

Title: The Phantom Menace: Modified Gravity as an Alternative to the Planet Nine Hypothesis - Harsh Mathur

Speakers:

Series: Cosmology & Gravitation

Date: July 11, 2022 - 9:00 AM

URL: <https://pirsa.org/22070025>

Abstract: An exciting development in outer solar system studies is the discovery of a new class of Kuiper belt objects with orbits that lie outside that of Neptune and have semimajor axes in excess of 250 A.U. The alignment of the major axes of these objects and other orbital anomalies are the basis for the Planet Nine hypothesis that an undiscovered giant planet orbits the sun at a distance of around 500 A.U. We show that a modified gravity theory known as MOND (Modified Newtonian Dynamics) provides an alternative explanation for the observed alignment, owing to significant quadrupolar and octupolar terms in the MOND galactic field within the solar system that are absent in Newtonian gravity. Using the well-established secular approximation of solar system dynamics we predict a population of Kuiper belt objects whose major axes are aligned with the direction to the center of the galaxy and that have additional clustering in orbital parameters. These features are exhibited by known Kuiper belt objects of the newly discovered class in support of the MOND hypothesis. Thus MOND, originally developed to explain galaxy rotation without invoking dark matter, may also be observable in the outer solar system.

phantom

View Zoom Add Slide Play Keynote Live Table Chart Text Shape Media Comment Collaborate Format Animate Document

Slide Layout

Blank Change Master

Appearance

Title  
 Body  
 Slide Number

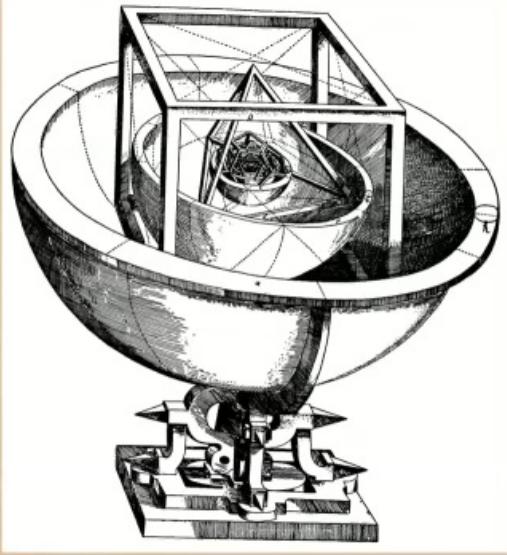
Background

Image Fill Scale to Fit Choose...

Scale 100%

Edit Master Slide

# Discovery of Planets



Kepler proved  
there are only  
six planets

Mysterium Cosmographicum (1596)

1

2

3

4

5

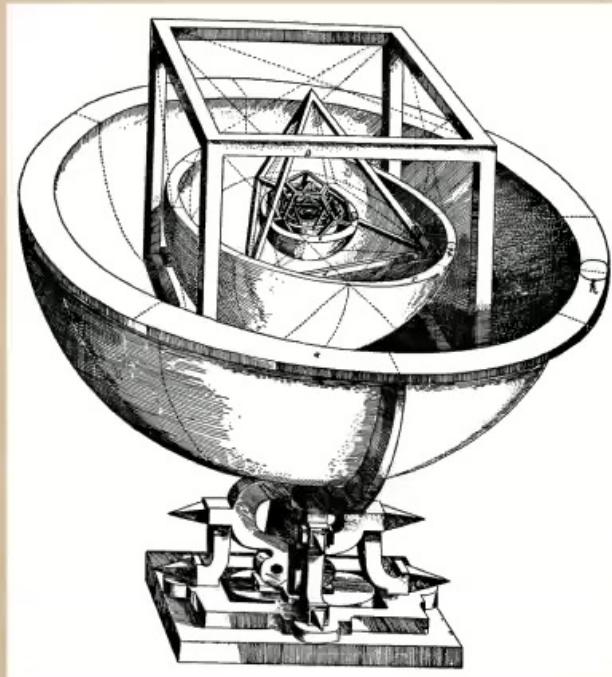
6

7

8

9

## Discovery of Planets



Kepler proved  
there are only  
six planets

Mysterium Cosmographicum (1596)

# The discovery of Uranus

Uranus imaged by  
Voyage 2 (NASA)

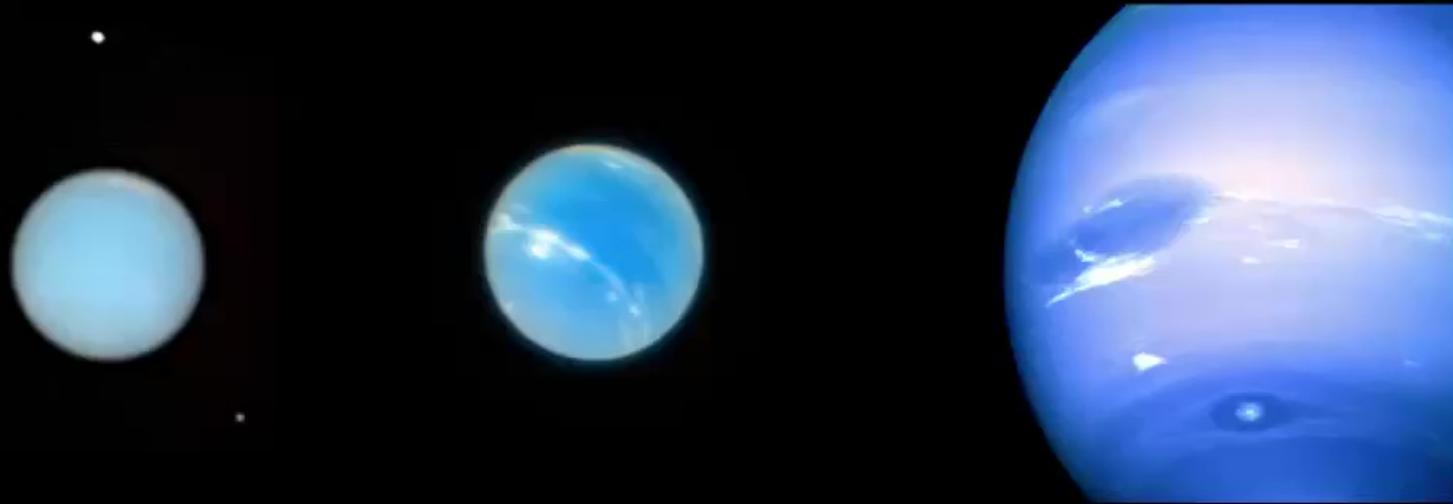
Planet 7

William Herschel (1781)



## The prediction of Neptune

Le Verrier (1846) Adams (1846)



Three views of Neptune: Hubble Space Telescope (left); ground based  
Very Large Telescope (center); Voyager 2 (right)

## Is there a Ninth Planet?

Vulcan

Problem: Anomalies in orbit of Mercury

Resolution: Einstein's theory of gravity

(1843–1916)

Pluto

Problem: Anomalies in the orbit of Uranus and Neptune

Resolution: Improved data shows no anomalies

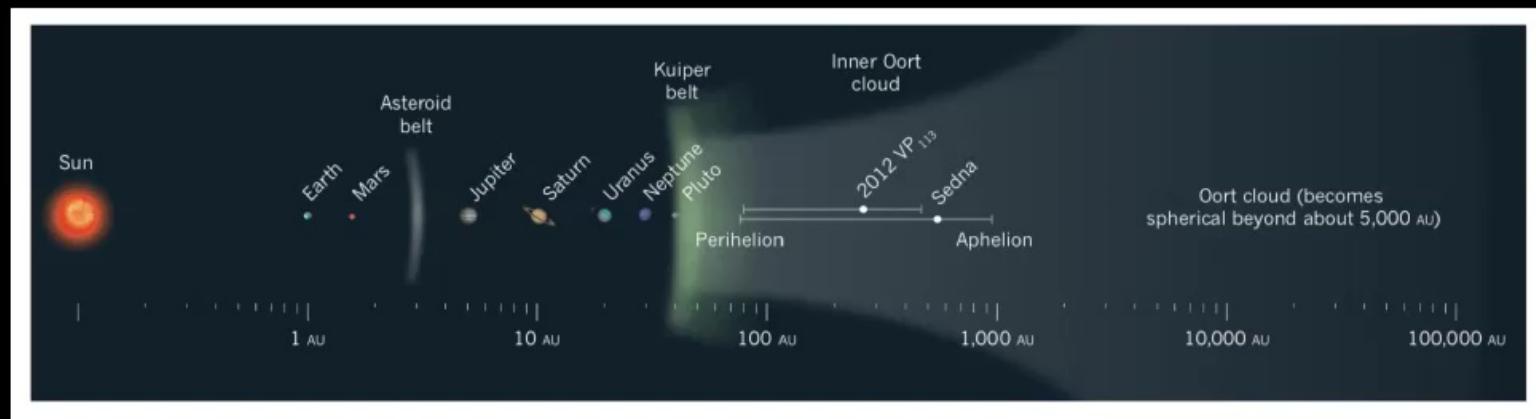
(1848–1993)

## The Kuiper belt

Artists view of Kuiper belt  
(NASA/ESA/ScSTI image)

# A brief survey of the Kuiper belt

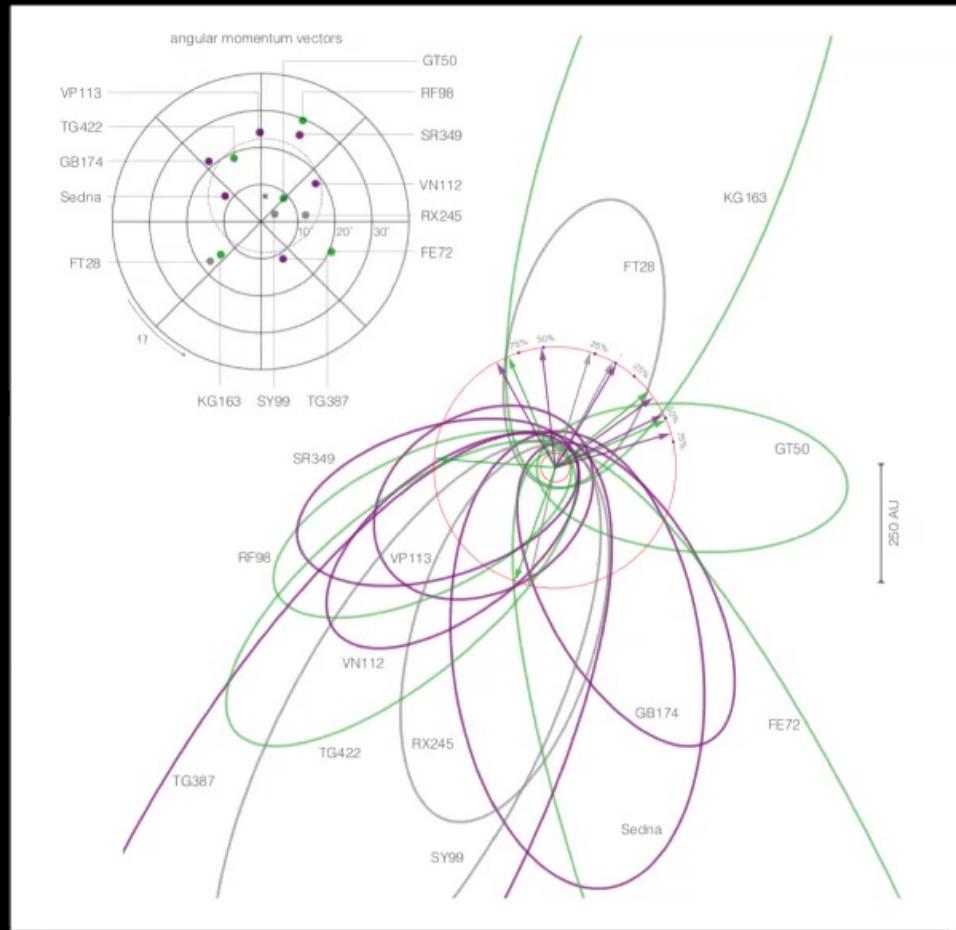
- ★ Resonant Kuiper belt      Historical Highlights
- ★ Classical Kuiper belt      Discovery of KBOs
- ★ Scattered disk      Discovery of Sedna
- ★ Centaurs      Discovery of Sedna's family



Schwamb, Nature 2014

# Anomalous alignment of distant Kuiper belt objects

Batygin and Brown (2016) Batygin et al (2019)

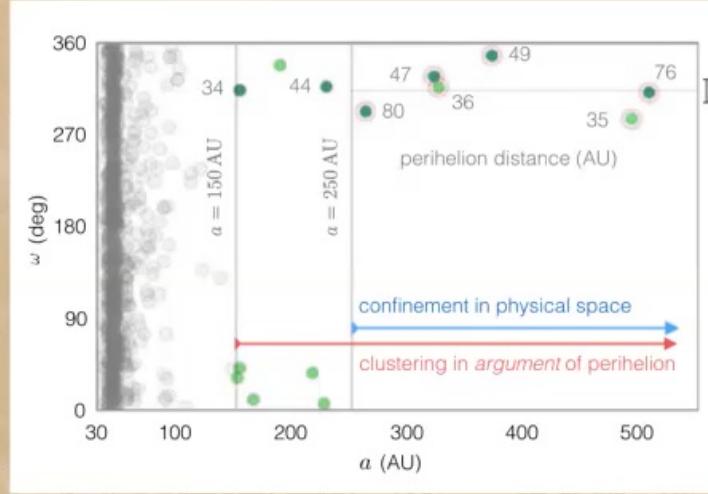
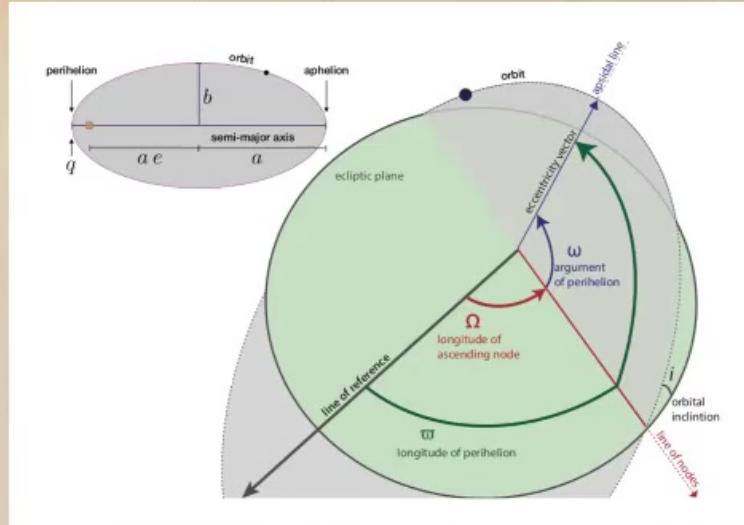


# Planet Nine Hypothesis

$$m_9 = 5 - 10 m_{\oplus}$$

500 AU from Sun

Batygin et al Phys Rep 2019



Batygin et al Phys Rep 2019

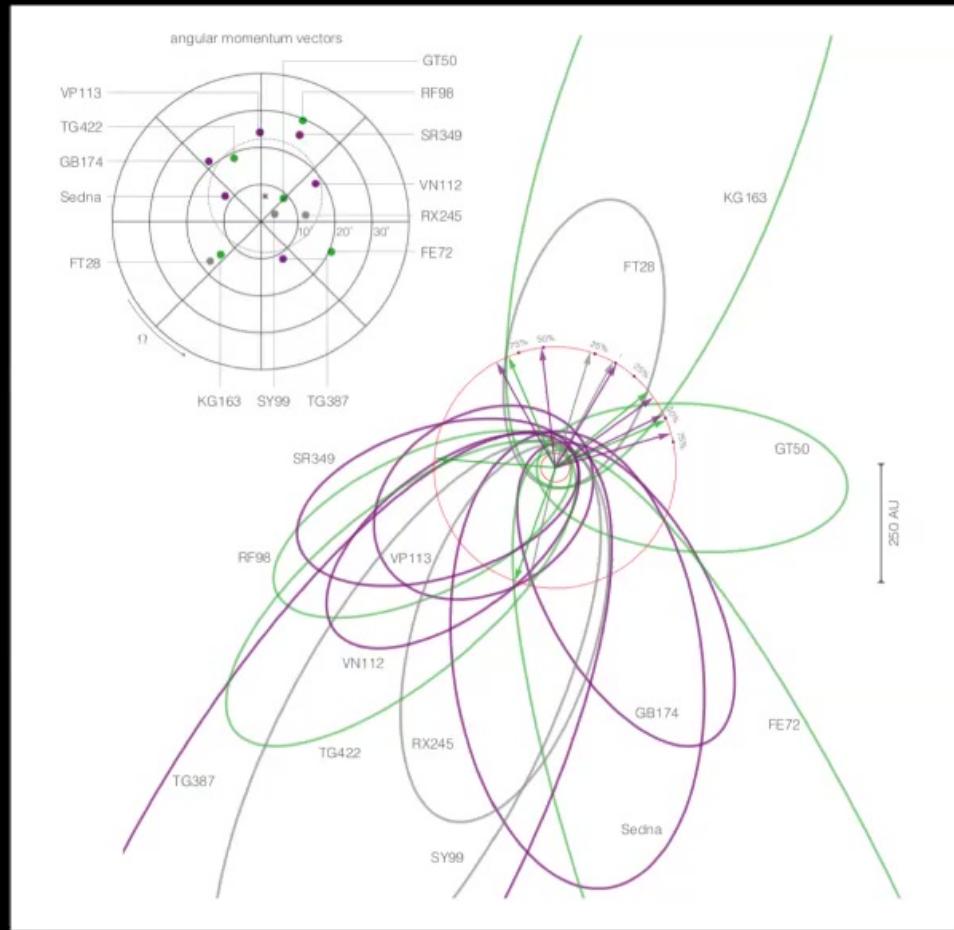
Six orbital elements

$$(a, e, \omega, i, \Omega, f)$$

$\Leftarrow$  Clustering in  $\omega$

# Anomalous alignment of distant Kuiper belt objects

Batygin and Brown (2016) Batygin et al (2019)

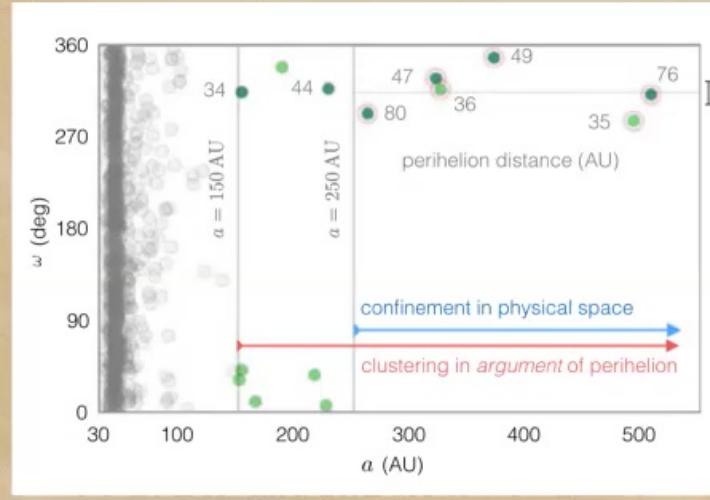
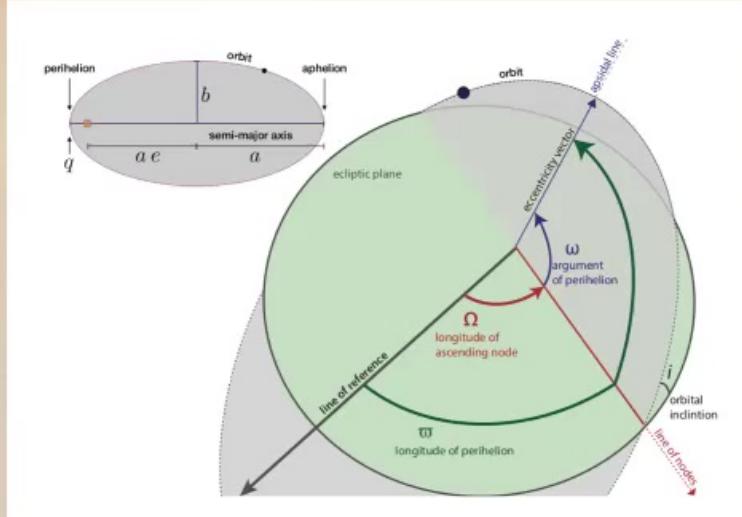


# Planet Nine Hypothesis

$$m_9 = 5 - 10 m_{\oplus}$$

500 AU from Sun

Batygin et al Phys Rep 2019



Batygin et al Phys Rep 2019

Six orbital elements

$$(a, e, \omega, i, \Omega, f)$$

$\Leftarrow$  Clustering in  $\omega$

## Part Three

### The Phantom Menace

## Newtonian Gravity as field theory

Field equations

$$\nabla \cdot \mathbf{g}_N = -4\pi G\rho$$

$$\nabla \times \mathbf{g}_N = 0$$

Force law

$$\mathbf{F} = m \mathbf{g}_N$$

## Quasilinear MOND

Milgrom 2011

$$\mathbf{g}_P = \mathbf{g}_N \nu \left( \frac{g_N}{a_0} \right)$$

$\mathbf{g}_Q$  is the curl free part of  $\mathbf{g}_P$

$\mathbf{g}_Q$  is the physical gravitational field

## Quasilinear MOND continued

The interpolating function  $\nu$

$$\nu(x) \rightarrow 1 \text{ as } x \rightarrow \infty$$

$$\nu(x) \rightarrow \frac{1}{\sqrt{x}} \text{ as } x \rightarrow 0$$

Not well constrained by data for intermediate  $x$

$$\nu(x) = \frac{1}{1 - \exp(-\sqrt{x})}$$

Newtonian field of a point mass

$$\mathbf{g}_N = -\hat{\mathbf{r}} \frac{GM_{\odot}}{r^2}$$

MOND field of a point mass

$$\mathbf{g}_Q = -\hat{\mathbf{r}} \frac{GM_{\odot}}{r^2} \nu \left( \frac{R_M^2}{r^2} \right)$$

The MOND radius

$$R_M = \sqrt{\frac{GM_{\odot}}{a_0}}$$

## A first clue

MOND radius of the sun  $R_M = 7000$  A.U.

$$a_0 = 1.2 \times 10^{-10} \text{ m/s}^2$$

(best fit to galaxy rotation data)

## The Galactic field in the Solar System: Newtonian Gravity

$$\mathbf{g}_N = -\hat{\mathbf{r}} \frac{GM_{\odot}}{r^2} + \boldsymbol{\gamma}_N$$

$\boldsymbol{\gamma}_N$  = Uniform field of galaxy

Galactic field has no effect on relative motion  
of solar system objects

# Galactic field in the solar system: MOND

(Milgrom 2009)

Define  $\mathbf{g}_A$ , the anomalous field via

$$\mathbf{g}_Q = -\hat{\mathbf{r}} \frac{GM_{\odot}}{r^2} + \mathbf{g}_A + \gamma_G$$

External field effect = existence of  $\mathbf{g}_A$

Galactic counterpart observed by Chae et al

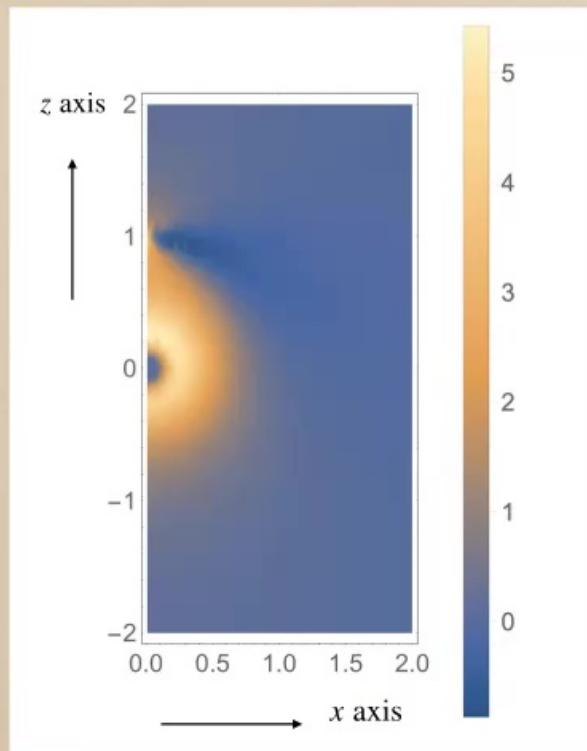
## The Phantom Mass

Can interpret  $\mathbf{g}_A$  = Newtonian field of  $\rho_{\text{ph}}$

$\rho_{\text{ph}}$  = phantom mass density

$$= -\frac{1}{4\pi G} \mathbf{g}_N \cdot \nabla \nu \left( \frac{g_n}{a_0} \right)$$

# The Phantom Mass Distribution



Sun is at origin;  
Galactic center is  
along pos z-axis

Rotationally symmetric about z-axis; Localized at  $\sim R_M$  from Sun

## Multipole Expansion

Quadrupole field of remote mass distribution

$$\psi = f \frac{GM}{R^3} r^2 P_2(\cos \theta)$$

Phantom Mass  $M \rightarrow M_{\odot}$ ,  $R \rightarrow R_M$ ,  $f = 0.0986\dots$

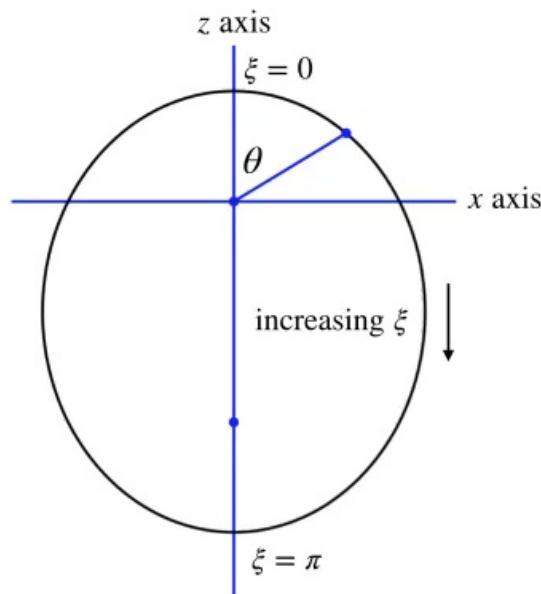
Planet Nine  $M \rightarrow m_9 = 5 m_{\oplus}$ ,  $R \rightarrow a_9$ ,  $f = -\frac{1}{(1 - e_9^2)^{3/2}}$

Second clue: Quadrupoles are comparable; note signs

## Planet Nine: Orbit averaged multipole moments

Mass of planet is assumed distributed along its orbit

Mass of an arc proportional to time spent on that arc



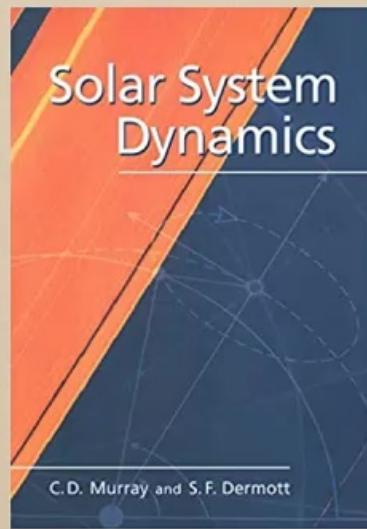
Kepler's anomaly equation

$$\frac{t}{2\pi T} = \xi - e \sin \xi$$

$$\Rightarrow dm = \frac{dt}{T} = \frac{m_9}{2\pi} (1 - e \cos \xi)$$

Multipole moments  
Rotate like rank  $\ell$  tensors

## Part 4 Orbital Dynamics



## Orbital Dynamics: Physical Idea

Under action of sun alone Kuiper object  
pursues Kepler orbit with orbital elements

$$a_K, e_K, \omega_K, i_K, \Omega_K, \xi_K$$

Under action of Planet Nine or MOND orbit  
slowly evolves

## Orbital Dynamics: Mathematical Implementation

Dynamical variables: work directly with orbital elements as  
dynamical variables

Contrast Lagrangian mechanics:

General coordinates and velocities are dynamical variables

Contrast Hamiltonian mechanics:

General coordinates and conjugate momenta are dynamical variables

## MOND Disturbing Function

Use Multipole Expansion; to quadrupole order

$$\mathcal{R}_Q = \frac{Gm_K M_{\odot}}{R_M} \left( \frac{a_K}{R_M} \right)^2 \frac{f}{8} \mathcal{S}_Q(e_K, \omega_K, i_K)$$

$$\mathcal{S}_Q = -2 - 3e_K^2 + 15e_K^2 \cos(2\omega_K) + 6 \cos^2 i_K + 9e_K^2 \cos^2 i_k - 15e_K^2 \cos(2\omega_K) \cos^2 i_K$$

Exact expression!

Independence of  $\Omega_K$  due to rotational invariance

## Coordinate System

Origin = Sun; Galactic center along pos z axis

Standard orientation: KBO orbit lies in x-y plane;  
perihelion along pos x axis

Orientation ( $\omega_K, i_K, \Omega_K$ ):

Twist by  $\omega_K$  about z;  $i_K$  about x;  $\Omega_K$  about z

## Conservation laws

- ♦ Angular momentum along symmetry axis z

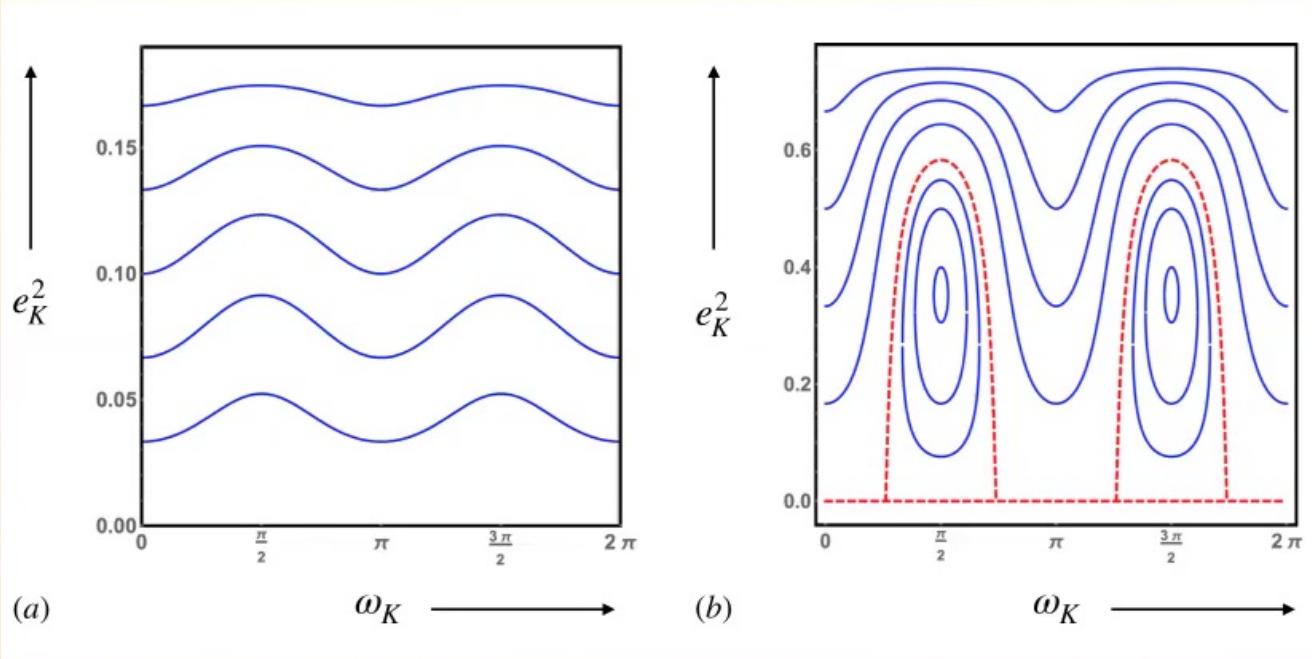
$$h = \sqrt{1 - e_K^2} \cos i_K$$

- ♦  $\mathcal{R}$  itself is conserved

Integrable model:

4 degrees of freedom 2 conservation laws

# MOND Dynamics in Quadrupole Approximation



$$\frac{3}{5} \leq h^2 \leq 1$$

Precession

$$0 \leq h^2 < \frac{3}{5}$$

Note fixed points!

## Fixed Point Analysis

Location of fixed points  $(e_K, \omega_K) = (e_C, \pm \pi/2)$

$$e_C^2 = 1 - \sqrt{5/3} |h|$$

Effect of octupole term: destabilizes + relative to -

Additional Perturbations: Giant planets, non-secular terms, motion of sun around galaxy

Their effect: chaotic flow except near stable fixed point

## Key Predictions

Expect a population of KBOs with orbits  
close to stable fixed point  $(e_C, -\pi/2)$

For small  $h$  find  $i_K \approx \pi/2$

- ♦ Orbits with  $i_K \approx \pi/2$  and  $\omega_K = -\pi/2$   
have  $\hat{\alpha}_K$  antiparallel to  $\hat{\mathbf{n}}_G$

$\hat{\alpha}_K$  = unit vector from sun to KBO perihelion

$\hat{\mathbf{n}}_G$  = unit vector from sun to galactic center

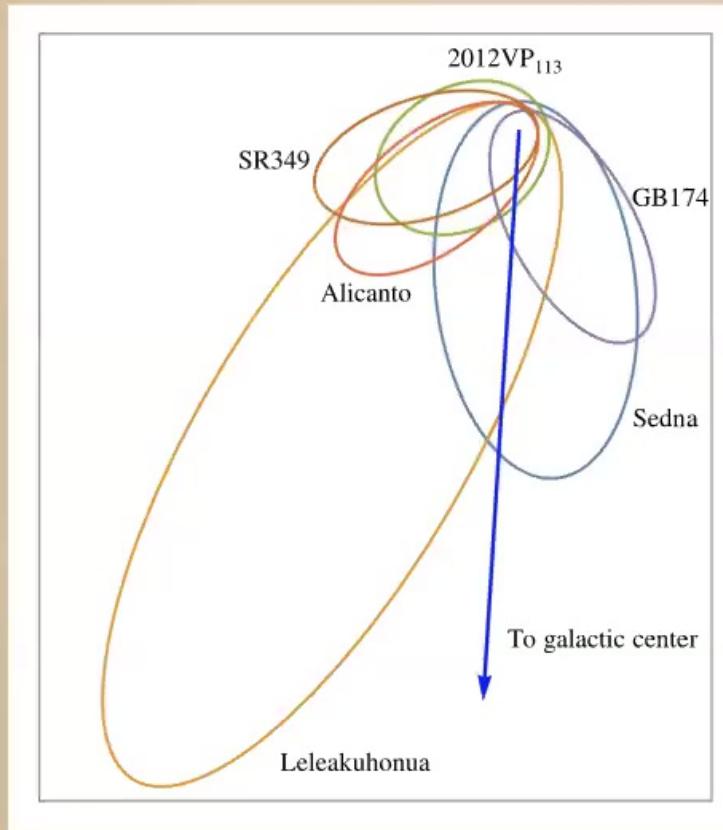
## Data: Sedna Family Orbital Parameters

Object	$\omega$	$\Omega$	$i$	$e$	$q$ (AU)	$a$ (AU)
Sedna	311.1°	144.2°	11.9°	0.85	76.34	499.70
TG387 <sup>a</sup>	118.0°	300.8°	11.7°	0.94	65.04	1031.49
2012 VP <sub>113</sub>	293.5°	90.7°	24.1°	0.69	80.39	258.27
VN112 <sup>b</sup>	326.8°	66.0°	25.6°	0.85	47.30	318.97
GB174	347.0°	130.9°	21.6°	0.86	48.61	336.67
SR349	340.0°	34.8°	18.0°	0.84	47.69	302.23

TABLE I. Orbital elements of six KBOs of the Sedna family. The data are from the Minor Planet Database of the International Astronomical Union. <sup>a</sup> TG387 is named Leleakuhonua. <sup>b</sup> VN112 is named Alicanto.

## Apsidal Alignment

Caveat: Observational bias



Mean value of

$$\hat{\alpha}_K \cdot \hat{\mathbf{n}}_G$$

is  $3\sigma$  away from

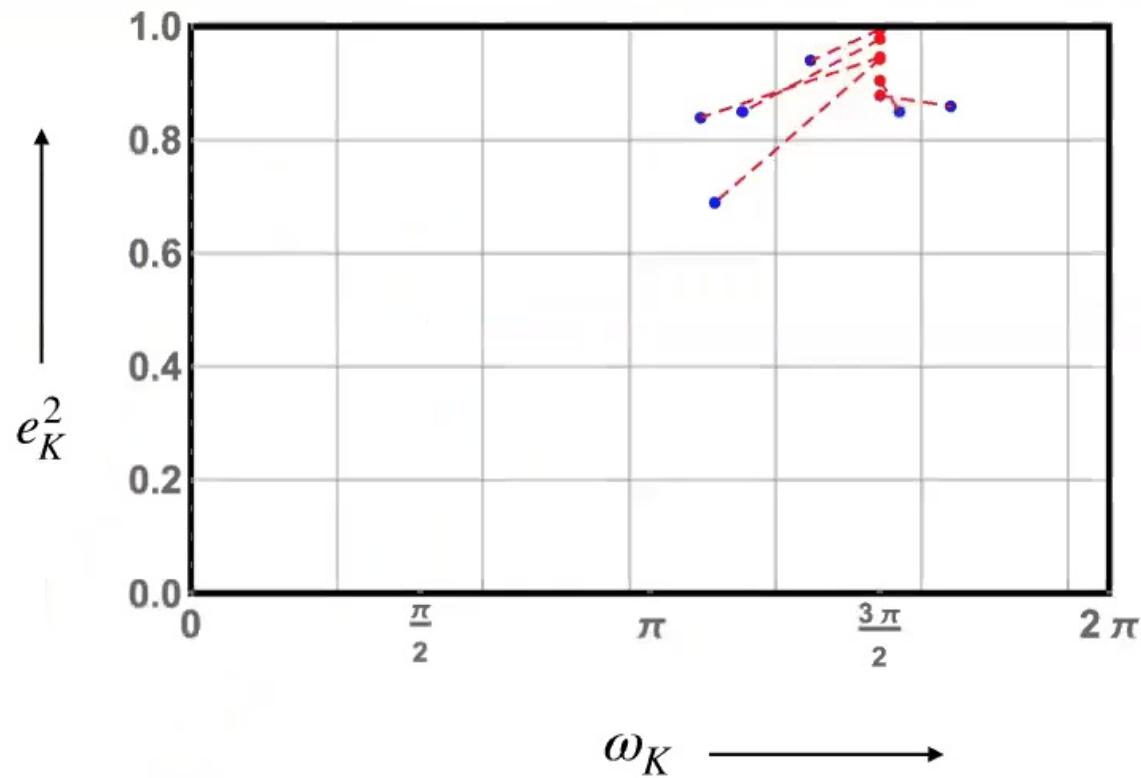
purely random  $\hat{\alpha}_K$

1 in 500 odds

relative to

pure random  $\hat{\mathbf{a}}_K$

## Phase Space Clustering



Future: Compare a larger sample of Sedna family objects to dynamical simulations to test MOND hypothesis

Future and ongoing surveys:

Dark energy survey

Transiting Exoplanet Survey Satellite (TESS)

Outer Solar System Origins Survey (OSSOS)

Vera Rubin Telescope

CMB-S4 ...

## Clustering - Spurious, Planet Nine or MOND?

Historically gravitational anomalies in solar system  
were often spurious but also led to  
discovery of Neptune and General Relativity

Kuiper belt is a laboratory for studying  
fundamental physics