Title: Quantum Theory - Lecture 220907

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Collection: Quantum Theory (2022-2023)

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→ Introduction to approximate methods of solving the Schrodinger equation.

→ Validity of perturbative approach to solve Schrodinger equation.

→ First & second order correction in energy.

→ First order correction in wavefunction.

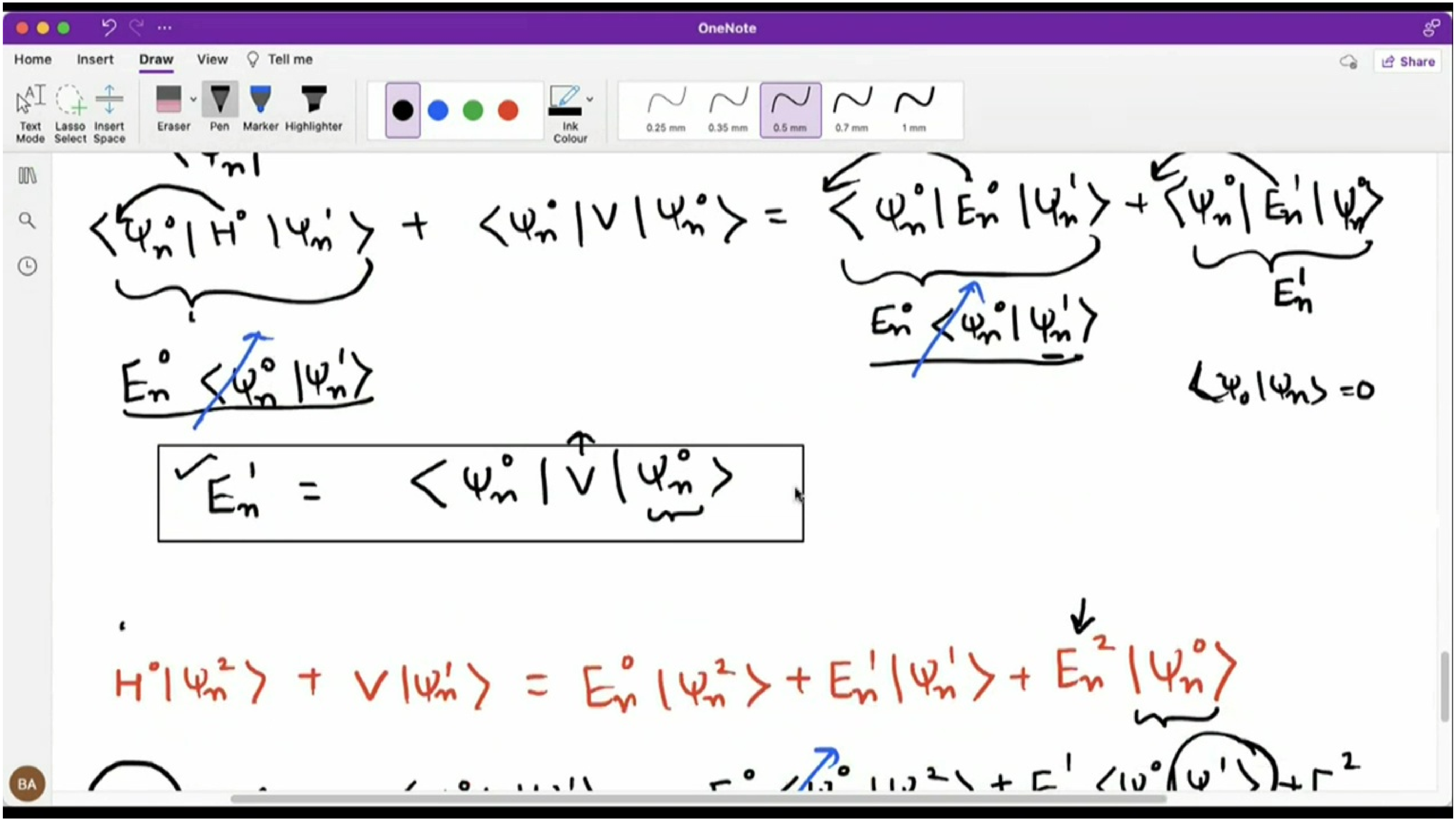
→ Analyzing the results.

→ Degenerate perturbation theory:

- Why can't we apply the non degenerate approach to degenerate problems?
- How to remove degeneracy from the system?

→ Revisiting: Balls-into-bins problem ✓

$$E^{(1)} = \langle n^0 | V | n^0 \rangle$$



$$\langle \psi_n^0 | H^0 | \psi_n^1 \rangle + \langle \psi_n^0 | V | \psi_n^0 \rangle = \langle \psi_n^0 | E_n^0 | \psi_n^1 \rangle + \langle \psi_n^0 | E_n^1 | \psi_n^0 \rangle$$

$$E_n^0 \langle \psi_n^0 | \psi_n^1 \rangle \qquad E_n^0 \langle \psi_n^0 | \psi_n^1 \rangle \qquad E_n^1$$

$$\langle \psi_0 | \psi_n \rangle = 0$$

$$E_n^1 = \langle \psi_n^0 | V | \psi_n^0 \rangle$$

$$H^0 | \psi_n^2 \rangle + V | \psi_n^1 \rangle = E_n^0 | \psi_n^2 \rangle + E_n^1 | \psi_n^1 \rangle + E_n^2 | \psi_n^0 \rangle$$

$$E^0 \langle \psi^0 | \psi^2 \rangle + E^1 \langle \psi^0 | \psi^1 \rangle + E^2$$

$$H^0|\psi_n^2\rangle + V|\psi_n^1\rangle = E_n^0|\psi_n^2\rangle + E_n^1|\psi_n^1\rangle + E_n^2|\psi_n^0\rangle$$

$$\langle\psi_n^0|H^0|\psi_n^2\rangle + \langle\psi_n^0|V|\psi_n^1\rangle = E_n^0\langle\psi_n^0|\psi_n^2\rangle + E_n^1\langle\psi_n^0|\psi_n^1\rangle + E_n^2$$

$$E_n^0\langle\psi_n^0|\psi_n^2\rangle$$

$$E_n^2 = \langle\psi_n^0|$$

$$H^0|\psi_n^2\rangle + V|\psi_n^1\rangle = E_n^0|\psi_n^2\rangle + E_n^1|\psi_n^1\rangle + E_n^2|\psi_n^0\rangle$$

$$\langle\psi_n^0|H^0|\psi_n^2\rangle + \langle\psi_n^0|V|\psi_n^1\rangle = E_n^0\langle\psi_n^0|\psi_n^2\rangle + E_n^1\langle\psi_n^0|\psi_n^1\rangle + E_n^2$$

$$E_n^0\langle\psi_n^0|\psi_n^2\rangle$$

$$E_n^2 = \langle\psi_n^0|V|\psi_n^1\rangle$$

$$H^0|\psi_n^2\rangle + V|\psi_n^1\rangle = E_n^0|\psi_n^2\rangle + E_n^1|\psi_n^1\rangle + E_n^2|\psi_n^0\rangle$$

$$\langle\psi_n^0|H^0|\psi_n^2\rangle + \langle\psi_n^0|V|\psi_n^1\rangle = E_n^0\langle\psi_n^0|\psi_n^2\rangle + E_n^1\langle\psi_n^0|\psi_n^1\rangle + E_n^2$$

$$E_n^0\langle\psi_n^0|\psi_n^2\rangle$$

$$E_n^2 = \langle\psi_n^0|V|\psi_n^1\rangle \qquad E_n^{(i)} = \langle\psi_n^0|V|\psi_n^{(i-1)}\rangle$$

$$E_n^1 = \langle \psi_n | V | \psi_n \rangle \quad \checkmark$$

$$H^0|\psi_n^2\rangle + V|\psi_n^1\rangle = E_n^0|\psi_n^2\rangle + E_n^1|\psi_n^1\rangle + E_n^2|\psi_n^0\rangle$$

$$\langle\psi_n^0|H^0|\psi_n^2\rangle + \langle\psi_n^0|V|\psi_n^1\rangle = E_n^0\langle\psi_n^0|\psi_n^2\rangle + E_n^1\langle\psi_n^0|\psi_n^1\rangle + E_n^2$$

$$E_n^0\langle\psi_n^0|\psi_n^2\rangle$$

$$E_n^2 = \langle\psi_n^0|V|\psi_n^1\rangle \qquad E_n^{(i)} = \langle\psi_n^0|V|\psi_n^{(i-1)}\rangle$$

$$E_n^2 = \langle \psi_n^0 | V | \psi_n^1 \rangle$$

$$E_n^{(i)} = \langle \psi_n^0 | V | \psi_n^{(i-1)} \rangle$$

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$E_n^1 \langle \psi_n^0 | \psi_n^2 \rangle$

$$E_n^2 = \langle \psi_n^0 | V | \psi_n^1 \rangle \qquad E_n^{(i)} = \langle \psi_n^0 | V | \psi_n^{(i-1)} \rangle$$

$$H^0 |\psi^1\rangle + H^1 |\psi_n^0\rangle = E_n^0 |\psi_n^1\rangle + E_n^1 |\psi^0\rangle$$

$$(H - E_n^0)|\psi$$

BA

$$H^0 |\Psi'\rangle + H' |\Psi_n^0\rangle = E_n^0 |\Psi_n'\rangle + E_n' |\Psi^0\rangle$$

$$(H^0 - E_n^0) |\Psi_n'\rangle = -(H - E_n') |\Psi_n^0\rangle$$

$$|\Psi_n'\rangle = \sum_{m \neq n} C_m^n |\Psi_m^0\rangle$$

$$|\Psi_n\rangle = |\Psi_n^0\rangle + \phi_n$$

$$\sum_{m \neq n}$$

$$\sum_{m \neq n} (H^0 - E_n^0)\, C_m^n |\Psi_m^0\rangle = (-H + E_n') |\Psi_n^0\rangle$$

$$|\psi_n^1\rangle = \sum_{m \neq n} c_m |\psi_m\rangle \qquad \sum_{m\neq n}$$

$$\sum_{m\neq n} (H^0 - E_n^0)\, c_m^n |\psi_m^0\rangle = (-H + E_n^1)|\psi_n^0\rangle$$

$$\Rightarrow \sum_{m\neq n} (E_m^0 - E_n^0)\, c_m^n |\psi_m^0\rangle = -(H - E_n^1)|\psi_n^0\rangle$$

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$$(H^0 - E_n^0)|\psi_n^1\rangle = -(V - E_n^1)|\psi_n^0\rangle$$

$$|\psi_n^1\rangle = \sum_{m\neq n} c_m^n |\psi_m^0\rangle$$

$$|\psi_n\rangle = |\psi_n^0\rangle + \phi_n \quad \sum_{m\neq n}$$

$$\sum_{m\neq n} (H^0 - E_n^0)\, c_m^n |\psi_m^0\rangle = (- \cdot + E_n^1)|\psi_n^0\rangle$$

$$\Rightarrow \sum_{m\neq n} (E_m^0 - E_n^0)\, c_m^n |\psi_m^0\rangle = -(\ - E_n^1)|\psi_n^0\rangle$$

$$\langle\psi_l^0|$$

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$$\sum_{m\neq n} (H^0 - E_n^0)\, C_m^n |\psi_m^0\rangle = (-V + E_n^1)|\psi_n^0\rangle$$

$$\Rightarrow \sum_{m\neq n} (E_m^0 - E_n^0)\, C_m^n |\psi_m^0\rangle = -(V - E_n^1)|\psi_n^0\rangle$$

$$\langle \psi_l^0 |$$

$$\Rightarrow \sum_{m\neq n} (E_m^0 - E_n^0)\, C_m^n \langle \psi_l^0 | \psi_m^0 \rangle = -\langle \psi_l^0 | V | \psi_n^0 \rangle + E_n^1 \langle \psi_l^0 | \psi_n^0$$

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$\langle \psi_l^0 |$

$$\Rightarrow \sum_{m \neq n} (E_m^0 - E_n^0) C_m^n \underbrace{\langle \psi_l^0 | \psi_m^0 \rangle}_{\delta_{lm}} = - \langle \psi_l^0 | V | \psi_n^0 \rangle + E_n^1 \underbrace{\langle \psi_l^0 | \psi_n^0 \rangle}_{\delta_{ln}}$$

Case I : if $l = n$.

$$\Rightarrow \sum_{m\neq n} (E_m^0 - E_n^0)\, C_m^{n1} \underbrace{\langle \psi_l | \psi_m \rangle}_{\delta_{lm}} = -\langle \psi_l | V | \psi_n \rangle + E_n^1 \underbrace{\langle \psi_l^0 | \psi_n^0 \rangle}_{\delta_{ln}}$$

Case I : if $l = n$ $\quad l \neq m$

$$E_n^1 = \langle \psi_l^0 | V | \psi_n^0 \rangle$$

Case II : if $l \neq n$, $l = m$

$$C_l^n = \frac{\langle \psi_l^0 | V | \psi_n^0 \rangle}{E_n^0 - E_l^0}$$

$$|\psi_n^1\rangle = \sum_{m \neq n} C_m^n \, |\psi_m^0\rangle$$

$$= \sum_{m \neq n} \langle \psi_m^0 |$$

$$C_l^n = \frac{\langle \psi_l^0 | V | \psi_n^0 \rangle}{E_n^0 - E_l^0}$$

$$|\psi_n^1\rangle = \sum_{m \neq n} C_m^n \, |\psi_m^0\rangle$$

$$= \sum_{m \neq n} \frac{\langle \psi_m^0 | V | \psi_n^0 \rangle}{E_n^0 - E_m^0} \, |\psi_m^0\rangle$$

$$\checkmark \; E_n^{(1)} = \langle n^0 | V | n^0 \rangle$$

$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle$$

$$\checkmark \; |\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle \, |k^0\rangle}{E_n^0 - E_k^0}$$

$$\checkmark \; E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{E_n^0 - E_k^0} \rightarrow \; ?$$

$$E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \cdots\cdots$$

$$V_{nk} = \langle n^0 | V | k^0 \rangle$$

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$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle$$

$$|\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle \, | k^0 \rangle}{E_n^0 - E_k^0}$$

? intermidiate

$$E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{E_n^0 - E_k^0} \rightarrow ?$$

$$E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \ldots\ldots$$

$$V_{nk} = \langle n^0 | V | k^0 \rangle$$

✓ $$E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k | V | n \rangle}{E_n^0 - E_k^0} \rightarrow ?$$

✓ $$E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \cdots$$

$$V_{nk} = \langle n^0 | V | k^0 \rangle$$

$$|n\rangle = |n^0\rangle + \sum_{k \neq n} \frac{V_{nk}}{E_n^0 - E_k^0} |k^0\rangle + \cdots$$

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$$V_{nk} = \langle n^0 | V | k^0 \rangle$$

$$|n\rangle = |n^0\rangle + \sum_{k \neq n} \frac{V_{nk}}{E_n^0 - E_k^0} |k^0\rangle + \cdots$$

$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle \qquad |\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle |k^0\rangle}{E_n^0 - E_k^0}$$

intermediate ?

$$\checkmark \; E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{E_n^0 - E_k^0} \rightarrow ?$$

$$\checkmark \; E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \cdots\cdots$$

$$V_{nk} = \langle n^0 | V | k^0 \rangle$$

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$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle$$

$$|\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle |k^0\rangle}{E_n^0 - E_k^0}$$

? intermediate

✓ $$E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{E_n^0 - E_k^0} \rightarrow ?$$

$n \rightarrow$ ground state

E_n^0

✓ $$E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \ldots\ldots$$

$$V_{nk} = \langle n^0 | V | k^0 \rangle$$

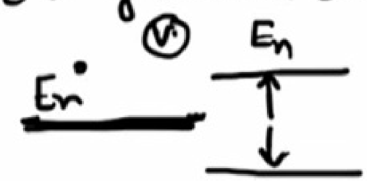
$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle$$

$$|\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle \, | k^0 \rangle}{E_n^0 - E_k^0}$$

? intermediate

$$\checkmark \; E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{E_n^0 - E_k^0} \rightarrow \, ?$$

$n \rightarrow$ ground state ✓



$$\checkmark \; E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \ldots\ldots$$

$$V_{nk} = \langle n^0 | V | k^0 \rangle$$

intermediate

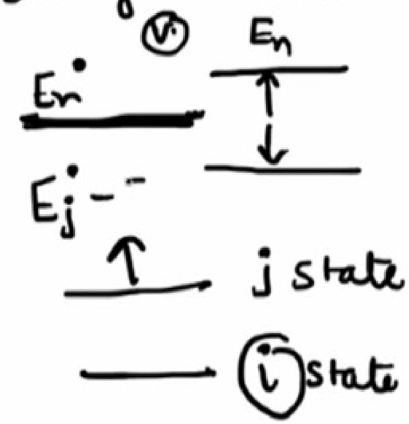
$$E_n^{(2)} = \sum_{k\neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{E_n^0 - E_k^0} \rightarrow ?$$

$k \neq n$ $\quad E_n - E_k^0$

$n \rightarrow$ ground state

$$E_n = E_n^0 + V_{nn} + \sum_{k\neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \cdots\cdots$$

$$V_{nk} = \langle n^0 | V | k^0 \rangle$$



$$E_n^{(2)} \quad \frac{}{E_i - E_j^0}$$

$$|n\rangle = |n^0\rangle + \sum \frac{V_{nk}}{} |k^0\rangle + \cdots\cdots$$

$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle$$

$$|\psi_n^1\rangle = \sum_{k\neq n} \frac{\langle k^0|V|n^0\rangle\, |k^0\rangle}{E_n^0 - E_k^0}$$

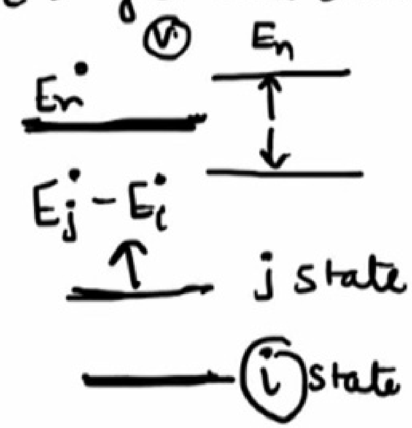
? intermediate

$$E_n^{(2)} = \sum_{k\neq n} \frac{\langle n^0|V|k^0\rangle\langle k^0|V|n^0\rangle}{E_n^0 - E_k^0} \to ?$$

n → ground state

$$E_n = E_n^0 + V_{nn} + \sum_{k\neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \dots\dots$$

$$V_{nk} = \langle n^0|V|k^0\rangle$$



$E_n^{(2)}$

$$\overline{E_i - E_j}$$

$E_n^{(1)} = \langle n^0 | V | n^0 \rangle = 0$ → permanent dipole momen

$E_n^{(2)} = \langle n^0 | V | n^1 \rangle$

$|\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle |k^0\rangle}{E_n^0 - E_k^0}$

intermidiate ?

$E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{E_n^0 - E_k^0}$ → ?

n → ground state

E_n^0 E_n

$E_j^0 - E_i^0$

j state

i state

$E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \ldots\ldots$

$V_{..} = \langle n^0 | V | k^0 \rangle$ $E^{(2)}$

$$E_n^{(2)} = \langle n^0|V|n^0\rangle$$

$$|\psi_n^1\rangle = \sum_{k\neq n} \frac{\langle k^0|V|n^0\rangle\,|k^0\rangle}{E_n^0 - E_k^0}$$

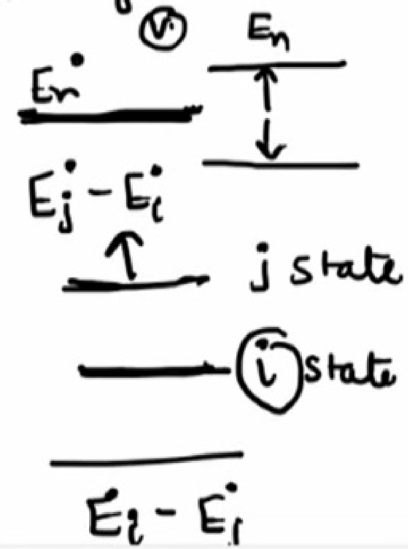
intermediate

$$E_n^{(2)} = \sum_{k\neq n} \frac{\langle n^0|V|k^0\rangle\langle k^0|V|n^0\rangle}{(E_n^0 - E_k^0)^2} \to ?$$

$n \to$ ground state

$$E_n = E_n^0 + V_{nn} + \sum_{k\neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \cdots$$

$$V_{nk} = \langle n^0|V|k^0\rangle$$



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$$E_n^{(1)} = \langle n^0 | V | n^0 \rangle = 0 \rightarrow \text{permanent dipole moment}$$

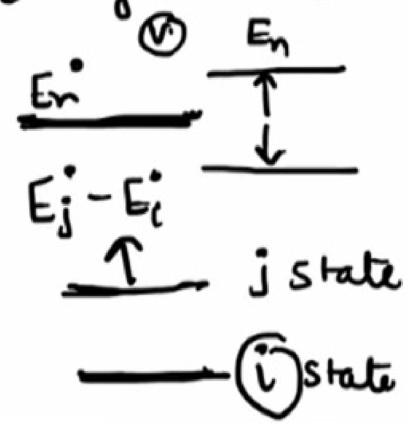
$$E_n^{(2)} = \langle n^0 | V | n^0 \rangle$$

$$|\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle \, | k^0 \rangle}{E_n^0 - E_k^0}$$

? intermediate

$$E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{(E_n^0 - E_k^0)^2} \rightarrow ?$$

$n \rightarrow$ ground state



$$E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \dots\dots$$

$V \;\; = \langle n^0 | V | k^0 \rangle$

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$$E_n^{(2)} = \langle n^0 | V | n^0 \rangle$$

$$|\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle |k^0\rangle}{E_n^0 - E_k^0}$$

intermidiate

$$E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{(E_n^0 - E_k^0)} \rightarrow ?$$

$n \rightarrow$ ground state

E_n^0 E_n

$$E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \dots\dots$$

$$V_{nk} = \langle n^0 | V | k^0 \rangle$$

$E_j^0 - E_i^0$

j state

i state

$$E_n^{(2)} \quad \frac{}{E_i^0 - E_j^0}$$

anti crossing or

$$E_n^{(1)} = \langle n^0 | V | n^0 \rangle = 0 \rightarrow \text{permanent dipole moment}$$

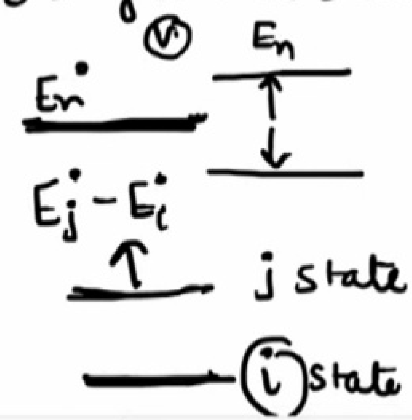
$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle \qquad |\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle |k^0\rangle}{E_n^0 - E_k^0}$$

? intermediate

$$E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{(E_n^0 - E_k^0)} \rightarrow ?$$

$n \rightarrow$ ground state

$$E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \ldots\ldots$$



$$E_n^{(1)} = \langle n^0 | V | n^0 \rangle = 0$$ → permanent dipole moment

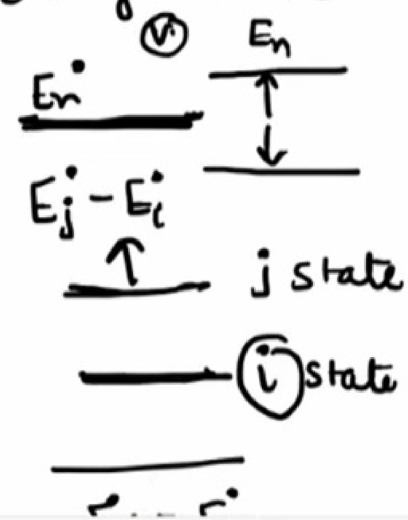
$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle$$

$$|\psi_n^1\rangle = \sum_{k \neq n} \frac{\langle k^0 | V | n^0 \rangle |k^0\rangle}{E_n^0 - E_k^0}$$

intermediate ?

$$E_n^{(2)} = \sum_{k \neq n} \frac{\langle n^0 | V | k^0 \rangle \langle k^0 | V | n^0 \rangle}{(E_n^0 - E_k^0)} \rightarrow ?$$

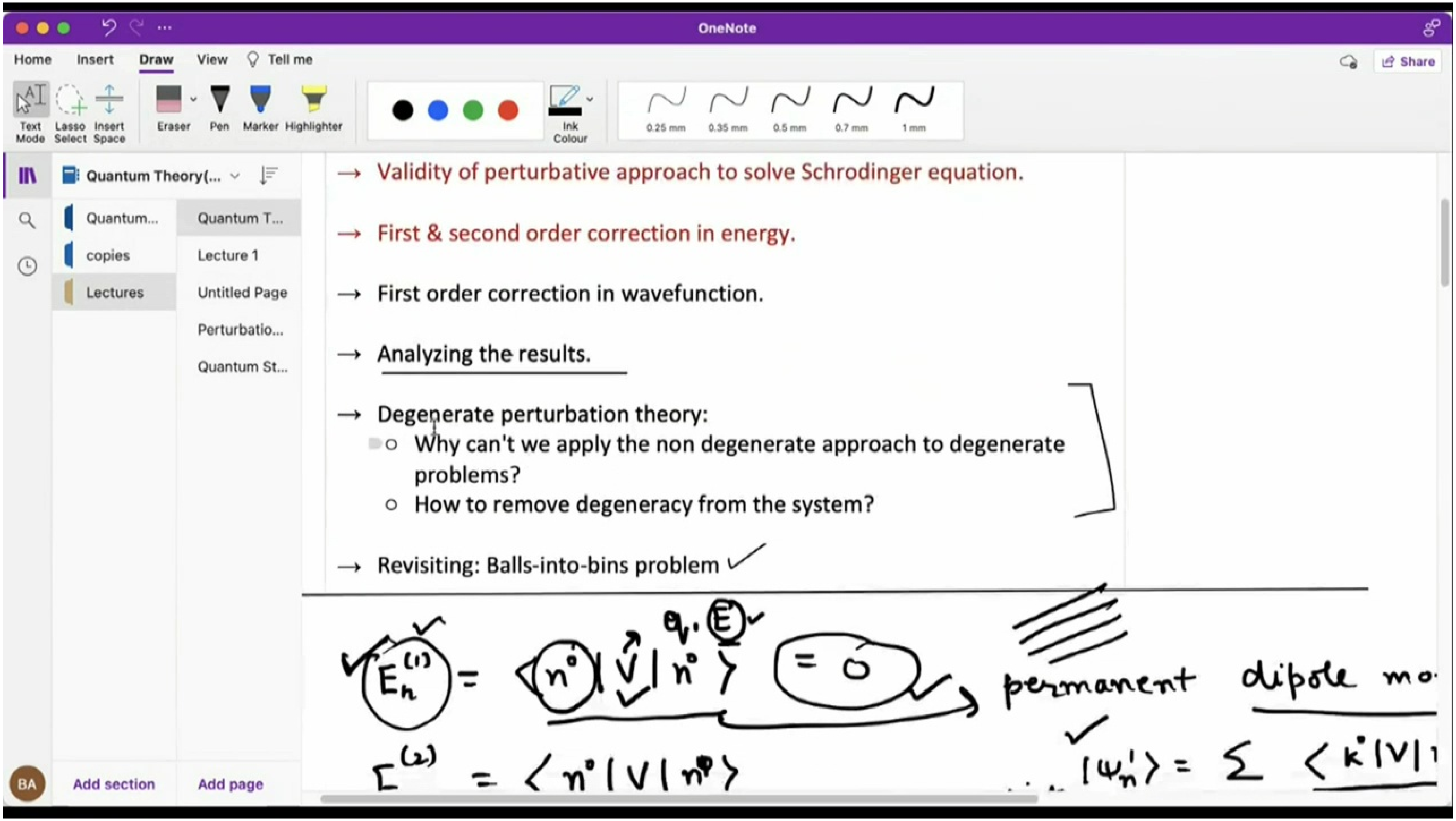
n → ground state



$$E_n = E_n^0 + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^0 - E_k^0} + \cdots\cdots$$

$$V_{nk} = \langle n^0 | V | k^0 \rangle$$

$E_n^{(2)}$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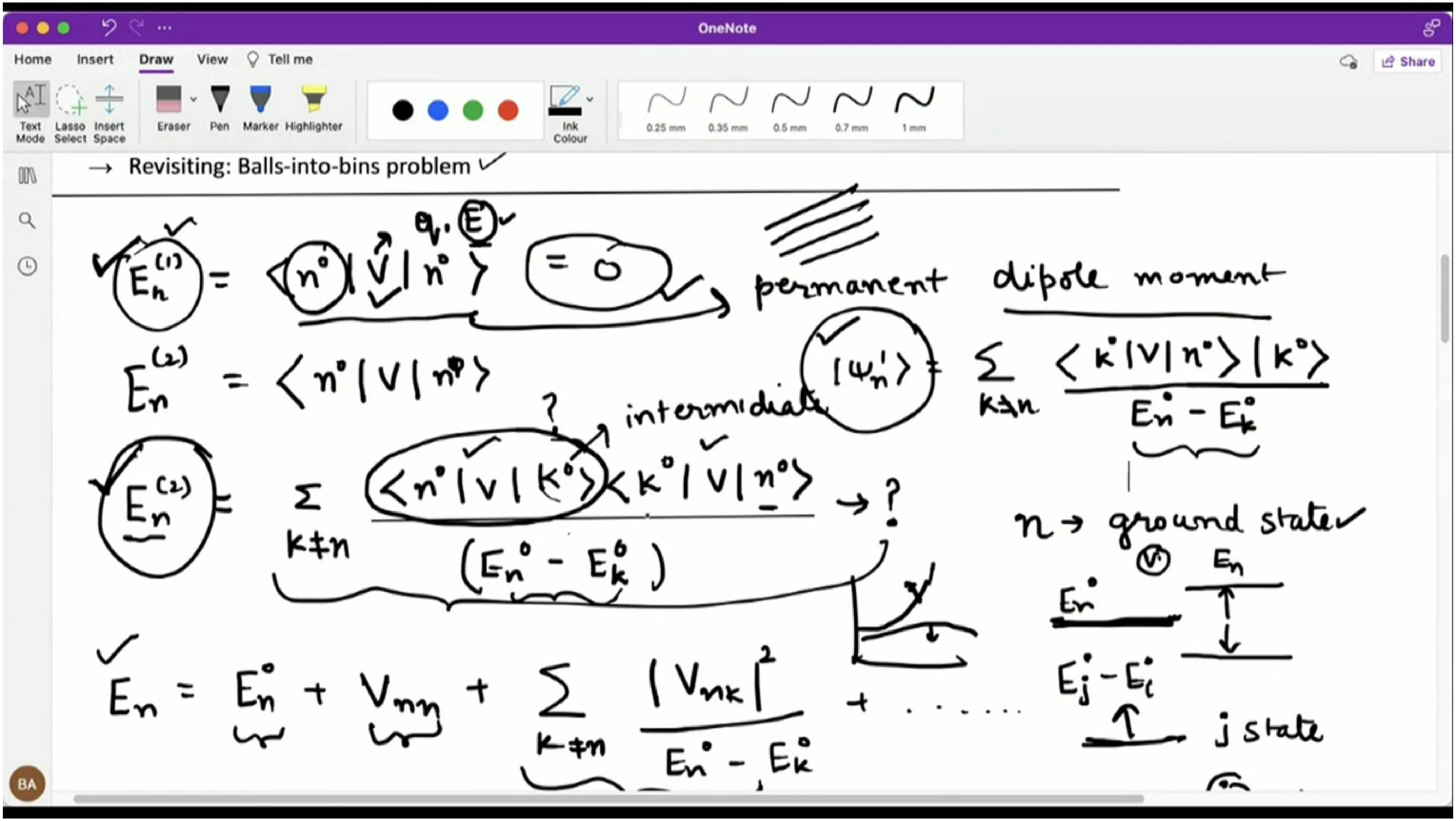
Add page
→ Validity of perturbative approach to solve Schrodinger equation.
→ First & second order correction in energy.
→ First order correction in wavefunction.
→ Analyzing the results.
→ Degenerate perturbation theory:
Why can't we apply the non degenerate approach to degenerate problems?
How to remove degeneracy from the system?
→ Revisiting: Balls-into-bins problem
permanent dipole mo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→ Revisiting: Balls-into-bins problem
E_n^(1) = ⟨n°| V |n°⟩ = 0 permanent dipole moment
q, E
E_n^(2) = ⟨n°| V |n°⟩
|ψ_n^1⟩ = Σ_{k≠n} ⟨k°|V|n°⟩ |k°⟩ / (E_n° − E_k°)
intermidiate
E_n^(2) = Σ_{k≠n} ⟨n°|V|k°⟩⟨k°|V|n°⟩ / (E_n° − E_k°) → ?
n → ground state
E_n = E_n° + V_nn + Σ_{k≠n} |V_nk|² / (E_n° − E_k°) + ...
E_j° − E_i°
j state

$|\psi_a^0\rangle \qquad |\psi_b^0\rangle$

$H^0|\psi_a^0\rangle = E_n^0|\psi_a^0\rangle \qquad H^0|\psi_b^0\rangle = E_n^0|\psi_b^0\rangle$

non-deg.

$|\psi_n\rangle = |\psi_n^0\rangle + O(\lambda) \qquad \lambda \to 0$

$|\psi_n\rangle \to |\psi_n^0\rangle$

Deg $\qquad \lambda \to 0$

non-deg.

$$|\psi_n\rangle = |\psi_n^0\rangle + O(\lambda)^{\to 0} \qquad \lambda \to 0$$

$$|\psi_n\rangle \to |\psi_n^0\rangle$$

Deg

$$\lambda \to 0$$

Correc:

Wednesday, 7 September 2022 8:46 PM

$|\psi_a^0\rangle$ ✓ $\quad |\psi_b^0\rangle$ ✓ $\quad \alpha|\psi_a^0\rangle + \beta|\psi_b^0\rangle$

✓ $H^0|\psi_a^0\rangle = E_n^0|\psi_a^0\rangle \quad H^0|\psi_b^0\rangle = E_n^0|\psi_b^0\rangle$

non-deg.

$$|\psi_n\rangle = |\psi_n^0\rangle + O(\lambda) \to 0 \qquad \lambda \to 0$$

$$|\psi_n\rangle \to |\psi_n^0\rangle$$

Deg

$\lambda \to 0$

'correct' combination of eigenkets

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Deg $\lambda \to 0$

'Correct' combination of eigenkets

$\langle \psi_n^0 |$

'Correct' combination of eigenkets ✓

$$\sum \frac{\langle \psi_n^0 | V | \psi_k^0 \rangle}{E_n^0 - E_k^0} \quad \rightarrow 0$$

$n \rightarrow k$

$\frac{0}{0}$

$$\langle \psi_n^{c0} | V | \psi_k^{c0} \rangle = 0$$

2 fold degenerate

$$H^0 |\psi_a^0\rangle = E_n^0 |\psi_a^0\rangle \qquad H^0 |\psi_b^0\rangle = E_n^0 |\psi_b^0\rangle$$

$$\langle \psi_a^0 | \psi_b^0 \rangle = 0 \qquad \langle \psi_b^0 | \psi_b^0 \rangle = \langle \psi_a^0 | \psi_a^0 \rangle = 1$$

H

$$H|\psi\rangle = E|\psi\rangle \quad , \text{ where } \quad H = H^0 + \lambda H'$$

$$E = E^0 + \lambda E' + \cdots \qquad |\psi\rangle = |\psi^0\rangle + \lambda|\psi'\rangle + \cdots$$

$$H^0|\psi'\rangle + H'|\psi^0\rangle = E^0|\psi'\rangle + E'|\psi^0\rangle$$

$$H^0|\psi'\rangle + H'|\psi^0\rangle = E^0|\psi'\rangle + E'|\psi^0\rangle$$

$\langle\psi_a^0|$ $\langle\psi_b^0|$

$$\langle\psi_a^0|H^0|\psi'\rangle + \langle\psi_a^0|H'|\psi^0\rangle = E^0\langle\psi_a^0|\psi'\rangle + E'\langle\psi_a^0|\psi^0\rangle$$

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$\langle \psi_a^0 |$ $\langle \psi_b^0 |$

$$\langle \psi_a^0 | H^0 | \psi' \rangle + \langle \psi_a^0 | H' | \psi^0 \rangle = E^0 \langle \psi_a^0 | \psi' \rangle + E' \langle \psi_a^0 | \psi^0 \rangle$$

$$\checkmark\ \alpha \langle \psi_a^0 | H' | \psi_a^0 \rangle + \beta \langle \psi_a^0 | H' | \psi_b^0 \rangle = \alpha E'$$

$$\langle \psi_a^0 | \qquad\qquad \langle \psi_b^0 |$$

$$\langle \psi_a^0 | H^0 | \psi' \rangle + \langle \psi_a^0 | H' | \psi^0 \rangle = E^0 \langle \psi_a^0 | \psi' \rangle + E' \langle \psi_a^0 | \psi^0 \rangle$$

$$\alpha \underbrace{\langle \psi_a^0 | H' | \psi_a^0 \rangle}_{W_{aa}} + \beta \underbrace{\langle \psi_a^0 | H' | \psi_b^0 \rangle} = \alpha E'$$

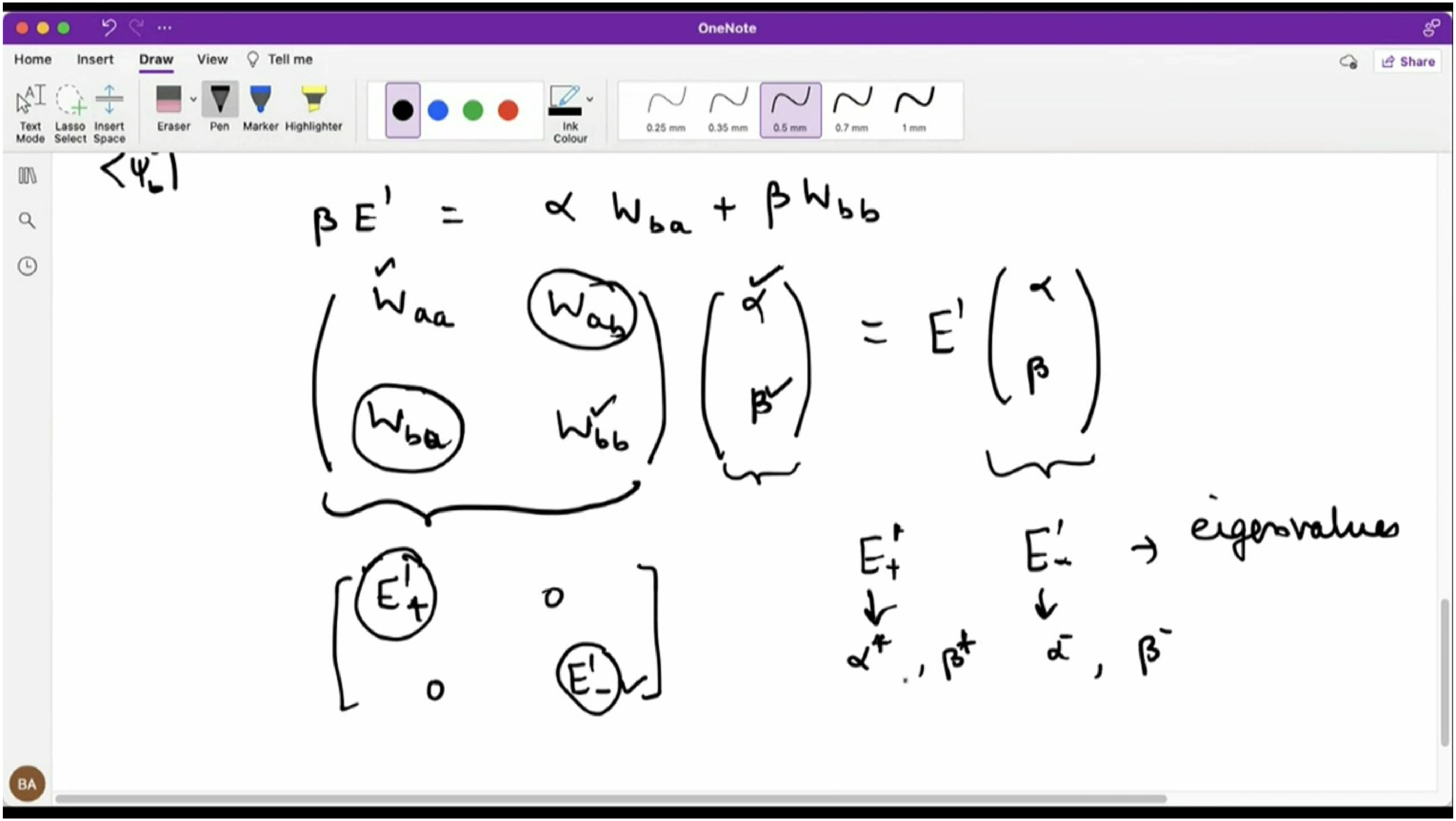
$$\alpha E' = \alpha W_{aa} + \beta W_{ab}$$

$$\langle \psi_b^0 |$$

$$\alpha E' = \alpha W_{aa} + \beta W_{ab}$$

$$\beta E' = \alpha W_{ba} + \beta W_{bb}$$

$$\begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E' \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$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β E' = α W_ba + β W_bb
W_aa W_ab / W_ba W_bb (α β) = E' (α β)
E'_+ 0 / 0 E'_-
E'_+ E'_- → eigenvalues
α^+, β^+ α^-, β^-

$\langle \psi_b^{\circ} |$

$$\beta E' = \alpha W_{ba} + \beta W_{bb}$$

$$\begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E' \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

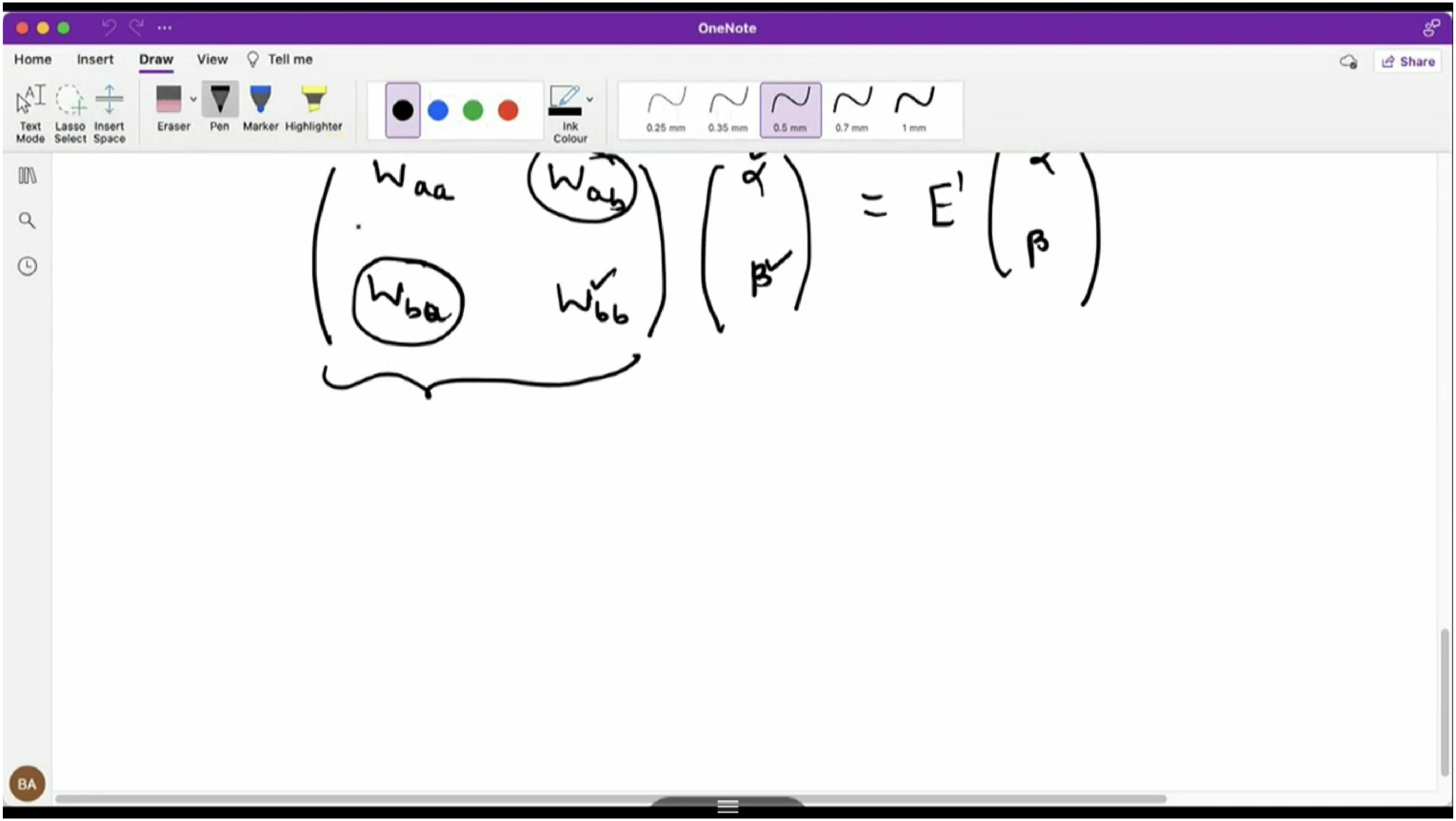
$$\begin{bmatrix} E'_+ & 0 \\ 0 & E'_- \end{bmatrix}$$

$E'_+ \quad E'_- \rightarrow$ eigenvalues

$\downarrow \qquad \downarrow$

$\alpha^+, \beta^+ \qquad \alpha^-, \beta^-$

$$|\psi^0\rangle = \alpha^+ |\psi_a^0\rangle + \beta^+ |\psi_b^0\rangle \rightarrow E'_+$$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$\begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E' \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$|\psi_n^c\rangle \rightarrow$ no more degenerate

$E_n^2 \qquad E_n^3$

<u>m - fold</u>. degener

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m - fold degeneracy. ρ^{α}

$$\begin{pmatrix} W_{11} & W_{12} & \cdots \\ W_{21} & W_{22} & \cdots \\ \vdots & \vdots & \end{pmatrix} \begin{pmatrix} \langle \psi_a^0 | \psi^0 \rangle \\ \langle \psi_b^0 | \psi^0 \rangle \\ \vdots \end{pmatrix} = E^1 \begin{pmatrix} \langle \psi_a^0 | \psi^0 \rangle \\ \langle \psi_b^0 | \psi^0 \rangle \\ \end{pmatrix}$$

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m - fold degeneracy.

$$\begin{pmatrix} W_{11} & W_{12} & \cdots \\ W_{21} & W_{22} & \cdots \\ \vdots & \vdots & \end{pmatrix} \begin{pmatrix} \langle \psi_a^0 | \psi^0 \rangle \\ \langle \psi_b^0 | \psi^0 \rangle \\ \vdots \end{pmatrix} = E^1 \begin{pmatrix} \langle \psi_a^0 | \psi^0 \rangle \\ \langle \psi_b^0 | \psi^0 \rangle \\ \vdots \end{pmatrix}$$

roots are same

either degeneracy is not lifted

partially degenerate

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$$\langle \psi_b^0 |$$

$$\alpha E^1 = \alpha W_{aa} + \beta W_{ab}$$

$$\beta E^1 = \alpha W_{ba} + \beta W_{bb}$$

$$\begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E^1 \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$E^1 = E^+ = E^-$$

$|\psi_n^c\rangle \rightarrow$ no more degenerate

$r^2 \qquad r^3$

$$\alpha E^1 = \alpha W_{aa} + \beta W_{ab}$$

$\langle \psi_b^0 |$

$$\beta E^1 = \alpha W_{ba} + \beta W_{bb}$$

$$\begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E^1 \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$E^1 = E^+ = E^-$$

$$E^2 \rightarrow |\psi\rangle$$

$|\psi_n^c\rangle \rightarrow$ no more degenerate

$E^2 \qquad E^3$

m - fold degeneracy. ρ^α

$$\begin{pmatrix} W_{11} & W_{12} & \cdots \\ W_{21} & W_{22} & \cdots \\ \vdots & \vdots & \end{pmatrix} \begin{pmatrix} \langle \psi_a^0 | \psi^0 \rangle \\ \langle \psi_b^0 | \psi^0 \rangle \\ \vdots \end{pmatrix} = E^1 \begin{pmatrix} \langle \psi_a^0 | \psi^0 \rangle \\ \langle \psi_b^0 | \psi^0 \rangle \\ \vdots \end{pmatrix}$$

roots are same

either degeneracy is not lifted

partially degenerate

E^2 ✓

roots are same

either degeneracy is not lifted

partially degenerate

E^2 ✓

non-degenerate

$$E_n^{(i)} = \langle n^0 | V | n^{i-1} \rangle$$

$$|n^{i-1}\rangle = \sum_{k \neq n} C_k^i |k^0\rangle$$

degenerate

$$\langle m^0 | n^0 \rangle E_n^{(i)} = \langle n^0 | V | n^{i-1} \rangle$$

$$\downarrow$$

$$|n^{i-1}\rangle = \sum_{k \notin D} C_k^{(i)} |k^0\rangle$$

potential well

$n_A \quad n_B \quad n_C$

$$E = E_A + E_B + E_C$$

$$E = \frac{\pi^2 \hbar^2}{2ma^2}\left(n_A^2 + n_B^2 + n_C^2\right)$$

$$E = \frac{\pi^2 \hbar^2}{2ma^2} \, 243$$

$$n_A^2 + n_B^2 + n_C^2 = 24$$

$$E = \frac{\pi^2 \hbar^2}{2ma^2} \; 243$$

	n_A	n_B	n_C
I	9	9	9
II	3	3	5
III	11	11	1
IV			

$$E = \frac{\pi^2 \hbar^2}{2ma^2} \, 243$$

$$\boxed{n_A^2 + n_B^2 + n_C^2 = 243}$$

	n_A	n_B	n_C
I	9	9	9
II	3	3	5
III	11	11	1
IV	5	7	3

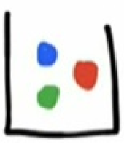
n_A	n_B	n_C

L.

$$E = \frac{\pi^2 \hbar^2}{2ma^2} \, 243$$

$$n_A^2 + n_B^2 + n_C^2 = 243$$

	$(n_A$	n_B	$n_C)$
I	9	9	9
II	3	3	5
III	11	11	1
IV	5	7	3



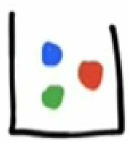
E_5 ? $n = 5$

n_A	n_B	n_C

$$E = \frac{\pi^2 \hbar^2}{2ma^2} \, 243$$

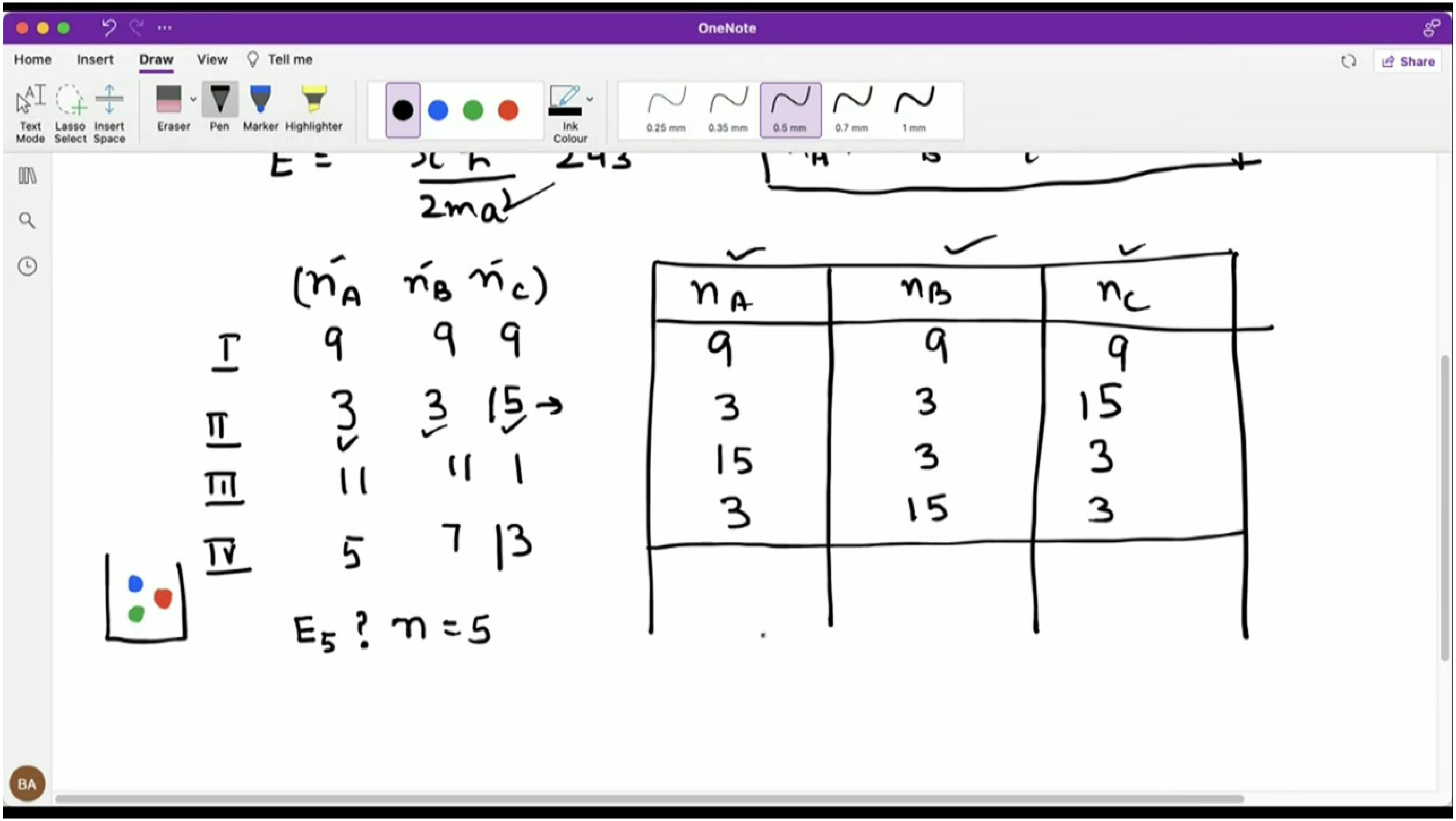
$$\boxed{n_A^2 + n_B^2 + n_C^2 = 243}$$

	n_A	n_B	n_C
I	9	9	9
II	3 ✓	3 ✓	5 ✓
III	11	11	1
IV	5	7	3



E_5 ? $n = 5$

n_A	n_B	n_C
9	9	9
3	3	5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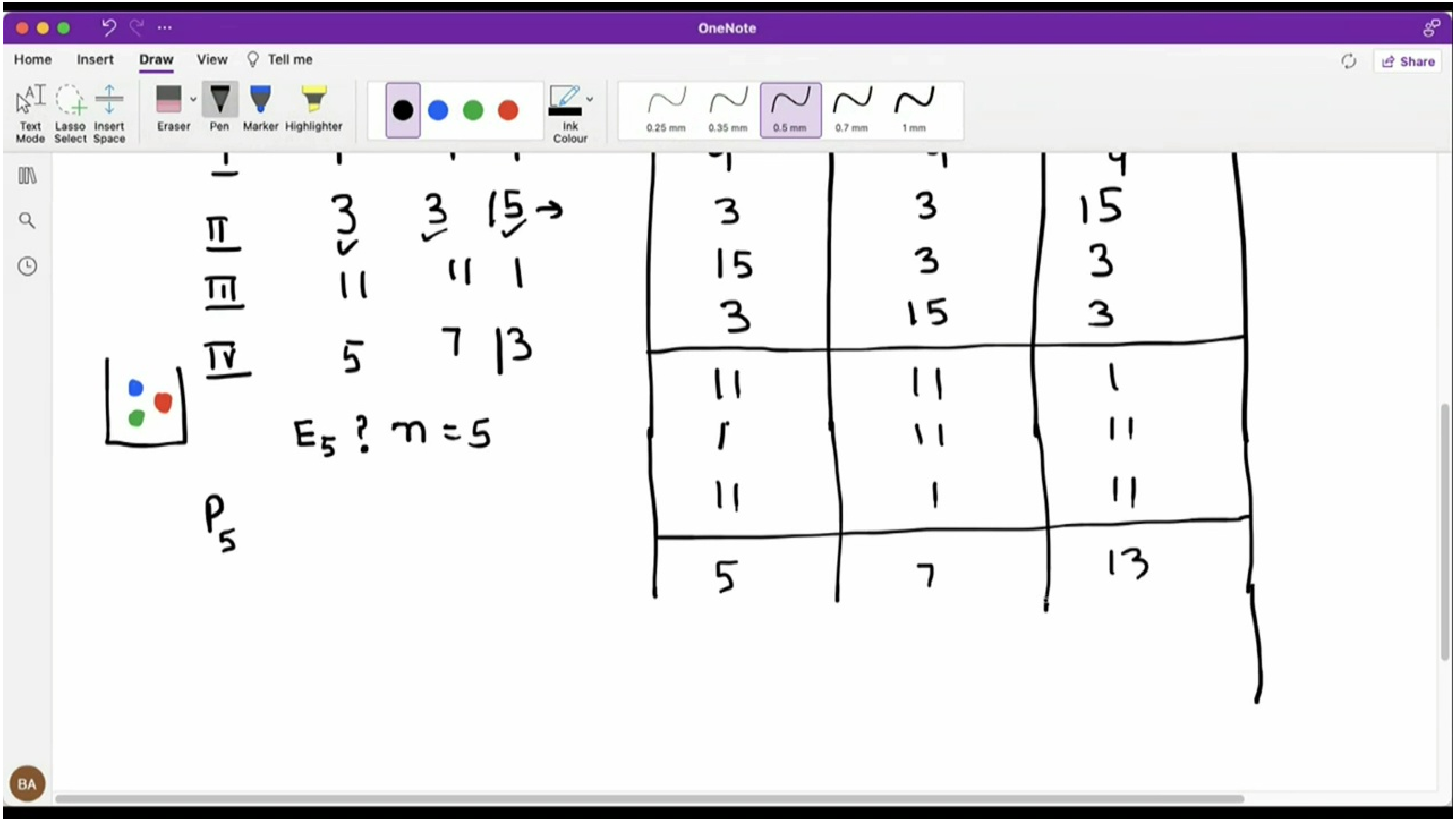
2ma²
(n_A n_B n_C)
I 9 9 9
II 3 3 15
III 11 11 1
IV 5 7 13
E_5 ? n = 5
n_A n_B n_C
9 9 9
3 3 15
15 3 3
3 15 3
BA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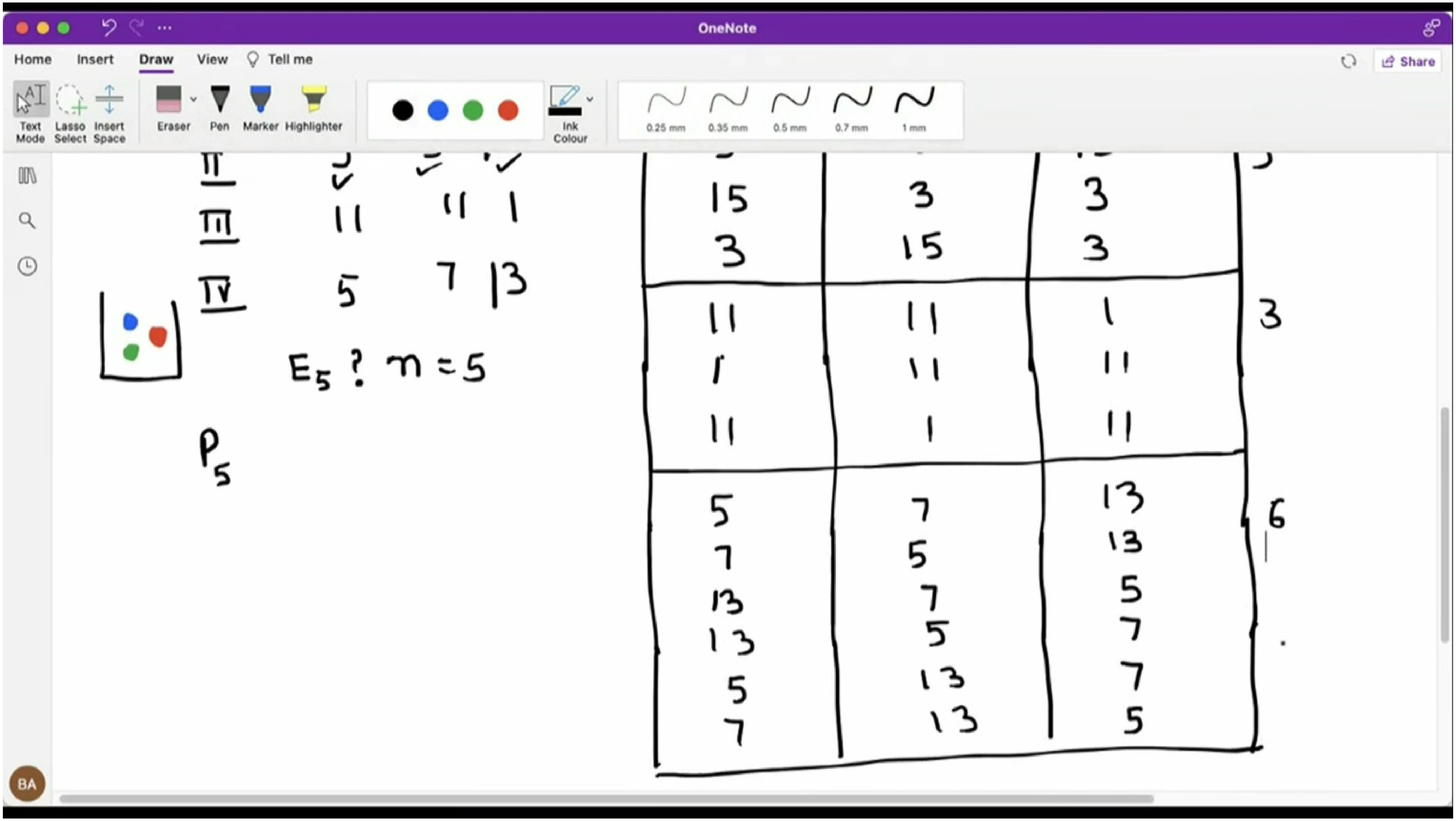
II 3 3 15 →
III 11 11 1
IV 5 7 13
E5 ? n = 5
P5
3 3 15
15 3 3
3 15 3
11 11 1
1 11 11
11 1 11
5 7 13
BA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E5 ? n = 5
P5

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$$(n_A, n_B, n_C) \checkmark = (7, 13, 5)$$

occupation number configuration

$$Q(\underset{-}{\overset{E_1}{N_1}}, \underset{-}{\overset{E_2}{N_2}}, \underset{-}{\overset{E_3}{N_3}} \ldots)$$

BA

Occupation number configuration

$$Q(\underset{E_1}{N_1}, \underset{E_2}{N_2}, \underset{E_3}{N_3} \; - - - \quad)$$

$$\checkmark\; Q(0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1) = (7, 13, 5)$$

$$(9, 9, 9)$$

$$Q($$

7	13	5

$$(n_A, n_B, n_C) = (7, 13, 5)$$

Occupation number configuration

$$Q(\underset{E_1}{N_1}, \underset{E_2}{N_2}, \underset{E_3}{N_3} \, - - - \quad)$$

$$Q(0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1) = (7, 13, 5)$$

$$(9, 9, 9)$$

$$Q(0, 0, 0, 0, 0, 0, 0, 0, \underset{E_9}{3}, 0, - - - - \quad)$$