

Title: Atom interferometry, atomic clocks

Speakers: Jason Hogan

Collection: School on Table-Top Experiments for Fundamental Physics

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Atom interferometry, atomic clocks

School on Table-Top Experiments for Fundamental Physics

Perimeter Institute

Jason Hogan
Stanford University
September 19, 2022



Outline

Lecture 1

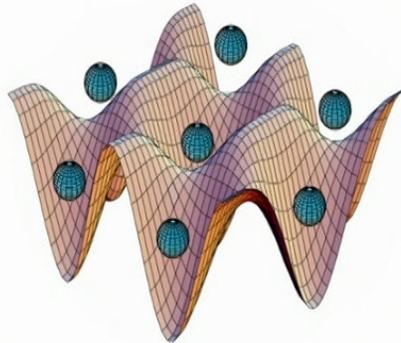
- General introduction and motivation
- Atom interferometer phase response
- Example applications: Spacetime curvature, EP tests, gravitational Aharonov-Bohm, atom charge neutrality

Lecture 2

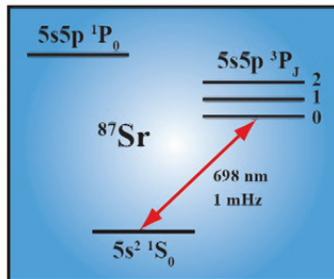
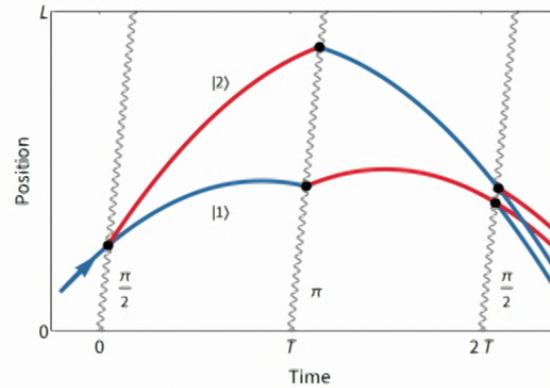
- Gravitational wave detection
- Clock atom interferometry and MAGIS
- MAGIS-100: design, systematic error mitigation, and construction
- Advanced atom optics (large momentum transfer techniques)

Atomic clocks and atom interferometers

Optical lattice clocks



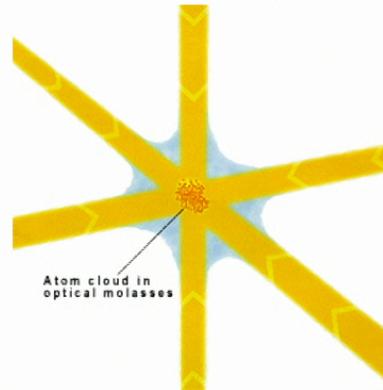
Atom interferometers



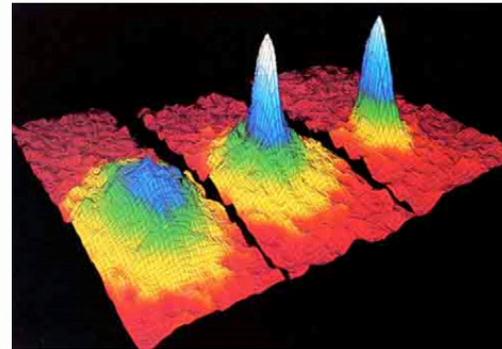
- How can we leverage the incredible gains in stability and accuracy of clocks for fundamental physics?
- Atomic clocks and interferometers offer the potential for gravitational wave detection in an unexplored frequency range
- Development of new "clock" atom interferometer inertial sensors based on narrow optical transitions

Cold atoms

Laser Cooling (MOT)



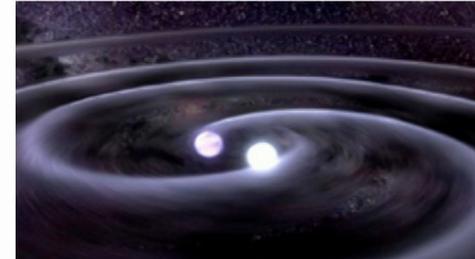
BEC



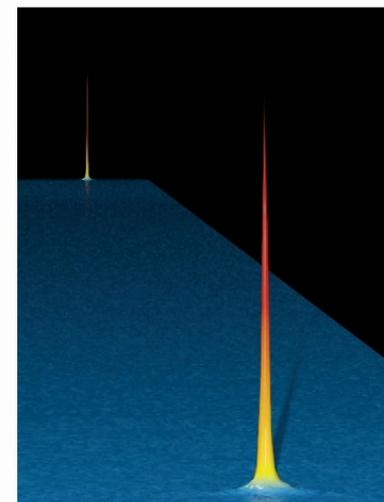
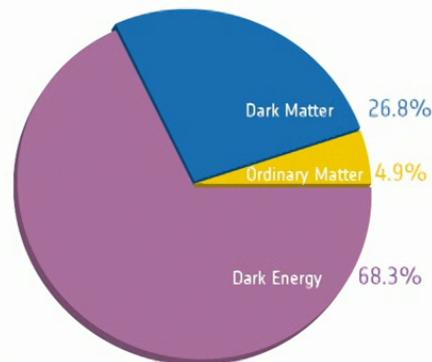
Images: <http://www.nobelprize.org>

Science applications

- Gravitational wave detection
- Quantum mechanics at macroscopic scales
- QED tests (alpha measurements)
- Quantum entanglement for enhanced readout
- Equivalence principle tests, tests of GR
- Short distance gravity
- Search for dark matter
- Atom charge neutrality



Compact binary inspiral



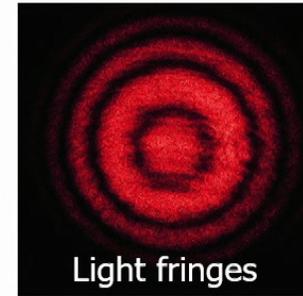
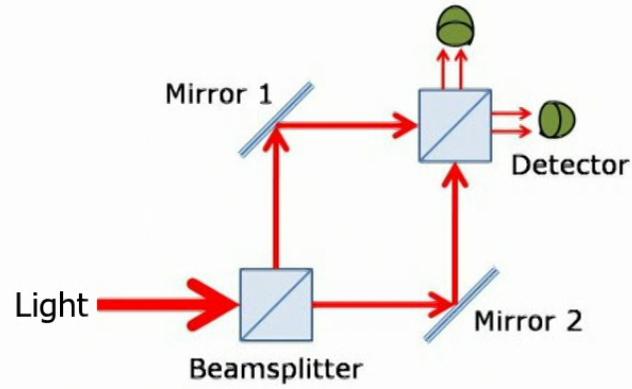
Rb wavepackets separated by 54 cm

Image: <https://www2.physics.ox.ac.uk/research/dark-matter-dark-energy>

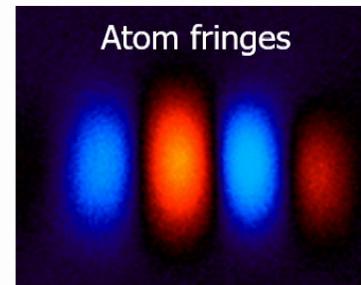
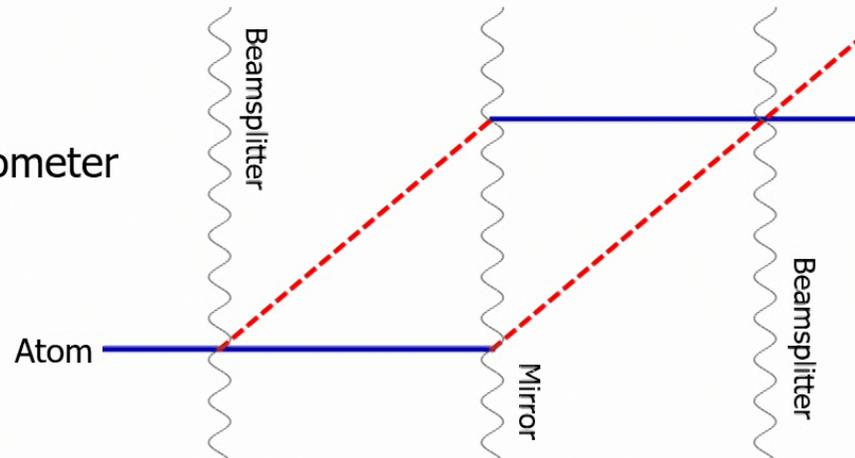
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Atom interference

Light interferometer



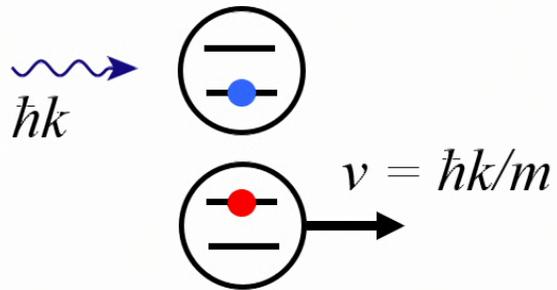
Atom interferometer



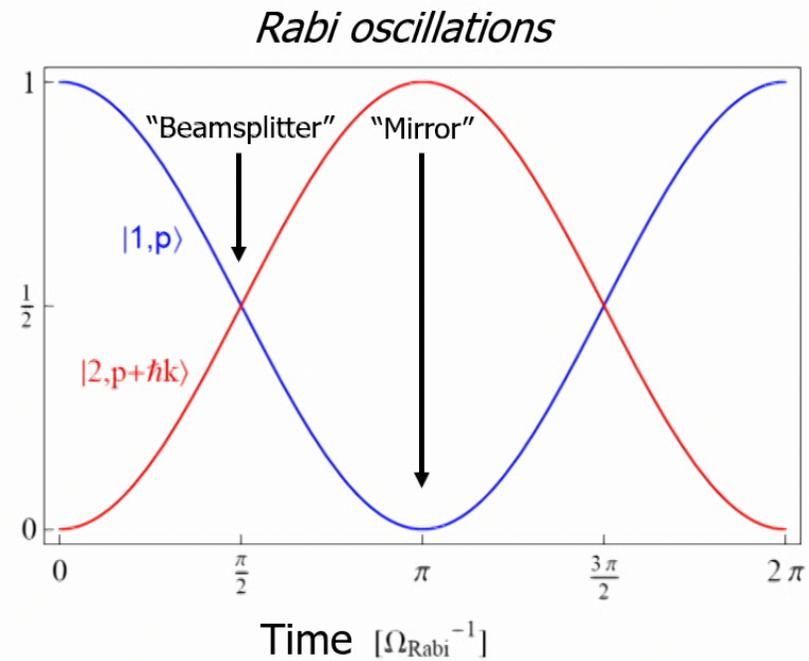
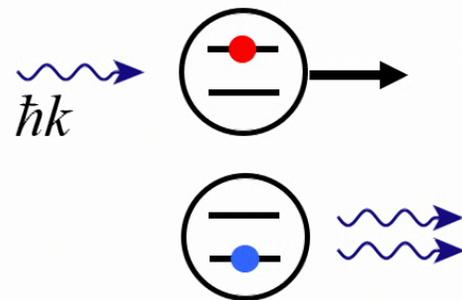
<http://scienceblogs.com/principles/2013/10/22/quantum-erasure/>
<http://www.cobolt.se/interferometry.html>

Atom optics using light

(1) Light absorption:



(2) Stimulated emission:



Light Pulse Atom Interferometry

$|\text{atom}\rangle = |p\rangle$

$\pi/2$ pulse "beamsplitter"

$$\frac{1}{\sqrt{2}}(|p\rangle + e^{i\Delta\phi}|p + \hbar k\rangle)$$

π pulse "mirror"

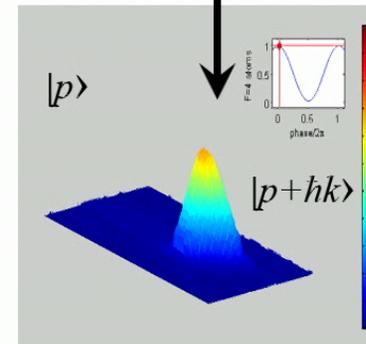
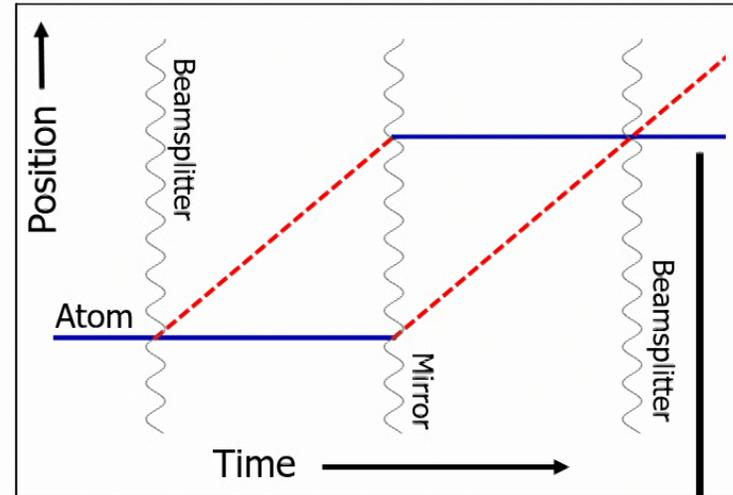
$$\frac{1}{\sqrt{2}}(|p + \hbar k\rangle + e^{i\Delta\phi}|p\rangle)$$

$\pi/2$ pulse "beamsplitter"

$$\frac{1}{2}((1 + e^{i\Delta\phi})|p\rangle + ((1 - e^{i\Delta\phi}))|p + \hbar k\rangle)$$

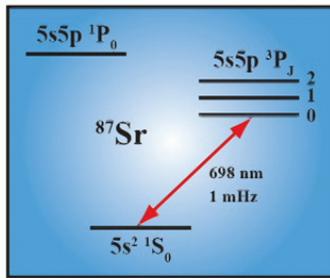
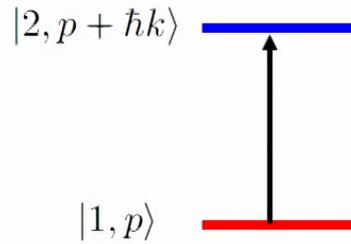
Probability in State $|p\rangle = \cos^2\left(\frac{\Delta\phi}{2}\right)$

Probability in State $|p + \hbar k\rangle = \sin^2\left(\frac{\Delta\phi}{2}\right)$

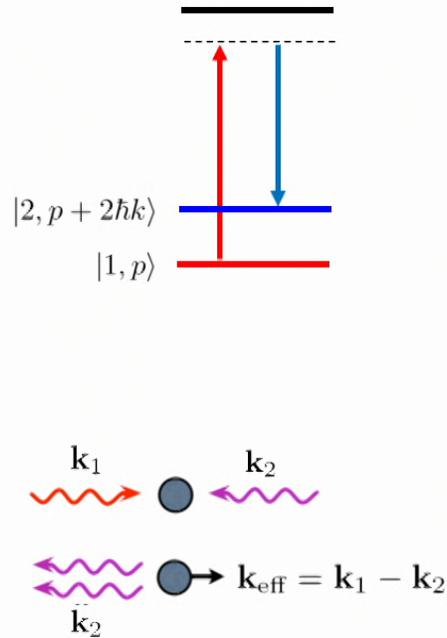


Common atom optics processes

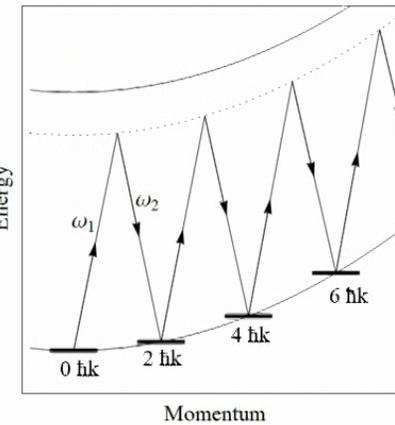
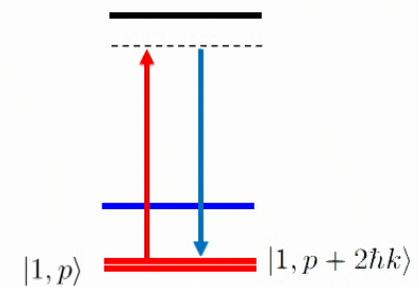
Single photon



Raman

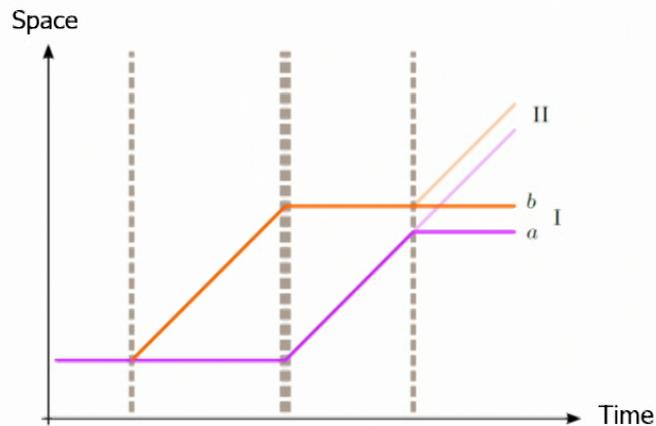


Bragg



Spontaneous emission in alkali atoms require 2-photon atom optics

Example interferometer geometries

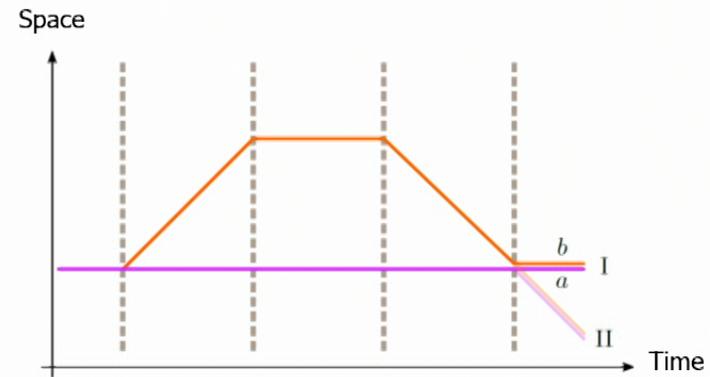


Mach-Zehnder interferometer

Phase shift measures acceleration

Example: Equivalence principle tests, inertial sensing

Also: Gyroscopes (space-space instead of space-time)



Ramsey-Borde interferometer

Phase shift measures kinetic energy difference (due to absorbed photons)

Example: fine structure constant measurements

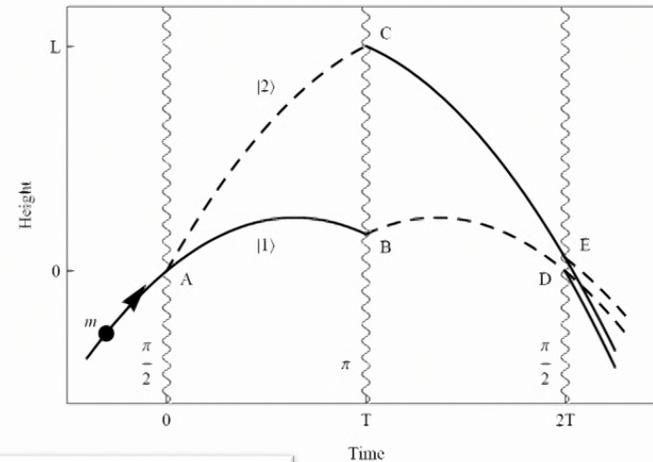
Figures from: A. Roura, Phys. Rev. X 10, 021014 (2020)

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Atom interferometer phase shift analysis

Phase shift can be decomposed as:

- **Laser** phase at each node
- **Propagation** phase along each path
- **Separation** phase at end of interferometer



$$\Delta\phi_{\text{tot}} = \Delta\phi_{\text{propagation}} + \Delta\phi_{\text{separation}} + \Delta\phi_{\text{laser}}$$

This approach mostly follows "Light-pulse atom interferometry" (2008), as well as Bongs/Kasevich (2006) and others.

Other approaches:

- C. Borde, ABCD formalism, midpoint theorem, e.g., Metrologia 39, 435-463, (2002)
- Storey, Cohen-Tannoudji. "The Feynman path integral approach to atomic interferometry. A tutorial" (1994)
- Representation-free approach: Kleinert (2015)
- Wigner function approach: Dubetsky (2006)

Propagation phase

Time evolution of atom's state between laser pulses:

Galilean transformation operator: $\hat{G}_c \equiv \hat{G}(\mathbf{x}_c, \mathbf{p}_c, L_c) = e^{i \int L_c dt} e^{-i \hat{\mathbf{p}} \cdot \mathbf{x}_c} e^{i \mathbf{p}_c \cdot \hat{\mathbf{x}}}$

Phase Translation Boost

$$\langle \mathbf{x} | \psi, A_i \rangle = \langle \mathbf{x} | \hat{G}_c | \phi_{CM} \rangle | A_i \rangle = e^{i \int_{t_I}^{t_F} L_c dt} e^{i \mathbf{p}_c \cdot (\mathbf{x} - \mathbf{x}_c)} \phi_{CM}(\mathbf{x} - \mathbf{x}_c) | i \rangle e^{-i E_i (t_F - t_I)}$$

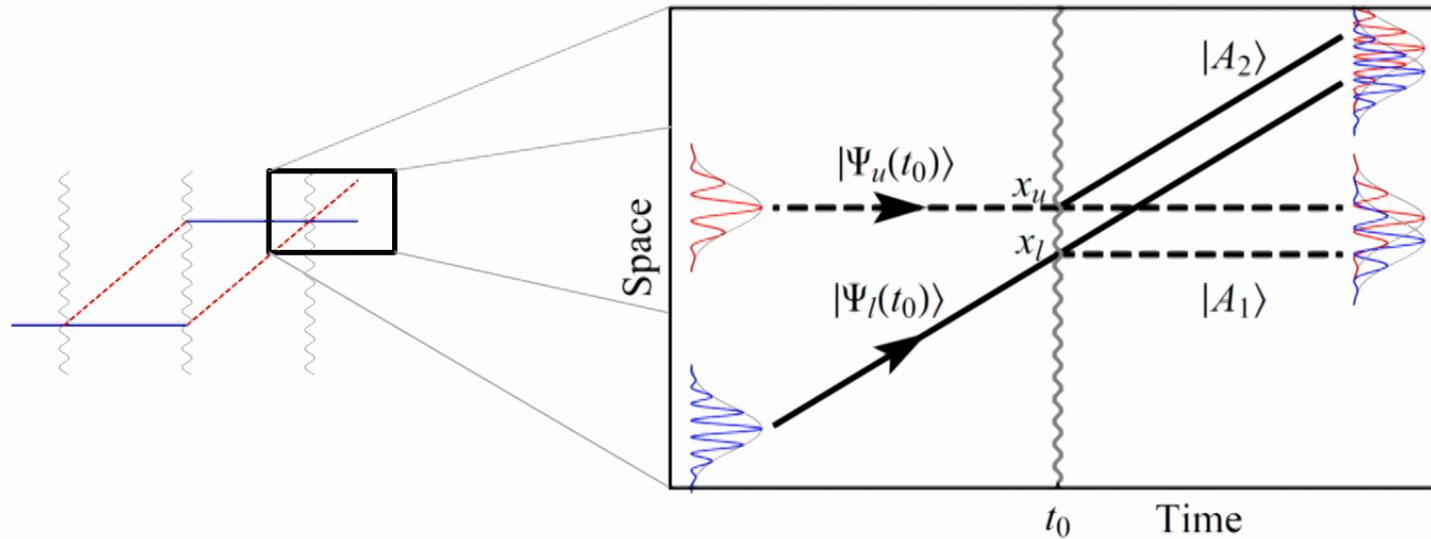
Propagation phase

- The phase of the center of the wavepacket is the classical action
- The carrier and wavepacket envelope move together along the classical path

$$\Delta \phi_{\text{propagation}} = \sum_{\text{upper}} \left(\int_{t_I}^{t_F} (L_c - E_i) dt \right) - \sum_{\text{lower}} \left(\int_{t_I}^{t_F} (L_c - E_i) dt \right)$$

Separation phase

Wavepackets do not always perfectly overlap at the final beamsplitter, due to tidal forces across wavepacket separation



$$\Delta\phi_{\text{separation}} = \bar{\mathbf{p}} \cdot \Delta\mathbf{x}$$

$$\Delta\mathbf{x} \equiv \mathbf{x}_l - \mathbf{x}_u$$

Conceptually similar to propagation phase; completes the loop.

Laser phase

$$|\Psi\rangle = \int d\mathbf{p} \sum_i c_i(\mathbf{p}) |\psi_{\mathbf{p}}\rangle |A_i\rangle$$

Atom-light interactions follow from Schrodinger equation (interaction picture):

$$\dot{c}_1(\mathbf{p}) = \frac{1}{2i} \Omega c_2(\mathbf{p} + \mathbf{k}) e^{-i\phi_L} e^{-i \int_{t_0}^t \Delta(\mathbf{p}) dt}$$

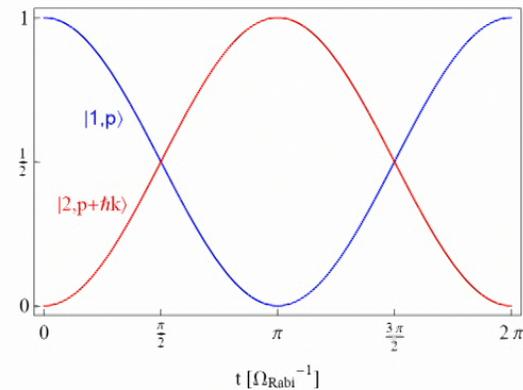
$$\dot{c}_2(\mathbf{p} + \mathbf{k}) = \frac{1}{2i} \Omega^* c_1(\mathbf{p}) e^{i\phi_L} e^{i \int_{t_0}^t \Delta(\mathbf{p}) dt}$$

Transition rules:	$ \mathbf{p}\rangle \rightarrow \mathbf{p} + \mathbf{k}\rangle e^{i\phi_L}$
	$ \mathbf{p} + \mathbf{k}\rangle \rightarrow \mathbf{p}\rangle e^{-i\phi_L}$

$$\phi_L \equiv \mathbf{k} \cdot \mathbf{x}_c(t_0) - \omega t_0 + \phi$$

Atom position

Rabi oscillations

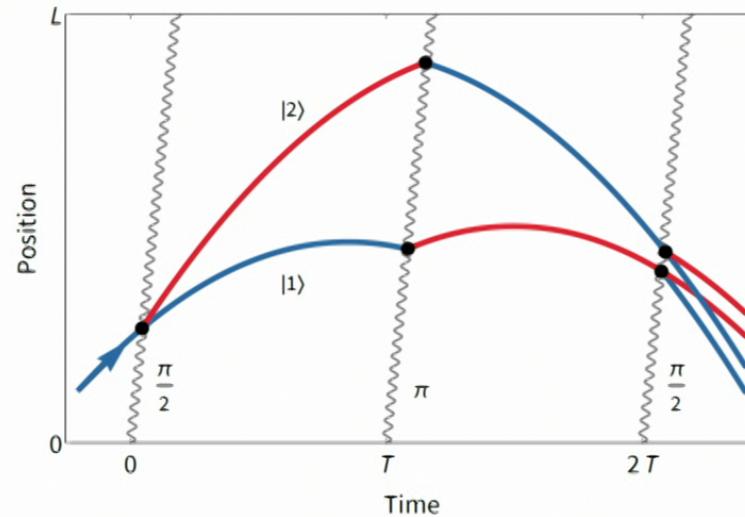


- Laser phase is imprinted on the wavefunction at each pulse
- The position of the atom (at time of pulse) is encoded in the atom's wavefunction
- "Measures" the atom position with a wavelength-scale "ruler" → *corresponding momentum kick* (uncertainty principle)

General relativistic phase shifts

Light-pulse interferometer phase shifts in GR:

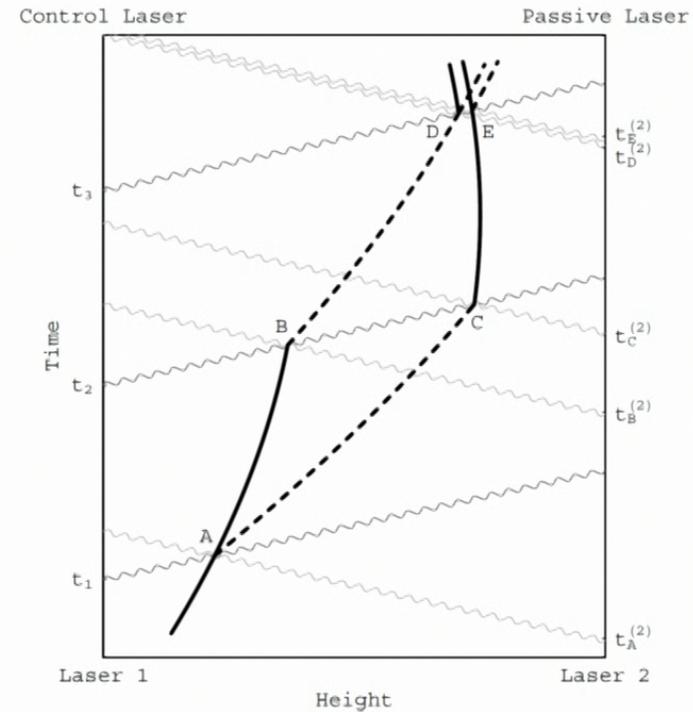
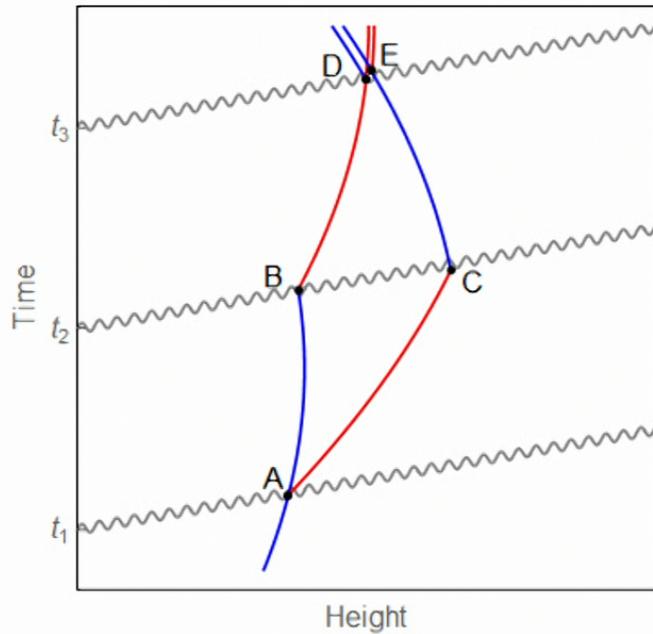
- Geodesic propagation for atoms and light.
- Path integral formulation to obtain quantum phases.
- Atom-field interaction at intersection of laser and atom geodesics.



Atom and photon geodesics

Dimopoulos et al., PRD 78, 042003 (2008)

Single photon vs two photon atom optics



Consequential differences:

- For 2-photon, light pulses must leave at different times
- By convention, "passive" laser is on before "control" pulse arrives
- Arrival of control pulse determines passive null geodesic (and its laser phase)
- No passive laser for single photon atom optics

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Atom interferometer phase shift calculation

The atom interferometer phase shift can be decomposed as

$$\Delta\phi_{\text{tot}} = \Delta\phi_{\text{propagation}} + \Delta\phi_{\text{separation}} + \Delta\phi_{\text{laser}}$$

non-relativistic

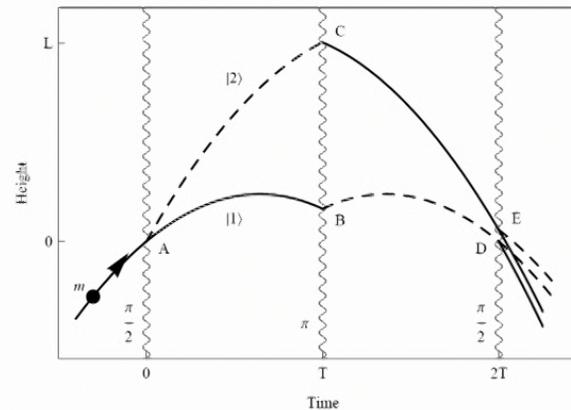
$$\left\{ \begin{array}{l} \Delta\phi_{\text{propagation}} = \sum_{\text{upper}} \left(\int_{t_i}^{t_f} (L_c - E_i) dt \right) - \sum_{\text{lower}} \left(\int_{t_i}^{t_f} (L_c - E_i) dt \right) \\ \Delta\phi_{\text{laser}} = \left(\sum_j \pm \phi_L(t_j, \mathbf{x}_u(t_j)) \right)_{\text{upper}} - \left(\sum_j \pm \phi_L(t_j, \mathbf{x}_l(t_j)) \right)_{\text{lower}} \\ \Delta\phi_{\text{separation}} = \bar{\mathbf{p}} \cdot \Delta\mathbf{x} \end{array} \right.$$

relativistic

$$\left\{ \begin{array}{l} \phi_{\text{propagation}} = \int L dt = \int m d\tau = \int p_\mu dx^\mu \\ \Delta\phi_{\text{laser}} = \left(\sum_j \pm \phi_L(t_j, \mathbf{x}_u(t_j)) \right)_{\text{upper}} - \left(\sum_j \pm \phi_L(t_j, \mathbf{x}_l(t_j)) \right)_{\text{lower}} \\ \phi_{\text{separation}} = \int_E^D \bar{p}_\mu dx^\mu \end{array} \right.$$

Mach-Zehnder as discrete derivative sensor

Simple picture: Atom interferometer records the positions of the atom with respect to a wavelength-scale “laser ruler”

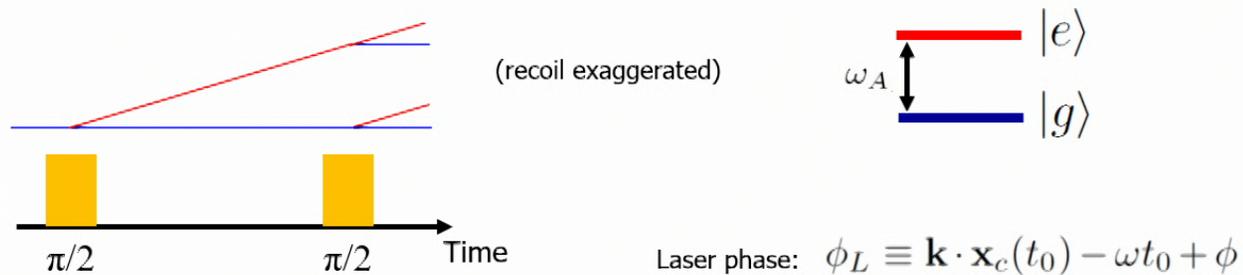


$$\phi_L \equiv \mathbf{k} \cdot \mathbf{x}_c(t_0) - \omega t_0 + \phi$$

- For Mach-Zehnder, propagation + separation phase tend to cancel
 ...Except for high-order potentials (e.g., Aharonov Bohm effects) | See Overstreet et al., Am. J. Phys. 89, 324 (2021)
- Laser phase records the position of the atom at each pulse
- Total phase encodes differences (motion) between pulses
- “Discrete derivative sensor”: Records any spatial (or temporal) variation of atom (or background fields).

Two pulse atomic clock sequence

Atomic clocks are closely related to atom interferometers
 Consider a *microwave* atomic clock (Ramsey sequence)



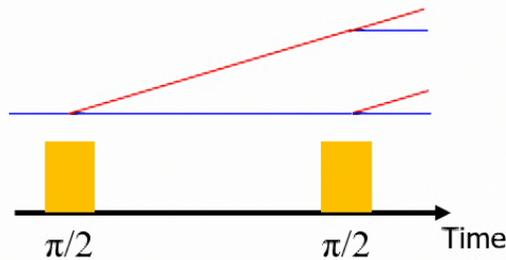
Can ignore separation phase (recoil is negligible for a microwave transition)

$$\begin{aligned} \Delta\phi &= \Delta\phi_{\text{prop}} + \Delta\phi_{\text{laser}} = -\omega_A T + \phi_1 - \phi_2 \\ &= (\omega - \omega_A)T + kx_1 - kx_2 \\ &= (\omega - \omega_A)T + \underline{kvT} \quad (\text{atom velocity } v) \end{aligned}$$

Sensitive to atom velocity (Doppler shift)

Atom interferometer as discrete derivative sensor

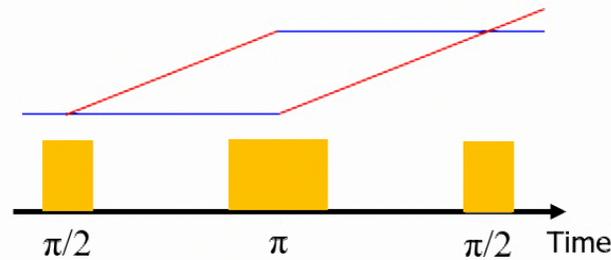
Ramsey sequence (clock)



$$\begin{aligned}\Delta\phi &= \phi_1 - \phi_2 = (\omega - \omega_A)T + kx_1 - kx_2 \\ &= (\omega - \omega_A)T + \underline{k v T}\end{aligned}$$

- Measures velocity

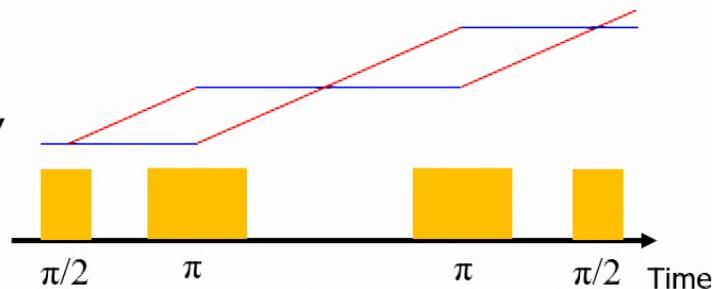
Mach-Zehnder



$$\begin{aligned}\Delta\phi &= (\phi_1 - \phi_2) - (\phi_2 - \phi_3) \\ &= k v_1 T - k v_2 T = \underline{k a T^2}\end{aligned}$$

- "Difference" of two Ramsey sequences
- Measures acceleration

"Double diamond"



$$\Delta\phi = k a_1 T^2 - k a_2 T^2 = k \delta a T^3$$

- Difference of two MZ loops
- Measures acceleration gradient (in space and/or time)

Tune response using pulse timing

Three pulse (accelerometer) gravity gradient phase shifts:

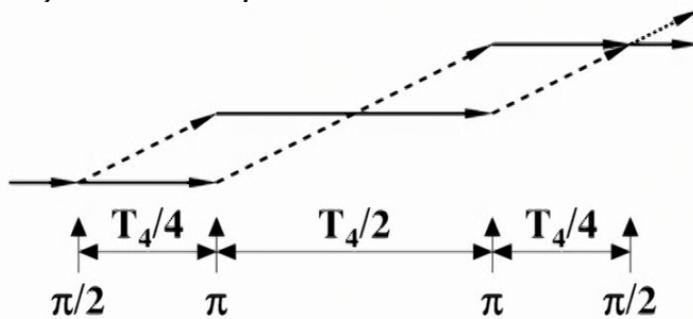
$$k_{\text{eff}} T_{zz} v_z T^3$$

$$\frac{\hbar k_{\text{eff}}^2}{2m} T_{zz} T^3$$

Can causes sensitivity to kinematic errors/noise

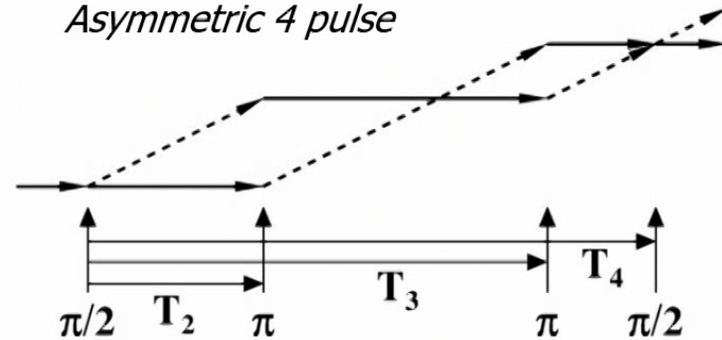
Can selectively suppress gravity gradient or acceleration response, etc.

Symmetric 4 pulse



Eliminates all T^2 terms

Asymmetric 4 pulse



Golden ratio diamonds: $T_{2,3} = \frac{\sqrt{5} \mp 1}{4} T_4$

Eliminates all T^3 terms

B. Dubetsky et al., PRA 74, 023615 (2006).

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Tune response using pulse timing

Three pulse (accelerometer) gravity gradient phase shifts:

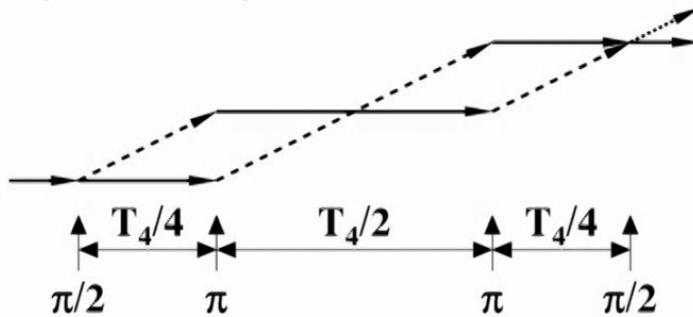
$$k_{\text{eff}} T_{zz} v_z T^3$$

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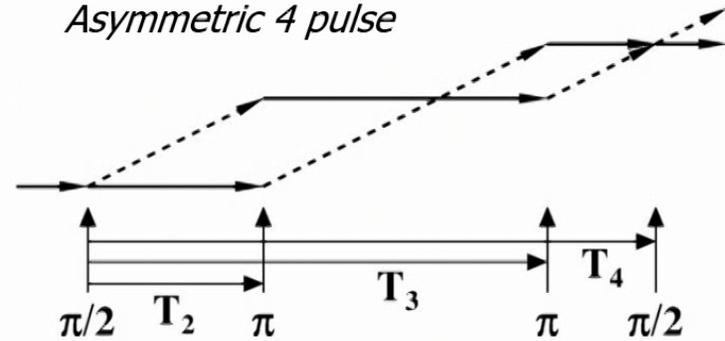
Can selectively suppress gravity gradient or acceleration response, etc.

Symmetric 4 pulse



Eliminates all T^2 terms

Asymmetric 4 pulse

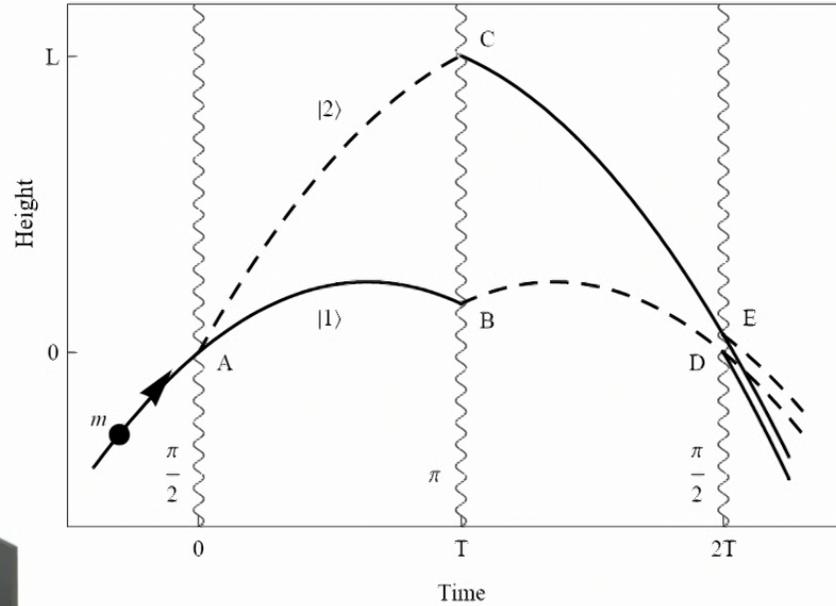
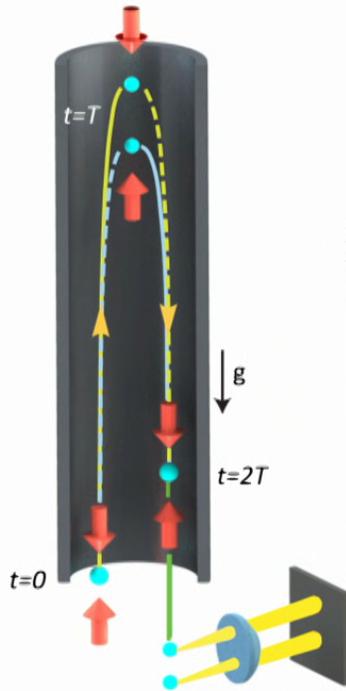


Golden ratio diamonds: $T_{2,3} = \frac{\sqrt{5} \mp 1}{4} T_4$

Eliminates all T^3 terms

B. Dubetsky et al., PRA 74, 023615 (2006).

Accelerometer sensitivity



$$\Delta\phi = k_{\text{eff}}gT^2$$

*Proportional to
spacetime
area enclosed.*

$$\frac{\delta g}{g} \sim \frac{\delta\phi}{k_{\text{eff}}gT^2}$$

Sensitivity

$$\delta\phi \sim \frac{1}{\sqrt{N}}$$

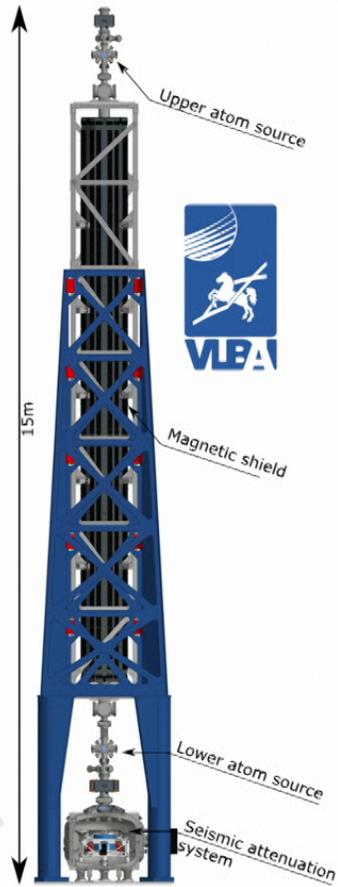
Shot noise

T : Long duration

k_{eff} : Large wavepacket separation

$\delta\phi$: High flux, spin squeezing

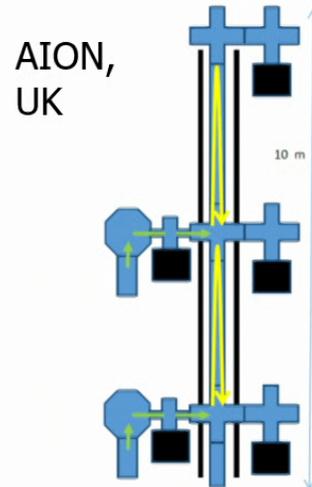
10-meter scale atom drop towers



Hannover, Germany

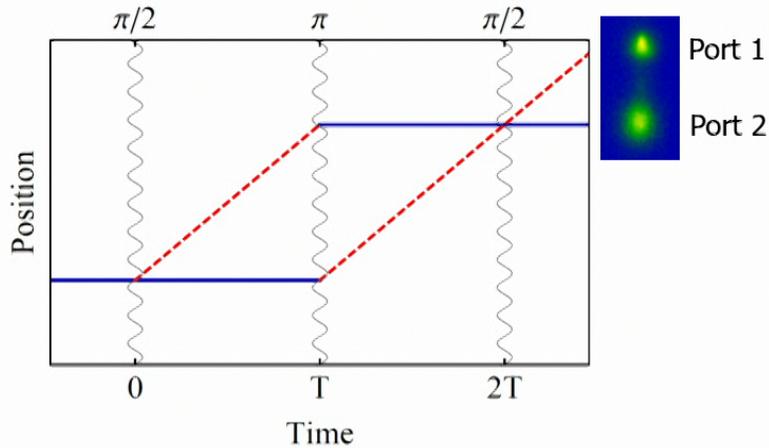


Wuhan, China



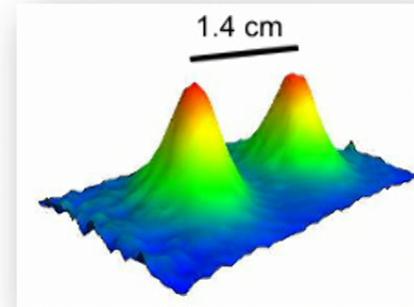
Stanford University

Interference at long interrogation time



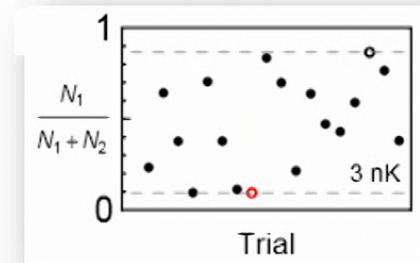
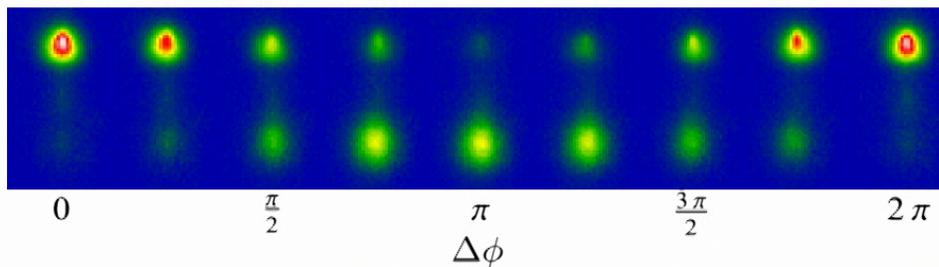
$2T = 2.3$ seconds

1.4 cm wavepacket separation



Wavepacket separation at apex (this data 50 nK)

Interference (3 nK cloud)



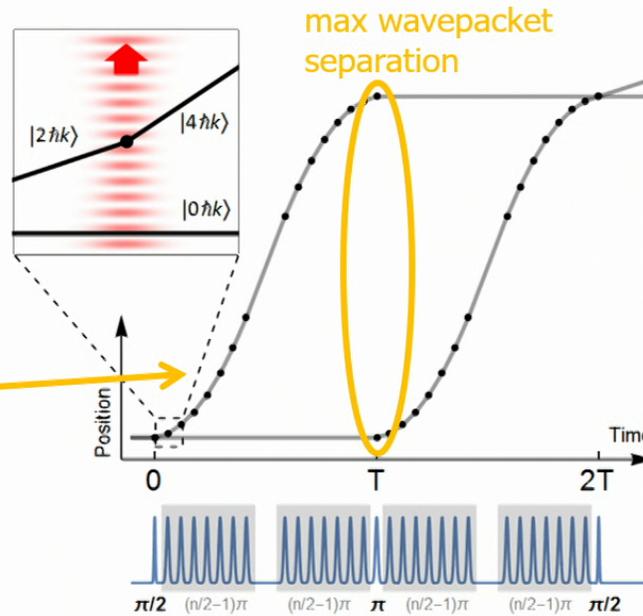
Dickerson, et al., PRL **111**, 083001 (2013).

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Large space-time area atom interferometry

Long duration (2 seconds),
large separation (>0.5 meter)
matter wave interferometer

90 photons worth
of momentum



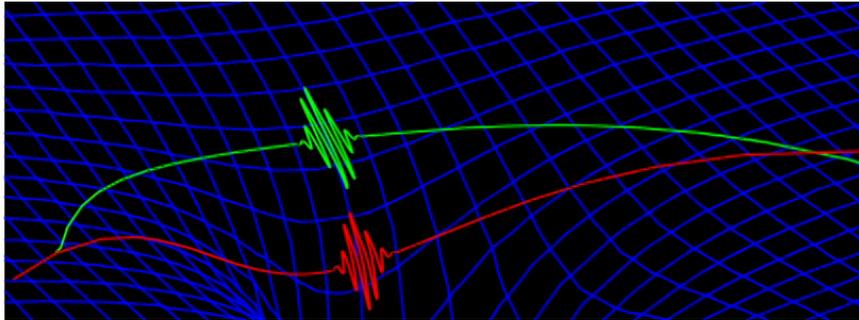
*World record wavepacket separation due to
momentum transfer from multiple laser pulses*

54 cm

Kovachy et al., *Nature* 2015

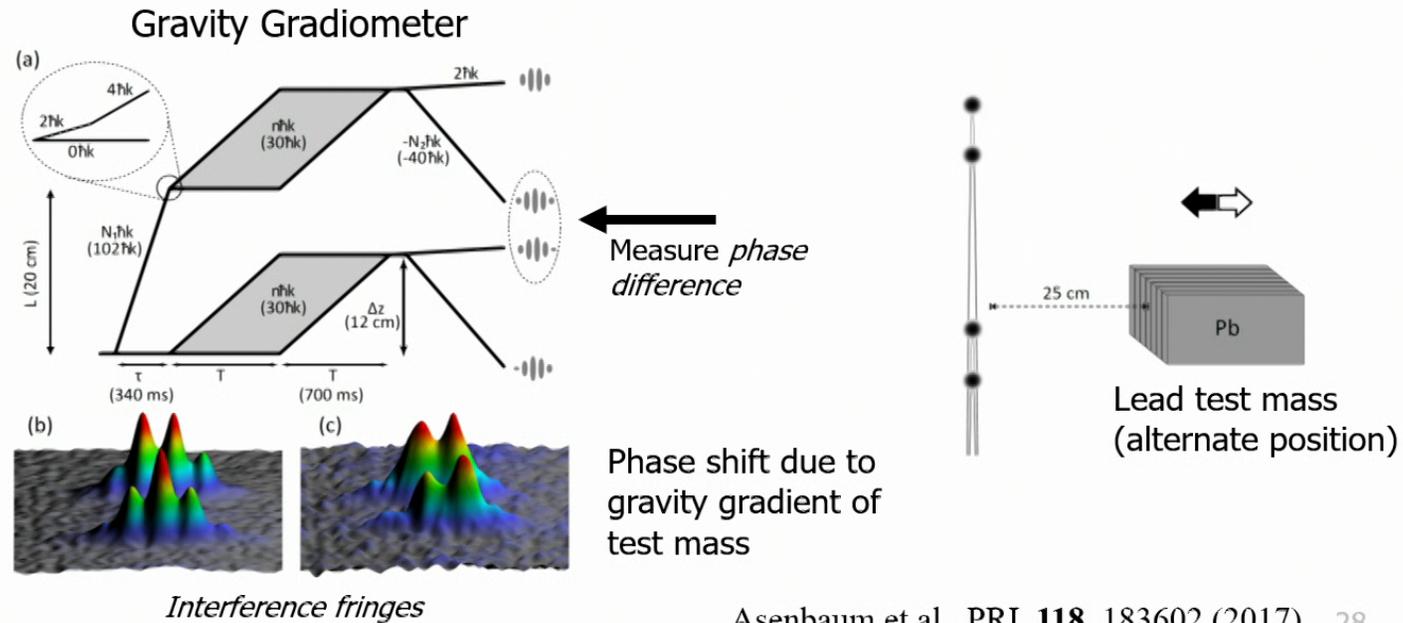
27

Phase shift from spacetime curvature



In general relativity, gravity is a result of *spacetime curvature*.

Observed the effect of spacetime curvature across a *single particle's wavefunction*.



Asenbaum et al., PRL **118**, 183602 (2017) 28