Title: 3d Gauge Theory and Elliptic Stable Envelopes IV

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Collection: QFT for Mathematicians 2022

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Stab: $H^*_T(X^A) \to H^*_T(X)$

$m_R \to \infty$ | $m_R = 0$

3d $\mathcal{N}=2$

(3d $\mathcal{N}=1$) χ

SQM into $\mathcal{X} = \mathrm{Maps}(E_\tau, X)$

$Q^2 = D_{\bar{z}}$

$Q = \boxed{\bar{Q}_+} + Q_+$

H_T

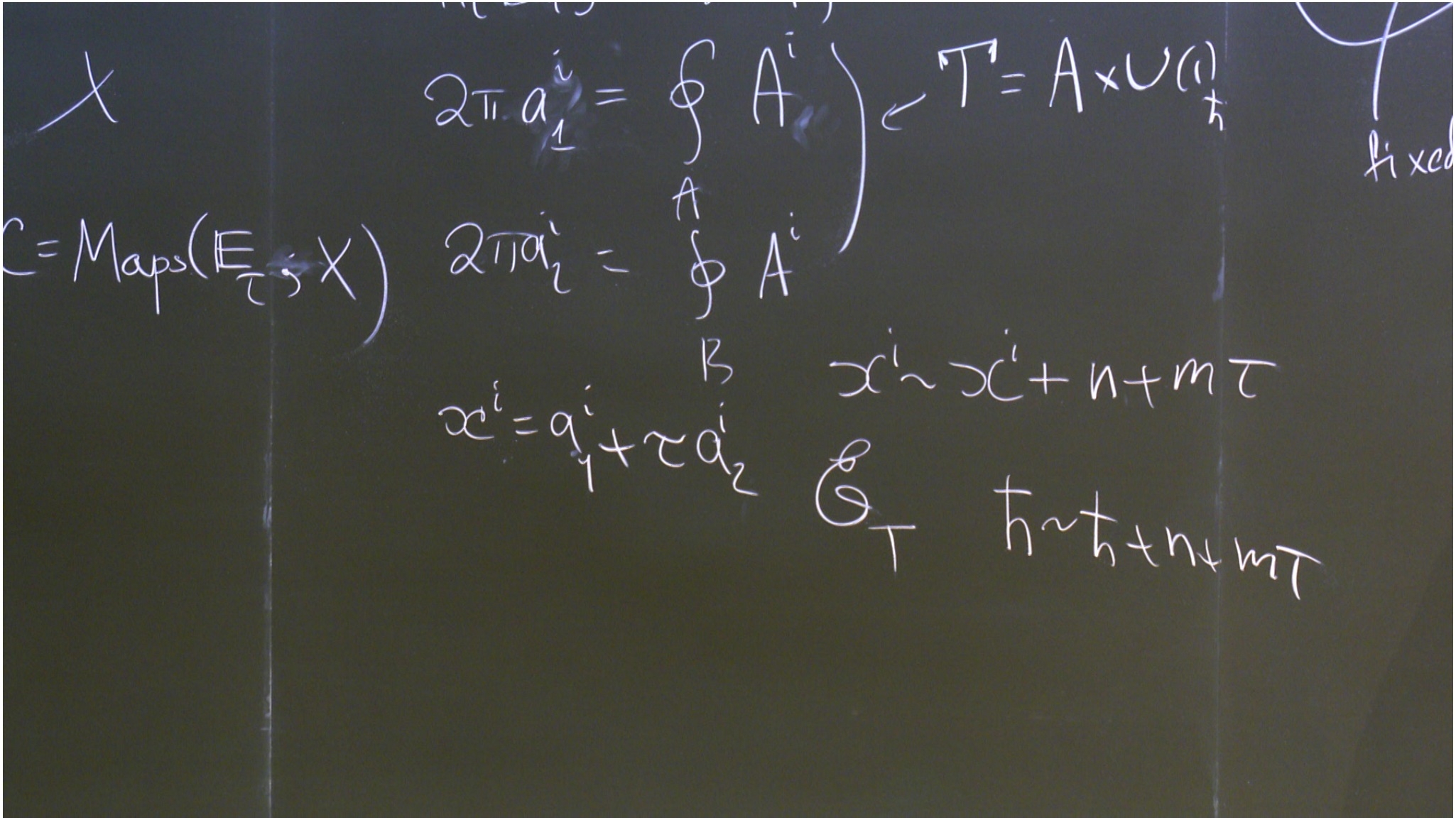
$\mathcal{H}[E_\tau] \to \mathcal{V}[E_\tau]$ $\quad G_f \to A$

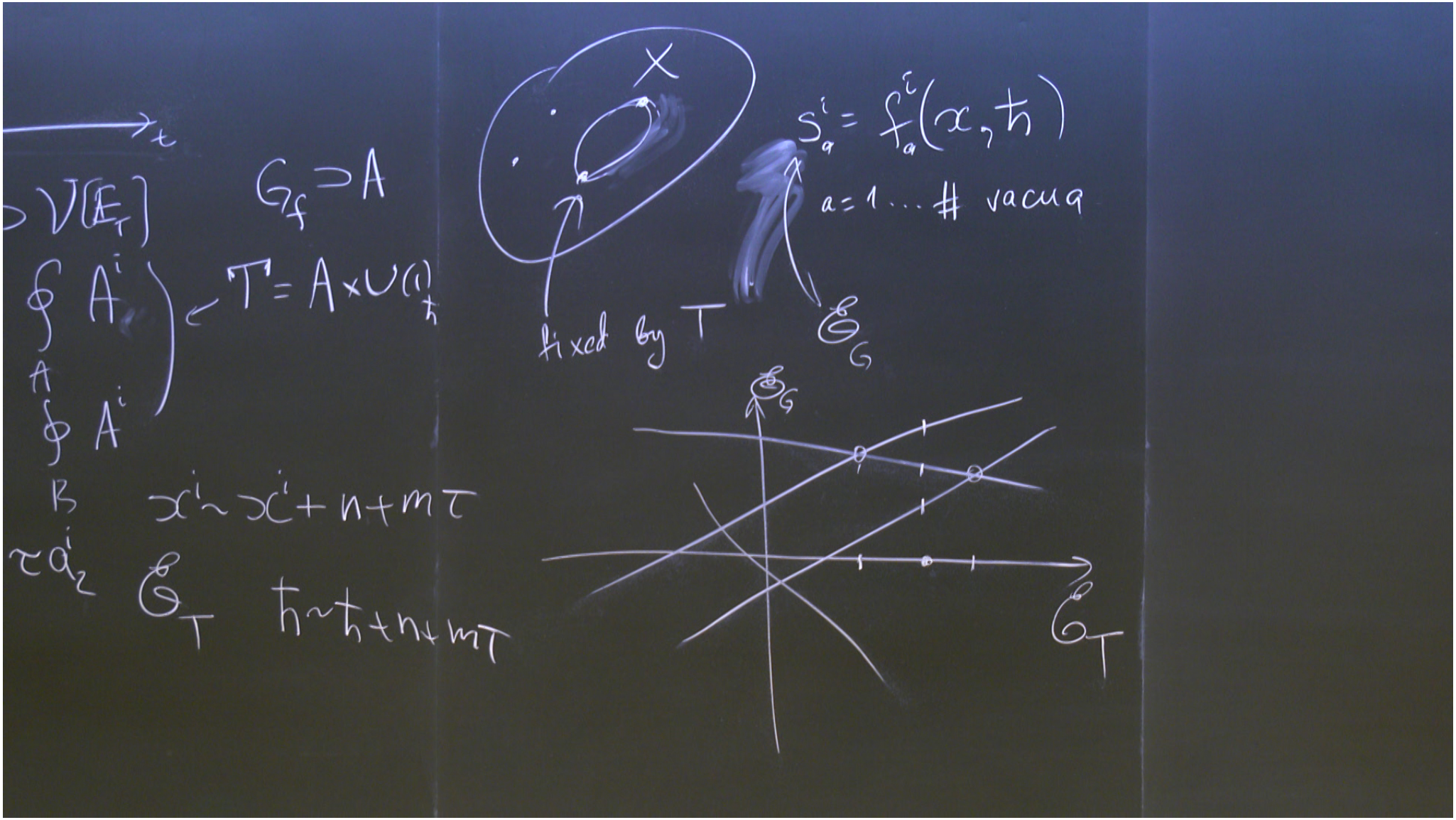
$2\pi a_1^i = \oint_A A^i$ $\Big)$ $\leftarrow T = A \times U(1)_\hbar$

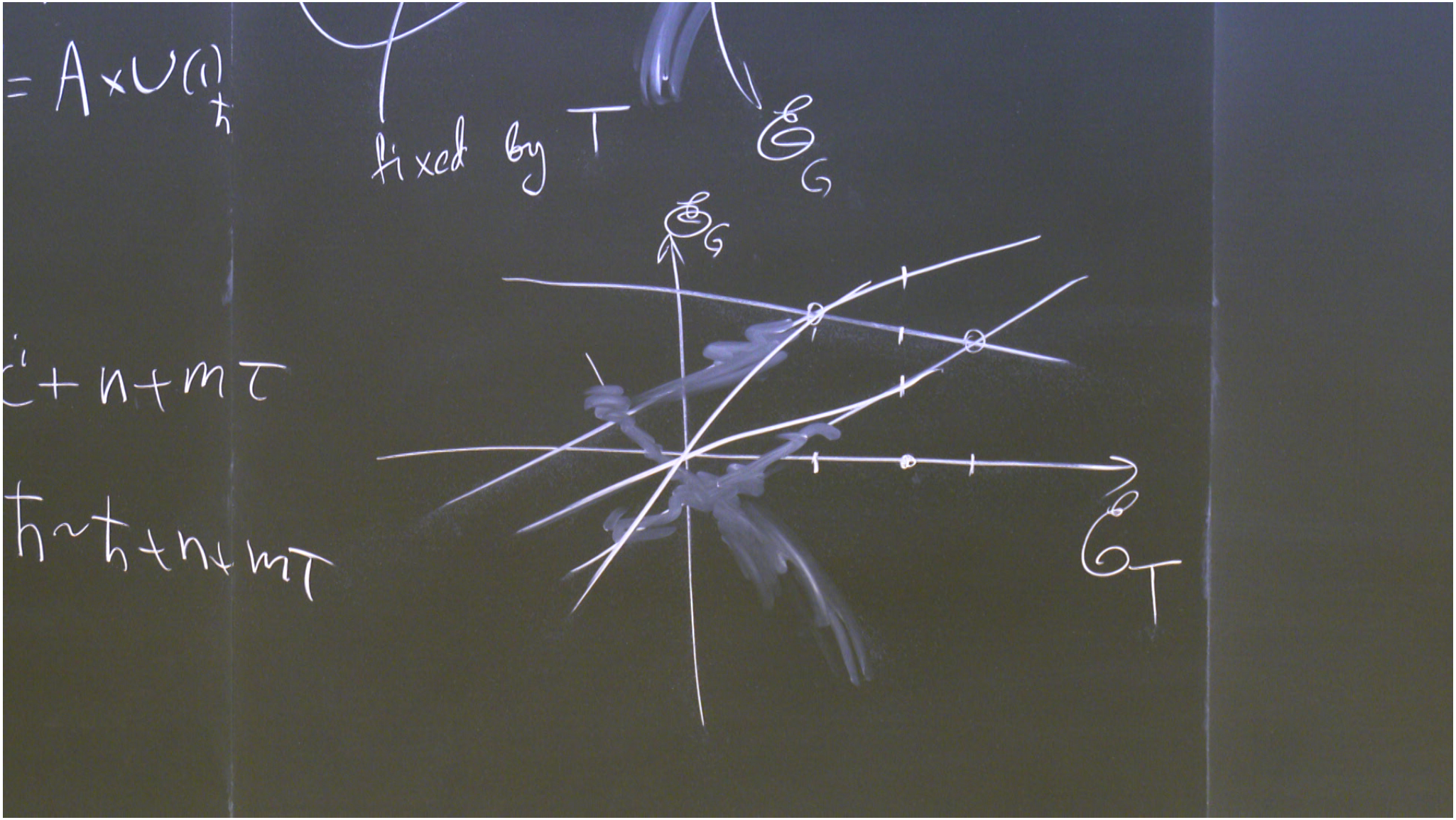
$2\pi a_2^i = \oint_B A^i$

$x^i = q_1^i + \tau q_2^i$ $\quad x^i \sim x^i + n + m\tau$

G_T


$2\pi a_1^i = \oint_A A^i$
$T = A \times U(1)_\hbar$
$C = \mathrm{Maps}(\mathbb{E}_\tau, X)$
$2\pi a_2^i = \oint_B A^i$
$x^i \sim x^i + n + m\tau$
$x^i = q_1^i + \tau q_2^i$
$\mathcal{G}_T$
$\hbar \sim \hbar + n + m\tau$


t
$G_f \supset A$
$V[E_\tau]$
$\oint_A A^i$
$\oint_B A^i$
$T = A \times U(1)_\hbar$
$x^i \sim x^i + n + m\tau$
τa_2^i
G_T
$\hbar \sim \hbar + n + m\tau$
X
fixed by T
$S_a^i = f_a^i(x, \hbar)$
a = 1 ... # vacua
G_G
G_G
G_T


= A × U(1)
fixed by T
$\mathcal{G}_G$
$\mathcal{G}_G$
$\mathcal{G}_T$
+ n + mτ
h ~ h + n + mτ

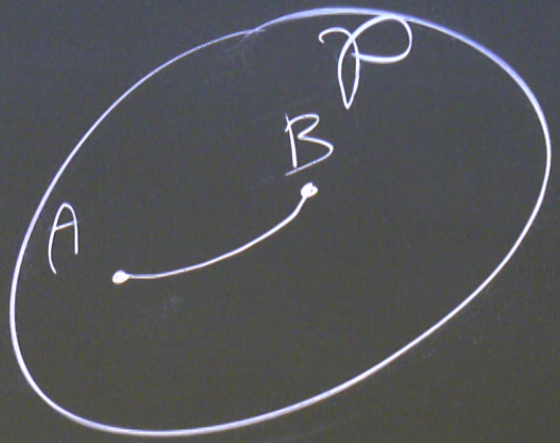
, th.)

vacua

T_p . H . $\sigma(H) = \{0\} \cup S$

eff. Hilb. space $S \subset (\Delta, \infty)$

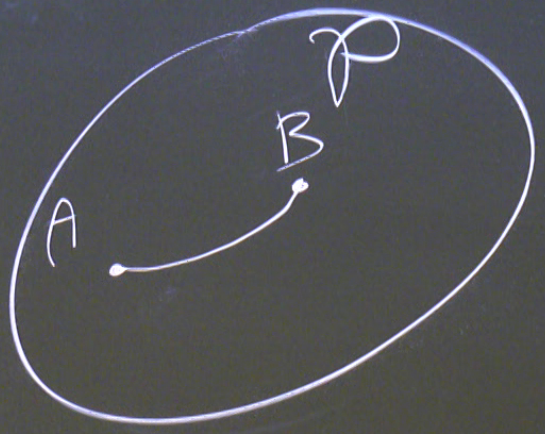
$\mathbb{C}|0\rangle$ Δ – large



T_p . H . $\sigma(H) = \{0\} \cup S$

eff. Hilb. space $\quad S \subset (\Delta, \infty)$

$|0\rangle$ $\quad \Delta$ – large



Effective action

$$\Gamma[A] \overset{\text{Adiab. limit}}{=\!=} \frac{k_{ij}}{4\pi} \int A^i_f \wedge dA^j_f$$

$k_{ij} = k_{ji} \in \frac{1}{2}\mathbb{Z}$

$= \{0\} \cup S$

$S \subset (\Delta, \infty)$

Δ – large

Effective action

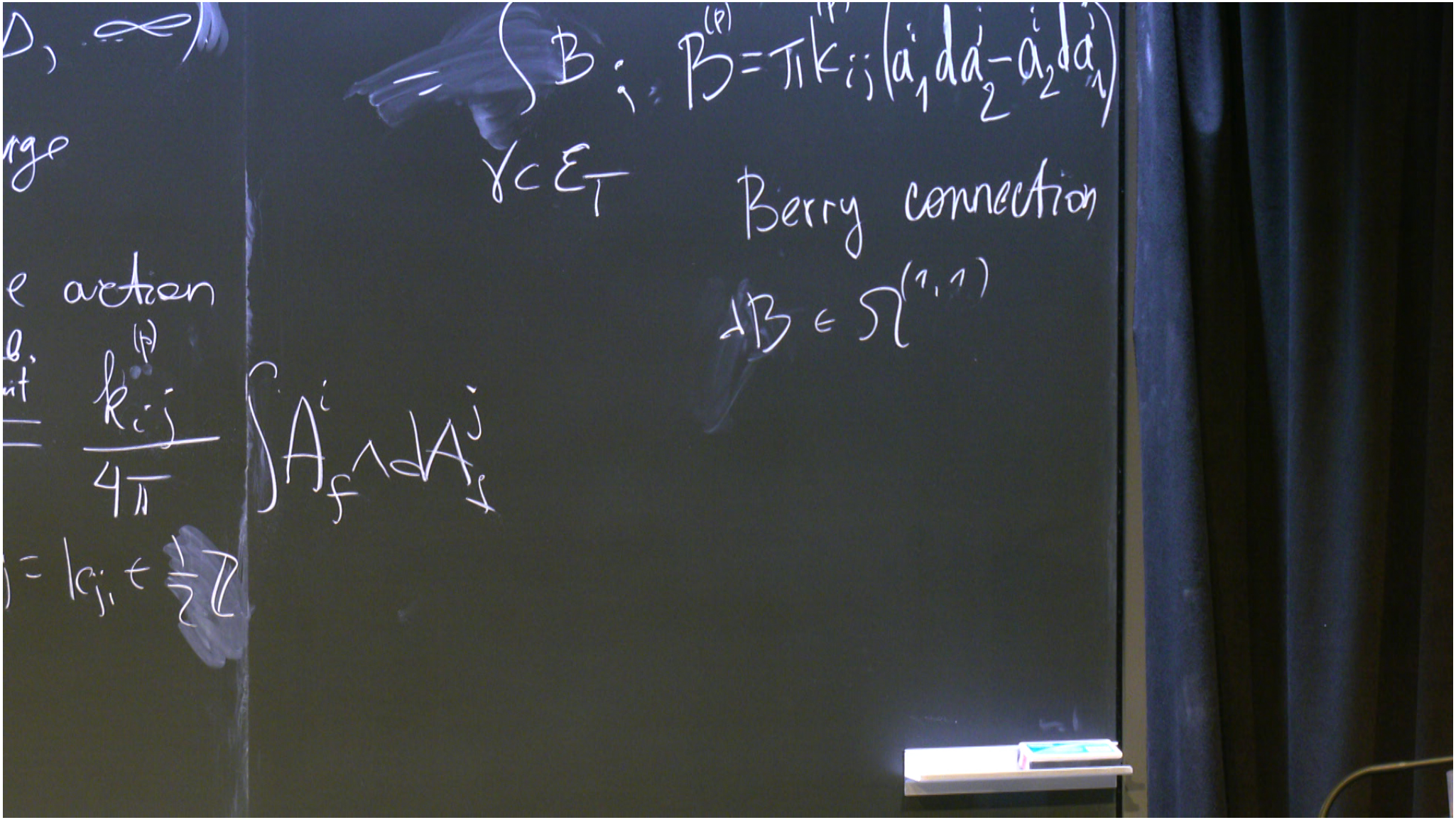
$$\Gamma[A] \overset{\text{Adiab. limit}}{=\!=} \frac{k^{(p)}_{ij}}{4\pi} \int A^i_f \wedge dA^j_f$$

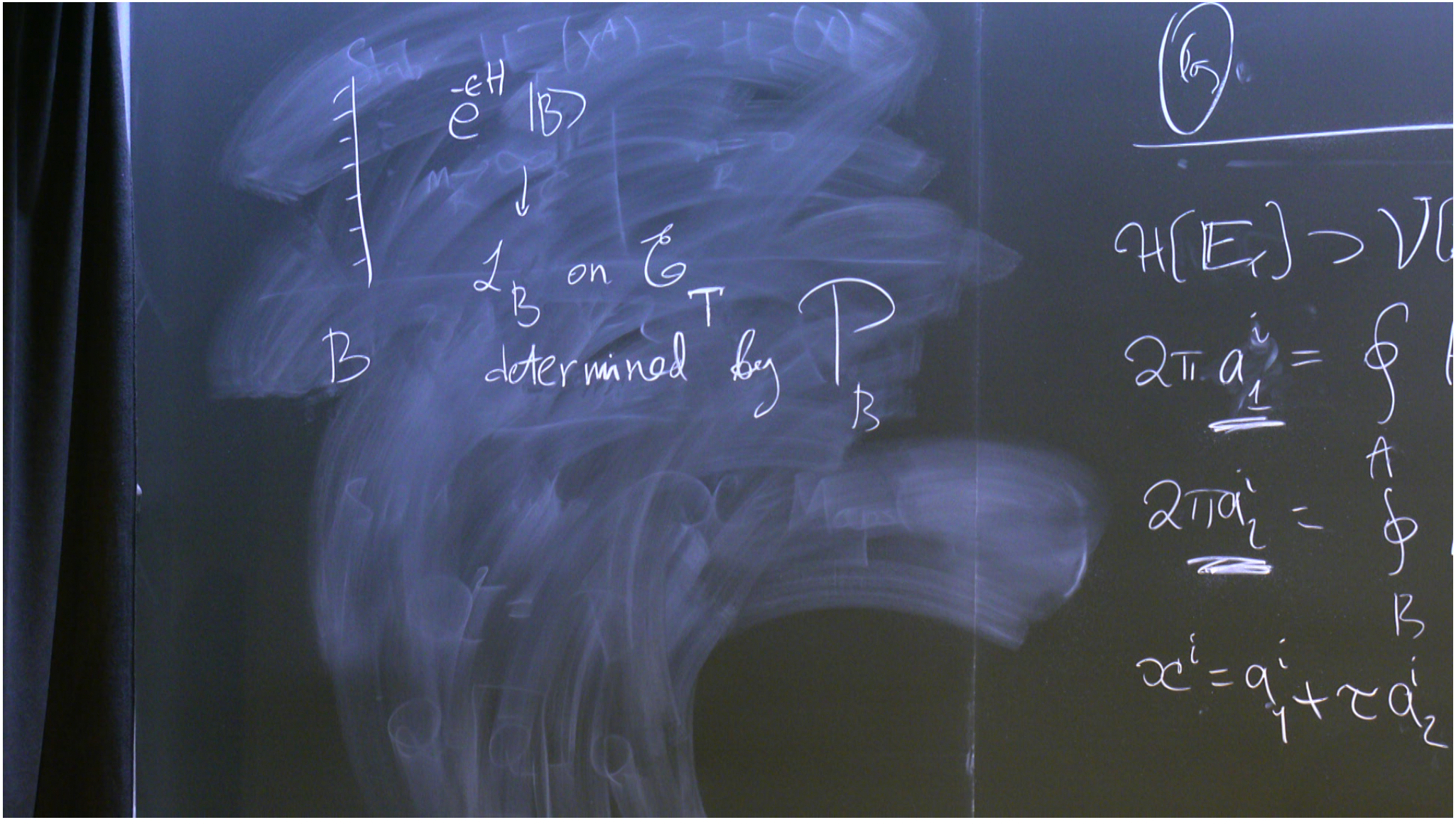
$k_{ij} = |c_{ij}| \in \mathbb{Z}$

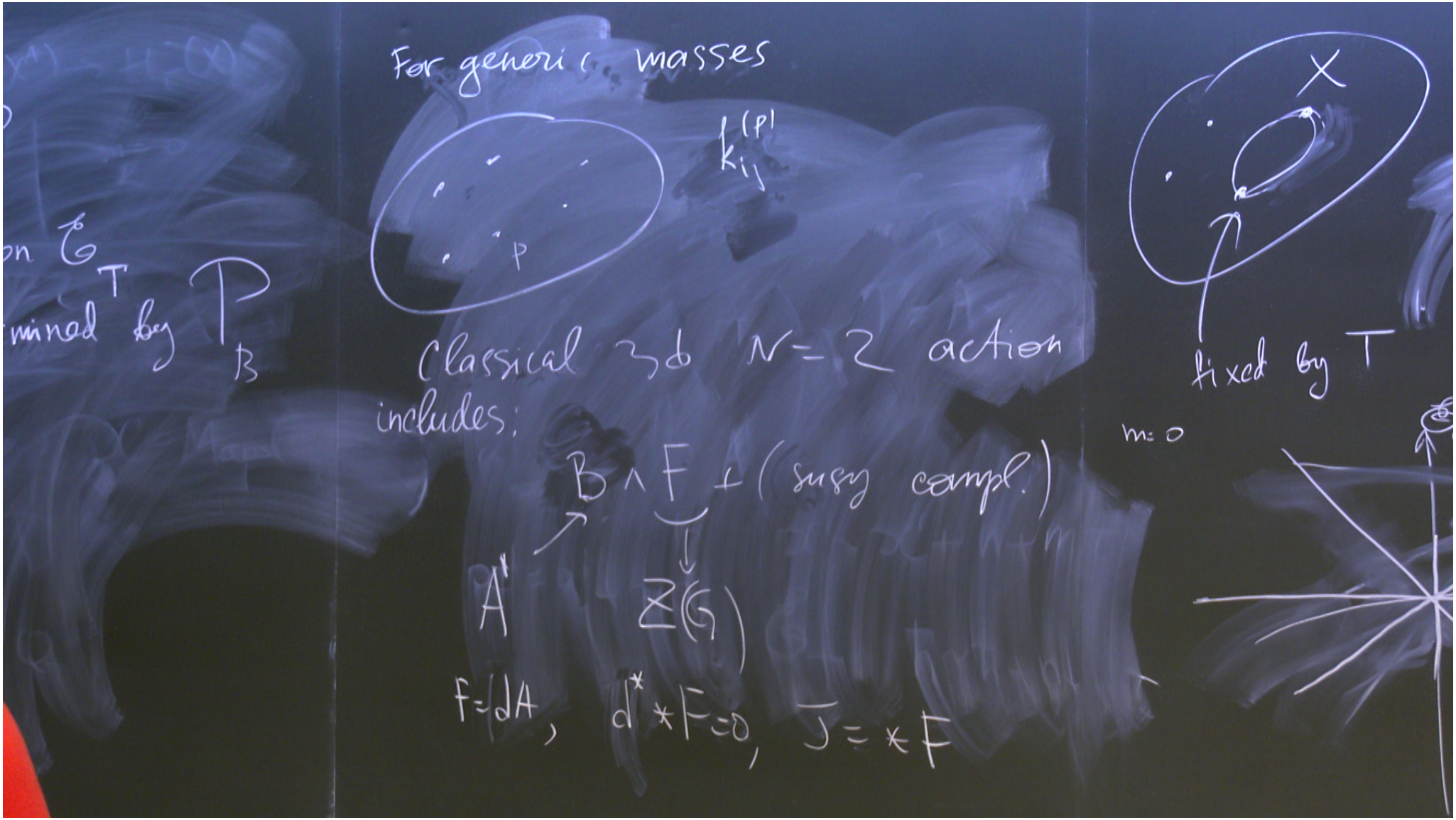
Ex. $\Gamma[A] = \pi k^{(p)}_{ij} \int (a^i_1 \ddot{a}^j_2 - a^i_2 \ddot{a}^j_1)\, dt$

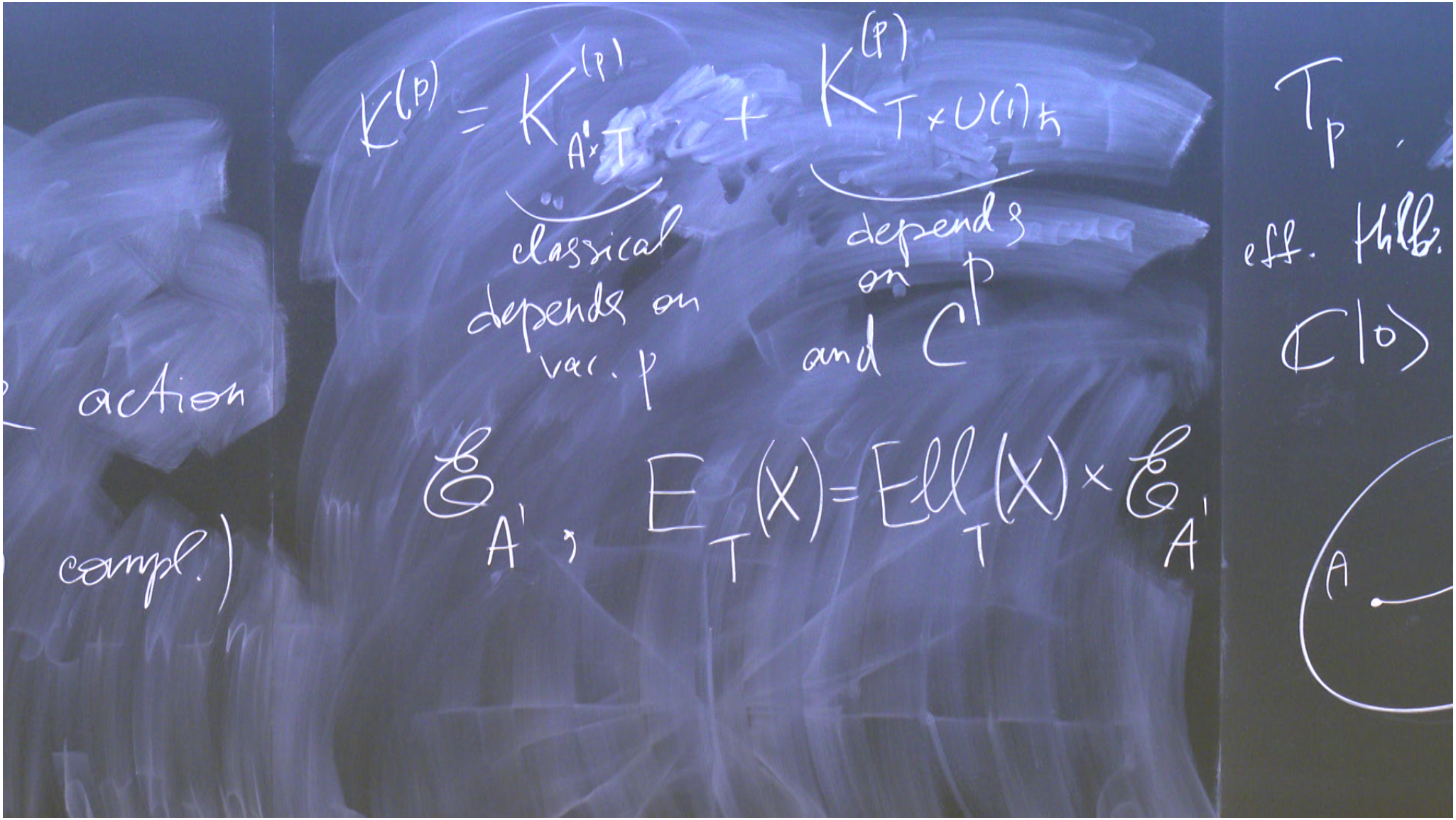
$$= \int_\gamma B; \quad B^{(p)} = \pi k^{(p)}_{ij} (a^i_1 da^j_2 - a^i_2 da^j_1)$$

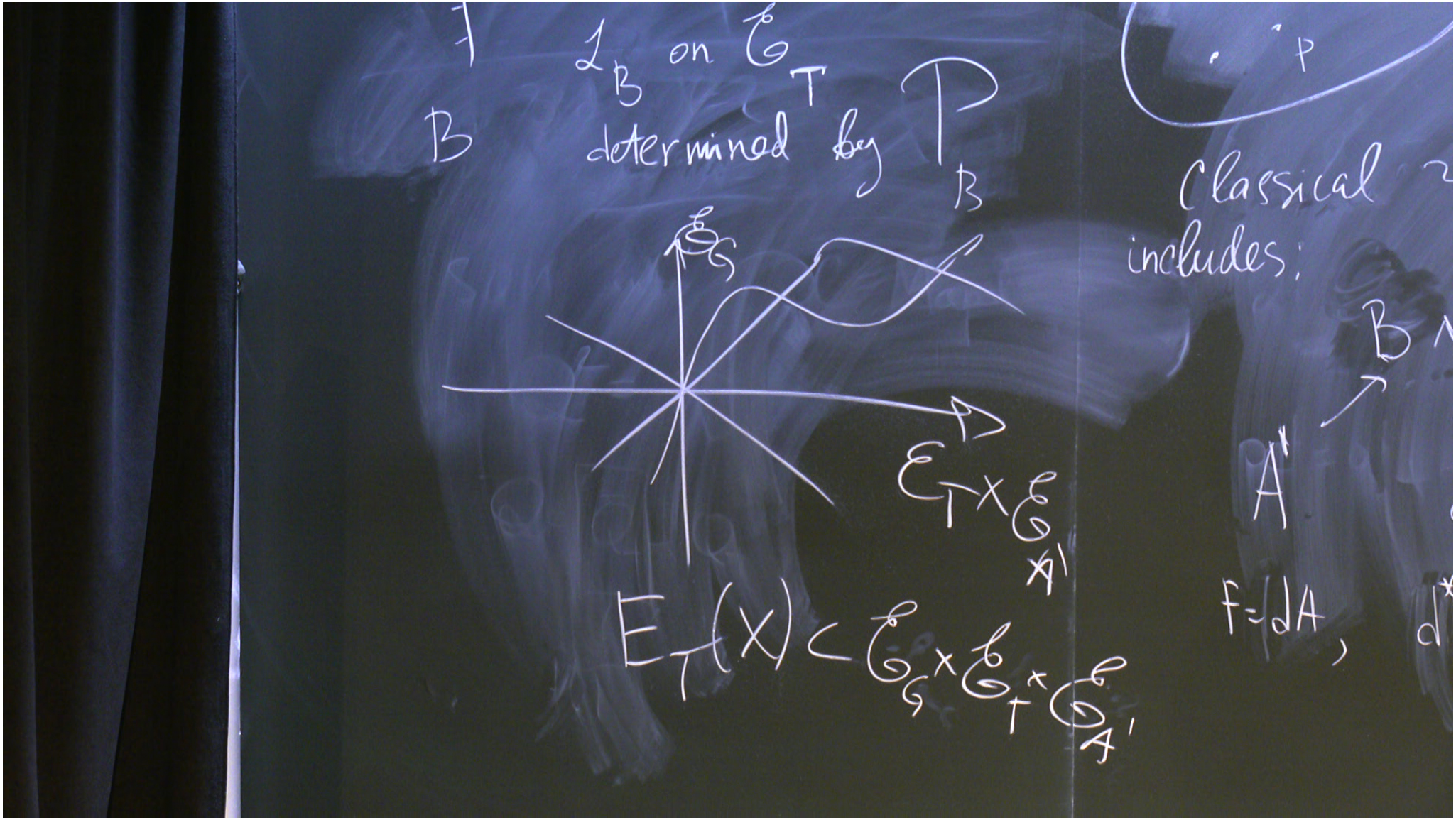
$\gamma \subset \mathcal{E}_T$ Berry connection

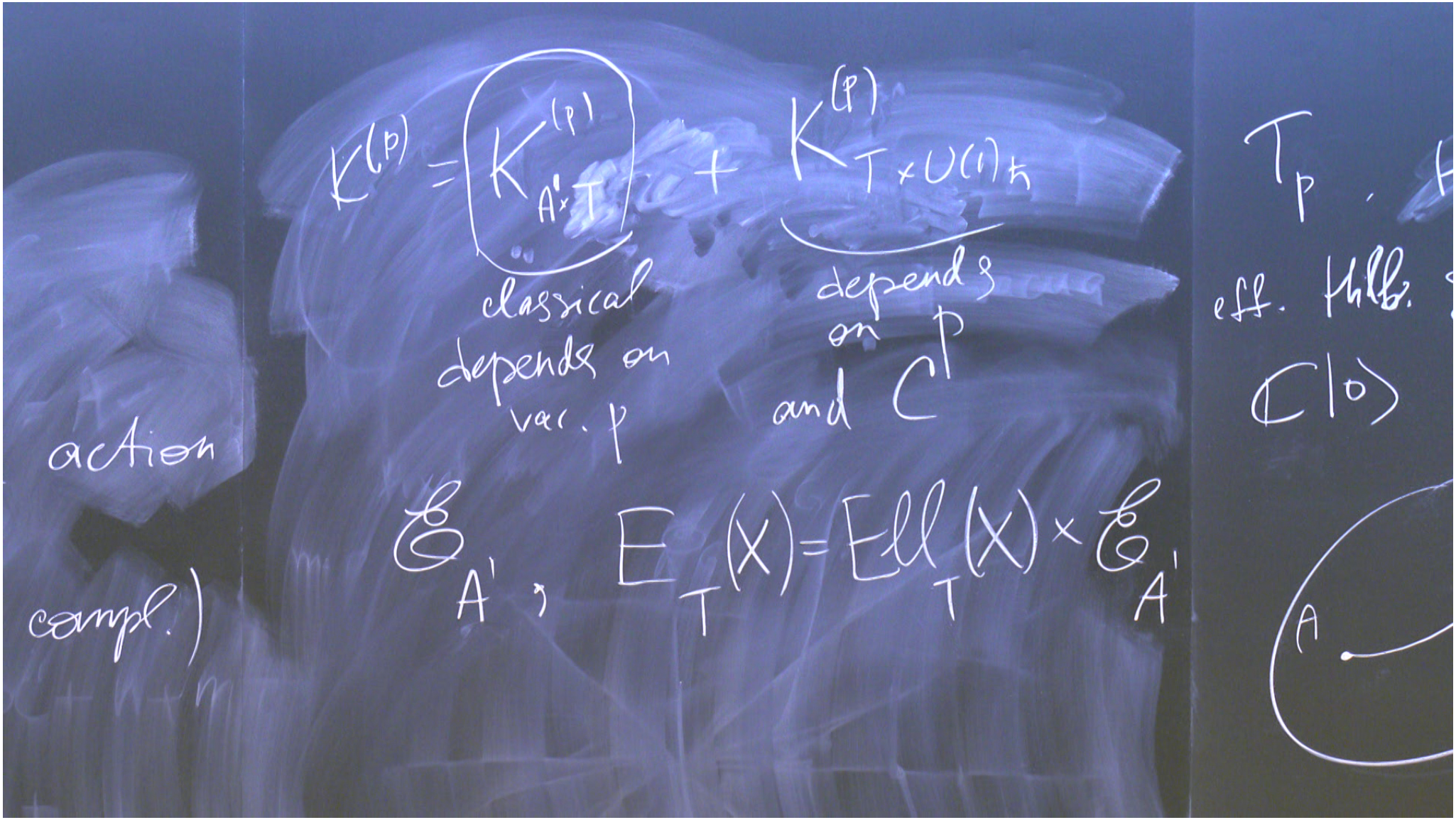

Berry connection

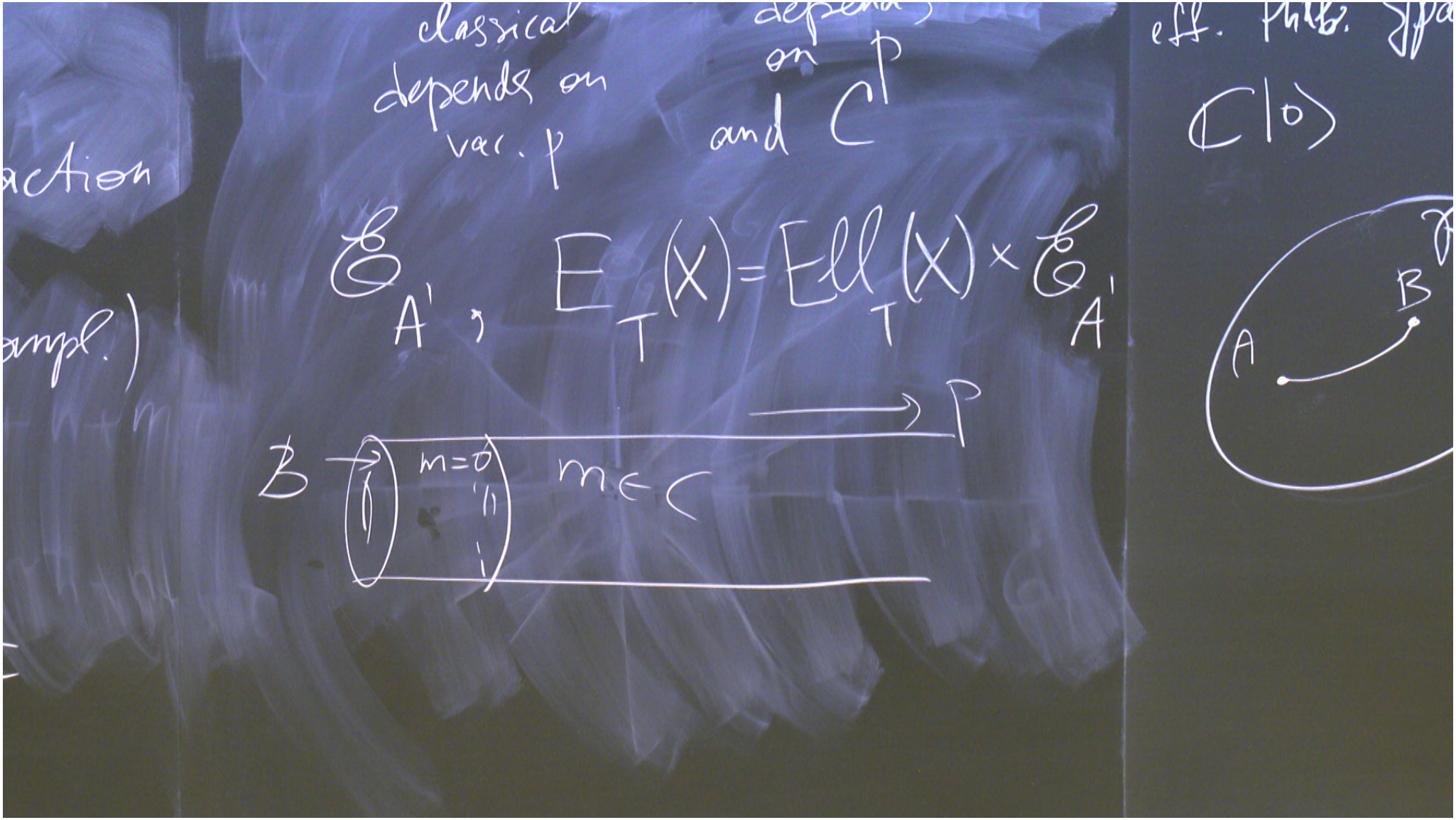

$e^{-\epsilon H}|B\rangle$
$\mathcal{L}_B$ on $\mathcal{C}_T$
B
determined by P_B
$\mathcal{H}(E_i) > V$
$2\pi a_1^i = \oint_A$
$2\pi a_2^i = \oint_B$
$x^i = q_1^i + \tau q_2^i$

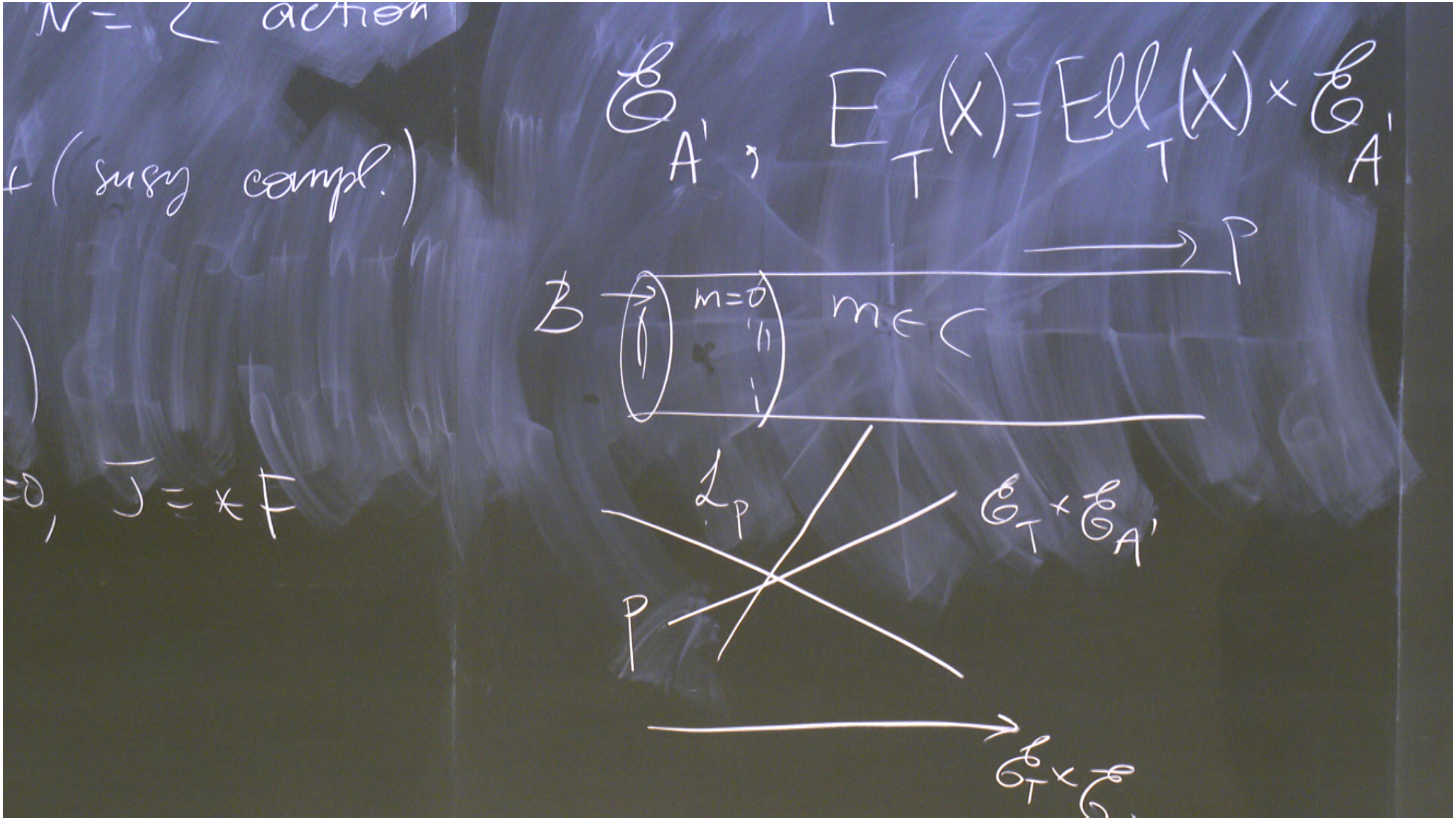

For generic masses
$k_{ij}^{(p)}$
p
X
fixed by T
Classical 3d $\mathcal{N}=2$ action
includes:
$B \wedge F$ + (susy compl.)
A^1
$Z(G)$
$F = dA$, $d^* * F = 0$, $\bar{J} = *F$
m = 0

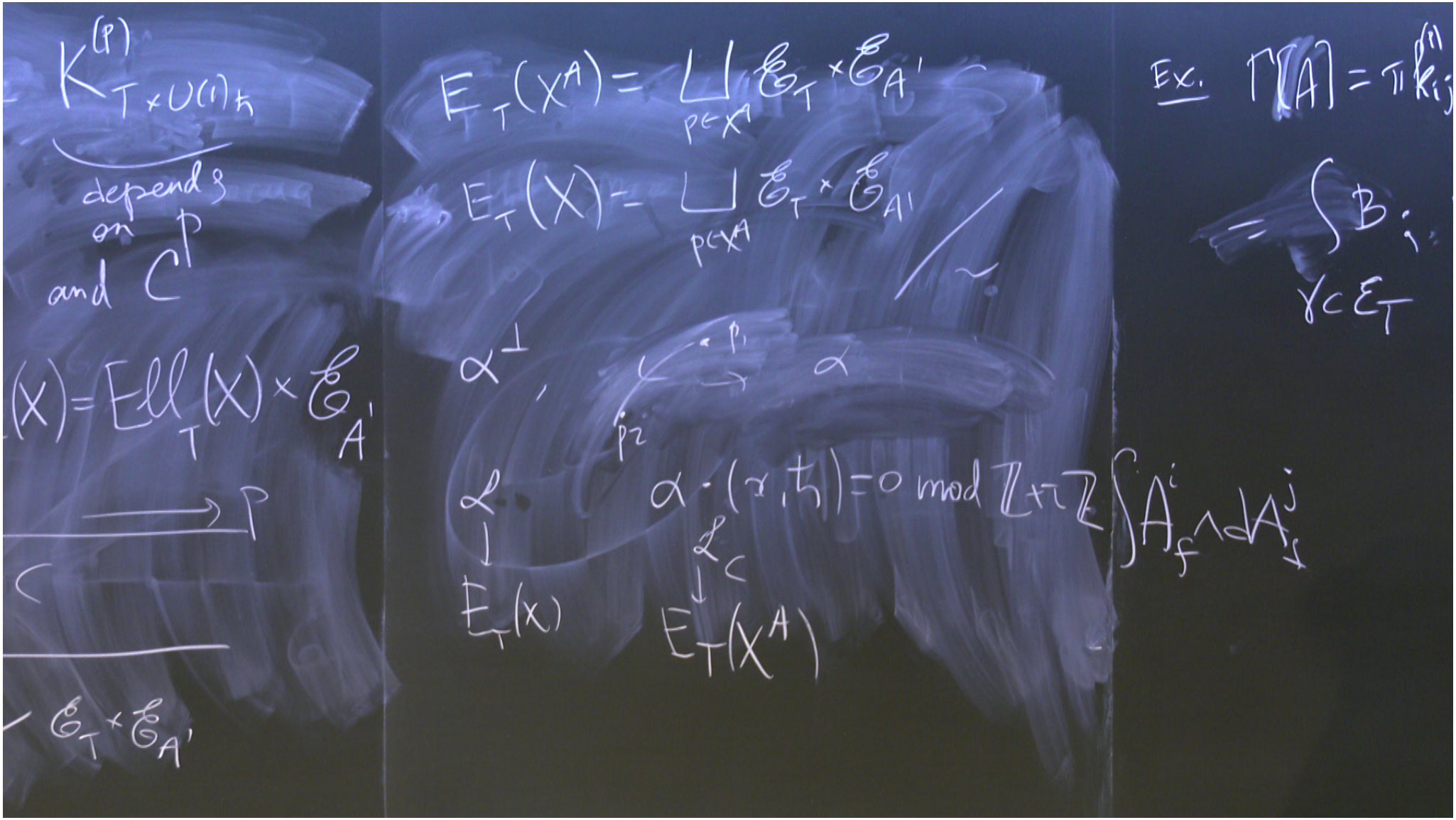

$K^{(p)} = K^{(p)}_{A' \times T} + K^{(p)}_{T \times U(1)_\hbar}$
classical depends on var. p
depends on p and C^p
T_p
eff. Hilb.
$C|0\rangle$
action
compl.)
$\mathcal{E}_{A'}$, $E_T(X) = Ell_T(X) \times \mathcal{E}_{A'}$
A

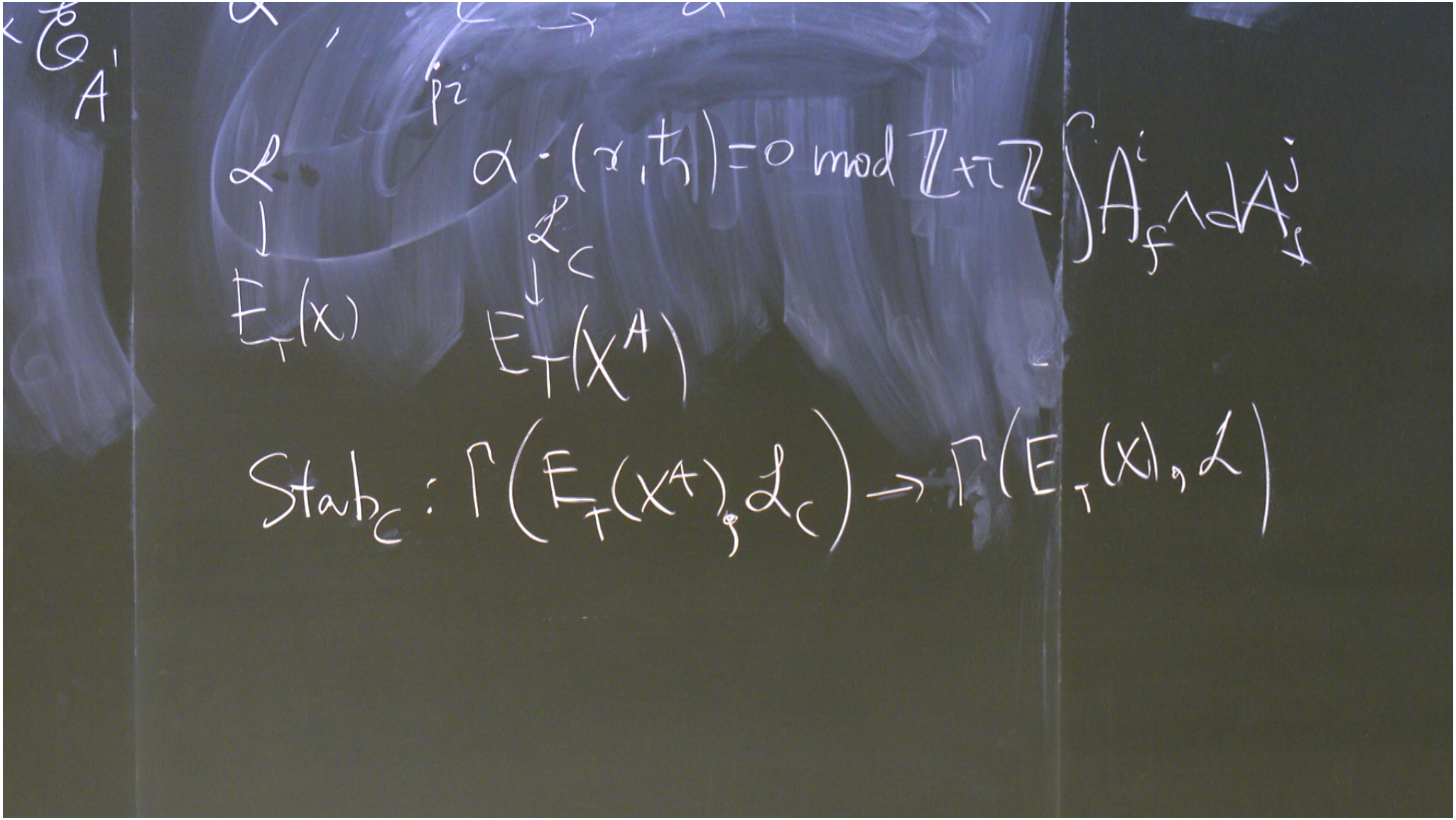

determined by
Classical
includes:
$E_T \times \mathcal{E}_{\mathcal{G}}$
A'
$F = dA$,
$E_T(X) \subset \mathcal{E}_{\mathcal{G}} \times \mathcal{E}_T \times \mathcal{E}_{A'}$

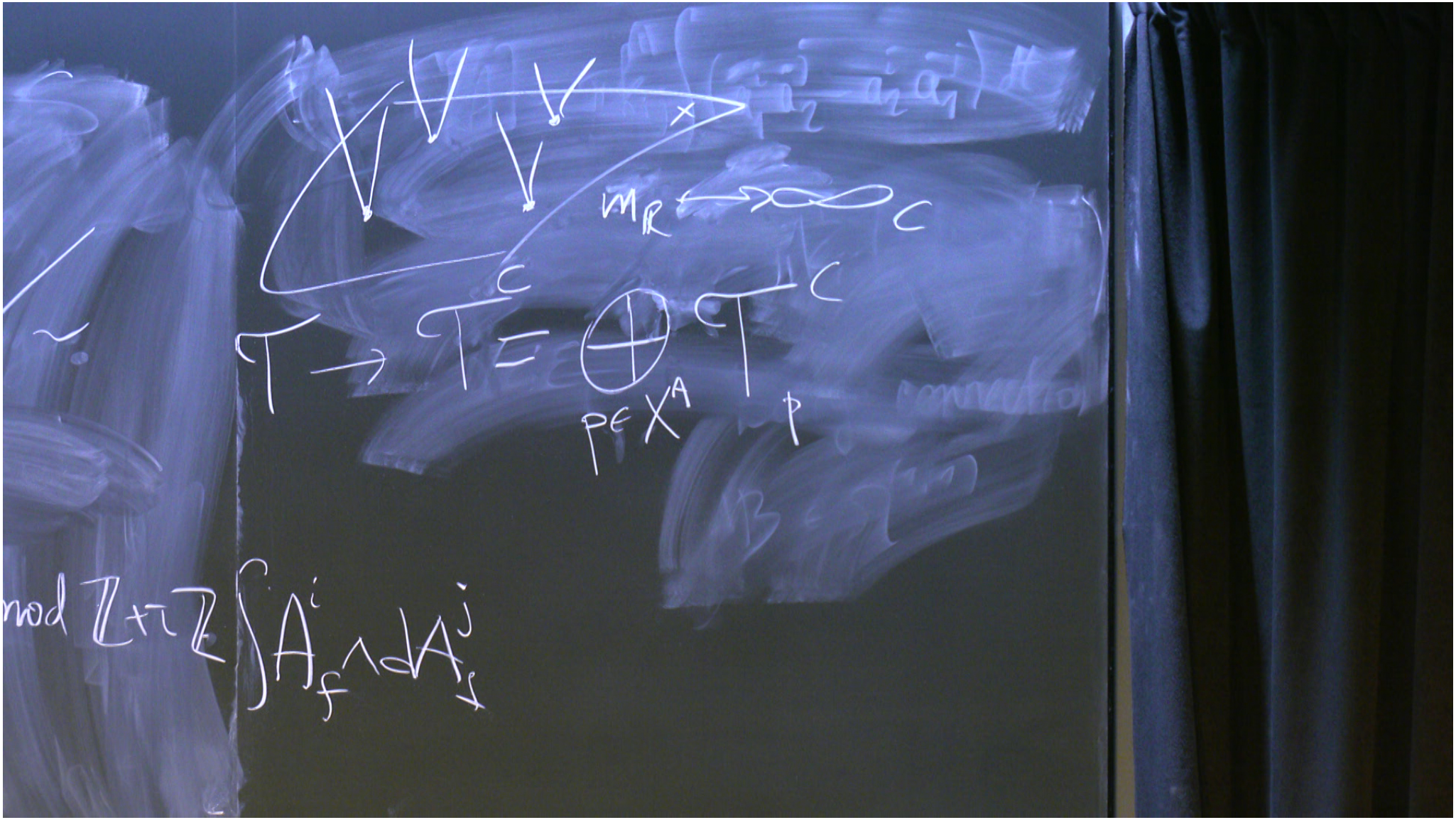

$K^{(p)} = K^{(p)}_{A' \times T} + K^{(p)}_{T \times U(1)_\hbar}$
classical
depends on var. p
depends on p and C^p
T_p .
eff. Hilb.
$C|0\rangle$
action
compl.)
$\mathcal{E}_{A'}$, $E_T(X) = Ell_T(X) \times \mathcal{E}_{A'}$
A

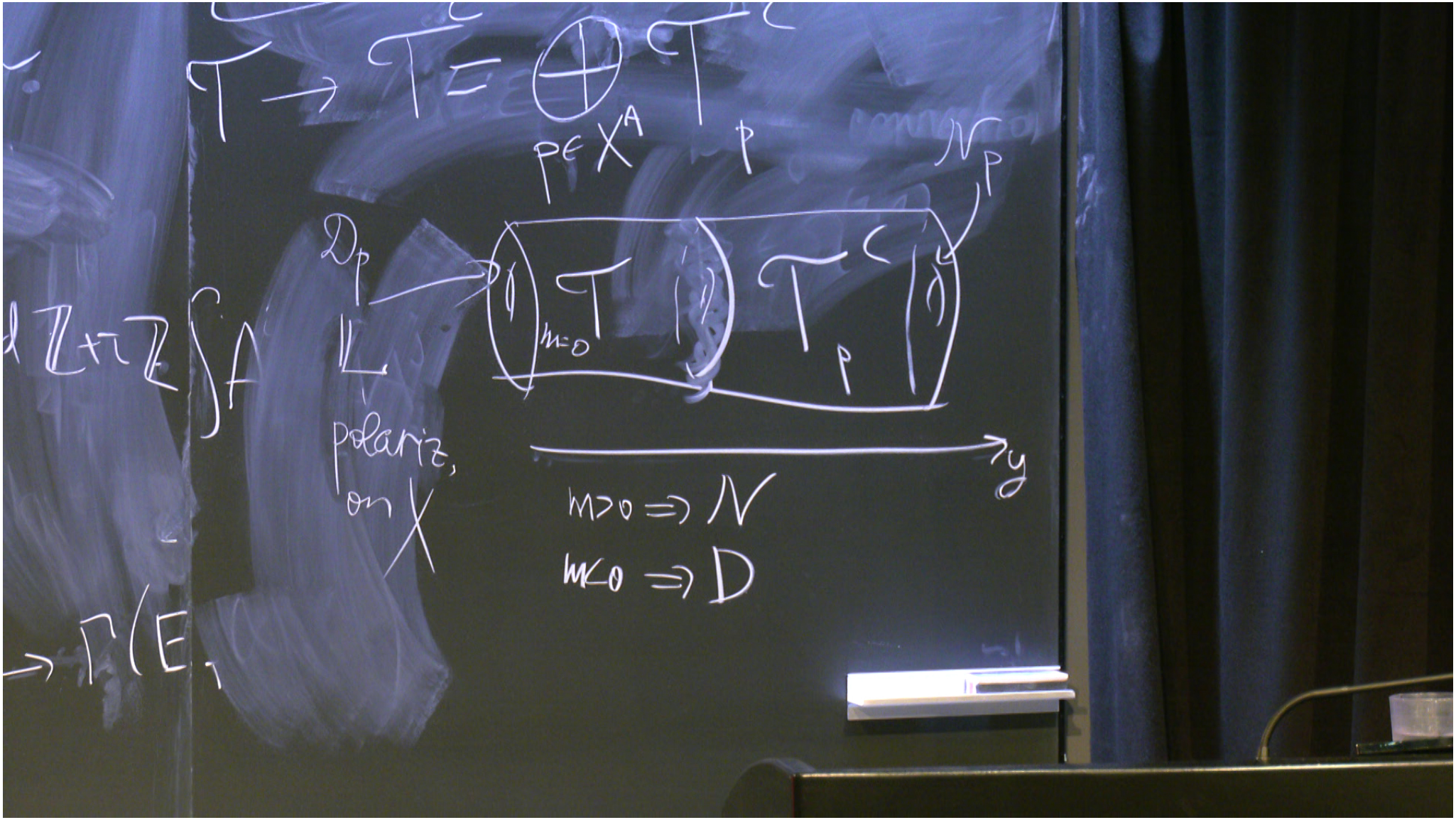

classical
depends on
var. p
depends on p
and C
eff. Phil. Appa
C|0⟩
action
ample.)
$\mathcal{E}_{A'}$, $E_T(X) = Ell_T(X) \times \mathcal{E}_{A'}$
p
B
m=0
m∈C
A
B

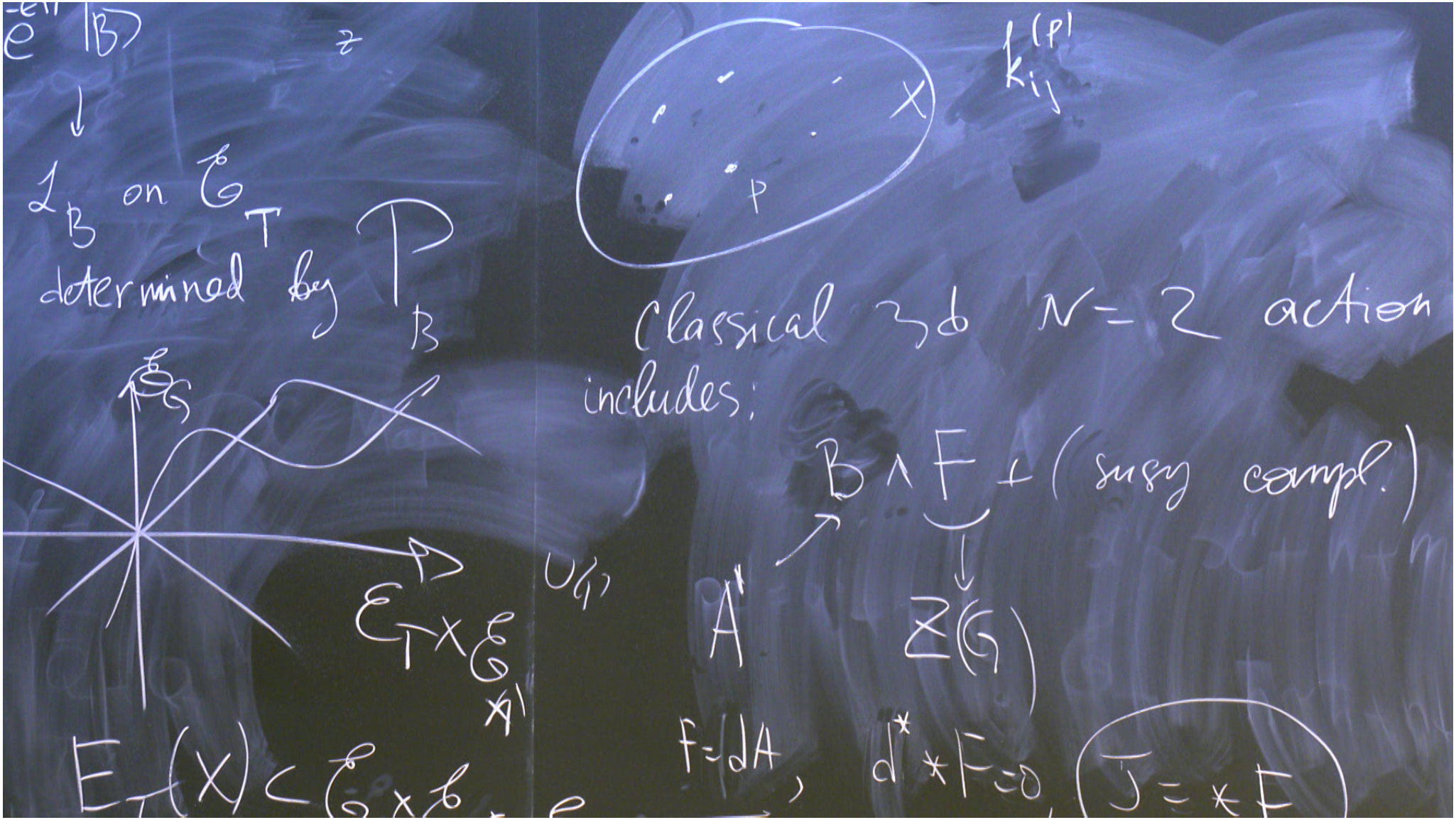

N = 2 action
(susy compl.)
$\mathcal{G}_{A'}$, $E_T(X) = Ell_T(X) \times \mathcal{G}_{A'}$
B
m = 0
m ∈ C
P
$\mathcal{L}_P$
$\mathcal{G}_T \times \mathcal{G}_{A'}$
P
$\mathcal{G}_T \times \mathcal{G}_{A'}$


depends on P and C
Ex.
$K^{(P)}_{T\times U(1)_\hbar}$
$E_T(X^A) = \coprod_{p\in X^A} \mathfrak{G}_T \times \mathfrak{G}_{A'}$
$E_T(X) = \coprod_{p\in X^A} \mathfrak{G}_T \times \mathfrak{G}_{A'}$
$\Gamma[A] = \pi \hbar^{(1)}_{ij}$
$= \int_{\gamma_C \in \Sigma_T} B_i$
$\alpha^\perp$
α
$\alpha \cdot (\gamma, \hbar) = 0 \bmod \mathbb{Z} + \tau\mathbb{Z}$
$\int A^i_f \wedge dA^j_f$
$E_T(X)$
$E_T(X^A)$
$\mathfrak{G}_T \times \mathfrak{G}_{A'}$

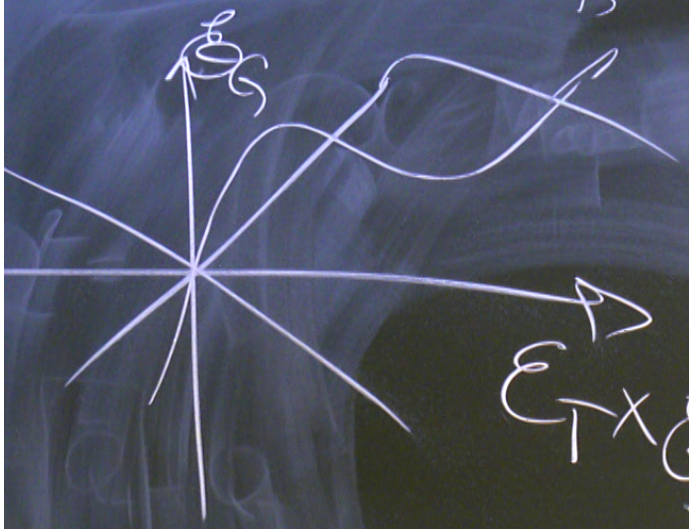

$\mathcal{L}$
$\alpha \cdot (x, t) = 0 \mod \mathbb{Z} + \tau\mathbb{Z} \int A^i_f \wedge dA^j_f$
$\mathcal{L}_C$
$E_T(X)$
$E_T(X^A)$
$\mathrm{Stab}_C : \Gamma(E_T(X^A), \mathcal{L}_C) \to \Gamma(E_T(X), \mathcal{L})$


$m_R \to \infty$
$T \to T = \bigoplus_{p \in X^A} T$
$\mathbb{Z} + \tau\mathbb{Z}$
$\int A^i_f \wedge dA^j_f$


$T \to T = \bigoplus_{p \in X^A} T_p$
D_p
N_p
$T_{m=0}$
T_p
polariz.
on X
$m > 0 \Rightarrow N$
$m < 0 \Rightarrow D$
y
$\Gamma(E,$


$e^{-\epsilon H}|B\rangle$
z
$\downarrow$
$\mathcal{L}_B$ on $\mathcal{C}_T$
determined by P_B
$\mathfrak{t}_G$
$\mathcal{E}_T \times \mathcal{E}_G$
$/\mathcal{W}$
$E_T(X) \subset \mathcal{E}_G \times \mathcal{E}_G$
X
p
$k_{ij}^{(p)}$
Classical 3d $\mathcal{N}=2$ action
includes:
$B \wedge F$ + (susy compl.)
$U(1)$
A^1
$\mathcal{Z}(G)$
$F = dA$, $d^* * F = 0$, $J = * F$

determined by



Classical 3d $\mathcal{N}=2$ action includes:

$$B \wedge F + (\text{susy compl.})$$

$$\mathcal{E}_T \times \mathcal{E}_{A'}$$

U_G

A'

$Z(G)$

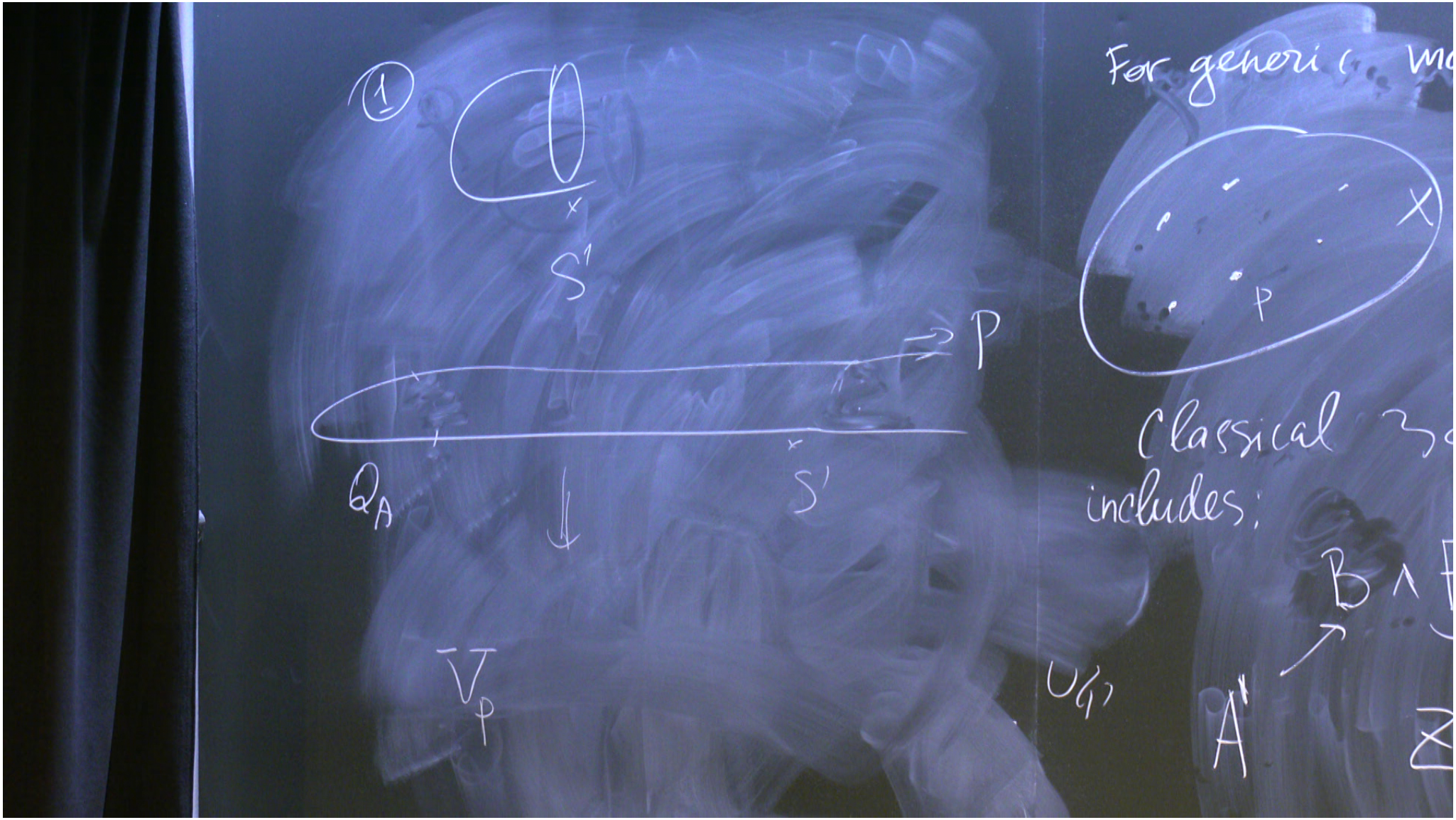
$$E_T(X) \subset \mathcal{E}_G \times \mathcal{E}_T \times \mathcal{E}_{A'}$$

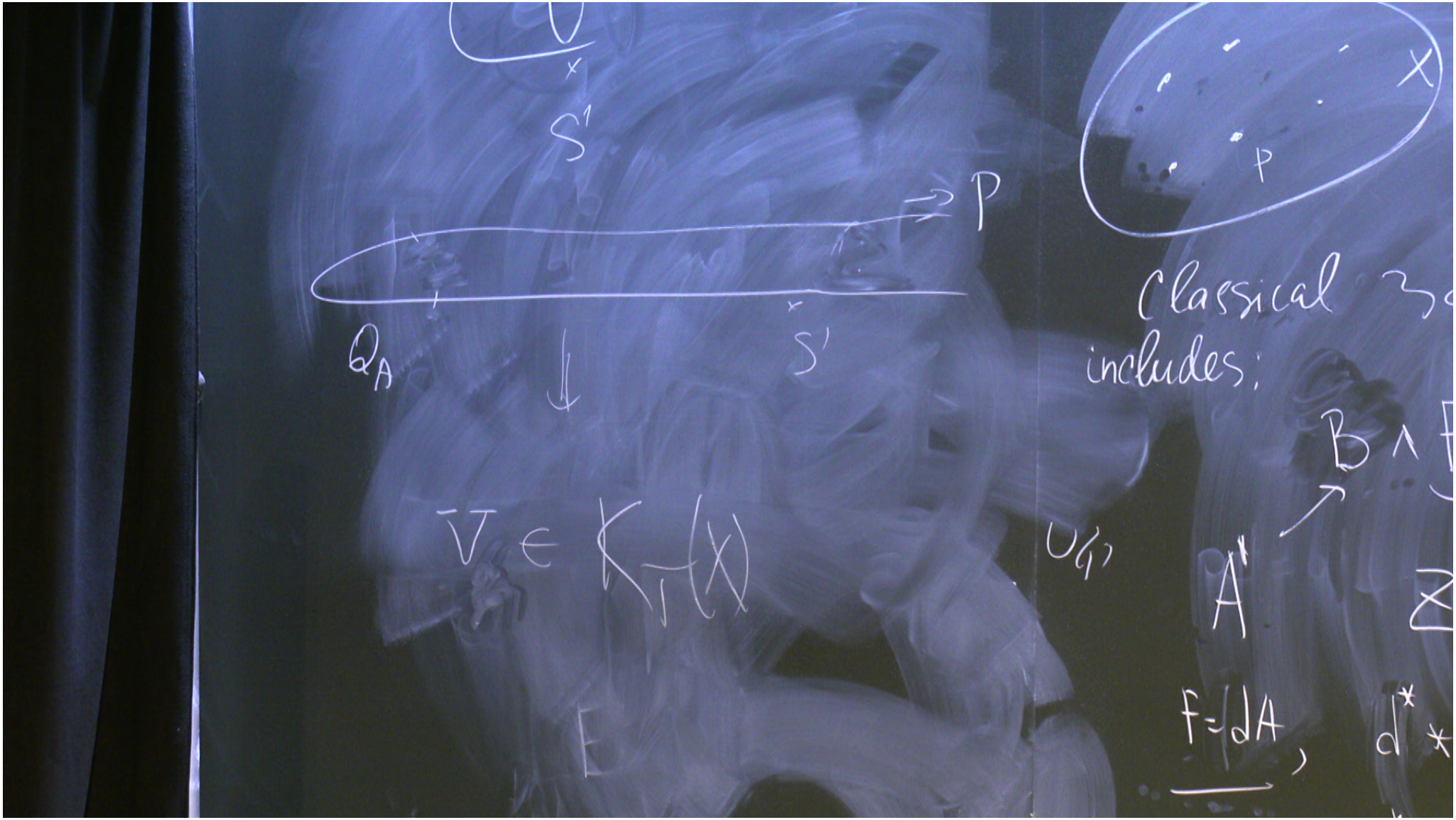
$F = dA$,

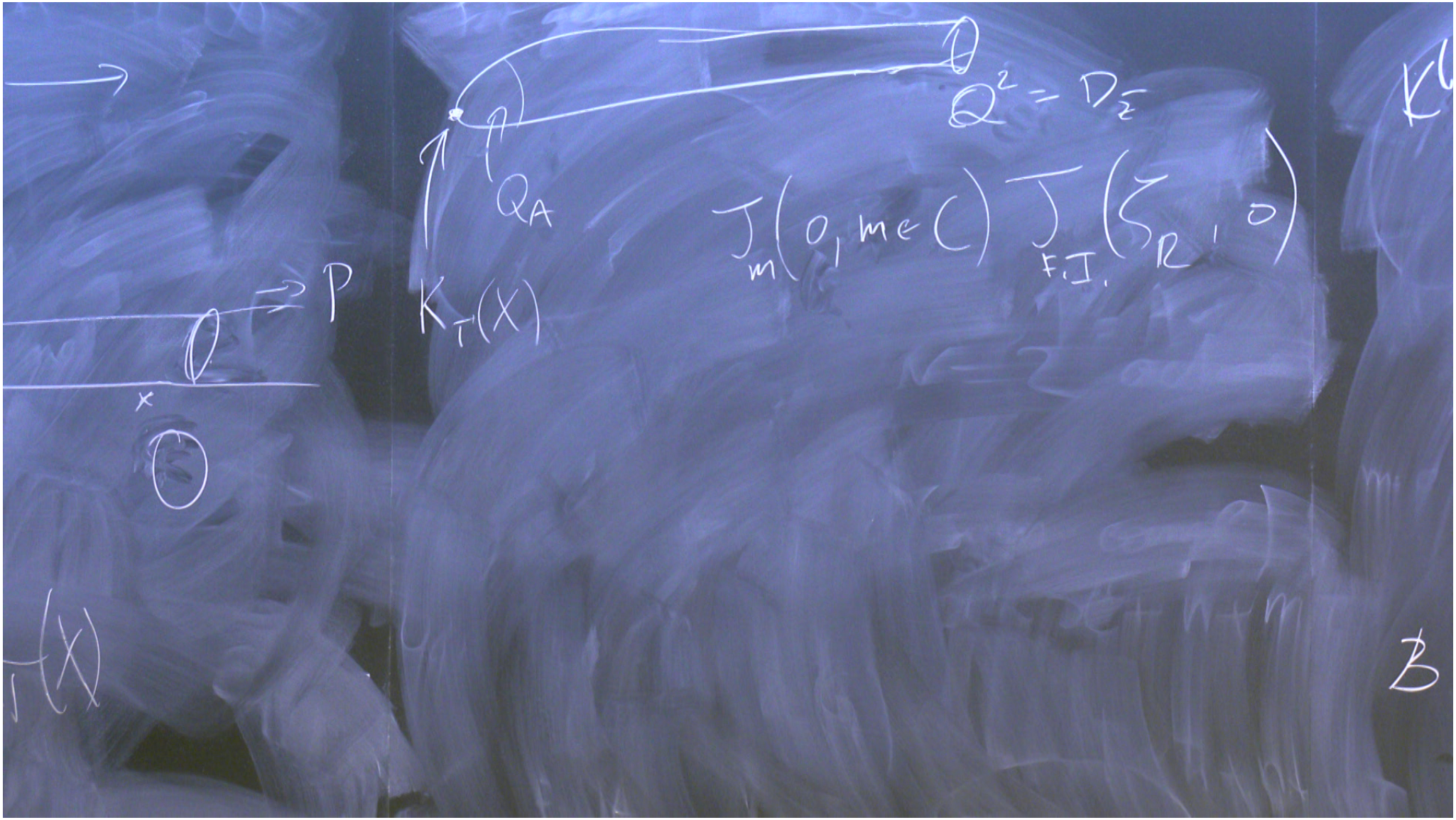
$d^* * F = 0$,

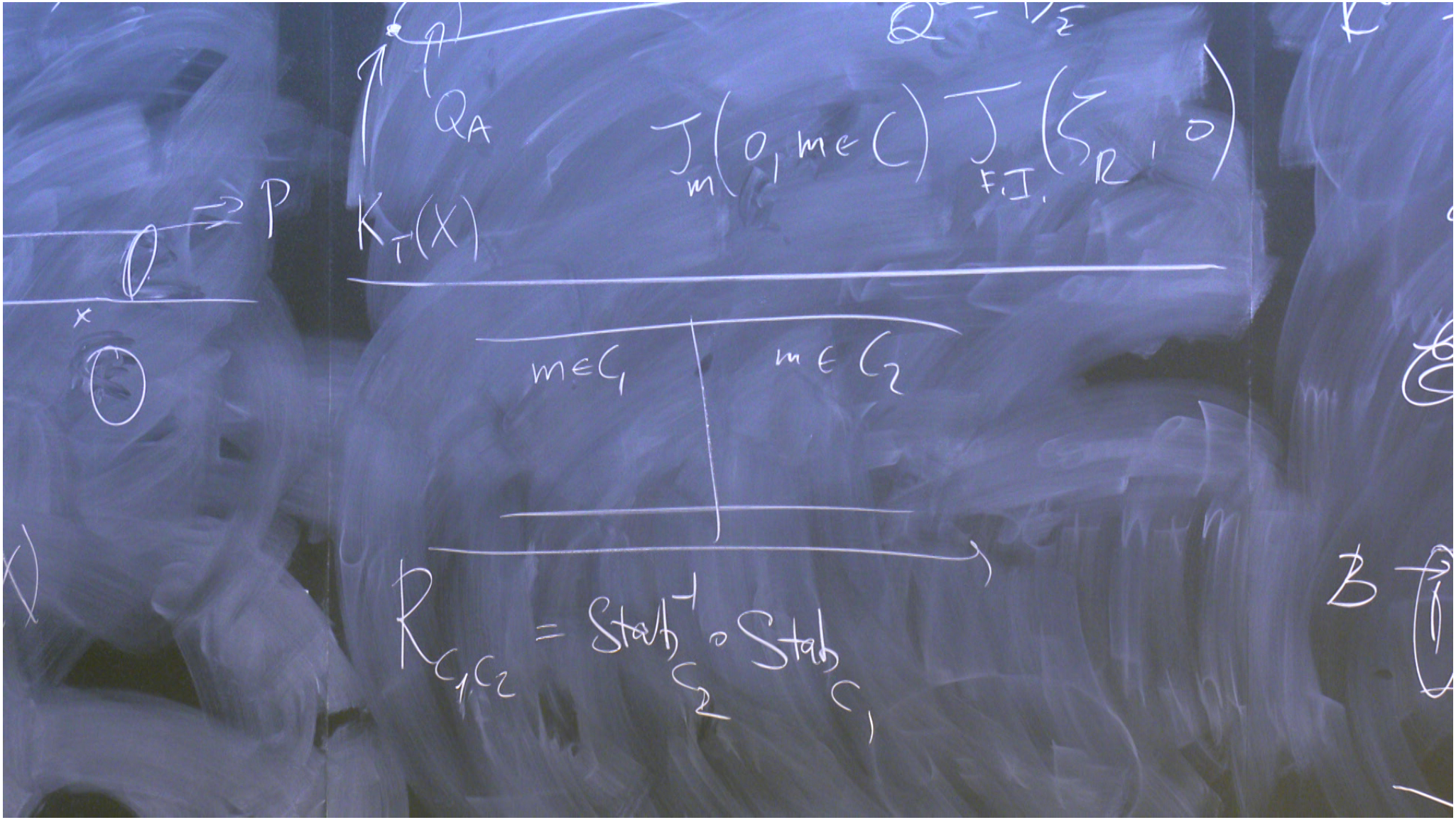
$J = * F$

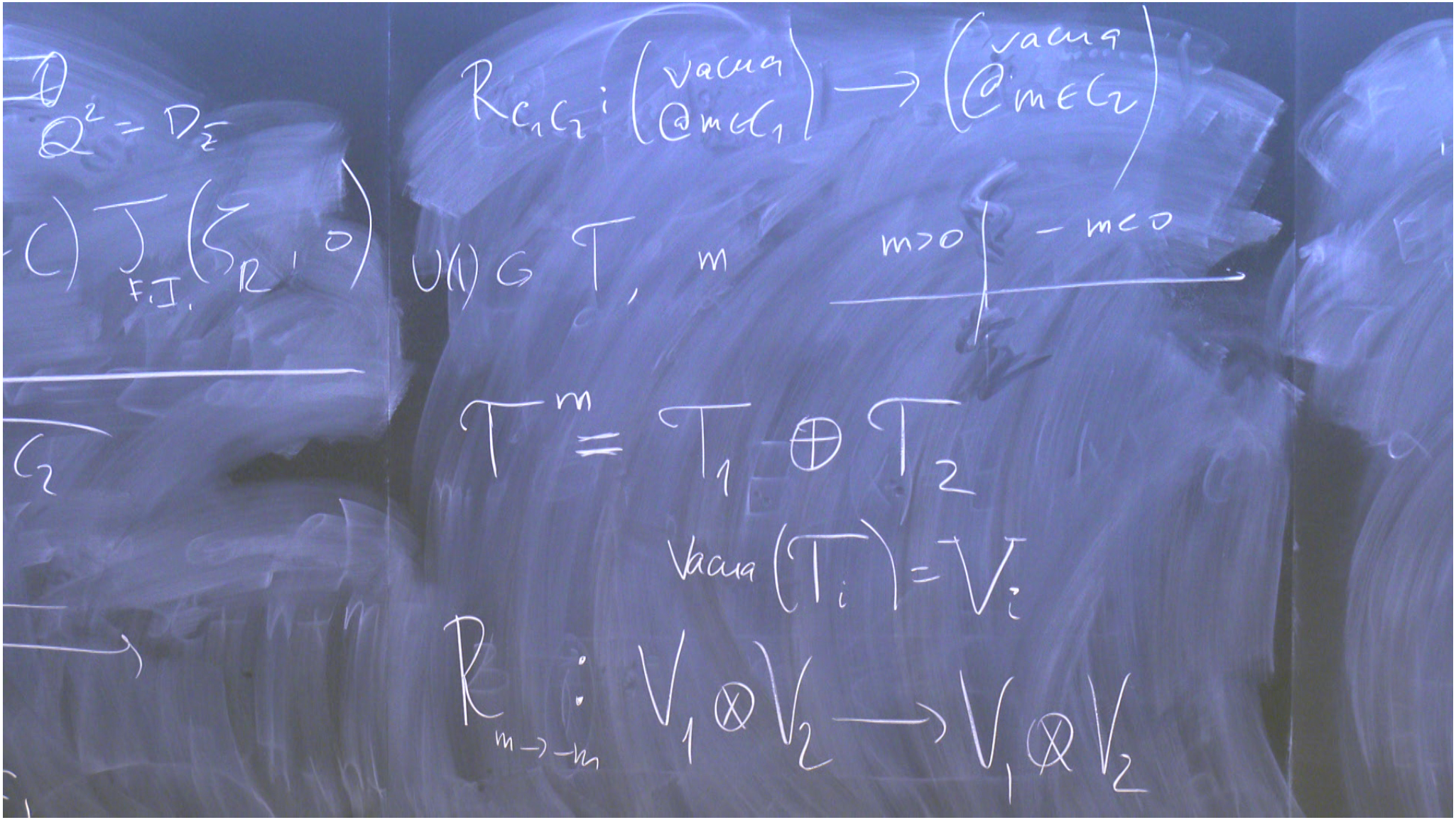
$d^* J = 0$

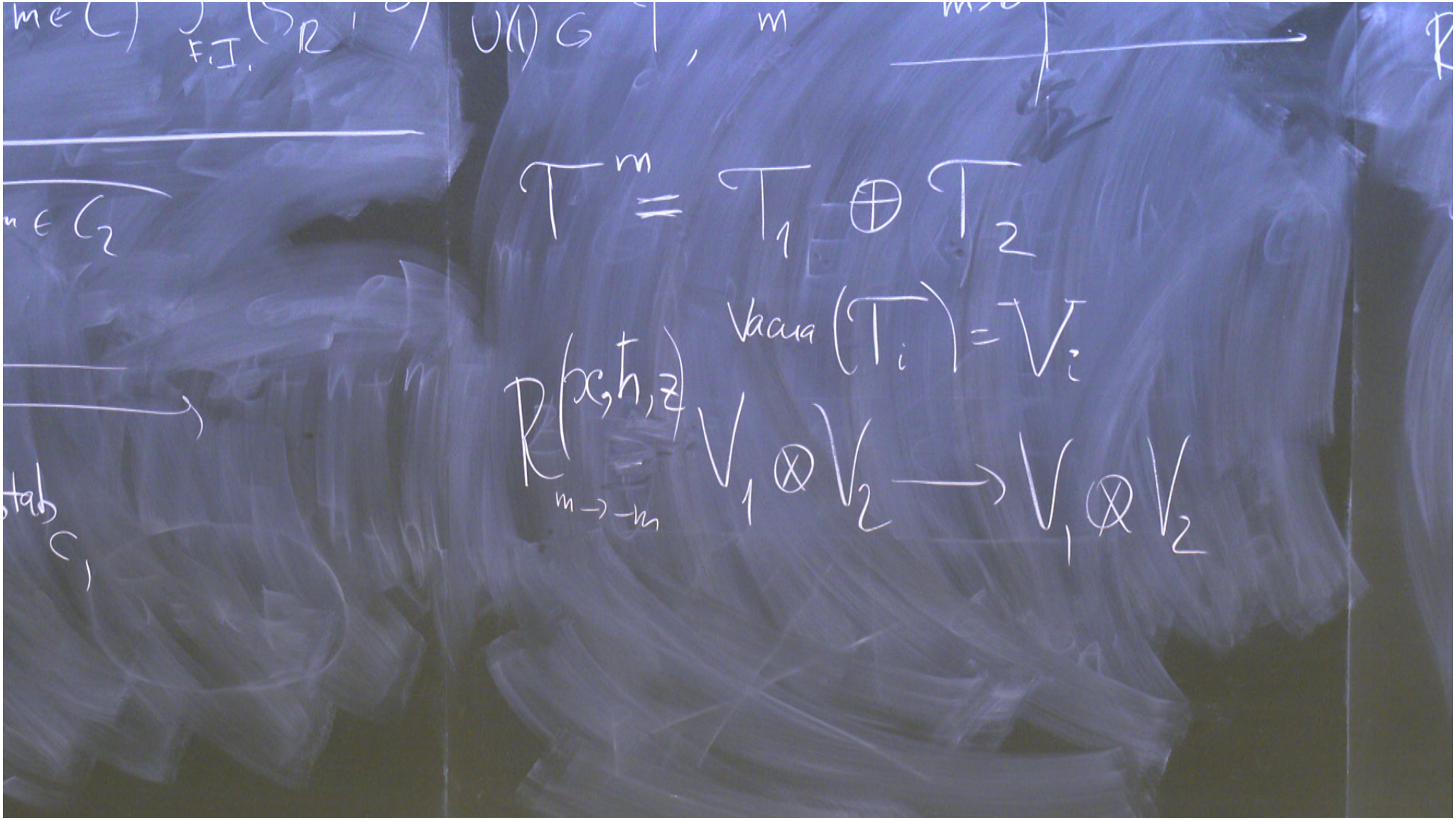

1
S¹
P
Q_A
S¹
V_p
For generi
X
p
Classical
includes:
B
A'

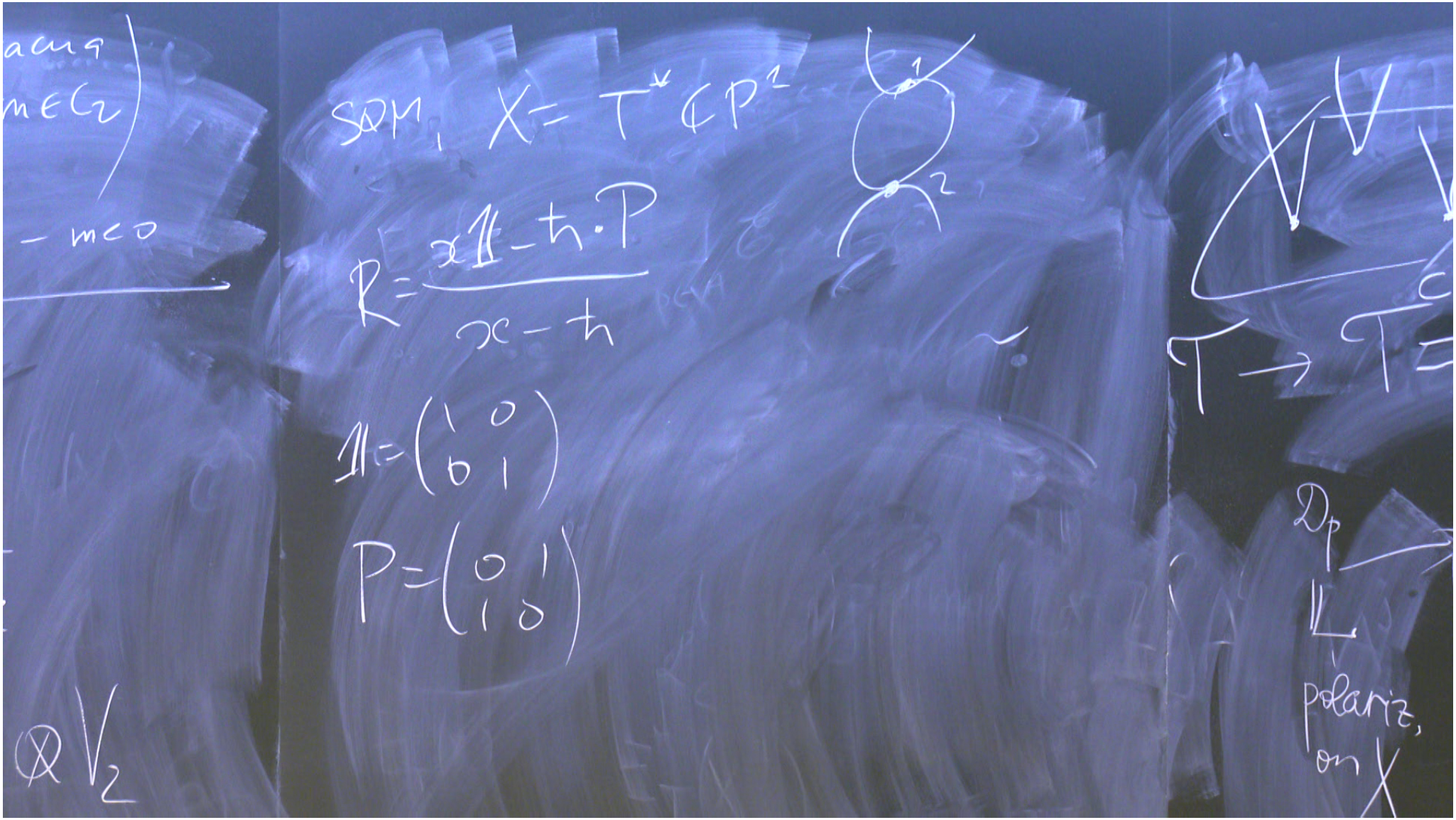

S¹
P
X
P
Classical
includes:
Q_A
S¹
$\bar{V} \in \Gamma(T(X))$
$U_{(i)}$
$A^{(i)}$
$B \wedge$
$F = dA$
d^*

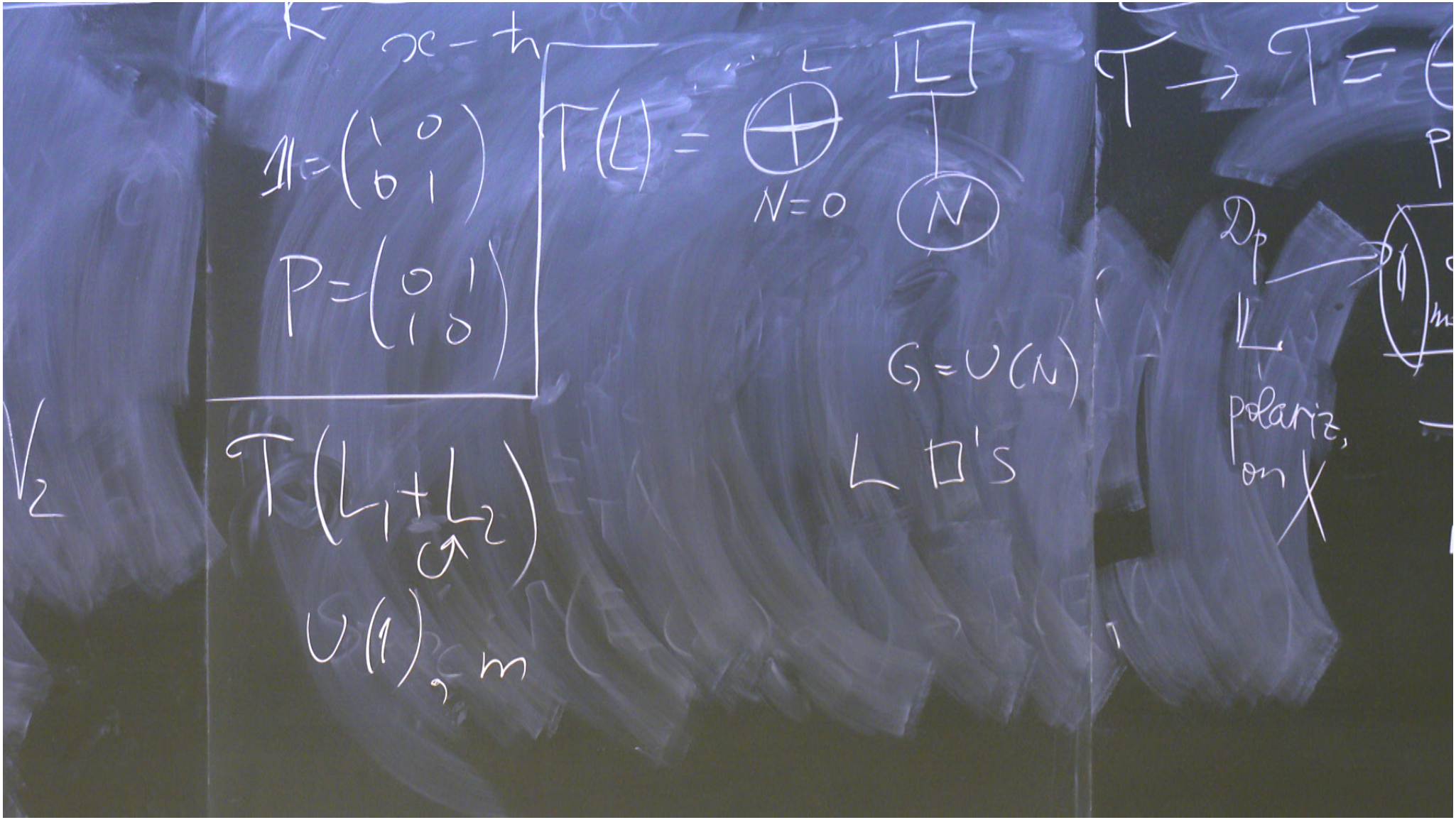

$Q^2 = D_z$
Q_A
$J_m(0, m_\epsilon)$
$J_{F.I.}(\zeta_R, 0)$
$K_T(X)$
P
O
x
B

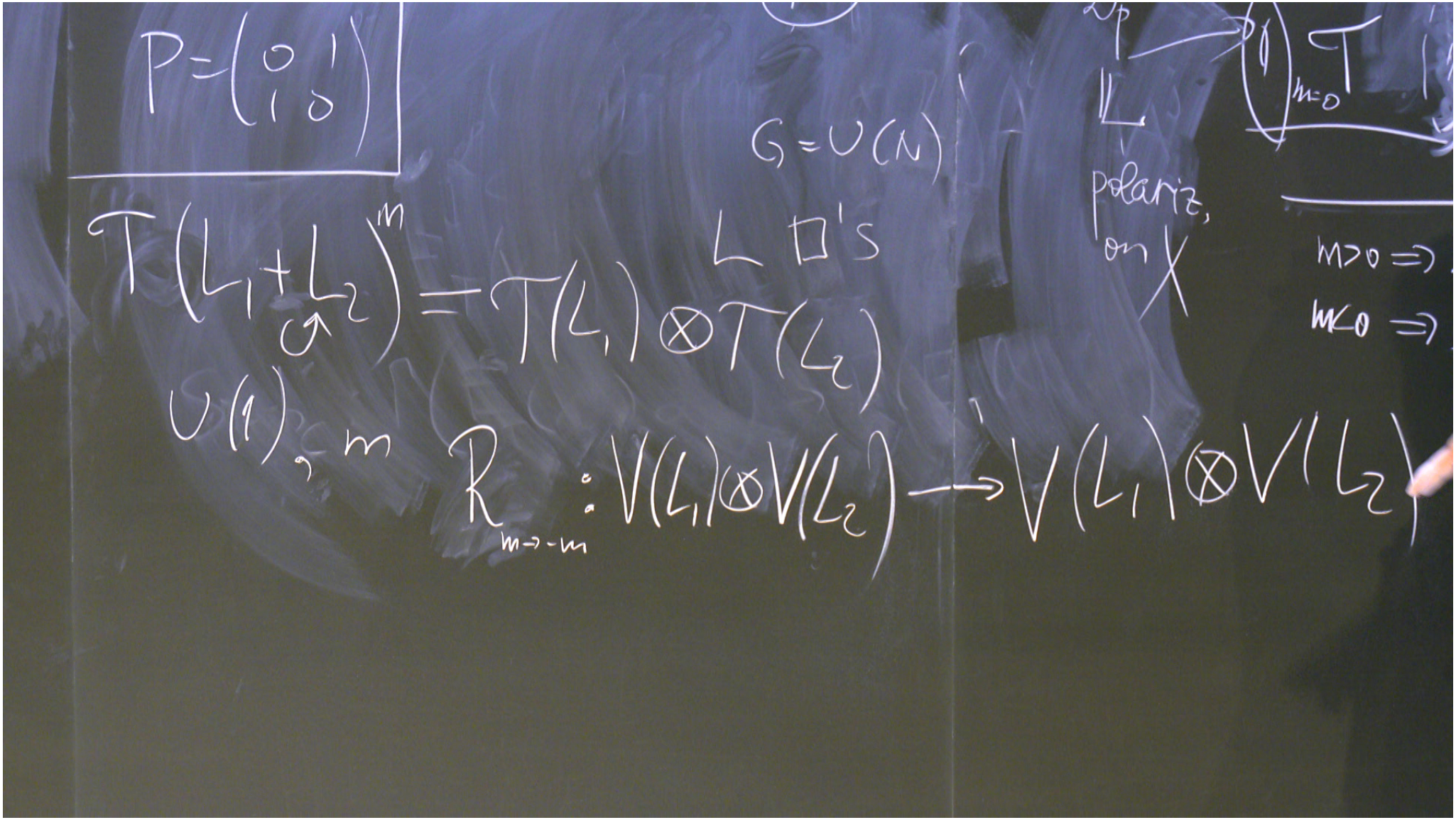

Q_A
$K_T(X)$
$m \in C_1$
$m \in C_2$
$R_{C_1,C_2} = Stab^{-1}_{C_2} \circ Stab_{C_1}$

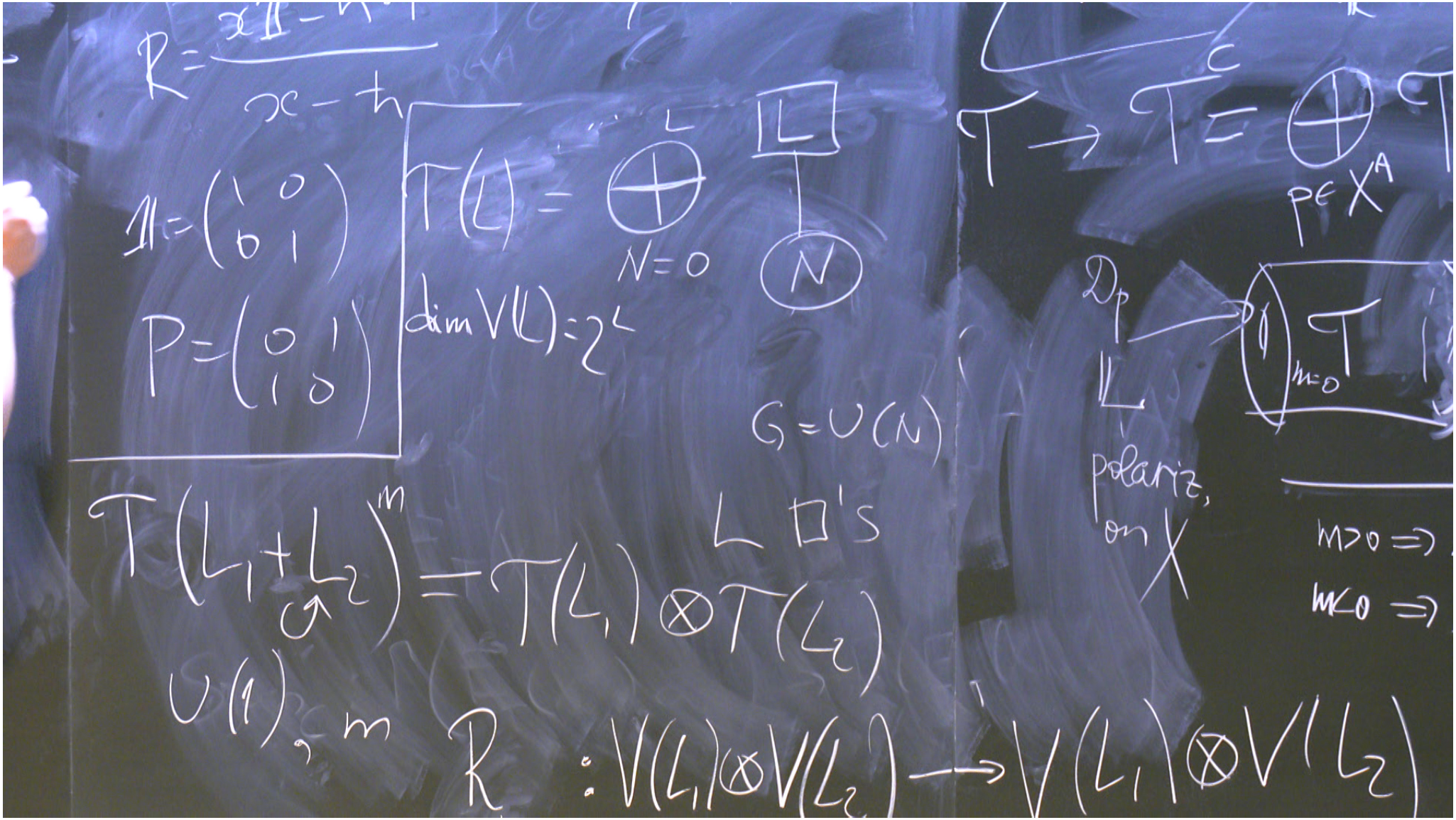

$Q^2 = D_2$
$R_{C_1 C_2} : \begin{pmatrix} \text{vacua} \\ @ m \in C_1 \end{pmatrix} \longrightarrow \begin{pmatrix} \text{vacua} \\ @ m \in C_2 \end{pmatrix}$
$U(1)_G$ T, m
$m > 0$ | $-m < 0$
$T^m = T_1 \oplus T_2$
$\text{Vacua}(T_i) = V_i$
$R_{m \to -m} : V_1 \otimes V_2 \longrightarrow V_1 \otimes V_2$

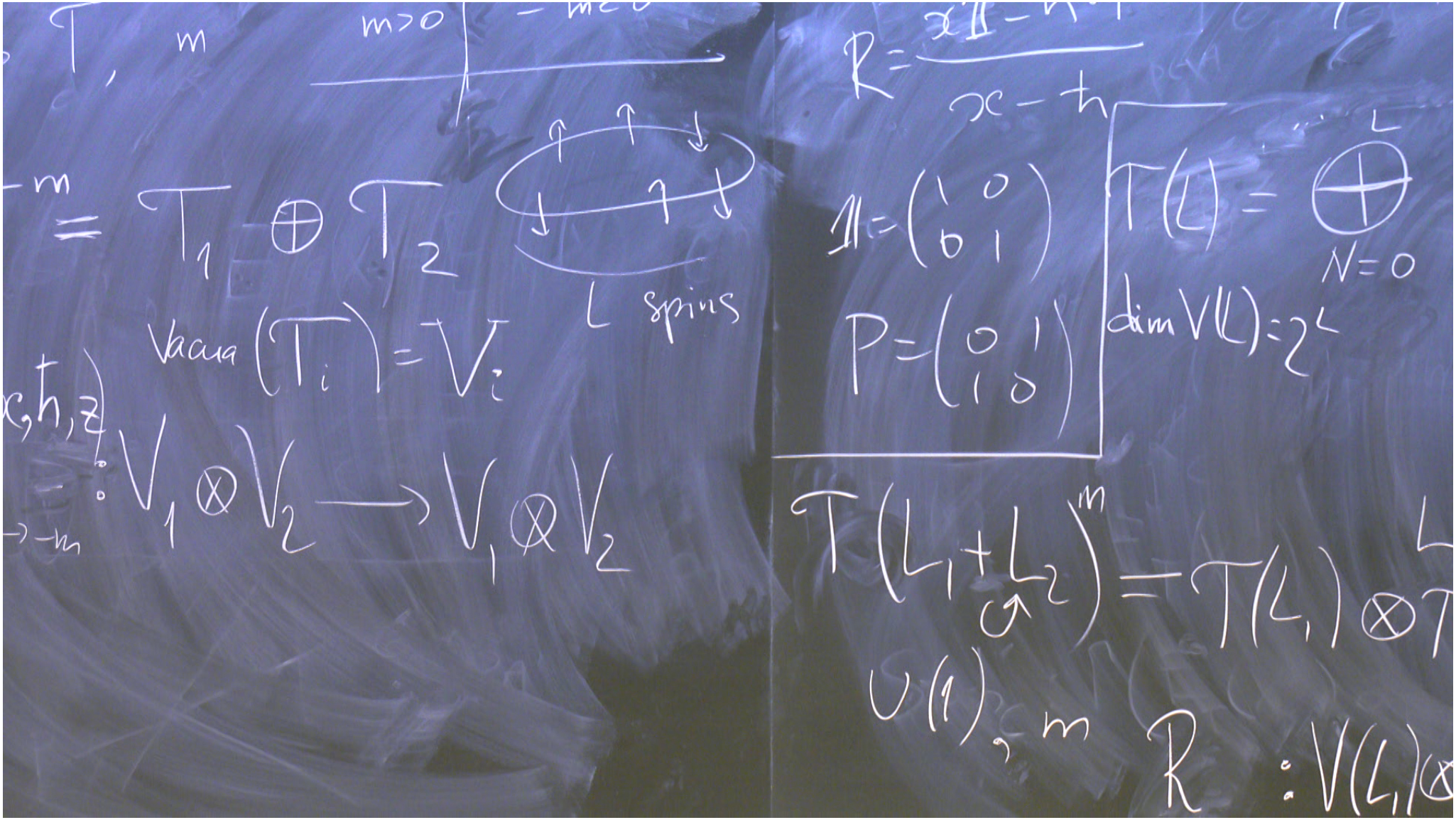

$T^m = T_1 \oplus T_2$
$\mathrm{Vacua}(T_i) = V_i$
$R^{(x,t,z)}_{m \to -m}$
$V_1 \otimes V_2 \longrightarrow V_1 \otimes V_2$

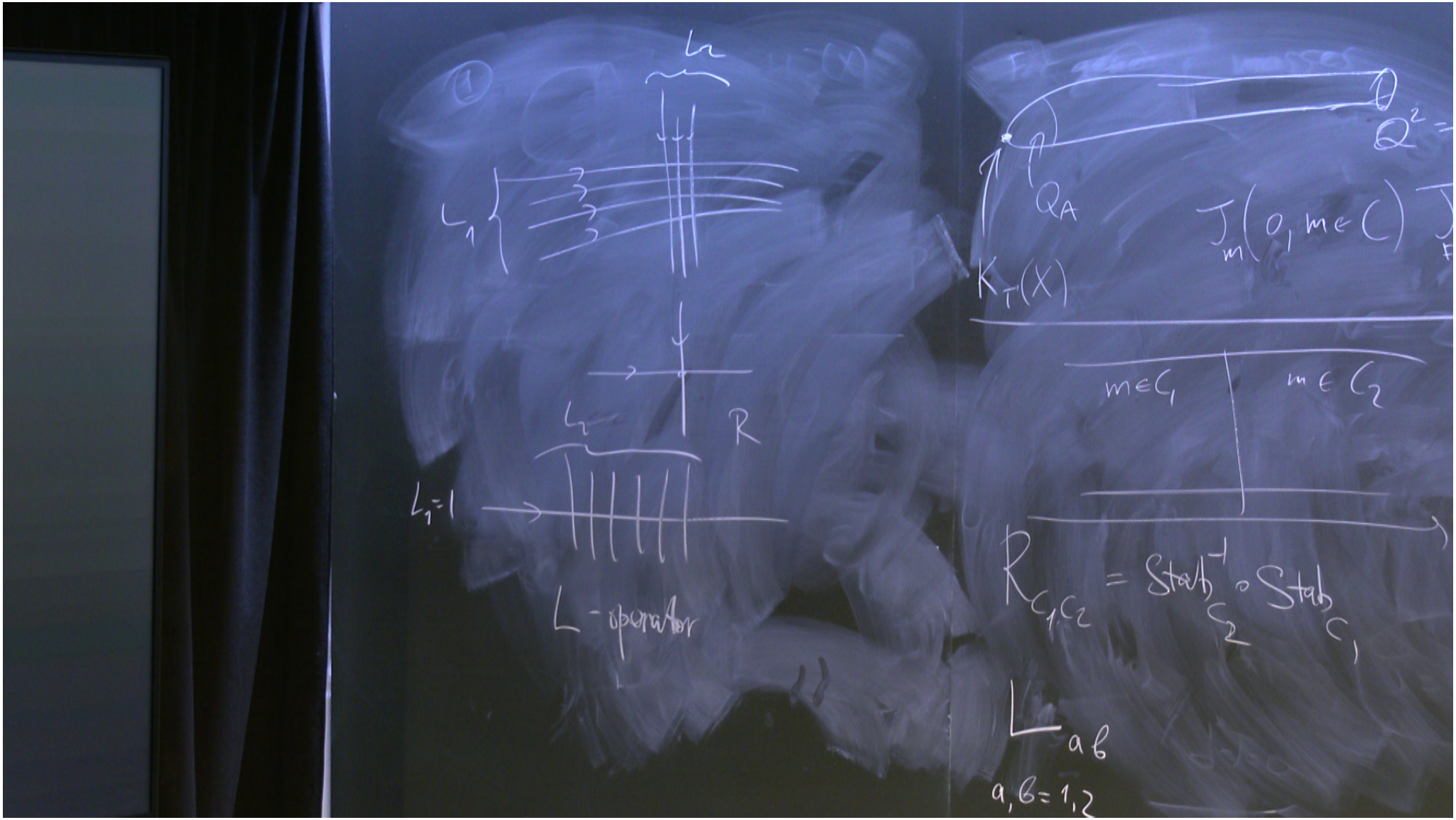

SQM, $X = T^* \mathbb{C}P^2$
$R = \frac{x\mathbb{1} - h \cdot P}{x - h}$
$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$P = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$
$\mathbb{C}^2$
$T \to T =$
D_p
polariz, on X
$\otimes V_2$

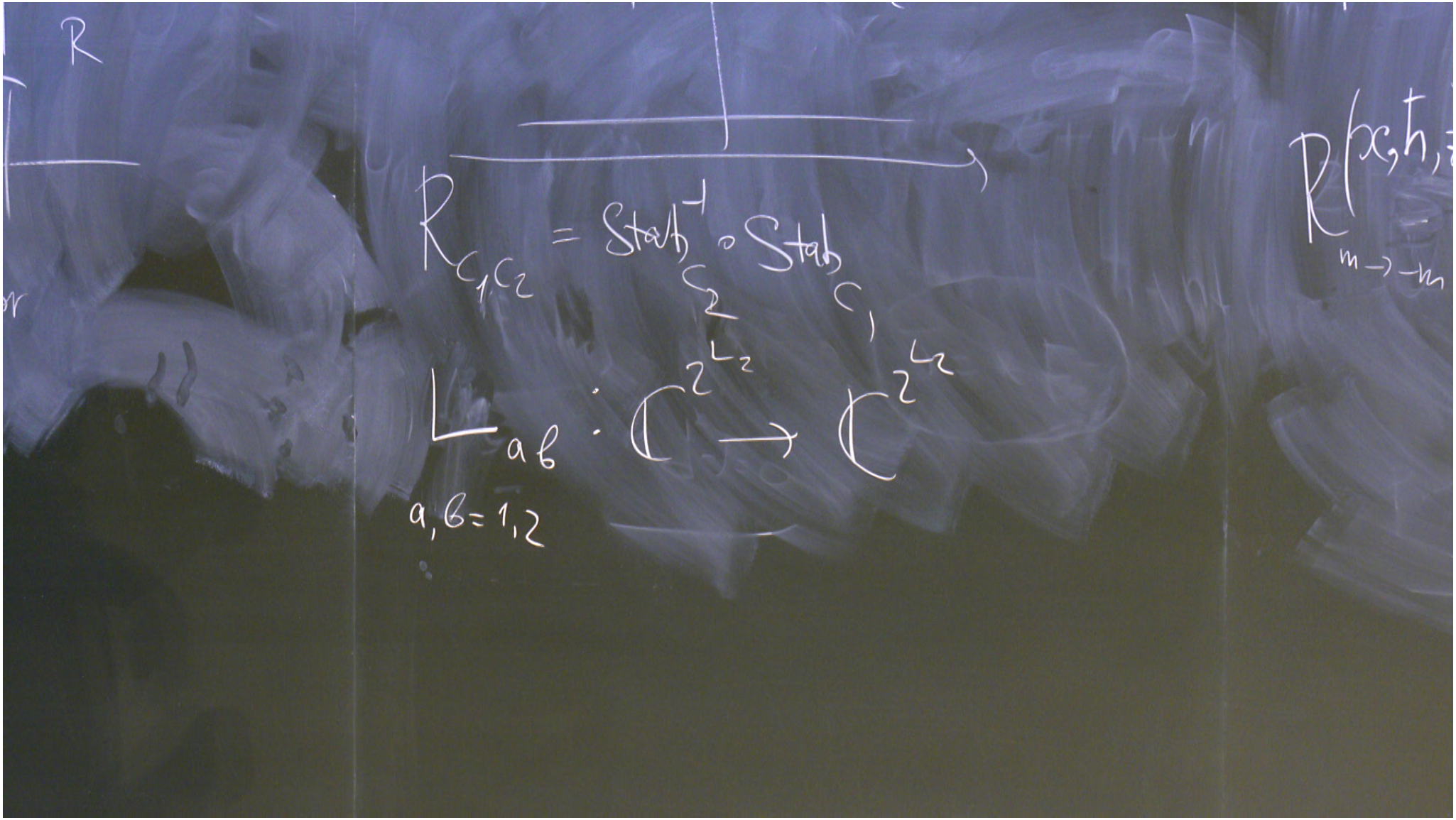

$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\mathcal{T}(L) = \bigoplus_{N=0}^{L}$
$G = U(N)$
L □'s
$\mathcal{T}(L_1 + L_2)$
$U(1)_q$, m
V_2
$\mathcal{T} \to \mathcal{T} =$
$\mathcal{D}_P$
L
polariz. on X

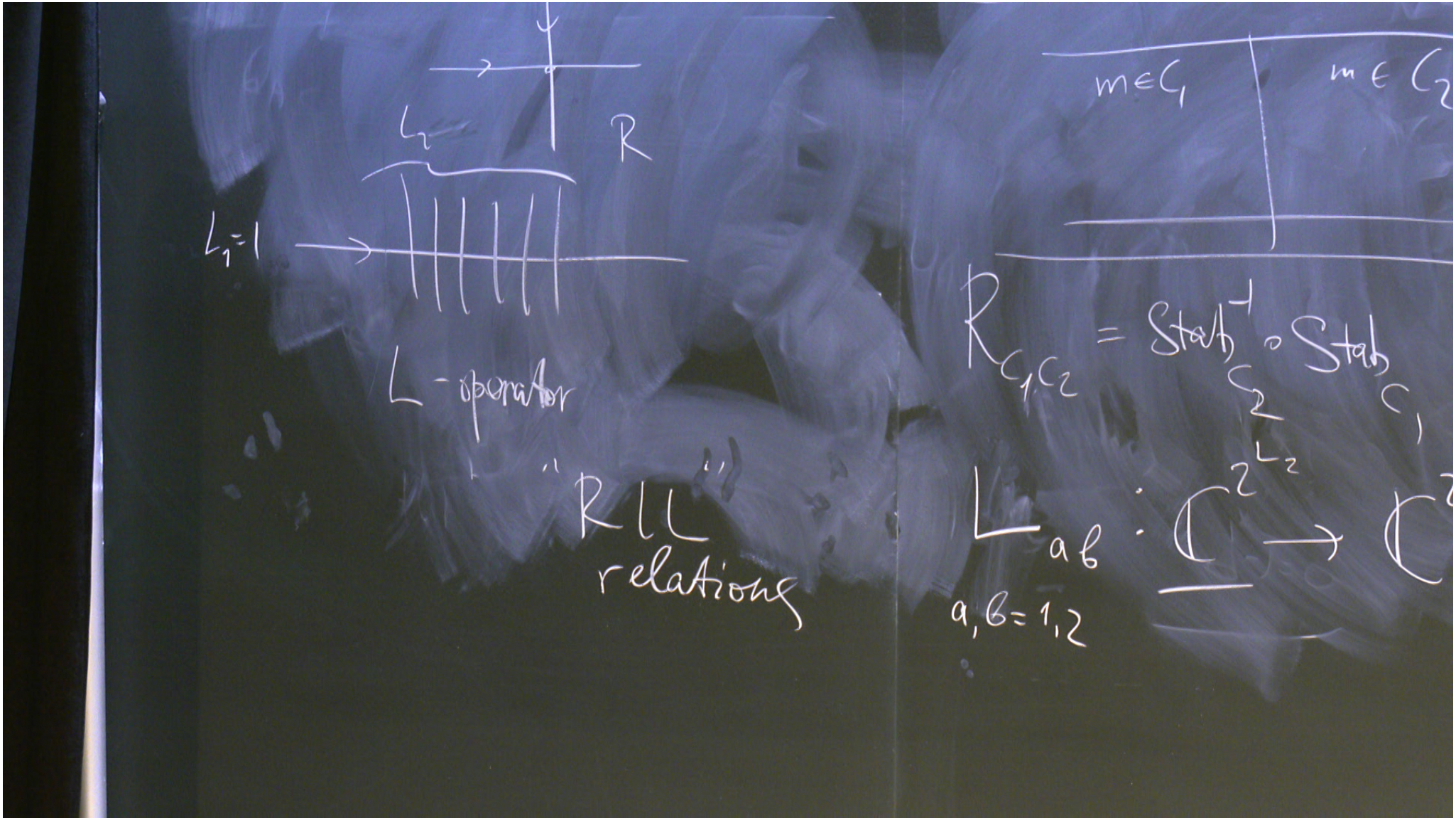

P=(0 1 / 1 0)
G=U(N)
L □'s
T(L1+L2) = T(L1) ⊗ T(L2)
U(1), m
R_{m→-m} : V(L1)⊗V(L2) → V(L1)⊗V(L2)
polariz, on X
m>0 ⇒
m<0 ⇒
m=0

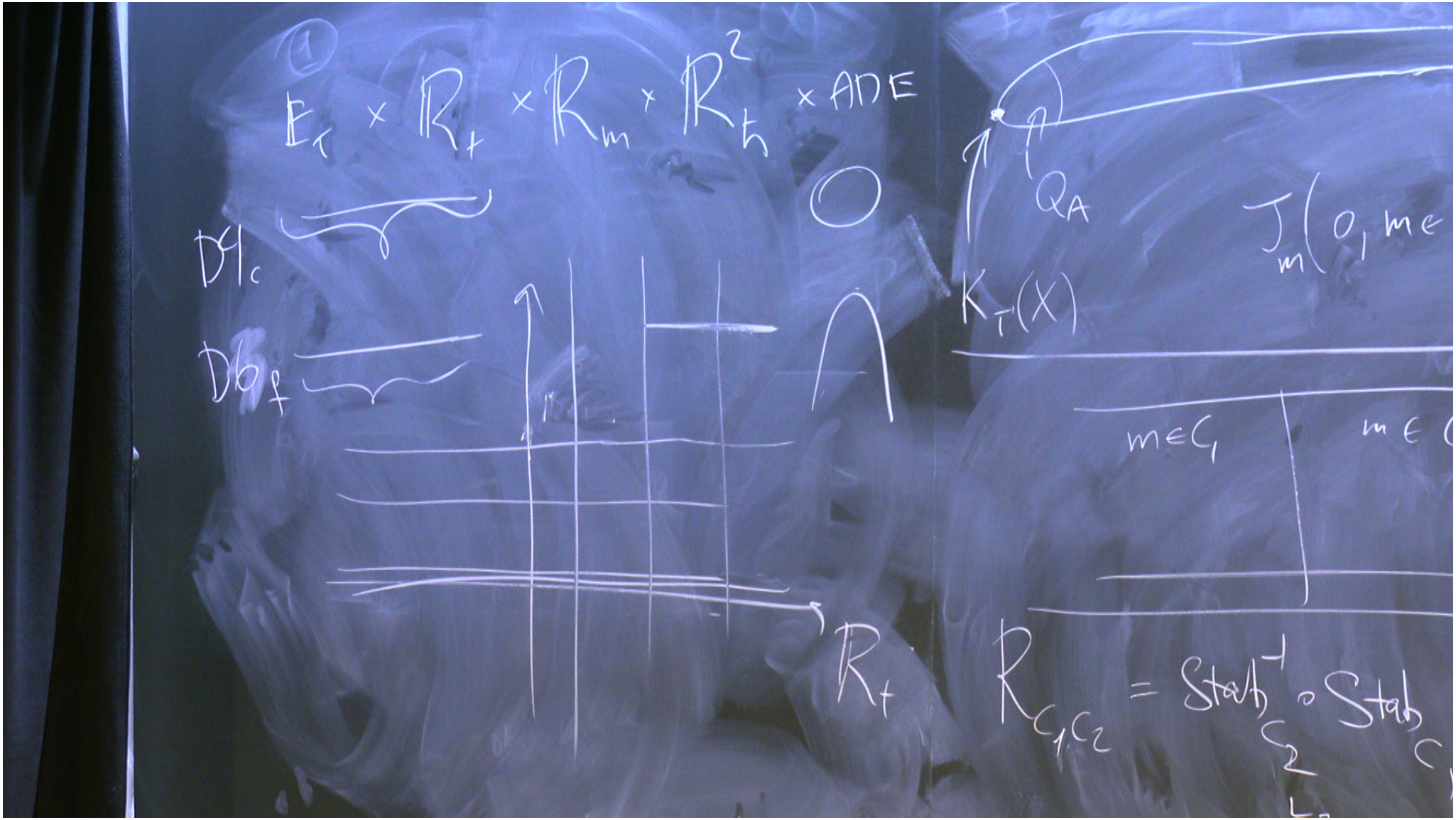

$R = \frac{\mathbb{H} - \mathbb{H}}{\mathbb{C} - \mathbb{H}}$
$\mathbb{H} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$T(L) = \bigoplus_{N=0}$
$\dim V(L) = 2^L$
$G = U(N)$
$T(L_1 + L_2)^m = T(L_1) \otimes T(L_2)$
$U(1)_m$
$R : V(L_1) \otimes V(L_2) \to V(L_1) \otimes V(L_2)$
$T \to \mathcal{T} = \bigoplus_{p \in X^A}$
$\mathcal{D}_p$
polariz. on X
$m=0$
$m>0 \Rightarrow$
$m<0 \Rightarrow$


m>0
L spins
$R = \frac{x\Pi - \Pi x}{x - h}$
$\Pi = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$T(L) = \bigoplus_{N=0}^{L}$
$\dim V(L) = 2^L$
$T(L_1 + L_2)$
$= T(L_1) \otimes T$
$U(1)$, m
$R : V(L_1) \otimes$


L - operator
$R_{C_1C_2} = \mathrm{Stab}_{C_2}^{-1} \circ \mathrm{Stab}_{C_1}$
L_{ab}
$a, b = 1, 2$


R
$R_{c_1,c_2} = \mathrm{Stab}_{c_2}^{-1} \circ \mathrm{Stab}_{c_1}$
$L_{ab} : \mathbb{C}^{2^{L_2}} \to \mathbb{C}^{2^{L_2}}$
$a, b = 1, 2$
$m \to -m$


L_1
R
$L_1 = 1$
L - operator
"RLL"
relations
$m \in C_1$
$m \in C_2$
$R_{C_1 C_2} = \mathrm{Stab}_{C_2}^{-1} \circ \mathrm{Stab}_{C_1}$
$L_{ab} : \mathbb{C}^{2^{L_2}} \to$
$a, b = 1, 2$


$E_\tau \times \mathbb{R}_t \times \mathbb{R}_m \times \mathbb{R}_h^2 \times ADE$
Q_A
$K_T(X)$
$m \in C_1$
$\mathbb{R}_t$
$R_{C_1,C_2} = Stab_{C_2}^{-1} \circ Stab_{C_1}$