Title: Time-optimal gates for quantum computing with Rydberg atoms

Speakers: Guido Pupillo

Collection: Cold Atom Molecule Interactions (CATMIN)

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Abstract: "Neutral atoms have emerged as a competitive platform for digital quantum simulations and computing. In this talk, we discuss recent results on the design of time-optimal two- and three-qubit gates for neutral atoms, where entangling gates are implemented via the strong and long-range interactions provided by highly excited Rydberg states. We combine numerical and semi-analytical quantum optimal control techniques to obtain theoretically laser pulses that are "smooth", time-optimal and "global" -- that is, they do not require individual addressability of the atoms. This technique improves upon current implementations of the controlled-Z (CZ) and the three-qubit C2Z gates with just a limited set of variational parameters, demonstrating the potential of quantum optimal control techniques for advancing quantum computing with Rydberg atoms."

UNIVERSITÉ DE STRASBOURG

Workshop – CATMIN 2022 – Perimeter Institute - Waterloo

Time-Optimal & Global Gates with Rydberg Atoms

Guido Pupillo

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Sven Jandura

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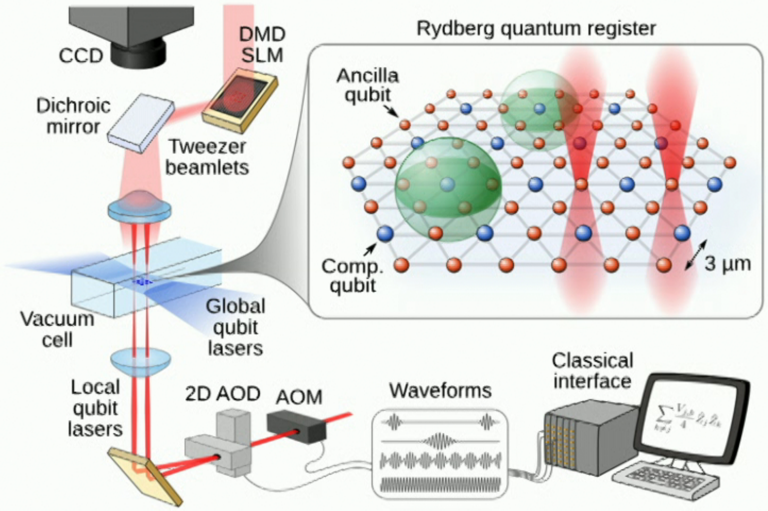
institut universitaire de France

QUSTEC

INVESTISSEMENTS D'AVENIR

MOQS Molecular Quantum Simulations

Rydberg Quantum Processors

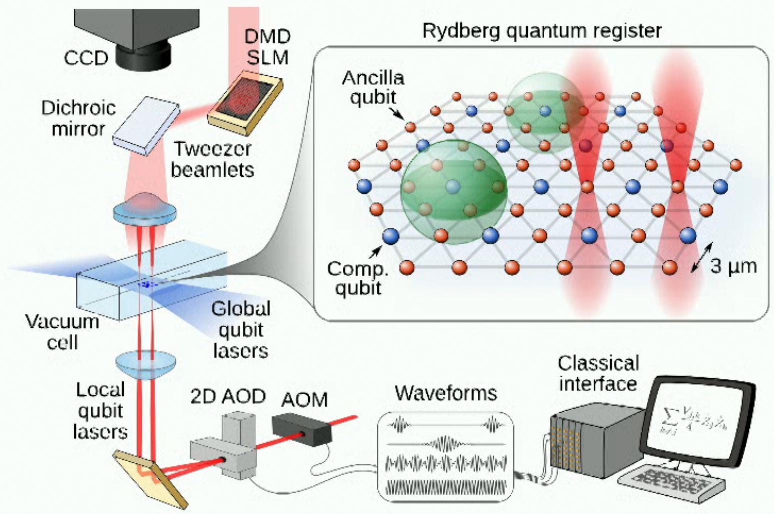


Morgado and Whitlock, AVS Quantum Sci. 3, 023501 (2021)

- Rydberg atoms: large principal quantum number ($n \gtrsim 50$)
- Alkali (K, Rb, Cs, ..) or alkaline earth-like (Sr, Yb..) atoms
- **Strong, long-range vdW / dipolar interactions**
- **"Long" lifetime of Rydberg states**

Rydberg Quantum Processors

"Fast quantum gates for Rydberg atoms",
Jaksch, Cirac, Zoller, Rolston, Côté, Lukin, PRL 85, 2208 (2000)



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Internal Hamiltonian

Rydberg-Rydberg interaction energy

Rabi frequency (single-atom addressability)

$$H^i(t, \mathbf{x}_1, \mathbf{x}_2) = u|r\rangle_1\langle r| \otimes |r\rangle_2\langle r|$$
$$+ \sum_{j=1,2} \left[(\delta_j(t) - i\gamma)|r\rangle_j\langle r| - \frac{\Omega_j(t, \mathbf{x}_j)}{2} (|g\rangle_j\langle r| + \text{h.c.}) \right]$$

Spontaneous emission

Laser detuning

- Rydberg atoms: large principal quantum number ($n \gtrsim 50$)
- Alkali (K, Rb, Cs, ..) or alkaline earth-like (Sr, Yb..) atoms
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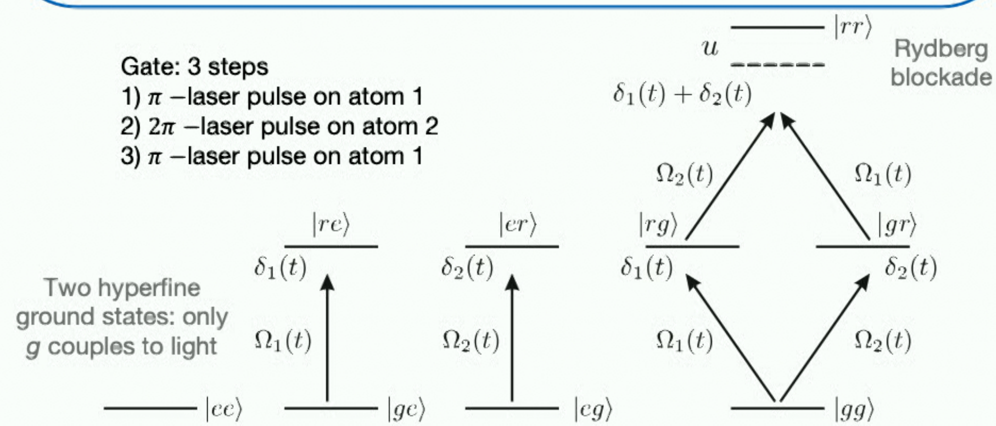
Gate: 3 steps
1) π –laser pulse on atom 1
2) 2π –laser pulse on atom 2
3) π –laser pulse on atom 1

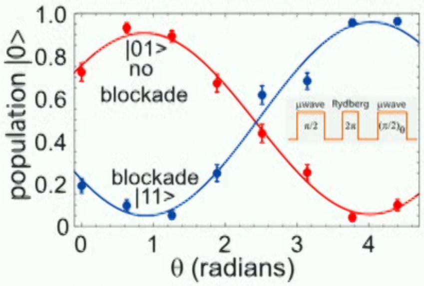


$$|gg\rangle \rightarrow e^{i(\pi - \tilde{\varphi})}|gg\rangle \qquad \varphi = \pi - \tilde{\varphi} \approx \pi \qquad \tilde{\varphi} \approx \pi\Omega_2/2u$$

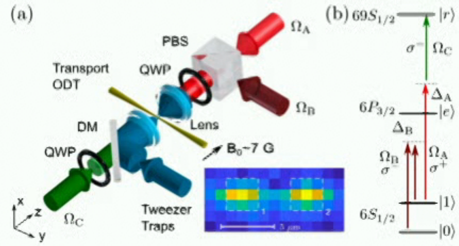
3

Quantum gates with Rydberg atoms: Recent Experiments

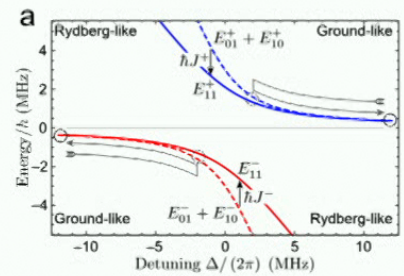
"Rydberg dressing"



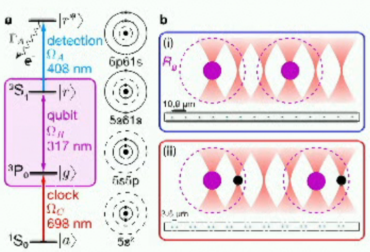
Saffman, PRL 123 (2019) 230501



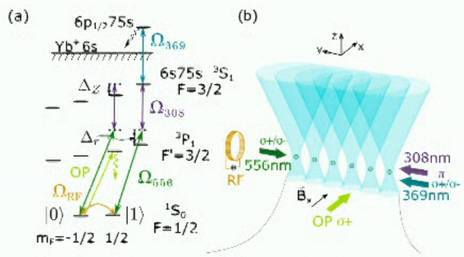
Pritchard, Quantum Sci. Technol. 4 (2019) 015011



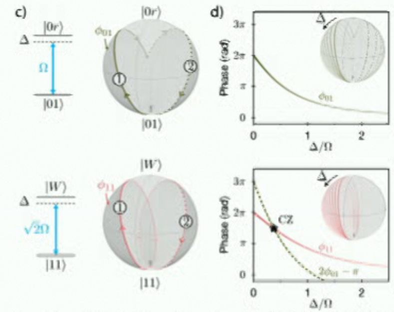
Biedermannm Deutsch, arXiv:2111.14677



Endres, Nature Physics 16, 857 (2020)



Thompson, PRX 12 (2022) 021028



Lukin, Vuletic, Greiner, PRL 123 (2019) 170503

Parallel gates!
3-qubit gates!

Best current gate fidelities *F*: two-qubit 99.1% (Endres); three-qubit >87% (Lukin)

Solutions?

Current limits to *F*: finite Rydberg blockade strength, Rydberg decay, light scattering, laser phase noise, spatial variations of laser intensity, Doppler shifts due to thermal motion, ….

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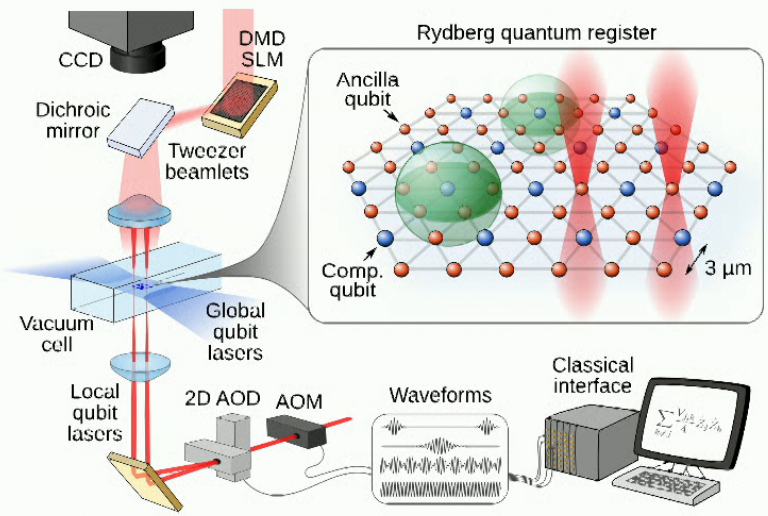

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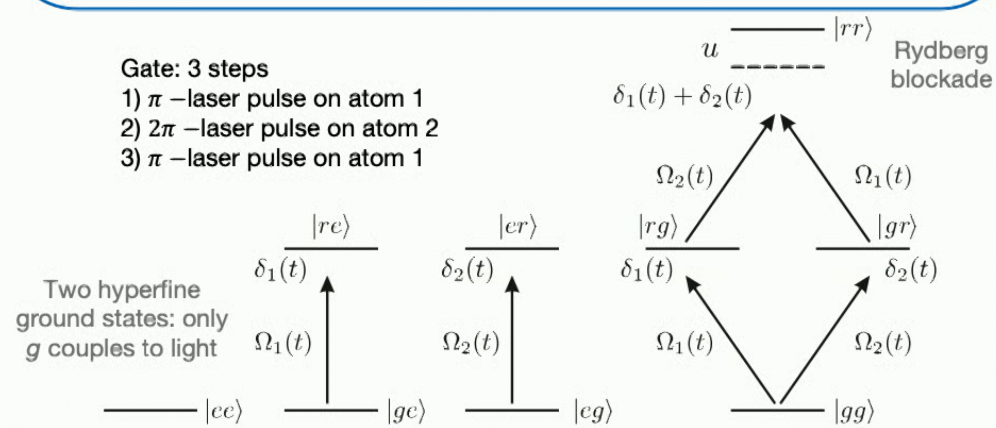

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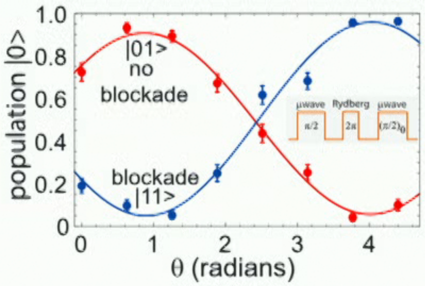

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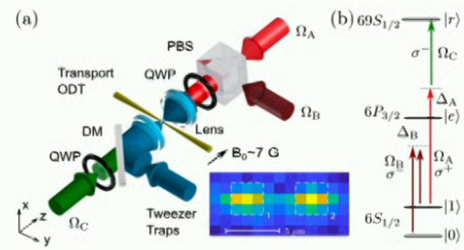

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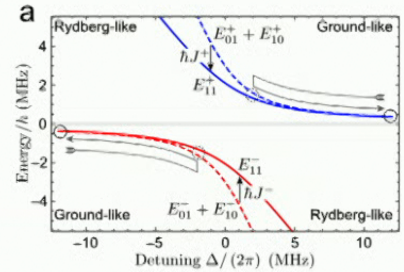

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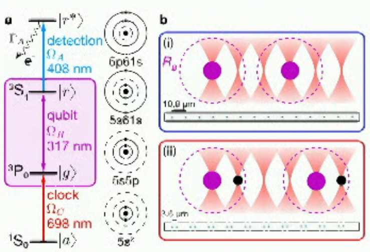

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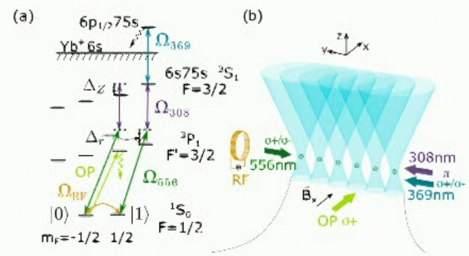

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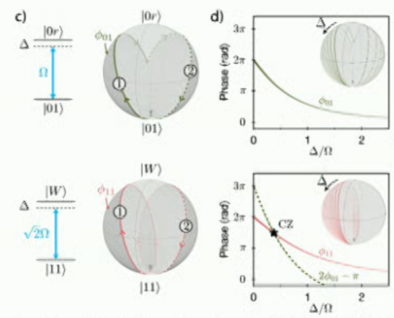

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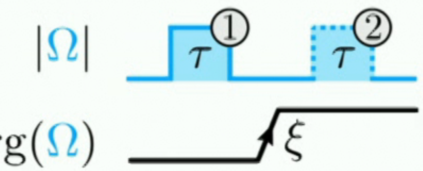
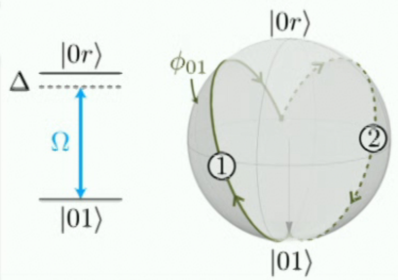
Current limits to *F*: finite Rydberg blockade strength, Rydberg decay, light scattering, laser phase noise, spatial variations of laser intensity, Doppler shifts due to thermal motion, ….

First steps: A CZ Gate with global addressability

Levine …. Greiner, Vuletic, Pichler, Lukin PRL 123, 170503 (2019)

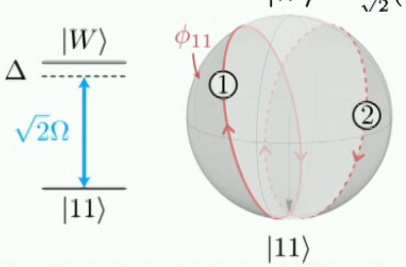
- **Global Laser addressing both atoms**: Advantage: Simpler experimental setup
- Two square pulses duration τ, detuning Δ and phase difference ξ



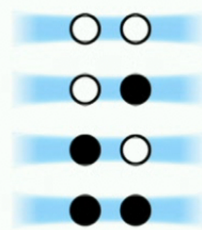


$$H_1 = \begin{pmatrix} 0 & \frac{\Omega}{2} \\ \frac{\Omega^*}{2} & 0 \end{pmatrix}$$

$$|W\rangle = \frac{1}{\sqrt{2}}(|1r\rangle + |r1\rangle)$$



$$H_2 = \begin{array}{c} \quad |11\rangle \qquad |W\rangle \\ \begin{pmatrix} 0 & \frac{\sqrt{2}\Omega}{2} \\ \frac{\sqrt{2}\Omega^*}{2} & 0 \end{pmatrix} \end{array}$$



$|00\rangle \to |00\rangle$

$|01\rangle \to |01\rangle e^{i\phi_{01}}$

$|10\rangle \to |10\rangle e^{i\phi_{10}}$

$|11\rangle \to |11\rangle e^{i\phi_{11}}$

- CZ Gate if $\phi_{11} - 2\phi_{01} = \pi$
- Duration $2\tau\Omega = 8.59 < 4\pi$

- Experimentally simpler…

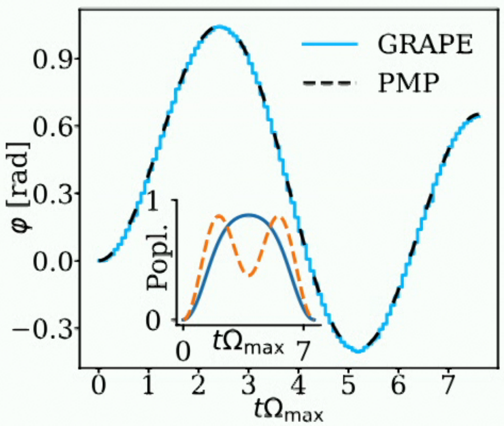
- Can we be even faster with a different shape of $\Omega(t)$?
- Can we generalise the pulse to three qubit gates?

Incomplete wish-list for Rydberg quantum gates

- **Gates should be "fast"**: many errors mitigated
- **Only global control lasers:** no single-site addressability, simpler experiments
- **Pulses should be smooth:** simpler experiments
- **Allow for three and more qubit gates natively:** shorten algorithms
- **Fidelities should be compatible with error correction**
- **….**

This Talk:

- **Time-optimal CZ Gate using a smooth, global laser pulse**
- **Time-optimal C2Z gate using a smooth laser pulse**
- **New method for semi-analytical description of the pulses**
- Realistic implementation & Different optimization objectives

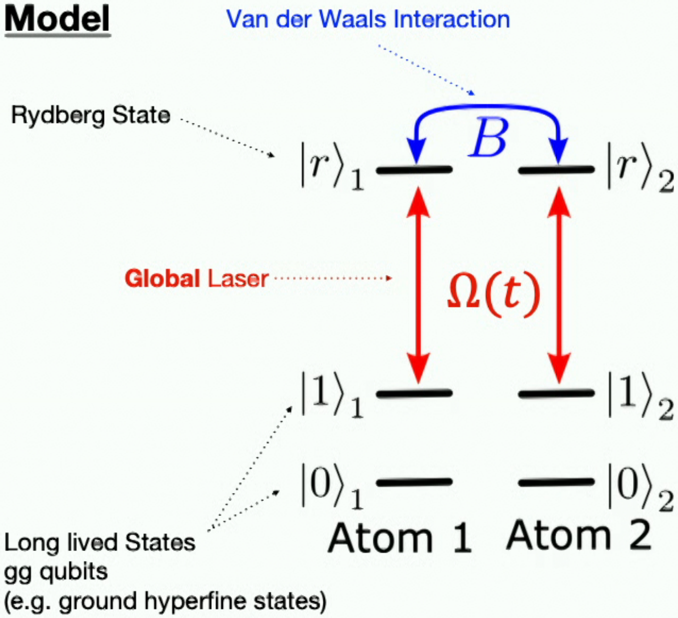


Time-optimal two- and three-qubit gates for Rydberg atoms

Svendura & Pupillo, Quantum 6, 712 (2022)

Problem Description

Model



Our Goal

- Fastest laser pulses $\Omega(t)$ for CZ and a C_2Z gate

Assumptions

- Blockade regime: $B \gg |\Omega|$, population of $|rr\rangle$ is suppressed
- Amplitude, phase of $\Omega(t) = |\Omega(t)|e^{i\varphi(t)}$ can be controlled
- $|\Omega(t)| \leq \Omega_{\max}$

$$H = \sum_{i=1}^{2} \frac{\Omega(t)}{2} |1_i\rangle\langle r_i| + \text{h.c.} + B|rr\rangle\langle rr|$$

Optimal Control Methods for quantum computing

- **Optimal Control algorithms: pulse (e.g. laser) control optimization**
  (many applications: e.g. NMR, *quantum metrology*, *control of chemical reactions*, ***quantum information***)

- **Optimal control for QIP**: *superconducting qubits, ions, neutral atoms*

  - **Improve** speed and fidelities of quantum gates
  - **Provide new solutions:** novel quantum gates

Pioneering works with neutral atoms: Saffman, Wilhelm, Koch, Lukin, Pichler, Montangero, Calarco, Deutsch, Jessen, ..

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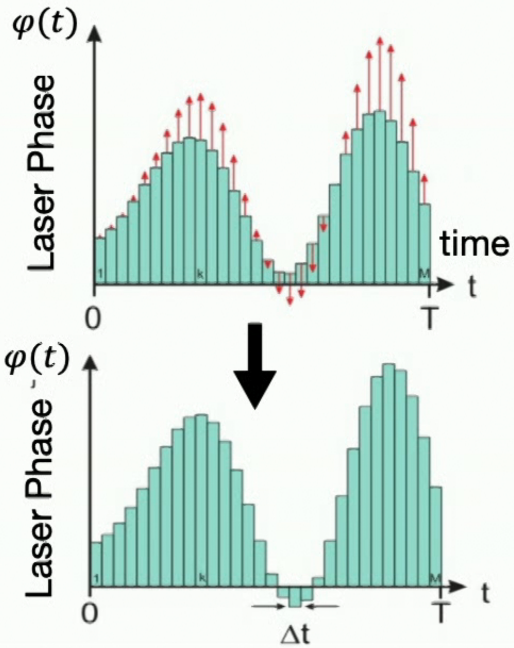
Many algorithms: GRAPE, CRAB, …

- Example: **GRAPE** = **Gr**adient **A**scent **P**ulse **E**ngineering
  - **Goal: Optimize gate fidelity *F***
  - **Control knob: e.g. laser phase $\varphi(t)$**
  - Make piecewise constant Ansatz for $\varphi(t)$
  - Maximize averaged fidelity F as a function of $\varphi_1, \dots \varphi_N$
  - Efficiently calculate gradient of F with respect to φ_j
  - Repeat until convergence to a "optimal" solution

"best solution" for Rydberg atoms?



"Problems": many parameters & many solutions
numerical solutions can be "local mininima"
numerical solutions may contain abrupt changes / unintelligible

GRAPE for Time-Optimal, global CZ gate (first case $B = \infty$)

- Time-optimal pulse: Many errors mitigated by faster gates
- **Time-optimality:** Ansatz $\Omega(t) = \Omega_{\max} e^{i\varphi(t)}$ $\Delta = \mathrm{d}\varphi / \mathrm{d}t$ Laser detuning and phase are equivalent!

  Fast gates: largest Rabi frequency
- Hamiltonian

$$H = H_1 \oplus H_2 = \begin{pmatrix} 0 & \Omega/2 & 0 & 0 \\ \Omega^*/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2}\Omega/2 \\ 0 & 0 & \sqrt{2}\Omega^*/2 & 0 \end{pmatrix}$$

Initial state $|\psi(0)\rangle = (1,0,1,0)$

(|01> in the first block and |11> in the second block)

- Perfect CZ gate if $|\psi(T)\rangle = (e^{i\theta}, 0, -e^{2i\theta}, 0)$ for some θ
- Objective function: Infidelity $1 - F$ with average fidelity F depending on $|\psi(T)\rangle$ and θ

$$F = \int \left| \left\langle \psi \right| U^{\dagger}_{desired} U_{real} \left| \psi \right\rangle \right|^2 \mathrm{d}\psi$$

- Random initial guess for $\varphi(t)$

Pedersen et.al. PRA 367, 47 (2007)

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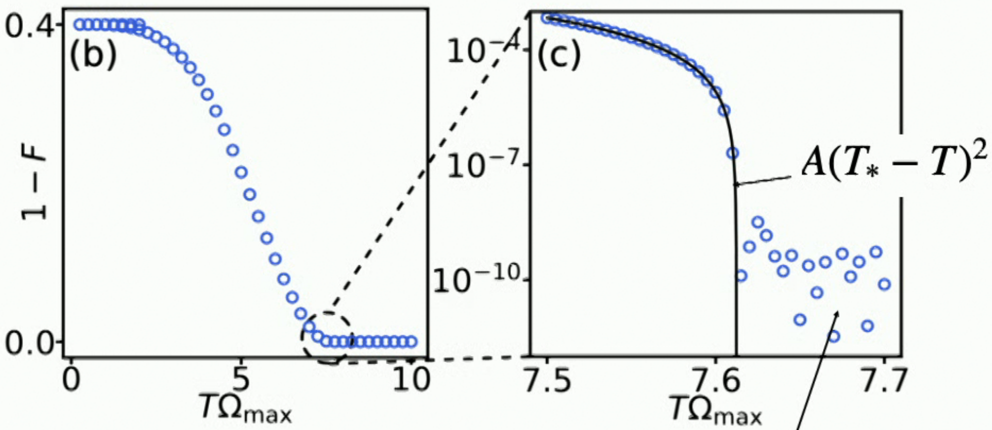
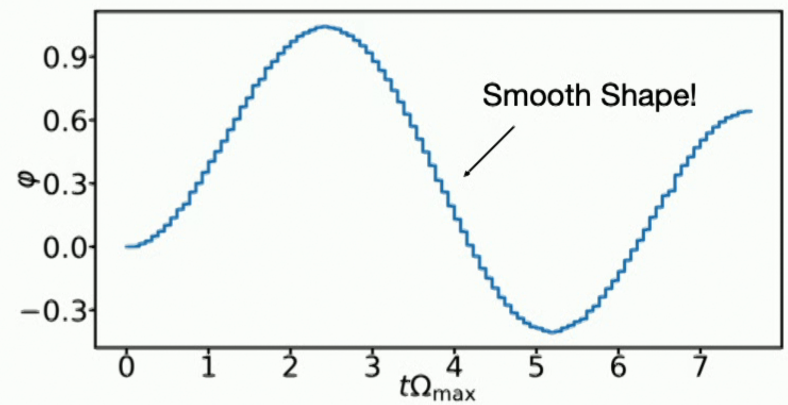
Time-Optimal Global CZ Gate: numerical GRAPE results

Minimal infidelity at different pulse durations T

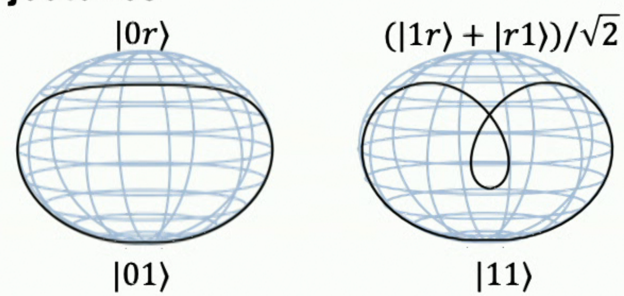


Remaining infidelity determined by convergence condition of optimisation

Optimal Laser Phase



Trajectories

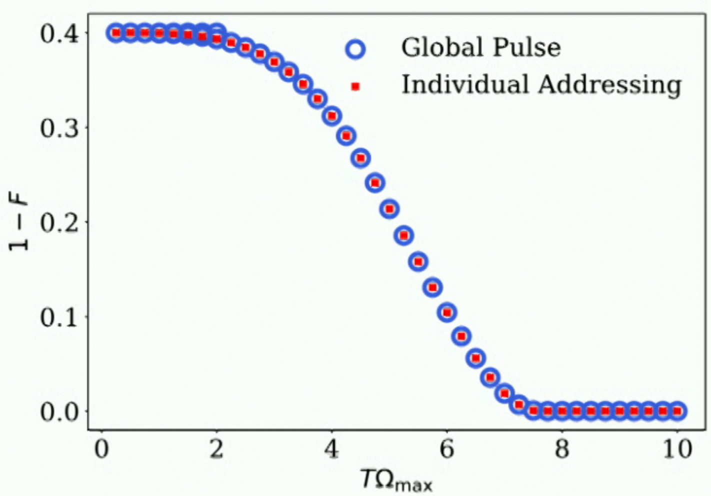


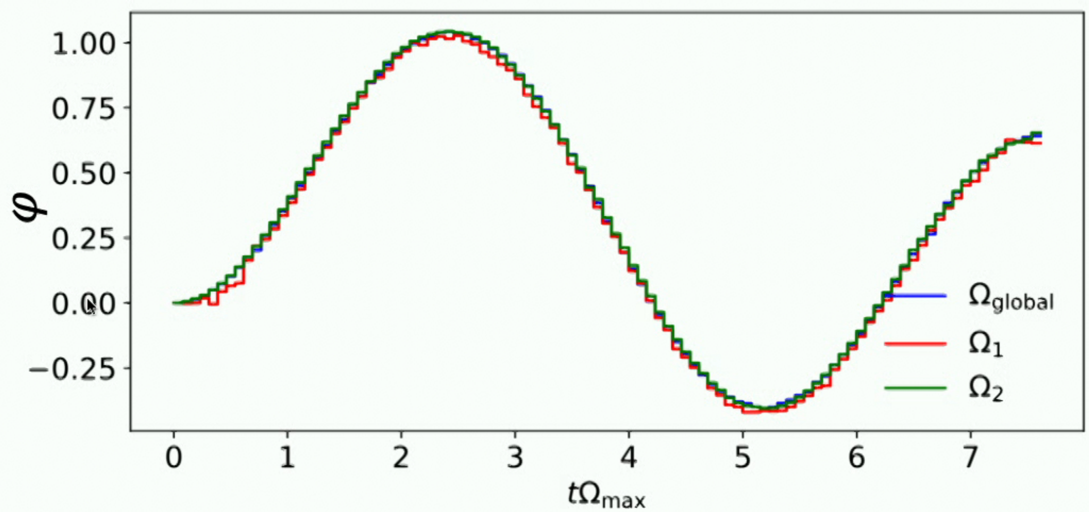
- Shortest Pulse duration: $T\Omega_{\max} = 7.612$
- **~10% faster** than pulse from Levine et. al. (PRL 123, 170503 (2019))
- Only **smooth** manipulation of laser phase needed

Similar to pulse from Pagano… Pfau, Montangero, Büchler, arXiv:2202.13849

Allowing for Individual Addressability

Now we allow $\Omega_1 \neq \Omega_2$ and optimise over phase and amplitude of both lasers





Individual addressability brings no speedup!

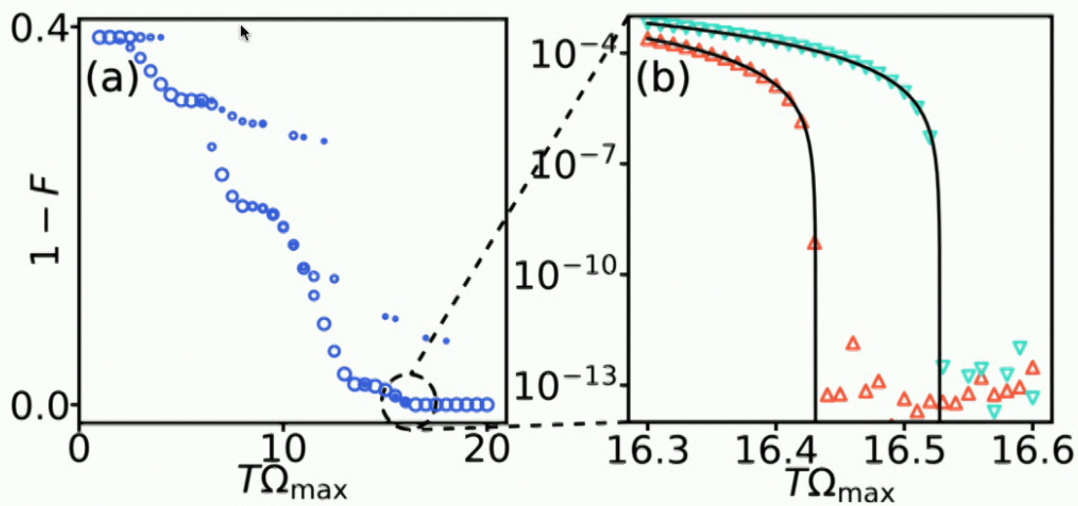
Time-Optimal Global C2Z Gate

- C2Z Gate: Three qubit gate with $|xyz\rangle \mapsto (-1)^{xyz}|xyz\rangle$
- Assume all three atoms are in each others' blockade radius
- Grape can be applied as for the CZ gate, with additional $H_3 = \begin{pmatrix} 0 & \frac{\sqrt{3}\Omega}{2} \\ \frac{\sqrt{3}\Omega^*}{2} & 0 \end{pmatrix}$ describing the coupling $|111\rangle \leftrightarrow (|11r\rangle + |1r1\rangle + |r11\rangle)/\sqrt{3}$
- We find two qualitatively different pulses

Minimal infidelity at different pulse durations

- Durations: Pulse 1: $T\Omega_{\max} = 16.43$

  Pulse 2: $T\Omega_{\max} = 16.53$



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Time-Optimal Global C_2Z Gate

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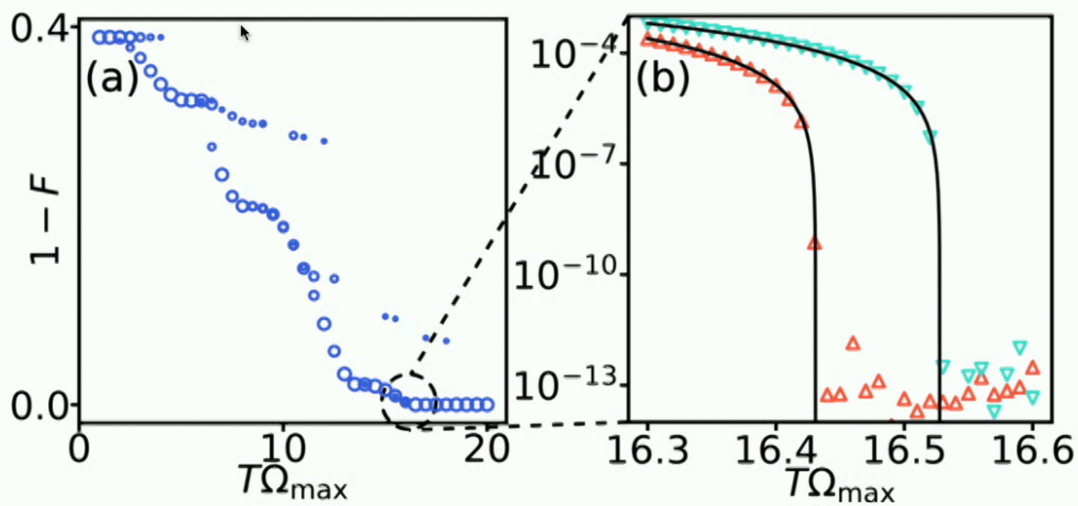
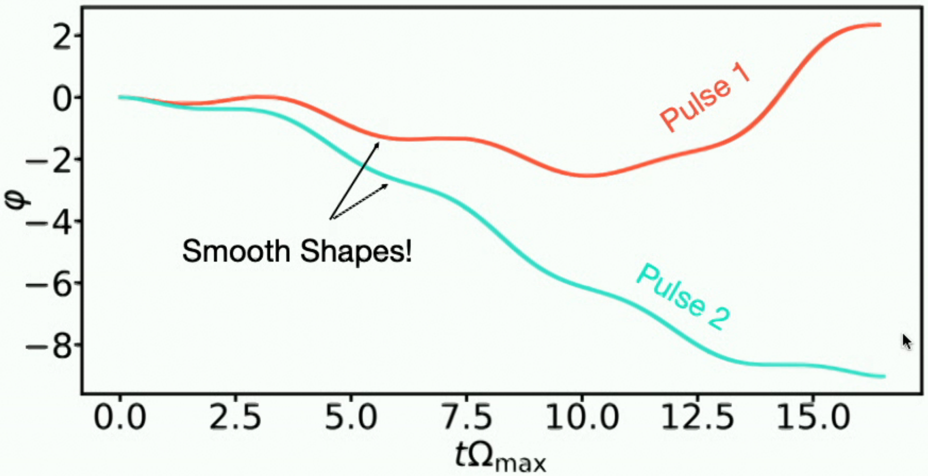
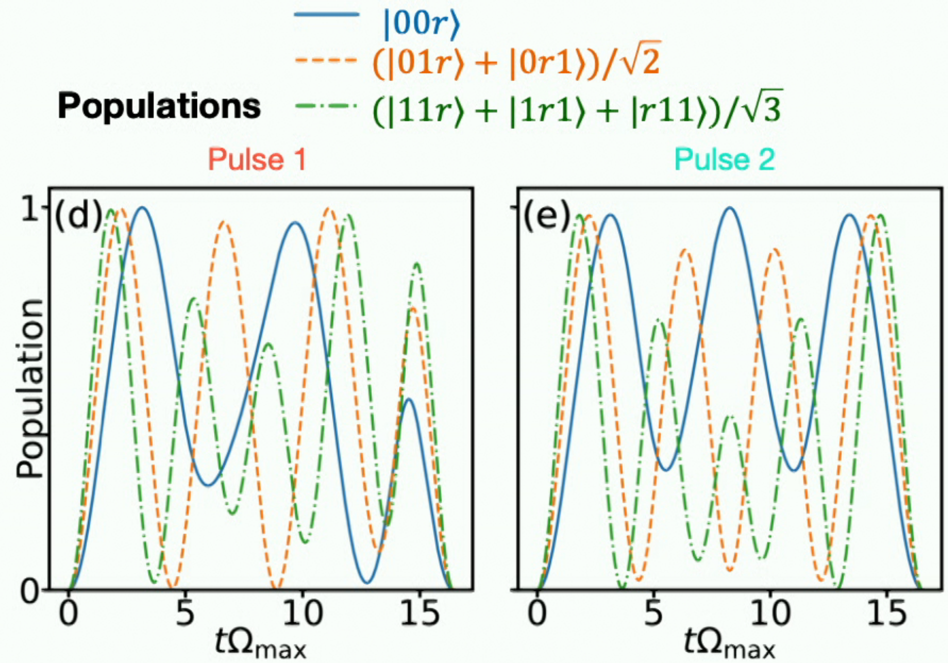


Time-Optimal Global C_2Z Gate

Optimal Laser Phase





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A semi-analytical approach: PMP method

- **GRAPE: 100 parameters for the CZ and 400 parameters for C_2Z gate**
- **Pontryagin's Maximum Principle (PMP): Description with just 4 (CZ) and 6(C_2Z) parameters**
  - Easier reproducibility
  - Reveals structure of time-optimal pulses
- **PMP for time-optimal parallel CZ and C_2Z gates:**

There are **co-states** $|\chi_k(t)\rangle$ such that the following system is satisfied

$$|\dot{\psi}_k(t)\rangle = -iH_k(\varphi(t))|\psi_k(t)\rangle \qquad H_k = \sqrt{k}\,\frac{\Omega_{\max}}{2}(\cos\varphi\sigma_x - \sin\varphi\sigma_y)$$
$$|\dot{\chi}_k(t)\rangle = -iH_k(\varphi(t))|\chi_k(t)\rangle$$

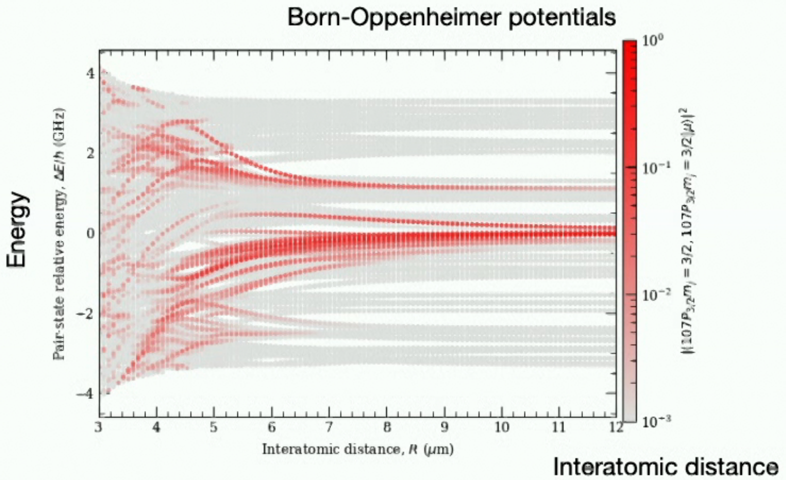
with $\varphi(t)$ given by

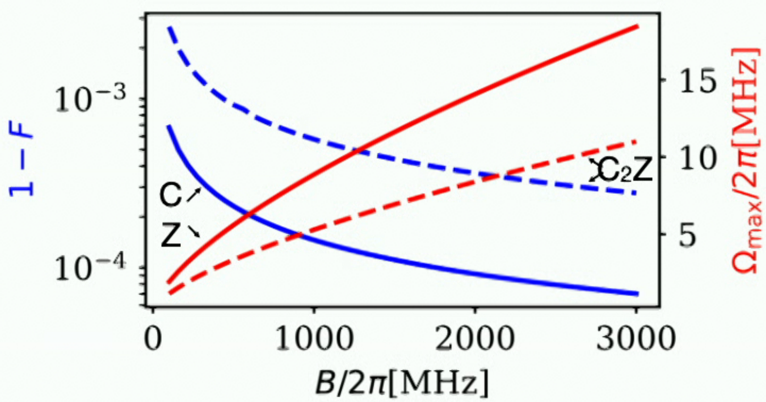
$$\varphi = \underset{\varphi'}{\operatorname{argmax}} \sum_k \operatorname{Im}(\langle\chi_k(t)|H_k(\varphi')|\psi_k(t)\rangle)$$

($k = 1,2$ for CZ and $k = 1,2,3$ for C_2Z gate)

Initial costates $|\chi_k(0)\rangle$ completely determine a pulse!

Experimental realization: Example





Experimental parameters:

- Cs atoms
- $|1\rangle = |6S_{1/2}F = 4m_f = 4\rangle |r\rangle = |107P_{3/2}m_j = 3/2\rangle$
- Error sources: Imperfect Blockade and Decay
- Interatomic distance: 7μm
- $B = 2\pi\times 180\mathrm{MHz}$

CZ Gate: Gate Error $1 - F = 4.6\times 10^{-4}$
Rabi Freq. $\Omega_{\max} = 2\pi\times 2.8\mathrm{MHz}$

C_2Z Gate: Gate Error $1 - F = 1.8\times 10^{-3}$
Rabi Freq. $\Omega_{\max} = 2\pi\times 1.7\mathrm{MHz}$

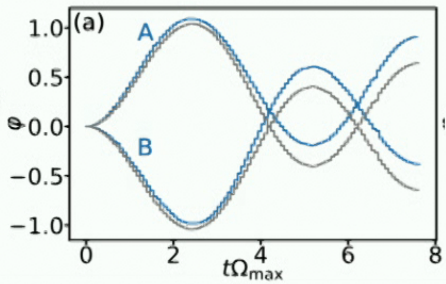
Good fidelities with reasonable experimental parameters

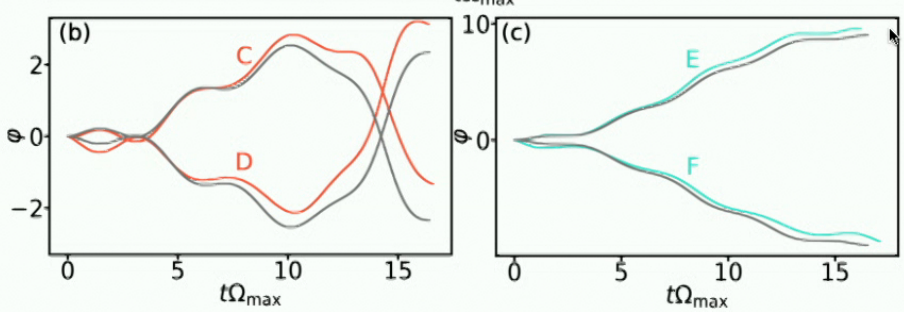
Other Optimization Objectives

Finite blockade strength B

Example $B/\Omega_{\max} = 10$

CZ Gate

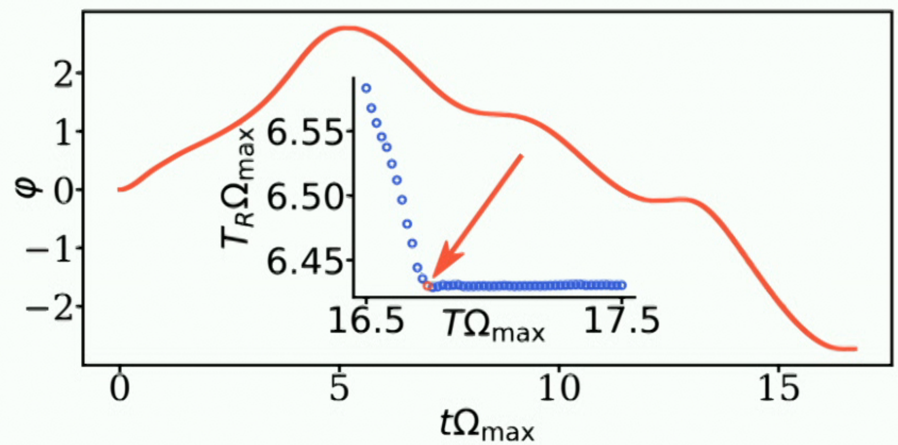




C$_2$Z Gate Pulse 1 C$_2$Z Gate Pulse 2

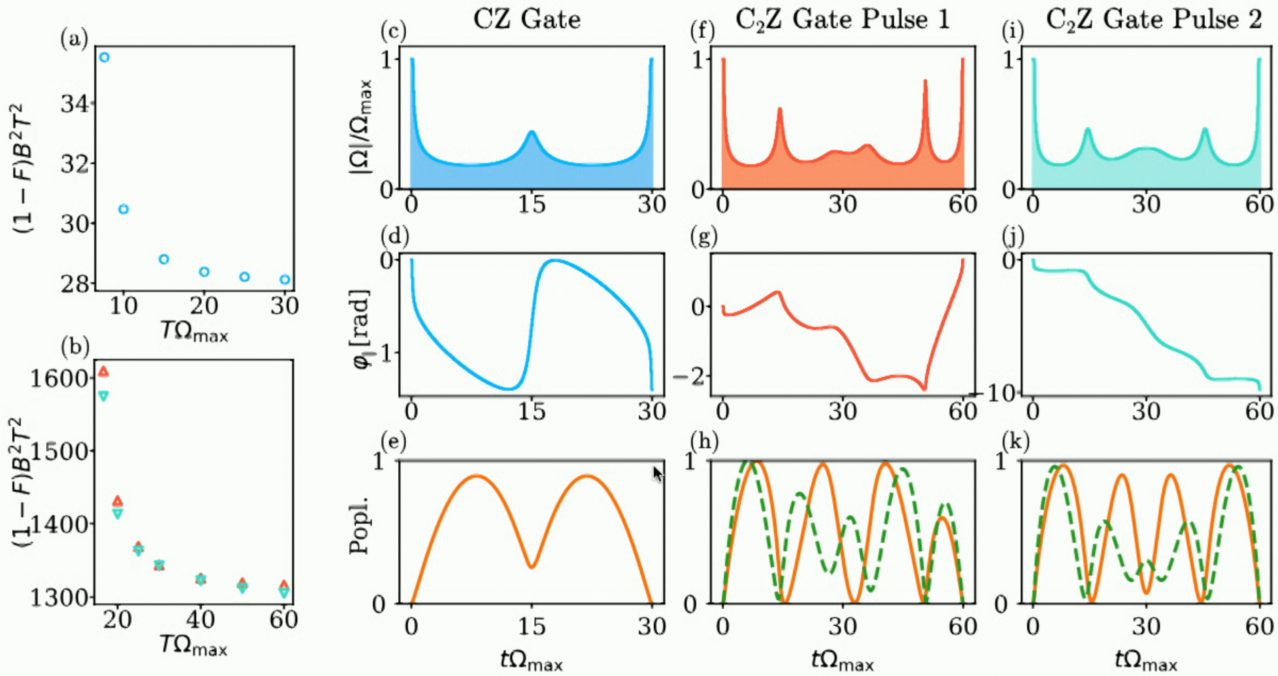
Minimizing Time T_R in Rydberg State

- CZ Gate: Time-Optimal pulse already has lowest possible T_R
- C$_2$Z Gate: 7% less time spent in Rydberg state with slightly longer pulse



Optimization vs other "imperfections" possible..

Optimization vs uncertainty on *B*



- Choose gates that have fidelity F=1 at infinite B
- Minimize $\frac{1}{2}\frac{\mathrm{d}^2(1-F)}{\mathrm{d}(1/B)^2}\Big|_{B=\infty}$
- ….

Robustness!

Conclusion

- **Time-optimal pulses for CZ and C_2Z g**ates using GRAPE
- Pulses: always maximal laser amplitude & ***modulate laser phase smoothly***
- Pontryagin's Maximum Principle (PMP): **semi-analytical description**
- Pulses can be optimized at finite B or to minimize the time spent in the Rydberg state instead of the total pulse duration

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What we're doing now

- Better understanding of Pulses
- Generalization to other phase gates
- Optimizing pulses for robustness

Thanks!

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MOQS Molecular Quantum Simulations