Title: Characterizing the non-linear evolution of dark energy and modified gravity models

Speakers: Farbod Hassani

Series: Cosmology & Gravitation

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Abstract: Understanding the reason behind the observed accelerating expansion of the Universe is one of the most notable puzzles in modern cosmology, and conceivably in fundamental physics. In the upcoming years, near future surveys will probe structure formation with unprecedented precision and will put firm constraints on the cosmological parameters, including those that describe properties of dark energy. In light of this, in the first part of my talk, I'm going to show a systematic extension of the Effective Field Theory of Dark Energy framework to non-linear clustering. As a first step, we have studied the k-essence model and have developed a relativistic N-body code, k-evolution.

I'm going to talk about the k-evolution results, including the effect of k-essence perturbations on the matter and gravitational potential power spectra and the k-essence structures formed around the dark matter halos. In the second part of my talk, I'm going to show that for some choice of parameters the k-essence non-linearities suffer from a new instability and blow up in finite time.

This talk is based on: arXiv:2204.13098, arXiv:2205.01055, arXiv:2107.14215, arXiv:2007.04968, arXiv:1910.01105, arXiv:1910.01104.

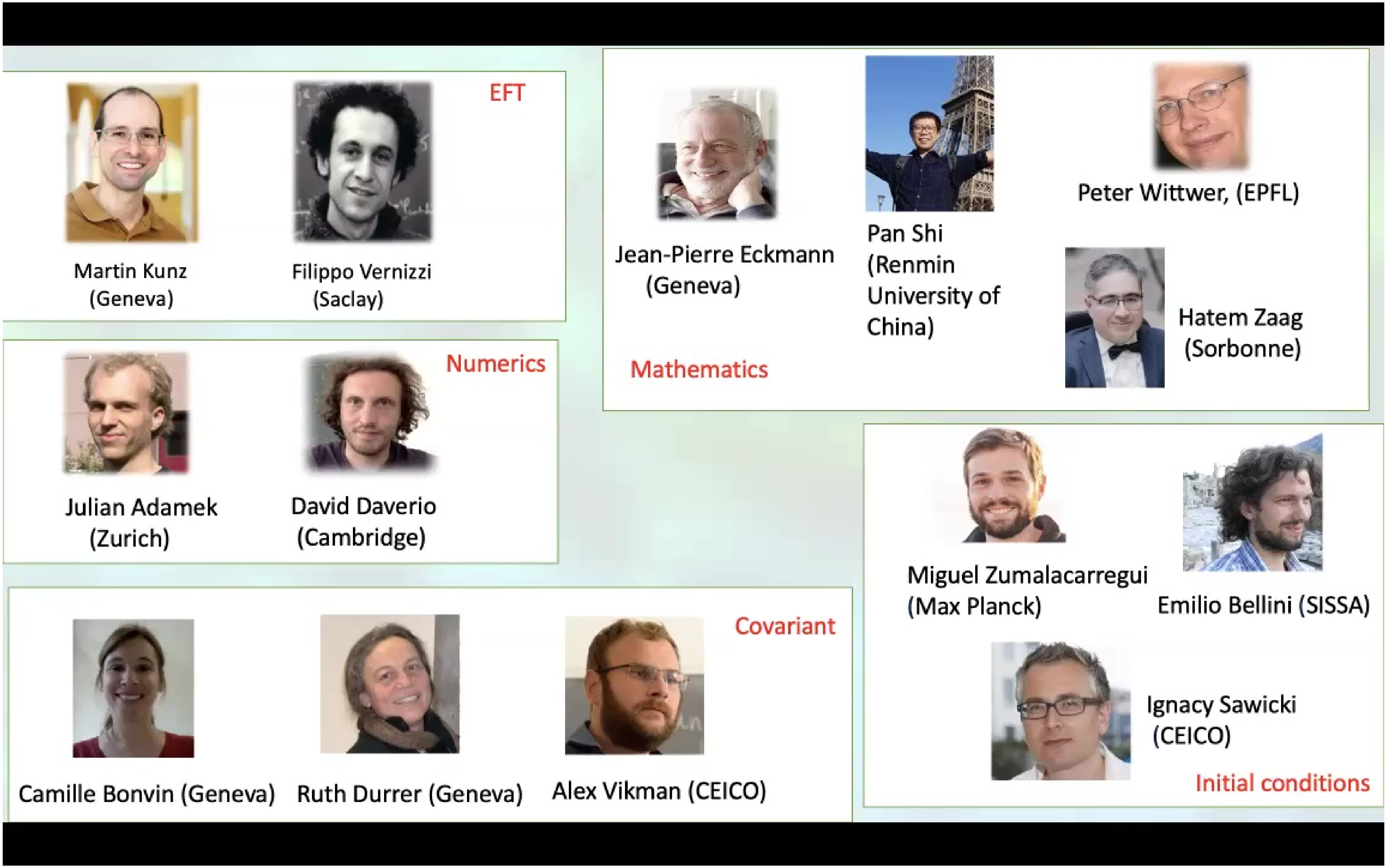
Zoom Link: https://pitp.zoom.us/j/99797451101?pwd=dituM2d2MDFCbDgyVXJ4c2s1NVoyUT09

Characterizing the non-linear evolution of dark energy models

Farbod Hassani

arXiv:2204.13098, arXiv:2205.01055, arXiv:2107.14215, arXiv:2007.04968, arXiv:1910.01105, arXiv:1910.01104

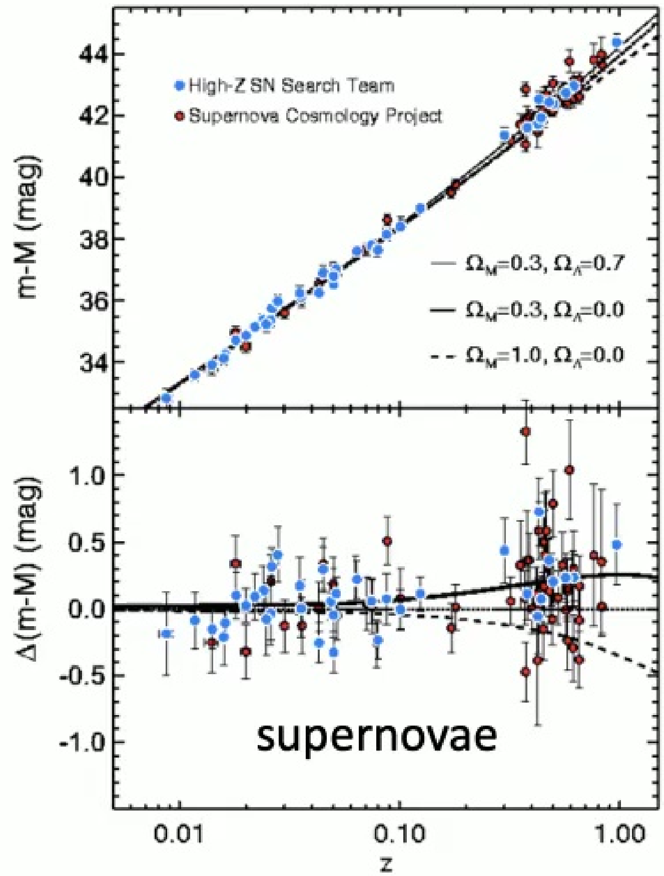
UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES
Département de physique théorique

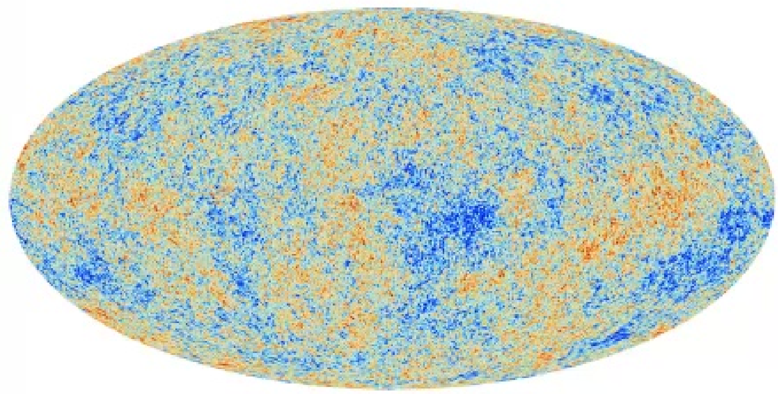

EFT
Martin Kunz (Geneva)
Filippo Vernizzi (Saclay)
Numerics
Julian Adamek (Zurich)
David Daverio (Cambridge)
Jean-Pierre Eckmann (Geneva)
Pan Shi (Renmin University of China)
Peter Wittwer, (EPFL)
Hatem Zaag (Sorbonne)
Mathematics
Miguel Zumalacarregui (Max Planck)
Emilio Bellini (SISSA)
Ignacy Sawicki (CEICO)
Initial conditions
Covariant
Camille Bonvin (Geneva)
Ruth Durrer (Geneva)
Alex Vikman (CEICO)

Outline

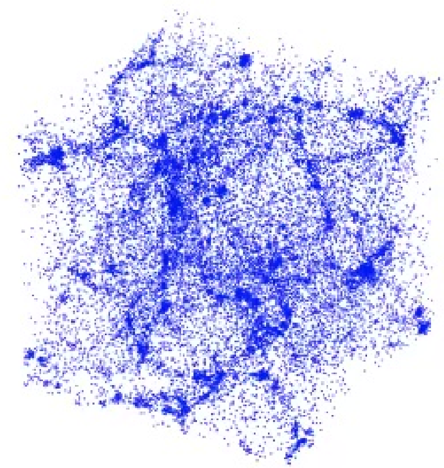
- Introduction
- EFT of dark energy
- k-evolution N-body code
- Non-linear instabilities appearing in the k-essence theories
  - 3+1D
  - 3+1D spherically symmetric and 1+1D scenarios

Accelerating expansion of the Universe:



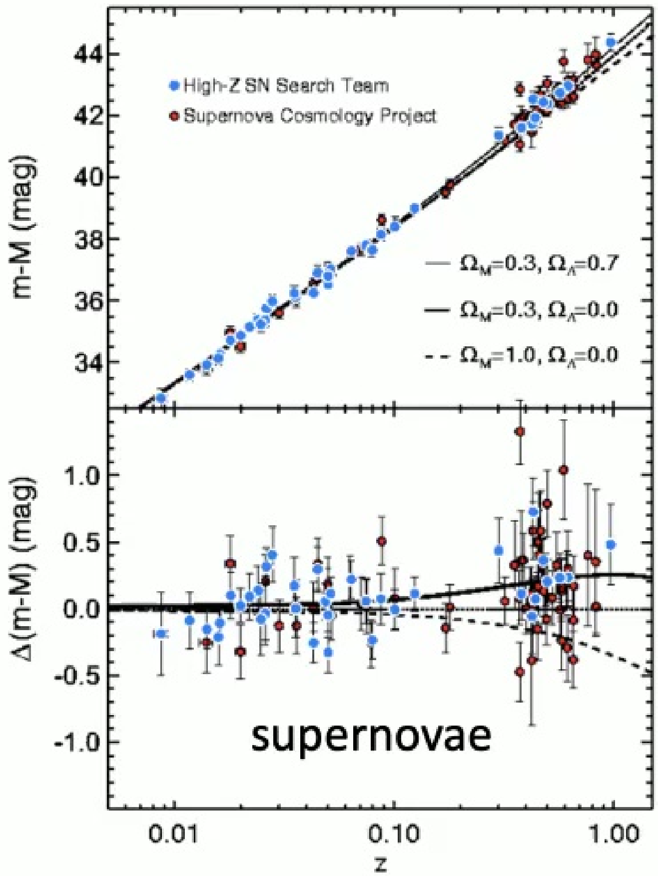


The CMB



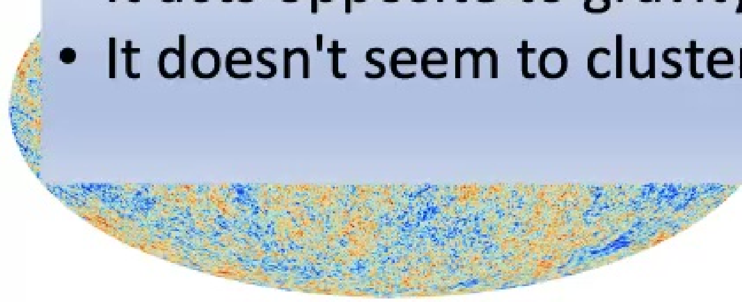
Large scale structure

Accelerating expansion of the Universe:

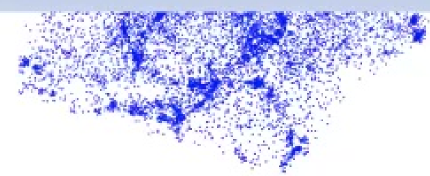


$$\Omega_{\rm DE} \sim 0.7$$

- It acts opposite to gravity at large scales
- It doesn't seem to cluster strongly



The CMB



Large scale structure

Basic facts

$$\nabla^2\Phi = 4\pi G\rho - \Lambda$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \; - V(\varphi) g_{\mu\nu}$$

"dark energy"

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Modified gravity

Dark energy

EFT of Dark Energy

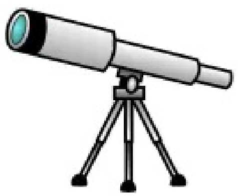
- In sum

Covariant theories ⟵ Effective field theory of dark energy ⟶ Observations

$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^4x\,\sqrt{-g}\left[\sum_{i=2}^{5}\frac{1}{8\pi G_{\mathrm{N}}}\mathcal{L}_i[g_{\mu\nu},\phi] + \mathcal{L}_{\mathrm{m}}[g_{\mu\nu},\psi_M]\right],$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(\phi, X)\,,\\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi\,,\\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right],\\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right],
\end{aligned}$$

$$\begin{aligned}
S =& \sqrt{-g}\Big[\frac{M_*^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} + \frac{M_2^4(t)}{2}\left(g^{00} + \frac{1}{N^2}\right) - \frac{m_3^3(t)}{2}\delta K\left(g^{00} + \frac{1}{N^2}\right)\\
&- m_4^2(t)\left(\delta K^2 - \delta K^{\nu}_{\mu}\delta K^{\mu}_{\nu}\right) + \frac{\tilde{m}_4^2(t)}{2}\,{}^{(3)}R\,\delta g^{00}\Big] \qquad (1)
\end{aligned}$$



convenient way to describe different of DE and MG models: Having a general action for class of theories

We can connect them to the observational predictions in terms of a **minimal number** of parameters

$$S = \frac{1}{16\pi G}\int\sqrt{-g}Rd^4x + \int\sqrt{-g}P(X,\varphi)d^4x \quad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

k-essence

Choosing the unitary gauge = time coincides with uniform-field hypersurfaces:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \Lambda(t) - c(t)g^{00} + \frac{M_2^4(t)}{2}\left(\delta g^{00}\right)^2 + \ldots\right]$$

We can restore the general covariance of the action by performing time diff:

$$t \to \tilde{t} = t + \xi^0(t,\vec{x}) \qquad \pi(t,\vec{x}) \doteq \xi^0(t,\vec{x})/a(t)$$

$$\begin{aligned} S = \int d^4x\sqrt{-g}\{&\frac{M_{\rm Pl}^2}{2}R - \Lambda[t(\eta)+a(\eta)\pi] - c[t(\eta)+a(\eta)\pi]a^2(\eta)\left[(1+\mathcal{H}\pi)^2g^{00}\right.\\ &\left.+2(1+\mathcal{H}\pi)g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi\right] + \frac{M_2^4[t(\eta)+a(\eta)\pi]}{2}\\ &\times a^4(\eta)\left[(1+\mathcal{H}\pi)^2g^{00} + 2(1+\mathcal{H}\pi)g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \bar{g}^{00}\right]^2\Big\}\end{aligned}$$

EFT of Dark Energy

- Evolution of the cosmological perturbations

- Evolution of perturbations to second order ⇒ $\frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta(\delta g_{\mu\nu})} = T^{\mu\nu}$
- Scalar field evolution ⇒ $\frac{\delta S}{\delta \pi} = 0$
- Perturbative scheme: short wave-corrections are considered

$$\pi, \pi', \pi'' \sim \epsilon \qquad \nabla\pi, \nabla\pi', \nabla\pi'' \sim \epsilon^{1/2} \qquad \nabla^2\pi, \nabla^2\pi', \nabla^2\pi'' \sim \epsilon^0$$

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Weak field regime

$\Phi \approx 10^{-6}$ $\quad$ $\delta \approx 10^{25}$

$\Phi \approx 10^{-12}$ $\quad$ $\delta \approx 10^{6}$

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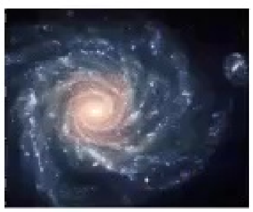

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Weak field regime

$\Phi \approx 10^{5}$

$\delta \approx 10^{45}$

$\Phi \approx 10^{9}$

$\delta \approx 10^{46}$

Strong field regime

EFT of Dark Energy

• Evolution of the cosmological perturbations

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Example:

Short wave correction (2nd order): $\nabla\Phi\nabla\pi + \nabla^2\pi\nabla^2\Phi$

Short wave correction (3d order): $(\nabla^2\Phi)^3$

$$\Phi'' + \mathcal{H}\Phi' + \boxed{\nabla\Phi\nabla\pi + \nabla^2\pi\nabla^2\Phi} + \cancel{\pi^2} + \cancel{\Phi^2} + \boxed{(\nabla^2\Phi)^3} + ... = 0$$

$$S = \frac{1}{16\pi G}\int \sqrt{-g} R d^4x + \int \sqrt{-g} P(X,\varphi) d^4x \qquad X = -\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

k-essence

Choosing the unitary gauge = time coincides with uniform-field hypersurfaces:

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2} R - \Lambda(t) - c(t) g^{00} + \frac{M_2^4(t)}{2}\left(\delta g^{00}\right)^2 + \ldots\right]$$

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$$S = \int d^4x \sqrt{-g}\Big\{\frac{M_{\rm Pl}^2}{2} R - \Lambda[t(\eta) + a(\eta)\pi] - c[t(\eta) + a(\eta)\pi] a^2(\eta)\left[(1+\mathcal{H}\pi)^2 g^{00}\right.$$
$$\left. + 2(1+\mathcal{H}\pi) g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi\right] + \frac{M_2^4[t(\eta) + a(\eta)\pi]}{2}$$
$$\times a^4(\eta)\left[(1+\mathcal{H}\pi)^2 g^{00} + 2(1+\mathcal{H}\pi) g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \bar{g}^{00}\right]^2\Big\}$$

EFT of Dark Energy

- Evolution of the cosmological perturbations

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$$\left.+2(1+\mathcal{H}\pi)g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi\right] + \frac{M_2^4[t(\eta)+a(\eta)\pi]}{2}$$
$$\times a^4(\eta)\left[(1+\mathcal{H}\pi)^2 g^{00} + 2(1+\mathcal{H}\pi)g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \bar{g}^{00}\right]^2\Big\}$$

$$S = \int \mathrm{d}^4 x \sqrt{-g}\, P(\phi, X) \;, \qquad X = -g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi \;.$$

$$\phi(t, \vec{x}) = \phi_0(t + \pi(t, \vec{x})) \;,$$

$$\begin{aligned}
\phi(t, \vec{x}) &= \phi_0 + \dot{\phi}_0 \pi + \frac{1}{2}\ddot{\phi}_0 \pi^2 + \dots \;, \\
X(t, \vec{x}) &= X_0 + \dot{X}_0 \pi + \frac{1}{2}\ddot{X}_0 \pi^2 + 2X_0 \dot{\pi} + 2\dot{X}_0 \pi \dot{\pi} + X_0 \left(\dot{\pi}^2 - \frac{(\nabla \pi)^2}{a^2} \right) + \dots \;,
\end{aligned}$$

$$S = \int \mathrm{d}^4 x\, a^3 \left[P_0 + \dot{P}_0 \pi + \frac{1}{2}\ddot{P}_0 \pi^2 + 2P_X X_0 \dot{\pi} + 2\,(P_X X_0)^{\cdot}\, \pi \dot{\pi} + P_X X_0 \left(\dot{\pi}^2 - \frac{(\nabla \pi)^2}{a^2} \right) + 2P_{XX} X_0^2 \dot{\pi}^2 \right] \;,$$

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$\Phi \approx 10^{-6}$
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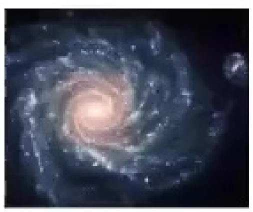

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Weak field regime

$\Phi \approx 10^{5}$
$\delta \approx 10^{45}$

$\Phi \approx 10^{9}$
$\delta \approx 10^{46}$

Strong field regime

k-evolution/gevolution

- Relativistic N-body code → $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

- Calculates all six metric degrees of freedom in Poisson gauge + k-essence scalar field

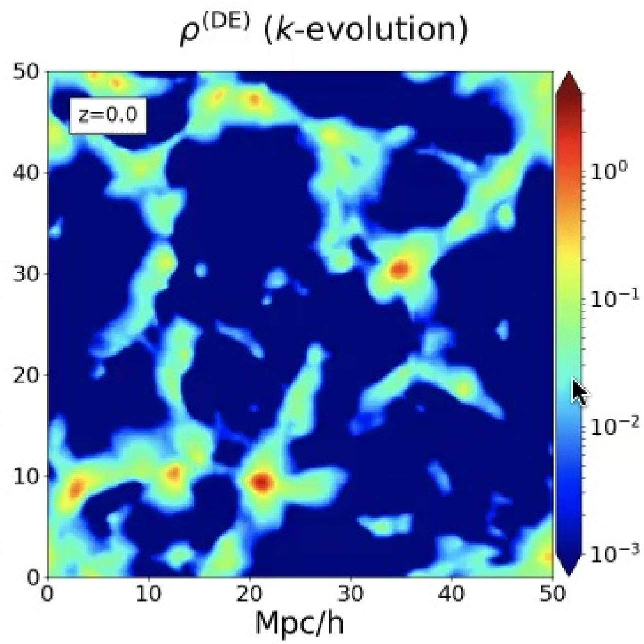
$$ds^2 = a^2(\eta)\left[-e^{2\Psi}d\eta^2 - 2B_i dx^i dt + (e^{-2\Phi}\delta_{ij} + h_{ij})dx^i dx^j\right]$$

- Background model should be close to FLRW

- N-body particles are evolved by solving the geodesic equation (CDM, neutrinos, photons)

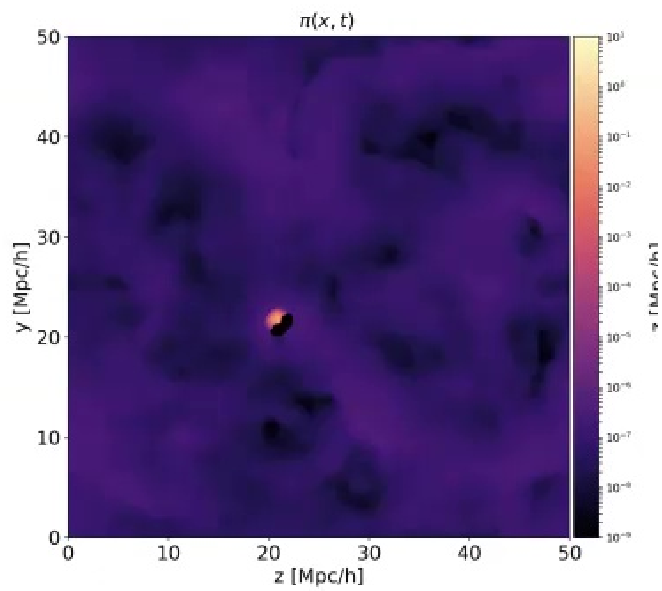
large speed of sound

$$c_s^2 \geq 10^{-4.7}$$

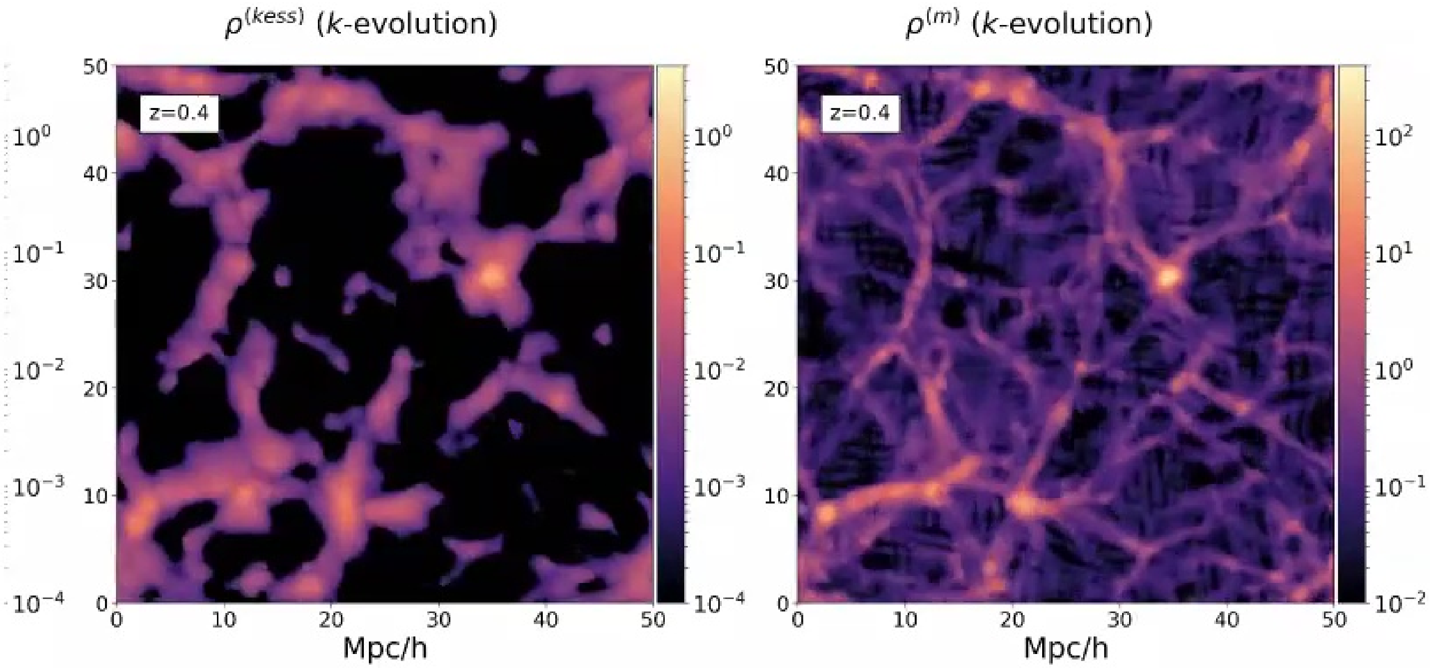


small speed of sound

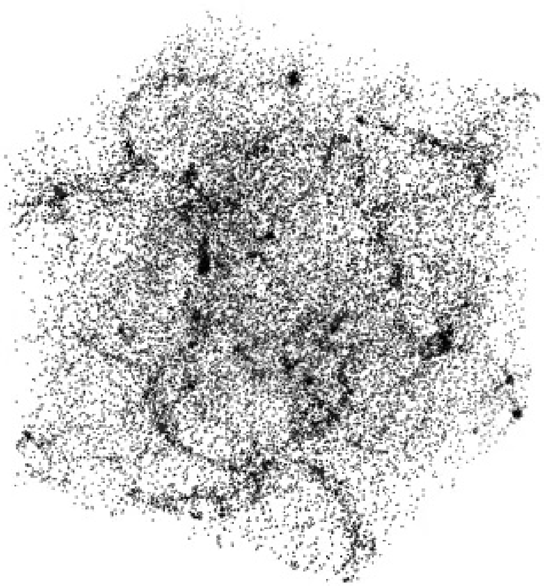
$$c_s^2 < 10^{-4.7}$$



Large speed of sound



DM particles



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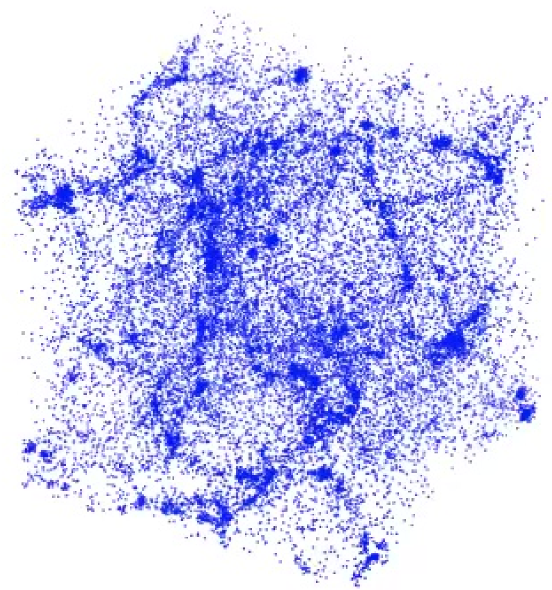
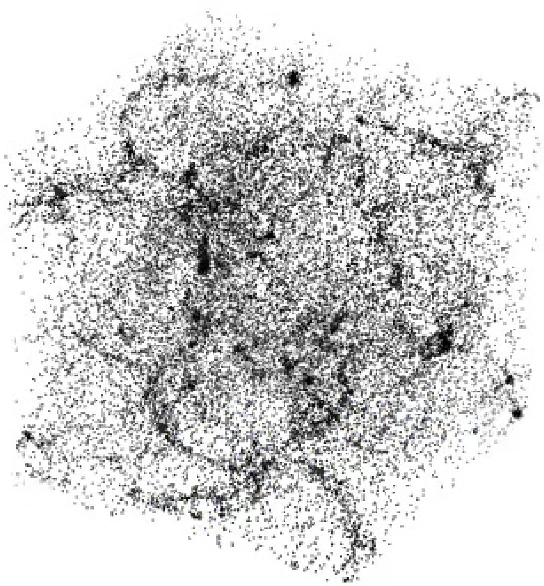
DM particles



MG

DM particles



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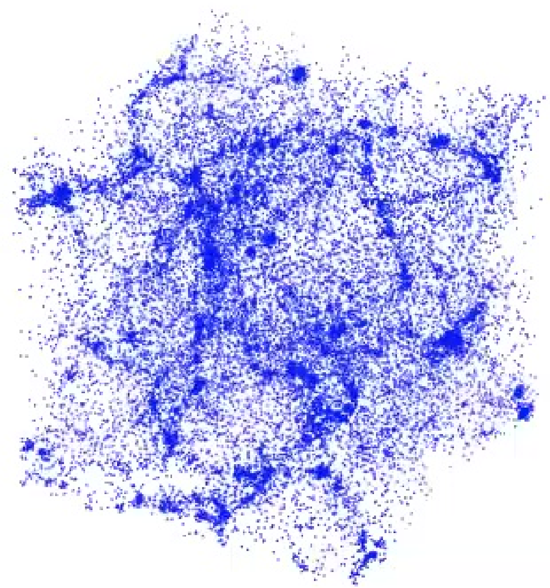
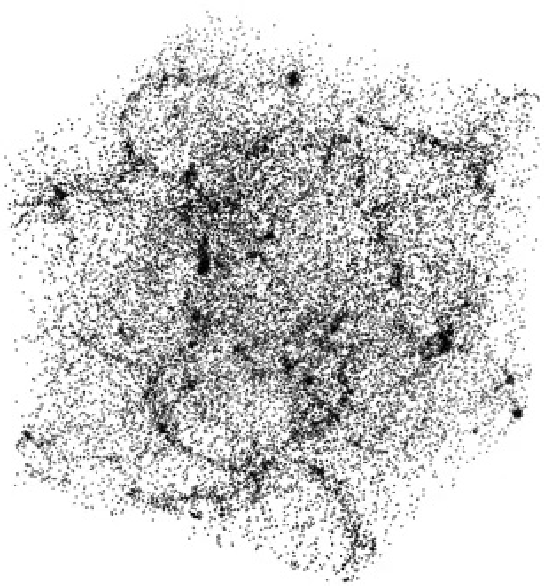
DM particles



MG

Matter power spectrum, redshift space distortions

DM particles



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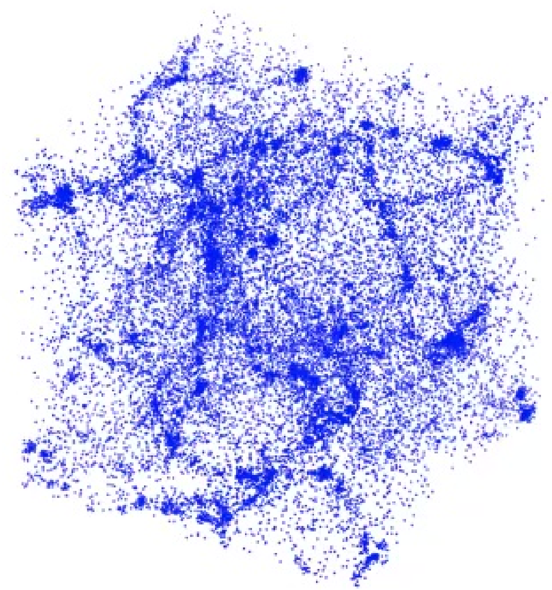
DM particles

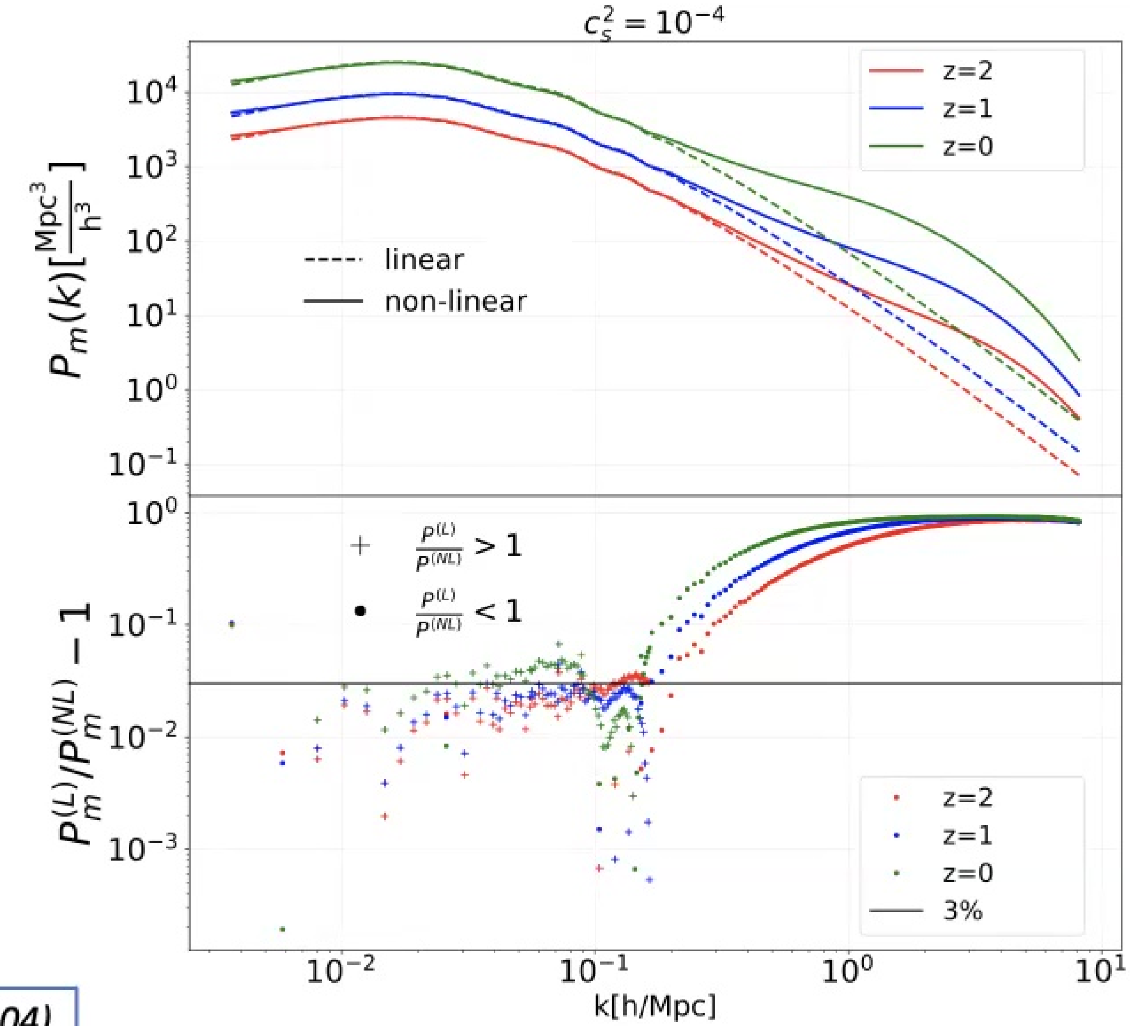


MG

Don't use CLASS/halo-fit to study clustering DE.

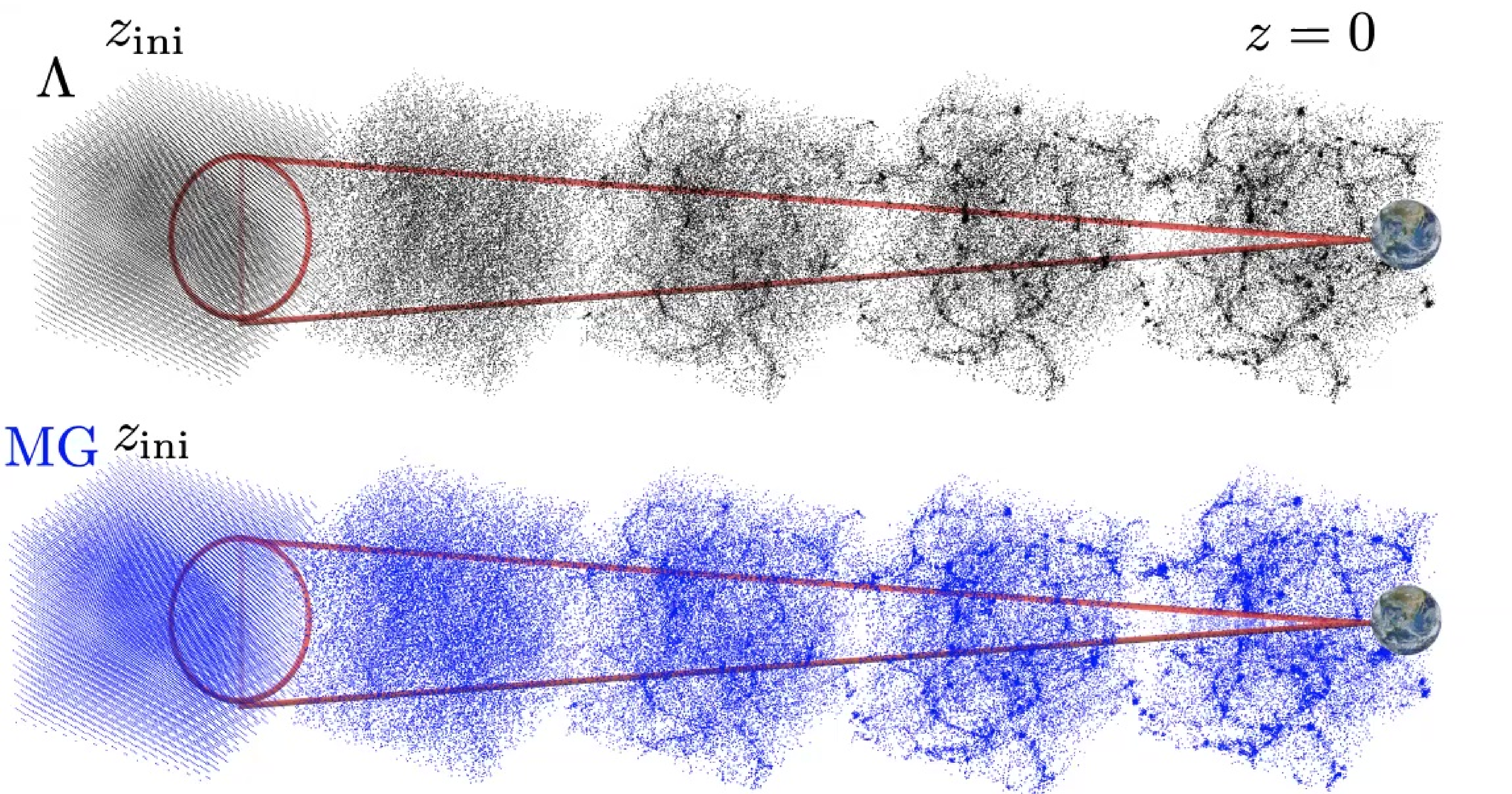
Matter power spectrum

To 5% difference between CLASS/halo-fit and k-evolution



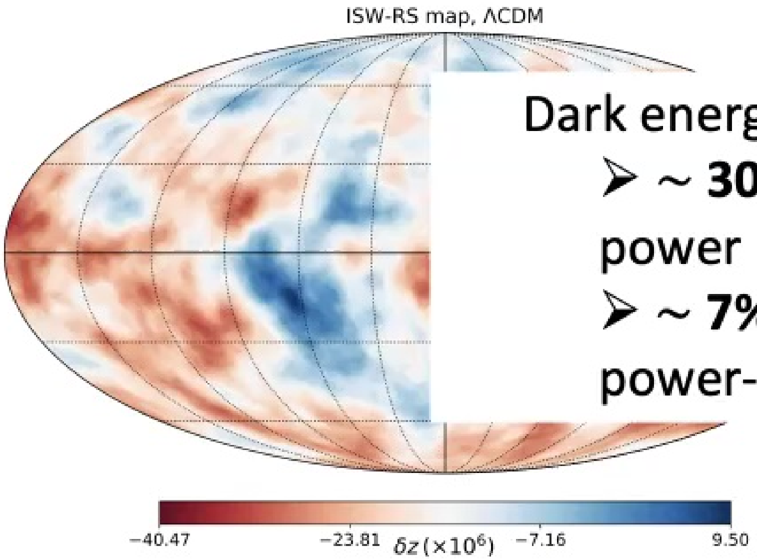
FH, J. Adamek, M. Kunz, F. Vernizzi (arXiv: 1910.01104)

FH, B. L'Huilier, A. Shafieloo, M. Kunz, J. Adamek (arXiv: 1910.01105)

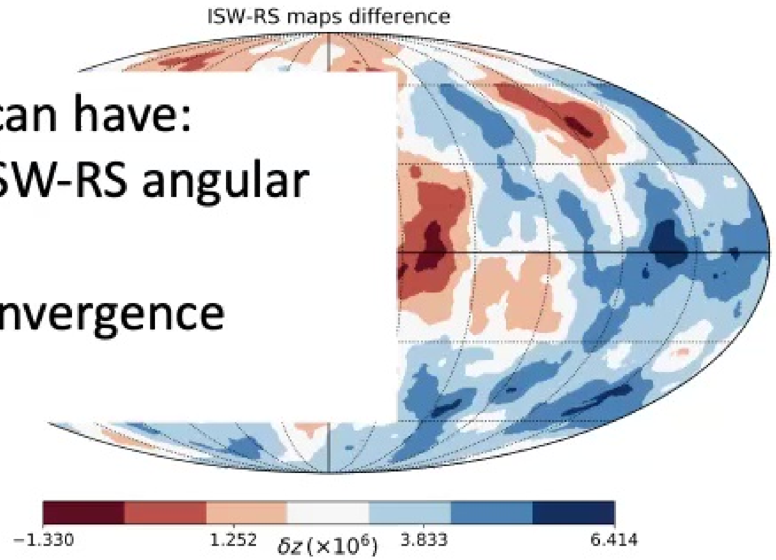

Λ
$z_{\rm ini}$
$z = 0$
MG
$z_{\rm ini}$

integrated Sachs-Wolfe

Map of ISW effect



Difference between maps of k-essence and LCDM

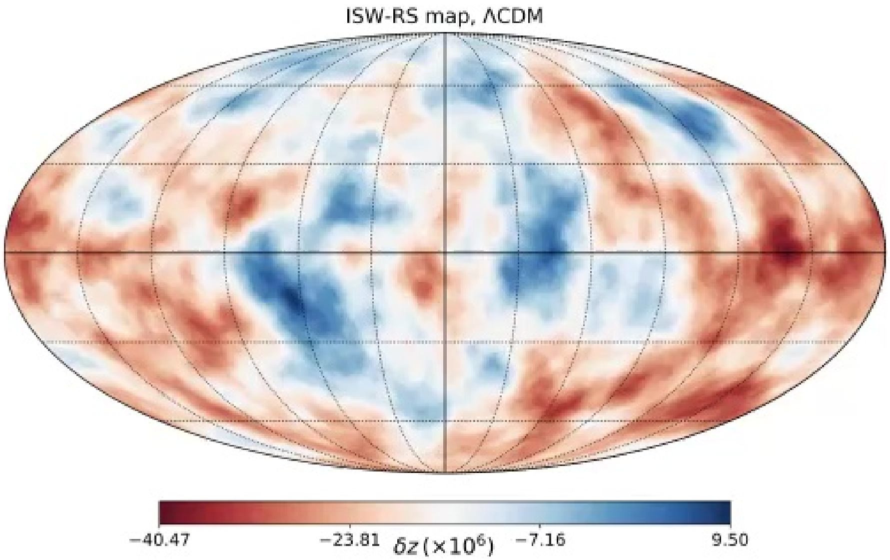


Dark energy clustering can have:

- $\sim$ **30%** effect on ISW-RS angular power
- $\sim$ **7%** effect on convergence power-spectrum

integrated Sachs-Wolfe

Map of ISW effect



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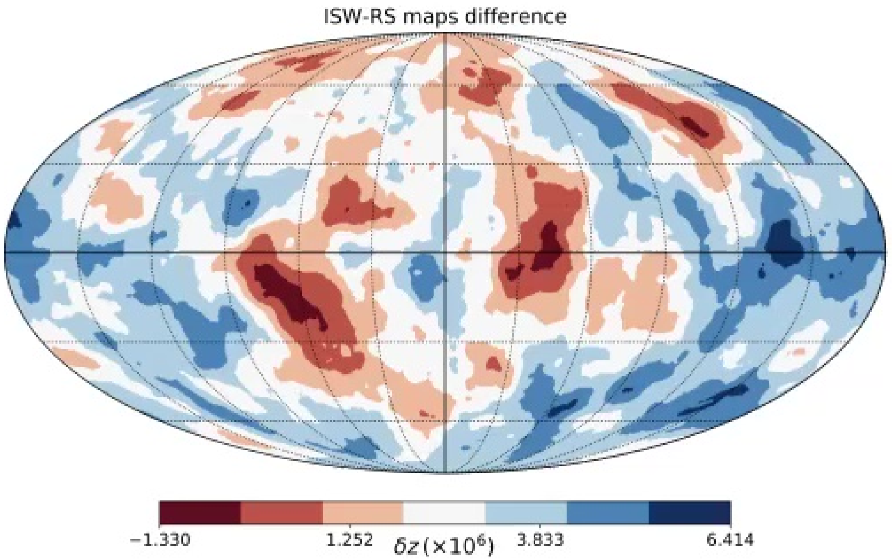

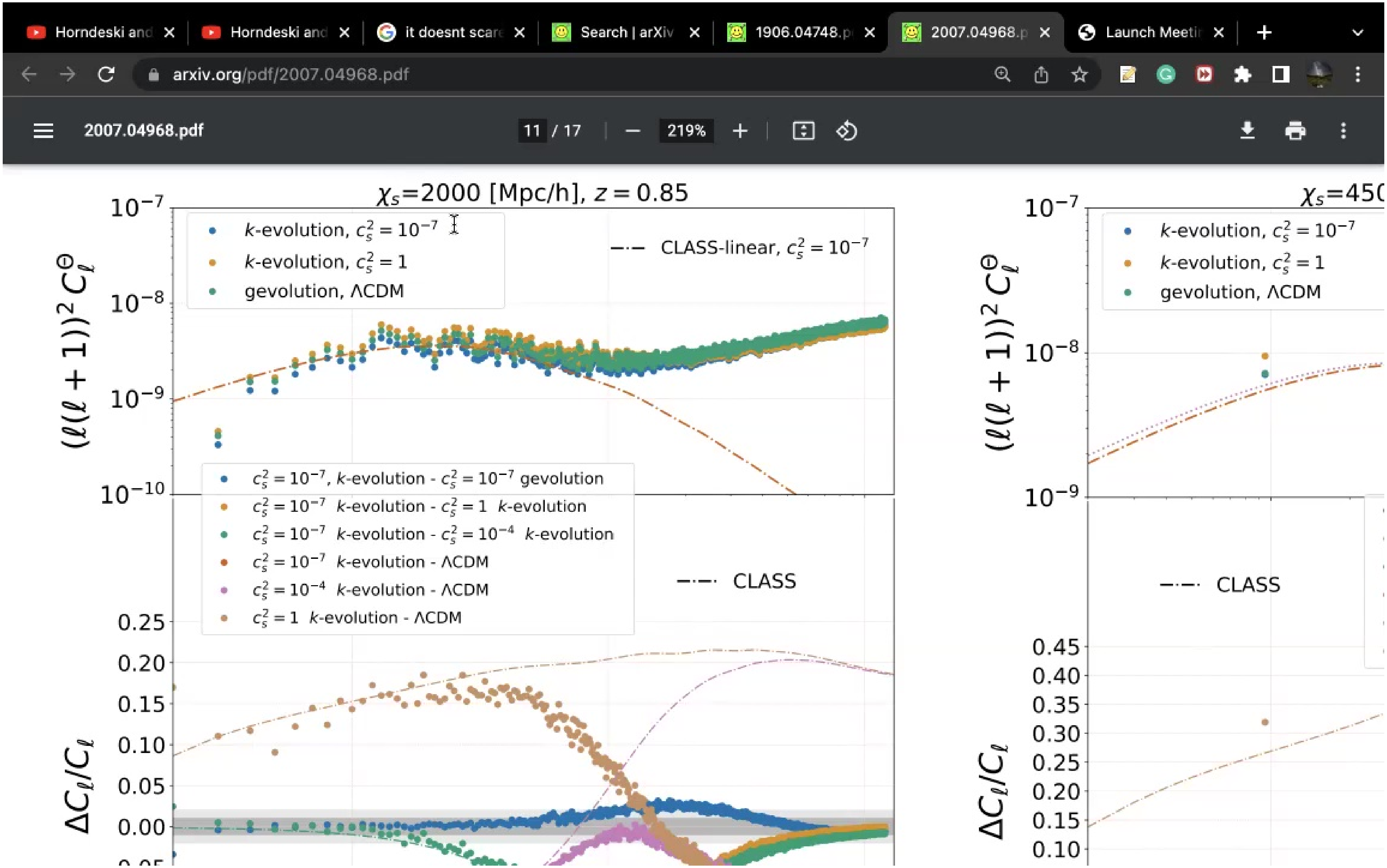
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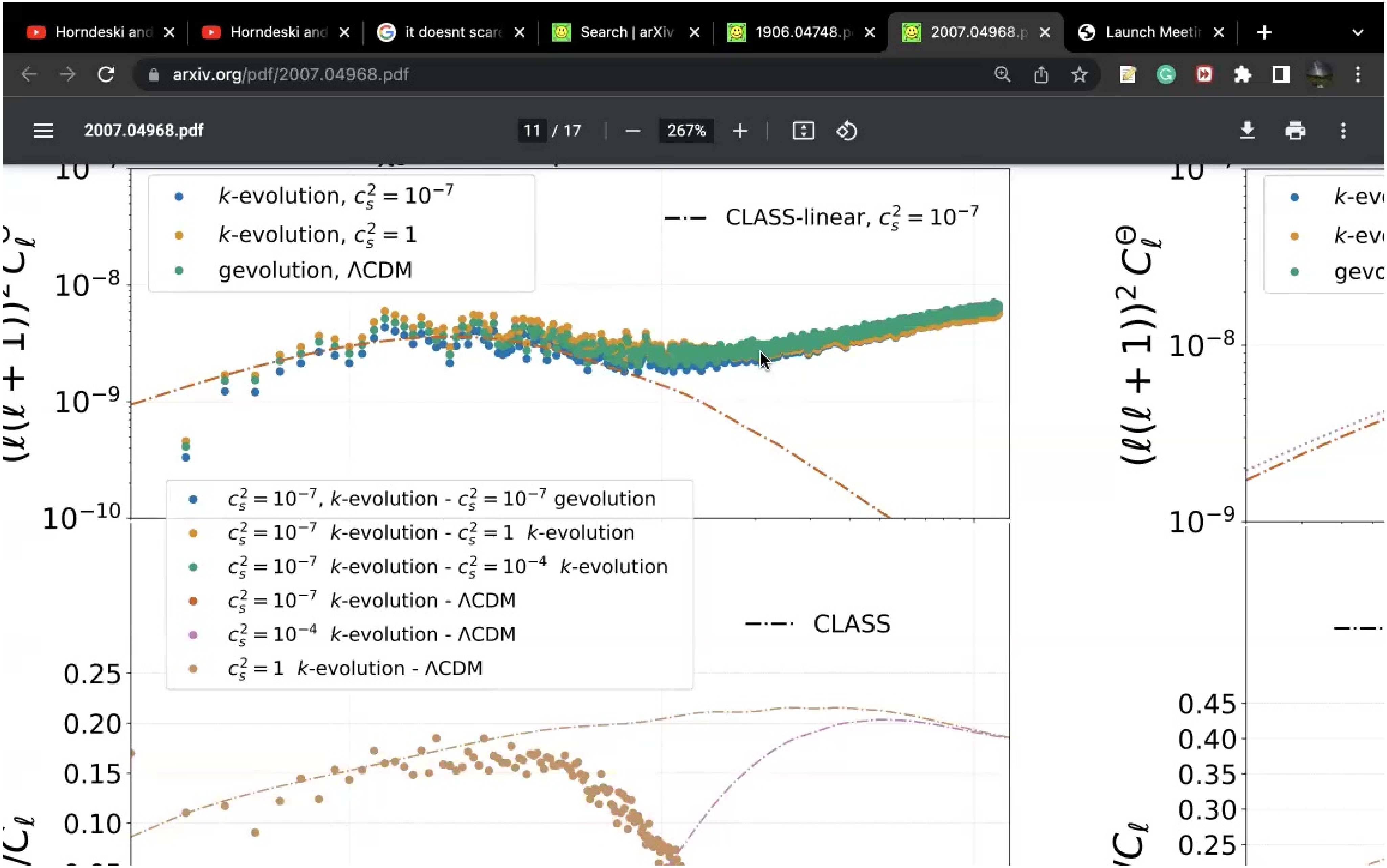
arxiv.org/pdf/2007.04968.pdf
2007.04968.pdf
11 / 17
219%
χ_s=2000 [Mpc/h], $z = 0.85$
k-evolution, $c_s^2 = 10^{-7}$
k-evolution, $c_s^2 = 1$
gevolution, ΛCDM
CLASS-linear, $c_s^2 = 10^{-7}$
$(\ell(\ell+1))^2 C_\ell^\Phi$
$c_s^2 = 10^{-7}$, k-evolution - $c_s^2 = 10^{-7}$ gevolution
$c_s^2 = 10^{-7}$ k-evolution - $c_s^2 = 1$ k-evolution
$c_s^2 = 10^{-7}$ k-evolution - $c_s^2 = 10^{-4}$ k-evolution
$c_s^2 = 10^{-7}$ k-evolution - ΛCDM
$c_s^2 = 10^{-4}$ k-evolution - ΛCDM
$c_s^2 = 1$ k-evolution - ΛCDM
CLASS
$\Delta C_\ell / C_\ell$
χ_s=450
k-evolution, $c_s^2 = 10^{-7}$
k-evolution, $c_s^2 = 1$
gevolution, ΛCDM
$(\ell(\ell+1))^2 C_\ell^\Phi$
CLASS
$\Delta C_\ell / C_\ell$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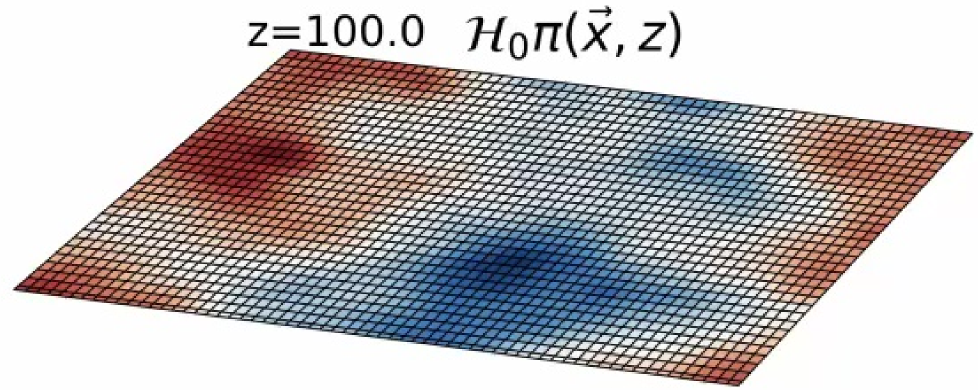
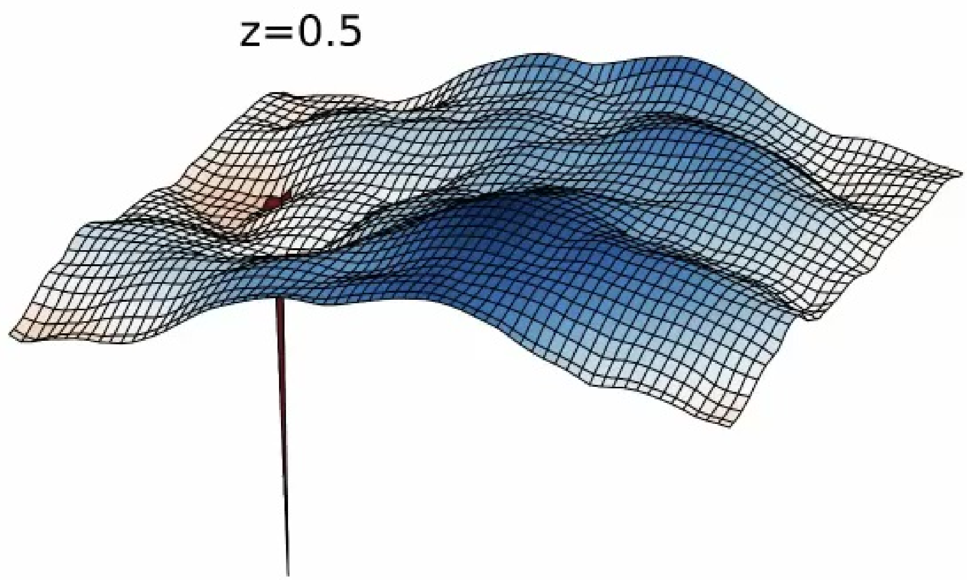
k-evolution, $c_s^2 = 10^{-7}$
k-evolution, $c_s^2 = 1$
gevolution, ΛCDM
CLASS-linear, $c_s^2 = 10^{-7}$
10^{-8}
10^{-9}
10^{-10}
$(\ell(\ell+1))^2 C_\ell$
$c_s^2 = 10^{-7}$, k-evolution - $c_s^2 = 10^{-7}$ gevolution
$c_s^2 = 10^{-7}$ k-evolution - $c_s^2 = 1$ k-evolution
$c_s^2 = 10^{-7}$ k-evolution - $c_s^2 = 10^{-4}$ k-evolution
$c_s^2 = 10^{-7}$ k-evolution - ΛCDM
$c_s^2 = 10^{-4}$ k-evolution - ΛCDM
$c_s^2 = 1$ k-evolution - ΛCDM
CLASS
0.25
0.20
0.15
0.10
$(\ell(\ell+1))^2 C_\ell^{\Theta}$
0.45
0.40
0.35
0.30
0.25

$c_s^2 < 10^{-4.7}$ $\qquad \mathcal{H}_0\pi$



$c_s^2 < 10^{-4.7}$ $\mathcal{H}_0\pi$



Non-linear scalar field equation: $\pi = \frac{\delta\phi}{\bar{\phi}}$

$$\partial_\tau^2\pi + \mathcal{H}(1-3w)\partial_\tau\pi + \Big(\partial_\tau\mathcal{H} - 3w\mathcal{H}^2 + 3c_s^2(\mathcal{H}^2 - \partial_\tau\mathcal{H})\Big)\pi$$
$$-\,\partial_\tau\Psi + 3\mathcal{H}(w - c_s^2)\Psi - 3c_s^2\partial_\tau\Phi - c_s^2\nabla^2\pi$$
$$= \mathcal{N}(\pi, \partial_\tau\pi, \vec{\nabla}\pi, \vec{\nabla}\partial_\tau\pi, \nabla^2\pi)$$

$$\mathcal{N}(\pi, \partial_\tau\pi, \vec{\nabla}\pi, \vec{\nabla}\partial_\tau\pi, \nabla^2\pi) = -\frac{\mathcal{H}}{2}\big(5c_s^2 + 3w - 2\big)\,\boxed{(\vec{\nabla}\pi)^2}$$
$$+\,2(1-c_s^2)\vec{\nabla}\pi\cdot\vec{\nabla}\partial_\tau\pi - \Big[(c_s^2-1)\big(\partial_\tau\pi + \mathcal{H}\pi - \Psi\big)$$
$$+\,c_s^2(\Phi - \Psi) + 3\mathcal{H}c_s^2(1+w)\pi\Big]\nabla^2\pi + (2c_s^2-1)\vec{\nabla}\Psi\cdot\vec{\nabla}\pi$$
$$-\,c_s^2\vec{\nabla}\Phi\cdot\vec{\nabla}\pi + \frac{3(c_s^2-1)}{2}\partial_i\Big(\partial_i\pi(\vec{\nabla}\pi)^2\Big)\,.$$

Non-linear scalar field equation:

$$\partial_\tau^2\pi + \mathcal{H}(1-3w)\partial_\tau\pi + \Big(\partial_\tau\mathcal{H} - 3w\mathcal{H}^2 + 3c_s^2(\mathcal{H}^2 - \partial_\tau\mathcal{H})\Big)\pi$$
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$$= \mathcal{N}(\pi, \partial_\tau\pi, \vec{\nabla}\pi, \vec{\nabla}\partial_\tau\pi, \nabla^2\pi)$$

3+1 D

Non-linear scalar field equation:

$$\partial_\tau^2\pi + \mathcal{H}(1-3w)\partial_\tau\pi + \Big(\partial_\tau\mathcal{H} - 3w\mathcal{H}^2 + 3c_s^2(\mathcal{H}^2 - \partial_\tau\mathcal{H})\Big)\pi$$
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$$= \mathcal{N}(\pi, \partial_\tau\pi, \vec{\nabla}\pi, \vec{\nabla}\partial_\tau\pi, \nabla^2\pi)$$

3+1 D

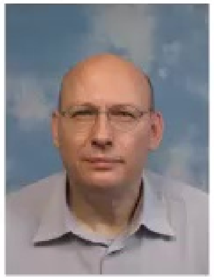
Spherical symmetry

$$\frac{d^2\pi}{d\tau^2} = +c_s^2\frac{\partial^2\pi}{\partial r^2} - \alpha\left(\frac{d\pi}{dr}\right)^2$$

$$c_s^2 \to 0 \qquad \frac{d^2\pi}{d\tau^2} = -\alpha\left(\frac{d\pi}{dr}\right)^2$$

Pan Shi

- ***Scale–invariant solutions to a Hamilton-Jacobi type equation issued from cosmology***
  Pan Shi, FH et al.
  in preparation



Peter Wittwer

- ***Stability of a blow-up solution for a second order in time Hamilton-Jacobi type equation***
  Pan Shi, FH et al.
  in preparation

$$\frac{d^2\pi}{d\tau^2} = -\alpha\left(\frac{d\pi}{dr}\right)^2$$

- ***On a second order in time Hamilton-Jacobi type equation issued from cosmology***
  Pan Shi, FH et al.
  in preparation

$$\frac{d^2\pi}{d\tau^2} = -\alpha\left(\frac{d\pi}{dr}\right)^2$$

One special solution $\pi_s(\tau, r) = \frac{\kappa(\tau)}{2} r^2$

$$\partial_\tau^2 \kappa(\tau) = \alpha\left[\kappa(\tau)\right]^2$$

We can think of this ODE as Newton's second law

$$F(x) = x^2$$

$$V(x) = -\frac{4}{3}x^3$$

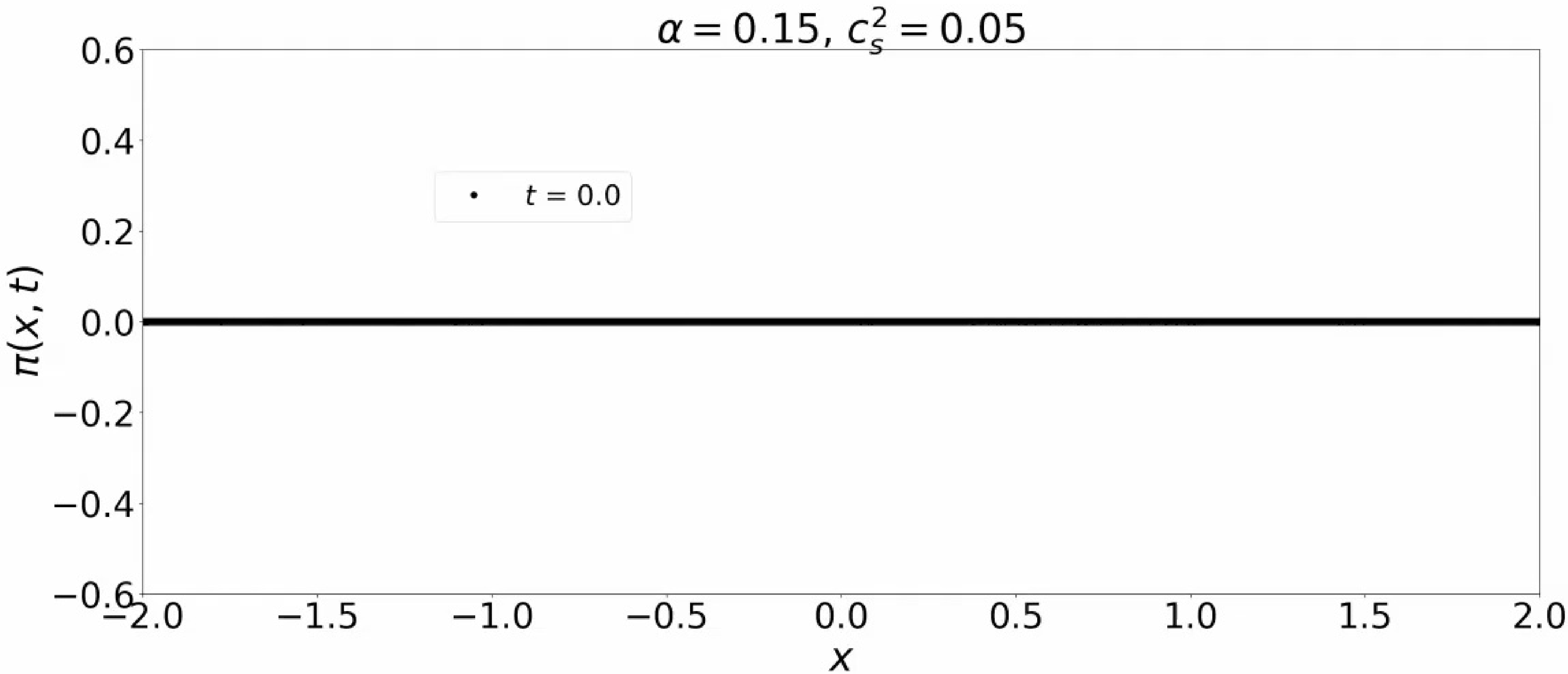
$$\kappa(\tau) = \frac{3}{2(\tau - \tau_b)^2}$$

$$\tau_b = \left(\frac{3}{8\alpha}\right)^{\frac{1}{3}}\left(\frac{1}{C}\right)^{\frac{1}{6}}\int_{s(\kappa_0)}^{\infty}\frac{ds}{\sqrt{1+s^3}}$$

$$C = \kappa'(0)^2 - \frac{8}{3}\kappa(0)^3$$

F. H, P. Shi, J. Adamek, M. Kunz, P. Wittwer; arXiv: 2107.14215

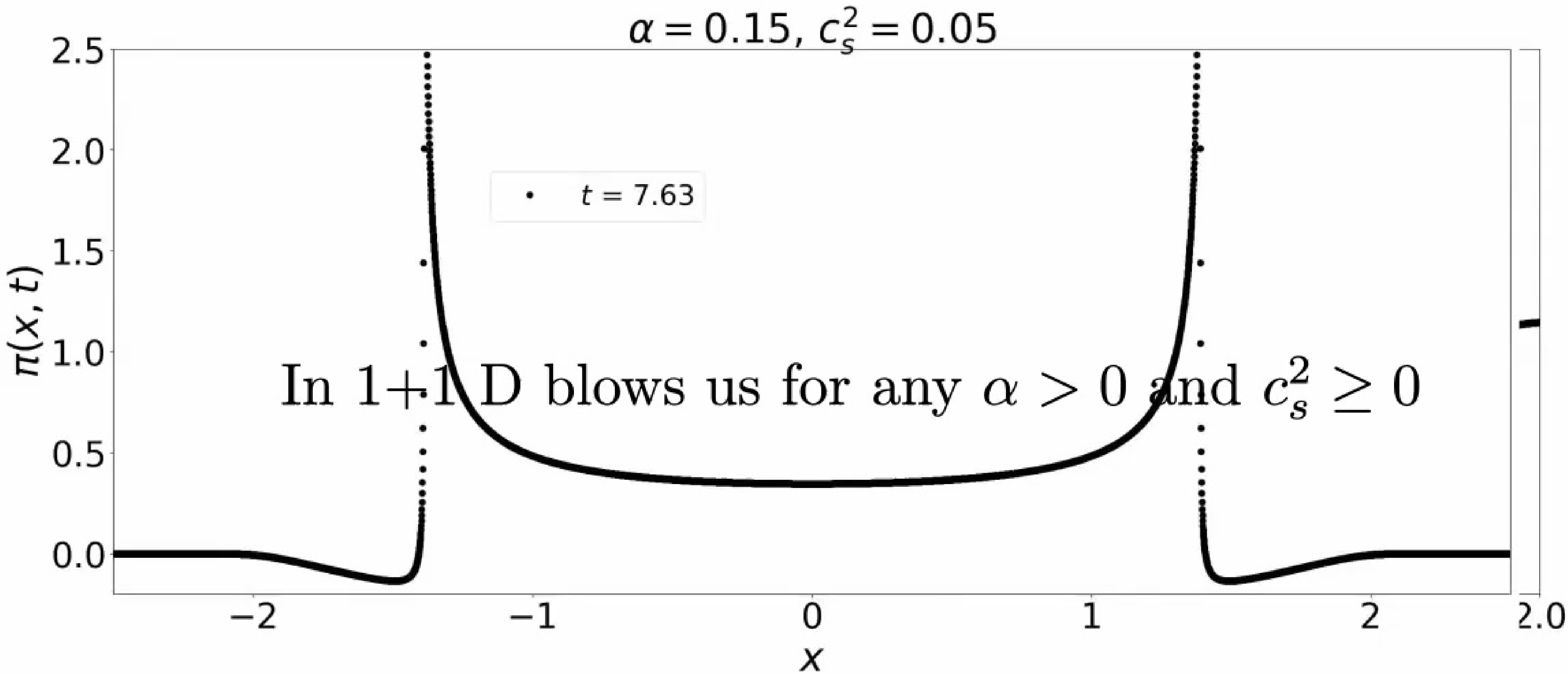
M-type blowup



$$\frac{d^2u}{d\tau^2} = +c_s^2 \frac{\partial^2 u}{\partial x^2} - \alpha\left(\frac{du}{dx}\right)^2$$

J. Eckmann, F. H, H. Zaag; arXiv: 2205.01055

M-type blowup



In 1+1 D blows us for any $\alpha > 0$ and $c_s^2 \geq 0$

$$\frac{d^2u}{d\tau^2} = +c_s^2 \frac{\partial^2 u}{\partial x^2} - \alpha \left(\frac{du}{dx}\right)^2$$

J. Eckmann, F. H, H. Zaag; arXiv: 2205.01055

large speed of sound:

- Linear predictions and halo-fit are not accurate enough especially for non-standard cosmologies

Dark energy clustering can have:

- ~ **30%** effect on ISW-RS angular power
- ~ **7%** effect on convergence power-spectrum

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- Non-linearities can become **VERY** important!
- Cosmological N-body simulations bound on speed of sound: $c_s^2 \geq 10^{-4.7}$
- From mathematical point of view any speed of sound leads to an instability!
- EFT of k-essence (DE) breaks down:
  - EFT expansion problem?
  - Strong field regime (BH)?
  - What about covariant k-essence?

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Thank you ☺