Title: The relative Langlands program via gauge theory

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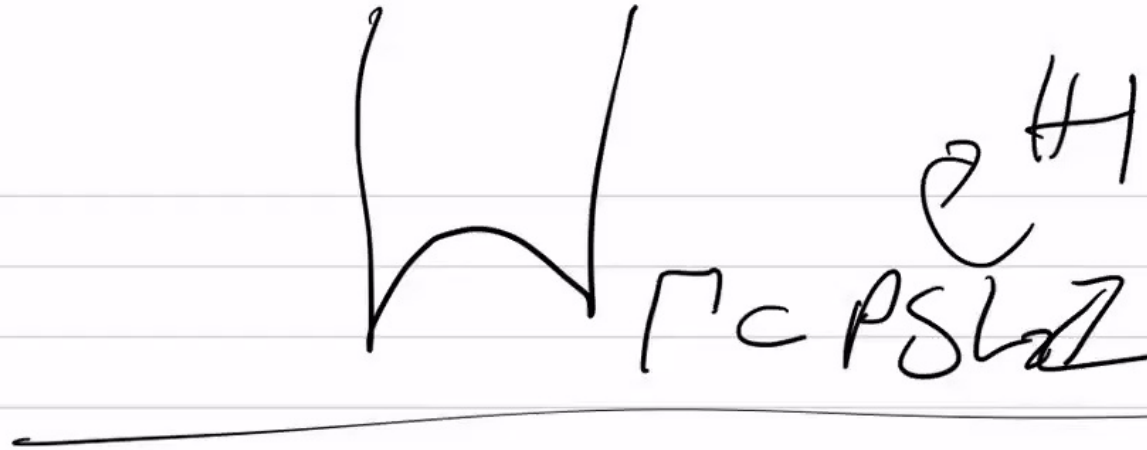
Abstract: I will discuss a close parallel between Gaiotto and Witten's S-duality for supersymmetric boundary conditions in 4d N=4 SYM and the relative Langlands program, an enhancement of the Langlands program that was developed to provide a framework for the theory of integral representations of L-functions. A special and conjecturally self-dual class of boundary conditions is provided by quantizations of "small" or "multiplicity-free" hamiltonian spaces called hyperspherical varieties. I'll explain how a hyperspherical variety produces objects of interest in all the different settings of the Langlands program (local / global, geometric / arithmetic) and a collection of conjectures providing S-dual descriptions of these objects. The talk is based on forthcoming joint work with Yiannis Sakellaridis and Akshay Venkatesh.

Langlands duality for periods & L-functions via E-M duality

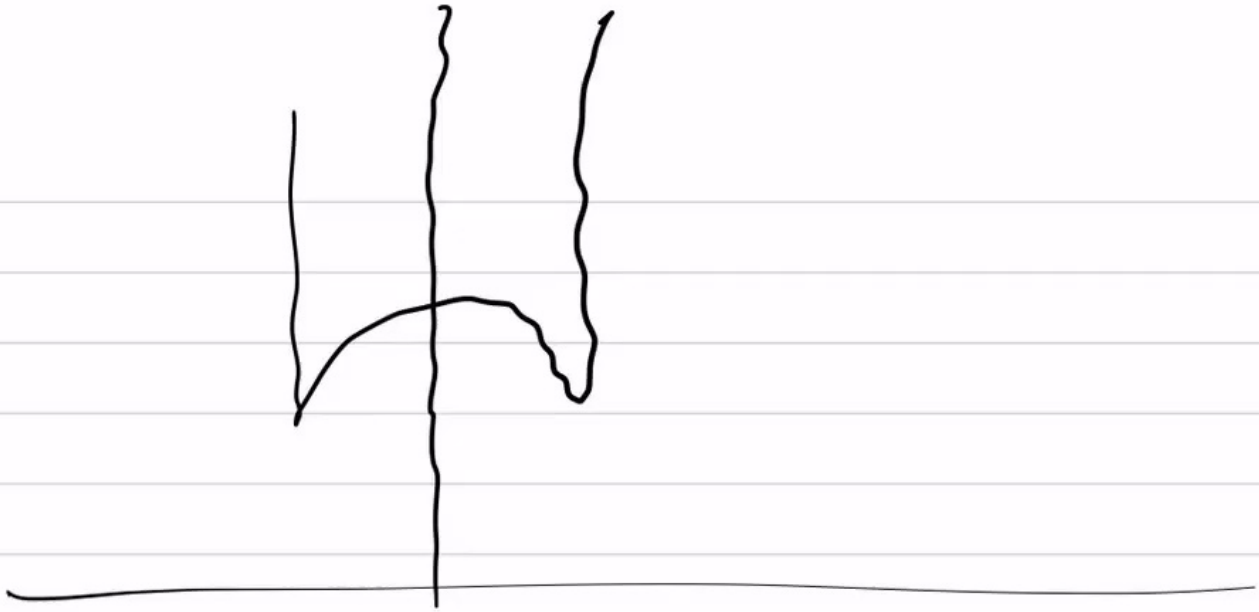
(w/ Yiannis Sakellaridis)



eg $\mathcal{H}$ $\Gamma \subset PSL_2\mathbb{Z}$ $e^{4\pi}$

φ hol. modular form

Period integrals



T - period

$$\int_{iy}^{S} \varphi(iy)\, dz$$

$a_0 \neq 0 \iff \rho$ reducible

$a_1 \longleftrightarrow$ Whittaker normalization $\underline{\quad}(\rho)$

$\sum a_n n^{-s}$ gives L-function of ρ

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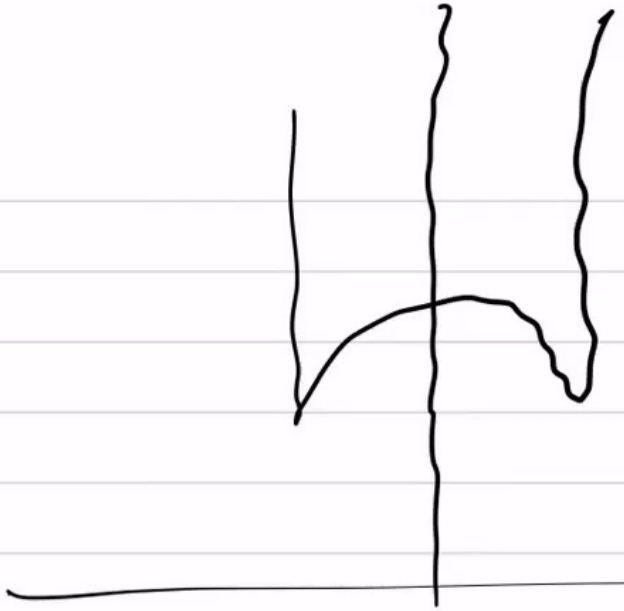
$a_1 \longleftrightarrow 1(\rho)$

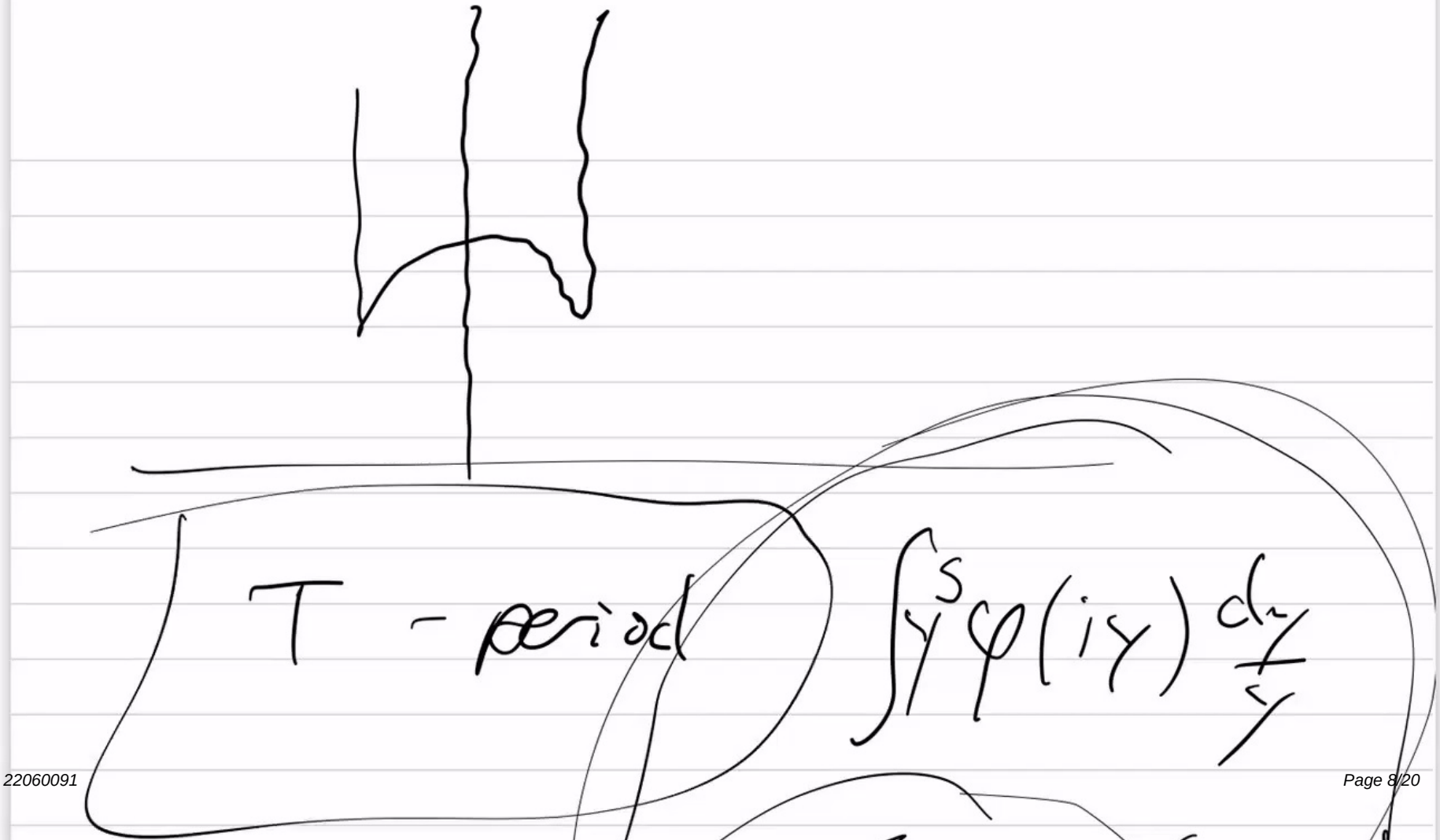
Whittaker normalization

$\sum a_n n^{-s}$ gives L-function of ρ



T - periodic

$$\int_{iy}^{S} \varphi(iy)\, dz$$



Q What is science behind matchy of L-functions & periods ?

A periods ⊂ 2 rankings

Langlands program d'après K-W

$G/\mathbb{C}$ reductive group $\Rightarrow$ A_G

$G^\vee/\mathbb{C}$ dual group $\Rightarrow$ $\mathcal{B}_{G^\vee}$

$A_G(S) \cong \mathrm{Sh}_{\mathcal{L}}(\mathcal{B}_{G^\vee}, S)$

Key features matches action of defect operators on two sides

- local operators (codim 4)

$H^*(BG)^{A6}$ EY

$\mathcal{O}(\mathfrak{g}^{*}[2]/G)$

Key features matches action of defect operators on two sides

- local operators (codim 4)

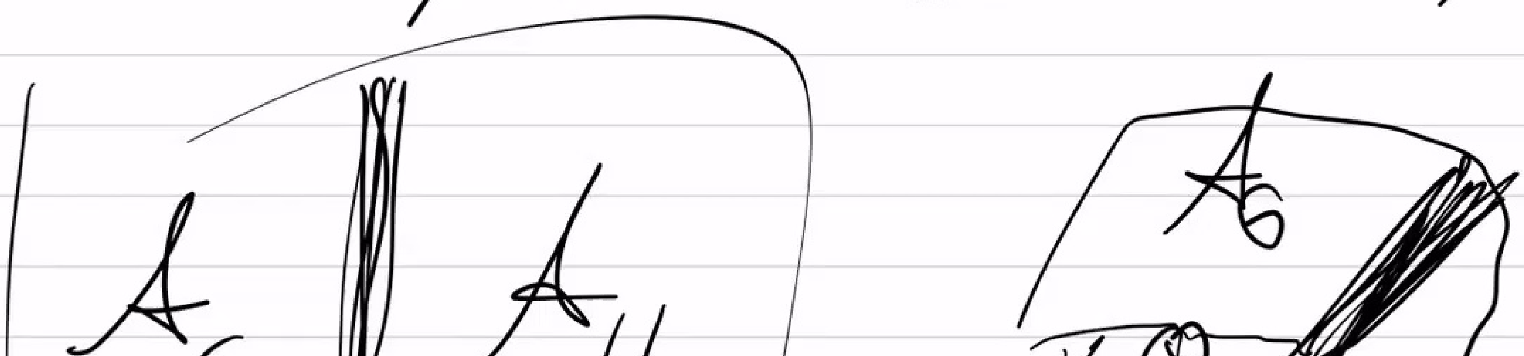
$H^*(BG)^{AG}$ EY

$\to O(\mathfrak{g}^{*}[2]/G)$

Richest story codim 1

Gaiotto-Witten, Hilburn-Yoo

boundary conditions & interfaces



Examples come from quantizations of hyperkähler $G/G^\vee$ spaces

3d A/B-model coupled to 4d

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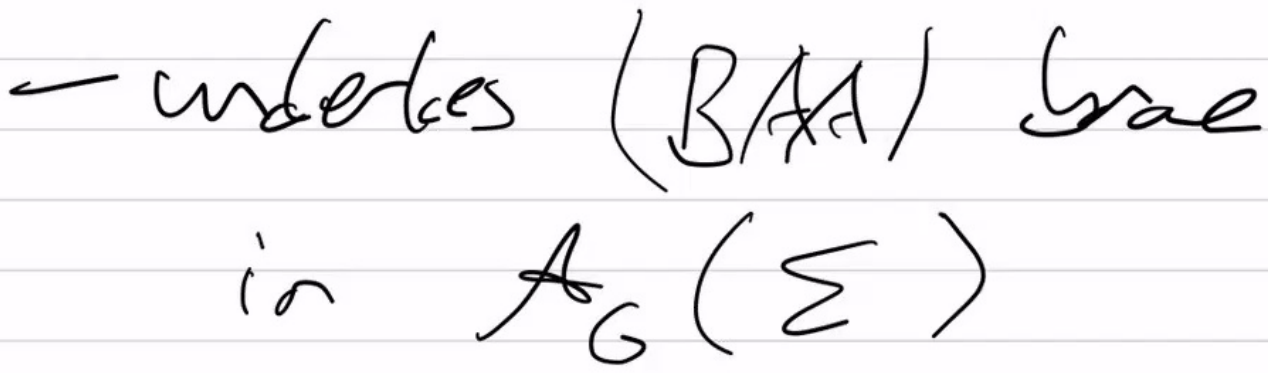
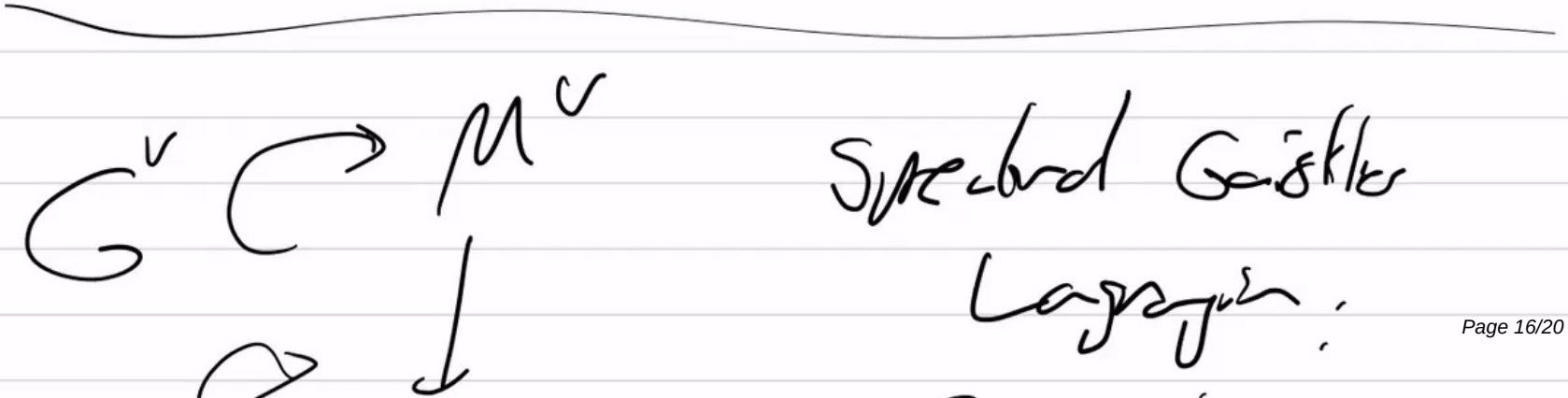
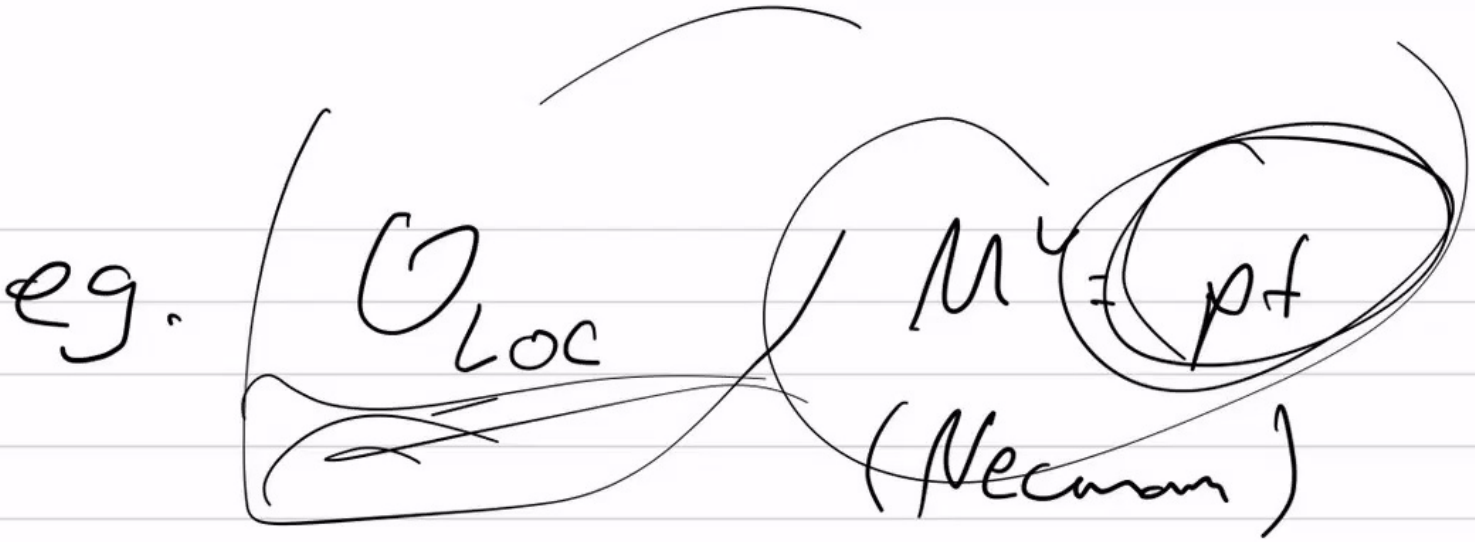
3d A/B-model coupled to 4d

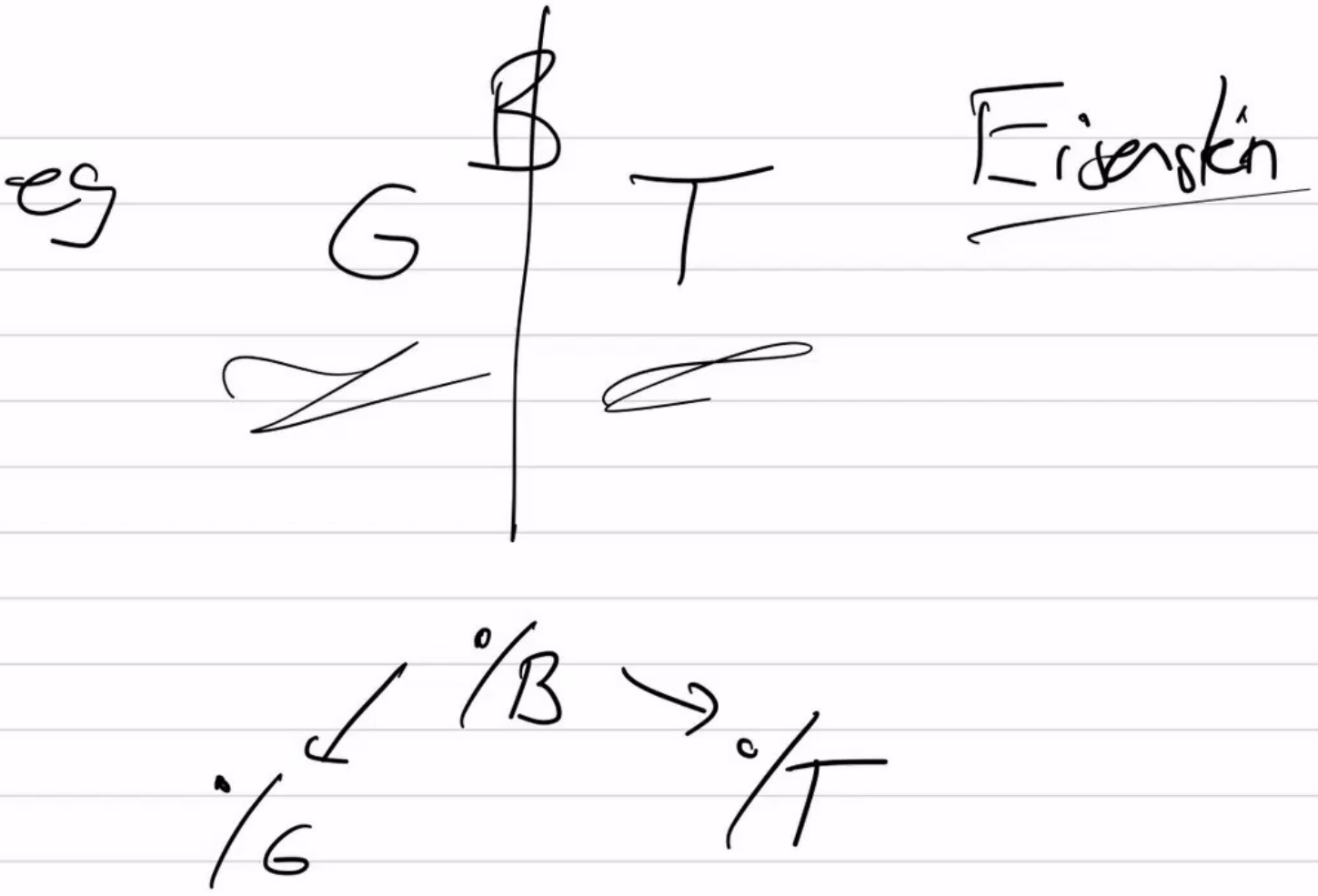






(ie. $\Gamma_{Loc}(-)$)

$\longleftrightarrow$ Hitchin section /


eg
G
B
T
Eisenstein
°/B
°/G
°/T

Dictionary:

MMMR...

F global field

local field

$\mathbb{Q}_p$

$\mathbb{F}_q((t))$