

Title: The relative Langlands program via gauge theory

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Abstract: I will discuss a close parallel between Gaiotto and Witten's S-duality for supersymmetric boundary conditions in 4d $N=4$ SYM and the relative Langlands program, an enhancement of the Langlands program that was developed to provide a framework for the theory of integral representations of L-functions. A special and conjecturally self-dual class of boundary conditions is provided by quantizations of "small" or "multiplicity-free" hamiltonian spaces called hyperspherical varieties. I'll explain how a hyperspherical variety produces objects of interest in all the different settings of the Langlands program (local / global, geometric / arithmetic) and a collection of conjectures providing S-dual descriptions of these objects. The talk is based on forthcoming joint work with Yiannis Sakellaridis and Akshay Venkatesh.



Langlands duality
for periods & L-functions
via E-M duality

(w/ Yiannis Sakellaridis)

(Handwritten notes at the bottom of the page, partially obscured)

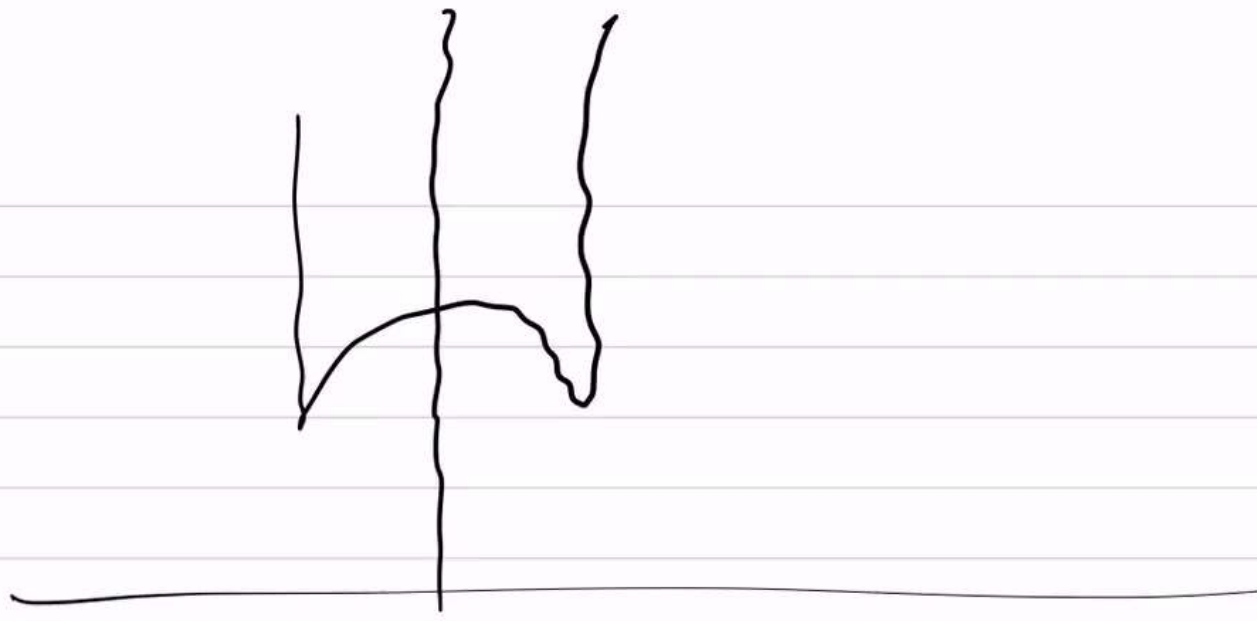


eg $W \Gamma \subset PSL_2 \mathbb{Z}$

e^H

φ hol. modular form

Period integrals



T - period $\int_{\gamma}^S \varphi(i\gamma) \frac{d\gamma}{\gamma}$



$a_0 \neq 0 \iff \rho$ reducible

$a_1 \iff \underline{1}(\rho)$

Whittaker normalization

$\sum a_n n^{-s}$ gives L-function of ρ

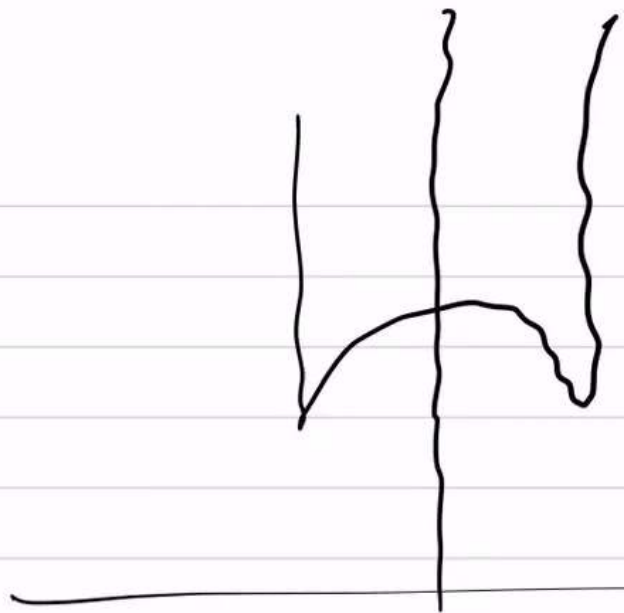


$a_0 \neq 0 \iff \rho$ reducible

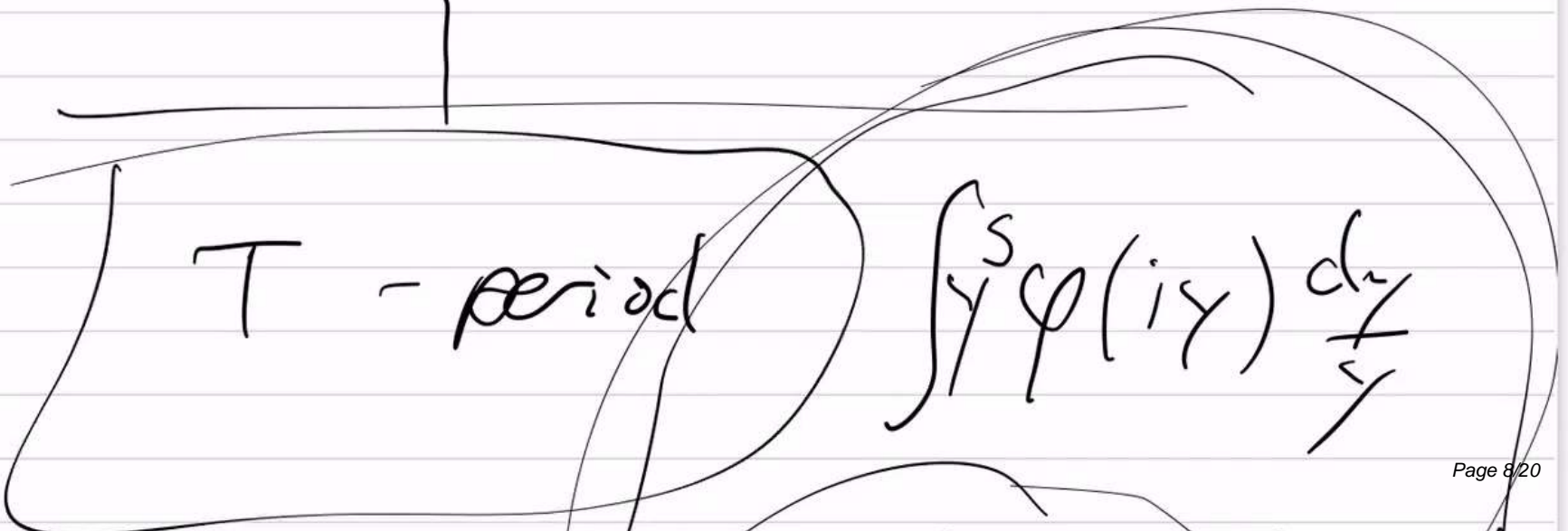
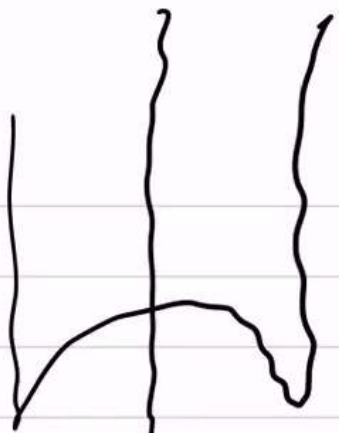
$a_1 \iff \underline{1}(\rho)$

Whittaker normalization

$\sum a_n n^{-s}$ gives L-function of ρ



T - period $\int_{\gamma}^S \varphi(i\gamma) \frac{d\gamma}{\gamma}$





Q

What is science behind
matching of L-functions
& periods?

A

periods \subset 2 rankers



Langlands program d'après K-W

G/\mathbb{C} reductive group $\Rightarrow A_G$

G^v/\mathbb{C} dual group $\Rightarrow B_G^v$

$A(S) \cong \text{Sh}_S(R, \Sigma)$



Key features matches action
of defect operators on
two sides

* local operators
Codim 4 $H^*(BG)^{AG} EY$
 $\downarrow \cup (op^{v*}[2]/G)$



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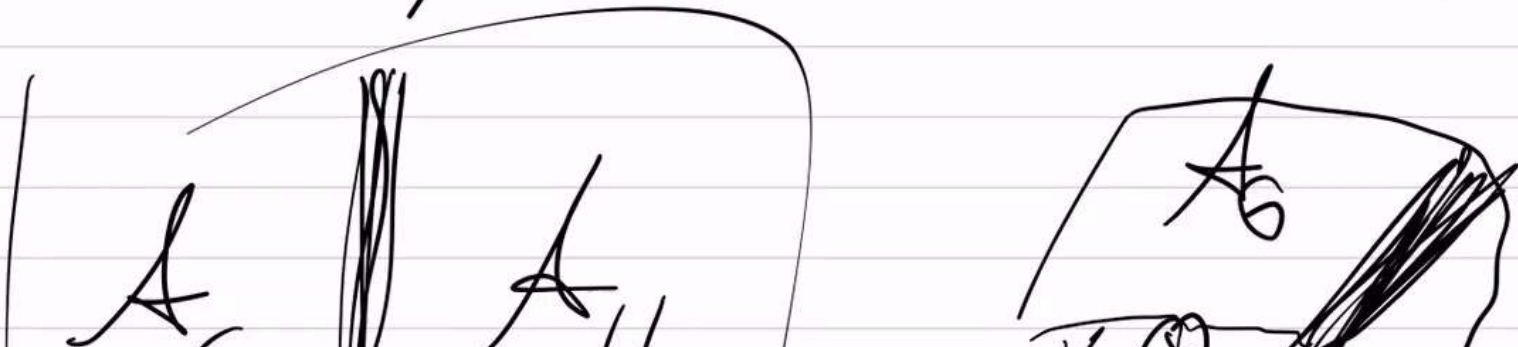
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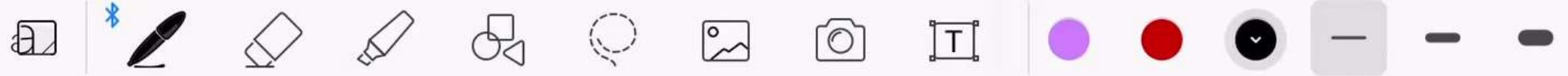


Richest story column 1

Gaiotto-Witten, Hilbert-Yoo

boundary conditions & interfaces





Examples come from quot. Zds
of hyperkähler G/G^v spaces
Zd A/B - model coupled
to Yd

Witten's conjecture

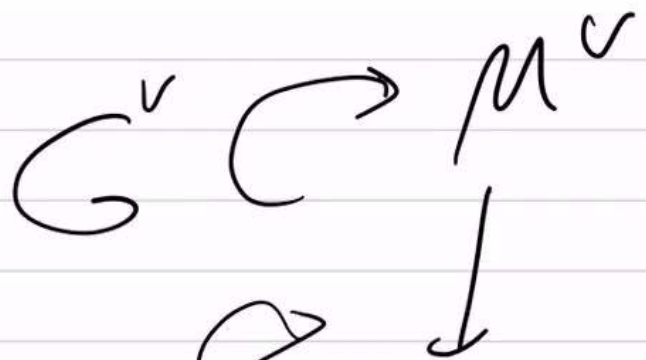


Examples come from quot. Zds
of hyperkähler G/G^v spaces
3d A/B - model coupled
to $4d$

Hamiltonian ...



— unteres (BAA) Gae
in $A_G(\Sigma)$



Sprechend Geister
Lagrange:



eg. \mathcal{O}_{Loc} / $M^u = \mathcal{P}^+$
 (Necman)

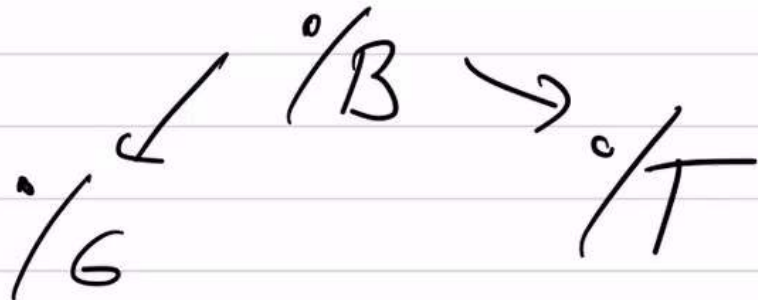
(ie. $\Gamma_{Loc}(-)$)

\longleftrightarrow Hitchin section



eg G B T Eisenstein

(Handwritten scribbles below G, B, and T)





Dictionary:

MMR . . .

F global field

number field

$TF(S)$



local field

\mathbb{Q}_p

$TF_q(f)$

\longleftrightarrow 2-months

