Title: 3d Gauge Theory and Elliptic Stable Envelopes IV

Speakers: Mykola Dedushenko

Collection: QFT for Mathematicians 2022

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Stab: $H^*_T(X^A) \to H^*_T(X)$

$m_R \to \infty$	$m_R = 0$
3d $\mathcal{N}=2$	
(3d $\mathcal{N}=1$)	χ

SQM into $\mathcal{X} = \mathrm{Maps}(E_\tau, X)$

$Q^2 = D_{\bar{z}}$

$Q = \boxed{\bar{Q}_+} + Q_+$

$\downarrow$

H_T

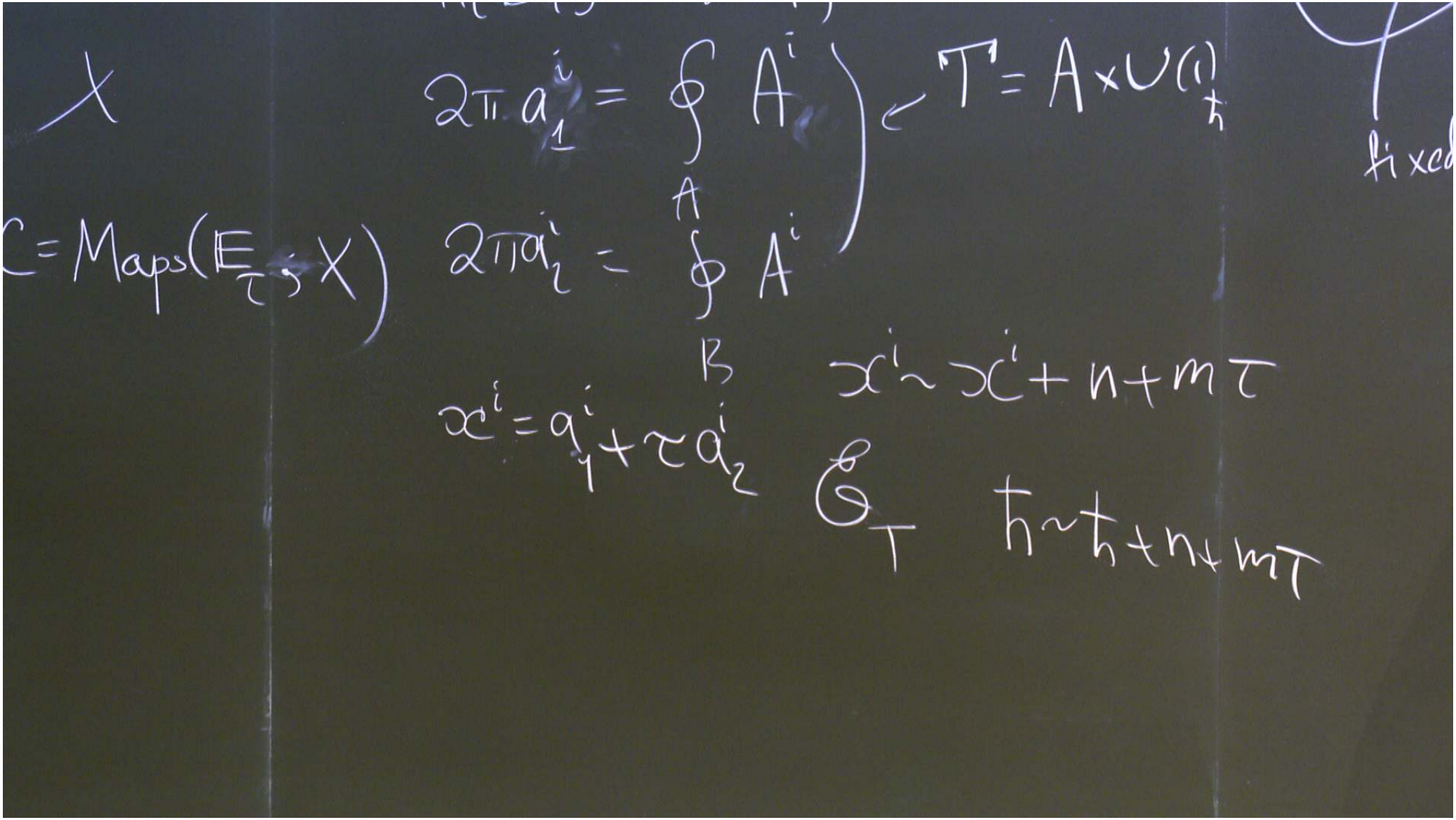
$\mathcal{H}[E_t] \supset V[E_t]$ $\quad G_f \supset A$

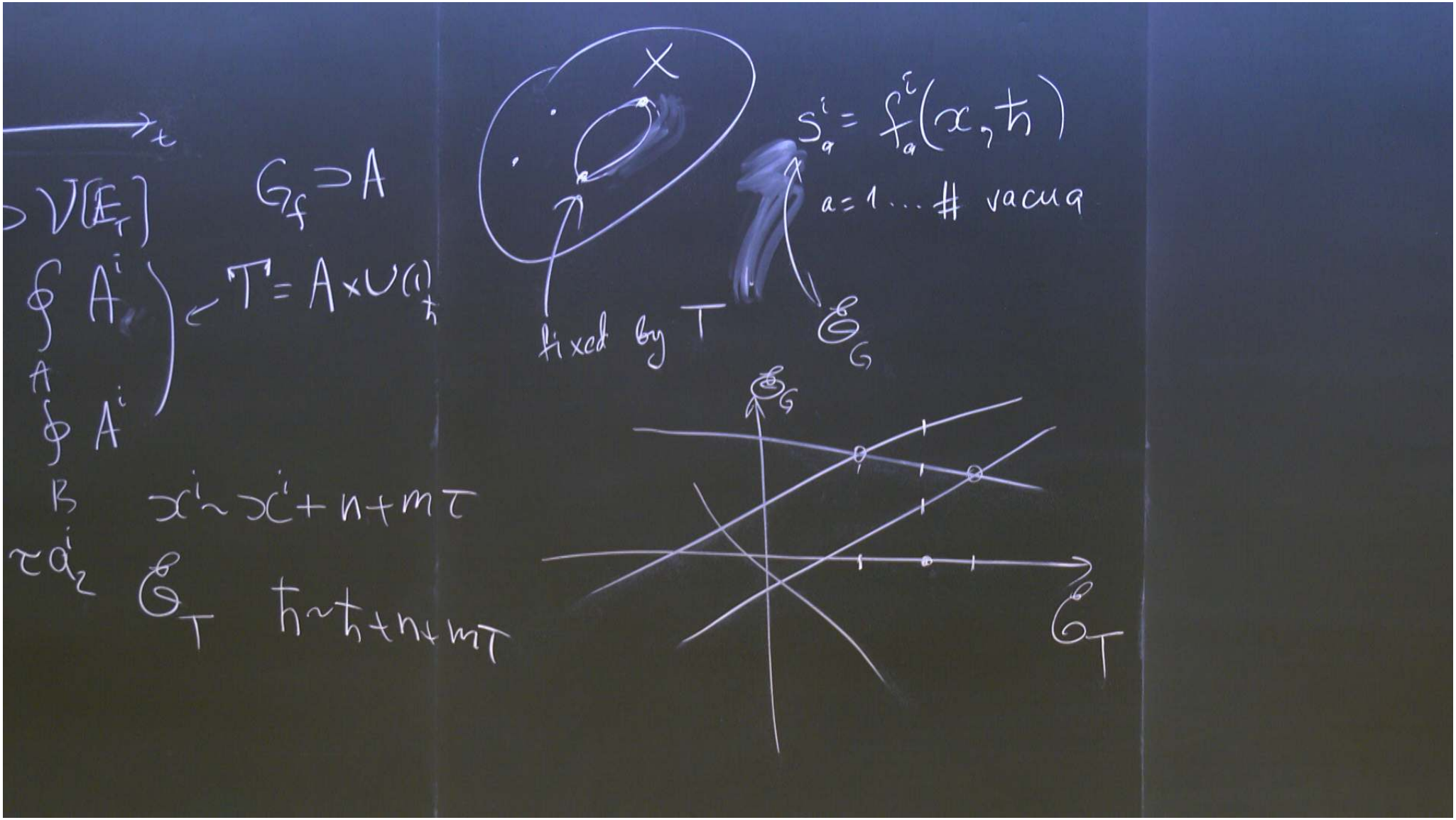
$2\pi a_1^i = \oint_A A^i$ $\Big) \leftarrow T = A \times U(1)_R$

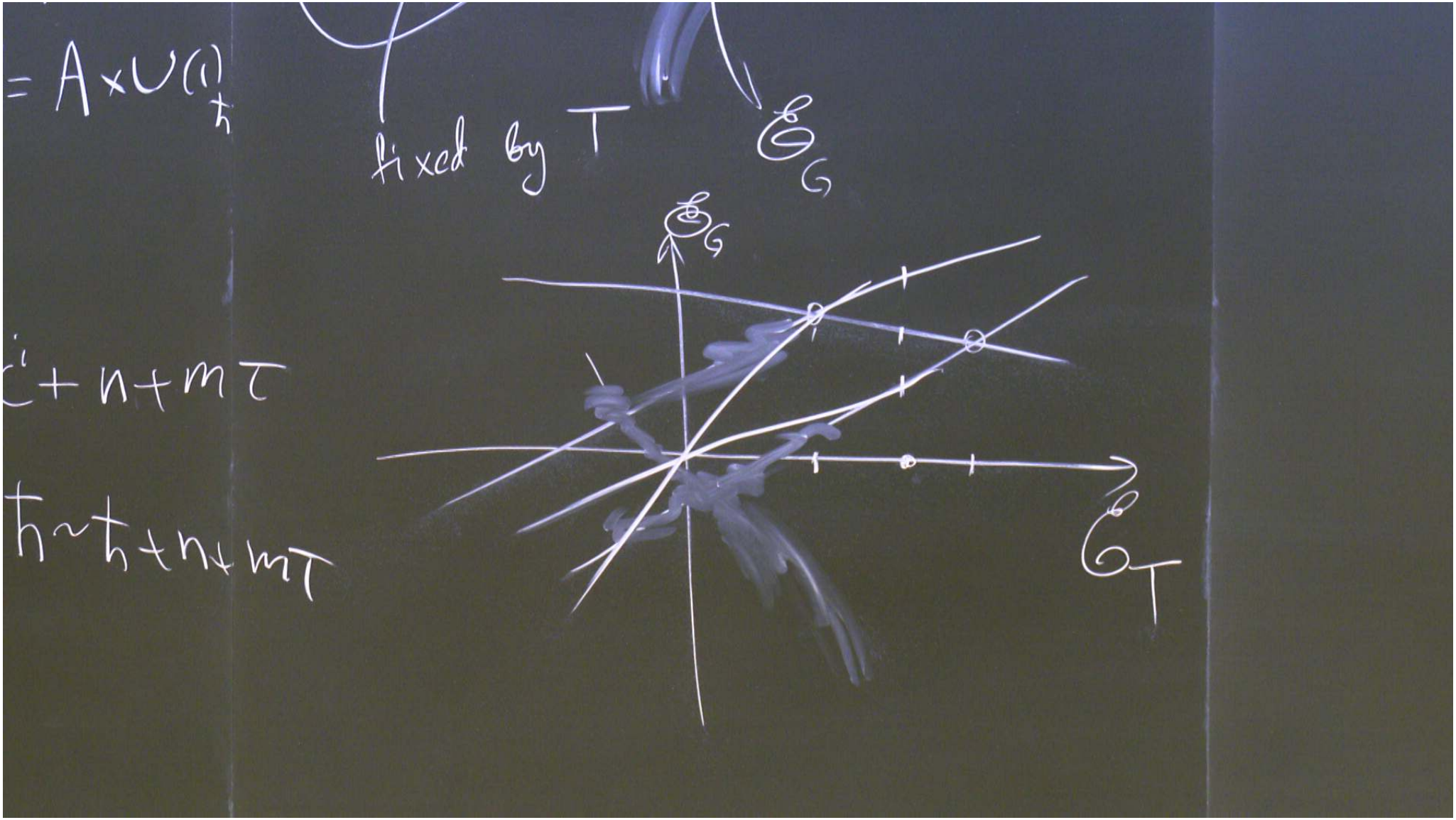
$2\pi a_2^i = \oint_B A^i$

$x^i \sim x^i + n + m\tau$

$x^i = q_1^i + \tau q_2^i$ $\quad G_T$


$2\pi a_1^i = \oint_A A^i$
$T = A \times U(1)_\hbar$
$C = \mathrm{Maps}(\mathbb{E}_\tau, X)$
$2\pi a_2^i = \oint_B A^i$
$x^i \sim x^i + n + m\tau$
$x^i = q_1^i + \tau q_2^i$
$\mathbb{Z}_T$
$\hbar \sim \hbar + n + m\tau$


t
V[E_T]
G_f ⊃ A
∮ A^i
A
∮ A^i
B
T = A × U(1)_h
x^i ~ x^i + n + mτ
τ a_2
G_T
h ~ h + n + mτ
X
fixed by T
G_G
S^i_a = f^i_a(x, h)
a = 1 ... # vacua
G_G
G_T


= A × U(1)
fixed by T
$\mathcal{O}_G$
$\mathcal{O}_G$
$\mathcal{O}_T$

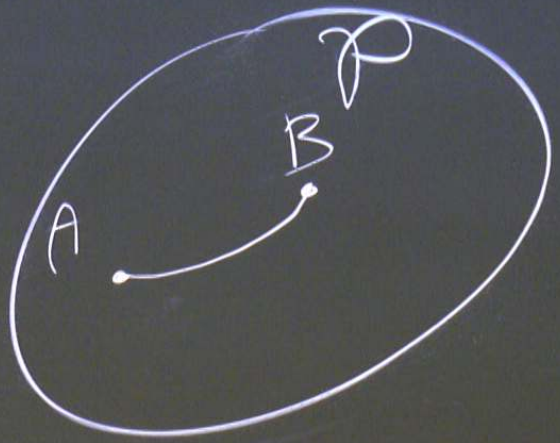
, th.)

vacua

T_p . H . $\sigma(H) = \{0\} \cup S$

eff. Hilb. space $S \subset (\Delta, \infty)$

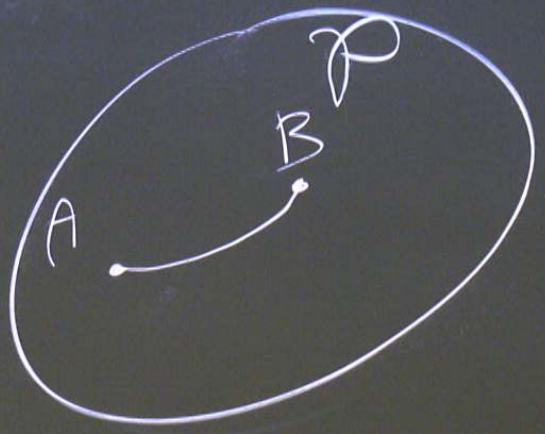
$\mathbb{C}|0\rangle$ Δ – large



T_p . H . $\sigma(H) = \{0\} \cup S$

eff. Hilb. space $S \subset (\Delta, \infty)$

$|0\rangle$ Δ – large



Effective action

$$\Gamma[A] \overset{\text{Adiab. limit}}{=\!=} \frac{k_{ij}}{4\pi} \int A^i_f \wedge dA^j_f$$

$k_{ij} = |q_i| \in \frac{1}{2}\mathbb{Z}$

$= \{0\} \cup S$

$S \subset (\Delta, \infty)$

Δ – large

Effective action

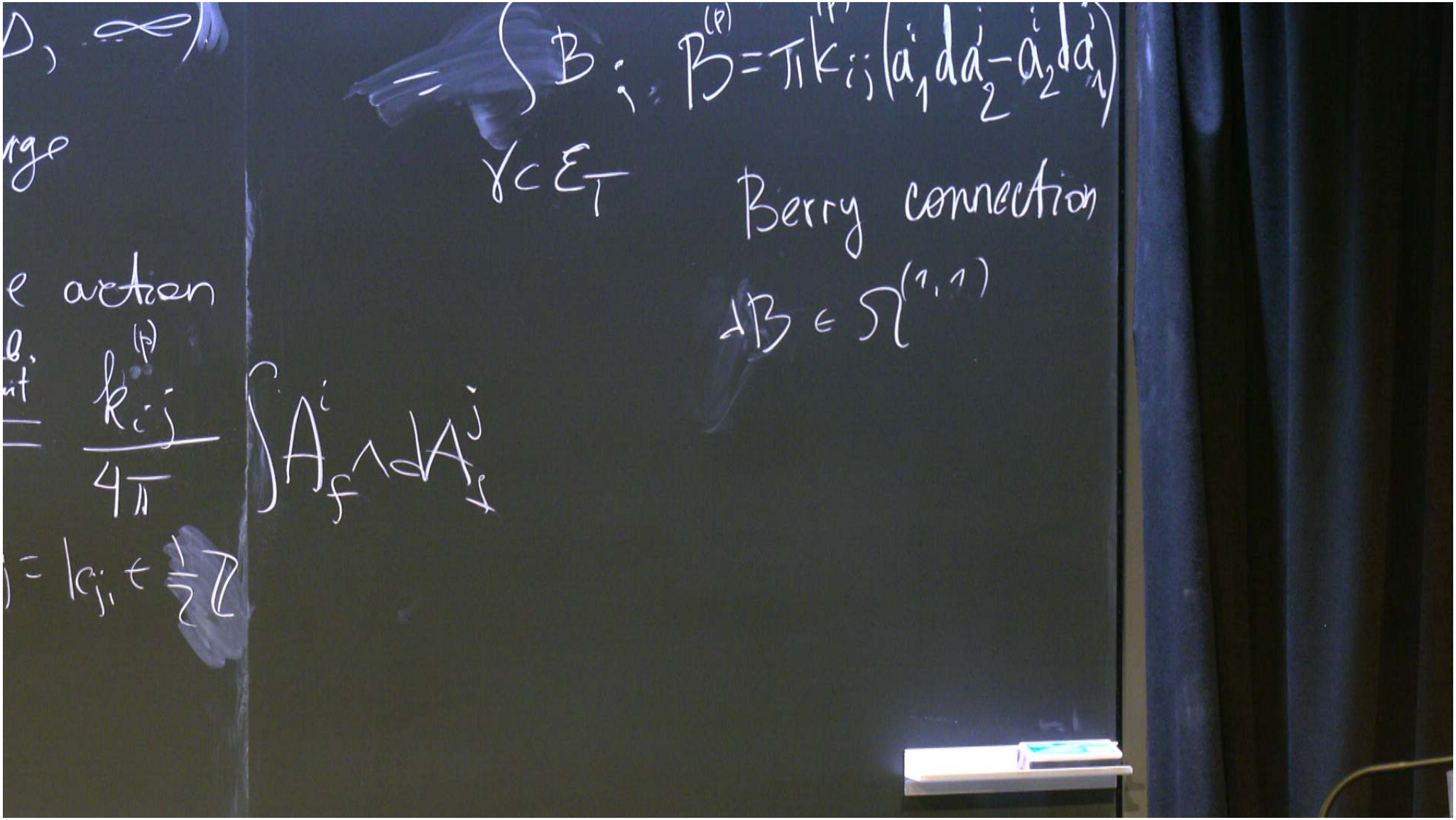
$$\Gamma[A] \overset{\text{Adiab. limit}}{=\!=} \frac{k^{(p)}_{ij}}{4\pi} \int A^i_f \wedge dA^j_f$$

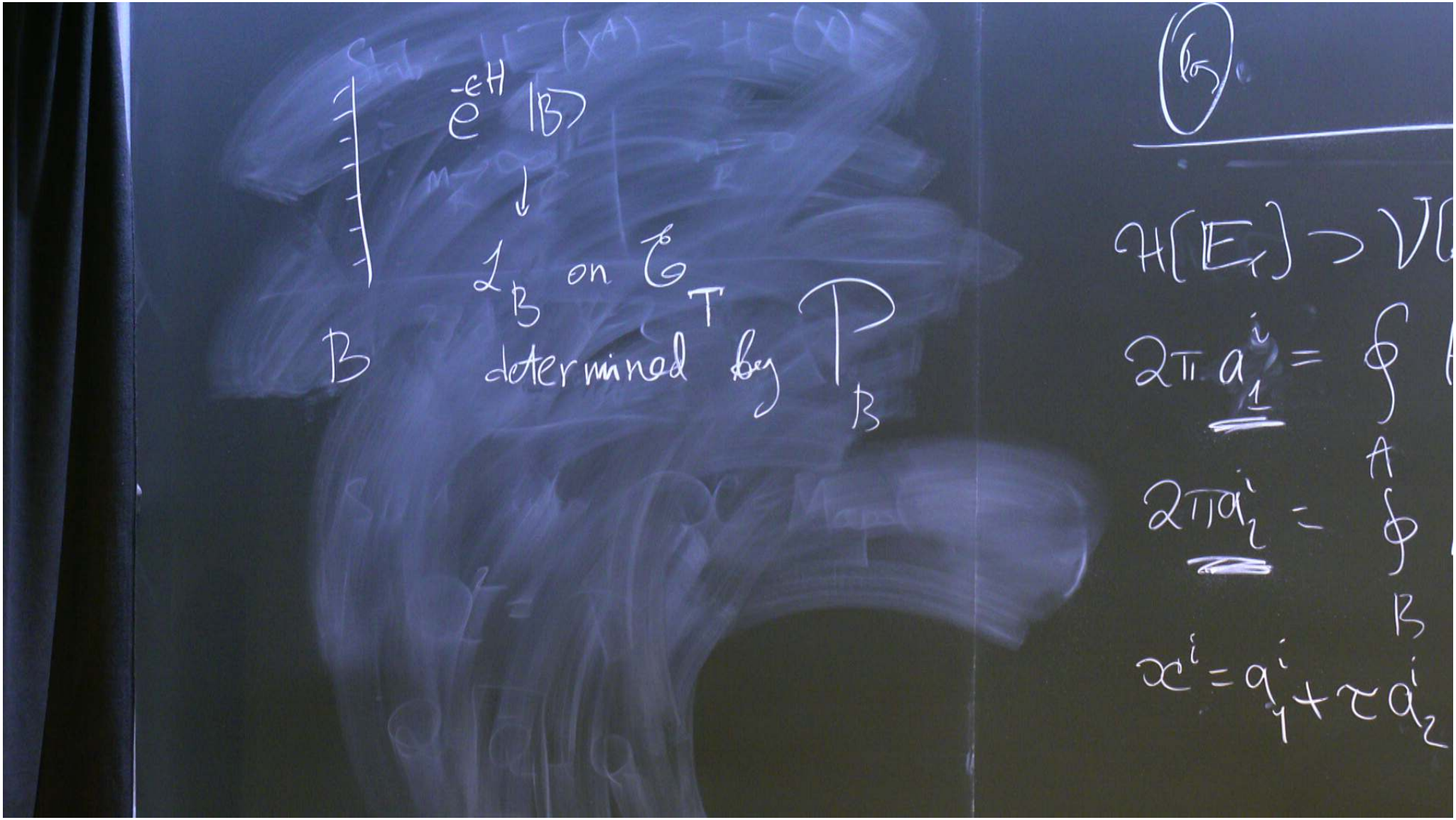
$k_{ij} = |c_{ij}| \in \frac{1}{2}\mathbb{Z}$

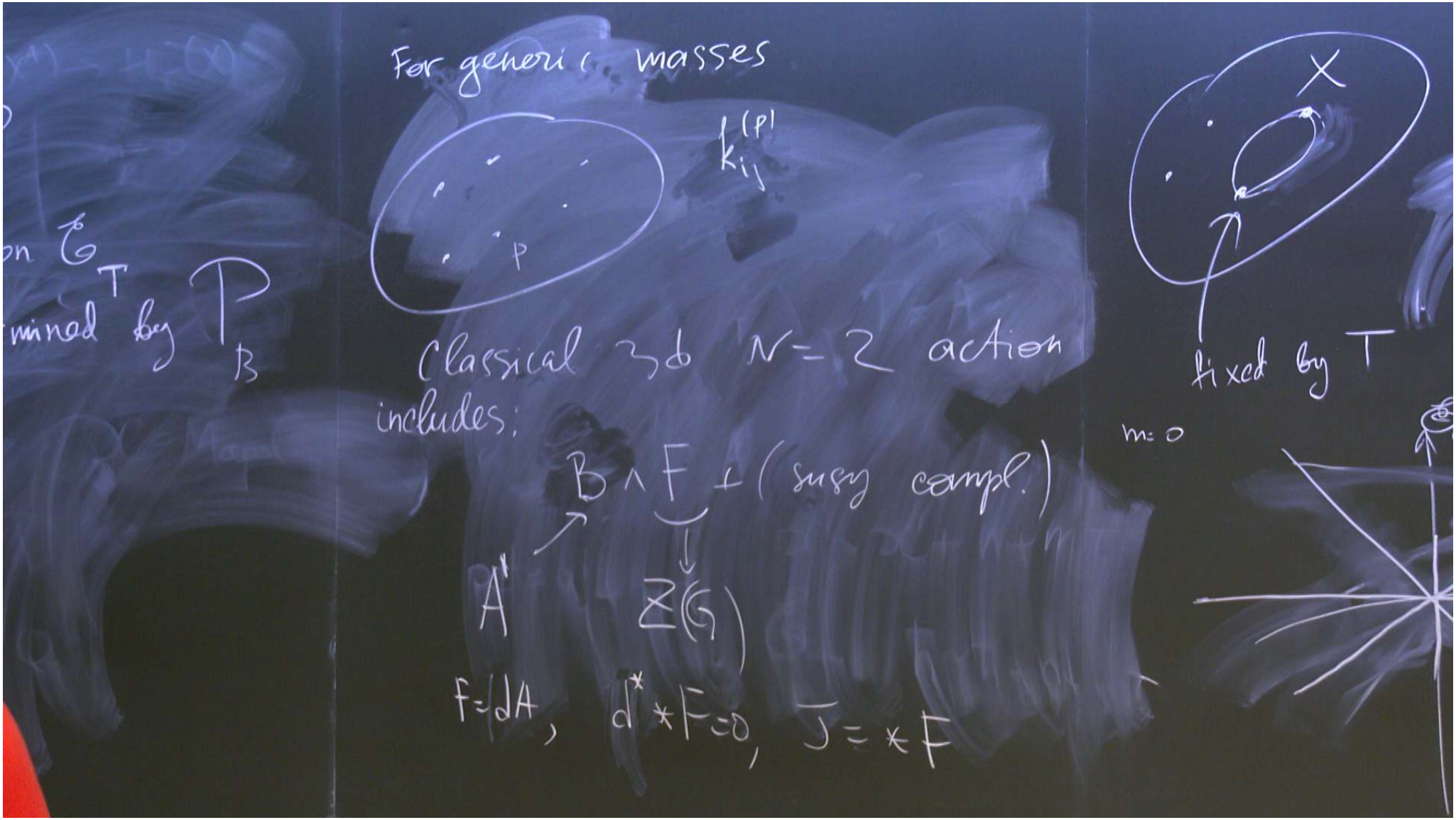
Ex. $\Gamma[A] = \pi k^{(p)}_{ij} \int (a^i_1 \ddot{a}^j_2 - a^i_2 \ddot{a}^j_1)\, dt$

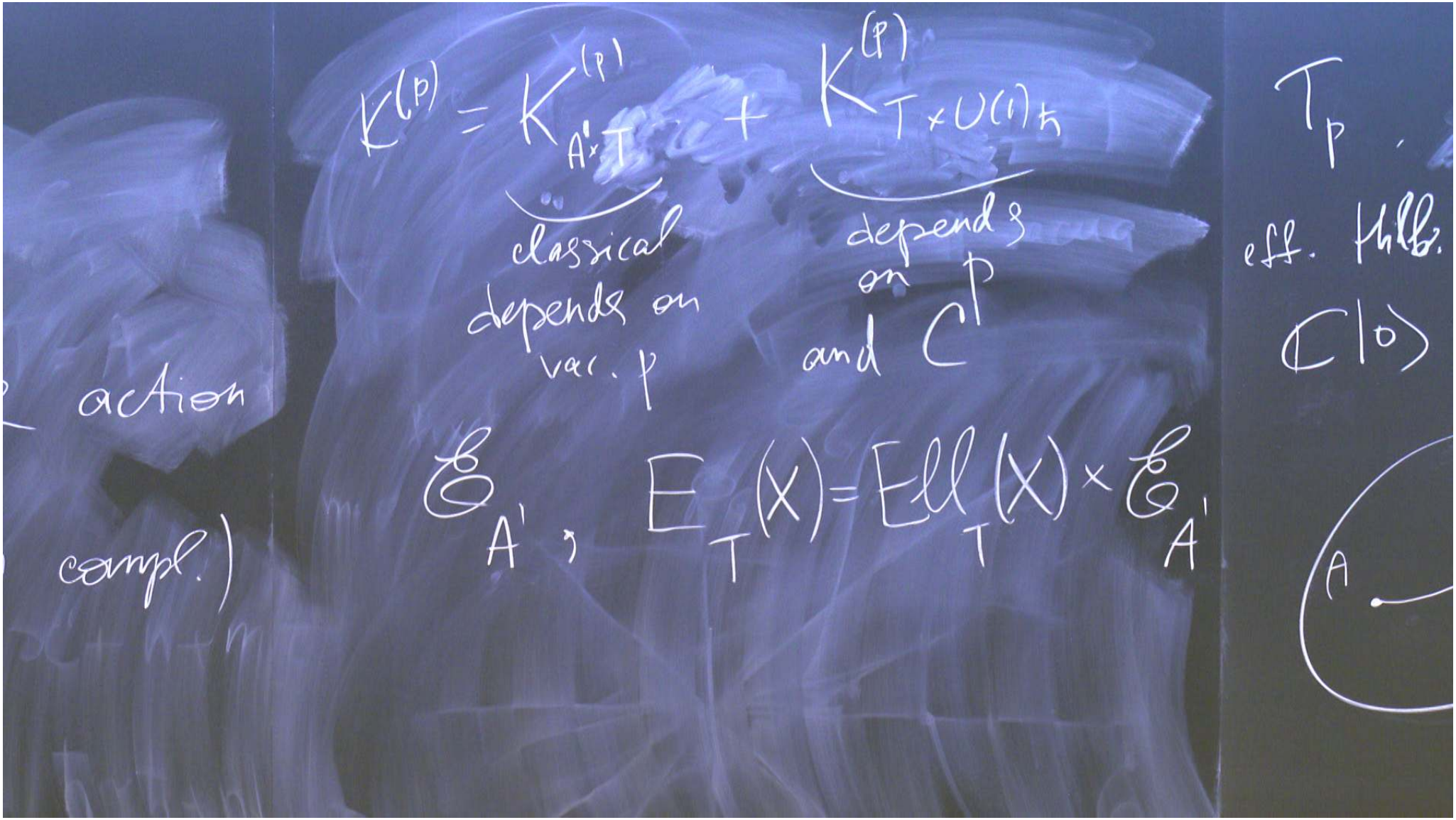
$$= \int_\gamma B \,; \quad B^{(p)} = \pi k^{(p)}_{ij} (a^i_1 da^j_2 - a^i_2 da^j_1)$$

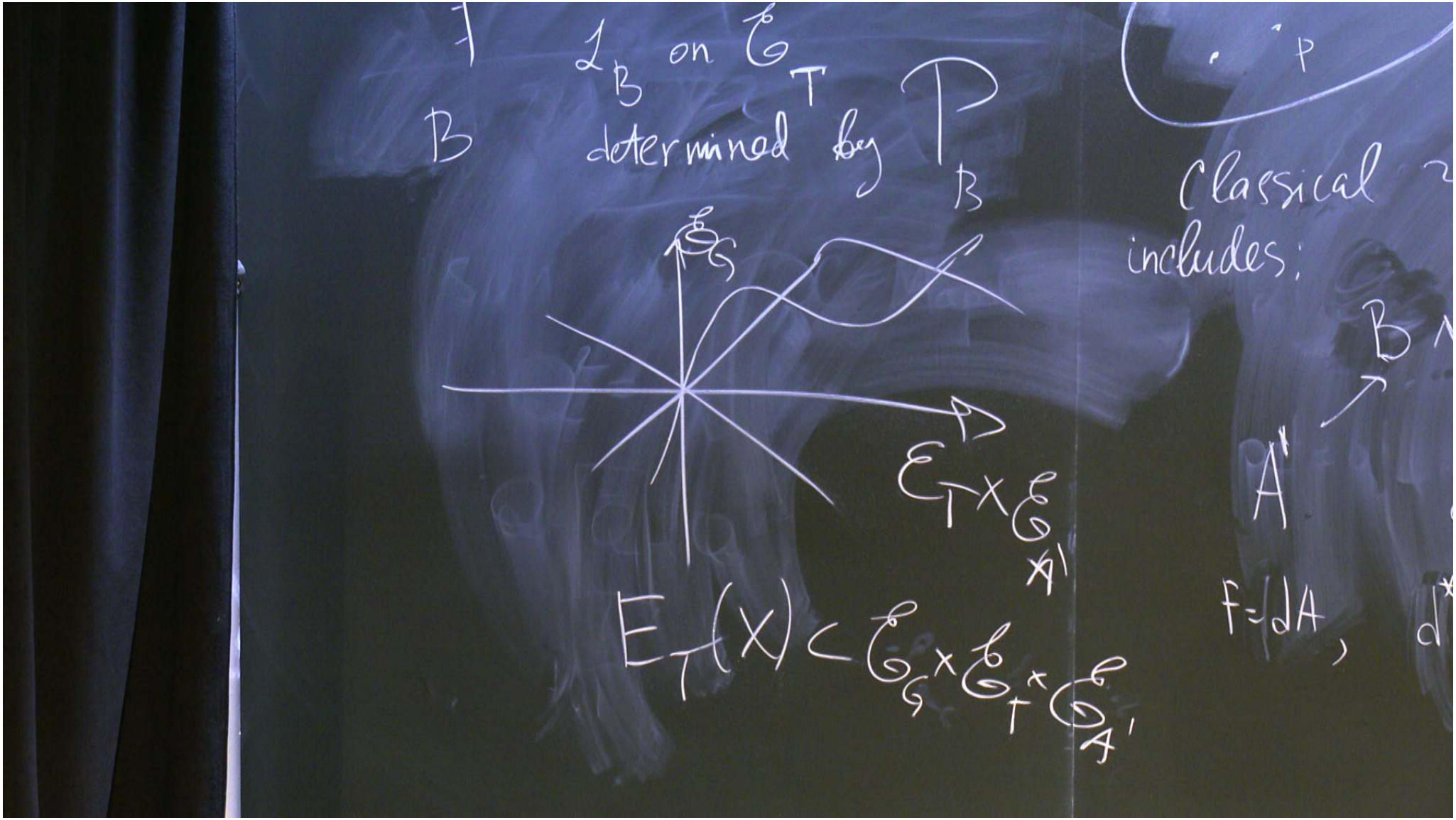
$\gamma \subset \mathcal{E}_T$ Berry connection

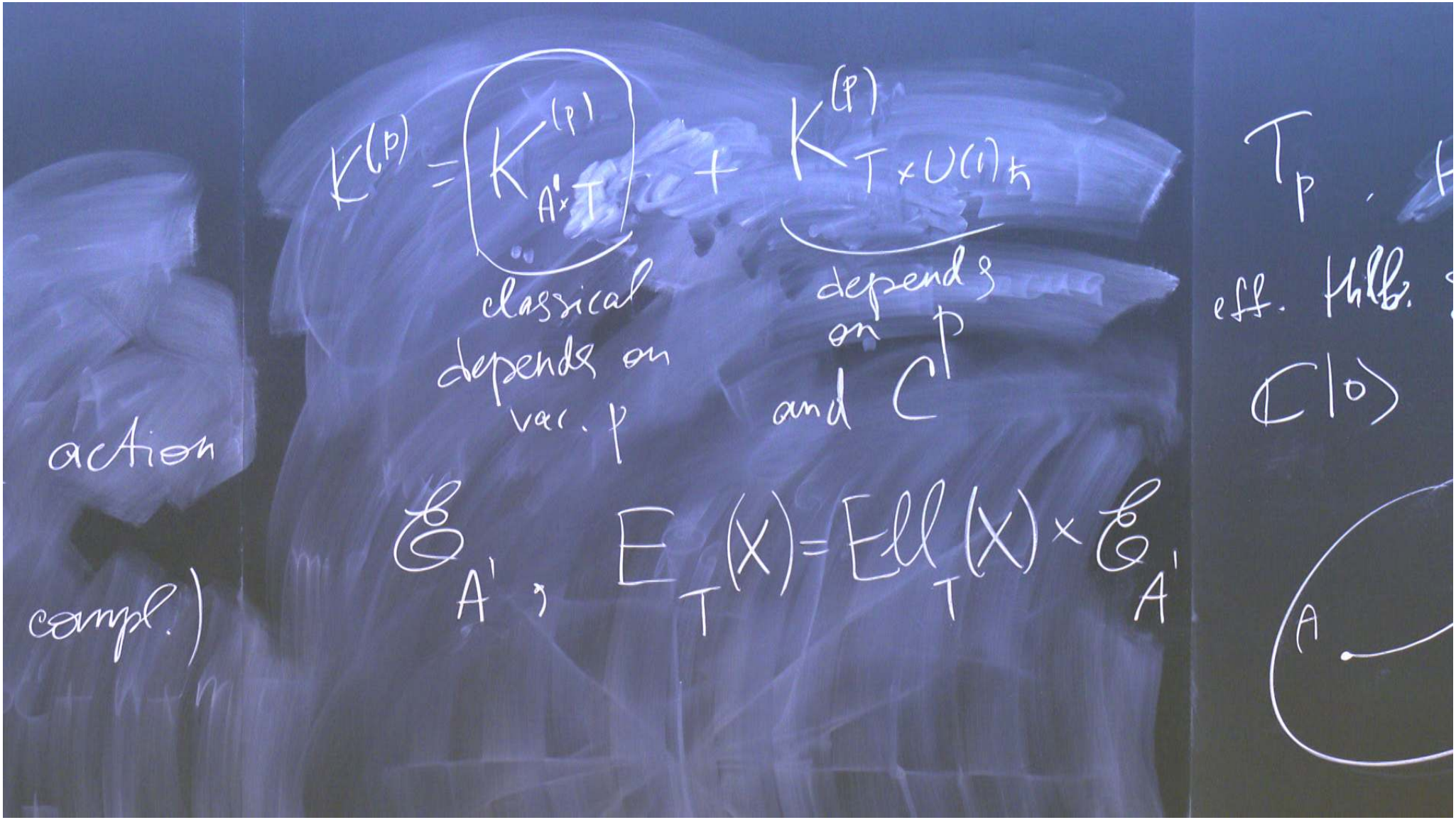

Berry connection

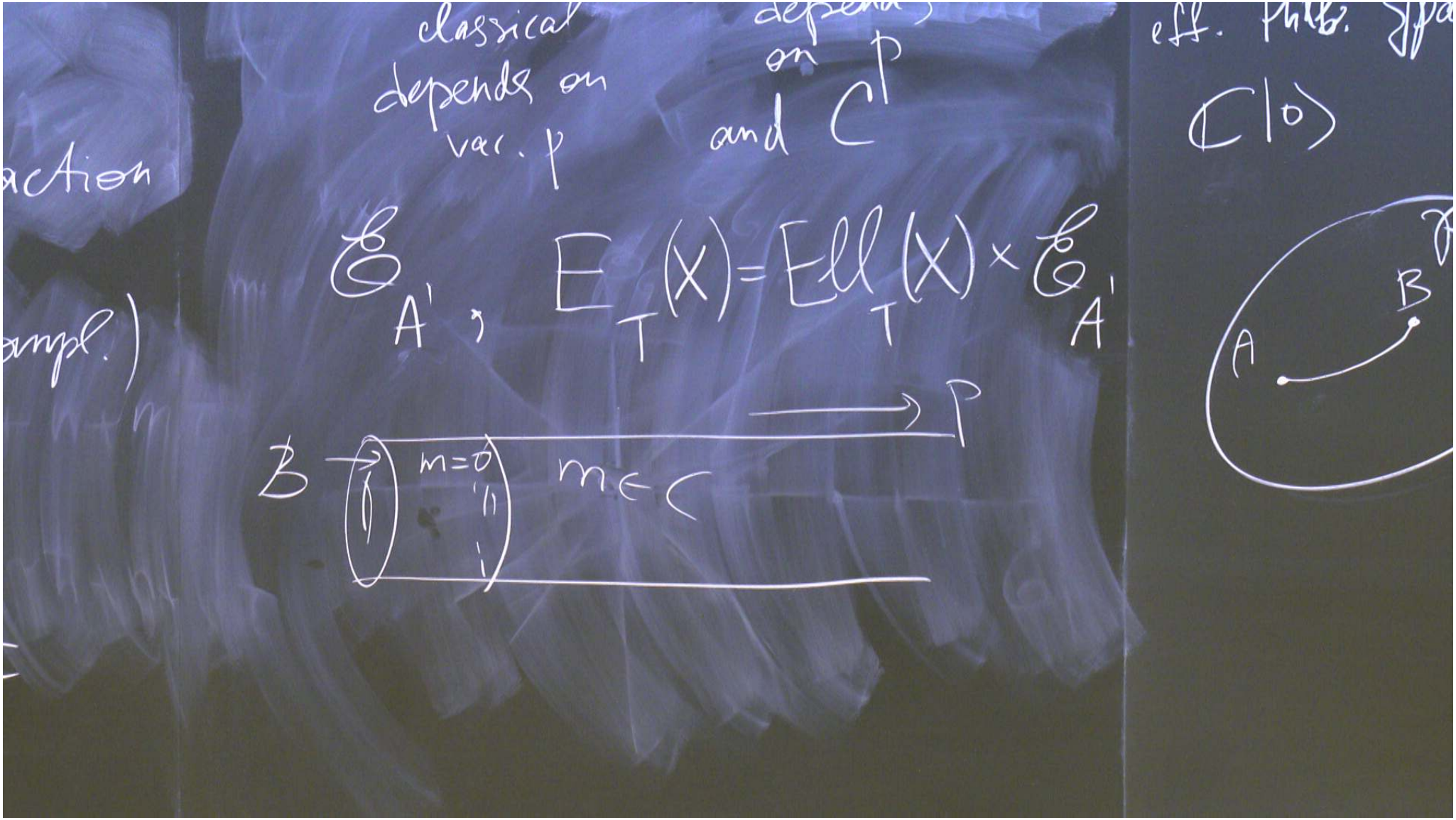

$e^{-\epsilon H}|B\rangle$
$\mathcal{L}_B$ on $\mathcal{C}_T$
B
determined by P_B
$\mathcal{H}[E_1] \supset V($
$2\pi a_1^i = \oint_A$
$2\pi a_2^i = \oint_B$
$x^i = a_1^i + \tau a_2^i$

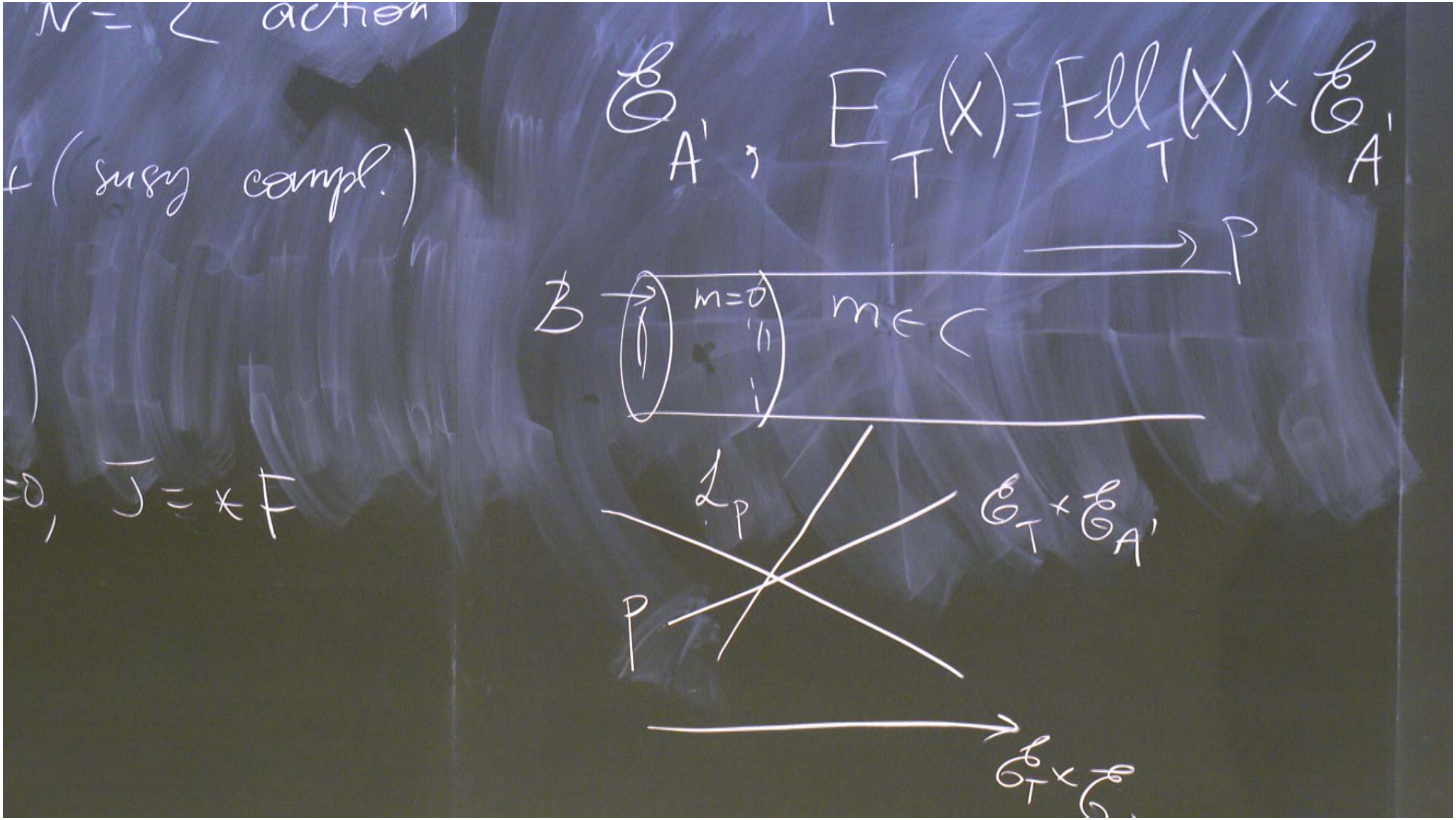

For generic masses
$k_{ij}^{(p)}$
p
X
fixed by T
Classical 3d N = 2 action
includes:
$B \wedge F$ + (susy compl.)
A'
$Z(G)$
$F = dA$, $d^* * F = 0$, $J = * F$
m = 0
on $\mathcal{C}_T$
mined by P_B

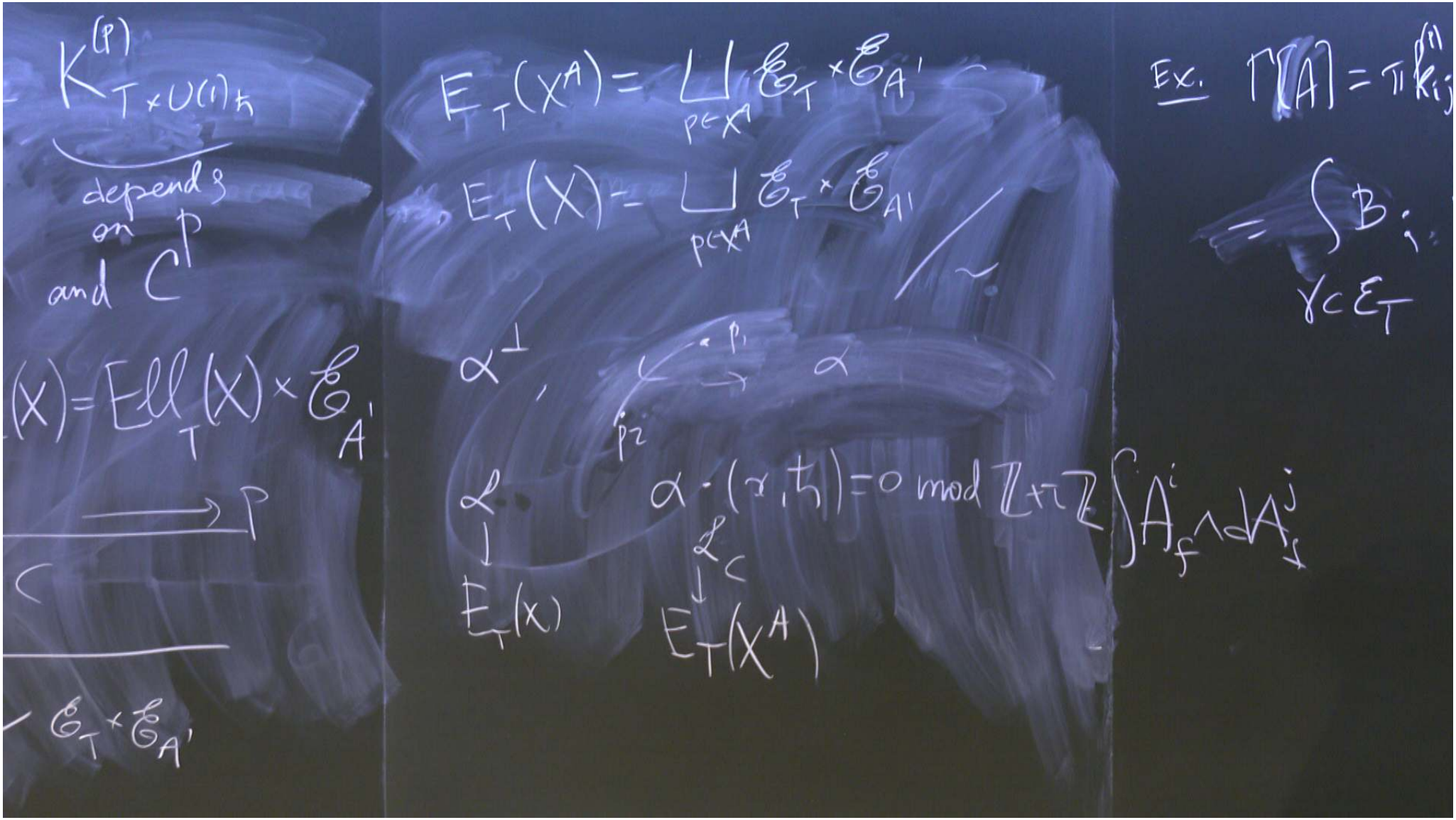

$K^{(p)} = K^{(p)}_{A' \times T} + K^{(p)}_{T \times U(1)_\hbar}$
classical depends on var. p
depends on p and C^p
T_p
eff. Hilb.
$C|0\rangle$
action
compl.)
$\mathcal{E}_{A'}$, $E_T(X) = Ell_T(X) \times \mathcal{E}_{A'}$
A

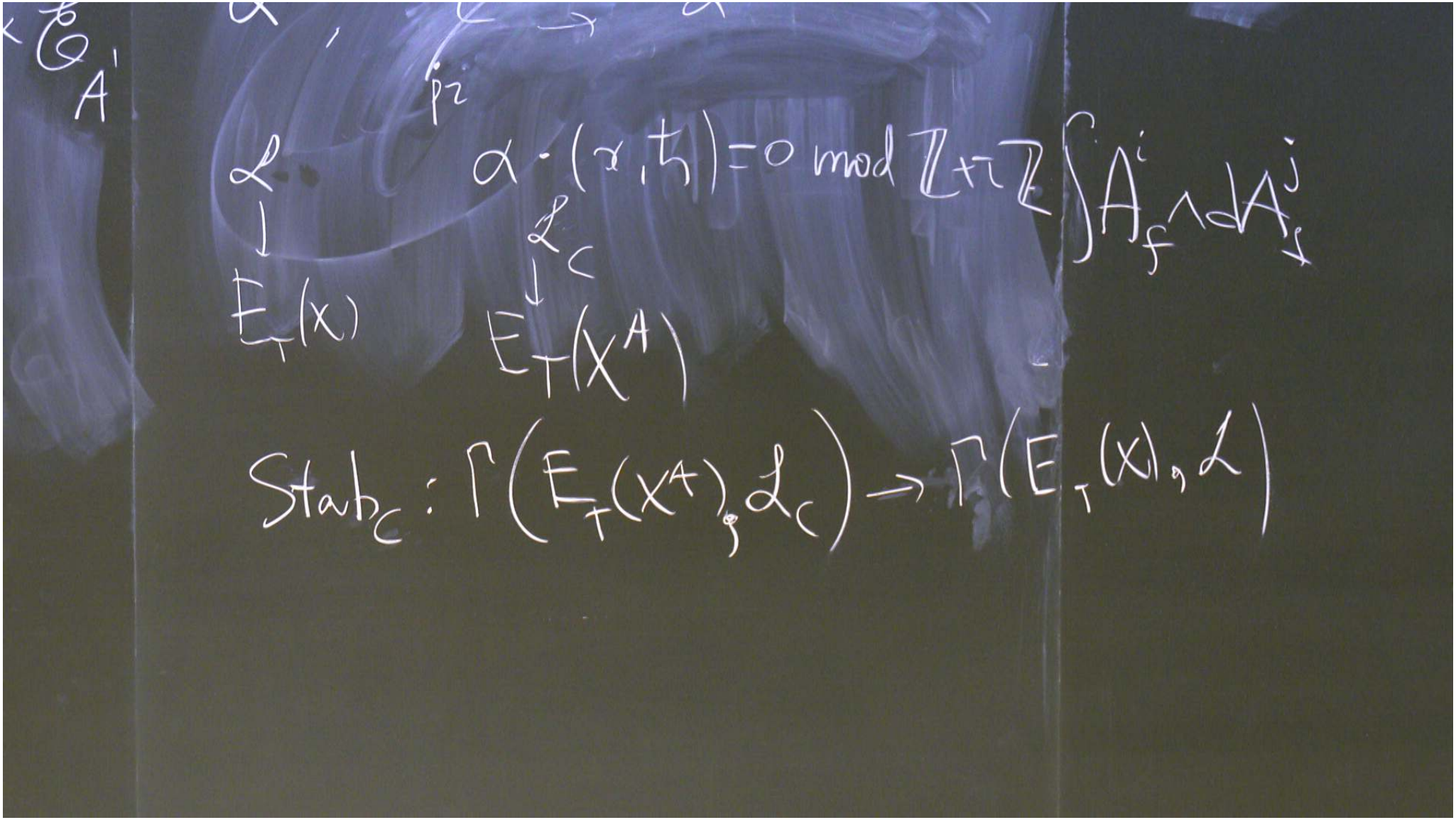

$\mathcal{L}_B$ on $\mathcal{C}_T$
determined by P_B
Classical
includes:
$\mathcal{E}_T \times \mathcal{E}_{\mathcal{G}} \times \mathcal{A}'$
$E_T(X) \subset \mathcal{E}_{\mathcal{G}} \times \mathcal{E}_T \times \mathcal{G}_{\mathcal{A}'}$
$\mathcal{A}'$
$F = dA$,

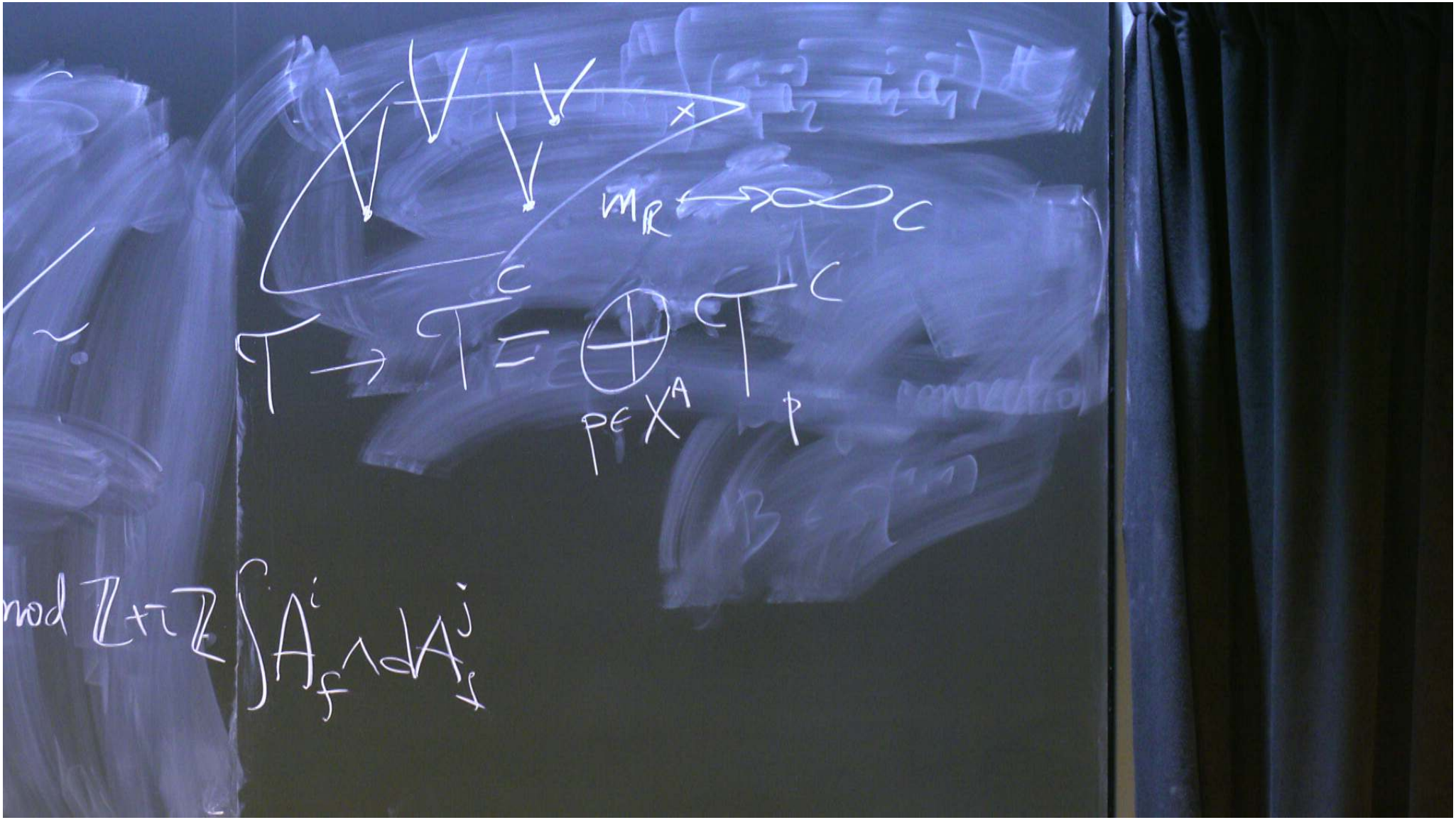

$K^{(p)} = K^{(p)}_{A' \times T} + K^{(p)}_{T \times U(1)_h}$
classical depends on var. p
depends on p and C^p
T_p .
eff. Hilb.
$C|0\rangle$
action
compl.)
$\mathcal{E}_{A'}$, $E_T(X) = Ell_T(X) \times \mathcal{E}_{A'}$
A

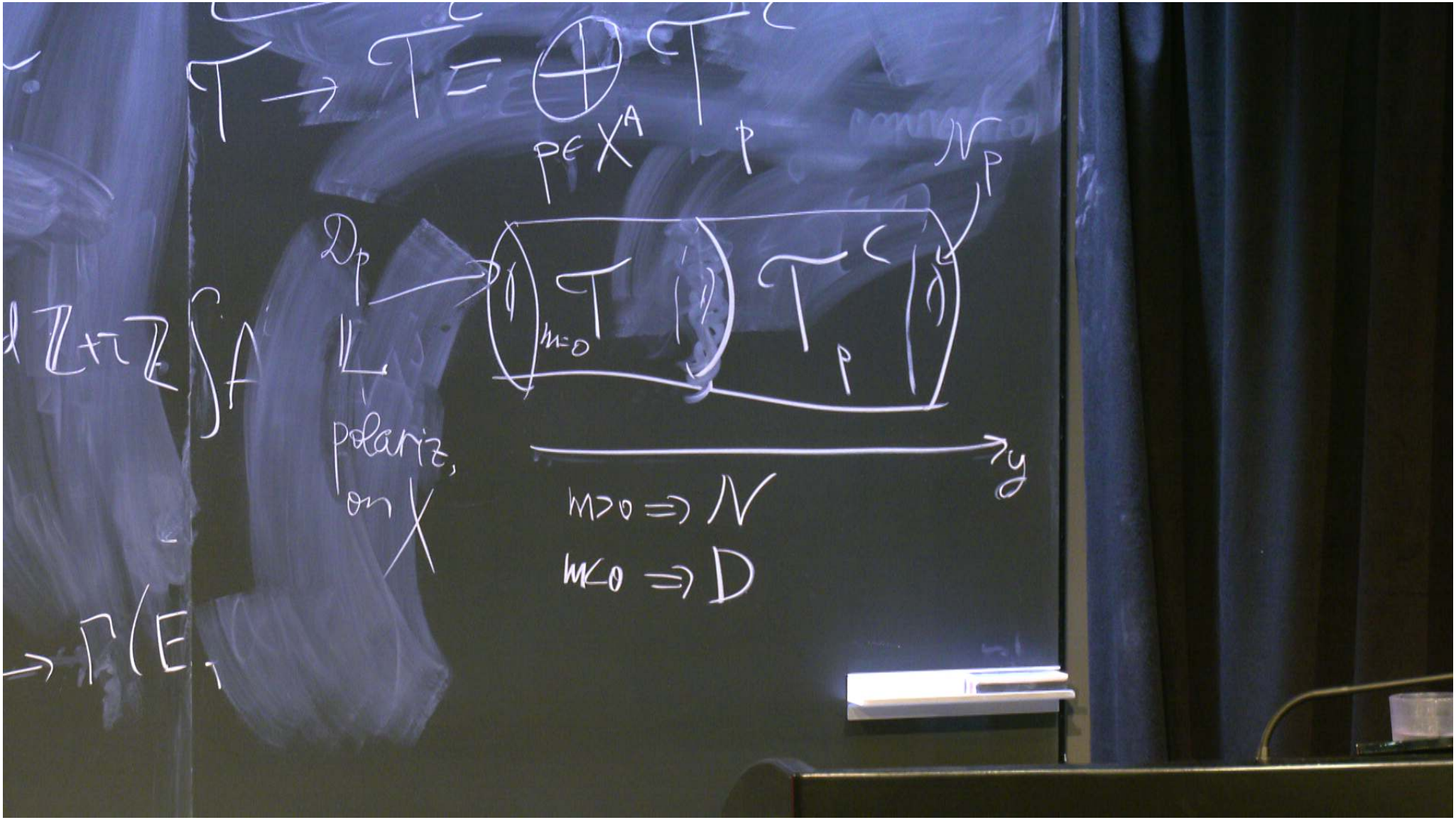

classical
depends on
vac. p
depends
on p
and C^p
eff. Phil. Spa
C|0>
action
ampl.)
E_T(X) = Ell_T(X) × E_A'
p
B
m=0
m ∈ C
A
B


$\mathcal{G}_{A'}$, $E_T(X) = Ell_T(X) \times \mathcal{G}_{A'}$
P
B
m=0
m∈C
$\mathcal{L}_P$
$\mathcal{G}_T \times \mathcal{G}_{A'}$
P
$\mathcal{G}_T \times \mathcal{G}_{A'}$


$K^{(p)}_{T\times U(1)_h}$
depends on p and C
$E_T(X^A) = \coprod_{p\in X^A} \mathcal{G}_T \times \mathcal{G}_{A'}$
$E_T(X) = \coprod_{p\in X^A} \mathcal{G}_T \times \mathcal{G}_{A'}$
Ex. $T[A] = \pi k^{(1)}_{ij}$
$= \int_{\gamma_C \mathcal{E}_T} B_i$
$\alpha^\perp$
p_1
p_2
α
$\alpha \cdot (x_1 - x_2) = 0 \bmod \mathbb{Z} + \tau\mathbb{Z}$
$\int A^i_f \wedge dA^j_i$
$E_T(X)$
$E_T(X^A)$
$\mathcal{G}_T \times \mathcal{G}_{A'}$


$\mathcal{L}$
$\alpha \cdot (x, t) = 0 \bmod \mathbb{Z} + \tau\mathbb{Z}$
$\int A^i_f \wedge dA^j_f$
$\mathcal{L}_C$
$E_T(X)$
$E_T(X^A)$
$\mathrm{Stab}_C : \Gamma(E_T(X^A), \mathcal{L}_C) \to \Gamma(E_T(X), \mathcal{L})$


$m_R \to \infty$
$T \to T = \bigoplus_{p \in X^A} T$
$\mod \mathbb{Z} + \tau\mathbb{Z} \int A^i_f \wedge dA^j_f$

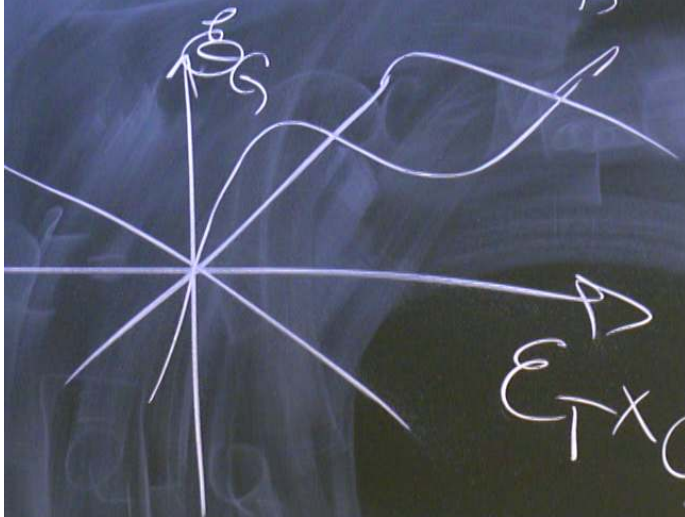

$T \to T = \bigoplus_{p \in X^A} T_p$
N_p
D_p
T
$m=0$
T_p
polariz.
on X
y
$m>0 \Rightarrow N$
$m<0 \Rightarrow D$

$e^{-\epsilon H}|B\rangle$

$\downarrow$

$\mathcal{L}_B$ on $\mathcal{C}_T$

determined by P_B



$E_T(X) \subset \mathcal{E}_G \times \mathcal{E}$

$k_{ij}^{(p)}$

Classical 3d $\mathcal{N}=2$ action includes:

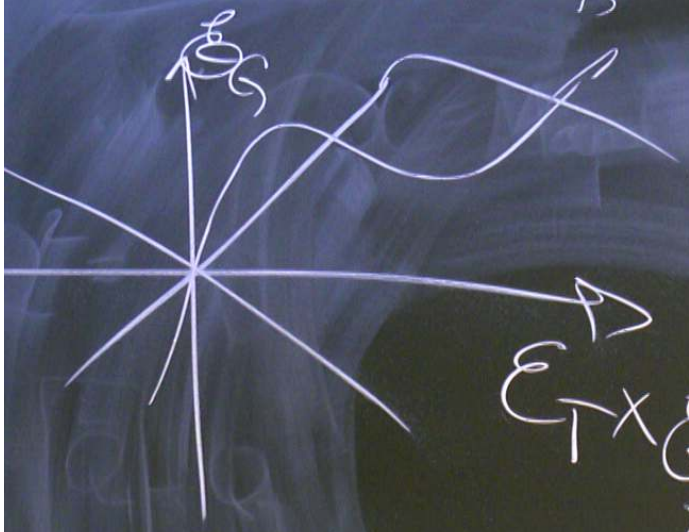
$B \wedge F + (\text{susy compl.})$

$U(1)$

A' $\quad$ $Z(G)$

$F = dA$, $\quad d^* * F = 0$, $\quad J = * F$

determined by B



$\mathcal{E}_T \times \mathcal{E}_{A'}$

$U(1)$

$E_T(X) \subset \mathcal{E}_G \times \mathcal{E}_T \times \mathcal{E}_{A'}$

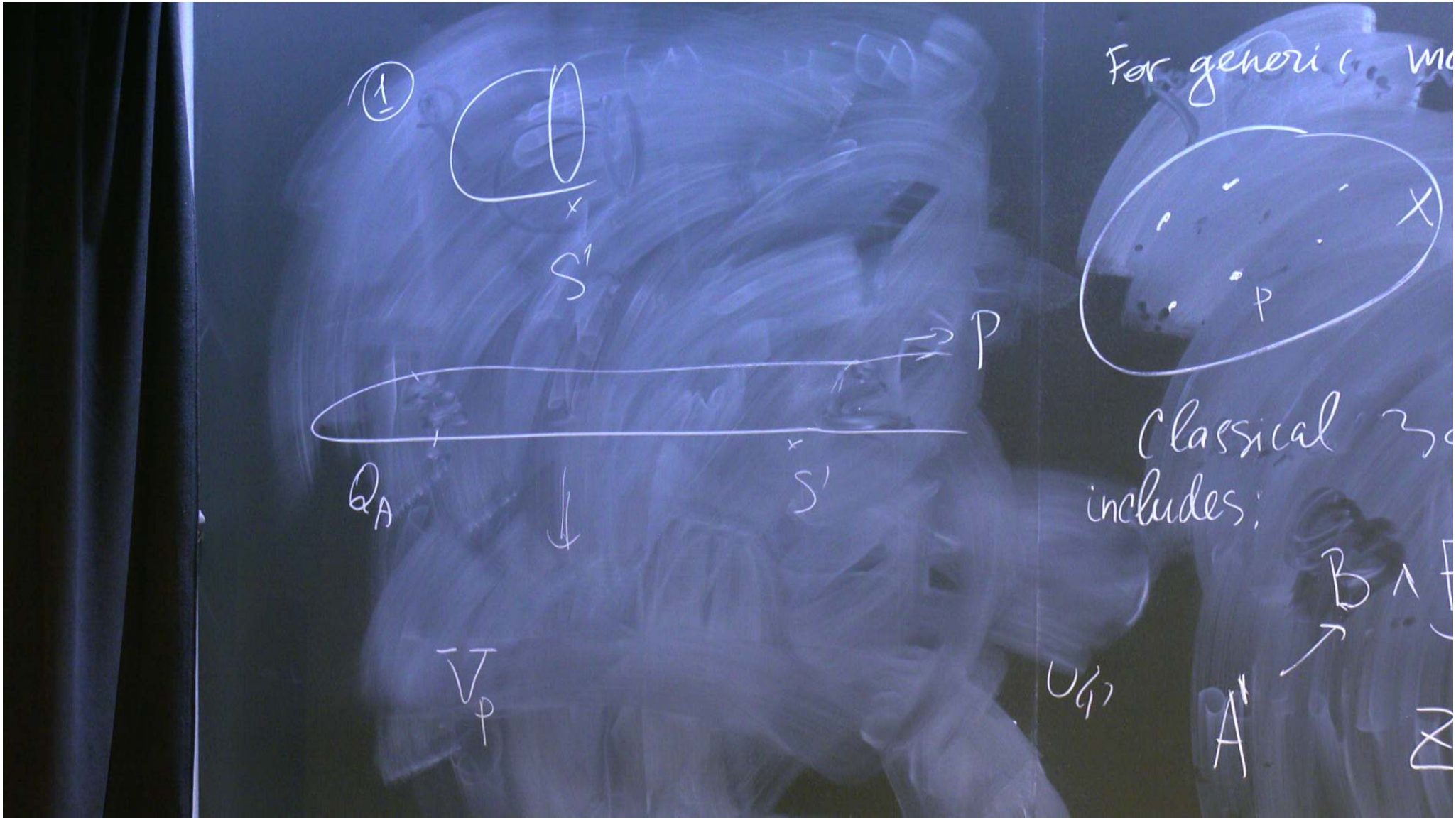
Classical 3d $N=2$ action includes:

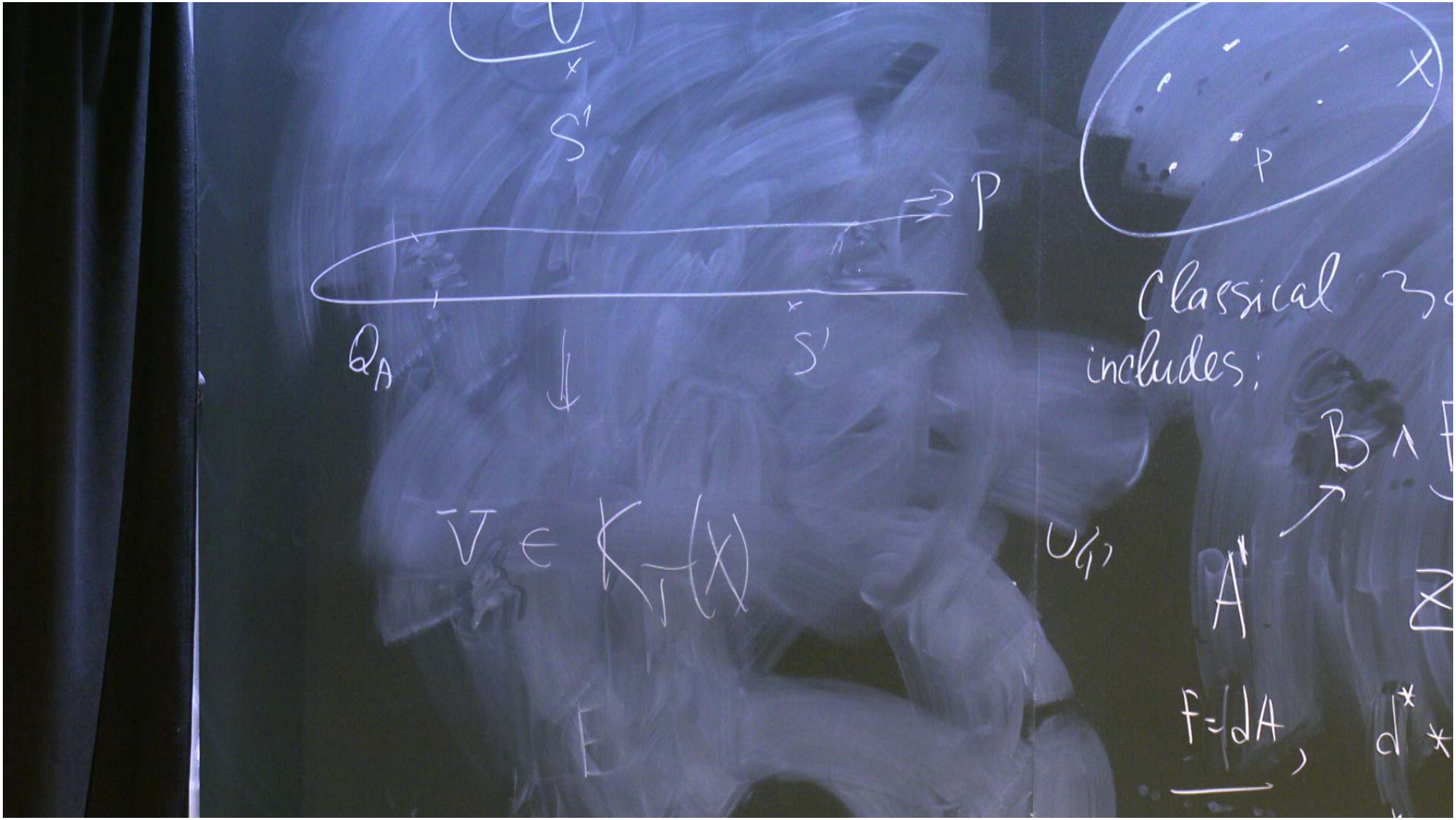
$$B \wedge F + (\text{susy compl.})$$

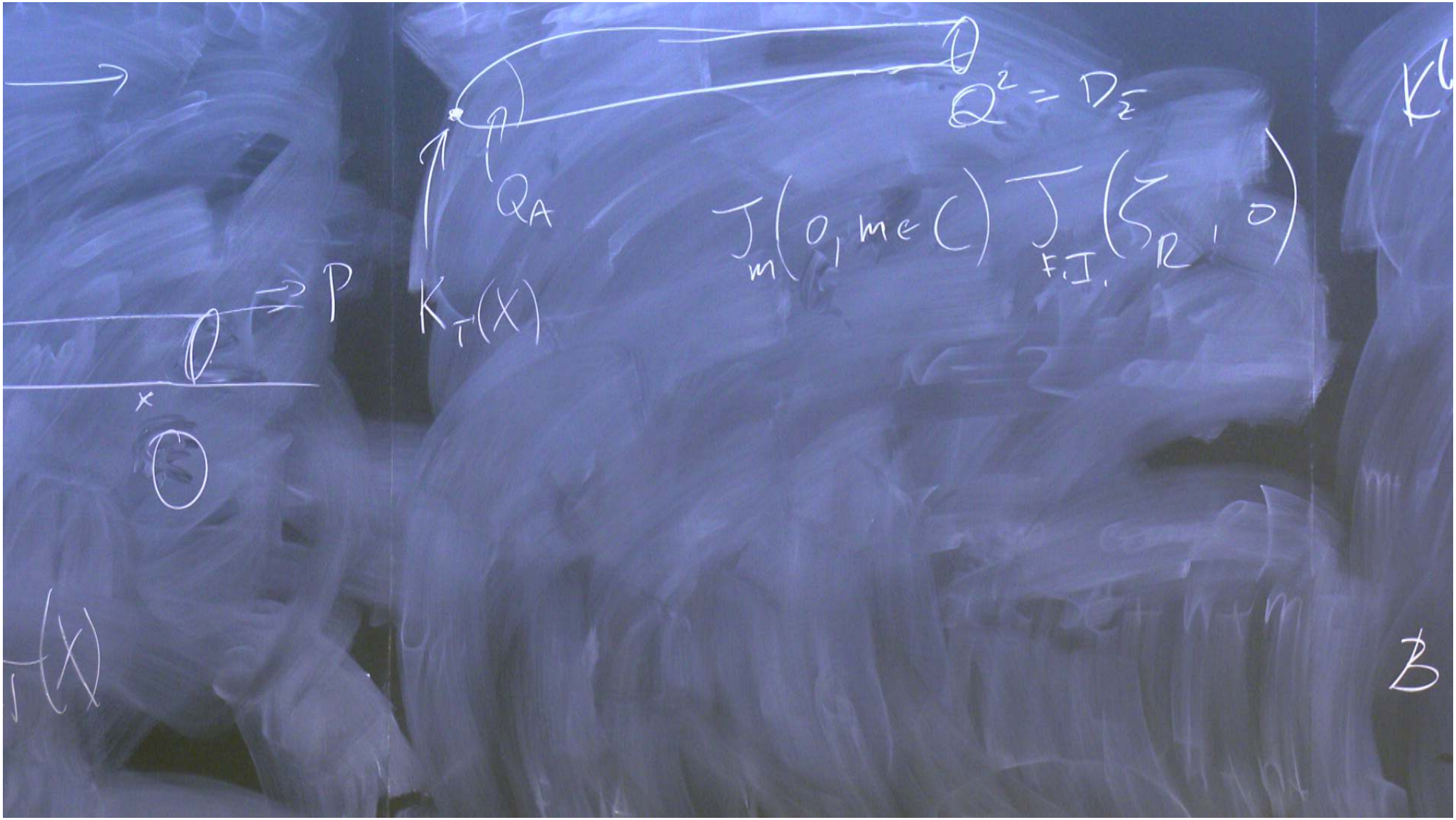
A' $\quad$ $Z(G)$

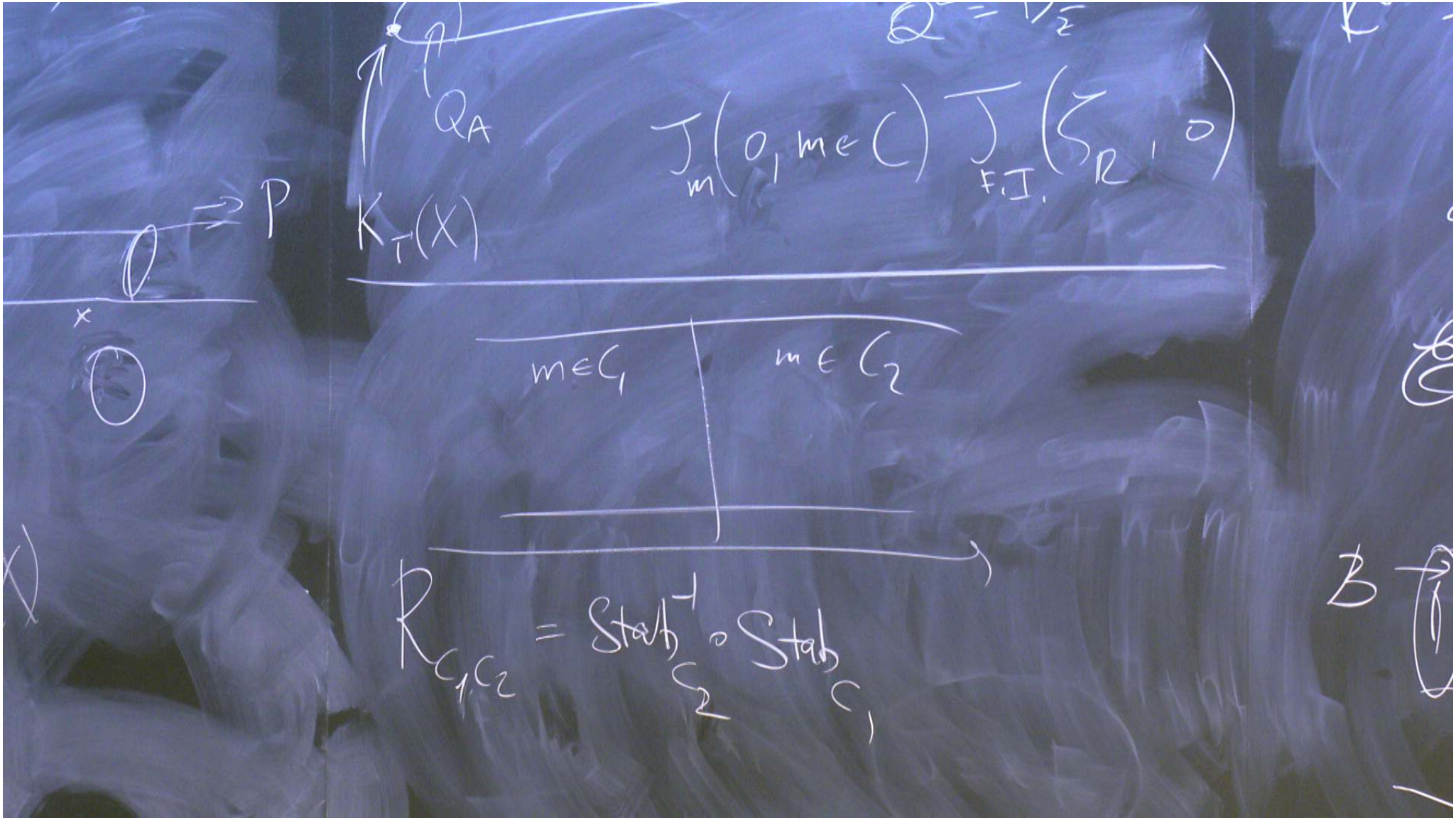
$F = dA$, $\quad d^* * F = 0$, $\quad J = * F$

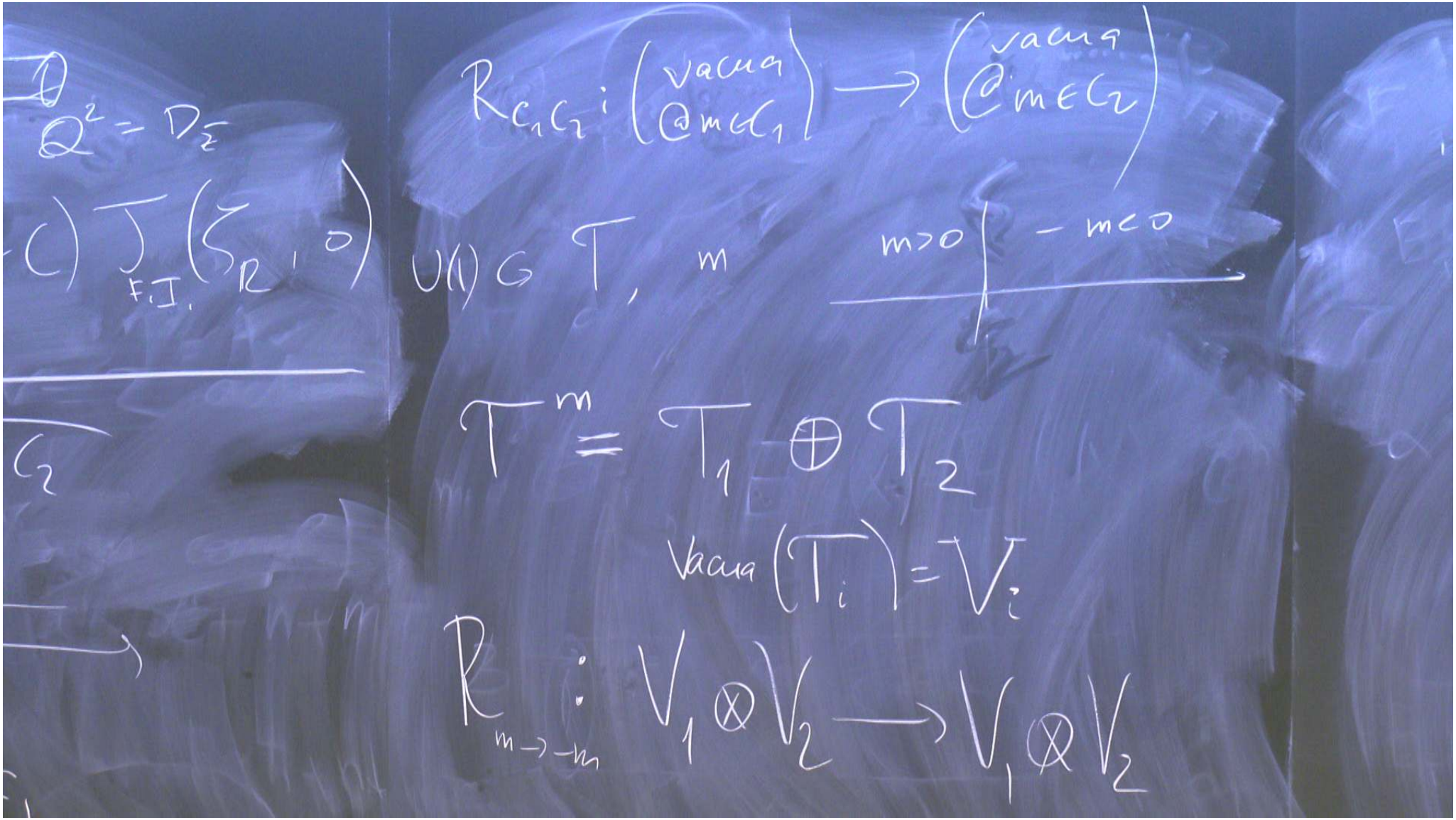
$d^* J = 0$

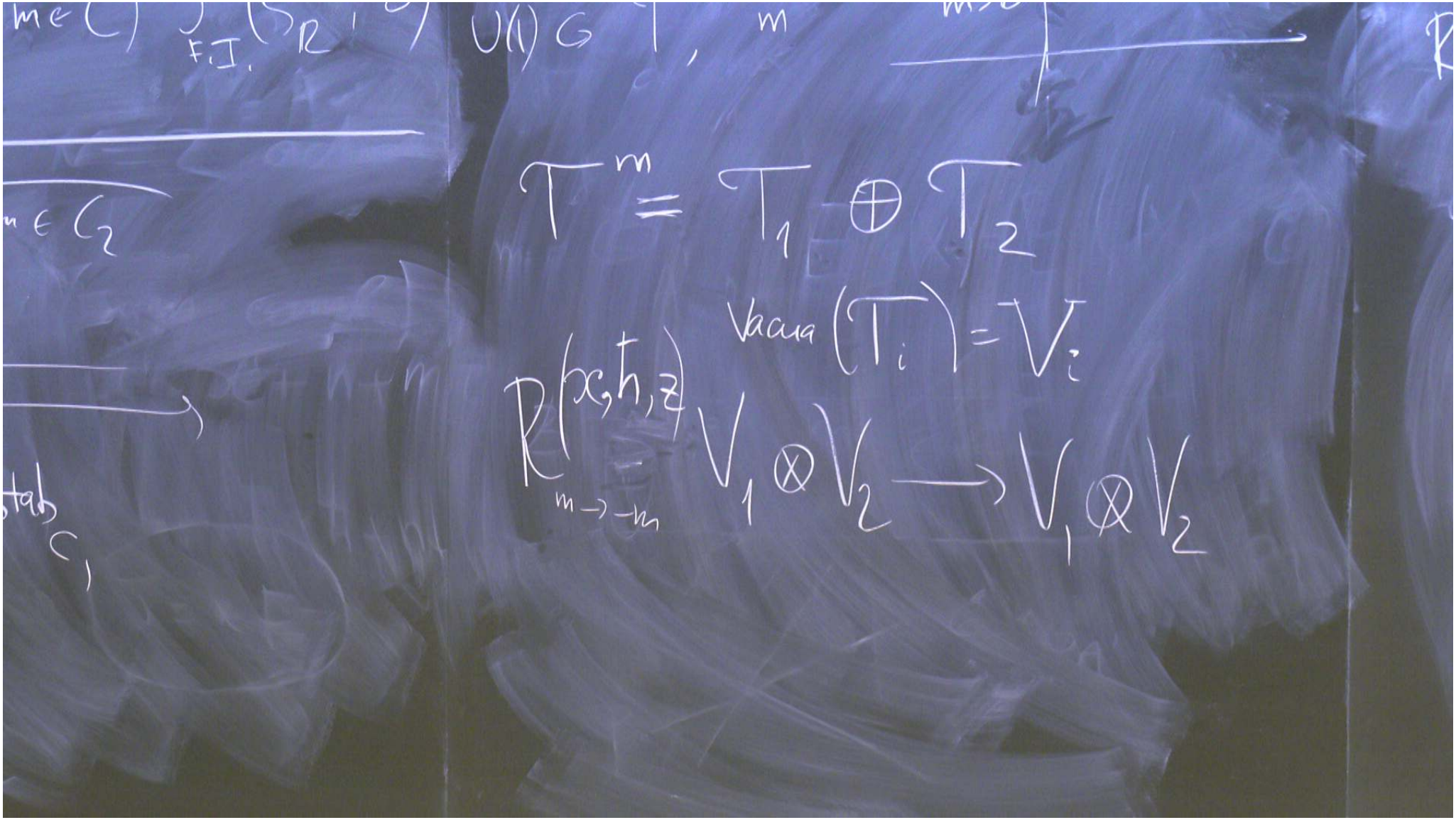

1
S'
P
Q_A
S'
V_p
For generic
X
p
Classical
includes:
B
A'

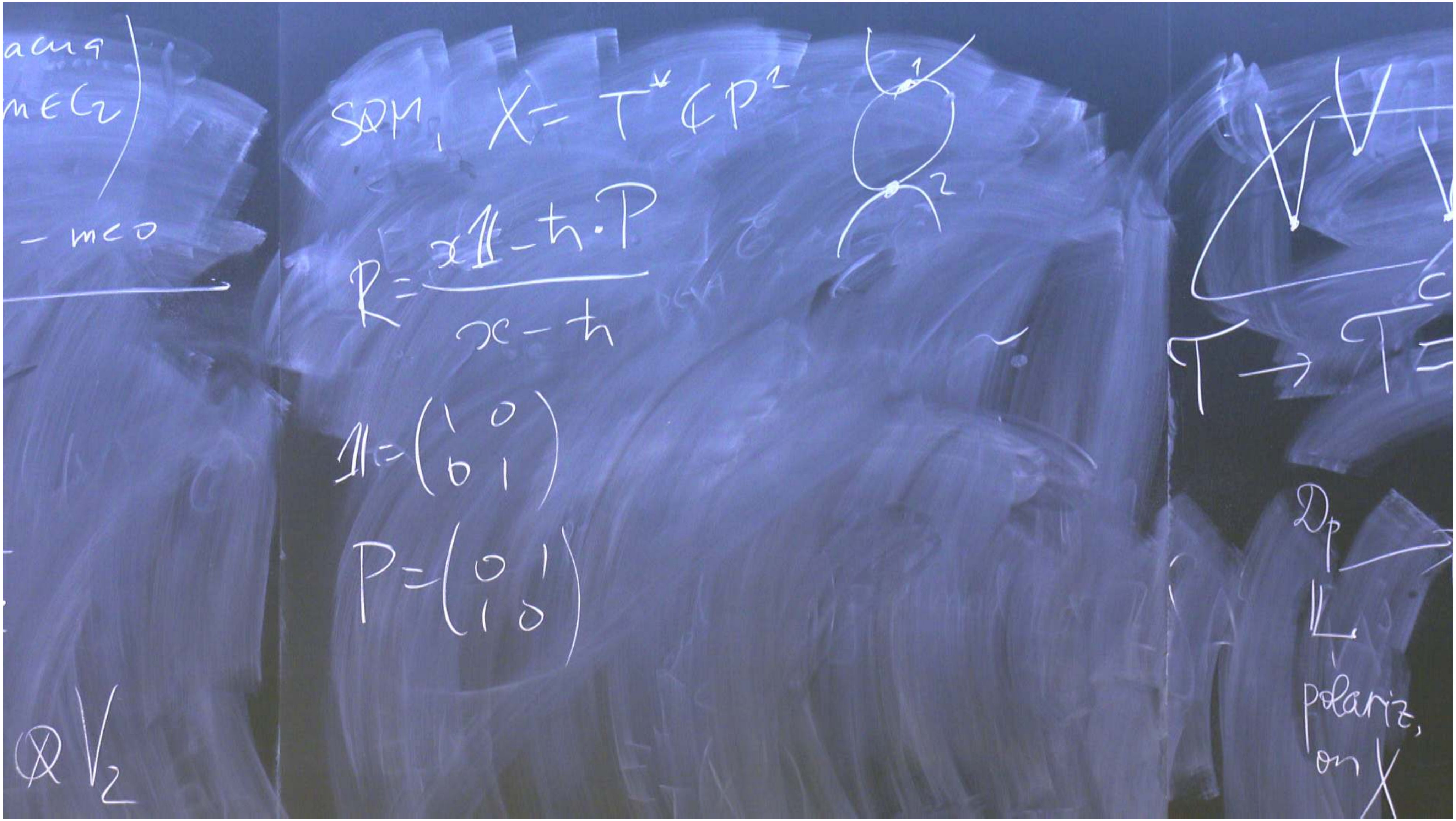

S'
P
Q_A
S'
V ∈ K_T(X)
X
p
Classical
includes:
B ∧
A'
f = dA
d* *

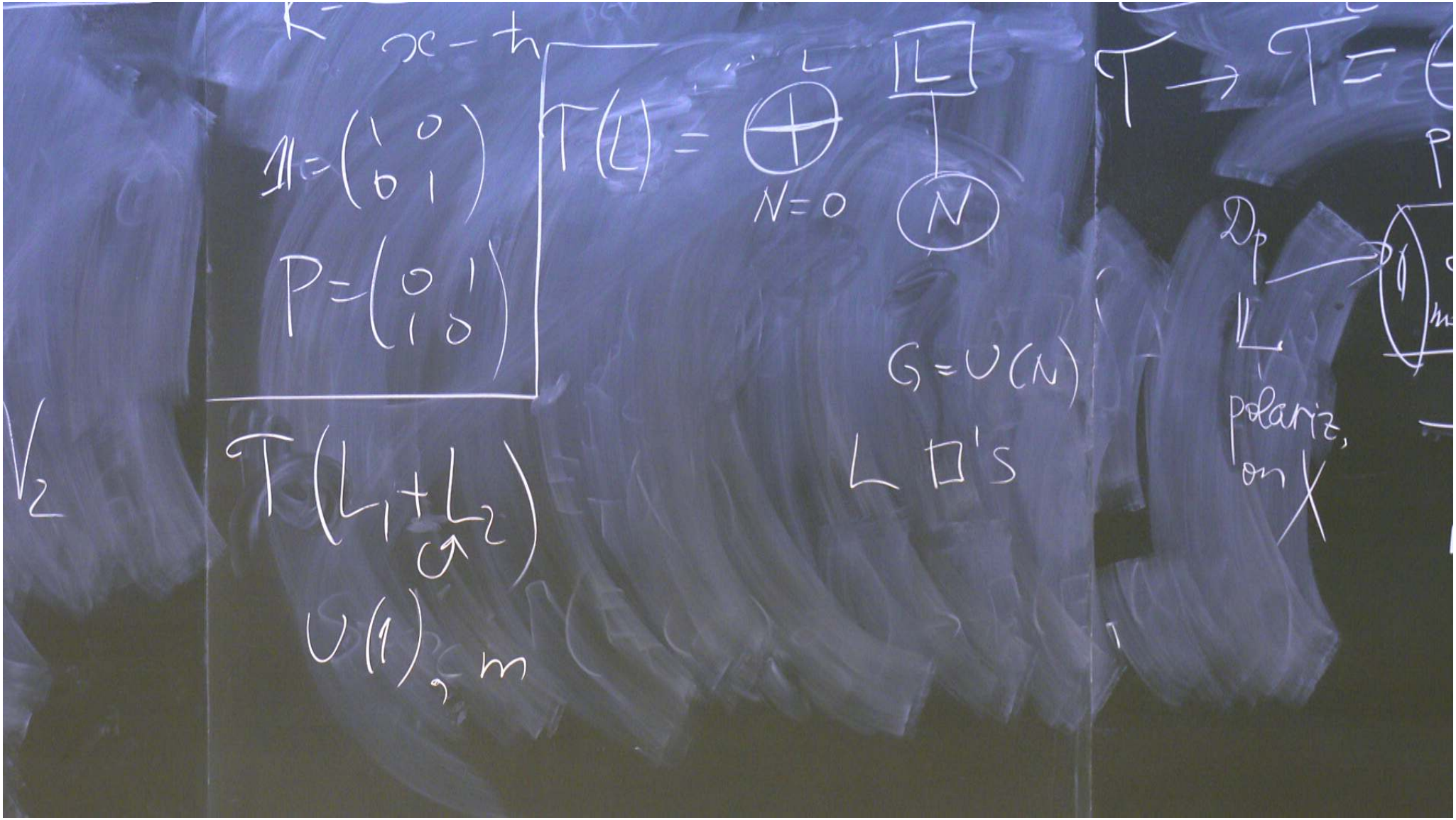

$Q^2 = D_z$
Q_A
$J_m(0, m\epsilon)$
$J_{F.I.}(\zeta_R, 0)$
$K_T(X)$
P
0
x
B

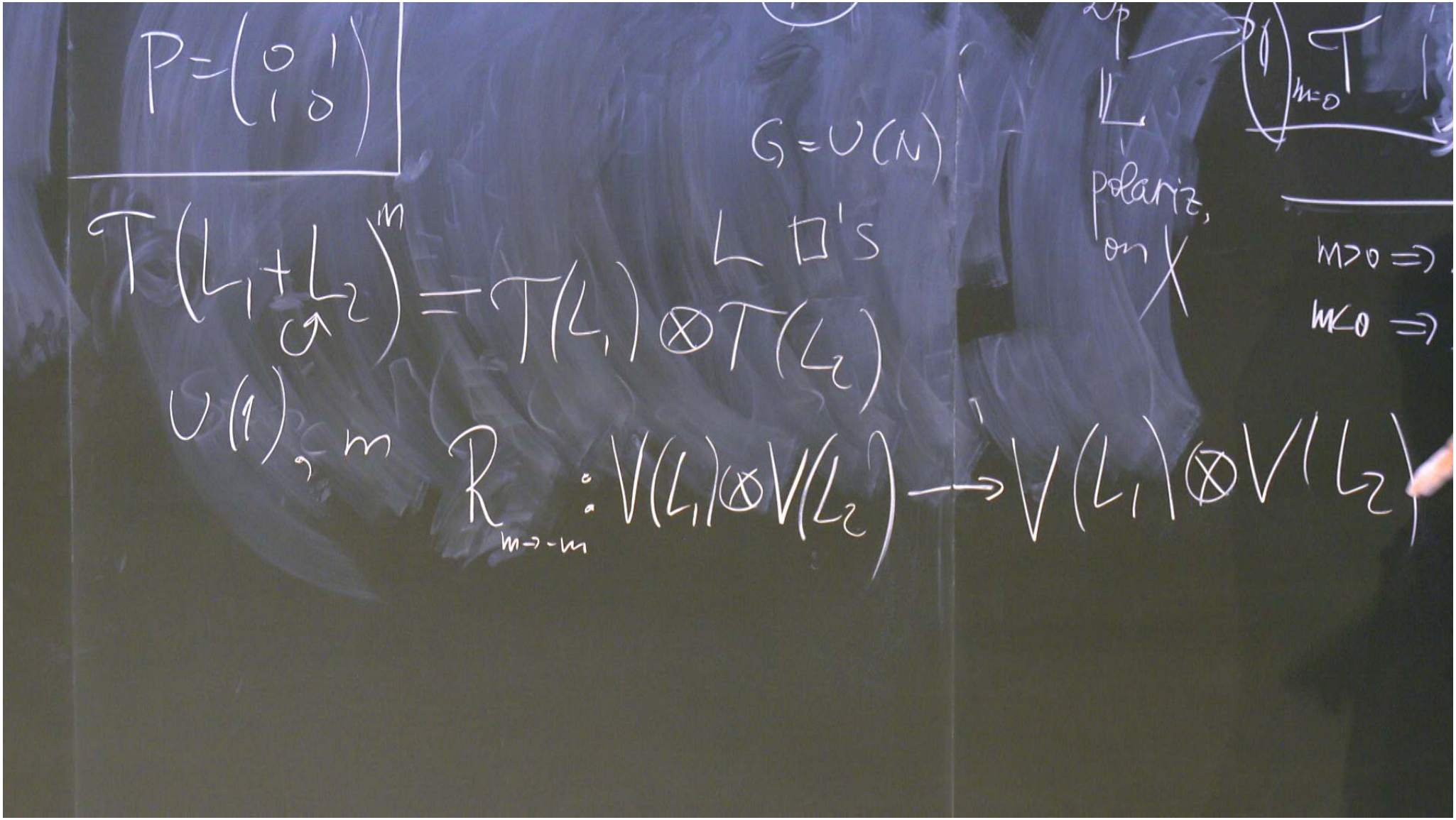

Q_A
$J_m(0, m \in C)$ $J_{F.I.}(\zeta_R, 0)$
$K_T(X)$
$m \in C_1$
$m \in C_2$
$R_{C_1,C_2} = \mathrm{Stab}_{C_2}^{-1} \circ \mathrm{Stab}_{C_1}$

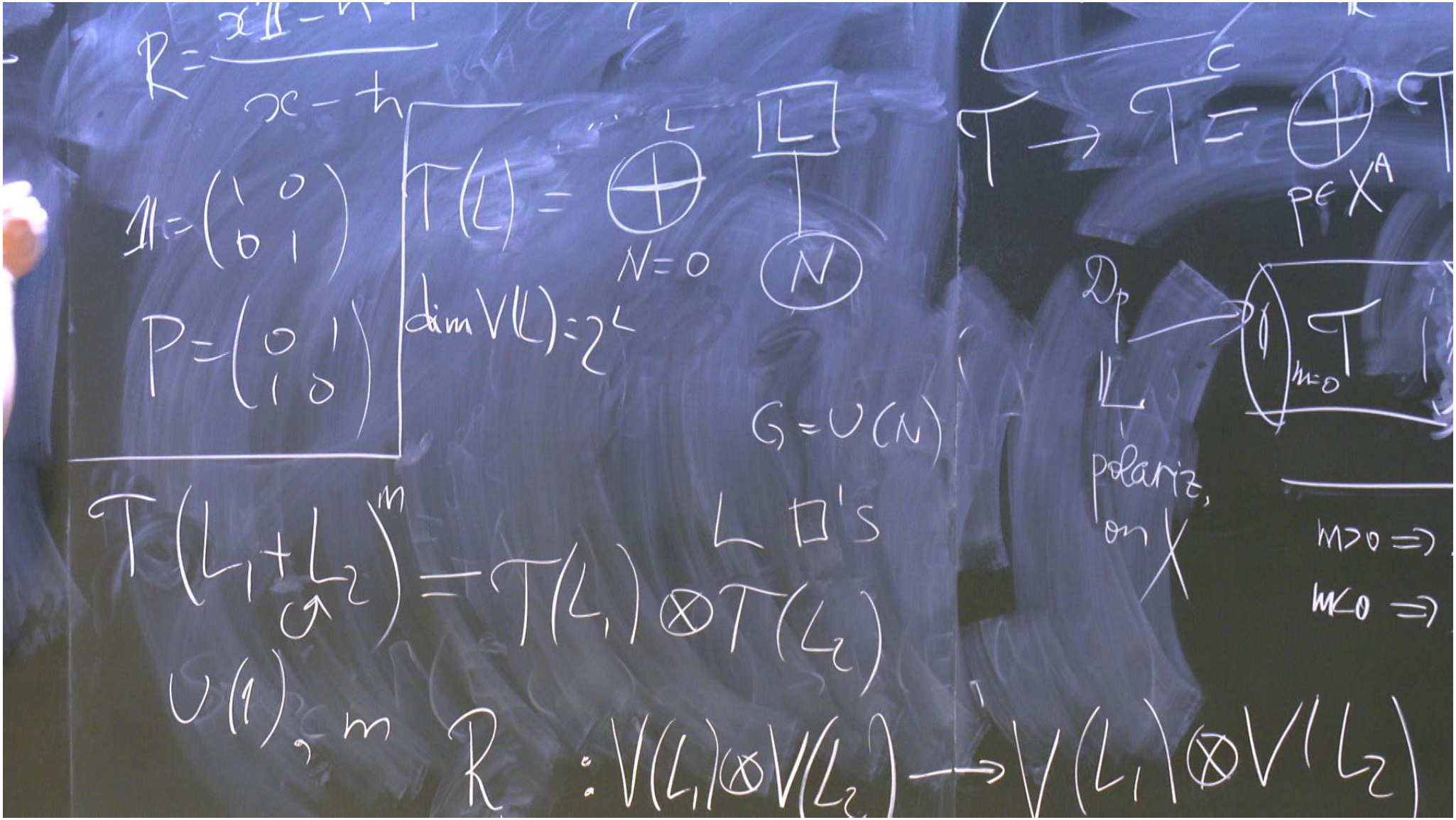

$Q^2 = D_2$
$R_{C_1C_2} : \begin{pmatrix} \text{vacua} \\ @ m \in C_1 \end{pmatrix} \longrightarrow \begin{pmatrix} \text{vacua} \\ @ m \in C_2 \end{pmatrix}$
$U(1)$ G T, m
$m > 0$ | $- m < 0$
$T^m = T_1 \oplus T_2$
$\text{Vacua}(T_i) = V_i$
$R_{m \to -m} : V_1 \otimes V_2 \longrightarrow V_1 \otimes V_2$


$T^m = T_1 \oplus T_2$
$\text{Vacua}(T_i) = V_i$
$R^{(x,t,z)}_{m \to -m} \; V_1 \otimes V_2 \longrightarrow V_1 \otimes V_2$


SQM, X = T* CP^1
R = (x𝟙 − ħ·P) / (x − ħ)
𝟙 = (1 0 / 0 1)
P = (0 1 / 1 0)
⊗ V_2
polariz, on X


x - ħ
1 = (1 0 / 0 1)
P = (0 1 / 1 0)
T(L) = ⊕
N = 0
L
N
G = U(N)
L □'s
T(L1 + L2)
U(1), m
V2
T → T =
Dp
L
polariz. on X

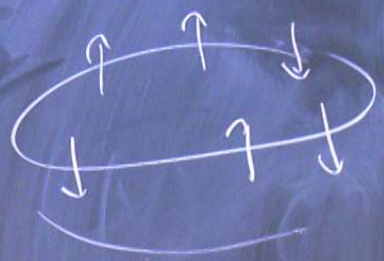

$P=\begin{pmatrix}0&1\\1&0\end{pmatrix}$
$G=U(N)$
L □'s
$T(L_1+L_2)=T(L_1)\otimes T(L_2)$
$U(1)_m$
$R_{m\to -m}: V(L_1)\otimes V(L_2)\to V(L_1)\otimes V(L_2)$
polariz. on X
$m=0$
$m>0\Rightarrow$
$m<0\Rightarrow$


R = (x 1 − ħ)/(x − ħ)
1 = (1 0; 0 1)
P = (0 1; 1 0)
T(L) = ⊕ L
N = 0
L
N
dim V(L) = 2^L
G = U(N)
L □'s
T(L1 + L2)^m = T(L1) ⊗ T(L2)
U(1), m
R : V(L1) ⊗ V(L2) → V(L1) ⊗ V(L2)
T → T^c = ⊕ T
p ∈ X^A
D_p
L
polariz. on X
m = 0
T
m > 0 ⇒
m < 0 ⇒

$T, \; m$ $\quad m>0 \quad -m<0$

$-m$

$= T_1 \oplus T_2$



L spins

$\mathrm{Vacua}(T_i) = V_i$

$x, \hbar, z$

$: V_1 \otimes V_2 \longrightarrow V_1 \otimes V_2$

$\to -m$

$R = \dfrac{x\mathbb{1} - \hbar \cdot T}{x - \hbar}$

$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

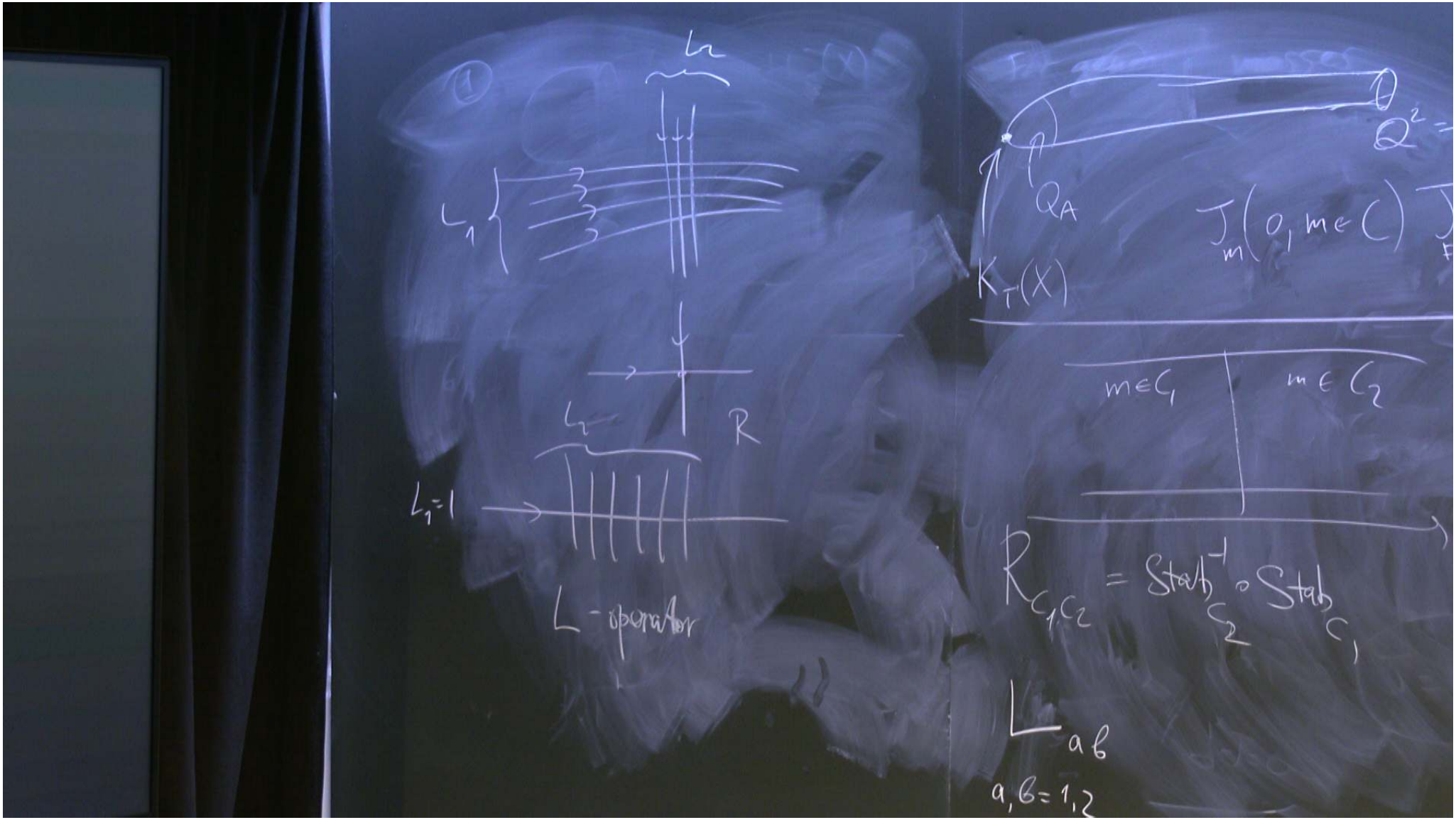
$T(L) = \bigoplus^{L}_{N=0}$

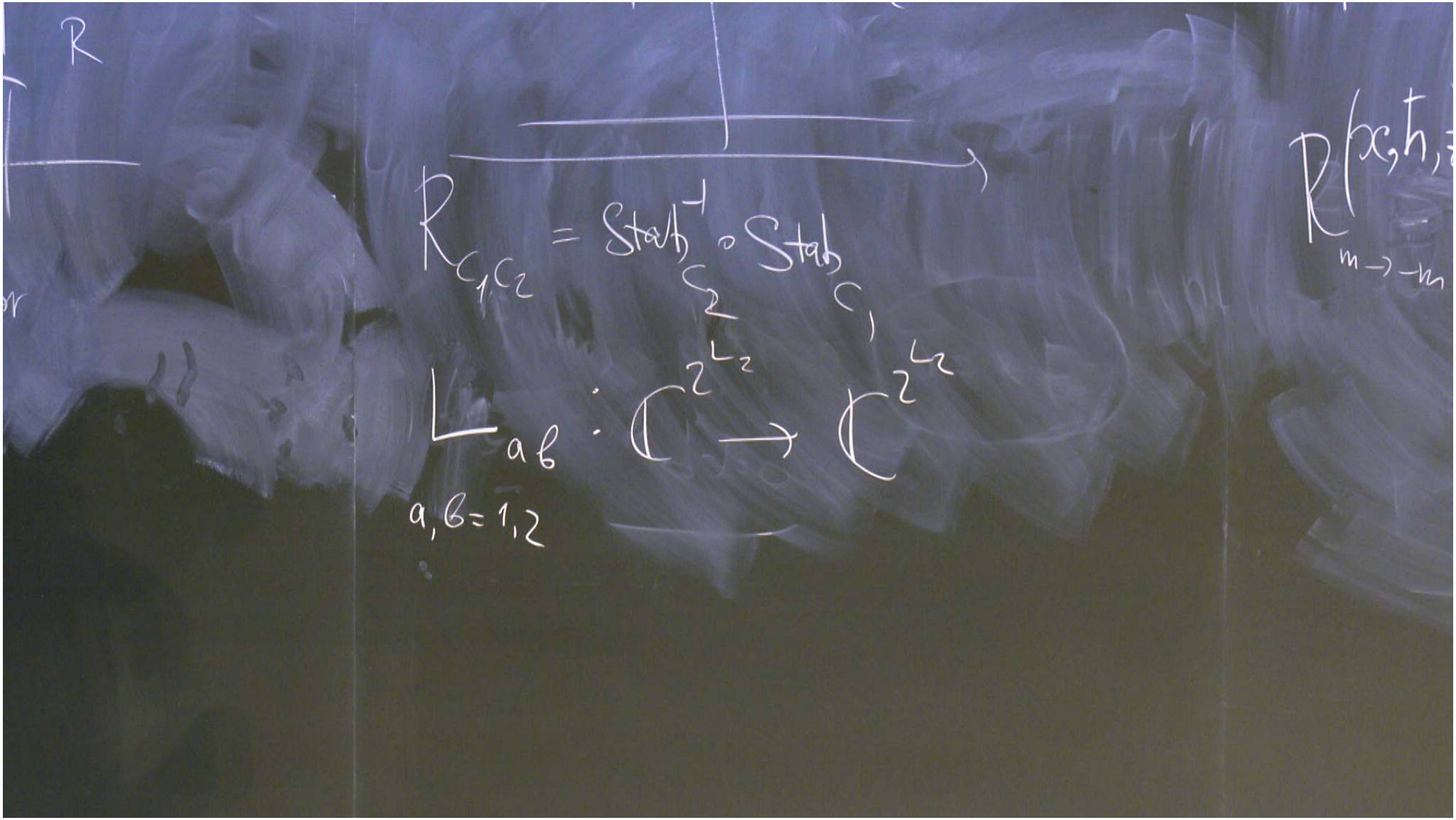
$\dim V(L) = 2^L$

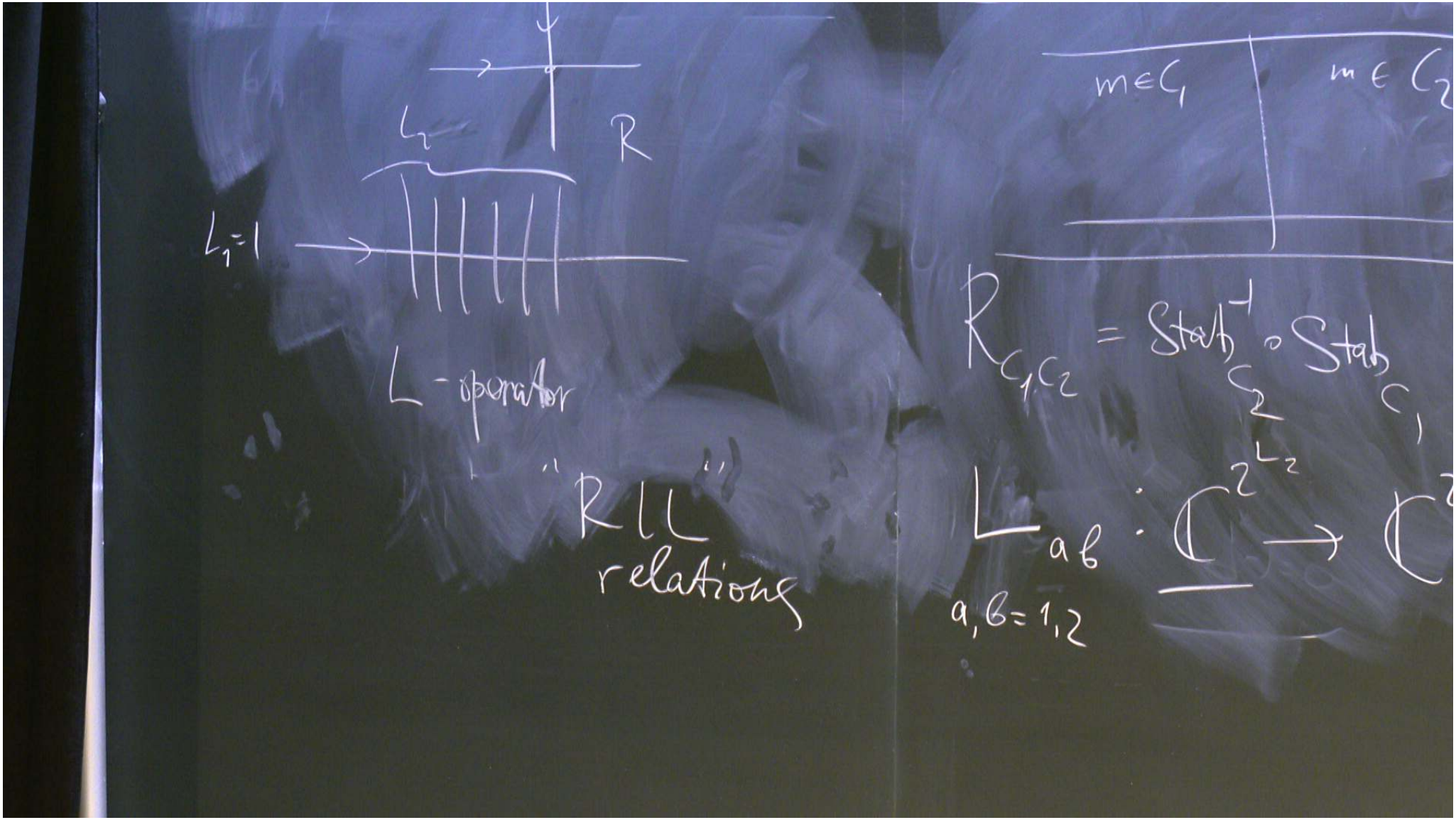
$T(L_1 + L_2)^m = T(L_1) \otimes T$

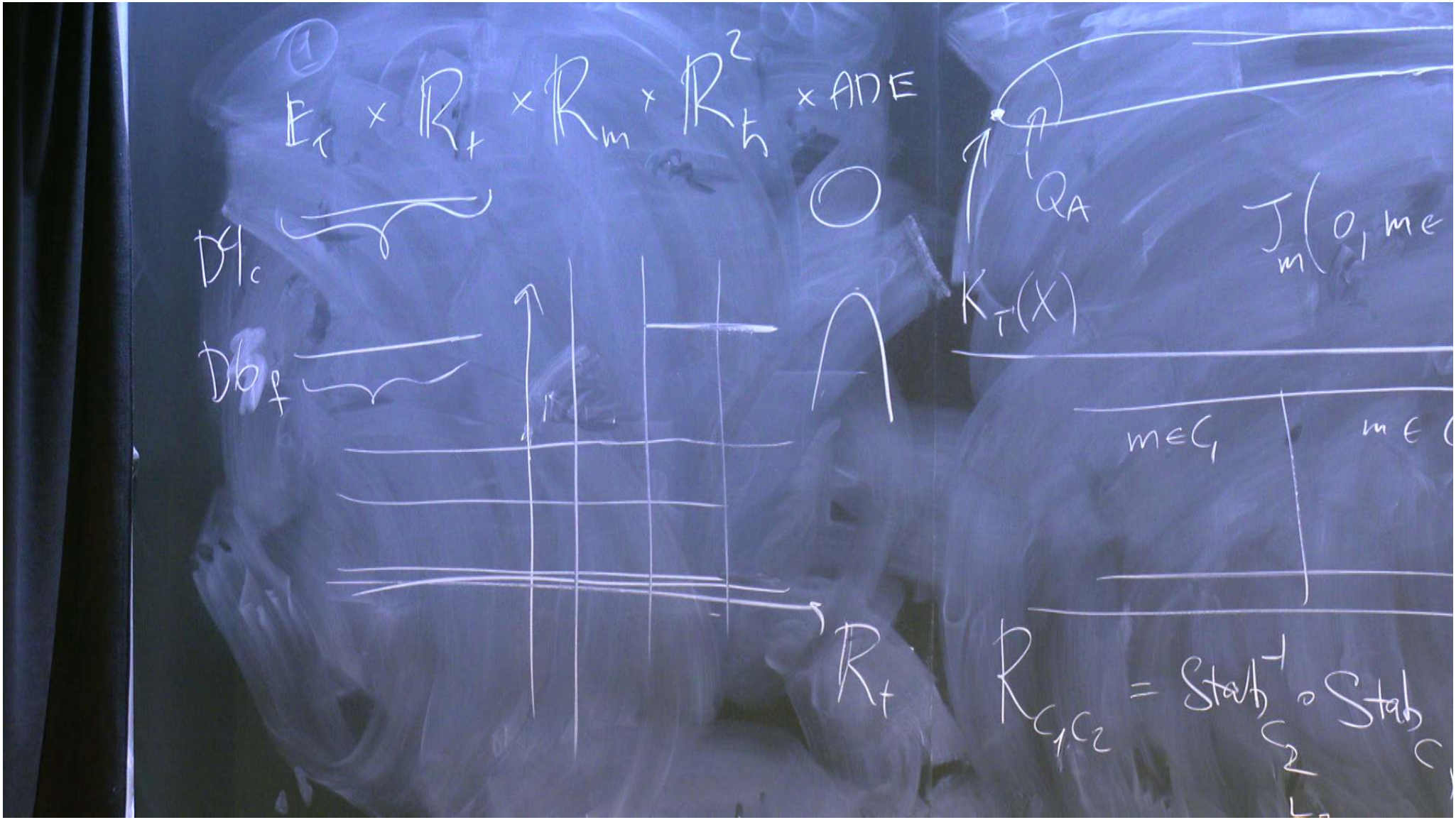
$U(1), \; m$

$R : V(L_1) \otimes$


L - operator
$R_{C_1C_2} = \mathrm{Stab}_{C_2}^{-1} \circ \mathrm{Stab}_{C_1}$
L_{ab}
$a,b = 1,2$


R
$R_{c_1,c_2} = \mathrm{Stab}_{c_2}^{-1} \circ \mathrm{Stab}_{c_1}$
$L_{ab} : \mathbb{C}^{2^{L_2}} \to \mathbb{C}^{2^{L_2}}$
$a, b = 1, 2$
$m \to -m$


L_1
R
$L_1 = 1$
L - operator
"RLL"
relations
$m \in C_1$
$m \in C_2$
$R_{C_1,C_2} = \mathrm{Stab}_{C_2}^{-1} \circ \mathrm{Stab}_{C_1}$
$L_{ab} : \mathbb{C}^{2^{L_2}} \to$
$a, b = 1, 2$


$E_\tau \times \mathbb{R}_t \times \mathbb{R}_m \times \mathbb{R}_h^2 \times ADE$
Q_A
$J_m(Q, m \in$
$K_T(X)$
$m \in C_1$
$m \in$
$\mathbb{R}_t$
$R_{C_1,C_2} = Stab_{C_2}^{-1} \circ Stab_{C}$