Title: Cosmological Bootstrap in Slow Motion

Speakers: Sadra Jazayeri

Series: Cosmology & Gravitation

Date: June 27, 2022 - 11:00 AM

URL: https://pirsa.org/22060095

Abstract: Inflation can be viewed as a natural "cosmological particle detector" which can probe energies as high as its Hubble scale. In this talk, I study the imprints of heavy relativistic particles during inflation on primordial correlators in situations where the scalar fluctuations have a reduced speed of sound. Breaking dS boosts allows new types of footprints of massive fields to emerge. In particular, I show that heavy particles that are lighter than Hubble divided by the speed of sound leave smoking gun imprints in the three-point function of curvature perturbations (due to the exchange of those fields) in the form of resonances in the squeezed limit which are vividly distinct from the previously explored signatures of heavy fields in de Sitter correlators. Throughout I use and extend the cosmological bootstrap techniques derived from locality, unitarity, and analyticity in order to find fully analytical formulae for the desired boost breaking correlators.

Cosmological Collider Physics at Low Speeds: A Bootstrap Approach

Sadra Jazayeri
(Institut d'Astrophysique de Paris)

Based on hep-th/2205.10340
SJ, Sébastien Renaux-Petel(IAP)

Disclaimer: references incomplete

European Research Council
Established by the European Commission

GEODESI

I. Cosmological Collider Physics at high and low speeds

II. Cosmological Bootstrap

III. Conclusions

- The current observational data relevant for inflation is consistent with a single canonical scalar field that is minimally coupled to gravity and rolls on an almost flat potential.

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$$

$$\mathrm{d}s^2 = \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{(H\eta)^2} = -dt^2 + \exp(2Ht)\, d\mathbf{x}^2$$

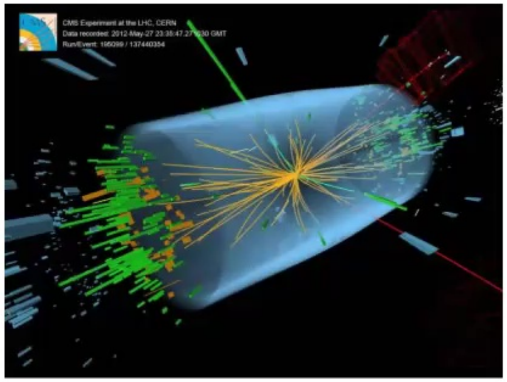
- The standard setup does not allow for sizeable primordial non-Gaussianities Maldacena 2002 (in the limit with exact de Sitter isometries the theory has to be free Pajer, Green 2020)

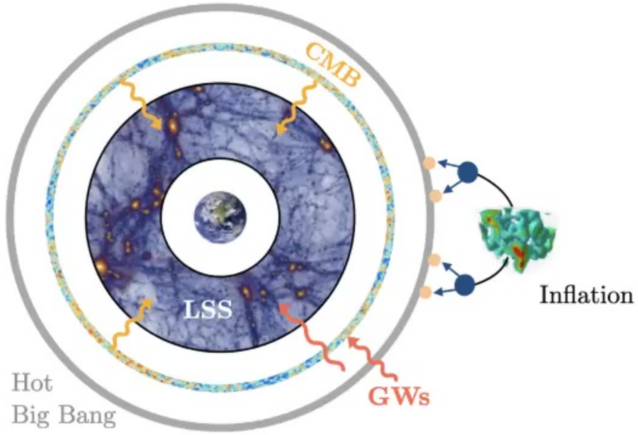
$$f_{\mathrm{NL}} = \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} = \underbrace{\mathcal{O}(\epsilon, \eta)}_{\text{mixing with gravity}}$$

- One way to enrich the phenomenology is to add new degrees of freedom to the field content during inflation

- In fact, any new field with a mass not far from the Hubble scale can potentially leave observable imprints in the correlation functions of curvature perturbations. This is the idea of "*Cosmological Collider Physics*"

$$\mathrm{E_{CME}} \lesssim 10^4\,\mathrm{Gev} \qquad\qquad H \lesssim 10^{14}\mathrm{Gev}$$



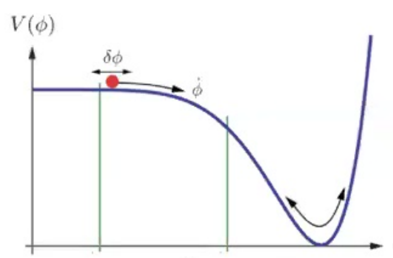


Chen, Wang 2009, Baumann Green 2011/ Noumi, Yamaguchi, Yokohama 2012, Arkani-Hamed, Maldacena 2015/ Lee, Bauman, Pimentel 2016/ Arkani-Hamed, Baumann, Lee, Pimentel 2018 /+ many works

de Sitter Collider

- Adding heavy fields: If we don't want to spoil de Sitter's isometries, especially boosts (only weakly broken due to the slowly-varying background of the scalar field), we should contract indices in a *Lorentz invariant* fashion. We better respect the shift symmetry of the inflaton as well.

$$\Delta\mathcal{L} = \underbrace{-\frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}m^2\sigma^2}_{\text{free theory}} \underbrace{- \frac{1}{\Lambda}(\partial_\mu\phi)^2\sigma}_{\text{interaction}} + \dots$$



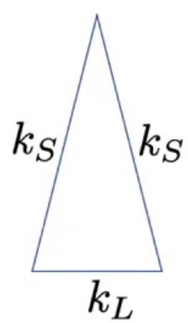
$\phi = \dot{\bar{\phi}}t + \delta\phi$

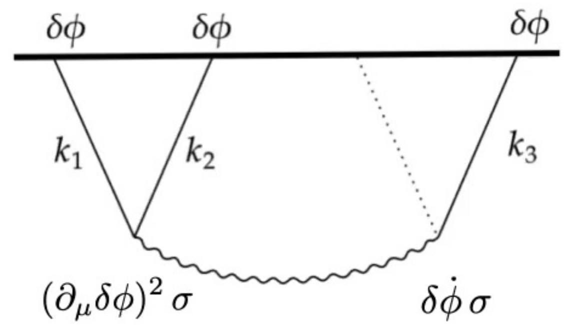
$$\frac{1}{\Lambda}(\partial_\mu\phi)^2\sigma \supset \frac{\dot{\bar{\phi}}}{\Lambda}\delta\dot{\phi}\sigma + \frac{1}{\Lambda}(\partial_\mu\delta\phi)^2\sigma$$

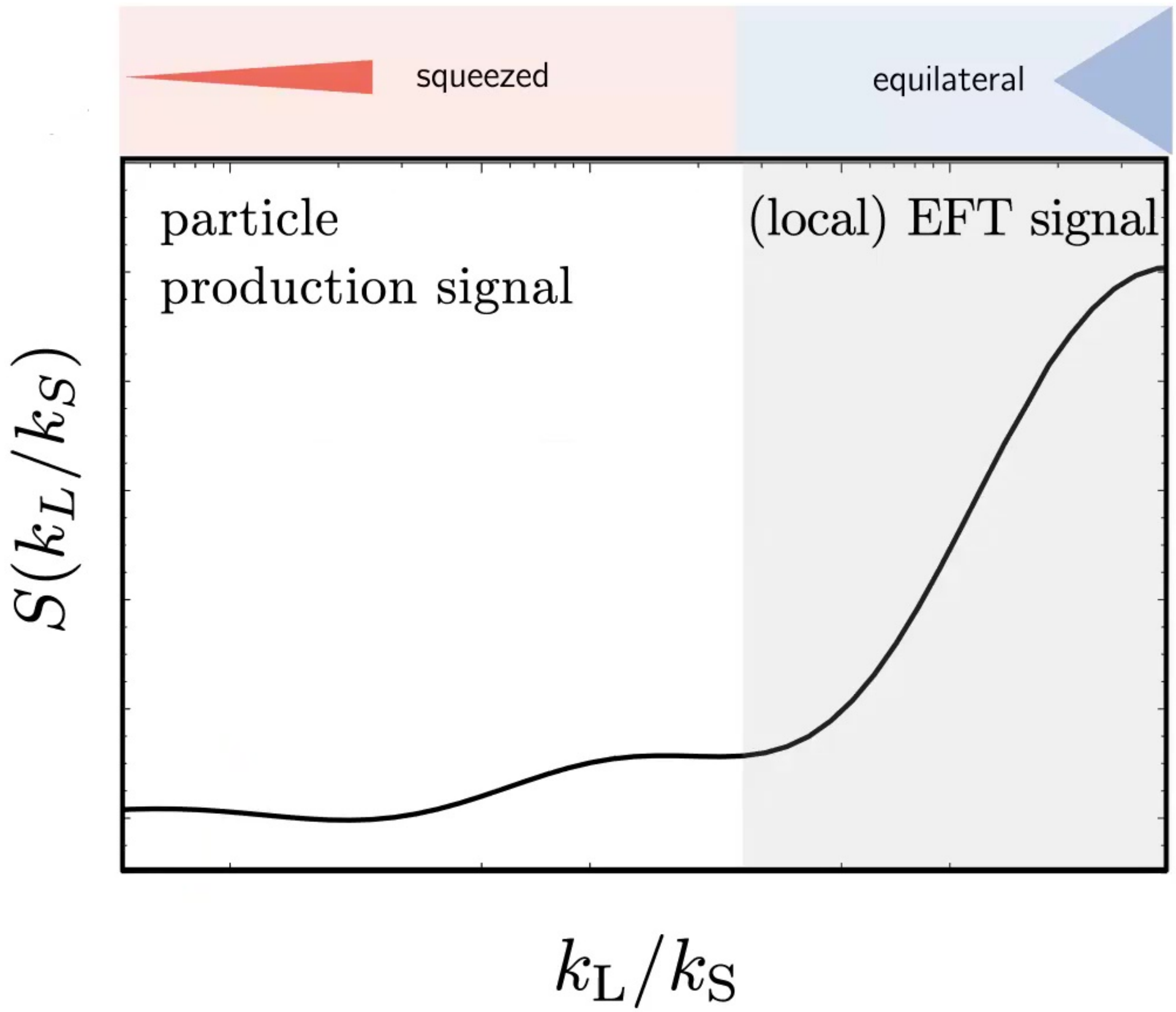
- We are interested in the impact of these interactions on the correlators of the curvature perturbation

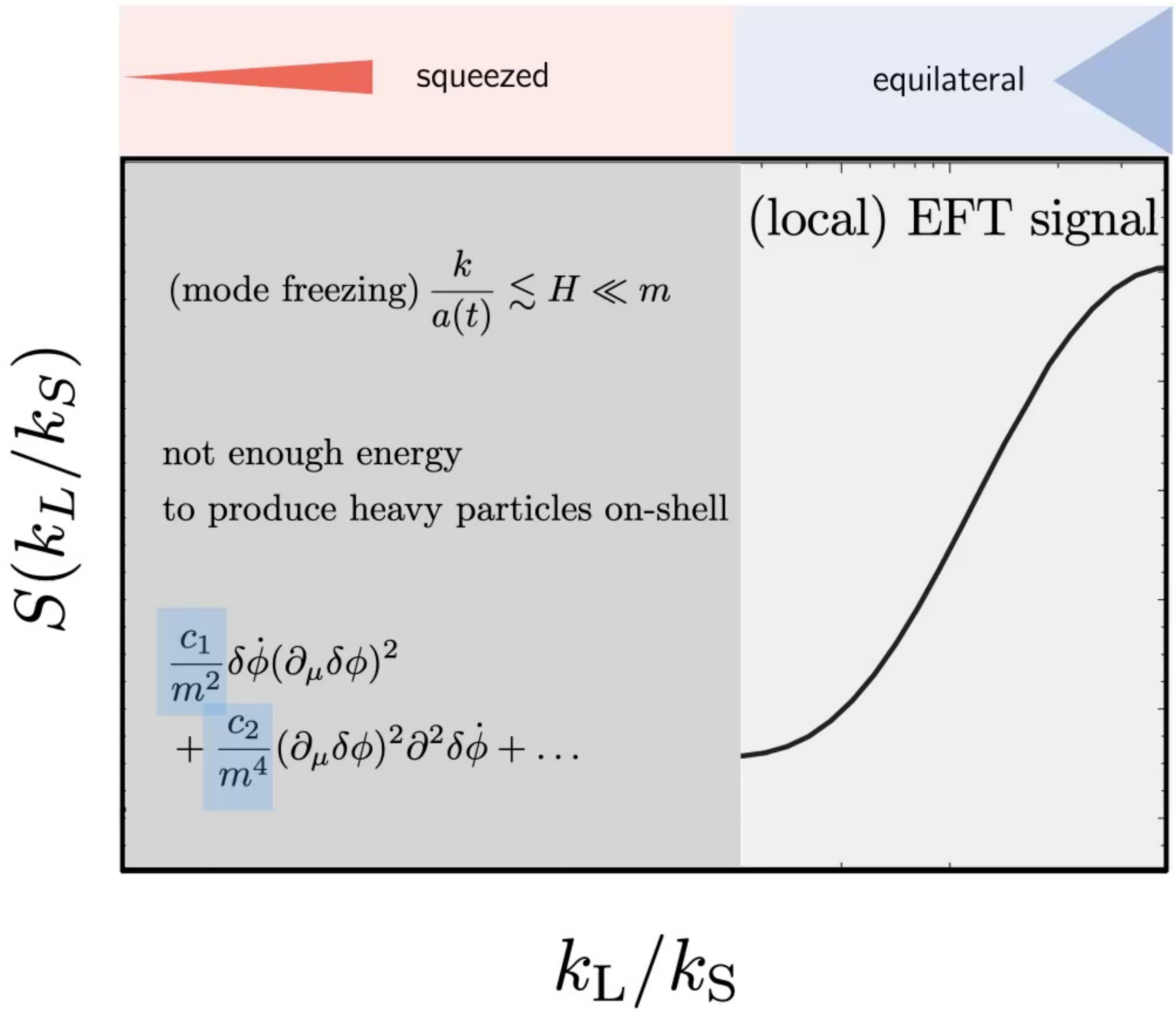
$$\zeta \sim -\frac{H}{\dot{\phi}}\delta\phi$$

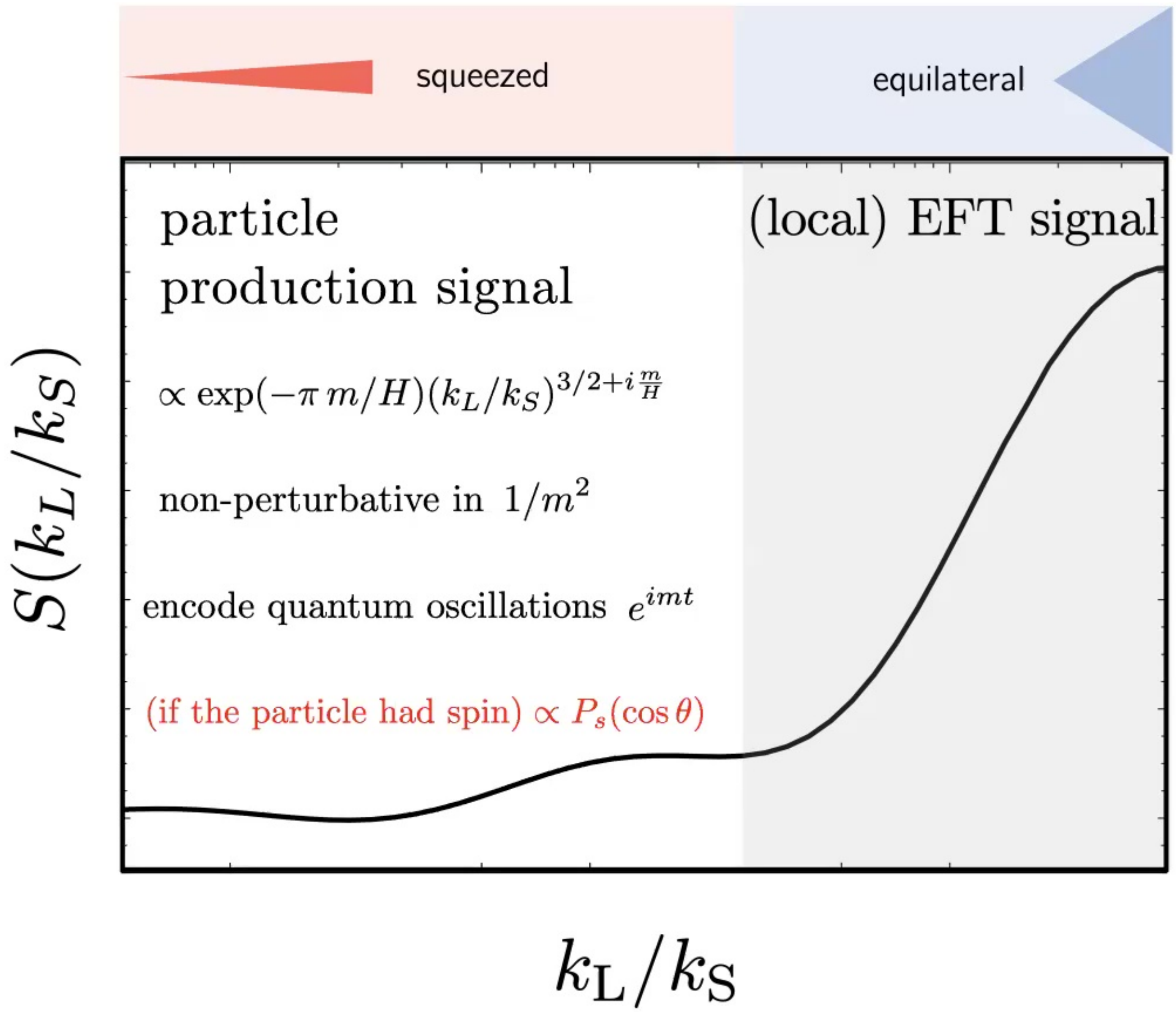
$$\langle\zeta(\mathbf{k}_1\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle' = (2\pi)^3\frac{1}{(k_1k_2k_3)^2}\underbrace{S(k_1,k_2,k_3)}_{\text{non-Gaussian shape}}\,.$$






squeezed
equilateral
particle
production signal
(local) EFT signal
$S(k_L/k_S)$
$k_{\rm L}/k_{\rm S}$


squeezed
equilateral
(local) EFT signal
(mode freezing) $\frac{k}{a(t)} \lesssim H \ll m$
not enough energy
to produce heavy particles on-shell
$\frac{c_1}{m^2}\delta\dot{\phi}(\partial_\mu\delta\phi)^2 + \frac{c_2}{m^4}(\partial_\mu\delta\phi)^2\partial^2\delta\dot{\phi} + \dots$
$S(k_L/k_S)$
$k_\mathrm{L}/k_\mathrm{S}$


squeezed
equilateral
particle production signal
(local) EFT signal
$\propto \exp(-\pi\, m/H)(k_L/k_S)^{3/2+i\frac{m}{H}}$
non-perturbative in $1/m^2$
encode quantum oscillations e^{imt}
(if the particle had spin) $\propto P_s(\cos\theta)$
$S(k_L/k_S)$
k_L/k_S

Low Speed Collider

- Adding heavy fields: If we give up de Sitter boosts we can have different sound speeds for different species. This situation can be effectively formulated within the framework of EFT of inflation

$$\phi = t + \pi(t, \mathbf{x}) \qquad \zeta \sim -H\pi$$

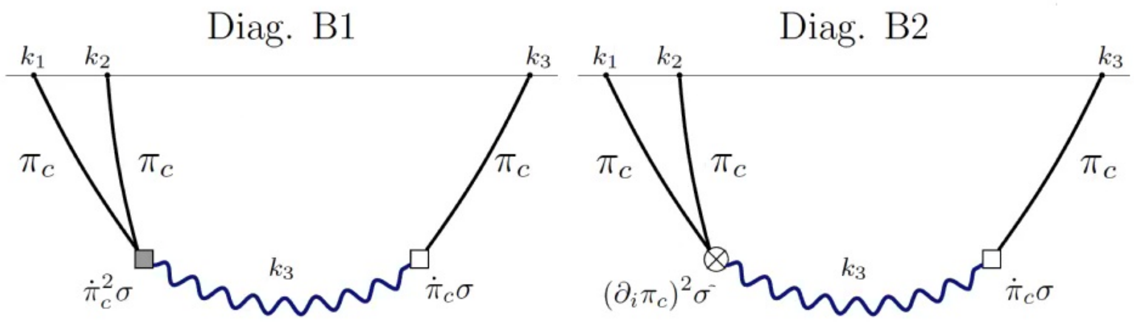
$$S_\pi = \int d\eta\, d^3\mathbf{x}\, a^2 \epsilon H^2 M_{\rm Pl}^2 \left[\frac{1}{c_s^2}\left(\pi'^2 - c_s^2(\partial_i\pi)^2\right) - \frac{1}{a}\left(\frac{1}{c_s^2} - 1\right)\left(\pi'(\partial_i\pi)^2 + \frac{A}{c_s^2}\pi'^3\right) + \dots \right]$$

(speed of sound) (large boost breaking interactions)

$$S_\sigma^{(2)} = \int d\eta d^3\boldsymbol{x}\, a^2 \left(\frac{1}{2}\sigma'^2 - \frac{1}{2}(\partial_i\sigma)^2 - \frac{1}{2}m^2 a^2 \sigma^2\right) \qquad \mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}} > 0$$

(unit sound speed)

$$S_{\pi\sigma} = \int d\eta d^3\mathbf{x}\, a^2 \left(\rho a \pi_c' \sigma + \frac{1}{\Lambda_1}\pi_c'^2\sigma + \frac{c_s^2}{\Lambda_2}(\partial_i \pi_c)^2 \sigma \right) \qquad \pi_c = \sqrt{2\epsilon} H M_{\rm Pl} c_s^{-1} \pi$$



Low Speed Collider

- Adding heavy fields: If we give up de Sitter boosts we can have different sound speeds for different species. This situation can be effectively formulated within the framework of EFT of inflation

$$\phi = t + \pi(t, \mathbf{x}) \qquad \zeta \sim -H\pi$$

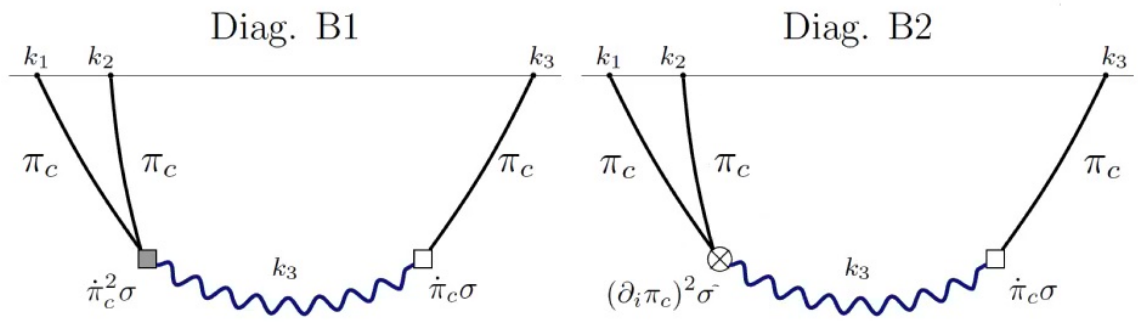
$$S_\pi = \int d\eta\, d^3\mathbf{x}\, a^2 \epsilon H^2 M_{\rm Pl}^2 \left[\frac{1}{c_s^2}\left(\pi'^2 - c_s^2(\partial_i\pi)^2\right) - \frac{1}{a}\left(\frac{1}{c_s^2} - 1\right)\left(\pi'(\partial_i\pi)^2 + \frac{A}{c_s^2}\pi'^3\right) + \dots\right]$$

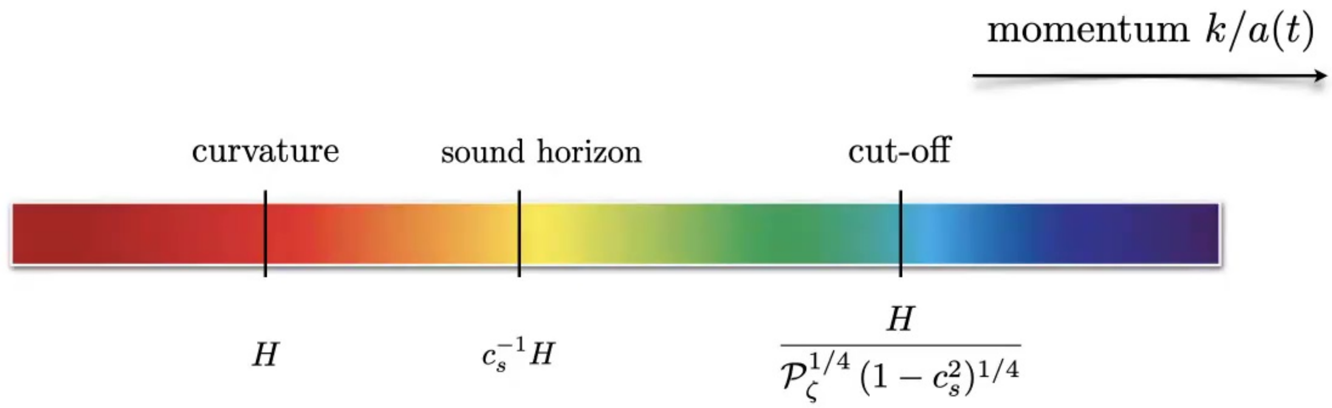
(speed of sound) (large boost breaking interactions)

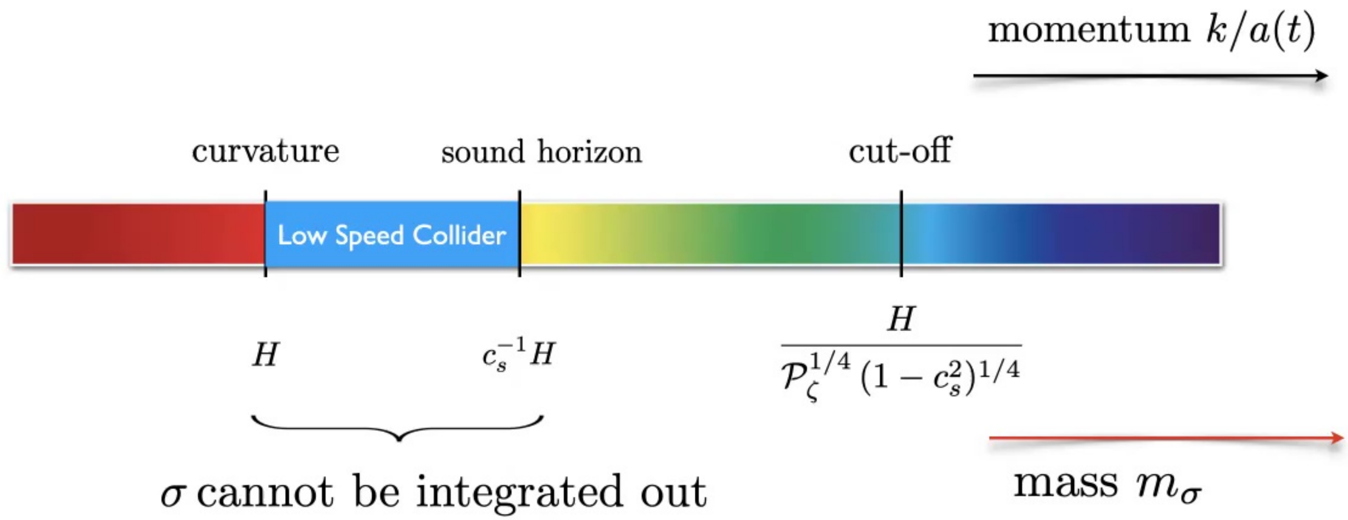
$$S_\sigma^{(2)} = \int d\eta d^3\boldsymbol{x}\, a^2 \left(\frac{1}{2}\sigma'^2 - \frac{1}{2}(\partial_i\sigma)^2 - \frac{1}{2}m^2 a^2 \sigma^2\right) \qquad \mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}} > 0$$

(unit sound speed)

$$S_{\pi\sigma} = \int d\eta d^3\mathbf{x}\, a^2 \left(\rho a \pi_c' \sigma + \frac{1}{\Lambda_1} \pi_c'^2 \sigma + \frac{c_s^2}{\Lambda_2} (\partial_i \pi_c)^2 \sigma \right) \qquad \pi_c = \sqrt{2\epsilon} H M_{\rm Pl} c_s^{-1} \pi$$

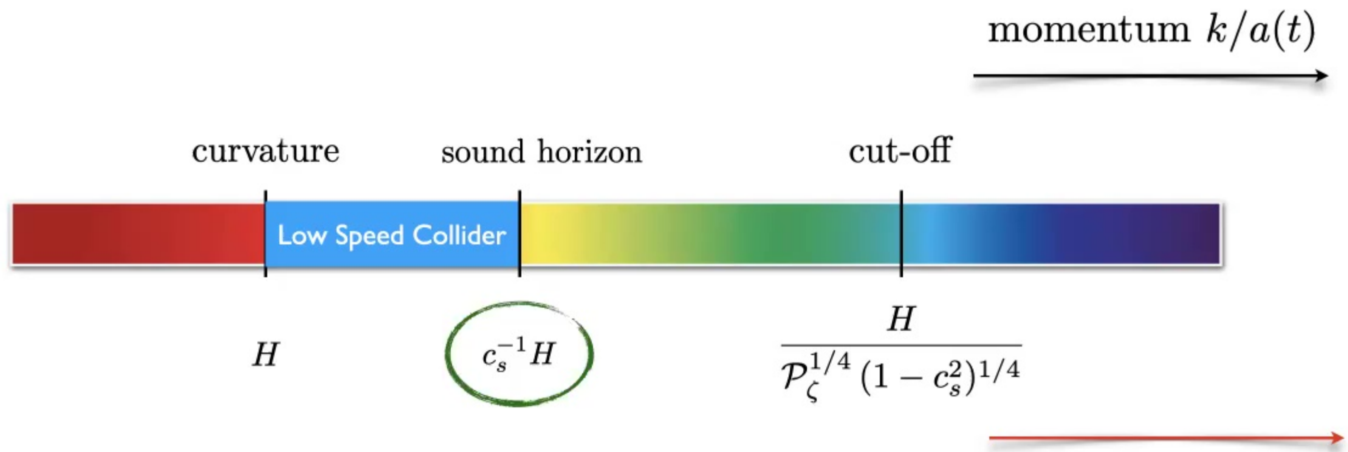



momentum $k/a(t)$
curvature
sound horizon
cut-off
H
$c_s^{-1}H$
$\frac{H}{\mathcal{P}_\zeta^{1/4}\,(1-c_s^2)^{1/4}}$

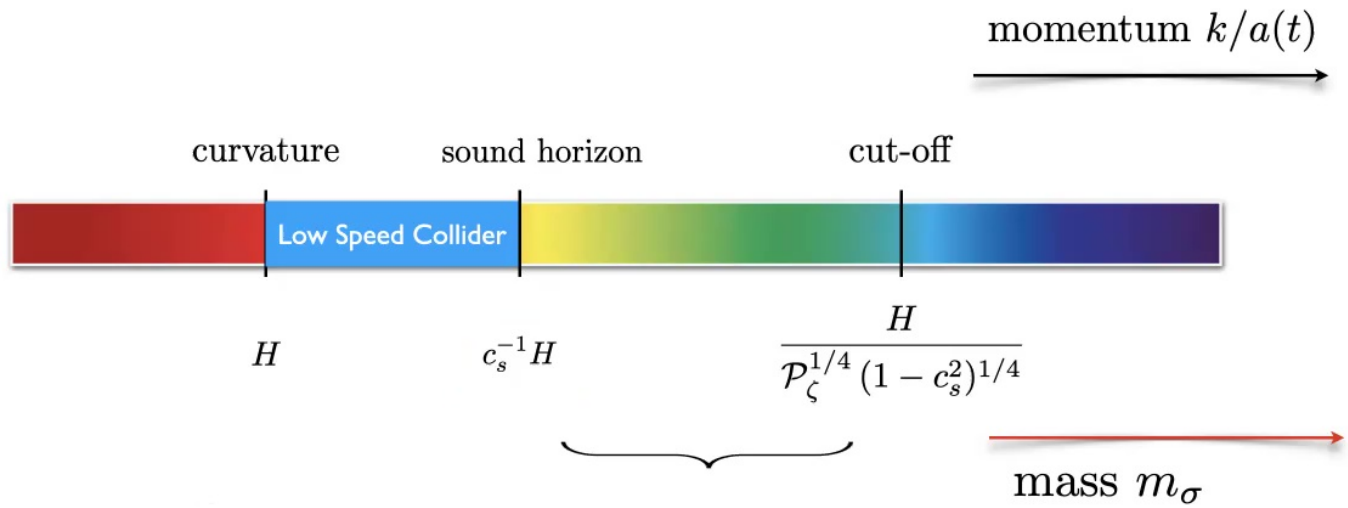


σ cannot be integrated out

(relativistic at sound horizon crossing)

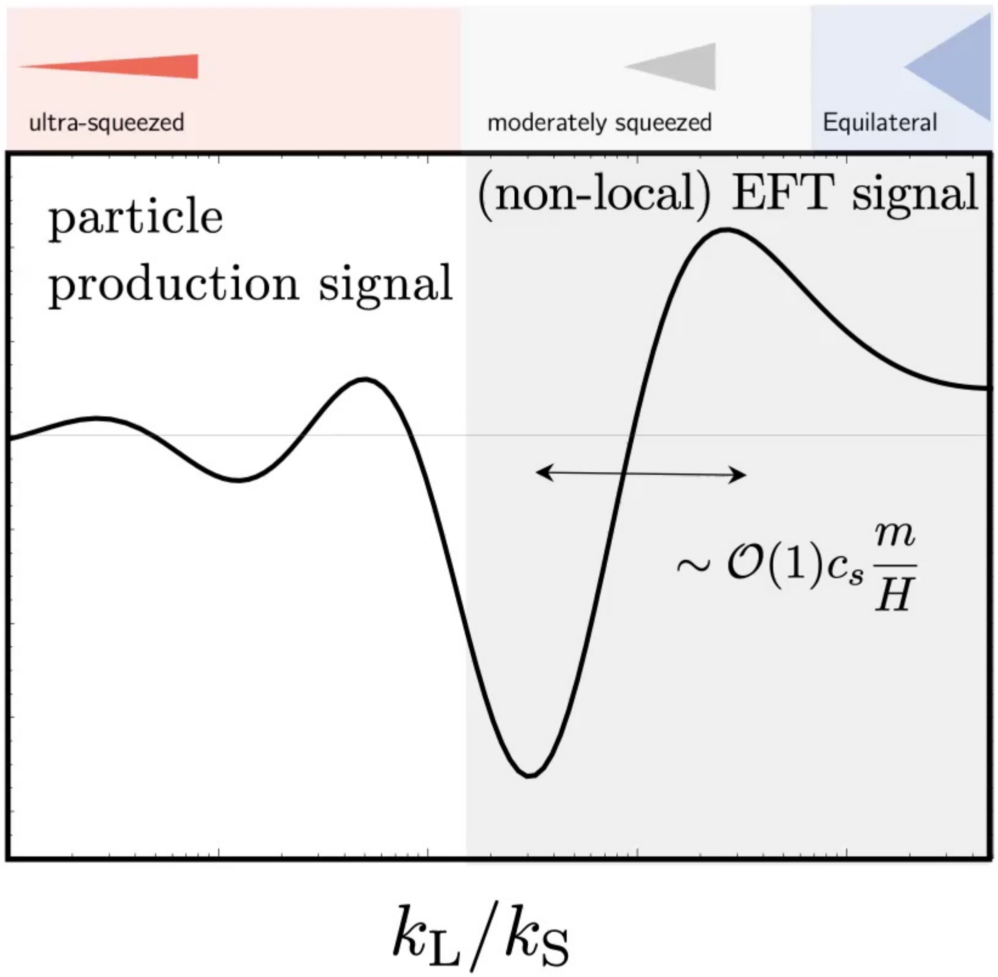


can be parametrically bigger than Hubble!
(Low speed collider is more UV sensitive)

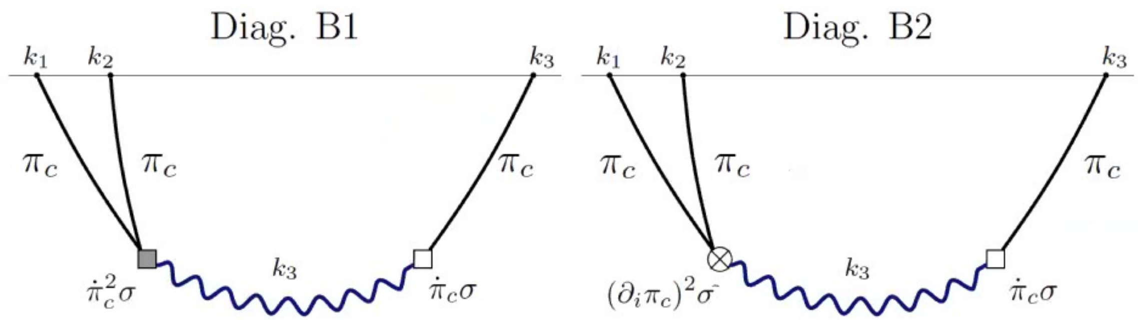


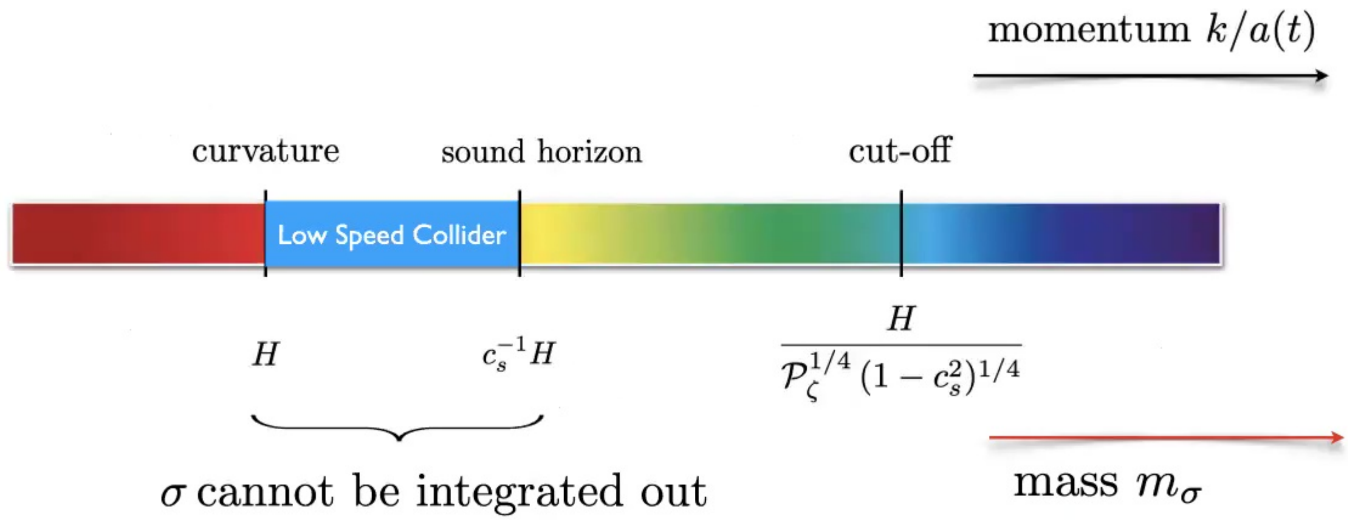
σ can be integrated out

(non-relativistic at sound horizon crossing)


ultra-squeezed
moderately squeezed
Equilateral
particle
production signal
(non-local) EFT signal
$\sim \mathcal{O}(1) c_s \frac{m}{H}$
k_L / k_S

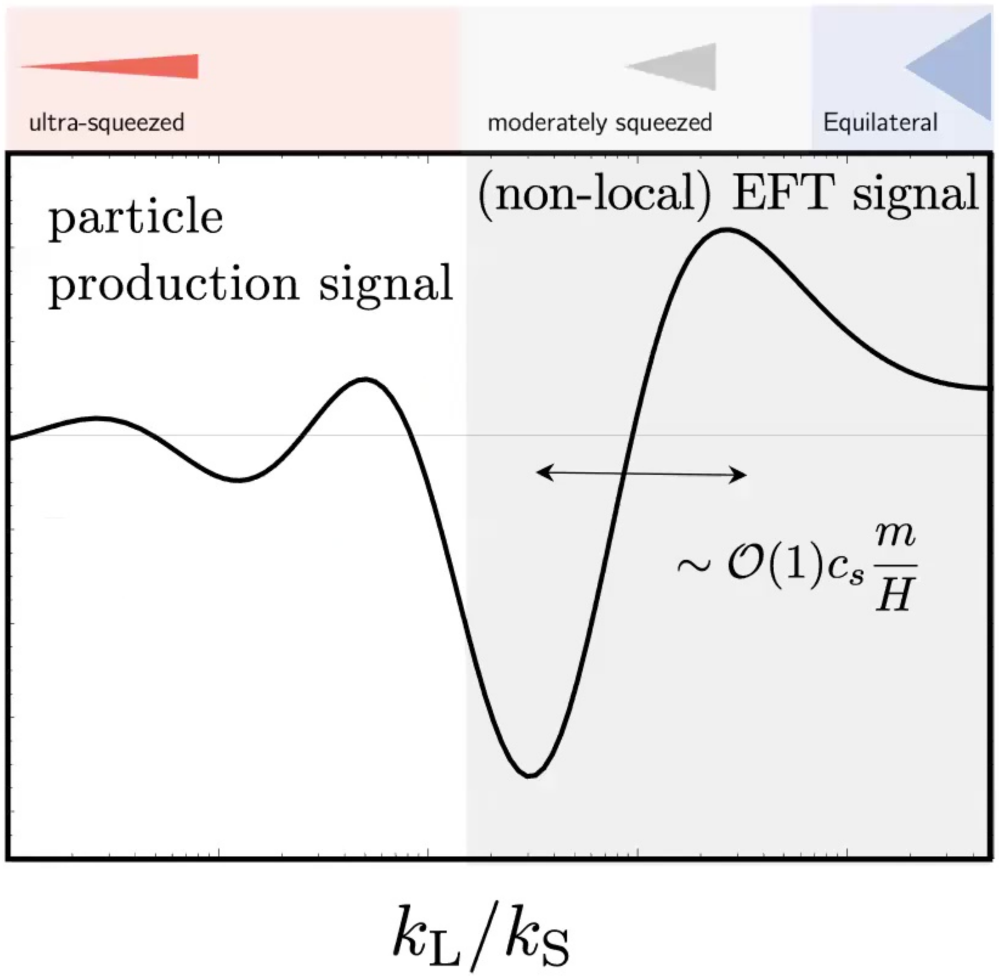
$$S_{\pi\sigma} = \int d\eta d^3\mathbf{x}\, a^2 \left(\rho a \pi_c' \sigma + \frac{1}{\Lambda_1}\pi_c'^2\sigma + \frac{c_s^2}{\Lambda_2}(\partial_i \pi_c)^2\sigma\right) \qquad \pi_c = \sqrt{2\epsilon} H M_{\rm Pl} c_s^{-1}\pi$$



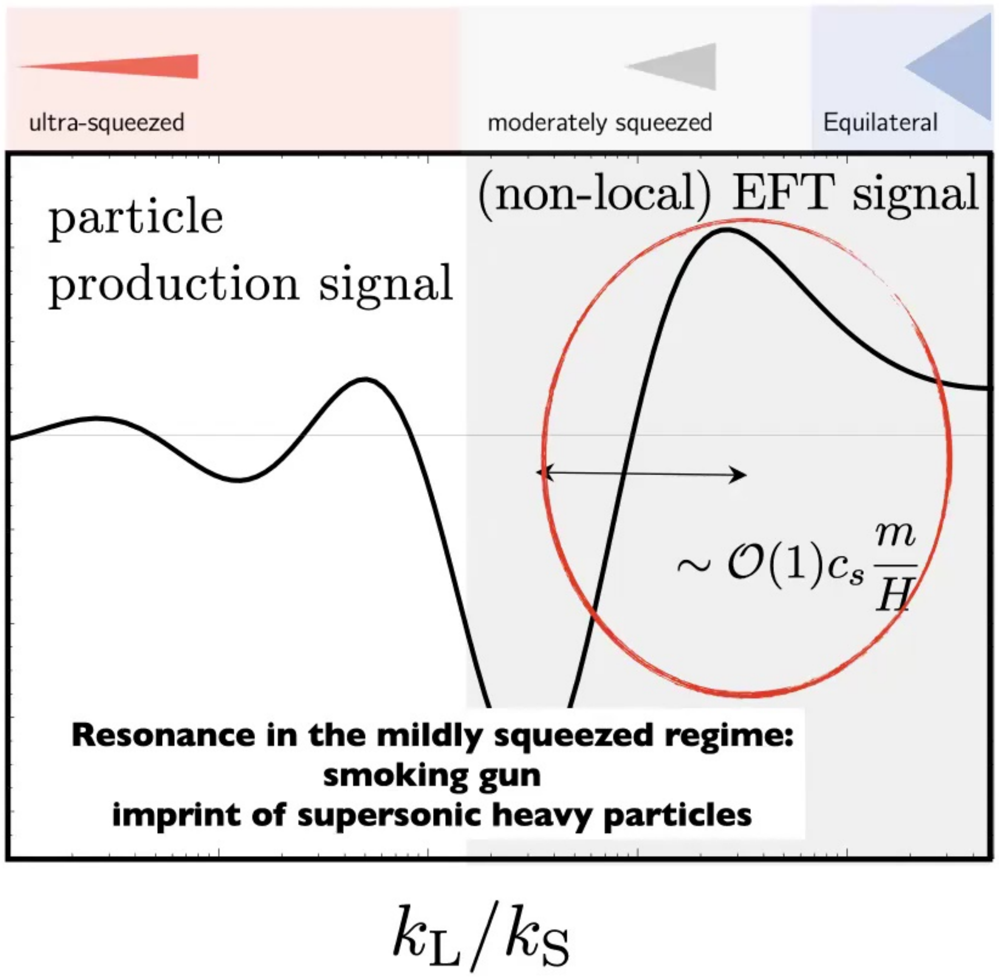


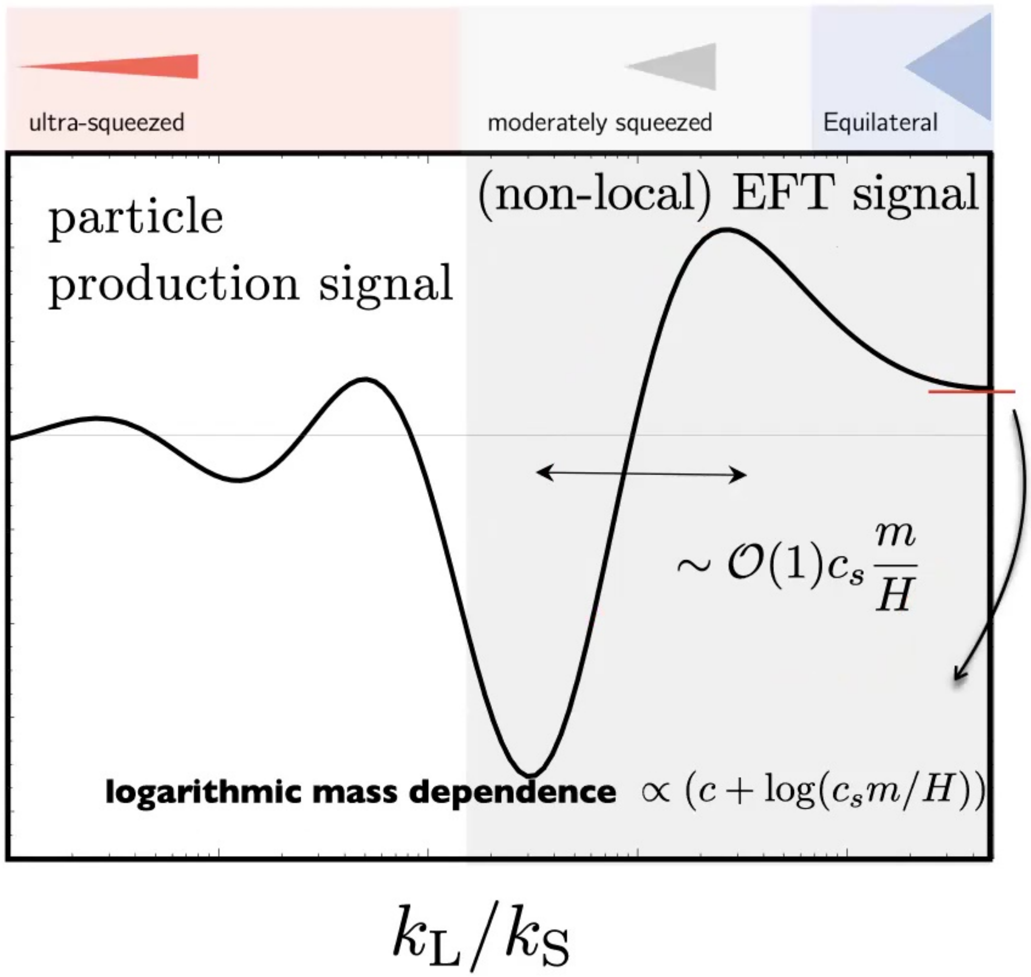
σ cannot be integrated out

(relativistic at sound horizon crossing)

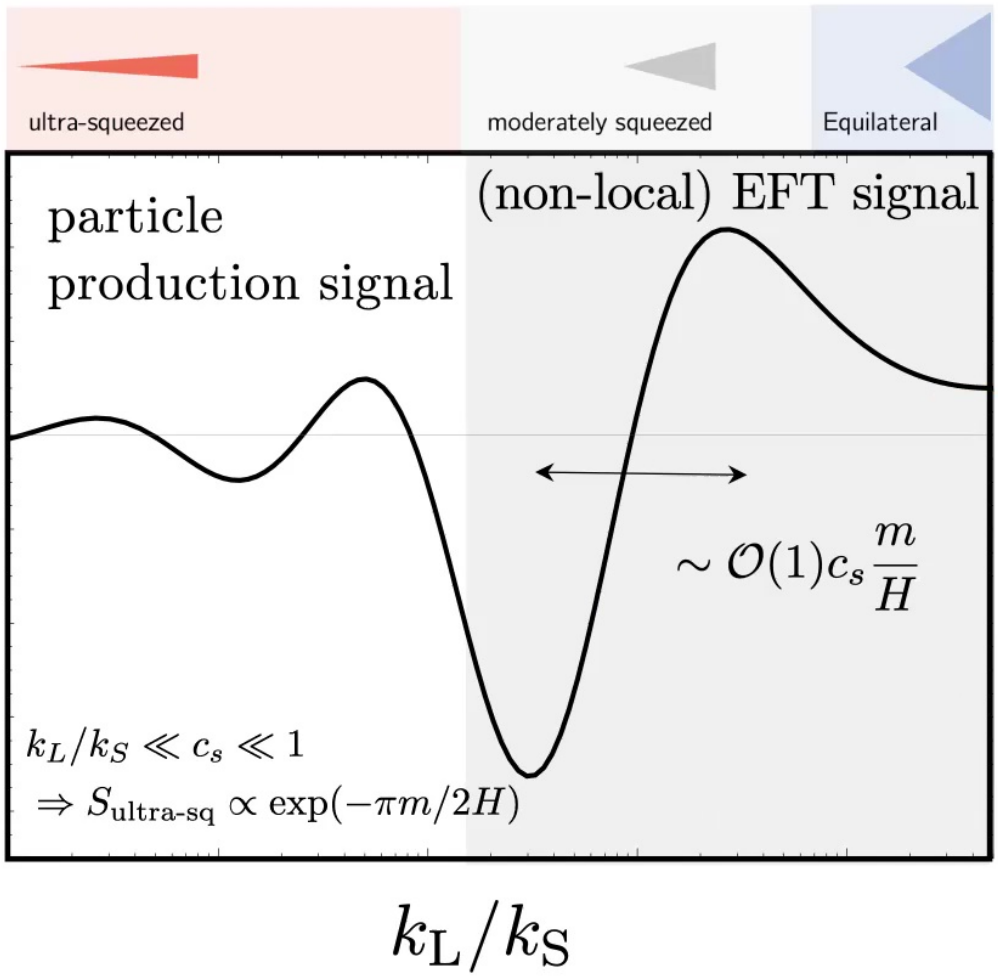


ultra-squeezed
moderately squeezed
Equilateral
particle
production signal
(non-local) EFT signal
$\sim \mathcal{O}(1) c_s \frac{m}{H}$
$k_\mathrm{L}/k_\mathrm{S}$

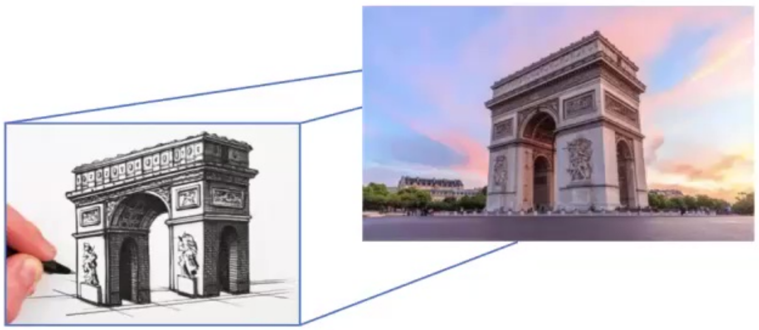

ultra-squeezed
moderately squeezed
Equilateral
particle production signal
(non-local) EFT signal
$\sim \mathcal{O}(1) c_s \frac{m}{H}$
Resonance in the mildly squeezed regime:
smoking gun
imprint of supersonic heavy particles
$k_{\mathrm{L}}/k_{\mathrm{S}}$

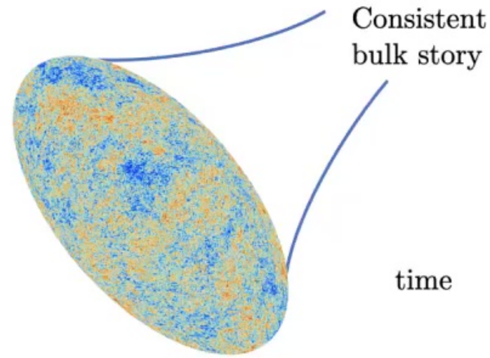


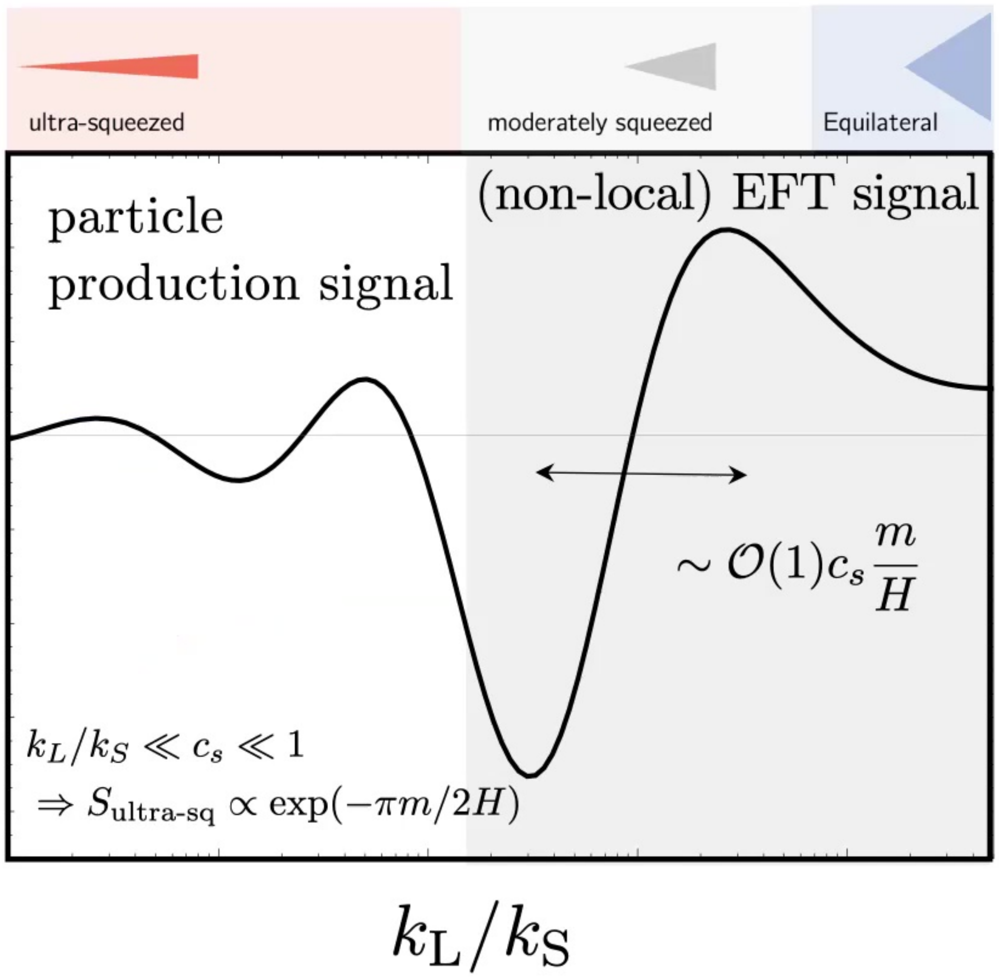
ultra-squeezed
moderately squeezed
Equilateral
particle production signal
(non-local) EFT signal
$\sim \mathcal{O}(1) c_s \frac{m}{H}$
logarithmic mass dependence $\propto (c + \log(c_s m/H))$
k_L/k_S


ultra-squeezed
moderately squeezed
Equilateral
particle production signal
(non-local) EFT signal
$\sim \mathcal{O}(1) c_s \frac{m}{H}$
$k_L/k_S \ll c_s \ll 1$
$\Rightarrow S_{\text{ultra-sq}} \propto \exp(-\pi m/2H)$
$k_{\mathrm{L}}/k_{\mathrm{S}}$

II Cosmological bootstrap

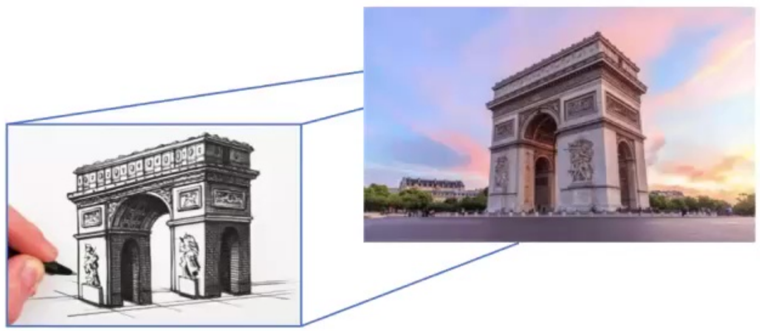


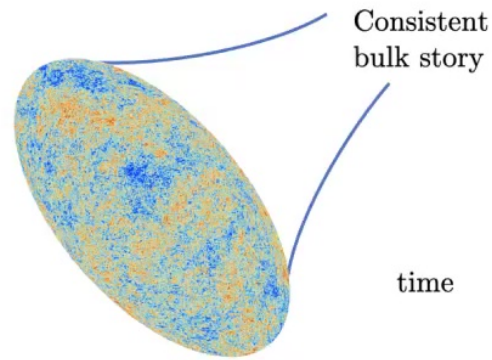




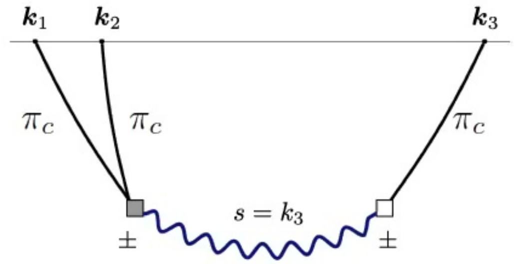
ultra-squeezed
moderately squeezed
Equilateral
particle production signal
(non-local) EFT signal
$\sim \mathcal{O}(1) c_s \frac{m}{H}$
$k_L/k_S \ll c_s \ll 1$
$\Rightarrow S_{\text{ultra-sq}} \propto \exp(-\pi m/2H)$
$k_\mathrm{L}/k_\mathrm{S}$

II Cosmological bootstrap





The In-in Computation is difficult



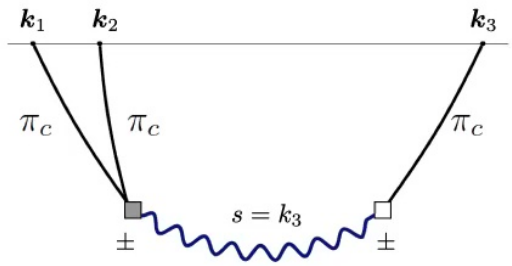
$$\langle \pi_c^3 \rangle = \sum_{\pm \text{ at each vertex}} \int \prod_{i=1}^{V} d\eta_i \ \text{vertex}_i(\boldsymbol{k}_i, \eta_i)$$

$$\prod_{j=1}^{n} \pi_c^{\mp}(k_j, \eta_i)\pi_c^{\pm}(k_j, \eta_0) \times \prod_{l=1}^{L} G_{\pm\pm}(s_l, \eta_{1l}, \eta_{2l})$$

bulk-to-boundary

bulk-to-bulk

The In-in Computation is difficult



$$\langle \pi_c^3 \rangle = \sum_{\pm \text{ at each vertex}} \int \prod_{i=1}^{V} d\eta_i \ \text{vertex}_i(\boldsymbol{k}_i, \eta_i)$$

$$\prod_{j=1}^{n} \pi_c^{\mp}(k_j, \eta_i) \pi_c^{\pm}(k_j, \eta_0) \times \prod_{l=1}^{L} G_{\pm\pm}(s_l, \eta_{1l}, \eta_{2l})$$

mode functions

$$\pi_c^{\pm}(k, \eta) = \frac{iH}{\sqrt{2c_s^3 k^3}}(1 \pm ic_s k\eta) \exp(\mp ic_s k\eta)$$

$$\sigma_+(k, \eta) = \frac{\sqrt{\pi} H}{2} \exp(-\pi\mu/2) \exp(i\pi/4)(-\eta)^{3/2} H_{i\mu}^{(1)}(-k\eta)$$

bulk-to-bulk propagators

$$G_{++}(s, \eta, \eta') = \sigma_+(s, \eta)\sigma_-(s, \eta')\theta(\eta - \eta') + \eta \leftrightarrow \eta'$$

$$G_{+-}(s, \eta, \eta') = \sigma_+(s, \eta)\sigma_-(s, \eta')$$

Pros	Cons
Explicitly unitary (Hermitian Hamiltonian)	**Complex nested time integrals** (due to the lack of time translation in cosmology)
Explicitly local (local interactions at vertices)	**Complex massive field mode functions**
Explicit invariance under putative symmetries (e.g. de Sitter isometries)	**Redundancies** field redefinitions Gauge/Diff transformations

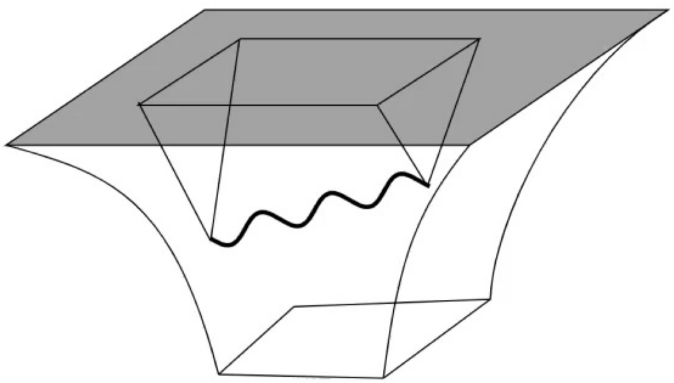
7

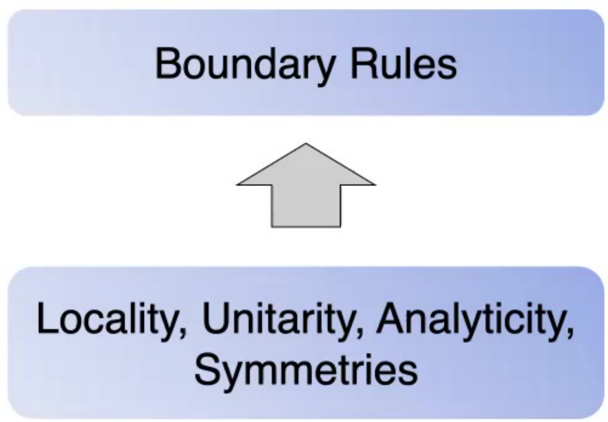
Cosmological bootstrap

Shifting the perspective on cosmological correlators: finding them without directly following the bulk time evolution. Active field.

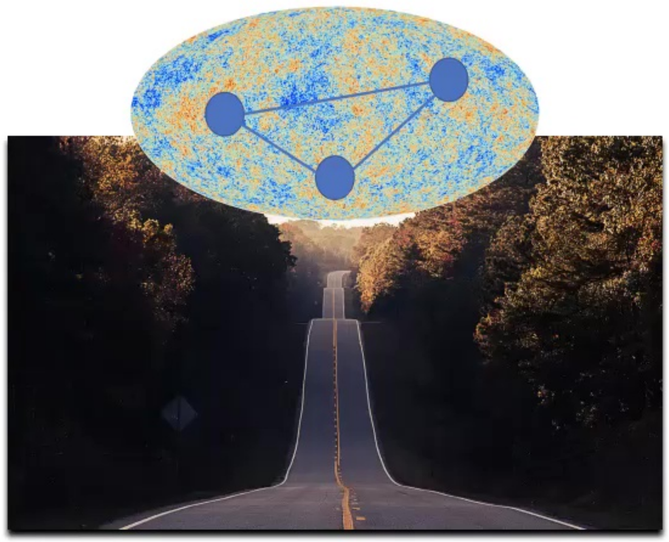
2017-2022: Arkani-Hamed, Baumann, Benincasa, Duaso Pueyo, Goodhew, Gorbenko, Jazayeri, Joyce, Lee, Meltzer, Melville, Pajer, Penedones, Pimentel, Renaux-Petel, Sleight, Salehi-Vaziri, Stefanyszyn, Tarona ….

Earlier works: Bzowski et al (2011,2012, 2013), Raju (2012), Kundo et al (2013, 2015), Maldacena and Pimentel(2011)

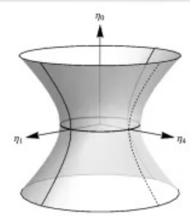




From a de Sitter seed four-point to inflationary three-points



with same method: 2 pt, 4 pt,
higher order derivatives (see paper)

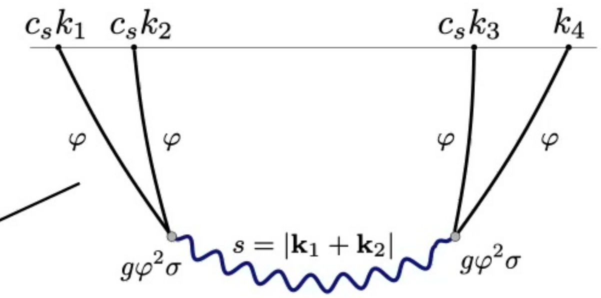


See 2205.00013 for another method,
for bispectrum only [no resonance]

Bispectra from a dS four-point

All our diagrams (2-,3-,4-pt) can be related to a **de Sitter-invariant seed four-point function** of a conformally coupled field, e.g.:

$$B(k_1, k_2, k_3) = W(k_i, \partial_{k_i}) \lim_{k_4 \to 0}$$



$$\varphi_\pm(k, \eta) = -\frac{H}{\sqrt{2k}} \eta \, \exp(\mp ik\eta)$$ (relativistic dispersion relation)

Boost breaking manifests itself both

- in the weight-shifting operators (*boost breaking vertices*)

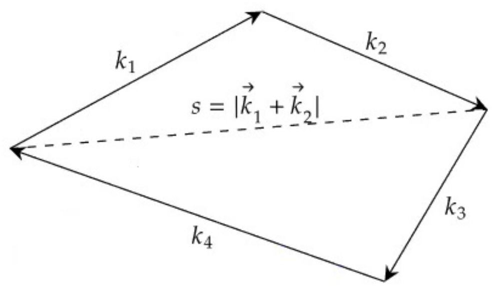
- and also in the argument of the four-point function (*different speeds of propagation*) $k_i \, (i = 1, 2, 3) \to c_s k_i$

Bispectra from a dS four-point

The seed correlator $\hat{F}(u,v)$ has been found in 1811.00024

$$u = \frac{s}{k_1 + k_2} \leq 1$$

$$v = \frac{s}{k_3 + k_4} \leq 1$$

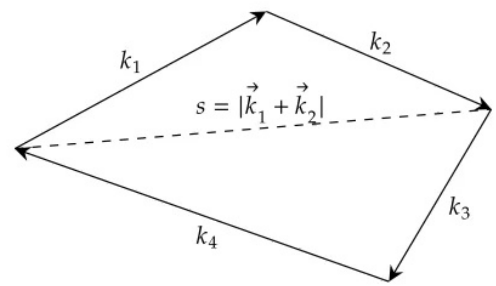


Bispectra from a dS four-point

The seed correlator $\hat{F}(u,v)$ has been found in 1811.00024

$$u = \frac{s}{k_1 + k_2} \leq 1$$

$$v = \frac{s}{k_3 + k_4} \leq 1$$



… but we need its analytical continuation beyond the kinematically allowed region

$$k_i(i=1,2,3) \to c_s k_i,\ k_4 \to 0,\ s = |\mathbf{k}_3 + \mathbf{k}_4| \to k_3$$

$$u \to \frac{k_3}{c_s(k_1+k_2)}, v \to \frac{1}{c_s}$$

Building blocks of the seed correlator

$$\hat{F} = \hat{F}_{++} + \hat{F}_{--} + \hat{F}_{+-} + \hat{F}_{-+}$$

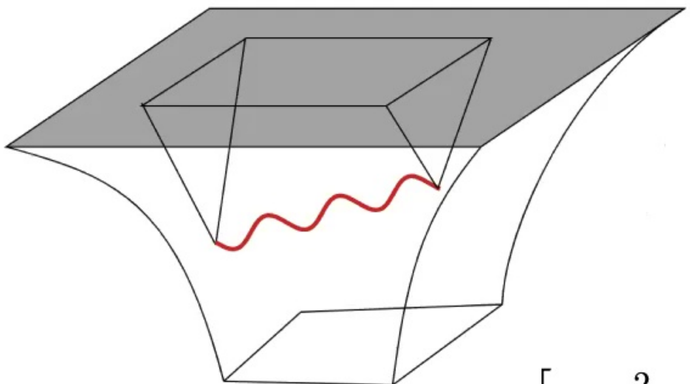
Difficult — Truly nested integrals ($\hat{F}_{++} + \hat{F}_{--}$)

Easiest part — Factorised time integrals ($\hat{F}_{+-} + \hat{F}_{-+}$)

$$F_{\pm\pm}(k_1, \ldots, k_4; s) = -\frac{g^2}{2H^2} \int_{-\infty(1\mp i\epsilon)}^{\eta_0} \frac{d\eta}{\eta^2} \int_{-\infty(1\mp i\epsilon)}^{\eta_0} \frac{d\eta'}{\eta'^2} e^{\pm i(k_1+k_2)\eta}\, e^{\pm i(k_3+k_4)\eta'} \times G_{\pm\pm}(s, \eta, \eta')$$

$$G_{++}(s, \eta, \eta') = \sigma_+(s, \eta)\sigma_-(s, \eta')\theta(\eta - \eta') + \eta \leftrightarrow \eta'$$

Bootstrap. I Locality



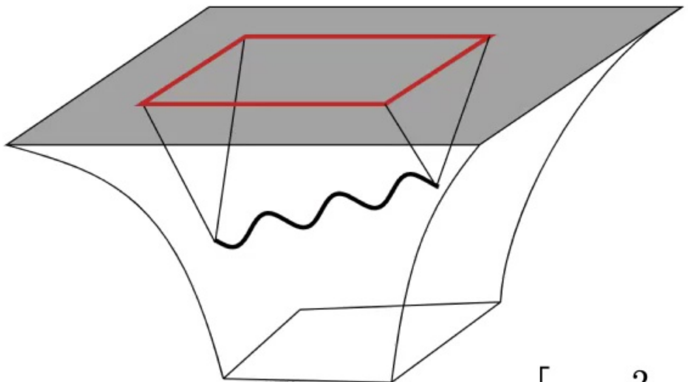
$$\left[\partial_\eta^2 - \frac{2}{\eta}\partial_\eta + k^2 + \frac{m^2}{\eta^2 H^2}\right] G_{\pm\pm}(s,\eta,\eta') = (\eta' H)^2 \delta(\eta - \eta')$$

Bulk local differential equation

Bootstrap. I Locality

$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \left(\mu^2 + \frac{1}{4}\right)\right]\hat{F}_{\pm\pm}(u,v) = g^2\frac{u\,v}{2(u+v)}$$

Boundary differential equation



$$\left[\partial_\eta^2 - \frac{2}{\eta}\partial_\eta + k^2 + \frac{m^2}{\eta^2 H^2}\right]G_{\pm\pm}(s,\eta,\eta') = (\eta' H)^2\delta(\eta-\eta')$$

Bulk local differential equation

Bootstrap. I Locality

$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \left(\mu^2 + \frac{1}{4}\right)\right]\hat{F}_{\pm\pm}(u,v) = g^2\frac{u\,v}{2(u+v)}$$

$$\hat{F}_{++}(u,v) = \sum_{m,n}\left(a_{m,n} + b_{m,n}\log(u)\right)\frac{1}{u^m}\left(\frac{u}{v}\right)^n + \sum_{\pm\pm}\beta_{\pm\pm}f_\pm(u)f_\pm(v)\,, \quad 1 < |u| < |v|$$

Suitable particular solution
"from 'infinity"

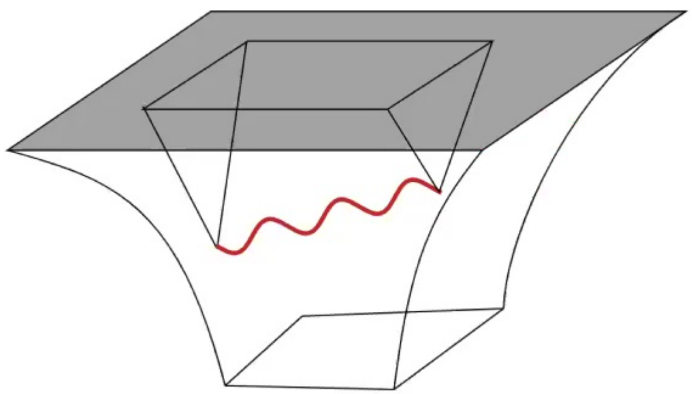
Homogeneous solution with
four free parameters to determine

$$f_+(u) = {}_2F_1\left(\frac{1}{4} - \frac{i\mu}{2}, \frac{1}{4} + \frac{i\mu}{2}; \frac{1}{2}; \frac{1}{u^2}\right)$$

$$f_-(u) = \frac{2}{u} \times {}_2F_1\left(\frac{3}{4} - \frac{i\mu}{2}, \frac{3}{4} + \frac{i\mu}{2}; \frac{3}{2}; \frac{1}{u^2}\right)$$

Series coefficients
and partial resummation

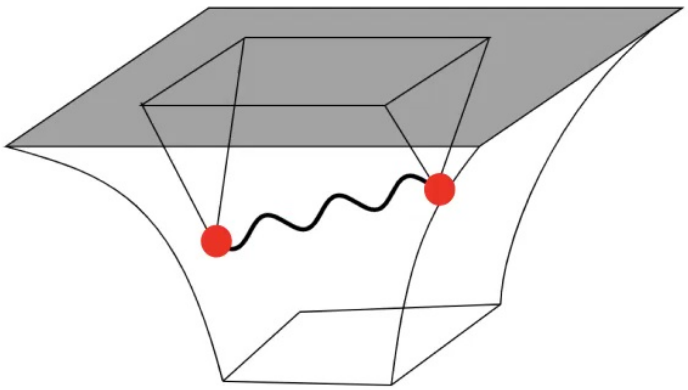
Bootstrap. 2 Unitarity



Cutting rule for the bulk to bulk propagator (energy positivity)

$$G^*_{++}(s,\eta,\eta') + G_{++}(s,\eta,\eta') = \sigma_-(s,\eta)\sigma_+(s,\eta') + \eta \leftrightarrow \eta'$$

Bootstrap. 2 Unitarity

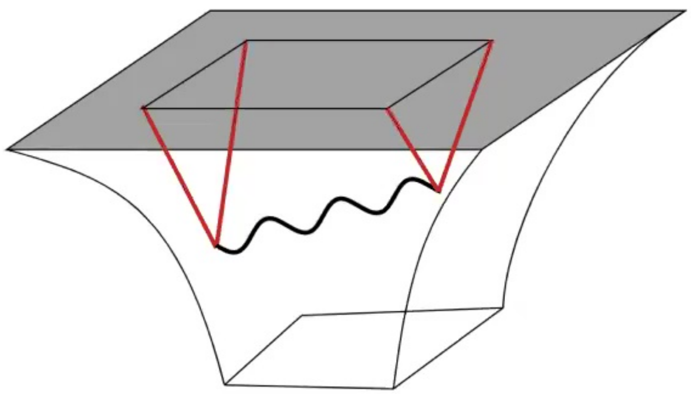


Reality of the couplings $g* = g$

Cutting rule for the bulk to bulk propagator (energy positivity)

$$G^*_{++}(s,\eta,\eta') + G_{++}(s,\eta,\eta') = \sigma_-(s,\eta)\sigma_+(s,\eta') + \eta \leftrightarrow \eta'$$

Bootstrap. 2 Unitarity



Hermitian analyticity of the bulk to boundary propagator

$$\varphi_+^*(k,\eta) = \varphi_+(-k,\eta)$$

Reality of the couplings $g* = g$

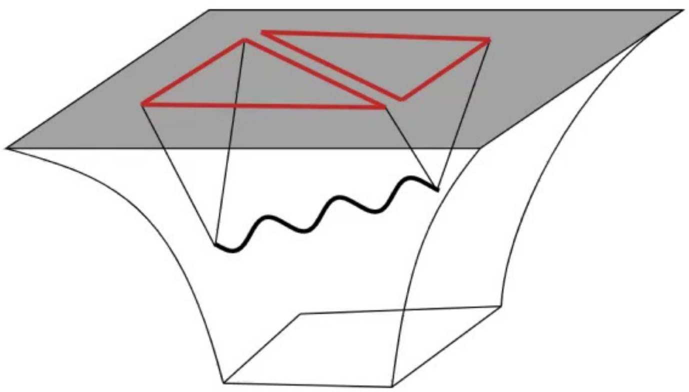
Cutting rule for the bulk to bulk propagator (energy positivity)

$$G_{++}^*(s,\eta,\eta') + G_{++}(s,\eta,\eta') = \sigma_-(s,\eta)\sigma_+(s,\eta') + \eta \leftrightarrow \eta'$$

Bootstrap. 2 Unitarity

Cosmological Cutting Rule

$$\hat{F}_{++}(u,v) + \hat{F}^*_{++}(-u^*, -v^*) = -\frac{1}{2}\hat{f}_3(u)\hat{f}^*_3(-v^*) - \frac{1}{2}\hat{f}_3(v)\hat{f}^*_3(-u^*)$$



Hermitian analyticity of the bulk to boundary propagator

$$\varphi^*_+(k,\eta) = \varphi_+(-k,\eta)$$

Reality of the couplings $g* = g$

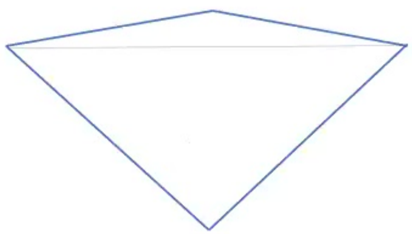
with $$\hat{f}_3(u) = \frac{ig}{2\sqrt{2\pi}}\left(|\Gamma(1/4 + i\mu/2)|^2 f_+(u) - |\Gamma(3/4 + i\mu/2)|^2 f_-(u)\right)$$

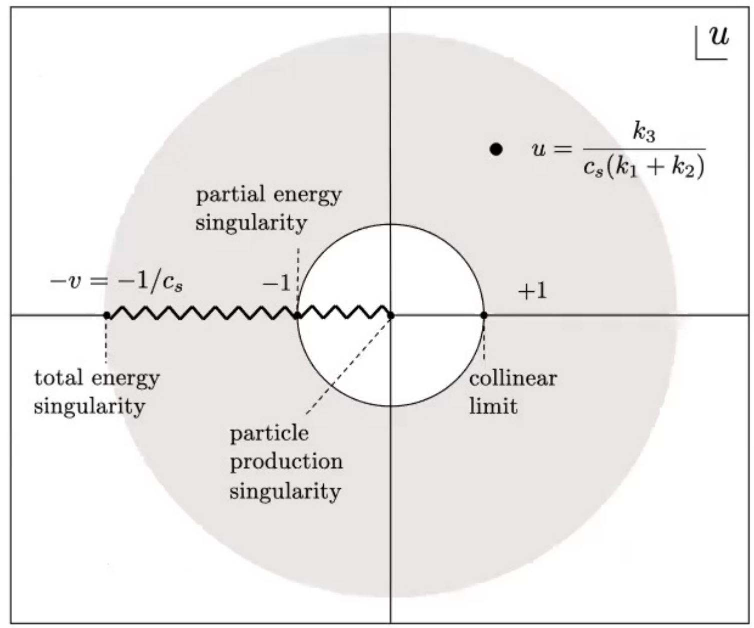
Bootstrap. 3 Analyticity

For Bunch-Davies initial conditions, no singularity can arise in the physical domain of the correlator

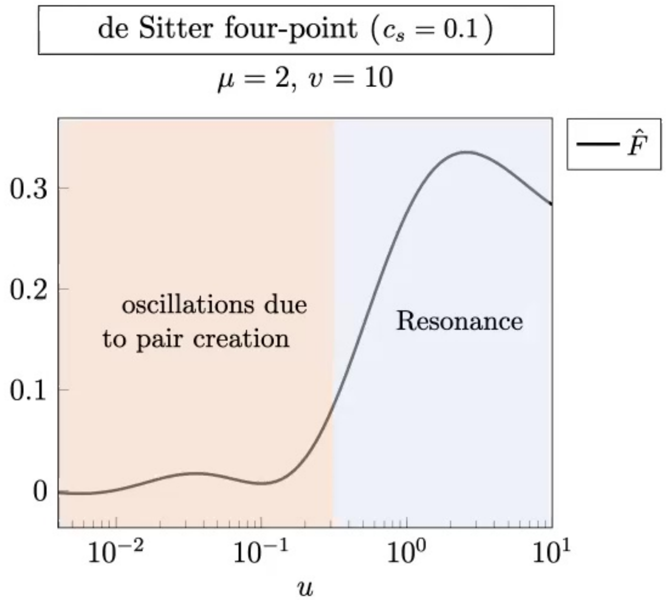
Requiring absence of singularity in the collinear (folded) limit

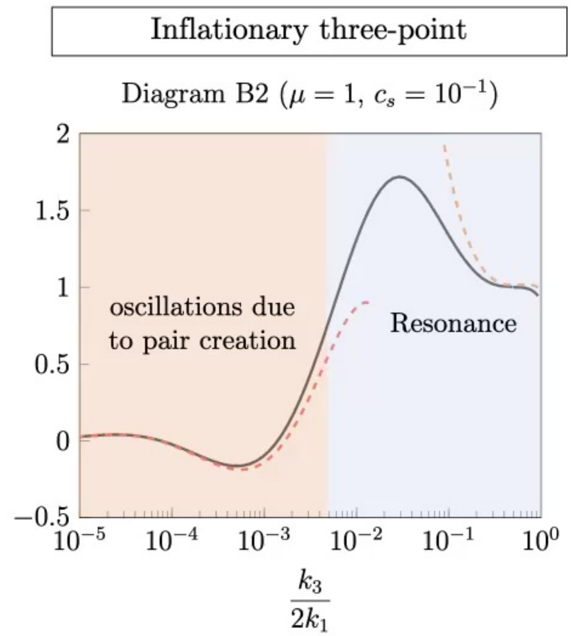
$$u \to 1$$





• Low speed collider $m < H/c_s$





Concluding remarks

- Cosmological bootstrap offers a powerful set of tools for computing the cosmological correlators.
- Using these methods one can enrich the menu of observational templates by computing novel analytical expressions for the correlators, some of which were not attainable until few years ago.
- In this work, we extended the reach of cosmological collider physics beyond the de Sitter invariant setup. We discovered new signatures of massive fields on primordial non-Gaussianity thanks to strongly broken dS boosts.
- Some fruitful future directions: more general diagrams (with multiple exchange) potentially with larger non-Gaussianity, incorporating boost-breaking massive spinning fields, etc.