Title: Field theory in the 1/2 Omega-background cont.

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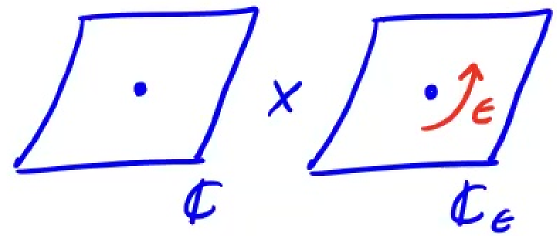
Going back to 4d $\mathcal{N}=2$ theory in $\frac{1}{2}\Omega$-background,

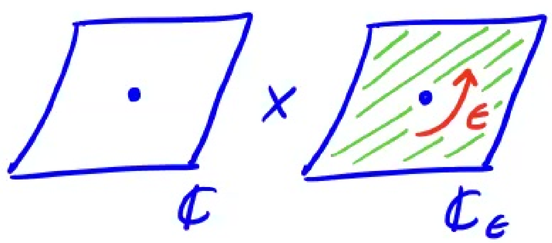
may enrich theory with defects:



categorical description??

- line defects: ones introduced by Fei together with $UV \to IR$ map

- surface defects: simplest ones have moduli space C



corr 2d BPS states encoded as trajectories ending on z in spectral networks W_u^ϑ

their vevs $\psi(z)$ are solns to oper eqn $D_\epsilon \psi(z) = 0$

(open-closed)

Lift to 5d = 5d $\mathcal{N}=1$ theory = topological string theory

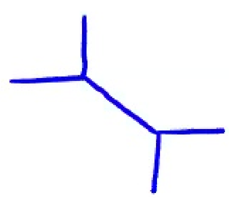
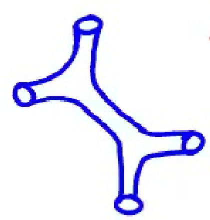
on $\mathbb{C} \times (\mathbb{C} \times_q S^1)$ in NS-background

$q = e^{i\epsilon}$

surface defects on $\mathbb{C} \times_q S^1$: couple 3d $\mathcal{N}=2$ thy to 5d $\mathcal{N}=1$ thy

moduli space of surface defect : $\Sigma \subset \mathbb{C}^*_X \times \mathbb{C}^*_Y$

Σ = 5d SW curve = mirror curve

example :  $\longleftrightarrow$ 

" resolved conifold "

$$\Sigma : \; Y = \frac{1 - QX}{1 - X} \xrightarrow[1:1]{} C = \mathbb{C}^*_X$$

$$\lambda = \log Y \, d \log X$$

$$Q = e^{2\pi i t}$$

NS partition function: $\widetilde{W}^{\text{eff}}(\epsilon, t) = -\frac{1}{2}\sum_{k=1}^{\infty}\frac{Q^k}{k^2 \sin(k\epsilon/2)}$

surface defect vev: sol'n to $D_\Sigma \psi(X) = 0$

with $D_\Sigma = (1-X)e^{\epsilon\partial_x/2\pi} - (1-QX)$ and $X = e^{2\pi i x}$

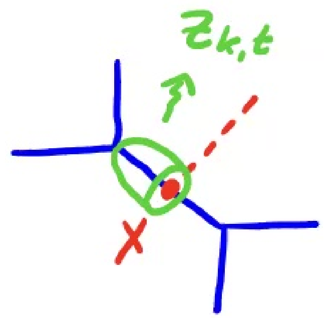
one sol'n: $\Psi_{GV}(\epsilon, x, t) = \dfrac{L(x,\epsilon)}{L(x+t,\epsilon)}$ with $L(x,\epsilon) = \prod_{j=0}^{\infty}(1-Xq^j)$

but more natural to consider the Borel sum $\Psi^{\Theta}(\epsilon, x, t) = B_\Theta \Psi^{\text{for}}(\epsilon, x, t)$

which encodes all solutions! *

[Garoufalides - Kashaev]

Indeed, Ψ^{ϑ} jumps in encode the 3d BPS particles coupled to the surface defect:



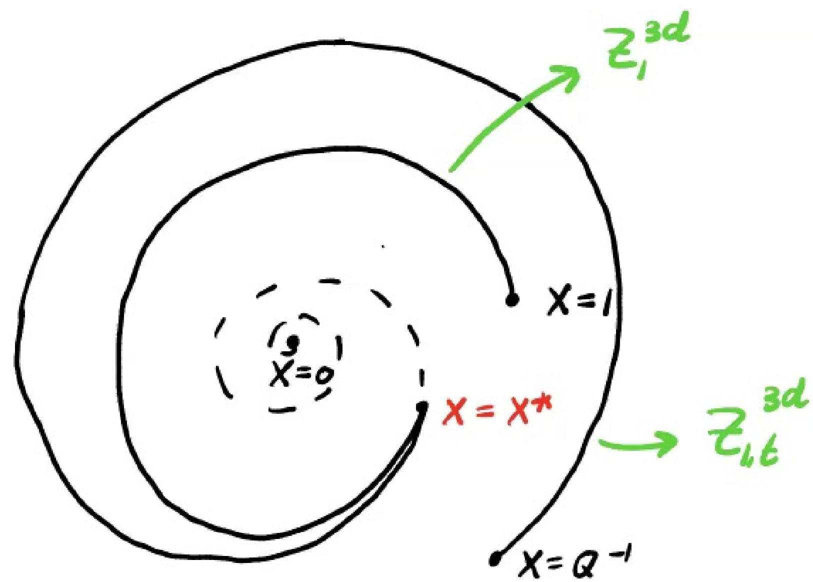
critical rays at $\vartheta^{3d} = \arg(Z^{3d})$

with either $Z_k^{3d} = \mp \frac{2\pi i}{\beta}(x+k)$

$$Z_{k,t}^{3d} = \mp \frac{2\pi i}{\beta}(t+x+k)$$

These may be visualized

in exponential spectral network:



[Banerjee-Longhi-Romo]
[Grassi-Hao-Neitzke]
[Alim-Hollands-Tulli]

Two special cases: $\vartheta_{GV} = \pi/2$ and $\vartheta_{np} = 0$

$$\Psi^{\vartheta_{GV}} = \Psi_{GV} \quad \text{and} \quad \Psi^{\vartheta_{np}} = \frac{S_2(x|\tilde{\epsilon},1)}{S_2(x+t|\tilde{\epsilon},1)}$$

↑ Fadeev's quantum dilog

$\tilde{W}^{\text{eff},\vartheta}$ can be computed in two ways:

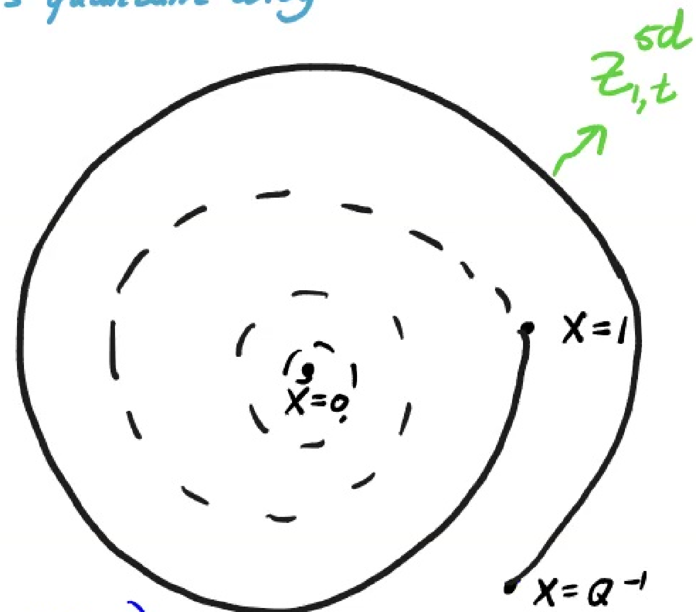
- $B^{\vartheta}\tilde{W}^{\text{eff,pert}}$

  critical rays at $\vartheta^{sd} = \arg Z^{sd}$

  with $Z^{sd}_{k} = \pm \frac{2\pi i}{\beta}(t+k)$

  corr to sd particles (D2 + kD0 or - D2 + kD0)

  in sd U(1) gauge thy



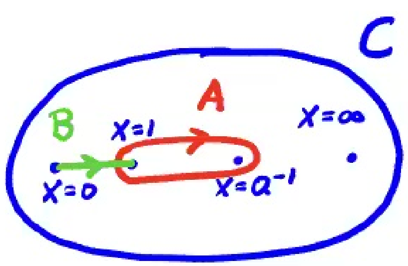
Again, two special cases: [Alim-Hollands-Tulli] [Grassi-Hao-Neitzke] similar to [Alim-Saha-Tulli-Teschner]

$$B_{\Theta_{GV}} \tilde{\widehat{W}}^{eff}_{5d} = \tilde{\widehat{W}}^{eff}_{GV}$$

$$B_{\Theta_{np}} \tilde{\widehat{W}}^{eff}_{5d} = \tilde{\widehat{W}}^{eff}_{np} = \tilde{\widehat{W}}^{eff}_{GV}(\epsilon, t) + \tilde{\widehat{W}}^{eff}_{GV}\left(\frac{4\pi^2}{\epsilon}, \frac{2\pi t}{\epsilon}\right)$$

- Other method: (important to go to more intricate geometries)

compute Borel sums of quantum periods:



$$B_{\Theta} \Pi_A = Q$$

$$B_{\Theta} \Pi_B = \text{"}\frac{\Psi^{\Theta}(x=0)}{\Psi^{\Theta}(x=-i\infty)}\text{"} = S_2(x|\tilde{\epsilon},1)^{-1} \frac{\Psi^{\Theta}(x=0)}{\Psi^{\Theta}(x=-i\infty)}$$

$$= e^{-\frac{1}{2\pi}\partial_t \tilde{W}^{eff,\Theta}}$$

[Alim-Hollands-Tulli], non-pert generalization [Aganagic-Cheng-Dijkgraaf-Krefl-Vafa]