

Title: Quantum Theory - Lecture 220919

Speakers: Dan Wohns

Collection: Quantum Theory (2022-2023)

Date: September 19, 2022 - 10:45 AM

URL: <https://pirsa.org/22090038>

The image shows a OneNote application window with a purple title bar. The ribbon includes Home, Insert, Draw, View, and Tell me. The Draw ribbon is active, showing tools like Eraser, Pen, Marker, and Highlighter, along with Ink Colour and line thickness options (0.25 mm, 0.35 mm, 0.5 mm, 0.7 mm, 1 mm). The main content area displays the title "Lecture 7" and the date "Monday, 19 September 2022 4:34 PM". Below this is a section titled "Outline" with a bulleted list of topics.

OneNote

Home Insert Draw View Tell me

Text Mode Lasso Select Insert Space Eraser Pen Marker Highlighter Ink Colour 0.25 mm 0.35 mm 0.5 mm 0.7 mm 1 mm

BA

Lecture 7

Monday, 19 September 2022 4:34 PM

Outline

- Description of the problem
 - Time dependent potential and what it will do
 - Try to solve it using what we already know (too complicated!!!)
- Changing frame of reference
- Solving new Schrodinger equation
- Exact solution (one example: constant perturbation)
- Perturbative solution (two example)
 - Constant perturbation (detail, Fermi's Golden rule)
 - Harmonic perturbation (touch)

Goal $H(t) = H_0 + V(t) \rightarrow$ tricky
 \downarrow
can be easily

$$i\hbar \frac{\partial}{\partial t} \underline{|\psi(t)\rangle} = \underline{H(t) |\psi(t)\rangle}$$

$$|\psi(t)\rangle = \sum_n \underline{c_n(t)} |n\rangle^x$$

$$e^{-iEt/\hbar}$$

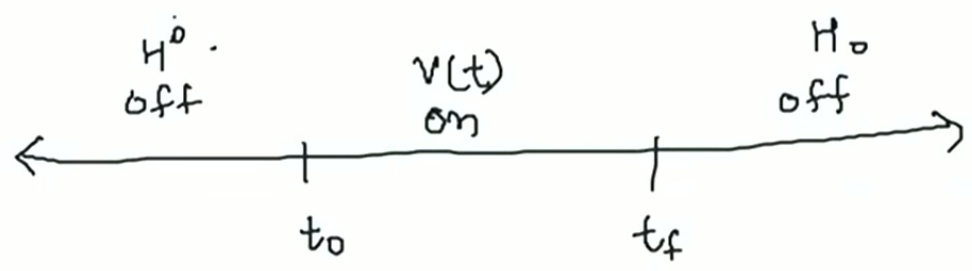
$$H |\psi\rangle = E |\psi\rangle$$

$$\downarrow$$
$$\textcircled{|n\rangle} \quad E_n$$

$$f(x) = \sum c_n |n\rangle$$

δt

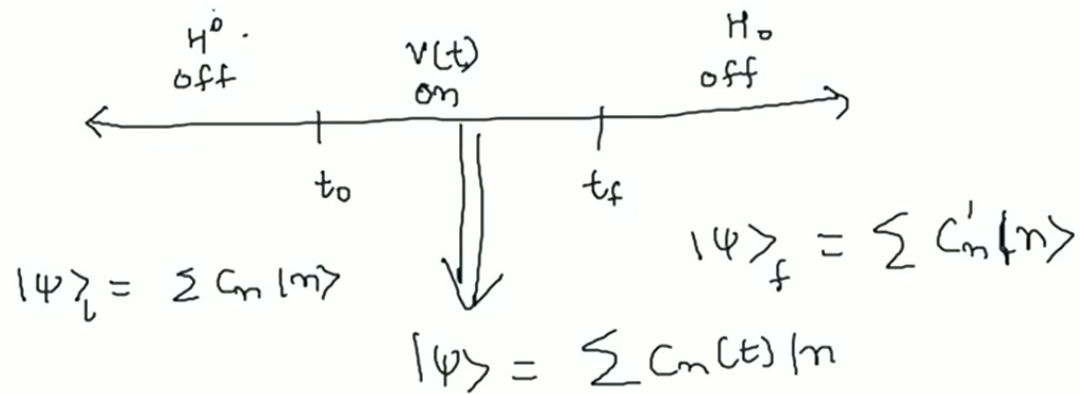
$$\xrightarrow{V(t)} |\psi(t)\rangle = \sum_n \underline{C_n(t)} |n\rangle$$



$$|\psi\rangle = \sum C_n |n\rangle$$

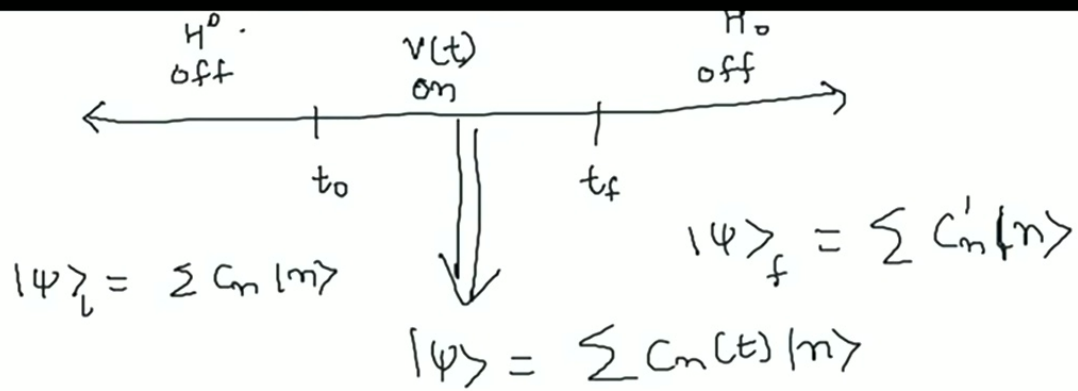
$$\delta t$$

$$\rightarrow |\psi(t)\rangle = \sum_n \underline{C_n(t)} |n\rangle$$



$$\begin{array}{ccc}
 & t_0 & t_f \\
 |\psi\rangle_i = \sum C_m |m\rangle & \Downarrow & |\psi\rangle_f = \sum C'_m |m\rangle \\
 |\psi\rangle = \sum C_m(t) |m\rangle & &
 \end{array}$$

• Rb 5s $|5s\rangle = |5s\rangle + 1$



• Rb 5s $|\psi_s\rangle = \underbrace{C_{5s}}_1 |\psi_s\rangle + \underbrace{C_{5p_{1/2}}}_{\downarrow} |\psi_{p_{1/2}}\rangle + \dots$

\leftarrow off \quad $\begin{matrix} \text{VCD} \\ \text{on} \end{matrix}$ \quad off \rightarrow
 t_0 \quad t_f

$$|\psi_i\rangle = \sum C_m |m\rangle$$

$$|\psi_f\rangle = \sum C'_m |m\rangle$$

$$|\psi\rangle = \sum C_m(t) |m\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

$$|\psi\rangle_i = \sum c_m |m\rangle \quad \Downarrow \quad |\psi\rangle_f = \sum c_m |m\rangle$$

$$|\psi\rangle = \sum c_m(t) |m\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

$$|\psi(t)\rangle = \sum_n \left(\frac{-i}{\hbar}\right)^n \int_0^t dt' \int_0^{t'} dt'' \dots \int_0^{t^{n-1}} dt^n \underbrace{H(t') H(t'') \dots H(t^n)}_{(H_0 + K(t))} |\psi(t^n)\rangle$$

$$|\psi\rangle_i = \sum c_m |m\rangle \quad \Downarrow \quad |\psi\rangle_f = \sum c_m |m\rangle$$

$$|\psi\rangle = \sum c_m(t) |m\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

$$|\psi(t)\rangle = \sum_n \left(-\frac{i}{\hbar}\right)^n \int_0^t dt' \int_0^{t'} dt'' \dots \int_0^{t^{n-1}} dt^n \underbrace{H(t') H(t'') \dots H(t^n)}_{(H_0 + V(t'))} |\psi(t)\rangle$$

$$|\psi\rangle_i = \sum c_m |m\rangle \quad \Downarrow \quad |\psi\rangle_f = \sum c_m |m\rangle$$

$$|\psi\rangle = \sum c_m(t) |m\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

$$|\psi(t)\rangle = \left(\sum_n \left(\frac{-i}{\hbar} \right)^n \int_0^t dt' \int_0^{t'} dt'' \dots \int_0^{t^{n-1}} dt^n \frac{H(t') H(t'') \dots H(t^n)}{(H_0 + V(t'))} \right) \underline{|\psi\rangle}$$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$


$$|\psi(t)\rangle = \left(\sum_n \left(\frac{-i}{\hbar}\right)^n \int_0^t dt' \int_0^{t'} dt'' \dots \int_0^{t^{n-1}} dt^n \frac{H(t') H(t'') \dots H(t^n)}{(H_0 + V(t'))} \right) |\psi(0)\rangle$$

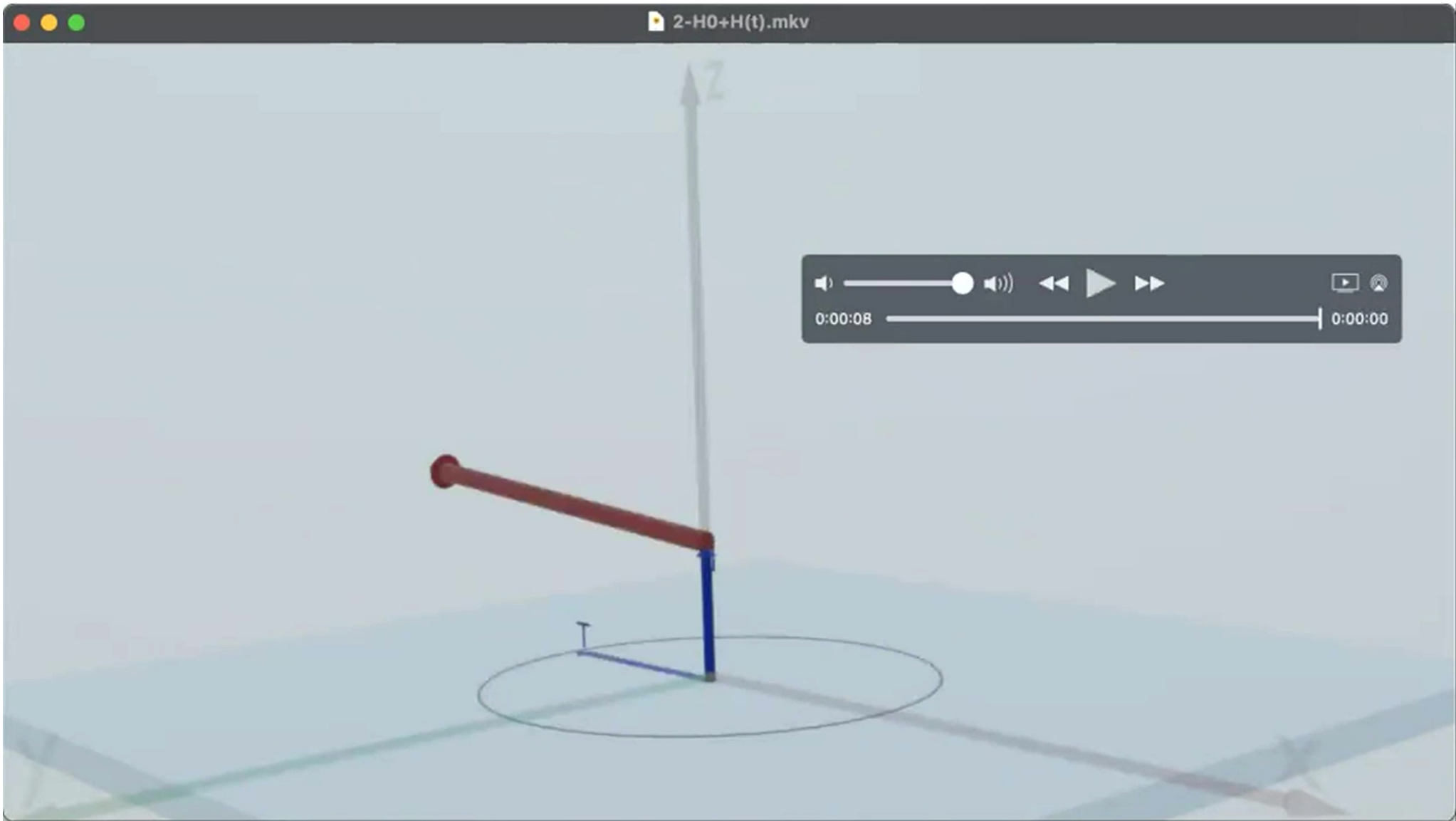
$$H(t) = \underline{H_0} + \underline{V(t)}$$

n \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
 $(H_0 + \underline{V(t)})$

$$H(t) = \underline{H_0} + \underline{V(t)}$$



 \rightarrow lab frame.



$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle_S = (H^0 + V(t)) |\psi(t)\rangle_S \quad U_0^\dagger |\psi(t_0)\rangle$$

$$|\tilde{\psi}(t)\rangle = R(t) |\psi(t)\rangle_S \quad \Rightarrow \quad |\psi(t)\rangle_S = R^\dagger(t) |\tilde{\psi}(t)\rangle$$

$$i\hbar |\dot{\tilde{\psi}}(t)\rangle = \underbrace{\left(\dot{R}(t) R^\dagger(t) i\hbar + R(t) H(t) R^\dagger(t) \right)}_{H'(t)} |\tilde{\psi}(t)\rangle$$

$$R(t) = U_0^\dagger = e^{iH_0 t/\hbar} \quad H'(t)$$

$$i\hbar |\dot{\tilde{\psi}}(t)\rangle = \left\{ i\hbar \left[\frac{iH_0}{\hbar} \right] e^{iH_0 t/\hbar} e^{-iH_0 t/\hbar} + \right.$$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle_S = (H^0 + V(t)) |\psi(t)\rangle_S \quad U_0^\dagger |\psi(t_0)\rangle$$

$$|\tilde{\psi}(t)\rangle = R(t) |\psi(t)\rangle_S \quad \Rightarrow \quad |\psi(t)\rangle_S = R^\dagger(t) |\tilde{\psi}(t)\rangle$$

$$i\hbar |\dot{\tilde{\psi}}(t)\rangle = \underbrace{\left(\dot{R}(t) R^\dagger(t) i\hbar + R(t) H(t) R^\dagger(t) \right)}_{H'(t)} |\tilde{\psi}(t)\rangle$$

$$R(t) = U_0^\dagger = e^{iH_0 t/\hbar}$$

$$i\hbar |\dot{\tilde{\psi}}(t)\rangle = \left\{ \frac{i\hbar \left[\frac{iH_0}{\hbar} \right] e^{iH_0 t/\hbar} e^{-iH_0 t/\hbar} + e^{iH_0 t/\hbar} \left[H^0 + V(t) \right] e^{-iH_0 t/\hbar}}{e^{iH_0 t/\hbar}} \right\} |\tilde{\psi}(t)\rangle$$

$$i\hbar |\dot{\tilde{\psi}}(t)\rangle = \underbrace{e^{iH_0 t/\hbar} V(t) e^{-iH_0 t/\hbar}}_{\tilde{V}(t)} |\tilde{\psi}(t)\rangle$$

$$|\tilde{\psi}(t)\rangle = e^{-iH_0 t/\hbar} |\psi(t)\rangle_S$$

$$V(t) = e^{iH_0 t/\hbar} V(t) e^{-iH_0 t/\hbar}$$

$$|\tilde{\psi}(t)\rangle = e^{i \int_{t_0}^t \frac{V(t')}{\hbar} dt'} |\tilde{\psi}(t_0)\rangle$$

↓

$$i\hbar |\dot{\tilde{\psi}}(t)\rangle = V(t) |\tilde{\psi}(t)\rangle - (1)$$

$$|\tilde{\psi}(t)\rangle = \sum_n C_n(t) |n\rangle - (2)$$

$$i\hbar \frac{\partial}{\partial t} \sum_n C_n(t) |n\rangle = V(t) \sum_n C_n(t) |n\rangle$$

$$i\hbar \dot{C}_m(t) = \sum_n C_n(t) \langle m | e^{iH_0 t/\hbar} V(t) e^{-iH_0 t/\hbar} |n\rangle$$

$$i\hbar \dot{C}_m(t) = \sum_n C_n(t) e^{-i\omega_{mn} t} V_{mn}$$

$$\omega_{mn} = E_m - E_n$$

$$|\tilde{\psi}(t)\rangle = e^{-iH_0 t/\hbar} |\psi(t)\rangle_S$$

$$V(t) = e^{iH_0 t/\hbar} V(t) e^{-iH_0 t/\hbar}$$

$$|\tilde{\psi}(t)\rangle = e^{i\int_0^t V(t') dt'/\hbar} |\tilde{\psi}(0)\rangle$$

↓

$$i\hbar |\dot{\tilde{\psi}}(t)\rangle = V(t) |\tilde{\psi}(t)\rangle - (1)$$

$$|\tilde{\psi}(t)\rangle = \sum_n C_n(t) |n\rangle - (2)$$

$$i\hbar \frac{\partial}{\partial t} \sum_n C_n(t) |n\rangle = V(t) \sum_n C_n(t) |n\rangle$$

$$i\hbar \dot{C}_m(t) = \sum_n C_n(t) \langle m | e^{iH_0 t/\hbar} V(t) e^{-iH_0 t/\hbar} |n\rangle$$

$$i\hbar \dot{C}_m(t) = \sum_n C_n(t) e^{-i\omega_{mn} t} V_{mn}$$

$$\omega_{mn} = \frac{E_m - E_n}{\hbar}$$

$$V_{mn} = \langle m | V(t) |n\rangle$$

$$|\tilde{\psi}(t)\rangle = e^{-iH_0 t/\hbar} |\psi(t)\rangle_S$$

$$|\tilde{\psi}(t)\rangle = e^{i\int_0^t \frac{V(t')}{\hbar} dt'} |\tilde{\psi}(0)\rangle$$

$$\tilde{V}(t) = e^{iH_0 t/\hbar} V(t) e^{-iH_0 t/\hbar}$$

↓

$$i\hbar |\dot{\tilde{\psi}}(t)\rangle = \tilde{V}(t) |\tilde{\psi}(t)\rangle - (1)$$

$$|\tilde{\psi}(t)\rangle = \sum_n C_n(t) |n\rangle - (2)$$

$$i\hbar \frac{d}{dt} \sum_n C_n(t) |n\rangle = \tilde{V}(t) \sum_n C_n(t) |n\rangle$$

$$i\hbar \dot{C}_m(t) = \sum_n C_n(t) \langle m | e^{iH_0 t/\hbar} V(t) e^{-iH_0 t/\hbar} |n\rangle$$

$$i\hbar \dot{C}_m(t) = \sum_n C_n(t) e^{i\omega_{mn} t} V_{mn}$$

$$\omega_{mn} = \frac{E_m - E_n}{\hbar}$$

$$V_{mn} = \langle m | V(t) |n\rangle$$

$$i\hbar \begin{bmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \\ \vdots \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} e^{i\omega_{12}t} & \dots \\ & V_{22} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \\ \vdots \end{bmatrix}$$

constant perturbation applied to two level problem

$$i\hbar \begin{bmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} e^{i\omega_{12}t} \\ V_{21} e^{i\omega_{21}t} & V_{22} \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix}$$

$\xrightarrow{|2\rangle}$
 $\xrightarrow{|1\rangle}$

$$i\hbar \dot{c}_1(t) =$$

$$i\hbar \begin{bmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \\ \vdots \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} e^{i\omega_{12}t} & \dots \\ & V_{22} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \\ \vdots \end{bmatrix}$$

constant perturbation applied to two level problem

$$i\hbar \begin{bmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} e^{i\omega_{12}t} \\ V_{21} e^{i\omega_{21}t} & V_{22} \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix}$$

$\xrightarrow{|2\rangle}$
 $\xrightarrow{|1\rangle}$

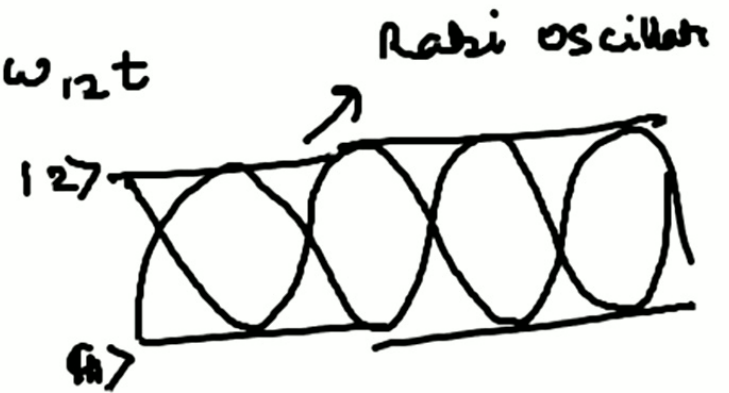
$$i\hbar \dot{c}_1(t) = V c_2(t) e^{i\omega_{12}t}$$

$$i\hbar \dot{c}_2(t) = V c_1(t) e^{-i\omega_{12}t}$$

$$i\hbar \dot{c}_1(t) = \sqrt{V} c_2(t) e^{i\omega_{12}t}$$

$$\Rightarrow i\hbar \dot{c}_2(t) = \sqrt{V} c_1(t) e^{-i\omega_{12}t}$$

$$\begin{bmatrix} 0 & \sqrt{V} e^{i\omega_{12}t} \\ \sqrt{V} e^{-i\omega_{12}t} & 0 \end{bmatrix}$$



$$\ddot{c}_2(t) + i\omega_{12} \dot{c}_2(t) + \frac{V^2}{\hbar^2} c_2(t) = 0$$

$$|c_2(t)|^2 \propto \sin^2(\omega_{21}t) \rightarrow$$

$$|c_1(t)|^2 \propto \cos^2(\omega_{21}t) \rightarrow$$

$$c_1(0) = 1 \quad c_2(0) = 0$$

Perturbative solution

$$i\hbar \dot{c}_m(t) = \sum_n e^{i\omega_{mn}t} V_{mn} c_n(t) \rightarrow$$

$$\begin{aligned} \underline{c_m(t)} = & c_m(0) + \frac{1}{i\hbar} \sum_n \int_0^t dt' c_n(0) V_{mn}(t') e^{i\omega_{mn}t'} \\ & + \left(\frac{1}{i\hbar}\right)^2 \sum_n \sum_l \int_0^t dt' \int_0^{t'} dt'' c_l(t'') V_{nl}(t'') e^{i\omega_{nl}t''} \\ & V_{mn}(t') e^{i\omega_{mn}t'} \\ & + \dots \end{aligned}$$

Perturbative solution

$$i\hbar \dot{c}_m(t) = \sum_n e^{i\omega_{mn}t} V_{mn} c_n(t) \rightarrow$$

$$c_m(t) = \delta_{m\mathbb{I}} + \frac{1}{i\hbar} \sum_n \int_0^t dt' c_n(0) \underline{V_{mn}(t')} e^{i\omega_{mn}t}$$

$$+ \left(\frac{1}{i\hbar}\right)^2 \sum_n \sum_l \int_0^t dt' \int_0^{t'} dt'' c_l(t'') \underline{V_{nl}(t'')} e^{i\omega_{nl}t''} \underline{V_{mn}(t')} e^{i\omega_{mn}t'} + \dots$$

\uparrow $c_m'(t)$ $\langle m | \underline{V(t)} | n \rangle$
 \downarrow $c_m(t)$

Perturbative solution

$$i\hbar \dot{c}_m(t) = \sum_n e^{i\omega_{mn}t} V_{mn} c_n(t) \rightarrow$$

$$c_m(t) = \delta_{m\mathbb{I}} + \frac{1}{i\hbar} \sum_n \int_0^t dt' c_n(0) \underline{V_{mn}(t')} e^{i\omega_{mn}t'}$$

$$+ \left(\frac{1}{i\hbar}\right)^2 \sum_n \sum_l \int_0^t dt' \int_0^{t'} dt'' c_l(t'') \underline{V_{nl}(t'')} e^{i\omega_{nl}t''} \underline{V_{mn}(t')} e^{i\omega_{mn}t'}$$

\uparrow $c_m^{(1)}(t)$ $\langle m | \underline{V(t)} | n \rangle$
 \downarrow $c_m^{(2)}(t)$ + ...

$$C_m^{(1)}(t) = \frac{1}{i\hbar} \sum_n \int_0^t C_n(0) V_{nm}(t') e^{i\omega_{nm}t'} dt'$$

\downarrow
 $\langle n | V | m \rangle$

⇒ Constant Perturbation Harmonic p.

$$V(t) = \begin{cases} 0 & \text{for } t \leq 0 \\ v & \text{for } t > 0 \end{cases} \quad i \rightarrow f$$

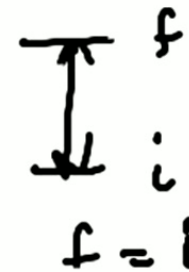
$$C_f^{(1)}(t) = \frac{1}{i\hbar} \int_0^t V_{fi} e^{i\omega_{fi}t'} dt'$$

$$= \frac{-2V_{fi}}{E_f - E_i} e^{i(\omega_{fi}t/2)} \sin\left(\frac{\omega_{fi}t}{2}\right)$$

Transition prob. $i \rightarrow f$

$$|c_f^{(u)}(t)|^2 = P_{i \rightarrow f} = \frac{|V_{fi}|^2}{(E_f - E_i)^2} \sin^2\left(\frac{\omega_{fi} t}{2}\right)$$

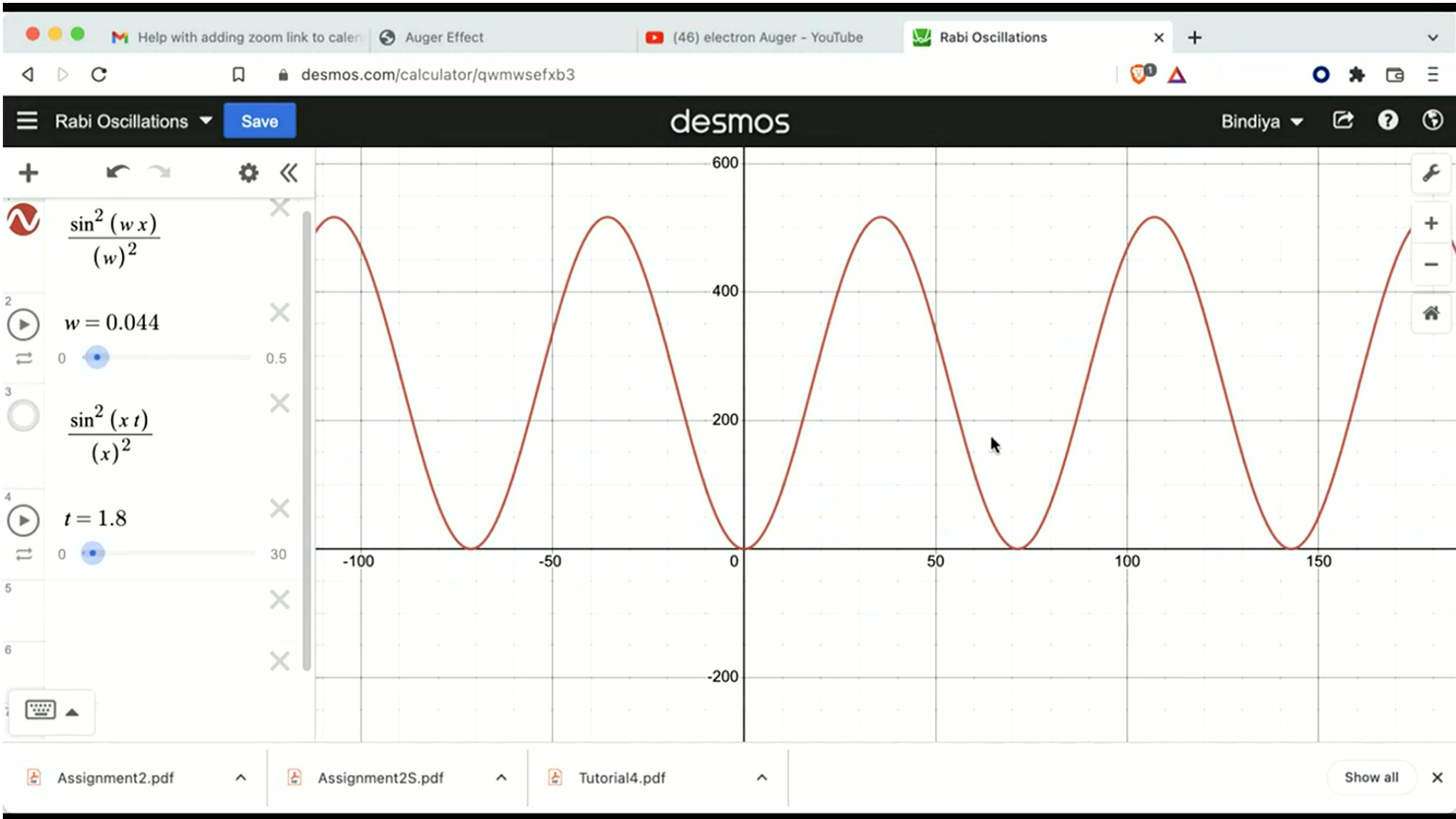
1) $V_{fi} = \langle f | V | i \rangle$



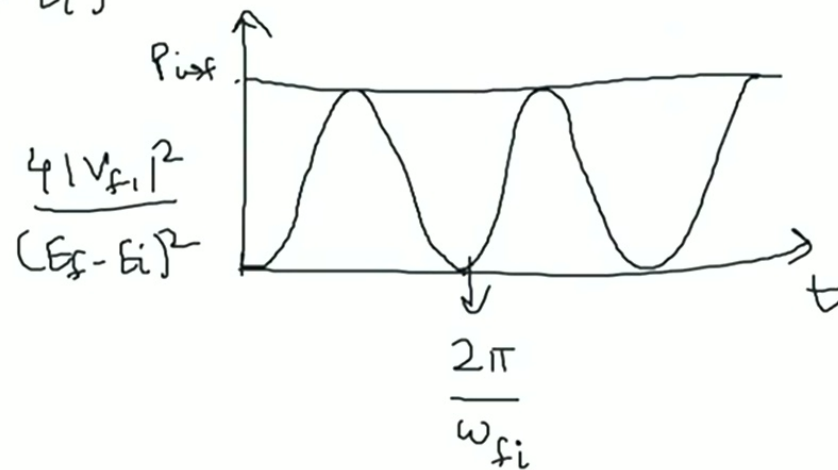
2) $(E_f - E_i)$

a) $E_f \neq E_i$ { energy non-conserving }

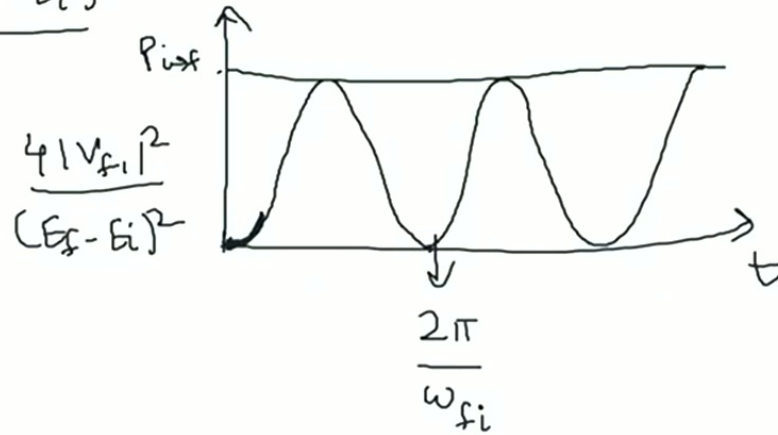
b) $E_f = E_i$ { energy conserving }



$$|C_f^{(1)}(t)|^2 = \frac{4|V_{fi}|^2}{(E_f - E_i)^2} \sin^2\left(\frac{\omega_{fi} t}{2}\right) \quad t = \frac{2\pi}{\omega_{fi}}$$

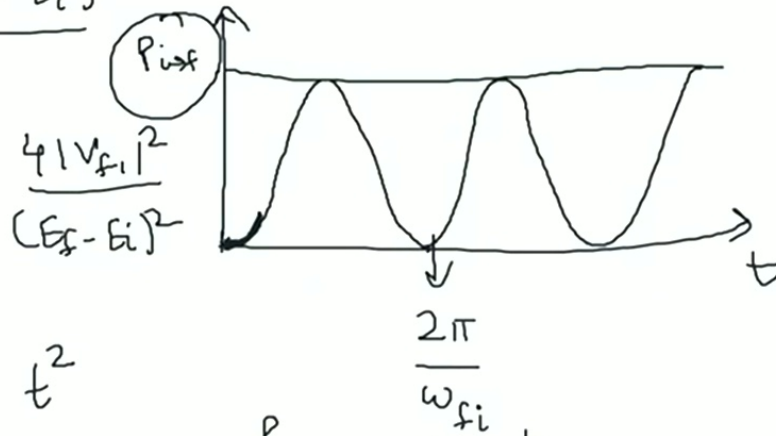


$$|C_f^{(1)}(t)| = \frac{4|V_{fi}|^2}{(E_f - E_i)^2} \sin^2\left(\frac{\omega_{fi} t}{2}\right) \propto t^2 \quad t = \frac{2\pi}{\omega_{fi}}$$



P_f

$$|C_f^{(1)}(t)| = \frac{4|V_{fi}|^2}{(E_f - E_i)^2} \sin^2\left(\frac{\omega_{fi} t}{2}\right) \propto t^2 \quad t = \frac{2\pi}{\omega_{fi}}$$



$$P_{f \to i} \propto t^2$$

$$w = \frac{P}{t} \propto t$$

$$P_{f \to i} \propto t$$

$$w \Rightarrow \frac{P}{t} \Rightarrow \text{constant.}$$

↓
transition

$$P_{f \rightarrow i} \propto t^{-1}$$

$$w = \frac{P}{t} \propto t$$

$$P_{f \rightarrow i} \propto \frac{w_{fi}}{t}$$

$$\textcircled{W} \Rightarrow \frac{P}{t}$$

↓
transition

⇒ constant.

$$b) E_f = E_i \quad E_f \rightarrow E_i$$

lim
E

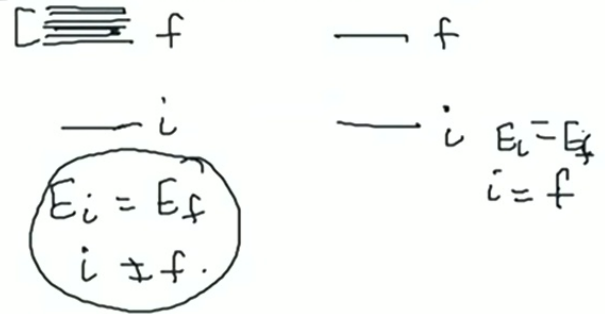
$$w = \frac{P}{t} \propto t$$

$$\frac{P_{f \rightarrow i} \propto \omega_{fi}^2 t}{\omega_{fi}^2 t} \Rightarrow \frac{P}{t} \Rightarrow \text{constant.}$$

$\textcircled{W} \Rightarrow \frac{P}{t}$
 \downarrow
 transition

b) $E_f = E_i$ $\textcircled{E_f} \rightarrow \textcircled{E_i}$

$$\lim_{E_f \rightarrow E_i} P_{f \rightarrow i}(t) = \frac{|V_{fi}|^2 t^2}{\hbar^2}$$



$$W = \frac{P}{t} \propto t$$

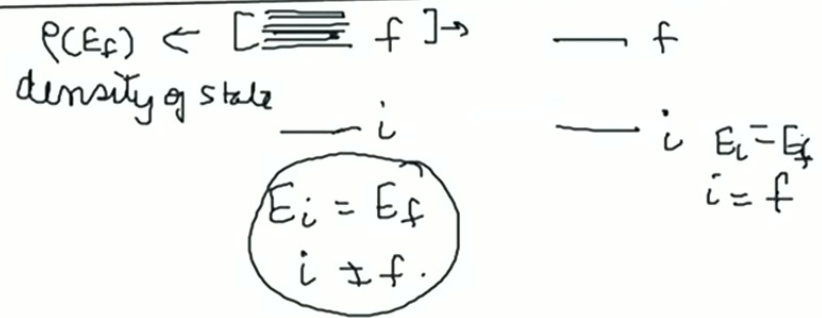
$$\frac{P_{f \rightarrow i} \propto t}{t} \Rightarrow \frac{P}{t} \Rightarrow \text{constant.}$$

\downarrow
 transition

b) $E_f = E_i$ $(E_f) \rightarrow (E_i)$

$$\lim_{E_f \rightarrow E_i} P_{f \rightarrow i}(t) = \frac{|V_{fi}|^2 t^2}{\hbar^2}$$

$\rho(E_f) \rightarrow$ no. of states per unit Energy around



$C_f \rightarrow 0$

\hbar

$\rho(E_f) \rightarrow$ no. of states per unit Energy around E

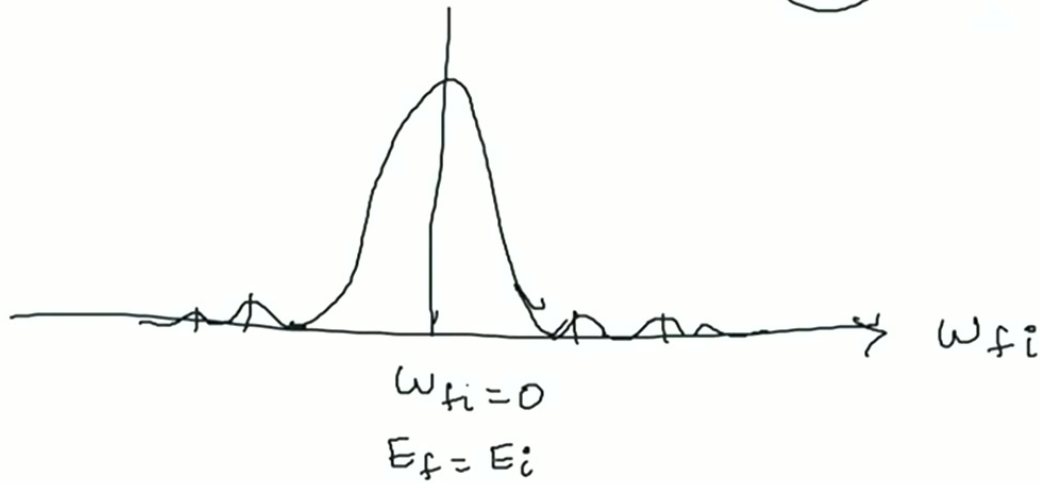
$$\sum P_{i \rightarrow f}(t) = \int \underbrace{P_{i \rightarrow f}(t)}_{\propto |V_{fi}|^2} \underbrace{\rho(E_f)}_{\text{no. of states per unit Energy around } E} dE_f$$

$$E_f - E_i < 0$$

$$= \frac{4 |V_{fi}|^2 \rho(E_f)}{\hbar^2} \int \frac{\sin^2(\omega_{fi} t/2)}{\omega_{fi}^2}$$

$$E_f - E_i < 0$$

$$= \frac{4 |V_{fi}|^2 \rho(E_f)}{\omega_{fi}} \int \frac{\sin^2(\omega_{fi} t/2)}{\omega_{fi}^2} d\omega_{fi}$$





$$\omega_{fi} = 0$$

$$E_f = E_i$$

$$\sum_f P_{i \rightarrow f}(t) = \frac{2\pi |V_{fi}|^2 \rho(E_f) t}{h}$$

$$\sum_f P_{i \rightarrow f}(t) = \frac{2\pi |V_{fi}|^2 \rho(E_f) t}{\hbar}$$

$$W = \frac{P}{t} = \text{const wrt } t$$

$$E_i = E_f$$

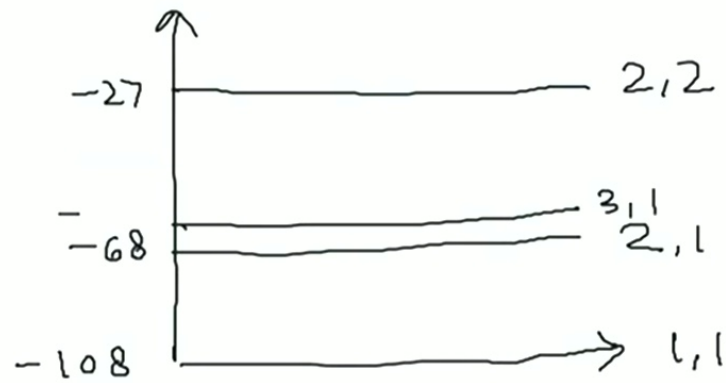
$$i \neq f$$

self ions

Self ionization

Auger

Autoionization

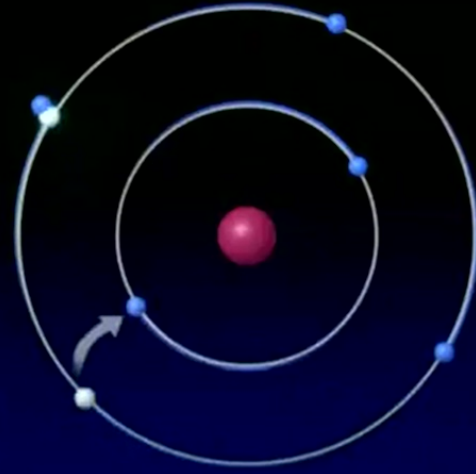


He

$$E_{n_1, n_2} = -54.4 \left[\frac{1}{n_1^2} + \frac{1}{n_2^2} \right]$$

(1s²)

Emission of an Auger electron



0:08 / 0:17 [Pause] [Play] [Volume] [Full Screen] [Settings] [Subtitles] [Share]

electron Auger

Assignment2.pdf

Assignment2S.pdf

Tutorial4.pdf

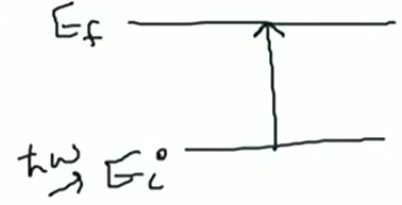
Show all X

Harmonic perturbation

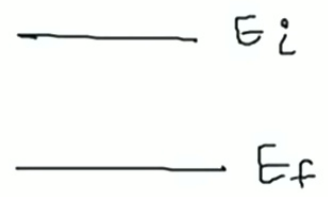
$$V(t) = \begin{cases} 0 & t \leq 0 \\ V_0 e^{i\omega t} + V_0^* e^{-i\omega t} & t > 0 \end{cases}$$

$$(C_f^{(+)}(t)) = \frac{-V_{fi}}{\hbar} \left[\frac{e^{i(\omega_{fi} + \omega)t} - 1}{\omega_{fi} + \omega} + \frac{e^{i(\omega_{fi} - \omega)t} - 1}{\omega_{fi} - \omega} \right]$$

$$2) \rightarrow \omega_{fi} - \omega \approx 0 \Rightarrow E_f = E_i + \hbar\omega$$

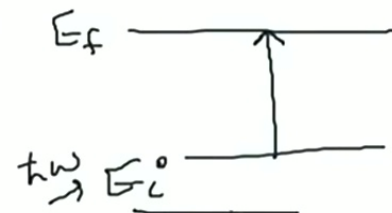


$$1) \rightarrow \omega_{fi} + \omega \approx 0 \Rightarrow E_f = E_i - \hbar\omega$$



$$2) \rightarrow \omega_{fi} - \omega \approx 0$$

$$\Rightarrow E_f = E_i + \hbar\omega$$



$$1) \rightarrow \omega_{fi} + \omega \approx 0$$

$$E_f = E_i - \hbar\omega$$

