Title: Holomorphic BF theory, 4d SYM and the analytic Geometric Langlands program

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Abstract: I will review the embedding of the analytic Geometric Langlands program in four-dimensional gauge theory.

Holomorphic BF theory and Geometric Langlands

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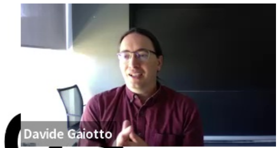
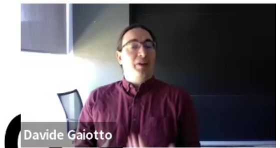
4d QFT and Geometric Langlands



- Global Geometric Langlands
  - Equivalence of categories
  - Flat connections on Bun[C,G] <—> Bundles on Flat[C,G']
- 4d A- and B- twisted N=4 SYM
  - 4d Topological Field Theories, related by S-duality
  - Attach categories Z_A[C,G] <—> Z_B[C,G] to Riemann surface C

4d QFT and Geometric Langlands



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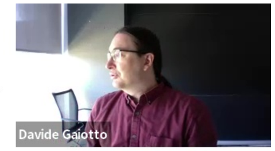
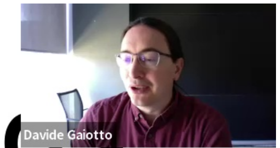


Holomorphic vs topological

- GL categories use complex structure of C
  - Technical? Bundle ~ unitary flat connections
  - Interesting objects use complex structure of C
- Defects in 4d TFT can be holomorphic on C
  - Holomorphic defects may be used to build a description of Z[C]
  - Holomorphic defects may be used to build nice objects in Z[C]

4d QFT and Geometric Langlands



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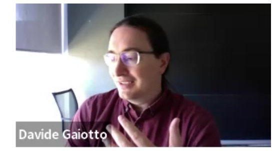

2d vs 4d primer

- B-twisted 2d theory of maps into M
    - Z[pt]: Derived category of coherent sheaves
- 4d Z_B[C,G] ~ 2d Z[pt] for M = Flat[C,G]
- A-twisted 2d theory of maps into T*M
    - Z[pt]: Derived category of D-modules on M
- 4d Z_B[C,G] ~ 2d Z[pt] for T*M ~ Higgs[C,G]

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2d vs 4d

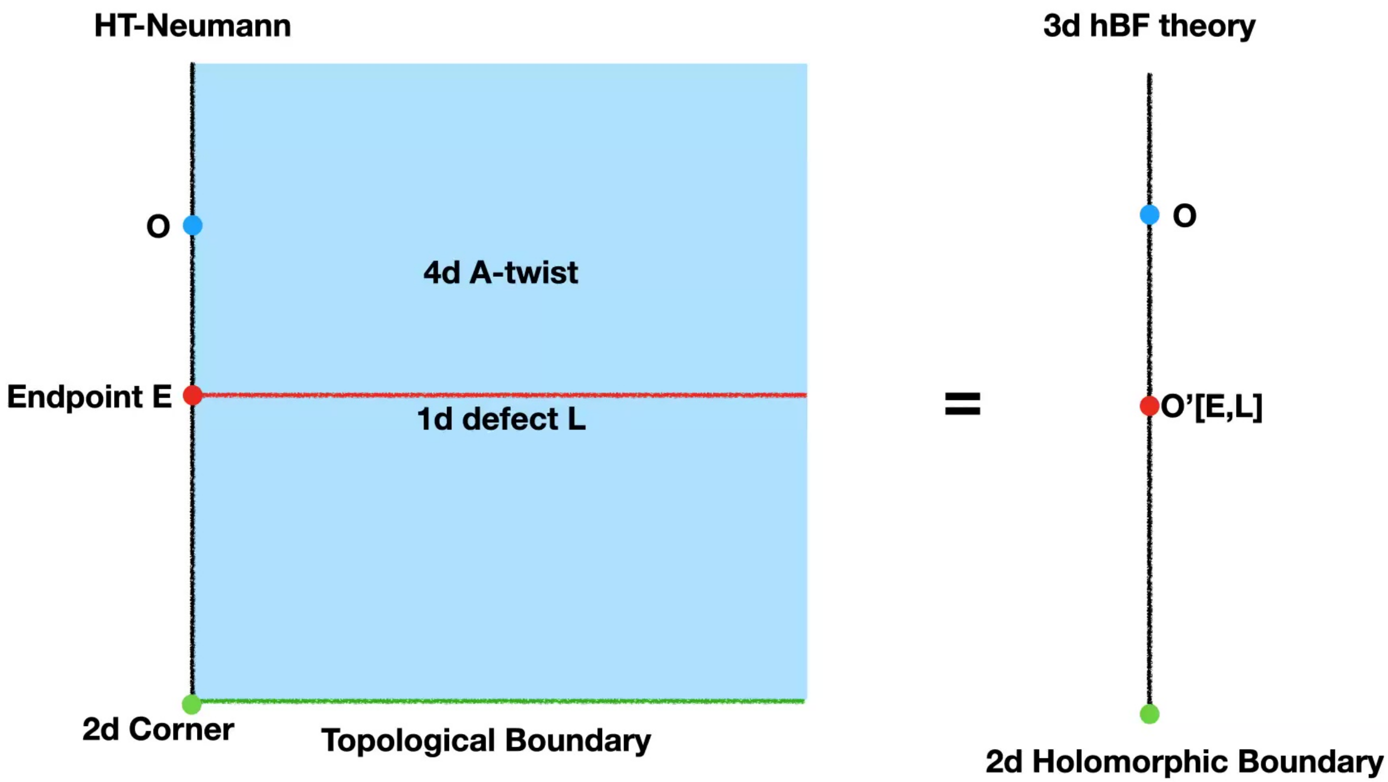
- 2d is poor approximation of 4d
- 4d is harder, but mathematically doable: semiclassical or perturbative
- 4d has extra structure: locality on C
- Special elements in Z[C], some known S-duals
  - 3d topological boundaries
  - 3d holomorphic-topological boundaries
  - $Z[U_3]$, C boundary of U_3

Half-BPS boundaries

- Preserve 3d N=4 in 4d N=4
- "Standard embedding": 4d A (B) twist —> 3d A (B) twist
    - topological boundary condition
- "Alternative embedding": holomorphic topological boundary
    - inherited from holomorphic twist
    - requires deformation, rarely available

A-twist, HT-Neumann

- 4d half space wt HT-Neumann <===> perturbative 3d HT BF theory
    - Many 4d calculations reduce to 3d BF theory
- Analogue of ``canonical coisotropic” brane in 2d
    - Phase space of BF theory <===> Higgs[C,G]
- 1d ’t Hooft lines ending on HT-Neu <===> local ops in 3d HT BF
    - Hecke correspondences on Higgs[C,G]


HT-Neumann
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4d A-twist
Endpoint E
1d defect L
2d Corner
Topological Boundary
=
3d hBF theory
O
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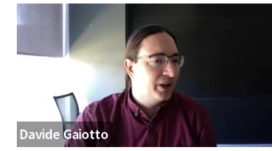
2d Holomorphic Boundary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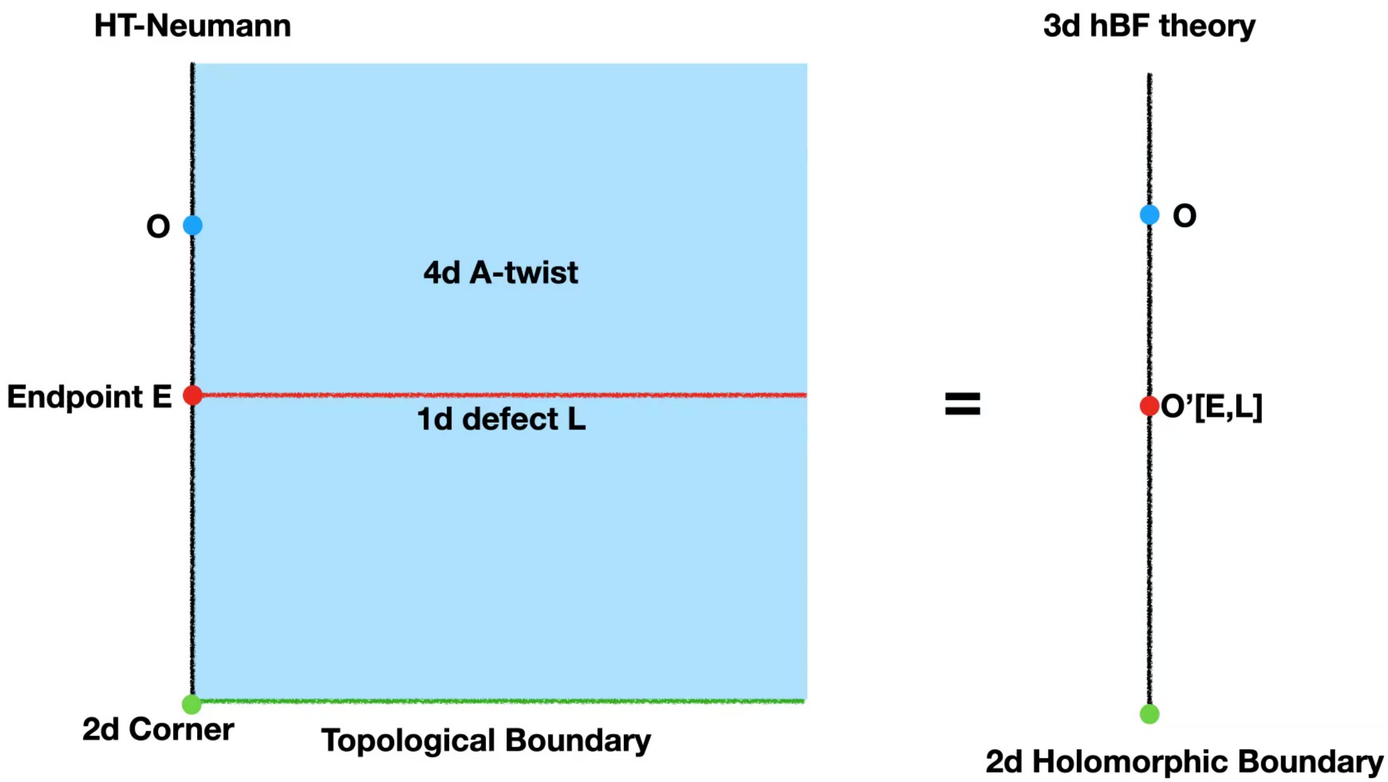
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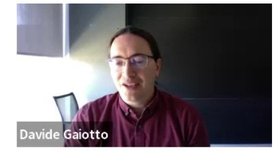
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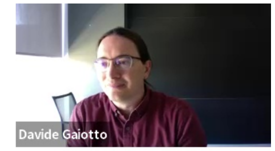
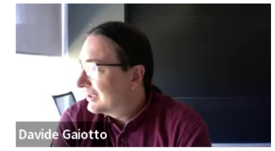
B-twist, HT-Nahm

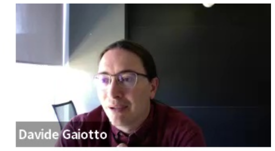


- Forces flat connection on C to be an “oper”
  - SL2: $D_t s = 0$ $D_{\bar{z}} s = 0$ $s \wedge D_z s = 1 \to D_z^2 s + t(z) s = 0$
  - Other groups: holomorphic-topological s with constraints on derivatives
- 1d Wilson lines ending on HT-Nahm
  - SL2, Fundamental: s(z) endpoint
  - Irrep R (derived Satake)—> D-module SR on C depending on t(z), etc.

S-duality I



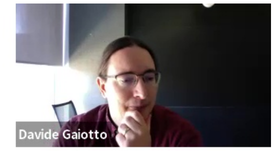
- HT-Neumann and HT-Nahm should be S-dual
  - Same spaces of local operators, endpoint of lines, etc.
- A: P[B] gauge-invariant local operators (and fermionic partners)
  - Quantize to $\hat{P}\left[\frac{\delta}{\delta A_{\bar{z}}}\right]$ quantum Hitchin Hamiltonians (“)
  - Not obvious, loop corrections could have obstructed
- B: t(z), …. functions on opers. BD: same as quantum Hitchin Hamiltonians



3d Holomorphic BF theory

- BV Action $$\int dz \, \mathrm{Tr}\, b \left(\mathrm{d}c - \frac{1}{2}[c,c] \right) = \int dz \, \mathrm{Tr}\, B \, F_A + \cdots$$
    - b, c are forms with $dt, d\bar{z}$ components, differential d acting on that
    - B scalar is 0 form part of b
    - A gauge field is 1-form part of c
- HT twist of 3d N=2 SYM or HT Neumann for 4d A-twisted N=4 SYM

S-duality II



- Endpoint of lines?
- B: S_R
- A: Phase space is $T^*Gr_G/G[[z]]$
  - Quantize to equivariant D-modules on affine Grassmanian
  - Functor to endpoints as Hecke transformations
  - Match as D-modules on C, HT factorization module structure?

Critical Kac-Moody

- Dirichlet boundary conditions for 3d BF theory
  - Lifts to HT-Neumann - topological Dirichlet corner
- Supports Kac-Moody at critical level: $B(z) \to \frac{\delta}{\delta A_{\bar{z}}} \to J(z)$
- HT-Neumann local ops <—> Center of critical Kac-Moody
- Boundary 't Hooft endpoints <—> D-modules on Gr_G
  - spectral flow modules
- 't Hooft endpoints <—> Averaged spectral flow modules

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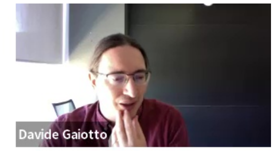
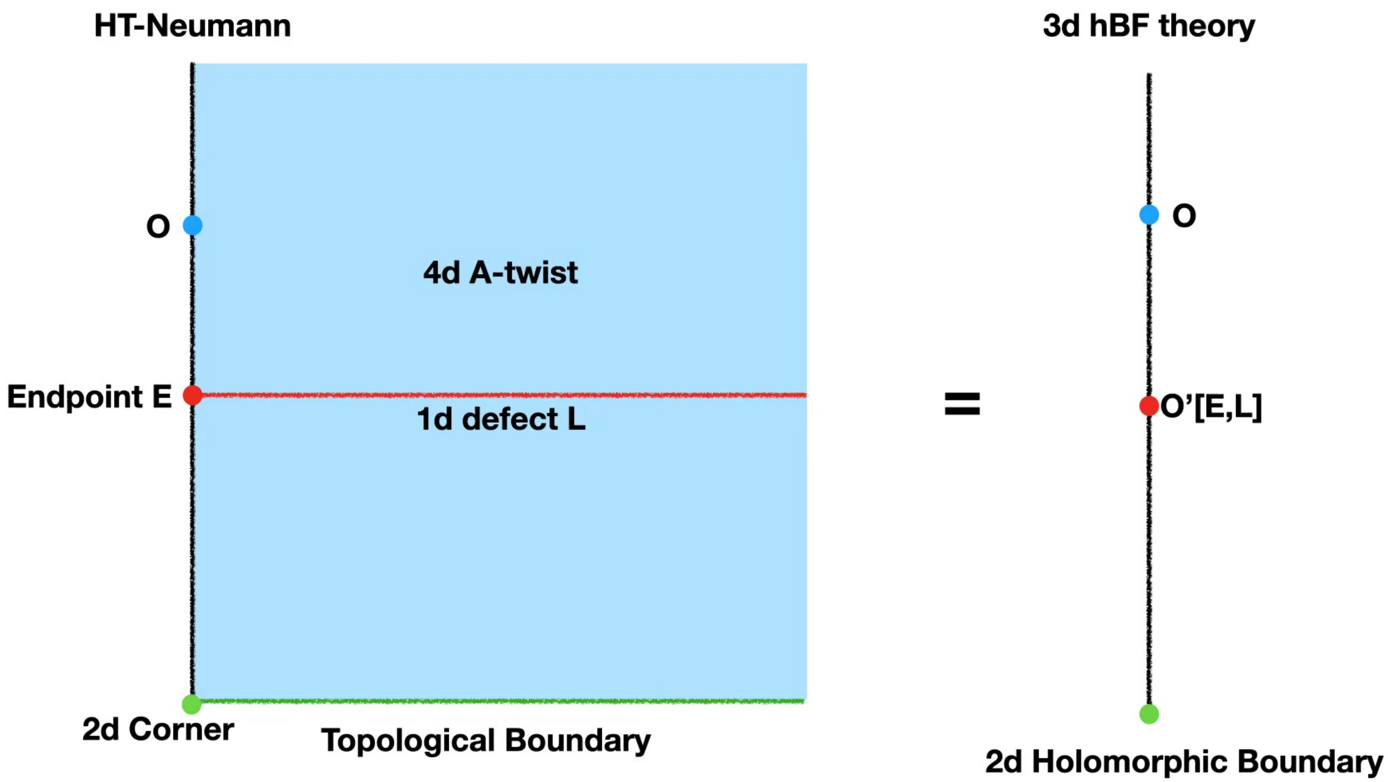


Topological Nahm

- Hitchin section boundary condition for 3d BF theory
    - SL2: $D_{\bar{z}} v = 0$ $\quad\quad v \wedge \Phi v = 1$
- Chiral algebra: DS reduction of critical KM
    - Classical W-algebra. See t(z) directly
- S-dual to structure sheaf of Flat[C,G’]
    - Corner: functions on Oper manifold
- Separation Of Variables: Dirichlet <—> Nahm + line defects

Boundary lines

- Boundary Wilson lines in HT-Neu <—> Wilson lines in 3d BF
    - Universal bundle on Higgs[G,C] —> Weyl modules for cKM
- Boundary ’t Hooft lines in HT-Nahm
    - Oper with singularity of trivial monodromy
- Gaudin model, etc.



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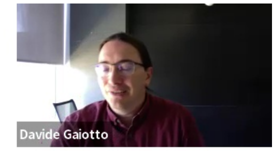

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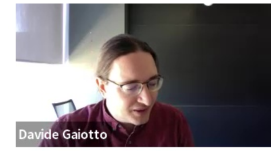


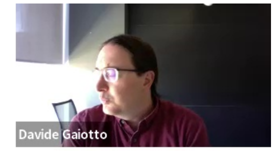
Local GL

- 2-categories Z[circle] of surface defects
- Surface defects meet HT-Neu along topological line
    - BF Wilson line with quantization of flag manifold or other
    - Higgs bundles with regular or wild ramification
- cKM modules? Surface - top Dirichlet lines?
- Dual Surface defects meet HT-Nahm along topological line
    - Ramified opers

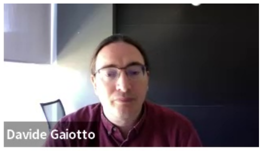
More HT boundaries

- Interfaces between U(N) and U(M)
- A: BF theory for U(N|M)
- B: N=M $A_{\bar{z}} = A'_{\bar{z}}$ $A_z = A'_z + XY$ $D_{\bar{z}} X = 0$ $D_{\bar{z}} Y = 0$
- B: N>M Partial oper N -> (N-M) + M, identify MxM block
- Mirabolic GL?
- OSp generalizations

Topological boundaries

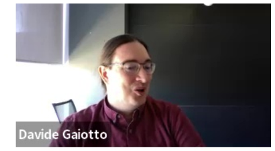


- Enriched Neumann: A(B)-twisted 3d N=4 with G action
- Conformal blocks of boundary VOA should give object in Z[C]
    - A: D-module on Bun, B: sheaf on Flat
- Many S-dual pairs from String theory
- More from spherical varieties?



Intermission

Analytic GL



- 4d A-twist on C x R x [0,1] with Holomorphic and anti-holomorphic Neu
  - Path integral $\int DBD\bar{B}DAD\bar{A}e^{\int \mathrm{Tr} B F_A dz - \mathrm{Tr} \bar{B} \bar{F}_A d\bar{z}}$
- Quantize BF phase space as a real symplectic manifold
  - Hilbert space $L^2[Bun(G,C)]$
- Action of quantum Hitchin and conjugate. Analysis to make self-adjoint.
- Action of Hecke operators from ’t Hooft along [0,1]
  - Choice of 2 endpoints

Analytic GL over reals

- Combine HT-Neumann and 3d manifold $(C \times [0,1])/Z_2$
    - Reflection of segment and anti-holomorphic involution on C
- Phase space: cotangent to “real” bundles
- S-duality: HT-Nahm and 3d manifold $(C \times [0,1])/Z_2$
    - Important subtleties if fixed lines
- Opers compatible with involution