Title: Abelian 3d mirror symmetry and boundary conditions

Speakers: Benjamin Gammage

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Abstract: 3d mirror symmetry predicts an equivalence between A- and B-twists of a pair of dual 3d N=4 theories. Essentially the strongest invariants one can produce of the resulting 3-dimensional topological field theories are their 2-categories of boundary conditions. The B-side 2-category was first described by Kapustin-Rozansky-Saulinas, but the 2-categorical structure on A-side boundary conditions has not previously been understood. For abelian gauge theories with matter, we propose a model for the 2-category of A-type boundary conditions using Kapranov-Schechtman's "perverse schobers," and we prove a 3d mirror equivalence between dual 2-categories. By reducing to lower-dimensions, we can recover both the BFN construction and the BLPW Koszul duality for hypertoric categories O. This is joint work with Justin Hilburn and Aaron Mazel-Gee.

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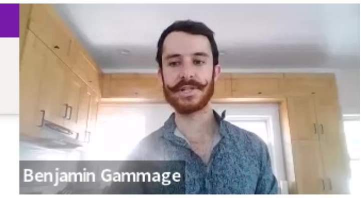


Starting point: $\mathfrak{X}$ holomorphic symplectic stack

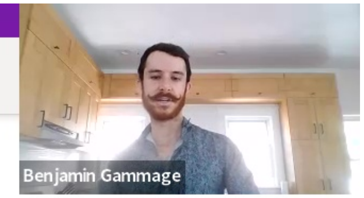
Ex: $\mathfrak{X} = T^*(V/G)$ reductive group

alg. variety

$\rightsquigarrow$ 3d $N=4$ theory

admitting 2 topological twists, the A-model & the B-model

3d mirror symmetry: Given a 3d $N=4$ theory T, there is a 3d dual theory $T^\vee$ such that the A-twist of T is equivalent to the B-twist of $T^\vee$, and vice versa.



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<u>3d mirror symmetry</u>: Given a 3d $N=4$ theory $\mathcal{T}$, there is a <u>3d dual</u> theory $\mathcal{T}^\vee$ such that the A-twist of $\mathcal{T}$ is equivalent to the B-twist of $\mathcal{T}^\vee$, and vice versa.

In good cases, each of these theories is specified by an $\mathfrak{X}$ as above.

<u>Ex</u>: • $\mathfrak{X} = T^*(BG)$ ← pure G-gerbe twisting $\Longrightarrow$ $\mathfrak{X}^\vee = U_\psi \backslash\backslash T^* G^L //_\psi U = \mathrm{Spec}(C_\cdot(G(\mathcal{O})\backslash \mathrm{Gr}_G))$

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(when $G = \mathbb{C}^\times$, this is just $\mathcal{X}^\vee = T^*\mathbb{C}^\times$.)

• $\mathcal{X} = T^*(\mathbb{C}/\mathbb{C}^\times) \Rightarrow \mathcal{X}^\vee = T^*\mathbb{C}$

How to make sense of this mathematically?

A-twist of T is equivalent to the B-twist of T,

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[BFN]: $\mathbb{E}_3$-alg of local ops in A-twist to $\mathcal{X} = T^*(V/G)$ is given by

$C_\cdot(\mathrm{Maps}(\mathbb{B}, V/G)) \rightsquigarrow$ holomorphic functions on $\mathcal{X}^\vee$

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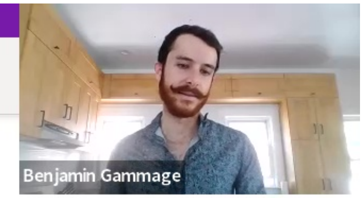

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Ex: • $\mathfrak{X} = T^*(BG) \Longrightarrow \mathfrak{X}^\vee = \mathcal{U} \backslash\backslash_\psi T^* G^L //_\psi \mathcal{U} = \mathrm{Spec}(C_\cdot(G(\mathcal{O}) \backslash Gr_G))$

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(G-grading)

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$\mathrm{Dmod}(\mathrm{Maps}(\mathring{\mathbb{D}}, V/G)) \rightsquigarrow$ coherent sheaves on $\mathcal{X}^\vee$ [Webster]

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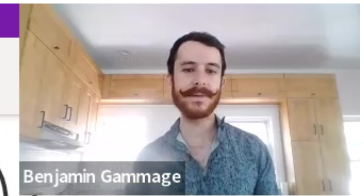

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G-gr ring

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$$\mathrm{Dmod}(\mathrm{Maps}(\mathring{\mathbb{D}}, V/G)) \rightsquigarrow \text{coherent sheaves on } \mathcal{X}^\vee$$ [Webster] (Hilburn-Raskin)

Better (and encompassing the above):

Study the 2-categories of boundary conditions for these theories.

In both twists, b.c.'s are given by holomorphic Lagrangian submanifolds of $\mathcal{X}$.

B-model: 2-category $\mathrm{KRS}(\mathcal{X})$. If $\mathcal{X} = T^*V$: some objects include V, Γ_{df},

$$\mathrm{Hom}(V, \Gamma_{df}) = \mathrm{MF}(V, f).$$

Also: $W \xhookrightarrow[\text{embedding}]{\text{closed}} V$ (smooth) $\rightsquigarrow T^*_W V$. $\quad \mathrm{Hom}(T^*_{W_1}V, T^*_{W_2}V) = \mathrm{Coh}(W_1 \times_V W_2)$

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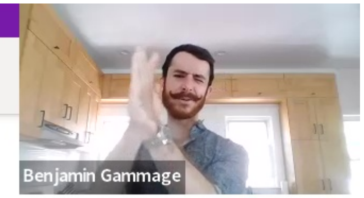

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- $W_1 = W_2 = pt \rightsquigarrow \mathrm{Hom}(T^*_{pt}V, T^*_{pt}V) = \mathrm{Coh}(pt \times_V pt) = \mathrm{Coh}(T_p[-1]V)$

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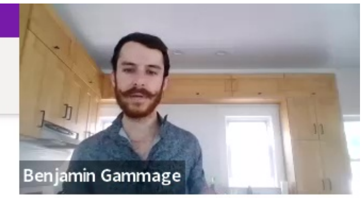

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$\rightsquigarrow$ For $\mathfrak{X} = T^*V$, $\quad KRS(\mathfrak{X}) = 2\mathrm{Coh}(V)$, the 2-category of coherent sheaves of categories on V [Arinkin, Stefanich]

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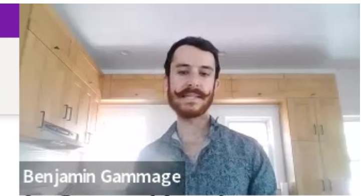

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$\mathbb{L} \subset T^*V$
conic Lagrangian

sing support $\rightsquigarrow$ consider $T^*_W V \subset \mathbb{L}$

Ex: $\mathbb{L} = V$ (and V 1-affine) $\Rightarrow 2\mathrm{Coh}_V(V) = 2\mathrm{QCoh}(V) \overset{\text{1-affine}}{=}$ [illegible]

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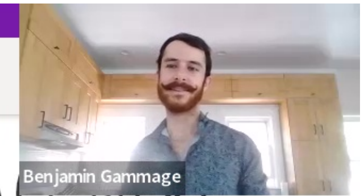

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$\mathbb{L} \subset T^*V$ conic Lagrangian

sing support → $\mathbb{L}$ $\rightsquigarrow$ consider $T^*_W V \subset \mathbb{L}$

Ex: $\mathbb{L} = V$ (and V 1-affine) $\Rightarrow 2\mathrm{Coh}_V(V) = 2\mathrm{QCoh}(V) \overset{\text{1-affineness}}{=} (\mathrm{QCoh}(V), \otimes)\text{-mod}$

A-model: 2-category $2\mathrm{Fuk}(\mathfrak{X})$ ← should be categorification of $\mathrm{Fuk}(\mathfrak{X})$:

1-morphisms are intersection points, 2-morphisms are holomorphic strips, categorical structure comes from count of solutions to

Fueter equation $I \cdot \partial_x + J \cdot \partial_y + K \cdot \partial_z = 0$

But cf. [Doan-Rezchikov]

This is hard! Easier: observe that when $\mathfrak{X} = T^*V$, $W\mathrm{Fuk}(\mathfrak{X})$ can be expressed

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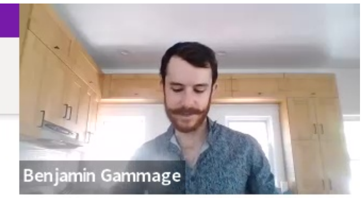

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Ex: $\mathbb{L} = V$ (and V 1-affine) $\Rightarrow 2Coh_V(V) = 2QCoh(V) \overset{1\text{-affine}}{\simeq} (QCoh(V), \otimes)\text{-mod}$

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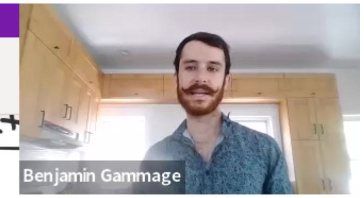

conic Lagrangian … support ⇝ consider $T_W V \subset \mathbb{L}$ … cat

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This is hard! Easier: observe that when $\mathfrak{X} = T^*V$, $W\mathrm{Fuk}(X)$ (FS-type stop & wrapping) can be expressed in terms of $\mathrm{Perv}_{\mathbb{L}}(V)$ ← category of perverse sheaves on V w/ $SS \subseteq \mathbb{L}$

(Ganatra-Pardon-Shende)

⇝ Idea: Model $2\mathrm{Fuk}(\mathfrak{X})$ by $2\mathrm{Perv}_{\mathbb{L}}(V)$ ← [Kapranov-Schechtman] Perverse schobers

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$(\mathrm{QCoh}(U), \otimes)$

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FS-type stop for wrapping

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Ex: $\mathbb{L} = V \Rightarrow 2\mathrm{Perv}_V(V) = 2\mathrm{Loc}(V)$ ← 2-category of local systems of categories on V

1-morphisms are intersection points, 2-morphisms are holomorphic strips,

categorical structure comes from count of solutions to

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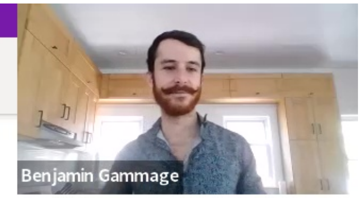
<u>Back to 3d mirror symmetry!</u> <u>Ex</u>: $T^*(B\mathbb{C}^\times) \longleftrightarrow T^*\mathbb{C}^\times$ [Teleman]



(Garath - Park - Shuhn) w/ $SS \subseteq \mathbb{L}$

$\rightsquigarrow$ **Idea:** Model $2\mathrm{Fuk}(\mathcal{X})$ by $2\mathrm{Perv}_{\mathbb{L}}(V)$ $\leftarrow$ [Kapranov-Schechtman] Perv schobers

Ex: $\mathbb{L} = V \Rightarrow 2\mathrm{Perv}_V(V) = 2\mathrm{Loc}(V)$ $\leftarrow$ 2-category of local system of categories on V

Back to 3d mirror symmetry!

Ex: $T^*(B\mathbb{C}^\times) \longleftrightarrow T^*\mathbb{C}^\times$ [Teleman]

- $2\mathrm{QCoh}(B\mathbb{C}^\times) = \mathrm{QCoh}(B\mathbb{C}^\times)\text{-mod}^2 = \mathrm{Rep}(\mathbb{C}^\times)\text{-mod}^2 = \{ \mathcal{C} + \mathcal{C} \xrightarrow{\sim} \mathcal{C} \} \simeq 2\mathrm{Loc}(\mathbb{C}^\times)$

  $\otimes$ w/ weight-1 representation

- $2\mathrm{Loc}(B\mathbb{C}^\times) = \{ \mathcal{C} + \underbrace{C_\cdot(\Omega\mathbb{C}^\times)}_{= \mathcal{O}(\mathbb{C}^\times)} \to HH^\cdot(\mathcal{C}) \} \simeq \mathrm{QCoh}(\mathbb{C}^\times)\text{-mod} \simeq 2\mathrm{QCoh}(\mathbb{C}^\times)$

Now: add matter. $T^*(\mathbb{C}/\mathbb{C}^\times) \longleftrightarrow T^*\mathbb{C}.$

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- $2\mathrm{Loc}(B\mathbb{C}^\times) = \{\mathcal{C} + \underbrace{C_\cdot(\Omega\mathbb{C}^\times)}_{\overset{\shortparallel}{\mathcal{O}(\mathbb{C}^\times)}} \to HH^\cdot(\mathcal{C})\} \simeq \mathrm{QCoh}(\mathbb{C}^\times)\text{-mod} \simeq 2\mathrm{QCoh}(\mathbb{C}^\times)$

Now: <u>add matter</u>. $\quad T^*(\mathbb{C}/\mathbb{C}^\times) \longleftrightarrow T^*\mathbb{C}$.

More generally, given $1 \to T \to (\mathbb{C}^\times)^n \to G \to 1$,

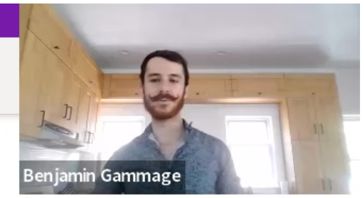
$$T^*(\mathbb{C}^n/T) \longleftrightarrow T^*(\mathbb{C}^n/G^\vee).$$

Abelian 3d HMS!

Thm [G.-Hilburn-Mazel-Gee]: $2\mathrm{Perv}_{\Bbbk}(\mathbb{C}^n/T) \simeq 2\mathrm{Coh}_{\Bbbk^\vee}(\mathbb{C}^n/G^\vee)$.

<u>Basic case</u>: $2\mathrm{Perv}_{\Bbbk}(\mathbb{C}) \simeq 2\mathrm{Coh}_{\Bbbk^\vee}(\mathbb{C}/\mathbb{C}^\times)$

More generally, given $1 \to T \to (\mathbb{C}^\times)^n \to G \to 1$,



$$T^*(\mathbb{C}^n/T) \longleftrightarrow T^*(\mathbb{C}^n/G^\vee).$$

Abelian 3d HMS!

Thm [G.–Hilburn–Mazel-Gee]: $2\mathrm{Perv}_{\mathbb{L}}(\mathbb{C}^n/T) \simeq 2\mathrm{Coh}_{\mathbb{L}^\vee}(\mathbb{C}^n/G^\vee)$.

Basic case: $2\mathrm{Perv}_{\mathbb{L}}(\mathbb{C}) \simeq 2\mathrm{Coh}_{\mathbb{L}^\vee}(\mathbb{C}/\mathbb{C}^\times)$

$\mathbb{L} = \mathbb{C} \cup T^*_0\mathbb{C} \subseteq T^*\mathbb{C}$ , $\mathbb{L}^\vee = \mathbb{L}/\mathbb{C}^\times$. (More generally, $\mathbb{L} = (\ldots)^n/T$).

What is $2\mathrm{Perv}_{\mathbb{L}}(\mathbb{C})$?

Recall that
$$\mathrm{Perv}_{\mathbb{L}}(\mathbb{C}) = \left\{ \underset{\circlearrowleft m_\Phi}{\Phi} \underset{\text{can}}{\overset{\text{var}}{\rightleftarrows}} \underset{\circlearrowleft m_\Psi}{\Psi} \;\middle|\; \begin{array}{l} m_\Phi := 1 - \text{can} \circ \text{var} \\ m_\Psi := 1 - \text{var} \circ \text{can} \end{array} \text{ are invertible} \right\}$$

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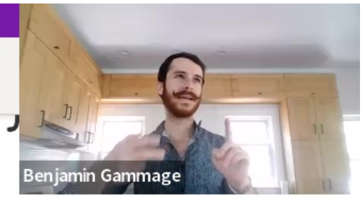


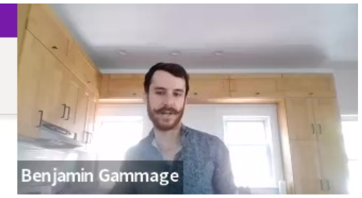
Basic case: $2\mathrm{Perv}_{\mathbb{L}}(\mathbb{C}) \simeq 2\mathrm{Coh}_{\mathbb{L}^\vee}(\mathbb{C}/\mathbb{C}^\times)$

$\mathbb{L} = \mathbb{C} \cup T^*_0\mathbb{C} \subseteq T^*\mathbb{C}$ , $\mathbb{L}^\vee = \mathbb{L}/\mathbb{C}^\times$. (More generally, $\mathbb{L} = \Big($  $\Big)^n / T$).

What is $2\mathrm{Perv}_{\mathbb{L}}(\mathbb{C})$?

Recall that
$$\mathrm{Perv}_{\mathbb{L}}(\mathbb{C}) = \left\{ \overset{\circlearrowleft m_\Phi}{\Phi} \underset{can}{\overset{var}{\rightleftarrows}} \overset{\circlearrowleft m_\Psi}{\Psi} \;\middle|\; \begin{array}{l} m_\Phi := 1 - can \circ var \\ m_\Psi := 1 - var \circ can \end{array} \text{ are } \underline{\text{invertible}} \right\}$$

[Kapranov - Schechtman]: Perverse schobers on $\mathbb{C}, \mathbb{L}$ are $\underline{\text{spherical functors}}$:
$$\mathrm{Sph} = \left\{ \mathcal{C}_\Phi \underset{F^R}{\overset{F}{\rightleftarrows}} \mathcal{C}_\Psi \;\middle|\; \begin{array}{l} T_\Psi := \mathrm{fib}(\mathrm{id}_{\mathcal{C}_\Phi} \to F^R F) \\ T_\Phi := \mathrm{cofib}(F F^R \to \mathrm{id}_{\mathcal{C}_\Psi}) \end{array} \text{ are invertible} \right\}$$



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Idea: $\{X \to \mathbb{C}/\mathbb{C}^\times\} = \left\{ \begin{array}{l} \text{Line bundle } \mathcal{L} \text{ on } X \\ + \text{ section } s \in \Gamma(X,\mathcal{L}) \end{array} \right\} = \{\text{Divisor } D = s^{-1}(0) \hookrightarrow X\} \rightsquigarrow \mathrm{Coh}(D) \xrightarrow{i_*} \mathrm{Coh}(X)$ spherical functor.

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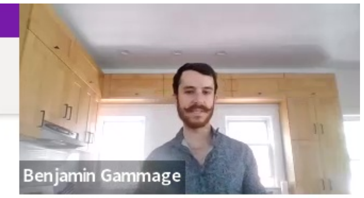

Recall that $\mathrm{Perv}_{\mathbb{L}}(\mathbb{C}) = \left\{ \overset{\circlearrowleft m_\Phi}{\Phi} \underset{can}{\overset{var}{\rightleftarrows}} \overset{\circlearrowleft m_\Psi}{\Psi} \;\middle|\; \begin{array}{l} m_\Phi := 1 - can \circ var \\ m_\Psi := 1 - var \circ can \end{array} \text{ are invertible} \right\}$

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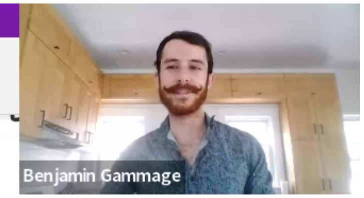
Universal case: $\{0\}/\mathbb{C}^\times \longrightarrow \mathbb{C}/\mathbb{C}^\times \rightsquigarrow \mathrm{Coh}(\mathbb{C}/\mathbb{C}^\times) \xrightarrow{i^*} \mathrm{Coh}(0/\mathbb{C}^\times)$

$$\begin{array}{ccc} \mathrm{Coh}(\mathbb{C}/\mathbb{C}^\times) & \xrightarrow{i^*} & \mathrm{Coh}(0/\mathbb{C}^\times) \\ \| & & \| \\ \mathrm{filtVect} & \xrightarrow{gr} & \mathrm{grVect} \end{array}$$

Thm [GHMG]: $\mathrm{Hom}_{\mathrm{Sph}}\left(\mathrm{filtVect} \overset{gr}{\rightleftarrows} \mathrm{grVect}, \; \mathcal{C} \rightleftarrows \mathcal{D} \right) = \mathcal{C}$



Recall ... $\mathbb{L}$ ... $\{\Phi \xleftarrow{can} \Psi \mid m_\Phi := 1 - var \circ can$

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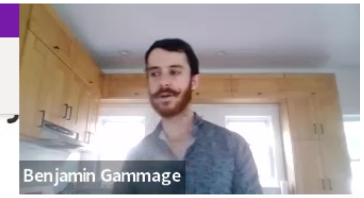

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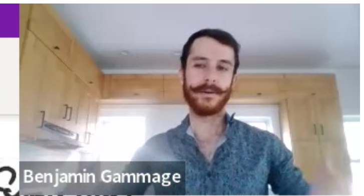

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Putting it all together:

$$\mathrm{Sph} = \left\{ \overset{\circlearrowleft}{\mathcal{C}_\Phi} \underset{F^R}{\overset{F}{\rightleftarrows}} \overset{\circlearrowleft}{\mathcal{C}_\Psi} \;\middle|\; \begin{matrix} T_\Psi := \mathrm{fib}(\mathrm{id}_{\mathcal{C}_\Phi} \to F^R F) \\ T_\Phi := \mathrm{cofib}(F F^R \to \mathrm{id}_{\mathcal{C}_\Psi}) \end{matrix} \text{ are invertible} \right\}$$

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Putting it all together:

$$2\mathrm{Rep}_{\mathcal{L}}(\mathbb{C}) \simeq \mathrm{Sph} \simeq \langle gr, gr^R \rangle \simeq \begin{pmatrix} \mathrm{filtVect} & \mathrm{grVect} \\ \mathrm{grVect} & \mathrm{filtVect} \end{pmatrix}$$

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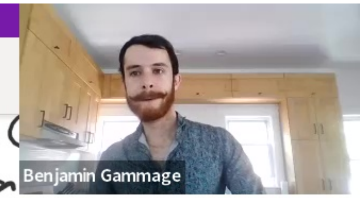

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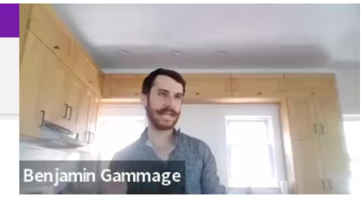

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Cor: Equivalence of 2-categories of A- & B-type boundary conditions for

$$(T^*\mathbb{C}^n /\!/_\alpha T, \mathbb{L}_\beta) \quad \text{and} \quad (T^*\mathbb{C}^n /\!/_\beta G^\vee, \mathbb{L}_\alpha)$$

↑ resolution/stability/FI parameter

↑ cocharacter/mass parameter specifying category or skeleton

$$\left(T^*\mathbb{C}^n /\!/_\alpha T \underset{\alpha \in \mathfrak{X}}{\overset{\text{open}}{\subseteq}} T^*(\mathbb{C}^n/T)\right)$$

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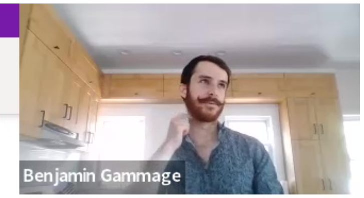

$$2\mathrm{Rep}_{\mathbb{L}}(\mathbb{C}) \simeq \mathrm{Sph} \simeq \langle gr, gr^R \rangle \simeq \begin{pmatrix} \cdots & \cdots \\ \mathrm{grVect} & \mathrm{filtVect} \end{pmatrix}$$

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resolution/ stability/ FI parameter

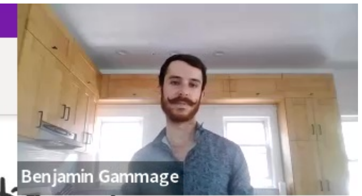
cocharacter/ mass parameter specifying category of skeleton

$\alpha \in \mathfrak{t}^\vee$

$\beta \in \mathfrak{g}$

(A & B-type)

$\rightsquigarrow$ 2-categories ☺ !



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Coulomb/ mass parameter specifying category co skeleton

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Cor: Via various kinds of decategorification, recover BFN construction, BLPW symplectic duality, Betti version of Hilburn-Raskin, . . .

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Benjamin Gammage

Betti version of Hilburn–Raskin, ...

Line operators

$\mathcal{X} = T^*Y$. Suppose $\mathbb{L} \subseteq \mathcal{X}$ is $\mathbb{L} = T^*_Z Y$, $Z \to Y$

E.g.: $Y = \mathbb{C}/\mathbb{C}^\times$, $Z = \mathbb{C}/\mathbb{C}^\times \sqcup 0/\mathbb{C}^\times$

$$ZCoh_{\mathbb{L}}(Y) = Coh(Z \times_Y Z)\text{-mod}^{(2)}$$

Thm (BZNP): $Tr(Coh(Z \times_Y Z)) = Coh_\Lambda(L_B Y)$

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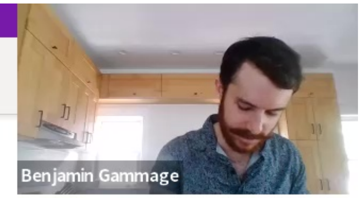


$2\mathrm{Coh}_{\Bbbk}(\;\ldots - \mathrm{Coh}(Z \times_Y Z) - \mathrm{mod}\ldots$

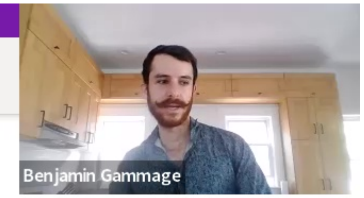
$\underline{\text{Thm}}$ (BZNP): $\mathrm{Tr}(\mathrm{Coh}(Z \times_Y Z)) = \mathrm{Coh}_\Lambda(L_B Y)$

$\underline{\text{Conj}}$: $\mathrm{Tr}(\mathrm{Perv}_{\Bbbk}(Y)) \simeq \mathrm{Perv}_\Lambda(\mathcal{L} Y)$

$\underline{\text{Thm}}$ (G.–Hilburn) $\mathrm{Coh}_\Lambda(L_B \mathbb{C}/\mathbb{C}^\times) = \mathrm{Perv}_\Lambda(\mathbb{C}((t)))$

$$\|$$

$\mathrm{Tr}(2\mathrm{Coh}) = \mathrm{Tr}$



$2\mathrm{Coh}_{\mathbb{L}}(\ \ \mathrm{Coh}(Z \times_Y Z)\text{-mod}$

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$$\begin{array}{ccc} \| & & \| \\ Tr(2\mathrm{Coh}(\mathrm{cat})) & = & Tr(2\mathrm{Perv}(\mathbb{C})) \end{array}$$

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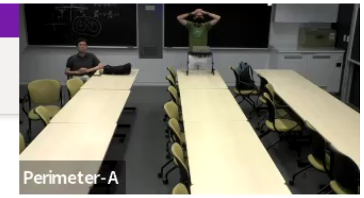

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$$Z(\;-\;) = Coh_{prop/Y}(L_B Y)$$

$\mathbb{E}_2$-cat.

$\underline{\text{Conj}}$: $Tr(2Perv_{\mathbb{C}}(Y)) \simeq Perv_{\Lambda}(\mathcal{L} Y)$

$\underline{\text{Thm}}$ (G.-Hilb) $Coh_{\Lambda}(L_B \mathbb{C}/\mathbb{C}^{\times}) = Sh_{\Lambda}(\mathbb{C}((t)))$

$$\| \qquad\qquad \|$$

$$Tr(2Coh(\mathbb{C}/\mathbb{C}^{\times})) = Tr(2Perv(\mathbb{C}))$$