

Title: Quantum Theory - Lecture 220914

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Collection: Quantum Theory (2022-2023)

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The image shows a OneNote application window with a purple title bar. The title bar contains the text "OneNote" and a share icon on the right. Below the title bar is a ribbon with tabs for "Home", "Insert", "Draw", "View", and "Tell me". The "Draw" tab is active, showing a toolbar with icons for "Text Mode", "Lasso Select", "Insert Space", "Eraser", "Pen", "Marker", "Highlighter", "Ink Colour" (with a color palette of black, blue, green, and red), and five pen tip styles labeled "0.25 mm", "0.35 mm", "0.5 mm", "0.7 mm", and "1 mm".

The main content area of the OneNote page has a title "Lecture-5 (14-September,2022)" followed by a horizontal line. Below the line is the text "Our learning goals:". Underneath this, there is a list of topics, each with a purple highlight box behind it:

- Questions & doubts
- Heisenberg picture
- Comparison
- Equation of motion

Below the list, there is a section titled "Introduction to density operators" followed by a list of sub-topics:

- Defining (with examples)
- Properties
- Pure & mixed states
- Examples
- Generic density matrix for a two level system

In the bottom-left corner of the OneNote window, there is a small circular icon with the letters "BA" inside.

$$\underbrace{\frac{-\hbar^2}{2m} \nabla_1^2 \psi - \frac{1}{r_1} \psi}_H = E \psi \quad \checkmark$$

$$H_1 \psi + H_2 \psi + \left(\frac{ze\psi}{|r_1 - r_2|} \right) = E \psi$$

$$\underbrace{H_1 \psi + H_2 \psi}_H + \frac{ze\psi}{|n_1 - n_2|} = E\psi \quad -4 \text{ au}$$

$$H_1 \psi + H_2 \psi + \frac{ze\psi}{|r_1 - r_2|} = E \psi$$

$$\langle n_1 | 1 \rangle \langle 1 | \rangle \langle 1 | \rangle$$

↑
↓

Cohen Tanoji

Bramsdem & Jochain ⇒ QM /

first

$$-4 \text{ au}$$

$$\Downarrow$$

$$-2.75$$

$$-2.9077$$

13'

$$\rightarrow 2.90372433$$

$$H_1 \psi + H_2 \psi + \frac{ze\psi}{|r_1 - r_2|} = E \psi$$

$$\langle n_1 | \psi \rangle \langle 11 | \rangle \langle 11 | \rangle$$

- 4 au
 ↓
 first - 2.75
 - 2.9077
 13' → 2.90372433

↕

Cohen Tanojji

Bremsden & Jochem → QM / AP.

Convergence of Dyson Series

$$i\hbar \frac{dU}{dt} = HU$$

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$$U = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{-i}{\hbar} \right)^n \int_0^t dt' \int_0^{t'} dt'' \dots \int_0^{t^{(n-1)}} dt^n H(t') H(t'') \dots H(t^{(n)}) t \dots$$

$$e^{iHT/\hbar} = \sum_{n=0}^{\infty} \int \left(\frac{i}{\hbar} \right)^n H(t') dt'$$

Convergence of ψ

$$i\hbar \frac{dU}{dt} = HU$$

$$U = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{-i}{\hbar} \right)^n \int_0^t dt' \int_0^{t'} dt'' \dots \int_0^{t^{n-1}} dt^n H(t') H(t'') \dots H(t^n) + \dots$$

$$e^{\int_0^t \frac{i}{\hbar} H(t') dt'} = \sum_{n=0}^{\infty} \left(\int_0^t \left(\frac{i}{\hbar} \right) H(t') dt' \right)^n \frac{1}{n!} \quad e^A =$$

$$U = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{-i}{\hbar} \int_0^t H(t') dt' \right)^n$$

$$e^{\int_0^t \frac{-i}{\hbar} H(t') dt'} = \sum_{n=0}^{\infty} \left(\int_0^t \left(\frac{-i}{\hbar} \right) H(t') dt' \right)^n \frac{1}{n!}$$

$$e^A =$$

$$U = 1 + \left(\frac{-i}{\hbar} \right) \int_0^t dt' H(t') + \underbrace{\left(\frac{-i}{\hbar} \right)^2 \int_0^t dt' \int_0^{t'} dt'' H(t') H(t'')} + \dots$$

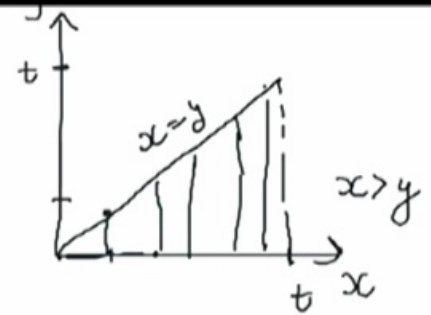
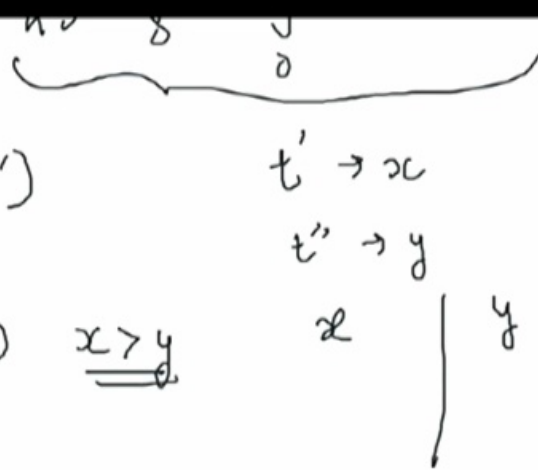
Consi

Consider

$$J_1 = \int_0^t dt' \int_0^{t'} dt'' H(t') H(t'')$$

$$= \int_0^t dx \int_0^x dy \underline{H(x)} \underline{H(y)} \quad \underline{x > y}$$

$$J_2 = \int_0^t dx$$



$$\sqrt{U} = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{-i}{\hbar}\right)^n \int_0^t dt' \int_0^{t'} dt'' \dots \int_0^{t^{(n-1)}} dt^n H(t') H(t'') \dots H(t^{(n)}) t \dots$$

$$e^{\int_0^t \frac{i}{\hbar} H(t') dt'} = \sum_{n=0}^{\infty} \left[\left(\frac{i}{\hbar}\right)^n H(t') dt' \right]^n \frac{1}{n!} \quad e^A =$$

$$\sqrt{U} = 1 + \left(\frac{-i}{\hbar}\right) \int_0^t dt' H(t') + \underbrace{\left(\frac{-i}{\hbar}\right)^2 \int_0^t dt' \int_0^{t'} dt'' H(t') H(t'')}_{t' \rightarrow \infty} + \dots$$

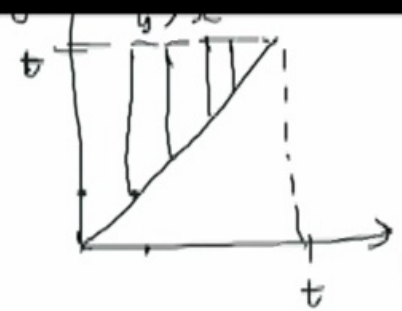
Consider $T = \int_0^t dt' \int_0^{t'} dt'' H(t') H(t'')$ $t' \rightarrow \infty$



$$J_2 = \int_0^t dx \int_x^t dy H(y) H(x) \quad y > x$$

$$J_2 = J_1 \int_0^t dx \left[\int_0^x dy [H(x) H(y)] + \int_x^t dy [H(x) H(y)] \right]$$

\uparrow $T[H(x) H(y)]$ $x > y$ time order 0-2 \uparrow $H(y) H(x)$



$$\sqrt{J_1} = \frac{1}{2!} \int_0^t dx \int_0^t dy H(x) H(y) =$$

$$\begin{aligned}
 & H(y) H(x) \\
 & y > x \\
 & T[H(x) H(y)] \\
 & = H(y) H(x)
 \end{aligned}$$

$$t=t_0 =$$

$$t=t$$

state kets

$$|\alpha(t_0)\rangle_H$$

$$|\alpha(t)\rangle_H = |\alpha(t_0)\rangle_H =$$

$$|\alpha(t_0)\rangle_S \Rightarrow U |\alpha(t)\rangle_S$$

$$|\alpha(t)\rangle_H = U^\dagger(t, t_0) |\alpha(t_0)\rangle_S$$

$$- \frac{i\hbar}{\hbar} U |\alpha\rangle$$

Operators

$$A^H(t_0) = A^{(S)}(t_0)$$

$$A^H(t) = U^\dagger(t, t_0) A^{(S)} U(t, t_0)$$

$$\frac{dA^H(t)}{dt} = \frac{i}{\hbar} [H^H A^H(t) - A^H(t) H^H]$$

$$\boxed{\frac{dA^{(H)}(t)}{dt} = \frac{i}{\hbar} [H, A^H]}$$

$$\rho = |\alpha\rangle\langle\alpha|$$

$$\rho_{\uparrow} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$|\uparrow\rangle\langle\uparrow| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$|\alpha\rangle \rightarrow$ State kets

$$|\alpha\rangle = \sum_m c_m |m\rangle$$

$$|\uparrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \checkmark \quad |\downarrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\langle \uparrow | e | \uparrow \rangle = 1$$

$$\langle \downarrow | e | \downarrow \rangle = 0$$

$N e^-$ s

N_{\uparrow} are in \uparrow

N_{\downarrow} are in \downarrow

$$W_{\uparrow} = \frac{N_{\uparrow}}{N_{\uparrow} + N_{\downarrow}}$$

$$W_{\downarrow} = \frac{N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$

$$\rho = \frac{1}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \text{mixture / mixed state.}$$

$$|\alpha\rangle = w_1 |\alpha_1\rangle + w_2 |\alpha_2\rangle + \dots$$

↑

$$|\alpha\rangle \quad \sim \quad w_1 |\alpha_1\rangle + w_2 |\alpha_2\rangle + \dots$$

$$|\alpha\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle) \rightarrow \text{Pure state}$$

$$w_1 |\alpha_1\rangle \quad w_2 |\alpha_2\rangle$$

$$|\alpha_1\rangle = \sum_n c_n |n\rangle$$

$$|\alpha_2\rangle = \sum_{n'} c_{n'} |n'\rangle$$

$$\langle n | n' \rangle = \delta_{nn'}$$

$$\begin{aligned}\text{Tr}(\rho) &= \sum_n \langle n | \rho | n \rangle \\ &= \sum_{n, \alpha} \langle n | \end{aligned}$$

1) Hermitians $\rho = \left(\sum_{\alpha} w_{\alpha} |\alpha\rangle \langle \alpha| \right)$ 1.

2) if $|\alpha\rangle$ are normalized. then $\text{trace}(\rho) = 1$

$$\text{Tr}(\rho) = \sum_n \langle n | \rho | n \rangle$$

$$= \sum_n \langle n | \sum_{\alpha} w_{\alpha} |\alpha\rangle \langle \alpha| n \rangle$$

$$= \sum_{n, \alpha} w_{\alpha} \underbrace{\langle n | \alpha \rangle \langle \alpha | n \rangle}_{|c_n|^2}$$

$$= \underline{\underline{\sum_{\alpha} w_{\alpha}}} = 1$$

$$\sum_n |c_n|^2 = 1$$

$$\rho = w_{\uparrow} |\uparrow\rangle\langle\uparrow| + w_{\downarrow} |\downarrow\rangle\langle\downarrow| \quad \sum w_{\alpha} = 1$$

$$w_{\uparrow} + w_{\downarrow} = 1$$

$$\rho = \frac{1}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \text{mixture / mixed state.}$$

$$|\alpha\rangle = w_1 |\alpha_1\rangle + w_2 |\alpha_2\rangle + \dots$$

$$|\alpha\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle) \rightarrow \text{Pure state}$$

$$w_1 |\alpha_1\rangle + w_2 |\alpha_2\rangle$$

$$\langle n|m\rangle = \langle n|\alpha_1\rangle = \sum_n c_n \langle n|m\rangle$$

$$= \delta_{nm}$$

$$|\alpha_2\rangle = \sum_{n'} c_{n'} |n'\rangle \quad \langle n'|m\rangle = \delta_{n'm}$$

$$\langle n|n'\rangle \neq \delta_{nn'}$$

3) Density

4) ✓ Trace of square of density matrix ≤ 1

$$\begin{aligned}\text{Tr}(\rho^2) &= \sum_m \langle m | \rho \rho | m \rangle \\ &= \sum_m\end{aligned}$$

4) ✓ Trace of square of density matrix = 1

$$\begin{aligned}
 \text{Tr}(\rho^2) &= \sum_m \langle m | \rho \rho | m \rangle \\
 &= \sum_m \sum_{\alpha \alpha'} w_\alpha w_{\alpha'} \langle m | \alpha \rangle \underbrace{\langle \alpha | \alpha' \rangle}_{\substack{\text{orthonormal} \\ \delta_{\alpha \alpha'}}} \langle \alpha' | m \rangle \\
 &= \sum_{m \alpha} |w_\alpha|^2 \langle m | \alpha \rangle \langle \alpha | m \rangle \\
 &= \sum_{m \alpha} 1
 \end{aligned}$$

$$\text{Tr}(\rho^2) = \sum_{\alpha} |w_{\alpha}|^2 \leq \left(\sum_{\alpha} w_{\alpha} \right)^2$$

$$\text{Tr}(\rho^2) \leq \left(\frac{\text{Tr}(\rho)}{1} \right)^2$$

$$\text{Tr}(\rho^2) \leq 1$$

Pure state τ

$$\rho = \frac{1}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \text{mixture / mixed state. } \text{Tr}(\rho^2)$$

$$|\alpha\rangle = w_1 |\alpha_1\rangle + w_2 |\alpha_2\rangle + \dots \quad \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$|\alpha\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle) \rightarrow \text{Pure state}$$

$$w_1 |\alpha_1\rangle + w_2 |\alpha_2\rangle$$

$$|\alpha_1\rangle = \sum_n c_n |n\rangle$$

$$|\alpha_2\rangle = \sum_{n'} c_{n'} |n'\rangle \quad \langle n' | m \rangle = \delta_{n'm}$$

mixed state $\text{Tr}(\rho) < 1$

1) $|\alpha\rangle$