Title: The relative Langlands program via gauge theory cont.

Speakers: David Ben-Zvi

Collection: QFT for Mathematicians 2022

Date: June 30, 2022 - 10:45 AM

URL: https://pirsa.org/22060092

G reductive group $/F$

$\rightsquigarrow \mathrm{Bun}_G = [G]_F$

[really: fix level structure along

(link $\subset$ 3-manifold)

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G reductive group $/F$

$\rightsquigarrow \mathrm{Bun}_G = [G]_F$

[really: fix level structure away

(ink $\leq$ 3-manifold)

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G reductive group $/F$

$\rightsquigarrow \mathrm{Bun}_G = [G]_F$

[really: fix level structure aby

(ink $\subset$ 3-manifold)

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A_G : study $\mathbb{C}$- linearization of

Bun_G (eg $H^*([G]_{\overline{\mathbb{F}}}, \mathbb{C})$)

$LOC_{G^\vee}$: character stack of

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Period

$H \subset G$

$$\boxed{Bun_H \subset Bun_G}$$

$$P_H(\varphi) = \int_{Bun_H} \varphi$$

$$G(F)\backslash G(\mathbb{A}) \times X(F)/G(\mathcal{O})$$

$$X(\mathbb{A})/G(\mathcal{O}_{\mathbb{A}}) \qquad\qquad \mathrm{Bun}_G$$

$$\rotatebox{90}{=} \qquad\qquad\qquad\qquad \rotatebox{90}{=}$$

$$\prod X(K)/G(\mathcal{O}) \qquad\qquad G(F)\backslash G(\mathbb{A})/G(\mathcal{O}_{\mathbb{A}})$$

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$$G(F)\backslash G(\mathbb{A}) \times X(F)/G(\mathcal{O})$$

$$X(\mathbb{A})/G(\mathcal{O}_{\mathbb{A}}) \qquad \mathrm{Bun}_G$$

$$\shortparallel \qquad \shortparallel$$

$$\prod X(K)/G(\mathcal{O}) \qquad G(F)\backslash G(\mathbb{A})/G(\mathcal{O}_{\mathbb{A}})$$

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Riemann

$$(\Gamma \cdot \pi)\,\xi(s) = \int_0^\infty y^{s/2} \sum e^{-n^2 \pi y} \, dy$$

Iwasawa, Tate :

Function field setting (unramified)

$\mathrm{Bun}_G X$ = G-bundles + ~~sectn~~ of X bdle

$$\mathrm{Bun}_G X \xrightarrow{\pi} \boxed{\mathrm{Bun}_G}$$

$P = \prod \frac{1}{\ldots}$

Functn field setting (unramified)

$\mathrm{Bun}_G X$ = G-bundles + sectn of X bdl

$\mathrm{Bun}_G X \xrightarrow{\pi} \boxed{\mathrm{Bun}_G}$

$P =$

Geometric version $\quad k = \bar{k} \quad \left(\mathbb{C} \text{ or } \bar{\mathbb{F}}_q\right)$

$$\mathrm{Bun}_{G,X} \xrightarrow{\pi} \mathrm{Bun}_G$$

$$P_X = \pi_! \, \mathbb{C}_{\mathrm{Bun}_{G,X}} \in \mathrm{Shv}(\mathrm{Bun}_G)$$

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Tate's thesis: functional eqn

for ξ $\longleftarrow$

Fourier transform on $\mathbb{A}^1$

$T^*\mathbb{A}^1$

Finiteness conditions

$G \curvearrowright X$ spherical varieties

$=$ nonabelian toric varieties

- $G \supset B$ has open orbit on X

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- toric
- flag $(G/N \circlearrowleft G \times T)$
- symmetric G/K
- $G \times G \circlearrowright G$

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- toric
- flag $(G/N \supset G \times T)$
- symmetric G/K
- $G \times G \circlearrowright G$

Microlocal version

$$\begin{tikzcd} X \arrow[r, squiggly] & M \arrow[d] & G \circlearrowright M \\ & \mathfrak{g}^* & G \times \mathbb{G}_m \circlearrowright \mathfrak{g}^* \end{tikzcd}$$

Spherical: coisotropic / multf. one

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Structure Theorem (B2SV)

M smooth affine hyperspherical

[+ generic stabilizer connected]

+ moment map

$\Rightarrow$ M obtained by

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Structure Theorem (BZSV)

M smooth affine (hyperspherical)

[+ generic stabilizer connected]

+ moment map

$\to$ M obtained by

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Knop Gaitsgory-Nadler

$\{S\}$ (SV) $\{SW\}$

$$\boxed{G \circlearrowright X} \Longrightarrow W_X \circlearrowright A_X^*$$

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Meta-Conjecture

M $\qquad\qquad$ $M^\vee$ dual

{hyperspherical variety}

ρ

	local	global
arithmetic		
geometric	$Shv(LX/LG)$ $\cong$	$Shv(Bun_G) \ni P_M$ $\cong$ $QC^!(Loc_G) \ni f$

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	local	global
arithmetic	$L^2(X(F))$ $\mathcal{O}(M^{\vee F})$	$P_M = L_{M^\vee}$
geometric	$Shv(LX/LG)$ $\wr\!\!\mid$ $\mathcal{O}(M^\vee)$	$Shv(Bun_G) \ni P_M$ $\wr\!\!\mid$ $QC^!(Loc_G) \ni f$