

Title: The relative Langlands program via gauge theory cont.

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G reductive group / F

$$\leadsto \text{Bun}_G = [G]_F$$

[really: fix level structure abg

(link $\subset 3\text{-manifold}$)



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A_G : Study ~~linearization~~ of B_m_G (eg $H^*(\mathbb{A}_G/\mathbb{F})$)

LOC_G : character stack of



Period

$$H \subset G$$

$$\text{Bun}_H \subset \text{Bun}_G$$

$$P_H(\varphi) = \int_{\text{Bun}_H} \varphi$$



$$G(F) \backslash G(A) \times X(F) / G(O)$$

Diagram showing two circles. The left circle contains $X(A) / G(O)$ and is labeled with $G(F) \backslash G(A) \times X(F) / G(O)$ above it. The right circle contains Bu_G and is labeled with $G(F) \backslash G(A) \times X(F) / G(O)$ above it.

$$G(F) \backslash G(A) / G(O)$$

$$\prod X(K) / G(O)$$



$$G(F) \backslash G(A) \times X(F) / G(O)$$

$$\downarrow$$

$$\text{X(A) / G(O)}$$

$$\downarrow$$

$$\Pi \text{ X(K) / G(O)}$$

$$B_{u_6}$$

$$\downarrow$$

$$G(F) \backslash G(A) / G(O)$$



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×

MITLieGroups

×

Perimeter Langlands

×

Langlands At Perimeter



Riemann

$$(\Gamma \cdot \pi) \zeta(s) = \int_0^\infty y^{s/2} \sum e^{-n^2 \pi y} dy$$

Iwasawa, Tate :



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Further field setting (unvaried)

$$\text{Bun}_G X = \text{G-bundles} + \text{Section of } X^{\text{ad}}$$

$\pi \rightarrow$

Bun_G

$$P = \pi_1 \frac{1}{f}$$



Further field setting (unvarified)

$$\text{Bun}_G X = G\text{-bundles} + \text{Section of } X^{\text{hd}}$$

$\pi \rightarrow$

Bun_G

$$P = \pi_1 \frac{1}{f}$$



Geometric version $k = \overline{k} \left(\begin{matrix} \mathbb{C} \\ \text{or} \\ \overline{\mathbb{F}_q} \end{matrix} \right)$

$$\text{Bun}_G X \xrightarrow{\pi} \text{Bun}_G$$

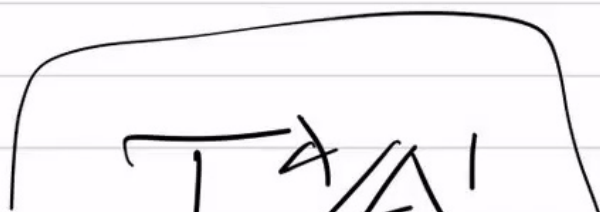
$$\mathcal{P}_X = \pi_! \mathbb{C}_{\text{Bun}_G X} \in \text{Sh}_v(\text{Bun}_G)$$



Tate's thesis: functional eqn

for $\xi \leftarrow$

T-convolution transform on A'





Finiteness results

$G \curvearrowright X$ spherical varieties

$=$ nonabelian toric varieties

$G \supset R$ has open orbit in X



- for: λ
- flag $(G/N \cong G \times T)$
- symmetric G/K
- $G \times G \hookrightarrow G$



- for:z

- flag

($G/N \cong G \times T$)

- Symmetric G/K

- $G \times G \hookrightarrow G$



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Microlocal version $X \rightsquigarrow M$

$G \xrightarrow{C} \downarrow$
 $\times \quad \curvearrowright \quad ag^*$
 G_m

Spherical: coisotropic / anisotropic one



Structure Near (BZSV)

M smooth affine hypersurface
 [+ generic stabilizer removed]
 + weak map

→ M obtained by



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Structure Near (BZSV)

M smooth affine hypersurface

[+ generic stabilizer connected]
+ weak map

→ M obtained by



Knap Gaitsgory - Noller

[S] (SV) [SW]

$$\boxed{G \hookrightarrow X} \Rightarrow W_X \hookrightarrow A_X^*$$



Mota - Conjecture

M

M^v ch-d

{hyperspherical
varieties}



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	local	global
arithmetic		
geometric	$\text{Sh}(G_X/G) \cong$ $\text{QC}^!(G) \ni \text{f}$	$\text{Sh}(Bun_G) \ni P_M$



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cr. theory

$$L^2(X(F))$$

y b b u l

$$P_M = L_M^\vee$$

$$G(M^\vee F)$$

geodesic

$$Sh(L_X/L_G)$$

$$Sh(Bun_G) \ni P_M$$

is

$$QC^!(G) \ni f$$

$$G/M^\vee$$