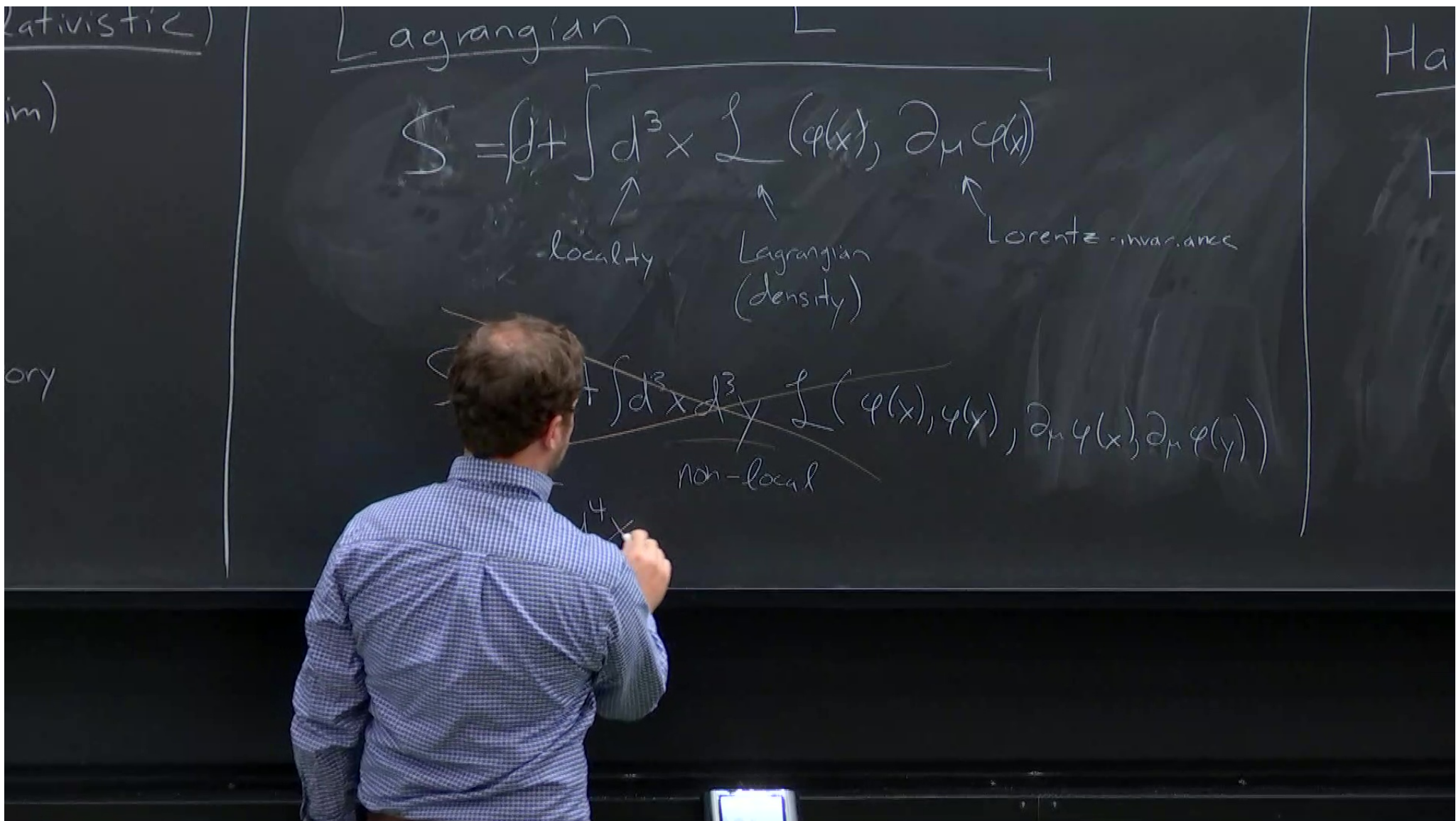
generalized
coordinates

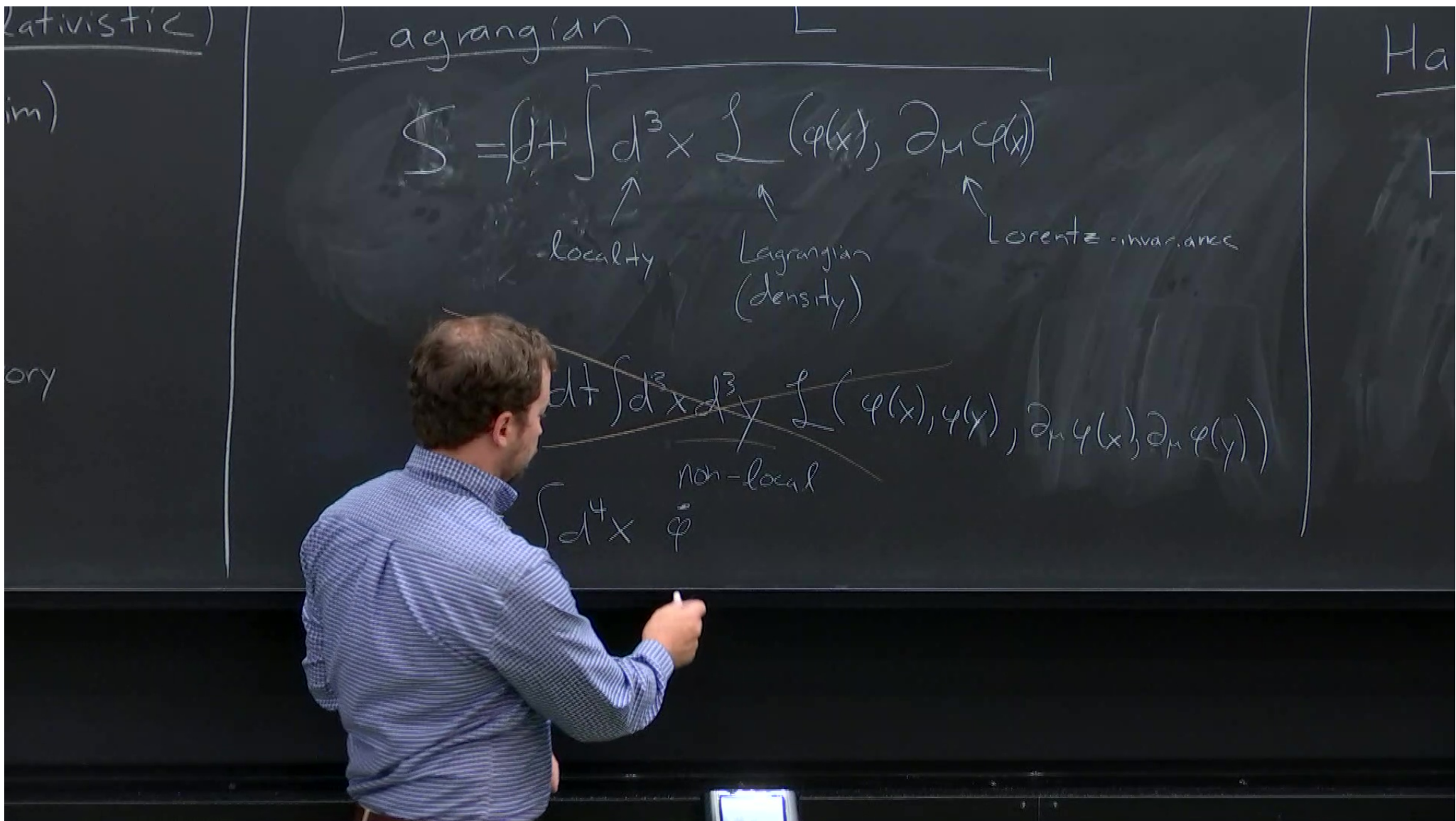
$$\varphi(\vec{x}, t) \leftrightarrow q_a(t)$$

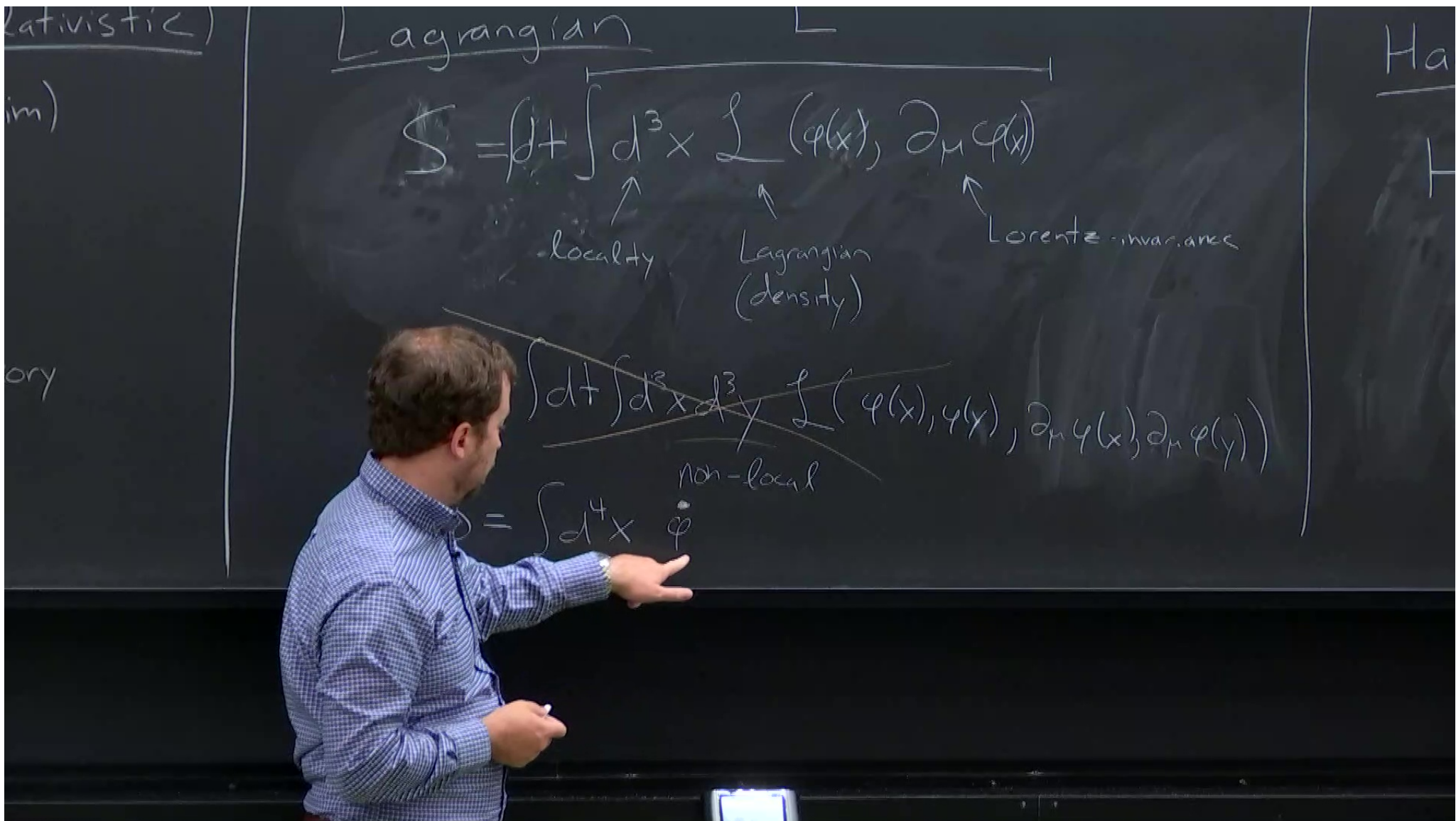
label

$$x^2 = t^2 - \vec{x}^2$$









relativistic)

im)

ory

Lagrangian

$$S = \int dt \int d^3x \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$$

locality

Lagrangian
(density)

Lorentz-invariance

~~$$S = \int dt \int d^3x \int d^3y \mathcal{L}(\varphi(x), \partial_\mu \varphi(x), \partial_\mu \varphi(y))$$~~

non-local

$S = \int d^4x \dot{\varphi}$

non-l

$$S = \int d^4x \partial_\mu \varphi \partial^\mu \varphi$$

Ha

relativistic)

im)

ory

Lagrangian

$$S = \int dt \int d^3x \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$$

$\uparrow$
 locality

$\uparrow$
 Lagrangian
(density)

$\uparrow$
 Lorentz-invariance

~~$S = \int dt \int d^3x \int d^3y \mathcal{L}(\varphi(x), \varphi(y), \partial_\mu \varphi(x), \partial_\mu \varphi(y))$~~

$S = \int d^4x \dot{\varphi}$

non-lo

$S = \int d^4x \partial_\mu \varphi \partial^\mu \varphi$

Ha

T

$\mu\phi(x)$
 ↑
 Lorentz-invariance

$\phi(y), \partial_\mu \phi(x), \partial_\mu \phi(y)$

$$S = \int d^4x \partial_\mu \phi \partial^\mu \phi$$

Hamiltonian

$$H = p\dot{q} - L$$

$$p = \frac{\partial L}{\partial \dot{q}}$$

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)}$$