Title: Gauge Invariance and Finite Counterterms in Chiral Gauge Theories

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Series: Particle Physics

Date: July 12, 2022 - 1:00 PM

URL: https://pirsa.org/22070001

Abstract: Any consistent regularization scheme induces an apparent violation of gauge invariance in non-anomalous chiral gauge theories. This violation shows up in perturbative calculations, and can be removed by including appropriate finite counterterms. In this talk I will discuss the derivation of such counterterms for a renormalizable chiral gauge theory. I will use the background field method, which ensures background gauge invariance in the quantized theory. As a concrete application, I will show the finite counterterm at one loop in the Standard Model, within dimensional regularization and the Breitenlohner-Maison-'t Hooft-Veltman prescription for gamma5.

Zoom Link: https://pitp.zoom.us/j/95507223984?pwd=Y0FDTEJpNDlVaE9Ba1ZNMURXeS9rdz09

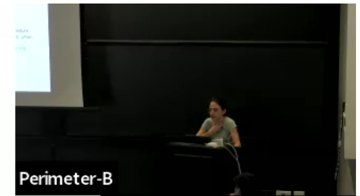
Introduction

Chiral gauge theories are everywhere (SM, SMEFT…..).
Their quantization and renormalization are well understood!

Gauge invariance is their defining feature.
However, it is explicitly broken in practical calculations:

- by gauge fixing
- by **regularization**

Introduction



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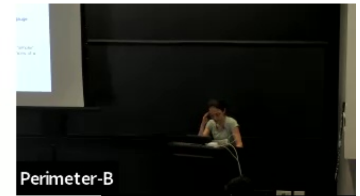
Their quantization and renormalization are well understood!

Gauge invariance is their defining feature.
However, it is explicitly broken in practical calculations:

- by gauge fixing
- by **regularization**

Problem: for chiral gauge theories there is no consistent regularization procedure preserving gauge invariance, even when the field content is anomaly-free, i.e. when:

$$D^{abc} = \mathrm{tr}(T_L^a\{T_L^b, T_L^c\}) - \mathrm{tr}(T_R^a\{T_R^b, T_R^c\}) = 0\,.$$ [Georgi-Glashow (1972)]



Introduction

Two possible strategies:

- enforce gauge invariance by hand, process by process

  Easier, but
  - need to identify the corrrect Ward Identities case by case
  - possible ambiguities when comparing different processes

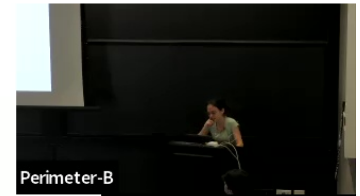
- enforce gauge invariance of the effective action $\Gamma[\phi]$ (generator of all 1PI graphs)

  .....all amplitudes handled at once!

Here: we pursue this for a (strictly) renormalizable theory with arbitrary gauge group and chiral fermion representations.

Remarks:

- we wanted to consider directly the SMEFT, but we realized that this "simpler", prerequisite case had only been partially treated in the literature (only fermions of a single chirality, not directly applicable to the SM)
- Scalars and higher dimensional operators are left for the future.



The theory

Very simple setup:

- Compact gauge group $\mathscr{G}$, gauge fields A_μ^a $(a = 1\ldots\text{dim}\mathscr{G})$, structure constants f_{abc},
- Massless chiral fermions f_L, f_R transforming under reps of $\mathscr{G}$ generated by T_L, T_R.

Classical action: $$S[A, f_X, \bar{f}_X] = \int d^4x\,(\mathscr{L}_{\text{YM}} + \mathscr{L}_{\text{Fermions}}) \qquad X = L, R$$

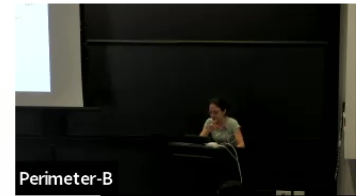
$$\mathscr{L}_{\text{YM}} = -\frac{1}{4\,G_{ab}} F^a_{\mu\nu} F^{b\mu\nu}\,, \qquad \frac{1}{G_{ab}} = \sum_G \frac{\delta_G^{ab}}{g_G^2}$$

$$\mathscr{L}_{\text{Fermions}} = \bar{f}_L i\gamma^\mu D_\mu f_L + \bar{f}_R i\gamma^\mu D_\mu f_R$$

Invariant under the infinitesimal gauge transformations

$$\delta_\alpha A_{a\mu} = \partial_\mu \alpha_a + f_{abc}\alpha_b A_{c\mu} \qquad \delta_\alpha f_X = -\,i\alpha_a T^a_X f_X$$

Ward Identities and Wess-Zumino condition



Ward identity expresses the gauge invariance of a functional $F[A, f_X, \bar{f}_X]$

infinitesimal gauge parameter

$$\delta_\alpha F[A, f_X, \bar{f}_X] = \int d^4x \; \boxed{\alpha_a(x)} \boxed{L_a(x)} \; F$$

differential operator:

$$L_a(x) = -\partial_\mu \frac{\delta}{\delta A_{a\mu}(x)} + f_{abc} A_{b\mu}(x) \frac{\delta}{\delta A_{c\mu}(x)} + \sum_{X=L,R} -i \frac{\overleftarrow{\delta}}{\delta f_X(x)} T^a_X f_X(x) + i \bar{f}_X(x) T^a_X \frac{\delta}{\delta \bar{f}_X(x)} \, .$$

$F[A, f_X, \bar{f}_X]$ gauge invariant $\longleftrightarrow \delta_\alpha F = 0 \longleftrightarrow$ $L_a(x) F = 0$ Ward identity for F

Wess-Zumino consistency conditions

$$\left[L_a(y), L_b(x)\right] F[A, f_X, \bar{f}_X] = -\delta^{(4)}(x-y) f_{abc} \, L_c(x) F[A, f_X, \bar{f}_X] \, .$$

- follow from the algebra of $\mathscr{G}$ and hold for any $F[A, f_X, \bar{f}_X]$.
- Reduce to 0=0 if F is gauge invariant, non-trivial constraint otherwise.

CP and P

- CP is a symmetry of our theory.
- P is not, unless the theory is vector-like ($T^a_L = T^a_R$).
  Still, we can define a generalized, “spurious” P symmetry.

Introduce both P and CP as spurious transformations:

$$
\begin{aligned}
&x^\mu \xrightarrow{\text{CP}} x_\mu\,, && x^\mu \xrightarrow{\text{P}} x^\mu_P = x_\mu\,,\\
&\partial_\mu \xrightarrow{\text{CP}} \partial^\mu\,, && \partial_\mu \xrightarrow{\text{P}} \partial^\mu\,,\\
&A_{a\mu}(x) \xrightarrow{\text{CP}} -A^\mu_a(x_P)\,, && A_{a\mu}(x) \xrightarrow{\text{P}} A^\mu_a(x_P)\,,\\
&f_{L,R}(x) \xrightarrow{\text{CP}} C f^*_{L,R}(x_P)\,, && f_{L,R}(x) \xrightarrow{\text{P}} \gamma^0 f_{R,L}(x_P)\,,\\
&T^a_{L(R)} \xrightarrow{\text{CP}} T^{aT}_{L(R)}\,, && T^a_{L(R)} \xrightarrow{\text{P}} T^a_{R(L)}\,.
\end{aligned}
$$

→ P and CP are formally symmetries of $S[A, f_X, \bar{f}_X]$.

By choosing a regularization preserving these symmetries, we can use them to restrict the structure of the WI-restoring counterterms.

Goal and plan

Our goal is to remove the unphysical gauge-violating contributions induced by regularization by adding appropriate local counterterms to the classical action $S[\phi]$.

In particular, we want to:

- characterize these counterterms at 1 loop for a wide class of regularization schemes, using functional methods
- derive explicit expressions in Dimensional Regularization (DimReg)

Starting from $S[\phi]$, we need to:

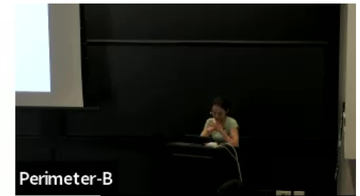
I. Regularize $S[\phi] \to S^{\text{reg}}[\phi]$

II. Quantize $S^{\text{reg}}[\phi] \to \Gamma^{\text{reg}}[\phi]$

III. Renormalize $\Gamma^{\text{reg}}[\phi] \to \Gamma[\phi]$

IV. Restore gauge invariance $\Gamma[\phi] \to \Gamma[\phi] + S_{\text{ct}}[\phi] \;\mid\; L_a(\Gamma[\phi] + S_{\text{ct}}[\phi]) = 0$

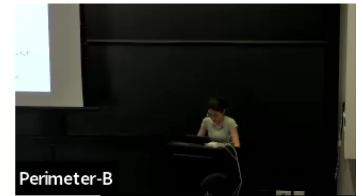
Regularize: $S[\phi] \to S^{\text{reg}}[\phi]$



We choose a regularization statisfying these Properties:

- invariance under **vectorial gauge** transformations
- invariance under 4-dimensional **Lorentz**
- invariance under generalized **P,CP**
- **quantum action principle** (q.a.p.)

Quantize: $S^{\text{reg}}[\phi] \to \Gamma^{\text{reg}}[\phi]$



* at intermediate steps!

Problem: when quantizing a non-abelian theory, gauge invariance is lost* to BRST.

→ $\Gamma[\phi]$ obeys non-linear Slavnov-Taylor identities instead of linear Ward Identities.

→ finding the counterterm is more complicated.

[Martin-Sanchez Ruiz (2000), Sanchez Ruiz (2003), Stöckinger et al (2020)]

The solution is the **Background Field Method:**

1. split fields in classical background + quantum fluctuation:

$$\phi \to \phi + \tilde{\phi}\,, \qquad S^{\text{reg}}[\phi] \to S^{\text{reg}}[\phi + \tilde{\phi}]$$

2. integrate over quantum fluctuations:

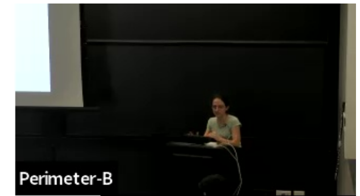
$$e^{i\Gamma^{\text{reg}}[\phi]} = \int_{1\text{PI}} \mathscr{D}\tilde{\phi}\; e^{\,i\, S^{\text{reg}}_{\text{full}}\,[\phi+\tilde{\phi}]} \qquad S^{\text{reg}}_{\text{full}} \equiv S^{\text{reg}} + S^{\text{reg}}_{\text{g.f.}} + S^{\text{reg}}_{\text{ghost}}$$

3. choose a smart gauge fixing: $\mathscr{L}_{\text{g.f.}}[\phi + \tilde{\phi}] = -\dfrac{1}{2\xi} f_a f_a \qquad f_a = \partial_\mu \tilde{A}^\mu_a - f_{abc} A_{b\mu} \tilde{A}^\mu_c$

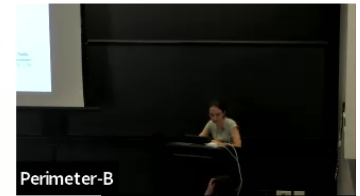
⇒ $S[\phi + \tilde{\phi}]$ is invariant under gauge transformations of the bckg fields,

$\Gamma[\phi]$ satisfies the linear Ward Identity (up to our regularization effects!)

Renormalize: $\Gamma^{\mathrm{reg}}[\phi] \to \Gamma[\phi]$



We assume the existence of a **consistent subtraction procedure** allowing to evaluate the renormalized $\Gamma[\phi]$ from the bare, regularized one, and is such that $\Gamma[\phi]$ is finite order by order in perturbation theory.



Restore gauge invariance

Let's prove that we can always add a finite counterterm $S_{ct}[\phi]$ to $\Gamma[\phi]$ such that the result is gauge invariant, i.e. $L_a(\Gamma[\phi] + S_{ct}[\phi]) = 0$.

Say $\Gamma[\phi]$ satisfies the WI up to loop order $n-1$: $\quad L_a(x)\Gamma[\phi]\Big|_{(k)} = 0 \qquad k \leq n-1\,,$

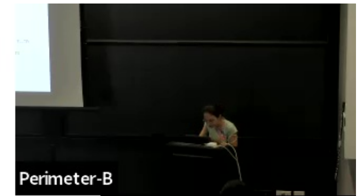
At order n, in general $L_a(x)\Gamma[\phi]\Big|_{(n)} \neq 0\,,$

but the q.a.p. implies

$$L_a(x)\Gamma[\phi]\Big|_{(n)} = (\Delta_a \cdot \Gamma)(x) = \Delta_a(x)\Big|_{(n)} + \mathcal{O}(\hbar^{n+1})$$

gen. functional of the 1PI Green functions with an insertion of $\Delta_a(x)|_{(n)}$

$\mathcal{O}(\hbar^n)$ local polynomial in the fields: Dimension 4, CP odd, P even, invariant under 4d Lorentz, vanishes for TL = TR



$\Delta_a(x)\,|_{(n)}$ must satisfy the **WZ condition**:

$$L_a(y)\Delta_b(x)\Big|_{(n)} - L_b(x)\Delta_a(y)\Big|_{(n)} = -\,\delta^{(4)}(x-y) f_{abc}\,\Delta_c(x)\Big|_{(n)}\,.$$

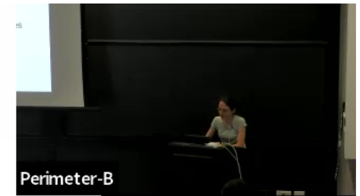
For non-anomalous theories, the solution is

$$\Delta_a(x)\Big|_{(n)} = -\,L_a(x) S_{\rm ct}[\phi]\Big|_{(n)}\,, \qquad S_{\rm ct}[\phi]\Big|_{(n)} = \int d^4y\ \mathscr{L}_{\rm ct}(y)\,|_{(n)}$$

Integrated $\mathcal{O}(\hbar^n)$ local polynomial in $\phi, \partial\phi$
invariant under 4d Lorentz, P, CP, vanishes for TL=TR

Adding this counterterm to the effective action, the result is gauge invariant:

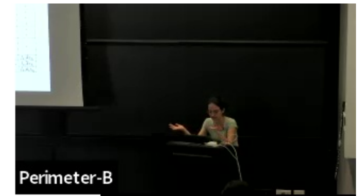
$$\Gamma_{\rm inv}[\phi]\,|_{(n)} = \Gamma[\phi]\,|_{(n)} + S_{\rm ct}[\phi]\,|_{(n)} \qquad\qquad L_a(x)\Gamma_{\rm inv}[\phi]\,|_{(n)} = \mathcal{O}(\hbar^{n+1})\,.$$



The explicit form of $\Delta_a(x)|_{(n)}$ and $S_{\rm ct}[\phi]|_{(n)}$ is regularization dependent.
However, we can deduce several of their features solely from the **Properties***.

One loop analysis
for well behaved* regularization schemes

A basis for $\Delta_a(x)\,|_{(1)}$ and $S_{\rm ct}[\phi]\,|_{(1)}$



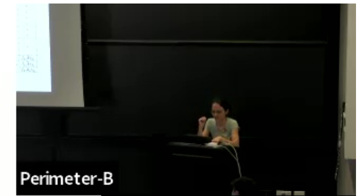
The one-loop gauge variation of $\Gamma[\phi]$ is a finite polynomial of dimension 4 in the fields.

We can write it as:

$$\Delta_a(x)\Big|_{(1)} = \sum_{k=0}^{14} C^k_{aA}\, I^k_A(x)$$

		CP	P
I^0_a	$\Box\partial^\mu A_{a\mu}$	$-$	$+$
I^1_{ab}	$\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_{a\mu})(\partial_\beta A_{b\nu})$	$-$	$-$
I^2_{ab}	$A_{a\mu}(\partial^\mu\partial^\nu - \Box g^{\mu\nu})A_{b\nu}$	$+$	$+$
I^3_{ab}	$A_{a\mu}\Box A^\mu_b$	$+$	$+$
I^4_{ab}	$(\partial_\nu A_{a\mu})(\partial^\nu A^\mu_b)$	$+$	$+$
I^5_{ab}	$(\partial_\nu A_{a\mu})(\partial^\mu A^\nu_b)$	$+$	$+$
I^6_{ab}	$(\partial^\mu A_{a\mu})(\partial^\nu A_{b\nu})$	$+$	$+$
I^7_{abd}	$(\partial_\mu A^\mu_a)A_{b\nu}A^\nu_d$	$-$	$+$
I^8_{abd}	$(\partial_\mu A^\nu_a)A_{b\mu}A^\nu_d$	$-$	$+$
I^9_{abd}	$\epsilon^{\mu\nu\alpha\beta}(\partial_\beta A_{a\mu})A_{b\nu}A_{d\alpha}$	$+$	$-$
I^{10}_{abde}	$A_{a\mu}A^\mu_b A_{d\nu}A^\nu_e$	$+$	$+$
I^{11}_{abde}	$\epsilon^{\mu\nu\rho\sigma}A_{a\mu}A_{b\nu}A_{d\rho}A_{e\sigma}$	$-$	$-$
I^{12}_{Xij}	$\bar{f}_{Xi}\overrightarrow{\not\partial} f_{Xj}$	$-\bar{f}_{Xj}\overleftarrow{\not\partial} f_{Xi}$	$\bar{f}_{\tilde{X}i}\overrightarrow{\not\partial} f_{\tilde{X}j}$
I^{13}_{Xij}	$\bar{f}_{Xi}\overleftarrow{\not\partial} f_{Xj}$	$-\bar{f}_{Xj}\overrightarrow{\not\partial} f_{Xi}$	$\bar{f}_{\tilde{X}i}\overleftarrow{\not\partial} f_{\tilde{X}j}$
I^{14}_{Xaij}	$\bar{f}_{Xi}\not{A}_a f_{Xj}$	$+\bar{f}_{Xj}\not{A}_a f_{Xi}$	$\bar{f}_{\tilde{X}i}\not{A}_a f_{\tilde{X}j}$

A basis for $\Delta_a(x)\,|_{(1)}$ and $S_{\rm ct}[\phi]\,|_{(1)}$



The one-loop gauge variation of $\Gamma[\phi]$ is a finite polynomial of dimension 4 in the fields.

We can write it as:

$$\Delta_a(x)\Big|_{(1)} = \sum_{k=0}^{14} C^k_{aA} I^k_A(x)$$

monomials

Coefficients with definite symmetry properties

$\Delta_a(x)\,|_{(1)}$ must satisfy the WZ condition
→ (many) relations between the C^k_{aA}:

$$\mathrm{WZ}[C^k_{aA}] = 0$$

		CP	P
I^0_a	$\Box\partial^\mu A_{a\mu}$	$-$	$+$
I^1_{ab}	$\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_{a\mu})(\partial_\beta A_{b\nu})$	$-$	$-$
I^2_{ab}	$A_{a\mu}(\partial^\mu\partial^\nu - \Box g^{\mu\nu})A_{b\nu}$	$+$	$+$
I^3_{ab}	$A_{a\mu}\Box A^\mu_b$	$+$	$+$
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I^7_{abd}	$(\partial_\mu A^\mu_a)A_{b\nu}A^\nu_d$	$-$	$+$
I^8_{abd}	$(\partial_\mu A^\nu_a)A_{b\mu}A^\nu_d$	$-$	$+$
I^9_{abd}	$\epsilon^{\mu\nu\alpha\beta}(\partial_\beta A_{a\mu})A_{b\nu}A_{d\alpha}$	$+$	$-$
I^{10}_{abde}	$A_{a\mu}A^\mu_b A_{d\nu}A^\nu_e$	$+$	$+$
I^{11}_{abde}	$\epsilon^{\mu\nu\rho\sigma}A_{a\mu}A_{b\nu}A_{d\rho}A_{e\sigma}$	$-$	$-$
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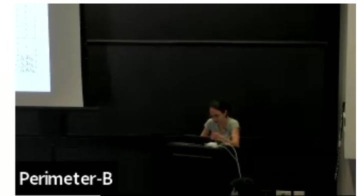
A basis for $\Delta_a(x)|_{(1)}$ and $S_{\rm ct}[\phi]|_{(1)}$



The one-loop gauge variation of $\Gamma[\phi]$ is a finite polynomial of dimension 4 in the fields.

We can write it as:

$$f_{pde}C^1_{c(eb)} + f_{cde}C^1_{p(eb)} + C^9_{pb[cd]} + C^9_{cb[pd]} = 0$$
$$(2f_{pde}C^1_{ceb} + 2C^9_{pb[cd]})\big|_{\text{symm. in bd}} = -f_{pce}C^1_{e(db)}$$
$$(4C^{11}_{c[pbdf]} + 4C^{11}_{p[cbdf]} + f_{pde}C^9_{ce[bf]} + f_{cde}C^9_{pe[bf]})\big|_{\text{antisymm. in bdf}} = 0$$
$$(2f_{pfe}C^9_{cb[ed]} - f_{cfe}C^9_{pe[bd]} - f_{cde}C^9_{pe[fb]} - 2f_{cfe}C^9_{pb[ed]})\big|_{\text{antisymm. in df}} + 12C^{11}_{p[cbdf]} + f_{pbe}C^9_{ce[fd]} = -f_{pce}C^9_{eb[fd]}$$
$$(4f_{phb}C^{11}_{c[badf]} - 4f_{chb}C^{11}_{p[badf]})\big|_{\text{antisymm. in adfh}} = -f_{pce}C^{11}_{e[afdh]}$$

		CP	P
	$\Box\partial^\mu A_{a\mu}$	$-$	$+$
	$\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_{a\mu})(\partial_\beta A_{b\nu})$	$-$	$-$
	$A_{a\mu}(\partial^\mu\partial^\nu - \Box g^{\mu\nu})A_{b\nu}$	$+$	$+$
	$A_{a\mu}\Box A^\mu_b$	$+$	$+$
	$(\partial_\nu A_{a\mu})(\partial^\nu A^\mu_b)$	$+$	$+$
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I^8_{abd}	$(\partial_\mu A^\nu_a)A_{b\mu}A^\nu_d$	$-$	$+$
I^9_{abd}	$\epsilon^{\mu\nu\alpha\beta}(\partial_\beta A_{a\mu})A_{b\nu}A_{d\alpha}$	$+$	$-$
I^{10}_{abde}	$A_{a\mu}A^\mu_b A_{d\nu}A^\nu_e$	$+$	$+$
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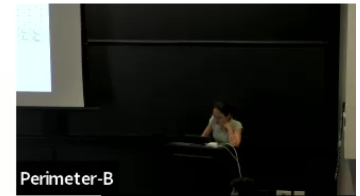
→ (many) relations between the C^k_{aA}:

$$\mathrm{WZ}[C^k_{aA}] = 0$$

A basis for $\Delta_a(x)|_{(1)}$ and $S_{\rm ct}[\phi]|_{(1)}$



Analogously:

$$S_{\rm ct}[\phi]|_{(1)} = \int d^4y\, \mathscr{L}_{\rm ct}(y)$$

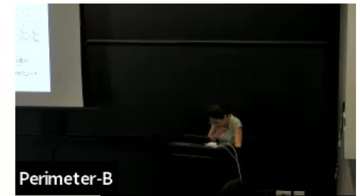
$$= \int d^4y \sum_{j=1}^{8} \xi^j_B\, \mathscr{I}^j_B(y)$$

monomials

coefficients
(with definite symmetry properties)

monomial	explicit expression	coefficient	CP	P
$\mathcal{I}^1_{ghl}$	$(\partial^\nu A^\mu_g) A_{h\nu} A_{l\mu}$	ξ^1_{ghl}	$-$	$+$
$\mathcal{I}^2_{gh}$	$A_{g\mu} \Box A^\mu_h$	ξ^2_{gh}	$+$	$+$
$\mathcal{I}^3_{gh}$	$A_{g\mu}\partial^\mu\partial^\nu A_{h\nu}$	ξ^3_{gh}	$+$	$+$
$\mathcal{I}^4_{ghl}$	$\epsilon^{\mu\nu\rho\sigma} A_{g\mu} A_{h\nu} (\partial_\rho A_{l\sigma})$	$\xi^4_{[gh]l}$	$+$	$-$
$\mathcal{I}^5_{ghlm}$	$\epsilon^{\mu\nu\rho\sigma} A_{g\mu} A_{h\nu}\ A_{l\rho} A_{m\sigma}$	$\xi^5_{[ghlm]}$	$-$	$-$
$\mathcal{I}^6_{ghlm}$	$A_{g\mu} A^\mu_h\ A_{l\nu} A^\nu_m$	$\xi^6_{(gh)(lm)}$	$+$	$+$
$\mathcal{I}^7_{Xij}$	$\bar{f}_{Xi} \overleftrightarrow{\not{\partial}} f_{Xj}$	ξ^7_{Xij}	ξ^7_{Xji}	$\xi^7_{\tilde{X}ij}$
$\mathcal{I}^8_{Xaij}$	$\bar{f}_{Xi} \not{A}_a f_{Xj}$	ξ^8_{Xaij}	ξ^8_{Xaji}	$\xi^8_{\tilde{X}aij}$

A basis for $\Delta_a(x)|_{(1)}$ and $S_{\rm ct}[\phi]|_{(1)}$



Analogously:

$$S_{\rm ct}[\phi]|_{(1)} = \int d^4y\ \mathscr{L}_{\rm ct}(y) = \int d^4y \sum_{j=1}^{8} \xi^j_B\, \mathscr{I}^j_B(y)$$

monomials

coefficients (with definite symmetry properties)

monomial	explicit expression	coefficient	CP	P
$\mathcal{I}^1_{ghl}$	$(\partial^\nu A^\mu_g)A_{h\nu}A_{l\mu}$	ξ^1_{ghl}	$-$	$+$
$\mathcal{I}^2_{gh}$	$A_{g\mu}\Box A^\mu_h$	ξ^2_{gh}	$+$	$+$
$\mathcal{I}^3_{gh}$	$A_{g\mu}\partial^\mu\partial^\nu A_{h\nu}$	ξ^3_{gh}	$+$	$+$
$\mathcal{I}^4_{ghl}$	$\epsilon^{\mu\nu\rho\sigma}A_{g\mu}A_{h\nu}(\partial_\rho A_{l\sigma})$	$\xi^4_{[gh]l}$	$+$	$-$
$\mathcal{I}^5_{ghlm}$	$\epsilon^{\mu\nu\rho\sigma}A_{g\mu}A_{h\nu}\ A_{l\rho}A_{m\sigma}$	$\xi^5_{[ghlm]}$	$-$	$-$
$\mathcal{I}^6_{ghlm}$	$A_{g\mu}A^\mu_h\ A_{l\nu}A^\nu_m$	$\xi^6_{(gh)(lm)}$	$+$	$+$
$\mathcal{I}^7_{Xij}$	$\bar f_{Xi}\overleftrightarrow{\not\partial} f_{Xj}$	ξ^7_{Xij}	ξ^7_{Xji}	$\xi^7_{\bar{X}ij}$
$\mathcal{I}^8_{Xaij}$	$\bar f_{Xi}\not{A}_a f_{Xj}$	ξ^8_{Xaij}	ξ^8_{Xaji}	$\xi^8_{\bar{X}aij}$

Take the gauge variation:

$$L_a(x)S_{\rm ct}[\phi]|_{(1)} = -(\xi^2_{ba}+\xi^2_{ab}+\xi^3_{ba}+\xi^3_{ab})\ I^0_a(x) + 2\xi^4_{[ab]c}I^1_{bc}(x) + \ldots\ldots = \sum_{k=0}^{14}\hat C^k_{aA}(\xi) I^k_A(x)$$

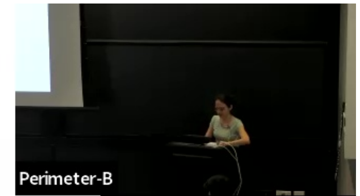
automatically satisfy $\mathrm{WZ}[\hat C^k_{aA}] = 0$

Solving the key equations

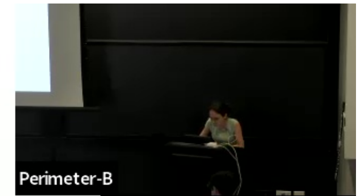
$$\Delta_a(x)\Big|_{(1)} + L_a(x)S_{\rm ct}[\phi]\Big|_{(1)} = \mathscr{A}_a(x)$$

$\updownarrow$

[M] $$\sum_{k=0}^{14}\left[C^k_{aA} + \hat{C}^k_{aA}(\xi)\right] I^k_A(x) = \mathscr{A}_a(x)$$



Solving the key equations



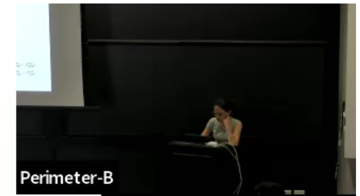
$$\Delta_a(x)\Big|_{(1)} + L_a(x)S_{\text{ct}}[\phi]\Big|_{(1)} = \mathscr{A}_a(x)$$

↕

[M] $$\sum_{k=0}^{14}\left[C^k_{aA} + \hat{C}^k_{aA}(\xi)\right] I^k_A(x) = \mathscr{A}_a(x)$$ & $$\text{WZ}[C^k_{aA}] = 0$$

We need to solve these two sets of equations:

1. Determine the most general form of the C^k_{aA} satisfying Wess Zumino
2. Find the counterterm coefficients ξ^j_B such that [M] is fulfilled.



Solving the key equations

At 1 loop, these tensors are linear combinations of single traces of generators:

$$C^k_{a_1\ldots a_n} = \sum c^k_{X_1\ldots X_n} T^{a_1\ldots a_n}_{X_1\ldots X_n}$$

$$\xi^j_{a_1\ldots a_n} = \sum \chi^j_{X_1\ldots X_n} T^{a_1\ldots a_n}_{X_1\ldots X_n}$$

$$T^{a_1\ldots a_n}_{X_1\ldots X_n} = \mathrm{tr}(T^{a_1}_{X_1}\ldots T^{a_n}_{X_n})$$

ξ

Several constraints restrict their form significantly:

- definite properties under P, CP
- definite symmetry transformations under exchange of indices
- C^k and $\hat{C}^k(\xi)$ must be 0 for vectorlike theories $\left(T^a_L = T^a_R\right)$

Example: $C^1_{a(bc)}$ three indices in $\mathrm{AD}_{\mathscr{G}}$ $\Rightarrow$ T^{abc}_{XYZ} $\quad X, Y, Z = L, R$

CP even and P odd $\Rightarrow$ $T^{abc}_{LLL} - T^{abc}_{RRR},\ T^{abc}_{RLL} - T^{abc}_{LRR}$

symmetric in bc $\Rightarrow$

$$C^1_{a(bc)} = c^1_{LLL}(T^{abc}_{LLL} + T^{acb}_{LLL} - T^{abc}_{RRR} - T^{acb}_{RRR}) + c^1_{RLL}(T^{abc}_{RLL} + T^{acb}_{RLL} - T^{abc}_{LRR} - T^{acb}_{LRR})$$

Solving the key equations

[M] consists of **three independent sets** of equations:

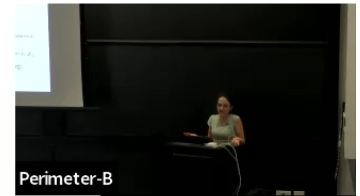
- fermionic
- bosonic, P-even
- bosonic, P-odd

Bosonic, P-even.	$\Delta_a(x)\|_{(1)}$	$S_{\rm ct}[\phi]\|_{(1)}$
# of independent parameters (before WZ)	61	13
- # of independent WZ conditions	−49	-
# of really independent parameters	12	13

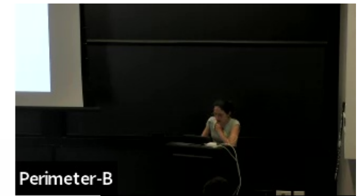
- In this sector we can **always** (even in an anomalous theory!) choose the counterterms in such a way to remove the “spurious” anomaly induced by the regularization

- the residual freedom amounts to the possibility of adding a gauge invariant term to $\mathcal{L}_{\rm ct}$

$$\mathcal{L}_{\rm ct} \supset \chi^2_{LL}\left(\mathcal{I}^2_{ab} - \mathcal{I}^3_{ab} - 2f_{ceb}\mathcal{I}^1_{eca} + \frac{1}{2} f_{dga} f_{ecb}\mathcal{I}^6_{egcd}\right)(T^{ab}_{LL} + T^{ab}_{RR}) = -\frac{\chi^2_{LL}}{2} F^a_{\mu\nu}F^{b\mu\nu}(T^{ab}_{LL} + T^{ab}_{RR})\,.$$



Solving the key equations



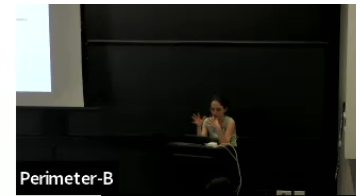
Bosonic, P-odd.	$\Delta_a(x)\,|_{(1)}$	$S_{\rm ct}[\phi]\,|_{(1)}$
# of independent parameters (before WZ)	27	3
- # of independent WZ parameters	-13	$-$
# of really independent parameters	4	3

- We don't have enough freedom to cancel $\Delta_a(x)\,|_{(1)}$ completely.

  We remain with:

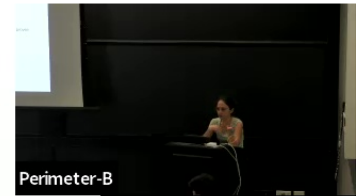
$$\Delta_a(x)\,|_{(1)} + S_{\rm ct}[\phi]\,|_{(1)} = -c^1_{LLL}\epsilon^{\mu\nu\rho\sigma}\partial_\mu\left(A^b_\nu\partial_\rho A^e_\sigma - \frac{i}{4}A^b_\nu A^c_\rho A^d_\sigma(if^{cde})\right)D^{abe} \equiv \mathscr{A}_a$$

- No more freedom in choosing the countertems $\Rightarrow$ $\mathscr{A}_a = 0 \longleftrightarrow D^{abc} = 0$



We found a general map **gauge variation of effective action → counterterm,** valid for any regularization respecting the properties.

Let's pick the most used one: **Dimensional Regularization**
= extend Lorentz indices analytically from $d = 4$ to $d = 4 - 2\epsilon$ complex dimensions.



Chirality in Dimensional Regularization

“The γ_5 problem”

Properties of γ_5 in $d = 4$:

i) $\{\gamma^\mu, \gamma_5\} = 0$

ii) $\mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) = 4i\epsilon^{\mu\nu\rho\sigma}$

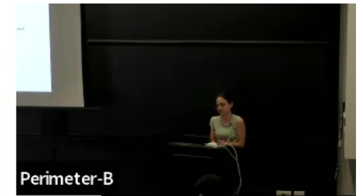
iii) cyclicity of the trace

No definition of γ_5 in $d \neq 4$ obeys all of them.

Several treatments have been proposed to deal with this.

Only the **Breitenlohner-Maison’t Hooft-Veltman (BMHV)** prescription has been proven to be consistent at all orders:

- split indices as: $\mu = \bar{\mu} \oplus \hat{\mu}$ (with $\bar{\mu}$ → 4, $\hat{\mu}$ → $d - 4$)
- γ_5 is an intrinsically 4d object: $\gamma_5 = \dfrac{i}{4!}\epsilon_{\bar{\mu}\bar{\nu}\bar{\alpha}\bar{\beta}}\gamma^{\bar{\mu}}\gamma^{\bar{\nu}}\gamma^{\bar{\alpha}}\gamma^{\bar{\beta}}$
- $\gamma_\mu = \gamma_{\bar{\mu}} + \gamma_{\hat{\mu}}$ $\quad \begin{cases} \{\gamma_{\bar{\mu}}, \gamma_5\} = 0 \\ [\gamma_{\hat{\mu}}, \gamma_5] = 0 \end{cases}$



Classical action in Dim Reg

$$S^{(d)}[A, f_X, \bar{f}_X] = \int d^d x \, (\mathscr{L}^{(d)}_{\text{YM}} + \mathscr{L}^{(d)}_{\text{Fermions}}) = \, ?$$

The bosonic part can be continued to d dimensions without violating any symmetry of the unregularized action:

$$\mathscr{L}_{\text{YM}} \to \mathscr{L}^{(d)}_{\text{YM}}, \qquad L_a S^{(d)}_{\text{YM}} = 0.$$

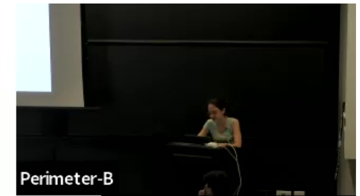
Fermionic Lagrangian less trivial:

- Kinetic term *must* be continued to d dimensions (else: propagator not regularized)
- More freedom in the interaction term: **scheme-dependent** choice

$$\mathscr{L}^{(d)}_{\text{Fermions}} = i\bar{f}\gamma^{\mu}\partial_{\mu}f - A^a_{\mu} \, \left(\bar{f} P_R \gamma^{\bar{\mu}} P_L T^a_L f + (L \to R) \right)$$

$$P_R \gamma^{\mu}, \gamma^{\mu} P_L \dots$$

Breaking of gauge invariance in DimReg



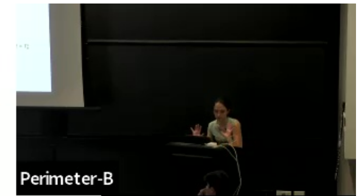
The d-dimensional kinetic terms necessarily mediates $f_L \leftrightarrow f_R$ transitions…

$$\mathscr{L}^{(d)}_{\text{Fermions}} = i\bar{f}\gamma^{\mu}\partial_{\mu}f - A^{a}_{\mu}\ \left(\bar{f}P_R\gamma^{\mu}P_L T^a_L f + (L \to R)\right)$$

$$= i\bar{f}_L\gamma^{\hat{\mu}}\partial_{\hat{\mu}}f_R + i\bar{f}_R\gamma^{\hat{\mu}}\partial_{\hat{\mu}}f_L + \text{LL, RR}$$

Hence breaks gauge invariance:

$$L_a S^{(d)} = L_a S^{(d)}_{\text{Fermions}} = 2\bar{f}(T^a_R - T^a_L)\gamma^{\hat{\mu}}\gamma_5\partial_{\hat{\mu}}f$$



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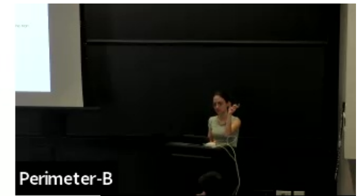
Hence breaks gauge invariance:

$$L_a S^{(d)} = L_a S^{(d)}_{\text{Fermions}} = 2\bar{f}(T^a_R - T^a_L)\gamma^{\hat{\mu}}\gamma_5\partial_{\hat{\mu}}f$$

- The (reg.) classical action is not invariant unless the theory is vector-like: $T^a_L = T^a_R$
- Evanescent: irrelevant at the tree level, but not beyond!

Breaking of gauge invariance in DimReg



Regularized action, including gauge fixing:

$$e^{i\Gamma^{(d)}[\phi]} = \int_{1\mathrm{PI}} \mathscr{D}\tilde{\phi}\; e^{iS^{(d)}_{\mathrm{full}}[\phi+\tilde{\phi}]} \qquad S^{(d)}_{\mathrm{full}} \equiv S^{(d)} + S^{(d)}_{\mathrm{g.f.}} + S^{(d)}_{\mathrm{ghost}}$$

In DimReg the path integral measure is invariant:

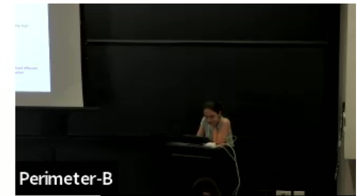
$$\ln \det \mathcal{J} = \delta^{(d)}(0) \int d^d x\, f(x) = 0 \qquad \Rightarrow \qquad \mathcal{J} = 1$$

scaleless

→ any regularization-induced breaking of gauge invariance comes solely from the non invariance of the classical action:

$$L_a \Gamma^{(d)}[\phi] = \frac{\int_{1\mathrm{PI}} \mathscr{D}\tilde{\phi}\; e^{iS^{(d)}_{\mathrm{full}}[\phi+\tilde{\phi}]}\; L_a S^{(d)}_{\mathrm{full}}[\phi+\tilde{\phi}]}{\int_{1\mathrm{PI}} \mathscr{D}\tilde{\phi}\; e^{iS^{(d)}_{\mathrm{full}}[\phi+\tilde{\phi}]}}\,.$$

Breaking of gauge invariance in DimReg



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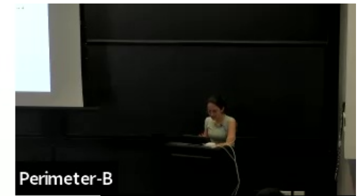
But to find for the WI restoring counterterm we want $L_a\Gamma[\phi]$ ← **renormalized** effective action

→ need to specify a subtraction scheme to evaluate $\Gamma[\phi]$ from $\Gamma^{(d)}[\phi]$!

Defining a renormalization scheme



Two types of contributions to the regularized effective action at 1 loop:

$$\Gamma^{(d)}|_{(1)} = \underbrace{\overline{\Gamma}^{\text{fin}}|_{(1)} + \cancel{\frac{1}{\epsilon}\overline{\Gamma}^{\text{div}}}|_{(1)}}_{\text{non-evanescent}} + \underbrace{\widehat{\Gamma}^{\text{fin}}|_{(1)} + \cancel{\frac{1}{\epsilon}\widehat{\Gamma}^{\text{div}}}|_{(1)}}_{\text{evanescent}},$$

Minimal subtraction: renormalized action defined by subtracting all divergent terms (evanescent and non evanescent) and taking the limit $d \to 4$

$$\Gamma|_{(1)} \equiv \lim_{d\to 4}\left\{\overline{\Gamma}^{\text{fin}}|_{(1)} + \widehat{\Gamma}^{\text{fin}}|_{(1)}\right\} = \overline{\Gamma}^{\text{fin}}|_{(1)}$$

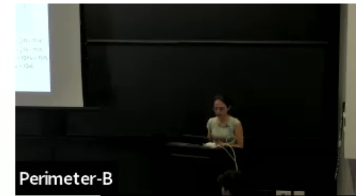
$$\Rightarrow \quad \boxed{\Delta_a|_{(1)}} = L_a\Gamma|_{(1)} = L_a\overline{\Gamma}^{\text{fin}}|_{(1)}.$$

This is the quantity we want!

We can find it directly by computing $L_a\Gamma^{(d)}[\phi]$, keeping only the finite piece:

$$L_a\Gamma^{(d)}|_{(1)} = \boxed{\overline{\Delta}_a^{\text{fin}}|_{(1)}} + \widehat{\Delta}_a^{\text{fin}}|_{(1)} + \frac{1}{\epsilon}\widehat{\Delta}_a^{\text{div}}|_{(1)}$$

Gauge variation of the effective action



$$\Delta_a\Big|_{(1)} = \Delta_a^{\text{Gauge}}\Big|_{(1)} + \Delta_a^{\text{Gauge+Fermions}}\Big|_{(1)}$$

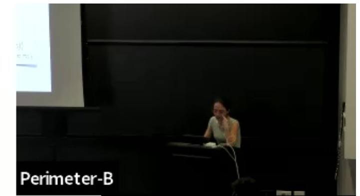
$$\Delta_a^{\text{Gauge}}\Big|_{(1)} = \frac{i}{8\pi^2}\,\text{tr}\left[T_A^a\gamma_5 a_2(x,x)\right] \equiv -\,\text{tr}\left[T_A^a(a_2^{\epsilon}(x) + a_2^{\not\epsilon}(x))\right]$$

$$a_2^{\epsilon} = \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2}\left[\mathcal{V}_{\mu\nu}\mathcal{V}_{\alpha\beta} + \frac{1}{3}\mathcal{A}_{\mu\nu}\mathcal{A}_{\alpha\beta} - \frac{8}{3}i\left(\mathcal{A}_\alpha\mathcal{A}_\beta\mathcal{V}_{\mu\nu} + \mathcal{A}_\alpha\mathcal{V}_{\mu\nu}\mathcal{A}_\beta + \mathcal{V}_{\mu\nu}\mathcal{A}_\alpha\mathcal{A}_\beta\right) - \frac{32}{3}\mathcal{A}_\mu\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right]$$

$$a_2^{\not\epsilon} = \frac{1}{16\pi^2}\left[\frac{4}{3}D_\nu^{\mathcal{V}}D_\nu^{\mathcal{V}}D_\mu^{\mathcal{V}}\mathcal{A}_\mu + \frac{8}{3}i[\mathcal{A}_\mu, D_\nu^{\mathcal{V}}\mathcal{V}_{\mu\nu}] - \frac{2}{3}i[\mathcal{A}_{\mu\nu}, \mathcal{V}_{\mu\nu}]\right]$$
$$+ \frac{1}{16\pi^2}\left[-8\mathcal{A}_\mu(D_\nu^{\mathcal{V}}\mathcal{A}^\nu)\mathcal{A}_\mu - \frac{8}{3}\left\{D_\mu^{\mathcal{V}}\mathcal{A}_\nu + D_\nu^{\mathcal{V}}\mathcal{A}_\mu, \mathcal{A}_\mu\mathcal{A}_\nu\right\} + \frac{4}{3}\left\{D_\mu^{\mathcal{V}}\mathcal{A}^\mu, \mathcal{A}_\nu\mathcal{A}_\nu\right\}\right].$$

$$\Delta_a^{\text{Gauge+Fermions}}\Big|_{(1)} = -\frac{2G_{aa}}{16\pi^2}\left(1 + \frac{\xi - 1}{6}\right)\bar{f}\gamma_5\gamma^\mu\left(\overrightarrow{\partial}_\mu + \overleftarrow{\partial}_\mu\right)T^aT_A^cT^af]]$$
$$+\frac{2G_{aa}}{16\pi^2}\left(1 + \frac{\xi - 1}{6}\right)A_\mu^m\,\bar{f}\gamma_5\gamma^\mu\left\{\left[T^c, T^aT_A^mT^a\right] - if^{cnm}T^aT_A^nT^a\right\}f$$

$$\mathcal{V}_\mu = \frac{1}{2}(T_R^a + T_L^a)A_\mu^a$$
$$\mathcal{A}_\mu = \frac{1}{2}(T_R^a - T_L^a)A_\mu^a$$
$$T^a = T_R^aP_R + T_L^aP_L$$
$$G_{aa} = \delta_{ab}^G g_G^2$$



Gauge variation of the effective action

$$\Delta_a\Big|_{(1)} = \Delta_a^{\text{Gauge}}\Big|_{(1)} + \Delta_a^{\text{Gauge+Fermions}}\Big|_{(1)}$$

A few comments on this result:

- computed with the path integral (heat kernel method)
- diagrammatic cross-checks in Feynman gauge

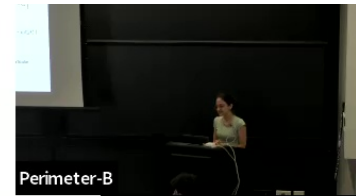
- It can be put in the form

$$\Delta_a(x)\Big|_{(1)} = \sum_{k=0}^{14} C^k_{aA}\, I^k_A(x)$$

the C^k_{aA} satisfy $\mathrm{WZ}[C^k_{aA}] = 0$

$$\begin{aligned}
C^0_{pa} &= \frac{1}{16\pi^2}\mathrm{tr}\, T^p_A \left\{-\frac{4}{3} T^a_A\right\} \\
C^1_{pab} &= \frac{1}{16\pi^2}\mathrm{tr}\, T^p_A \left\{4\left(T^a_V T^b_V + \frac{1}{3} T^a_A T^b_A\right)\right\} \\
C^2_{pab} &= \frac{1}{16\pi^2}\mathrm{tr}\, T^p_A \left\{-\frac{8}{3} i \left([T^a_A, T^b_V] + [T^a_V, T^b_A]\right)\right\} \\
&\dots
\end{aligned}$$

Counterterm



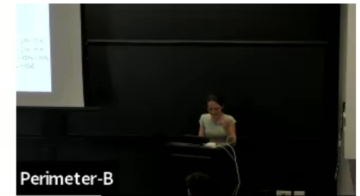
$$S_{\rm ct}[\phi]\,|_{(1)} = \int d^4y \mathscr{L}_{\rm ct}(y)\,|_{(1)}$$

$$\begin{aligned}\mathcal{L}_{\rm ct}|_{(1)} \;=\; & \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2}{\rm Tr}\left\{\frac{8}{3}\partial_\mu\mathcal{V}_\nu\left\{\mathcal{V}_\alpha,\mathcal{A}_\beta\right\}+4i\mathcal{V}_\mu\mathcal{V}_\nu\mathcal{V}_\alpha\mathcal{A}_\beta+\frac{4}{3}i\mathcal{V}_\mu\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right\}\\ + & \frac{1}{16\pi^2}{\rm Tr}\left\{-\frac{4}{3}(D^{\mathcal{V}}_\mu\mathcal{A}_\nu)^2+2(D^{\mathcal{V}}_\mu\mathcal{A}^\mu)^2-\frac{4}{3}[\mathcal{A}_\mu,\mathcal{A}_\nu]^2+\frac{4}{3}(\mathcal{A}_\mu\mathcal{A}_\nu)^2+\mathcal{A}^2_{\mu\nu}\right\}\\ - & \frac{2}{16\pi^2}\left(1+\frac{\xi-1}{6}\right)G_{aa}\bar{f}\gamma_5\gamma^\mu T^a\mathcal{A}_\mu T^a f.\end{aligned}$$

[Rescaling needed to normalize canonically the kinetic terms: $A^a_\mu \to g_G\delta^G_{ab}A^b_\mu$]

- not gauge invariant (by definition)
- vanishes manifestly for a vector-like theory
- consistent with the general anomaly → counterterm map we derived, for a particular choice for the gauge-invariant counterterm.

Gauge variation of the effective action



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$$\begin{aligned}
a_2^{\epsilon} &= \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2}\left[\mathcal{V}_{\mu\nu}\mathcal{V}_{\alpha\beta} + \frac{1}{3}\mathcal{A}_{\mu\nu}\mathcal{A}_{\alpha\beta} - \frac{8}{3}i\left(\mathcal{A}_\alpha\mathcal{A}_\beta\mathcal{V}_{\mu\nu} + \mathcal{A}_\alpha\mathcal{V}_{\mu\nu}\mathcal{A}_\beta + \mathcal{V}_{\mu\nu}\mathcal{A}_\alpha\mathcal{A}_\beta\right) - \frac{32}{3}\mathcal{A}_\mu\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right] \\
a_2^{\not\epsilon} &= \frac{1}{16\pi^2}\left[\frac{4}{3}D_\nu^{\mathcal{V}}D_\nu^{\mathcal{V}}D_\mu^{\mathcal{V}}\mathcal{A}_\mu + \frac{8}{3}i[\mathcal{A}_\mu, D_\nu^{\mathcal{V}}\mathcal{V}_{\mu\nu}] - \frac{2}{3}i[\mathcal{A}_{\mu\nu}, \mathcal{V}_{\mu\nu}]\right] \\
&+ \frac{1}{16\pi^2}\left[-8\mathcal{A}_\mu(D_\nu^{\mathcal{V}}\mathcal{A}^\nu)\mathcal{A}_\mu - \frac{8}{3}\left\{D_\mu^{\mathcal{V}}\mathcal{A}_\nu + D_\nu^{\mathcal{V}}\mathcal{A}_\mu, \mathcal{A}_\mu\mathcal{A}_\nu\right\} + \frac{4}{3}\left\{D_\mu^{\mathcal{V}}\mathcal{A}^\mu, \mathcal{A}_\nu\mathcal{A}_\nu\right\}\right].
\end{aligned}$$

$$\begin{aligned}
\Delta_a^{\text{Gauge+Fermions}}\Big|_{(1)} = &-\frac{2G_{aa}}{16\pi^2}\left(1 + \frac{\xi - 1}{6}\right)\bar{f}\gamma_5\gamma^\mu\left(\overrightarrow{\partial}_\mu + \overleftarrow{\partial}_\mu\right)T^aT_A^cT^af]] \\
&+\frac{2G_{aa}}{16\pi^2}\left(1 + \frac{\xi - 1}{6}\right)A_\mu^m\,\bar{f}\gamma_5\gamma^\mu\left\{\left[T^c, T^aT_A^mT^a\right] - if^{cnm}T^aT_A^nT^a\right\}f
\end{aligned}$$

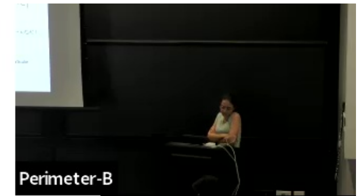
$$\mathcal{V}_\mu = \frac{1}{2}(T_R^a + T_L^a)A_\mu^a$$

$$\mathcal{A}_\mu = \frac{1}{2}(T_R^a - T_L^a)A_\mu^a$$

$$T^a = T_R^aP_R + T_L^aP_L$$

$$G_{aa} = \delta_{ab}^G g_G^2$$

Counterterm



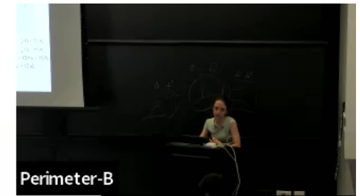
$$S_{\rm ct}[\phi]\,|_{(1)} = \int d^4y\, \mathscr{L}_{\rm ct}(y)\,|_{(1)}$$

$$\begin{aligned}
\mathcal{L}_{\rm ct}|_{(1)} \;=\;& \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2}\mathrm{Tr}\left\{\frac{8}{3}\partial_\mu \mathcal{V}_\nu \left\{\mathcal{V}_\alpha, \mathcal{A}_\beta\right\} + 4i\mathcal{V}_\mu\mathcal{V}_\nu\mathcal{V}_\alpha\mathcal{A}_\beta + \frac{4}{3}i\mathcal{V}_\mu\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right\} \\
&+ \frac{1}{16\pi^2}\mathrm{Tr}\left\{-\frac{4}{3}(D^{\mathcal{V}}_\mu\mathcal{A}_\nu)^2 + 2(D^{\mathcal{V}}_\mu\mathcal{A}^\mu)^2 - \frac{4}{3}[\mathcal{A}_\mu,\mathcal{A}_\nu]^2 + \frac{4}{3}(\mathcal{A}_\mu\mathcal{A}_\nu)^2 + \mathcal{A}^2_{\mu\nu}\right\} \\
&- \frac{2}{16\pi^2}\left(1 + \frac{\xi - 1}{6}\right) G_{aa}\bar{f}\gamma_5\gamma^\mu T^a \mathcal{A}_\mu T^a f.
\end{aligned}$$

[Rescaling needed to normalize canonically the kinetic terms: $A^a_\mu \to g_G \delta^G_{ab} A^b_\mu$]

- not gauge invariant (by definition)
- vanishes manifestly for a vector-like theory
- consistent with the general anomaly → counterterm map we derived, for a particular choice for the gauge-invariant counterterm.

Gauge variation of the effective action



$$\Delta_a \Big|_{(1)} = \Delta_a^{\text{Gauge}} \Big|_{(1)} + \Delta_a^{\text{Gauge+Fermions}} \Big|_{(1)}$$

$$\Delta_a^{\text{Gauge}} \Big|_{(1)} = \frac{i}{8\pi^2} \operatorname{tr}\left[T_A^a \gamma_5 a_2(x,x)\right] \equiv - \operatorname{tr}\left[T_A^a (a_2^{\epsilon}(x) + a_2^{\not\epsilon}(x))\right]$$

$$a_2^{\epsilon} = \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2}\left[\mathcal{V}_{\mu\nu}\mathcal{V}_{\alpha\beta} + \frac{1}{3}\mathcal{A}_{\mu\nu}\mathcal{A}_{\alpha\beta} - \frac{8}{3} i \left(\mathcal{A}_\alpha \mathcal{A}_\beta \mathcal{V}_{\mu\nu} + \mathcal{A}_\alpha \mathcal{V}_{\mu\nu}\mathcal{A}_\beta + \mathcal{V}_{\mu\nu}\mathcal{A}_\alpha\mathcal{A}_\beta\right) - \frac{32}{3}\mathcal{A}_\mu\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right]$$

$$a_2^{\not\epsilon} = \frac{1}{16\pi^2}\left[\frac{4}{3} D_\nu^{\mathcal{V}} D_\nu^{\mathcal{V}} D_\mu^{\mathcal{V}} \mathcal{A}_\mu + \frac{8}{3} i [\mathcal{A}_\mu, D_\nu^{\mathcal{V}} \mathcal{V}_{\mu\nu}] - \frac{2}{3} i [\mathcal{A}_{\mu\nu}, \mathcal{V}_{\mu\nu}]\right]$$

$$+ \frac{1}{16\pi^2}\left[-8\mathcal{A}_\mu (D_\nu^{\mathcal{V}}\mathcal{A}^\nu)\mathcal{A}_\mu - \frac{8}{3}\left\{D_\mu^{\mathcal{V}}\mathcal{A}_\nu + D_\nu^{\mathcal{V}}\mathcal{A}_\mu, \mathcal{A}_\mu\mathcal{A}_\nu\right\} + \frac{4}{3}\left\{D_\mu^{\mathcal{V}}\mathcal{A}^\mu, \mathcal{A}_\nu\mathcal{A}_\nu\right\}\right].$$

$$\Delta_a^{\text{Gauge+Fermions}} \Big|_{(1)} = -\frac{2G_{aa}}{16\pi^2}\left(1 + \frac{\xi - 1}{6}\right) \bar{f}\gamma_5\gamma^\mu \left(\overrightarrow{\partial}_\mu + \overleftarrow{\partial}_\mu\right) T^a T_A^c T^a f]]$$

$$+ \frac{2G_{aa}}{16\pi^2}\left(1 + \frac{\xi - 1}{6}\right) A_\mu^m \; \bar{f}\gamma_5\gamma^\mu \left\{ \left[T^c, T^a T_A^m T^a\right] - i f^{cnm} T^a T_A^n T^a \right\} f$$

$$\mathcal{V}_\mu = \frac{1}{2}(T_R^a + T_L^a) A_\mu^a$$

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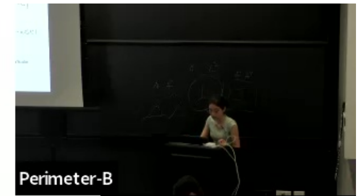
Counterterm



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&+ \frac{1}{16\pi^2}\text{Tr}\left\{-\frac{4}{3}(D^{\mathcal{V}}_\mu \mathcal{A}_\nu)^2 + 2(D^{\mathcal{V}}_\mu \mathcal{A}^\mu)^2 - \frac{4}{3}[\mathcal{A}_\mu, \mathcal{A}_\nu]^2 + \frac{4}{3}(\mathcal{A}_\mu\mathcal{A}_\nu)^2 + \mathcal{A}^2_{\mu\nu}\right\} \\
&- \frac{2}{16\pi^2}\left(1 + \frac{\xi - 1}{6}\right) G_{aa}\bar{f}\gamma_5\gamma^\mu T^a \mathcal{A}_\mu T^a f.
\end{aligned}$$

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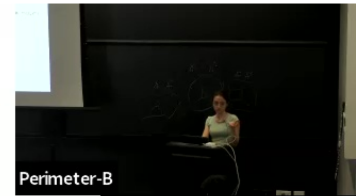


In the Standard Model

As an application we can derive the WI-restoring counterterm for the SM, using DimReg and the BMHV scheme for γ_5.

Partial result: valid in the limit of vanishing Yukawa couplings.

VVDD: $D_\mu W^-_\nu D^\mu W^{+\nu}$ $\quad$ $\partial_\mu Z_\nu \partial^\mu Z^\nu$ $\qquad$ $* D_\mu W^\pm_\nu = (\partial_\mu \pm ieA_\mu) W^\pm_\nu$

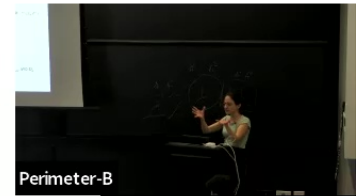
VVVD: $iF^{\mu\nu} W^+_\mu W^-_\nu$ $\quad$ $iD^\mu W^-_\mu W^+_\nu Z^\nu$ $\quad$ $iD^\nu W^-_\mu W^+_\nu Z^\mu$ $\quad$ $iD_\nu W^-_\mu W^{+\mu} Z^\nu$ $\quad$ $+\text{hc}$

VVVV: $(W^-_\mu W^{+\mu})^2$ $\quad$ $(W^-_\mu W^{-\mu})(W^+_\nu W^{+\nu})$ $\quad$ $(Z_\mu Z^\mu)^2$ $\quad$ $(W^+_\mu Z^\mu)(W^-_\nu Z^\nu)$ $\quad$ $(W^+_\mu W^{-\mu})(Z_\nu Z^\nu)$

ffW: $W^+_\mu \overline{f_u} \gamma^\mu P_L f_d$ $\quad$ $W^+_\mu \overline{f_u} \gamma^\mu P_R f_d$ $\quad$ $+\text{hc}$

ffZ: $Z_\mu \overline{f} \gamma^\mu P_L f$ $\quad$ $Z_\mu \overline{f} \gamma^\mu P_R f$ $\quad$ $+\text{hc}$ $\qquad$ [©Luca Vecchi]

Potentially relevant for NLO calculations of Drell-Yan, WW scattering….



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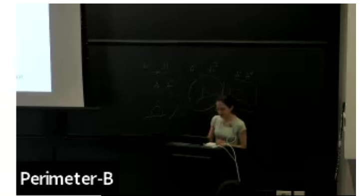
Potentially relevant for NLO calculations of Drell-Yan, WW scattering….

- $\mathscr{L}_{\mathrm{ct}}$ has to be invariant under the “vector” groups $SU(3)_c$ and $U(1)_{\mathrm{em}}$
  $\rightarrow$ does not contain gluons, photons only via $F_{\mu\nu}$ and D_μ
- No counterterms with $\epsilon^{\mu\nu\rho\sigma}$ are need; peculiarity of the EW gauge group.



Conclusion and outlook

Any consistent regularization scheme induces an apparent violation of gauge invariance in chiral gauge theories.

These theories require a **two-step renormalization procedure,**to be reiterated order by order:
1. remove infinities by adding a set of local, divergent counterterms
2. remote finite, gauge-violating contributions by adding a set of local, finite counterterms

Our main result is an **analytic expression of the finite one-loop counterterms for renormalizable chiral gauge theories including fermions**:

- general result for a wide class of regularization schemes
- explicit calculation for using DimReg, MS & BHMV for γ_5
- application to the SM

Main differences with respect to previous approaches:

- use of the Background Field Methord
- inclusion of charged fermions of both chiralities
- no need for additional sterile fermions

Possible extensions to theories with **scalars**, and to the **SMEFT**.
Output could be implemented in the **automated** one-loop computations in the SMEFT.