Title: Discussion on knot homology

Speakers: Tudor Dimofte, Lev Rozansky, Ben Webster

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discussion

Monday, June 27, 2022 1:57 PM

M-theory / string theory setup

"duality frame" ⇝ way of computing link homology

Setup: $CY_3 \times HK \times \mathbb{R}_t$ (or S^1)

CY_3: T^*S^3 — deformed conifold; $\to X$

HK: $TN = \mathbb{C}^2 = T^*\mathbb{C}$ with a funny metric

"duality frame" ⇝ way of computing link homology

Setup: $CY_3 \times HK \times \mathbb{R}_t$ (or S^1)

$TN = \mathbb{C}^2 = T^*\mathbb{C} = \mathbb{C}\times\mathbb{C} \supset \mathbb{C}^\times \times \mathbb{C}^\times$ with a funny metric

T^*S^3 — deformed conifold — Witten

$\mathcal{O}(-1)\oplus\mathcal{O}(-1)$ — Vafa

M+N M5 branes

$S^3 \times$  $\times \mathbb{R}^1$

cyls

$TN = \mathbb{C}^2 = T^*\mathbb{C} = \mathbb{C}\times\mathbb{C} \supset \mathbb{C}^\times \times \mathbb{C}^\times$

with a funny metric

T^*S^3

deformed conifold

Witten

$\mathcal{O}(-1)\oplus\mathcal{O}(-1)$

Vafa

volume $\mathbb{P}^1$

$M+N$ M5 branes

$S^3 \times$  $\times \mathbb{R}^1$

eyes

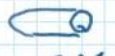
$GL(M/N)$

5dim theory on $TN \times \mathbb{R}$

BPS particles on the cigar

volume $\hbar$

M+N M5 branes

$S^3 \times$  $\times \mathbb{R}^1$

cyl

$GL(M/N)$

5dim theory on $TN \times \mathbb{R}$

BPS particles on the cigar

Witten - Aganagic : $S^3 \rightsquigarrow$  $\times$  $\rightsquigarrow$ Nakajima var.

$W(\otimes V_{\alpha_i})$

volume $\hbar$

M+N M5 branes

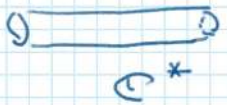
$S^3 \times$  $\times \mathbb{R}^1$

cyl

$GL(M/N)$

5dim theory on $TN \times \mathbb{R}$

BPS particles on the cigar

Witten - Aganagic : $S^3 \rightsquigarrow$  $\times$  $\rightsquigarrow$ Nakajima var.

$\mathbb{C}^*$ $\mathbb{R}$

$W(\otimes V_{\alpha_i})$

$\mathbb{C}^*$ 1

$W(\otimes V_{\alpha_i})$

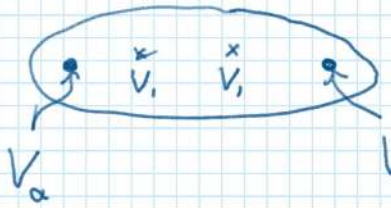
$K \subset S^3 \rightsquigarrow N^\vee K \subset T^* S^3$

$M5: \quad N^\vee K \times \text{cyg} \times \mathbb{R}$

$\mathcal{K} \subset S^3 \rightsquigarrow N^\vee \mathcal{K} \subset T^* S^3$

$M\tilde{5}$: $N^\vee \mathcal{K} \times cyg \times \mathbb{R}$

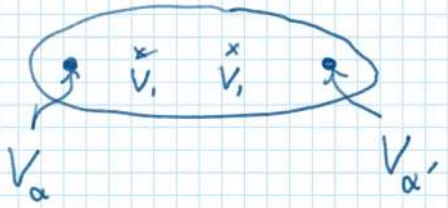
$SU(2)$



$Inv(V_\alpha \otimes V_1 \otimes V_1 \otimes V_{\alpha'})$

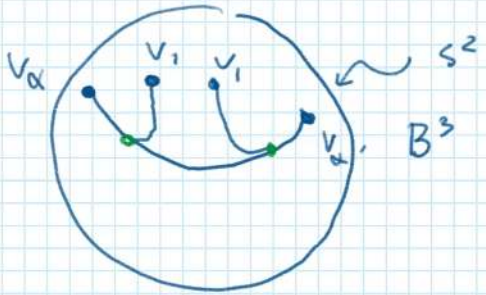
$\cong V_1 \otimes V_1$

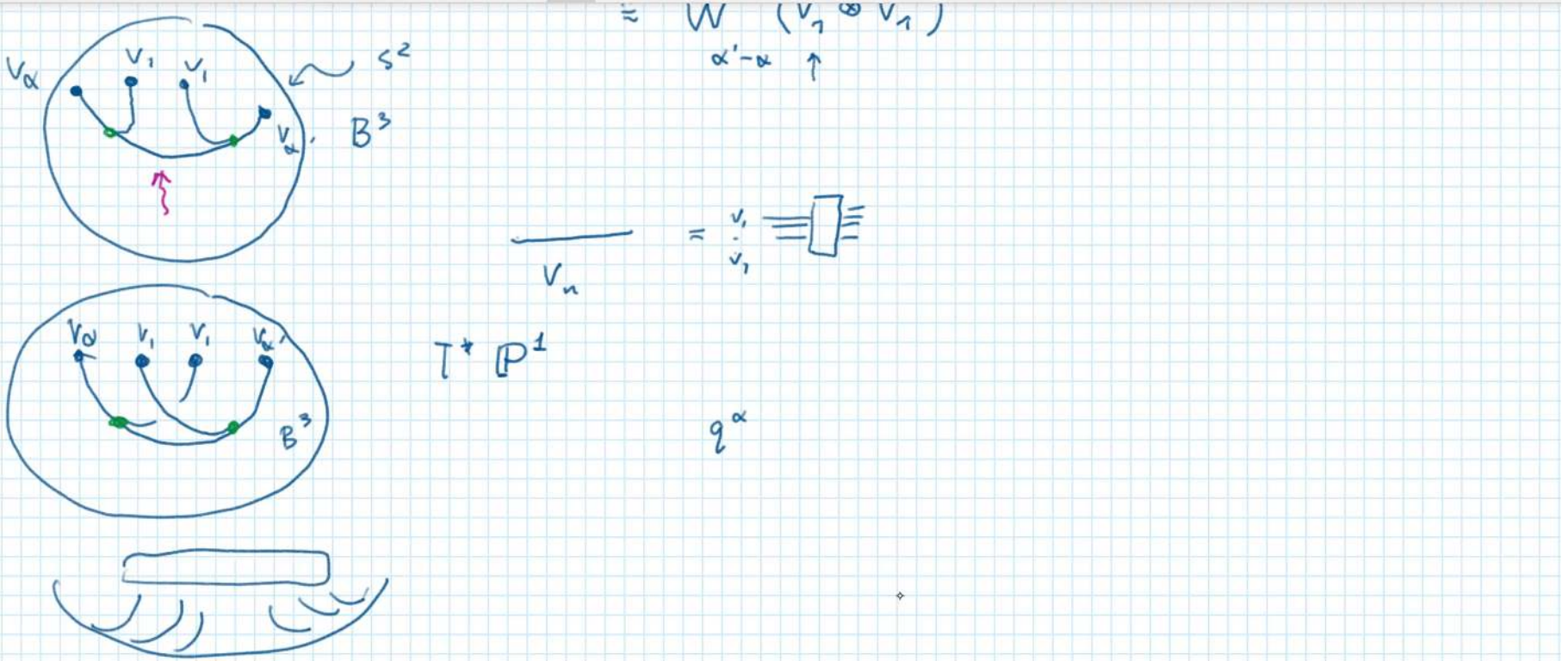
$SU(2)$

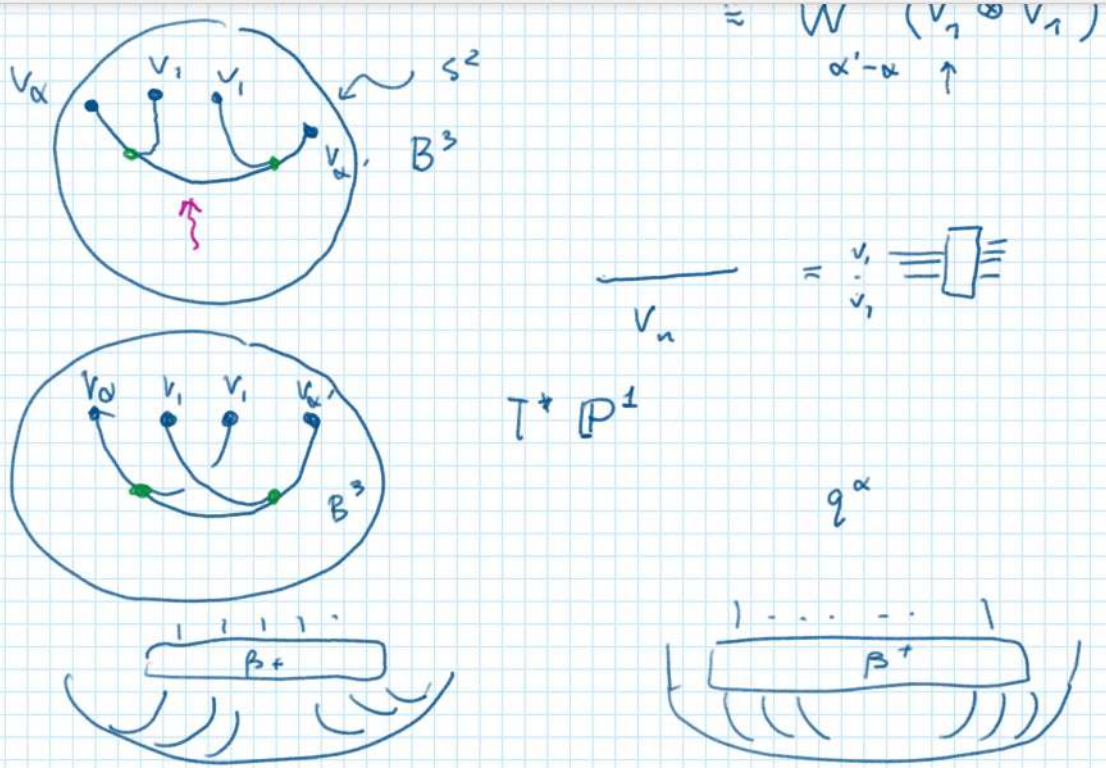


$$Inv\left(V_\alpha \otimes V_1 \otimes V_1 \otimes V_{\alpha'}\right) \cong W_{\alpha'-\alpha}\left(V_1 \otimes V_1\right)$$

($V_{\alpha+1} \oplus V_{\alpha-1}$)




V_α
V_1
V_1
$V_{\alpha'}$
S^2
B^3
V_n
V_α
V_1
V_1
$V_{\alpha'}$
B^3
$T^* \mathbb{P}^1$
q^α
$\alpha' - \alpha$


V_α
V_1
V_1
$V_{\alpha'}$
S^2
B^3
V_n
V_1
V_1
$T^*\mathbb{P}^1$
q^α
β_+
β^+