

Title: Collective effects when photons interact with many atoms

Speakers: Francis Robicheaux

Collection: Cold Atom Molecule Interactions (CATMIN)

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Abstract: Atoms separated by distances comparable to or less than the wavelength of a photon for a dipole allowed transition can manifest collective effects. These effects can be apparent in the radiative lifetime of an excited state, how the atoms recoil when a photon is emitted, the transmission/reflectivity of an atomic cloud, etc. This talk will describe some of these processes and the theoretical machinery used to model them. This work is supported by the National Science Foundation under Award No. 2109987-PHY.

Collective effects when photons interact with many atoms

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Deepak Suresh, Shihua Huang , R.T. Sutherland

Superradiance

Recoil of atoms due to photon momentum

Support from NSF 2109987-PHY

Some Many Atom Effects

Clausius-Mossotti (Lorentz-Lorenz) corrections to χ of gas

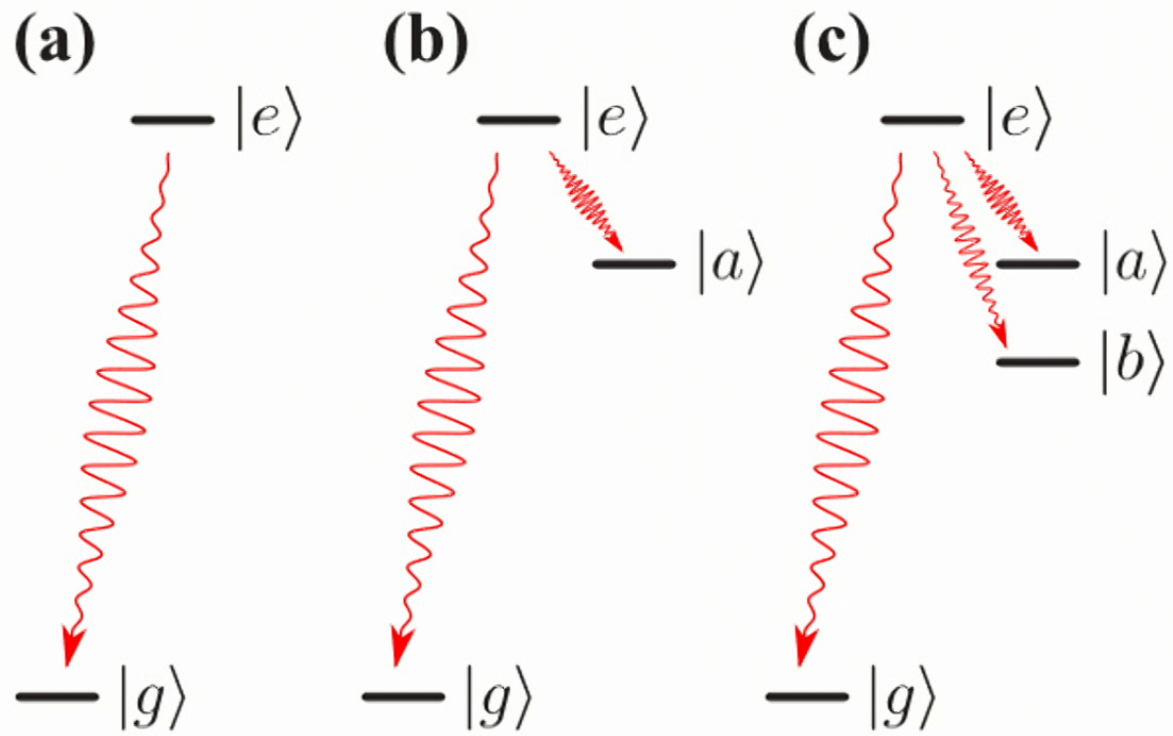
Superradiance: many atoms emit photons with rate $\gg N \times$ single atom

Subradiance: many atoms inhibit photon emission

Atom array can give 100% reflection or change propagation direction

Control emission direction using phase relation between atoms

Many others



PHYSICAL REVIEW A **75**, 033802 (2007)

Superradiance in ultracold Rydberg gases

T. Wang,¹ S. F. Yelin,^{1,2} R. Côté,¹ E. E. Eyler,¹ S. M. Farooqi,¹ P. L. Gould,¹ M. Koštrun,^{1,2} D. Tong,¹ and D. Vranceanu³

PHYSICAL REVIEW A **77**, 052712 (2008)

PHYSICAL REVIEW A **93**, 033407 (2016)

Dynamics of low-density ultracold Rydberg gases

Absence of collective decay in a cold Rydberg gas

J. O. Day, E. Brekke, and T. G. Walker^{*}

Tao Zhou, B. G. Richards, and R. R. Jones

PHYSICAL REVIEW A **94**, 032514 (2016)

Fermi's golden rule for N -body systems in a blackbody radiation

Massimo Ostilli¹ and Carlo Presilla^{2,3,*}

PHYSICAL REVIEW A **95**, 033839 (2017)

Superradiance in inverted multilevel atomic clouds

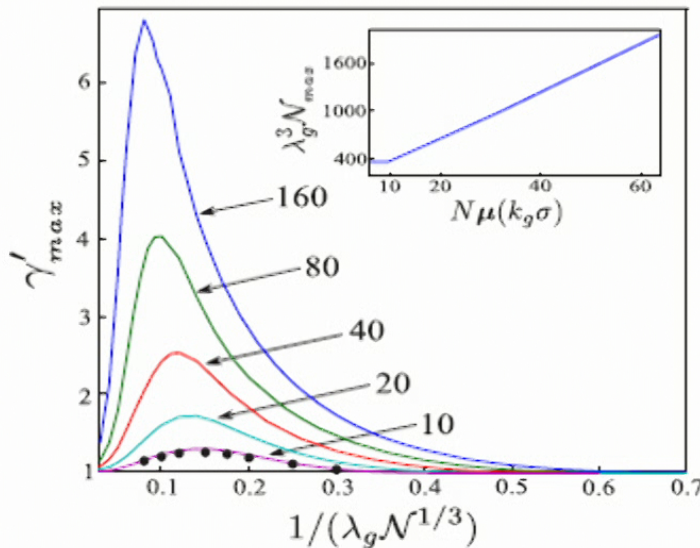
R. T. Sutherland^{1,*} and F. Robicheaux^{1,2,†}

PHYSICAL REVIEW A **96**, 053409 (2017)

Spontaneous avalanche dephasing in large Rydberg ensembles

T. Boulier,^{1,2} E. Magnan,^{1,2} C. Bracamontes,¹ J. Maslek,¹ E. A. Goldschmidt,³ J. T. Young,¹ A. V. Gorshkov,^{1,4} S. L. Rolston,¹
and J. V. Porto^{1,*}

Optimum density: 2 state model



Maximum decay rate vs.
 $\sim \text{avg}(\text{separation})/\lambda$

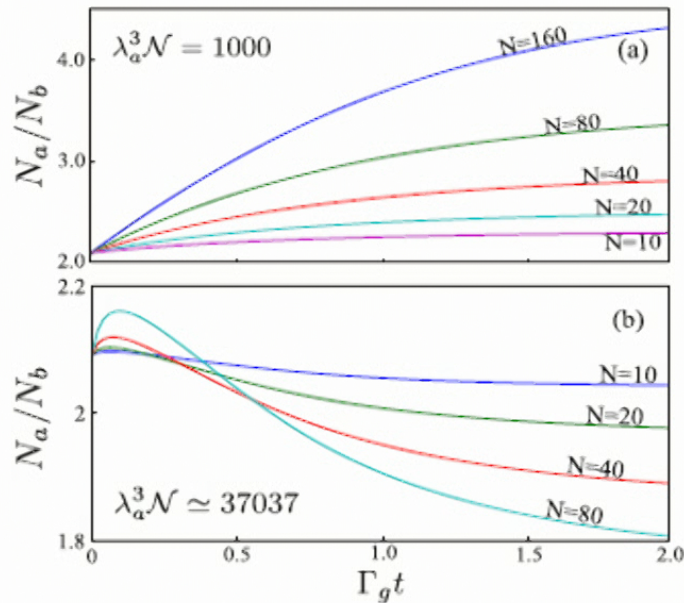
Find the maximum photon
emission rate during the time
dependent decay

Scaled emission rate: divide by N
and Γ

More atoms give larger decay rate, but not linear in N

Best density of atoms increases with number of atoms

Branching ratio: 4 state model



Decay to a, b, and g

Think of 26p decaying to 26s, 25s, or 5s (rates: 169, 81, 3500 s⁻¹)

At low density, superradiance causes more to go into 26s than from ratio of rates

Dephasing suppresses decay into 26s relative to 25s at higher density (random spacings of atoms)

Most population still decay to ground state

Early time Superradiance for Atom Arrays

Use early time properties of decay to determine superradiance


Inspired by [Stuart J. Masson and Ana Asenjo-Garcia](#), “Universality of Dicke superradiance in arrays of quantum emitters,” *Nat Comm* 13, 1 (2022). [arXiv:2106.02042 \(2021\)](#).

Apply to various arrays and random atom cloud

Similar results to [E. Sierra, S.J. Masson, and A. Asenjo-Garcia](#), “Dicke superradiance in ordered lattices: dimensionality matters,” *Phys. Rev. Res.* 4 023207 (2022). [arXiv:2110.08380 \(2021\)](#)

PHYSICAL REVIEW A 104, 063706 (2021)

Theoretical study of early-time superradiance for atom clouds and arrays

F. Robicheaux 

Masson & Asenjo-Garcia: Use the $t=0$ properties to predict superradiance: $g^{(2)}(0) > 1$ (variance in decay eigenvalues)

Equivalent to slope of photon emission rate > 0 at $t=0$

$$\dot{\gamma}(0) = -N\Gamma^2 + \sum_{n,m \neq n} \Gamma_{mn} \Gamma_{nm} = -2N\Gamma^2 + \text{Tr}[\underline{\Gamma} \underline{\Gamma}].$$

Photon emission in particular directions k_f gives

$$\begin{aligned} \dot{\gamma}(0, k_f) &= -2N\Gamma^2 + \Gamma \sum_{mn} \Gamma_{mn} \cos \varphi_{nm} \\ &= -2N\Gamma^2 + \Gamma \text{Tr}[\underline{\Gamma} \underline{\cos \varphi}], \end{aligned}$$

Can have directional superradiance even when there isn't total superradiance

Directional photoemission rate

The directional photoemission rate depends on correlations within the atom cloud

Scale out the $N \Gamma$ part

$$\bar{\gamma}(t, \mathbf{k}) = \frac{1}{N} \sum_n \left[\langle \hat{e}_n \rangle(t) + \sum_{m \neq n} e^{i\mathbf{k} \cdot (\mathbf{R}_m - \mathbf{R}_n)} \langle \hat{\sigma}_m^+ \hat{\sigma}_n^- \rangle(t) \right]$$

First term from average number of excitations

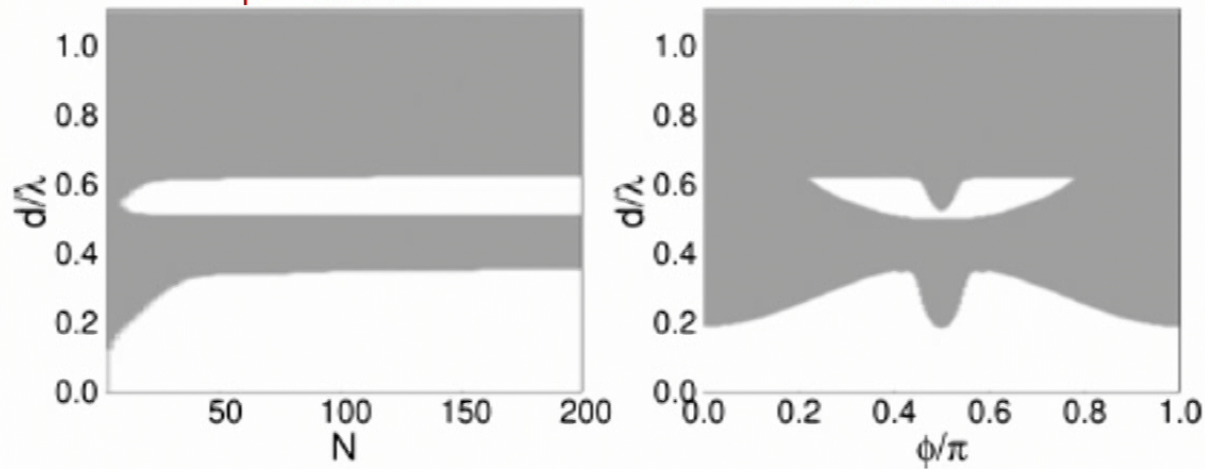
Second term is from entanglement between atom excitations

1 Line, Directional decay

$$\phi = 0.4 \pi$$

$$N = 100$$

$d = \text{spacing of atoms}$



Left plot for emission into 0.4π , Right plot for fixed number

White indicates superradiant region

For left plot the upper region only exists for $N > 8$

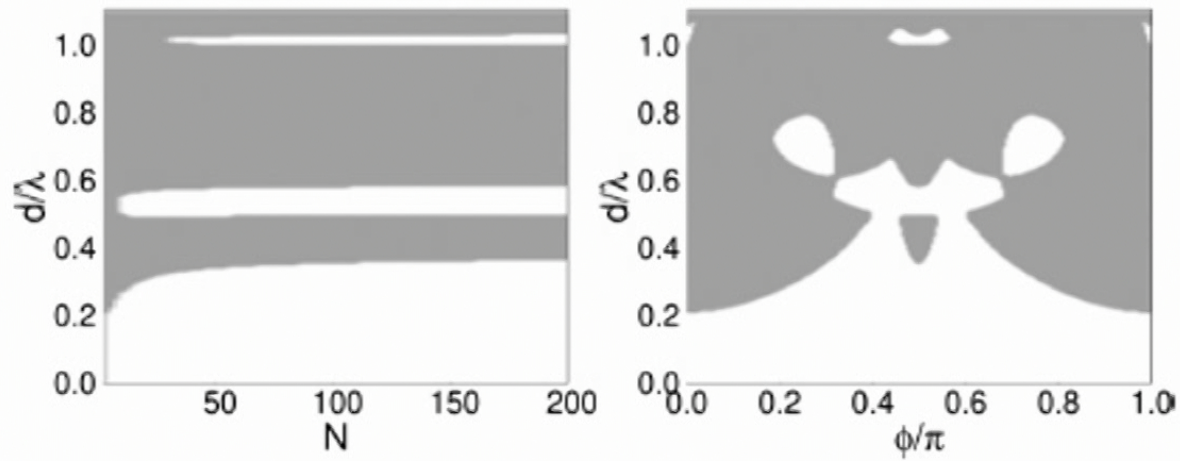
Fairly quick convergence with respect to atom number

2 Line, Directional decay

$$\phi = 0.5 \pi$$

$$N = 100$$

d = spacing of atoms



Left plot for emission into 0.5π , Right plot for fixed number

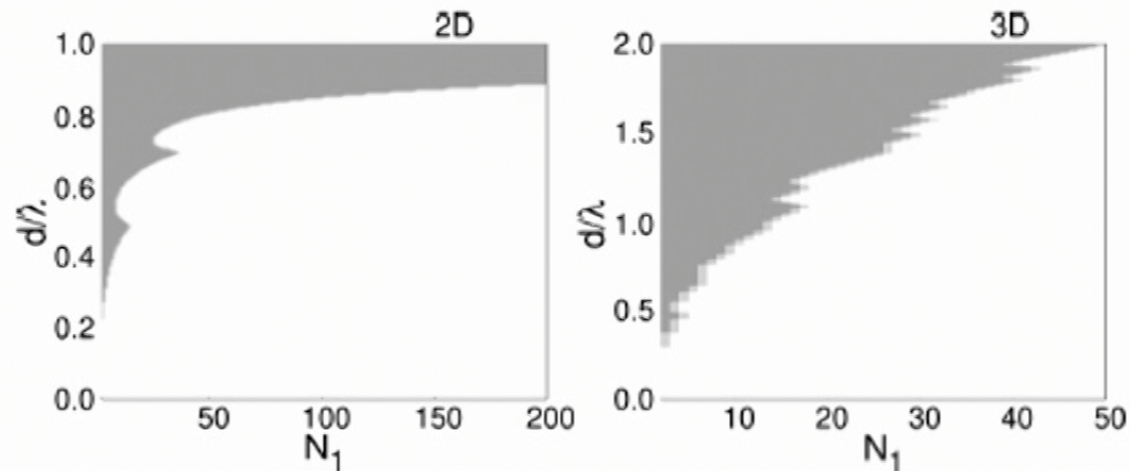
For left plot the middle (upper) region only exists for $N > 8$ (28)

Upper region is superradiant for $d > \lambda$

Dipole moment perp to pair of lines, photon direction perp to dipole moment

Total decay

$d = \text{spacing of atoms}$



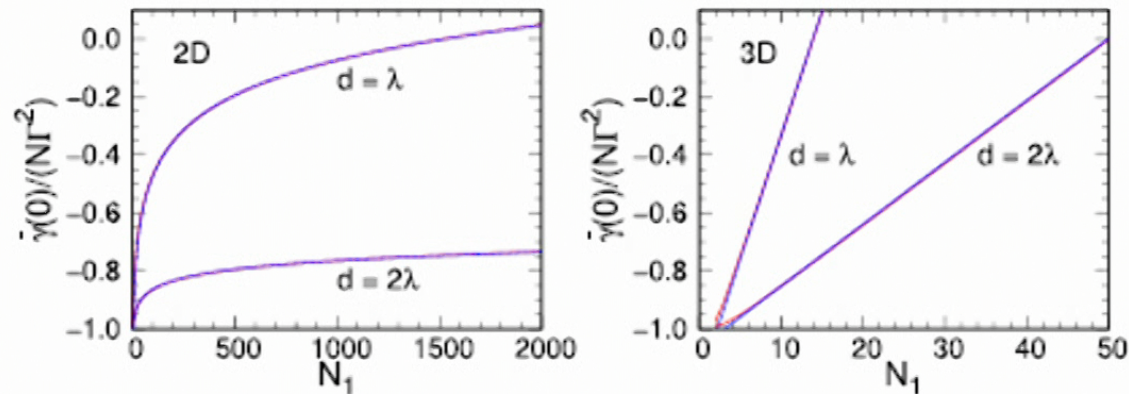
Left plot for a square array, Right plot for a cubic array

Dipole moment in z

For square array $\max(d/\lambda) \sim C + D \sqrt{\ln(N_1)}$

For cubic array $\max(d/\lambda) \sim C + D \sqrt{N_1}$

Asymptotic: Total rate



For square array: $\frac{\dot{\gamma}(0)}{N\Gamma^2} \sim C + D\frac{\lambda^2}{d^2} \ln N_1$ cubic: $\frac{\dot{\gamma}(0)}{N\Gamma^2} \sim C + D\frac{\lambda^2}{d^2} N_1$


The slope of photon emission rate increases with increasing number of atoms

Each line is 2 lines: full calculation and asymptotic form

Asymptotic form comes from dimensionality and number of atoms per area (volume)

PHYSICAL REVIEW A 104, 063706 (2021)

Theoretical study of early-time superradiance for atom clouds and arrays

F. Robicheaux ^{*}

What happens if there are many possible final states?

Rate at $t=0$ is simple:

$$\gamma_{\alpha_f}(0) = N\Gamma_{\alpha_f}$$

Initial slope of the photon emission rate has similar form as two state but needs more atoms to go positive because 1st term is relatively larger

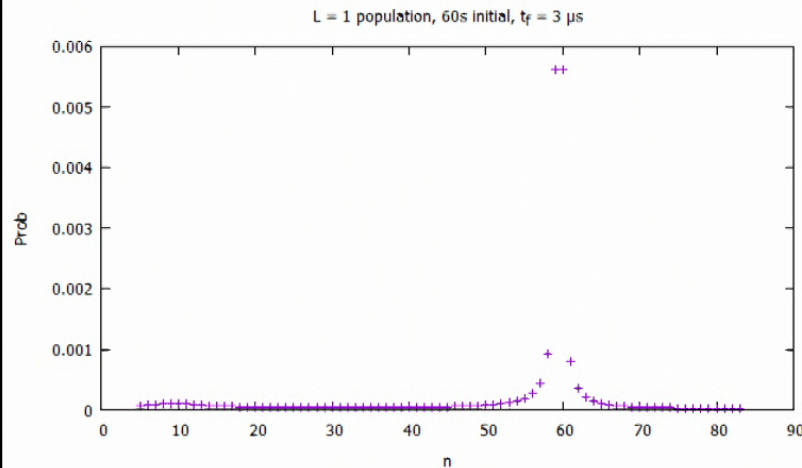
$$\dot{\gamma}_{\alpha_f}(0) = -N\Gamma_{\alpha_f}\Gamma + \sum_n \sum_{m \neq n} \frac{1}{9} \sum_{ii'} (\Gamma_{nm}^{\alpha_f ii'})^2$$
$$\Gamma_{nm}^{\alpha_f ii'} = \Gamma_{\alpha_f} \left[j_0(k_{\alpha_f} R) + \frac{3\hat{R}_i \hat{R}_{i'} - 1}{2} j_2(k_{\alpha_f} R) \right]$$

1st term proportional to N , 2nd term proportional to N^2 so always “superradiant” in 3D for enough atoms, but ...

$60P_{3/2}$ can decay to $nS_{1/2}$, $nD_{3/2}$, $nD_{5/2}$ means need a lot more

Dephasing will suppress the peak rate

Rb 60s, 300 K blackbody



$$\Gamma = 9800 \text{ s}^{-1}$$

Population in various p-levels

$t_f = 3 \mu\text{s}$ (approx. 3% decay)

Most population in nearby states because larger dipole and larger BB

Final population in 60s = 0.971

Superradiance?

For large N, atom cloud collectively absorbs/emits BB

Concluding Remarks

Early time behavior of photon emission rate can determine whether “superradiant” but not the peak rate

Atom arrays give less dephasing: implications for Rydberg arrays?

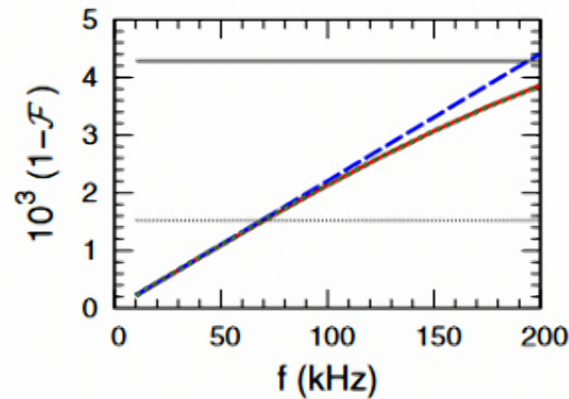
For 2 states, the initial photon emission slope increases with number of atoms (doesn't converge) for 2D & 3D

Photon-recoil and laser-focusing limits to Rydberg gate fidelity

F. Robicheaux,^{1,2,3,*} T. M. Graham,³ and M. Saffman^{3,4}

Role of photon momentum for entangling COM degrees of freedom with internal states leading to loss of fidelity

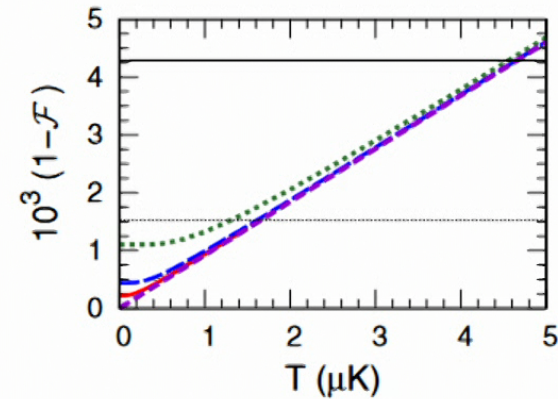
$\pi - 2\pi - \pi C_z$ gate ($\sim 1 \mu\text{s}$ between π pulses leads to Δx of COM)



$T = 0 \text{ K}$

Red (full density matrix), blue (impulse approx.), green (better impulse)

Horizontal lines – Ryd lifetime



Red (10 kHz trap), blue (20 kHz trap), green (50 kHz trap)

PHYSICAL REVIEW A **99**, 013410 (2019)

Atom recoil during coherent light scattering from many atoms

F. Robicheaux^{1,2,*} and Shihua Huang¹

PHYSICAL REVIEW A **103**, 043722 (2021)

Photon-induced atom recoil in collectively interacting planar arrays

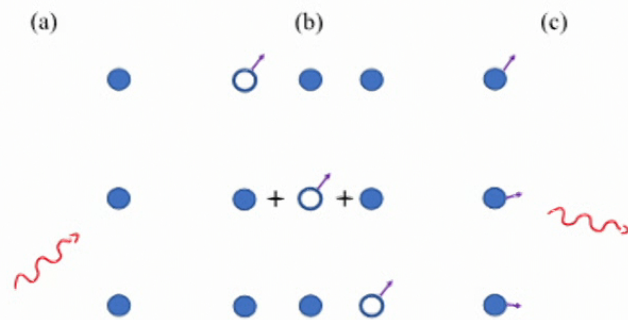
Deepak A. Suresh¹ and F. Robicheaux^{1,2,*}

PHYSICAL REVIEW A **105**, 033706 (2022)

Atom recoil in collectively interacting dipoles using quantized vibrational states

Deepak A. Suresh¹ and F. Robicheaux^{1,2,*}

Basic idea



(a) Ground state atoms initially “at rest” with an incoming photon

(b) Photon is temporarily absorbed by an atom leading to a superposition of excited states with a recoil

(c) Photon emitted leaving ground state atoms but with each (possibly) having different recoil

Highly simplistic picture

Example: interaction between excited and ground state atoms

Is recoil in 1 atom but with probability? (1 atom gets all E_r)

Is recoil spread over atoms? (Many atoms get less than E_r)

Calculation

Two state approximation for each atom's internal states

Density matrix for the COM position and internal states

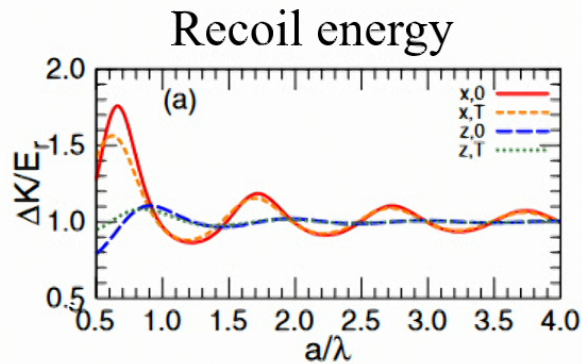
Too many states for practical calculation => weak laser ($N+1$ internal states if only allow up to 1 excitation)

Still too many states: (number of vibrational states)^(number of atoms)

Lifetime internal states \ll COM oscillation period implies the impulse approximation will work well

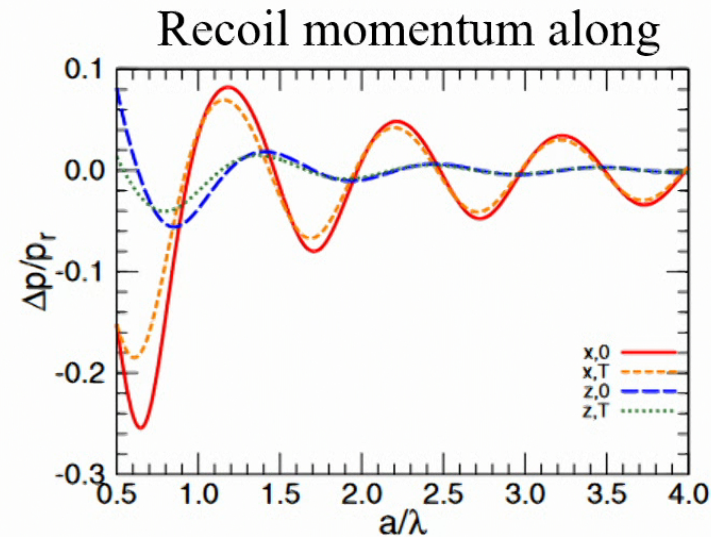
For other limit, use limited number of vibrational states and do full quantum

Two atoms: symmetric excitation



0 = ground state

$T = 1 \mu\text{K}$



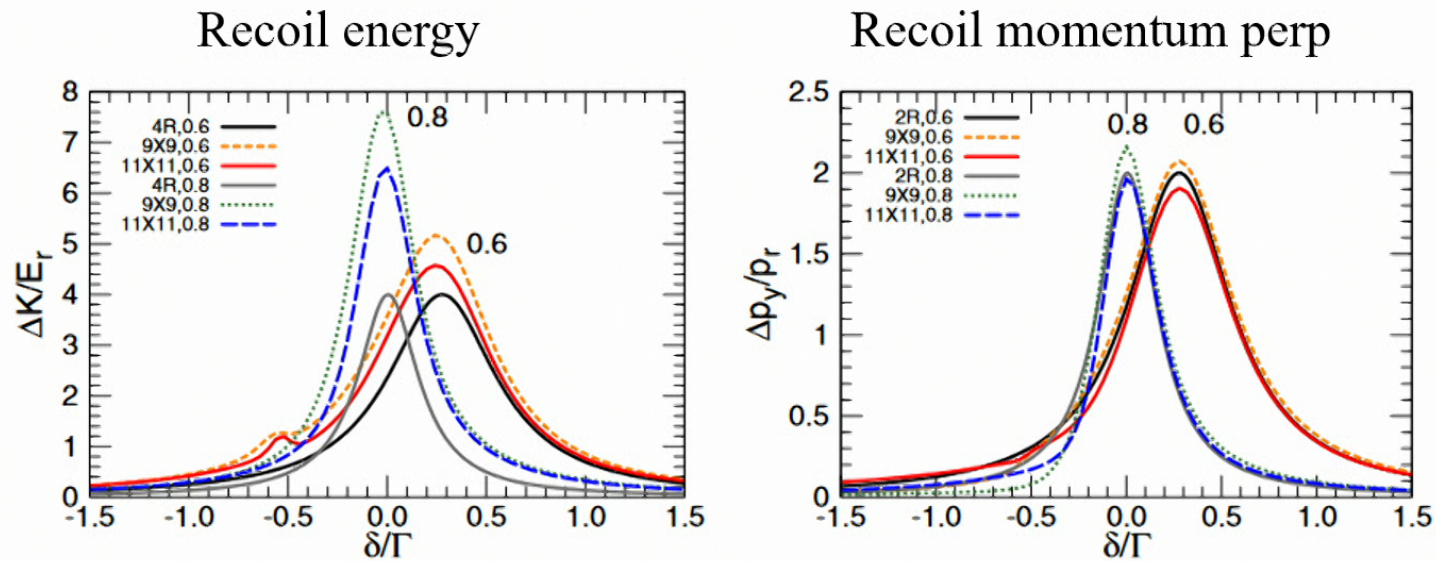
Start with symmetric excitation $|eg\rangle + |ge\rangle$ (no incoming photon)

Total recoil energy is larger than 1 for some separations

Not due to the force between atoms

Average recoil momentum can only be along atom separation

Atom array: recoil of center per photon



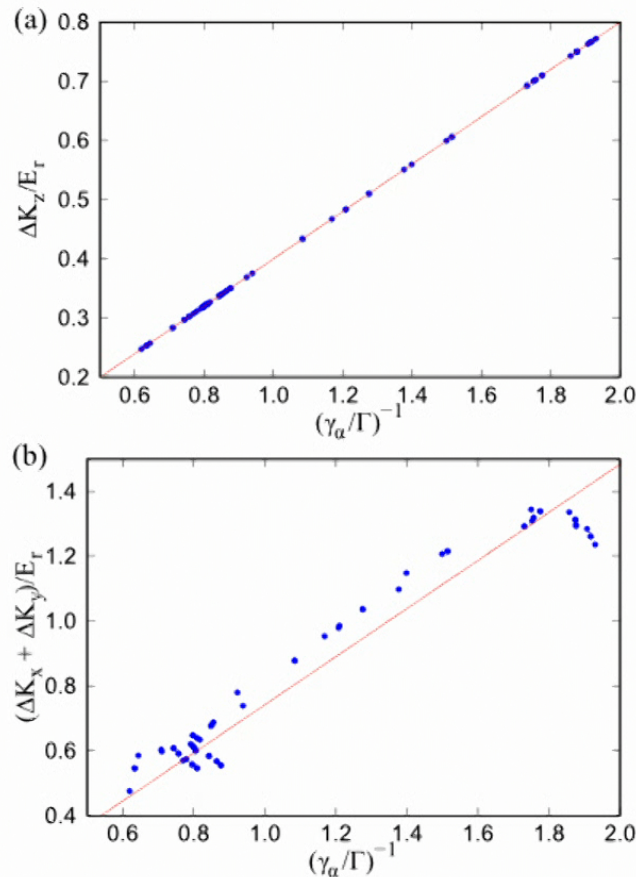
Beam normally incident on atom array with separation 0.6, 0.8 λ

Recoil momentum makes sense: ~ 2 (Reflect Prob) p_r

Recoil energy roughly OK for 0.6 λ but too large for 0.8 λ

What's going on?

8X8 array: start in excitation eigenstate



Start with single excitation
in superposition = eigenstate
of interaction (Green's fct)

Let decay to ground state

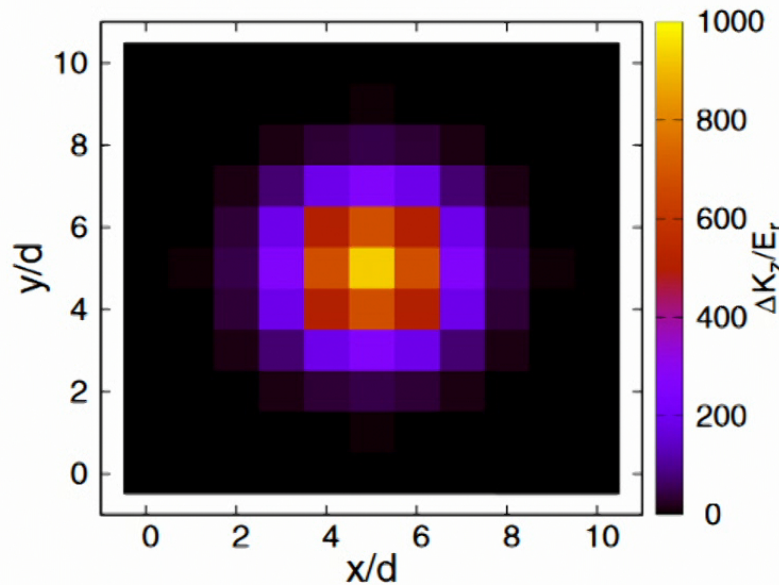
Perpendicular to array recoil
energy is $(2/5) \Gamma/\gamma_a$

In plane recoil is roughly
proportional to lifetime

In plane makes “sense”: forces
between atoms

Perpendicular: no forces
between atoms, only photon
emission

Two 11X11 arrays



Start with single excitation in most subradiant state ($\gamma_\alpha \sim \Gamma/10^4$)

Let decay to ground state

What about impulse approx.?

Total energy change of array $\sim 10,400 E_r$

Center atom: in plane $33 E_r$, out of plane $920 E_r$

Edge atoms hardly recoil

Physical sense? Bouncing photon?

Concluding Remarks

Subradiant states can have large recoil energy per photon

Superradiant states can have less than expected recoil

Amount of recoil could affect utility of atom arrays (most subradiant states are most interesting => largest recoil)

Size of an atom's recoil roughly follows excitation probability (as expected)

What happens when not in the weak laser limit?