Title: 3d Theories and Twists II

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$$
\begin{tikzcd}
\text{Physics} \arrow[r, leftrightarrow, "\text{duality}"] \arrow[d, Rightarrow] \arrow[dr, phantom, "\Downarrow"] & \text{Physics}' \arrow[d, Rightarrow] \\
\text{Math} \arrow[r, leftrightarrow, "\text{conjecture}"'] & \text{Math}'
\end{tikzcd}
$$

← want to factor through a mathematical field/string thy

$\Downarrow$ $\Downarrow$ $\Downarrow$ $\leftarrow$ want to factor through a mathematical field/string thy

$$\text{Math} \xleftrightarrow[\text{conjecture}]{} \text{Math}'$$

4d $\mathcal{N}=4$ $\circlearrowright$ EM $\Rightarrow$ Langlands

4d $\mathcal{N}=2$ $\circlearrowright$ $\Rightarrow$ SW geometry, Hitchin sys

3d $\mathcal{N}=4$ $\circlearrowright$ 3d MS $\Rightarrow$ 3d HMS?

Knot homology

symplectic duality

top twists

Math $\xleftrightarrow{\text{conjecture}}$ Math   field/string thy

4d $\mathcal{N}=4$ $\circlearrowright$ EM $\Rightarrow$ Langlands   top

4d $\mathcal{N}=2$ $\circlearrowright$ $\Rightarrow$ SW geometry, Hitchin sys

3d $\mathcal{N}=4$ $\circlearrowright$ 3d MS $\Rightarrow$ 3d HMS?
knot homology
symplectic duality   top twists

elliptic coh (HK)   HT twist

3d $\mathcal{N}=2$

conjecture

$4d\ \mathcal{N}=4$ $\circlearrowright$ EM $\Rightarrow$ Langlands — top

$4d\ \mathcal{N}=2$ $\circlearrowleft \circlearrowright \circlearrowleft$ $\Rightarrow$ SW geometry, Hitchin sys — HT — $T[\Sigma]$

$3d\ \mathcal{N}=4$ $\circlearrowright$ 3d MS $\Rightarrow$ 3d HMS?
Knot homology
symplectic duality
— top twists

elliptic coh (HK) — HT twist

$3d\ \mathcal{N}=2$ $\circlearrowleft \circlearrowright \circlearrowleft$ — $T[M^3]$ — HT
3d-3d corresp.

$$\text{Math} \xleftrightarrow[\text{conjecture}]{} \text{Math}'$$

field/string thy

4d $\mathcal{N}=4$ ↻ EM ⇒ Langlands — top

4d $\mathcal{N}=2$ ↻ ↻ ↻ ⇒ SW geometry, Hitchin sys — HT

$T[$

3d $\mathcal{N}=4$ ↻ 3d MS ⇒ 3d HMS?

knot homology

symplectic duality

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elliptic coh (HK) — HT twist

3d $\mathcal{N}=2$ ↻ ↻ ↻

$T[M^3]$ — HT

3d-3d corresp.

3d $\mathcal{N}=2$

Spinors in 3d are 2-dim^l

SUSY alg is generated by two:

$$Q_\alpha, \quad \bar{Q}_\alpha \qquad \alpha = 1,2$$

$$\{Q_\alpha, \bar{Q}_\beta\} = \sigma^\mu_{\alpha\beta} \partial_\mu$$

$$S \otimes S \longrightarrow V$$

$\mathbb{C}_z \times \mathbb{R}_t$

$$\{Q_+, \bar{Q}_+\} = \partial_{\bar{z}}$$

$$\{Q_-, \bar{Q}_-\} = \partial_z$$

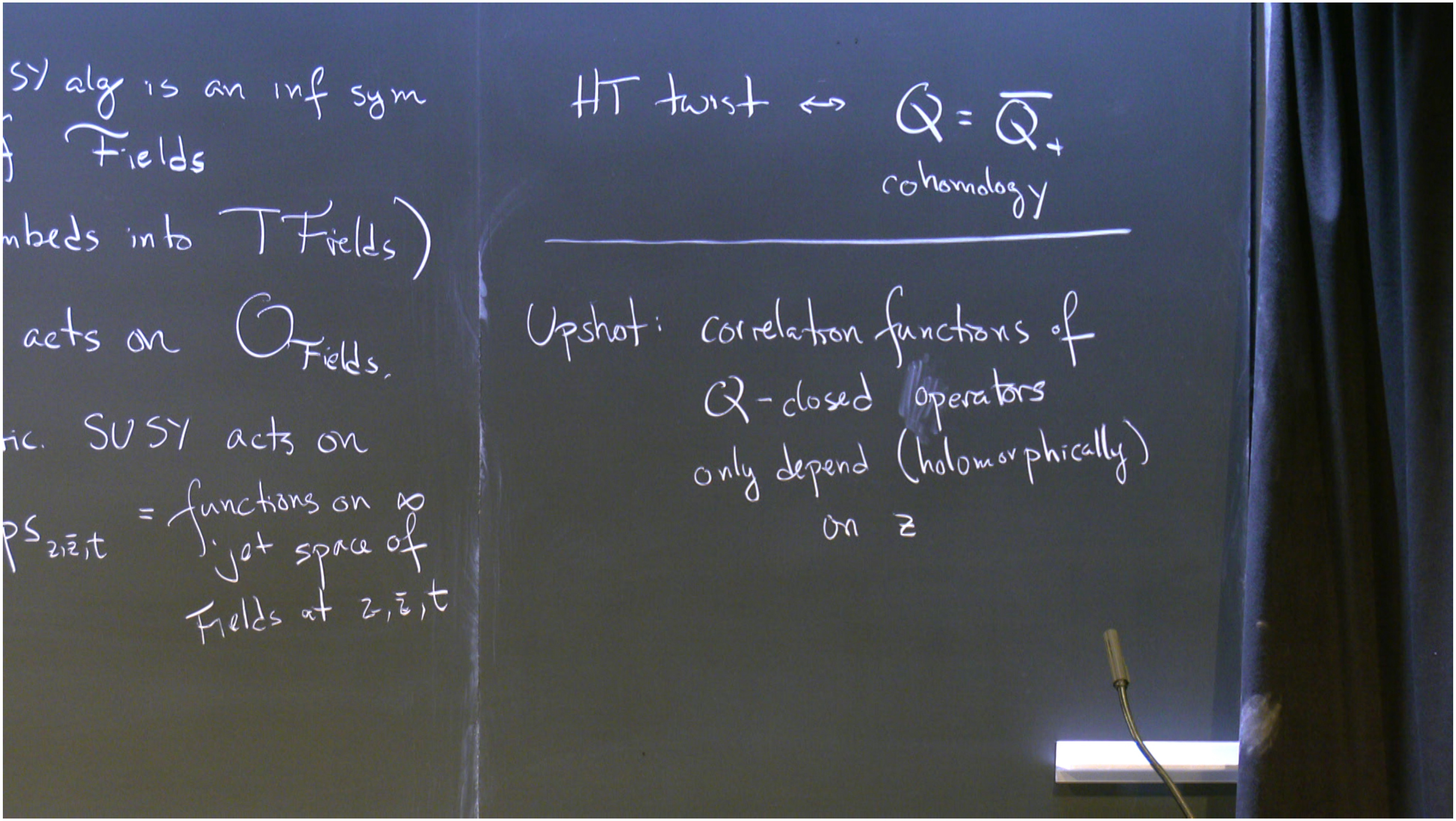
$$\{Q_+, \bar{Q}_-\} = \{Q_-, \bar{Q}_+\} = \partial_t$$

SUSY alg is an inf sym of Fields (embeds into T Fields)

thus acts on $\mathcal{O}_{\text{Fields}}$.

In ptic. SUSY acts on

$\text{Ops}_{z,\bar{z},t}$ = functions on ∞ jet space of Fields at $z, \bar{z}, t$


SY alg is an inf sym
f Fields
nbeds into T Fields)
acts on $\mathcal{O}_{Fields}$.
ic. SUSY acts on
$ps_{z,\bar{z},t}$ = functions on ∞ jet space of Fields at $z, \bar{z}, t$
HT twist $\leftrightarrow$ $Q = \bar{Q}_+$ cohomology
Upshot: correlation functions of Q-closed operators only depend (holomorphically) on z

Q-coh of

no singularities

- Ops becomes a commutative vertex algebra

$$\langle \varphi_1(z_1, \bar{z}_1, t_1)\, \varphi_2(z_2, \bar{z}_2, t_2) \ldots \rangle$$

- Ops becomes a commutative, shifted Poisson vertex algebra

$$\langle \varphi_1(z_1, \bar{z}_1, t_1)\, \varphi_2(z_2, \bar{z}_2, t_2) \ldots \rangle$$

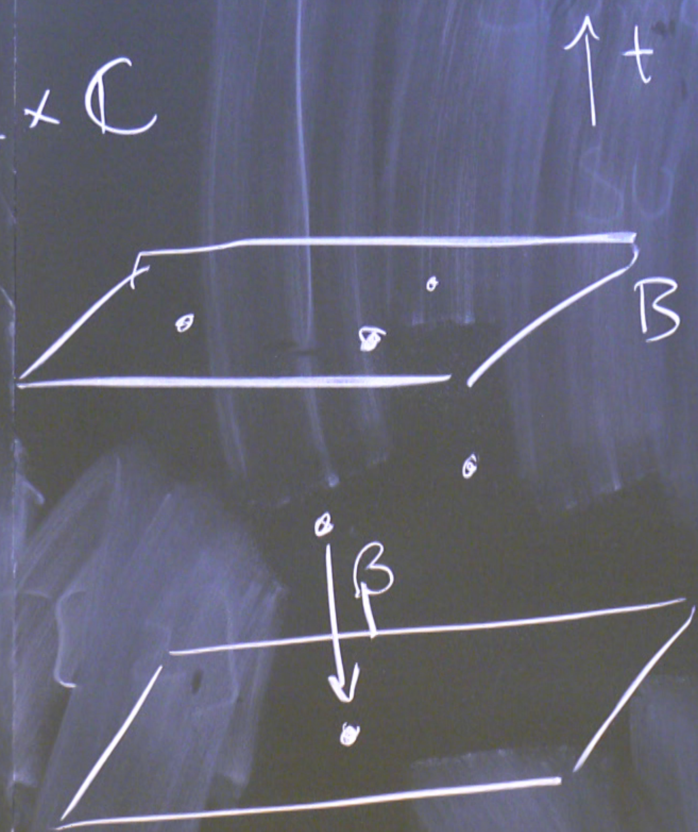
$$\lim_{z \to w} \varphi_1(z)\, \varphi_2(w) \quad \text{exists}$$

$\leftarrow \bullet \varphi_2$

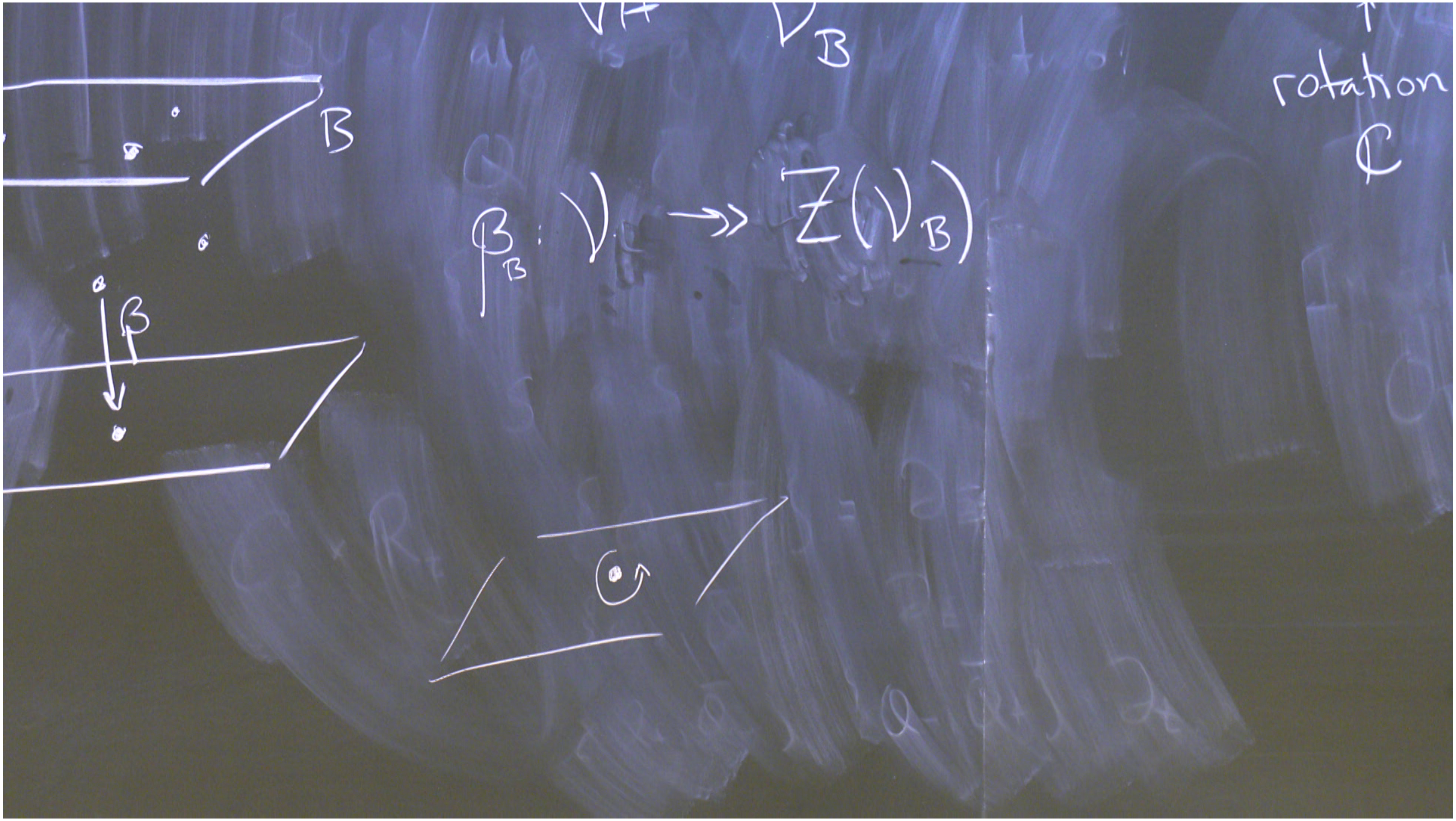
$t \uparrow$ $\varphi_1 \bullet \rightarrow$

• Given a bdy condition $\mathcal{B}$, get a potentially non-comm'l VA $\mathcal{V}_{\mathcal{B}}$

$\mathbb{R}_+ \times \mathbb{C}$



$$\beta_{\mathcal{B}} : \mathcal{V} \twoheadrightarrow Z(\mathcal{V}_{\mathcal{B}})$$


B
β
β_B : V ↠ Z(V_B)
rotation
ℂ

omm

(V_B)

• V & V_B are graded by

spin $\mathbb{Z}$ & R-charge $\mathbb{Z}$

↑

rotation in $\mathbb{C}$

HT

Upshot

comm

spin $\mathbb{Z}$ & $\boxed{\text{R-charge } \mathbb{Z}}$

$\uparrow$

rotation in $\mathbb{C}$

cohomological

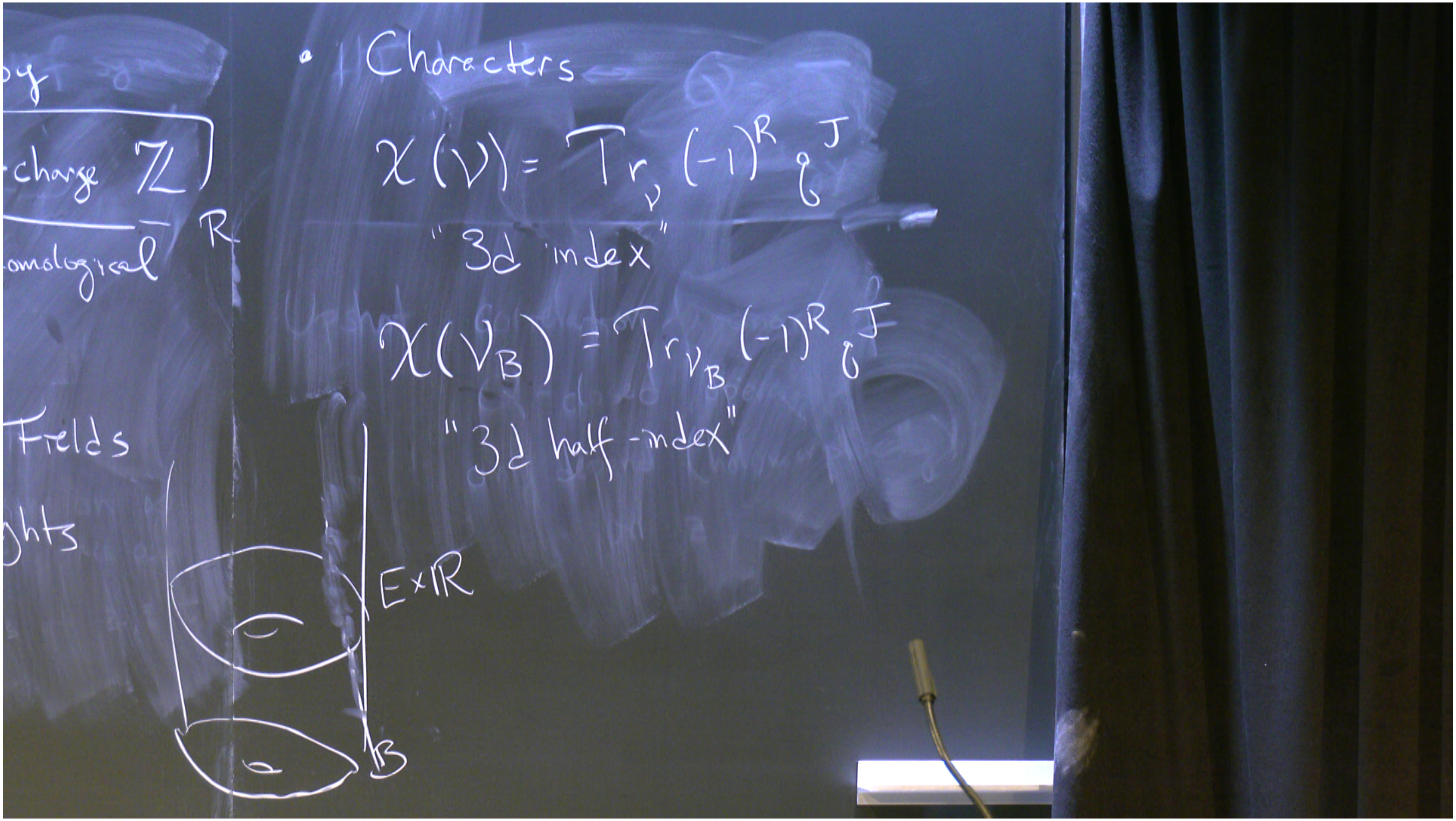
(V_B)

Upshot

additional $\mathbb{C}^*$ sym of Fields

s.t. Q_α $\bar{Q}_\alpha$ have weights

-1 $\quad$ 1


Characters
χ(V) = Tr_V (-1)^R q^J
"3d index"
χ(V_B) = Tr_{V_B} (-1)^R q^J
"3d half-index"
E×ℝ
B
charge ℤ
R
omological
Fields
ghts

• Line operators form a dg category $\mathcal{C}$

L'

$\in \mathrm{Hom}(L, L')$

L

composition

z

$\mathbb{1}$

$V = \mathrm{End}_{\mathcal{C}}(\mathbb{1})$

expect $HH_*^{spin}(\mathcal{C}) \overset{?}{=}$ equiv^t quantum K-thy

Line operators form a dg category $\mathcal{C}$

Given a

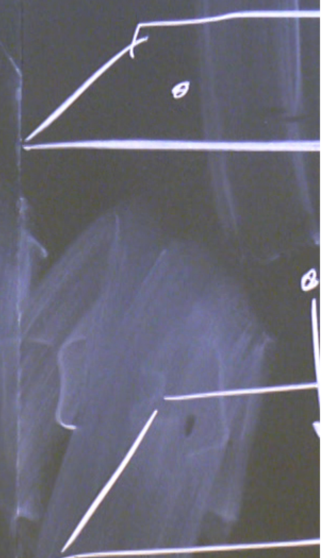
L'

$\in \mathrm{Hom}(L, L')$

L

composition

$\mathbb{R}_+ \times \mathbb{C}$



z

$\mathbb{1}$ $\quad V = \mathrm{End}_{\mathcal{C}}(\mathbb{1})$

expect $HH_*^{spin}(\mathcal{C}) \overset{?}{=}$ equiv^t quantum K-thy (X)

$$X = V/\!/^{stab} G$$

$$\lim_{z \to w} \Big\langle \underset{z}{\Big|} \quad \underset{w}{\Big|} \Big\rangle \, ?$$

Corr f^{n}'s of Configs that are t-independent

are also indep^{t} of z

y (X)

Configs that are t-independent

are also indep^t of z

ops preserve $\overline{Q}_+$ $= Q$ and Q_-

$$\{\overline{Q}_+, Q_-\} = \partial_t$$

$$\widetilde{Q} = \overline{Q}_+ + Q_- \quad \widetilde{Q}^2 = \partial_t$$

$$\partial_z = \{\widetilde{Q}, \overline{Q}_-\}$$

x) ops preserve Q_+ and Q_-

$$\{\bar{Q}_+, Q_-\} = \partial_t$$

$$\tilde{Q} = \bar{Q}_+ + Q_- \qquad \tilde{Q}^2 = \partial_t$$

$$\partial_{\bar{z}} = \{\tilde{Q}, \bar{Q}_-\}.$$

Expect: meromorphic $\otimes$ category

$$\left\langle L \,\Big|_{z} \quad \Big|_{w}\!\!\overset{L''}{\underset{L'}{\bullet}} \right\rangle = \sum_n \frac{1}{(z-w)^n} \left\langle \Big|\!\overset{L\otimes L''}{\underset{L\otimes L'}{\bullet\, a^{(n)}}} \right\rangle$$

Examples come from 3d $\mathcal{N}=2$ gauge thys

data: $\underset{\text{red}}{G}$, $\underset{\text{cx}}{V}$, $W: V/\!\!/G \to \mathbb{C}$, $k \in H^4(BG)$

$z \qquad w$

Examples come from 3d $\mathcal{N}=2$ gauge thys

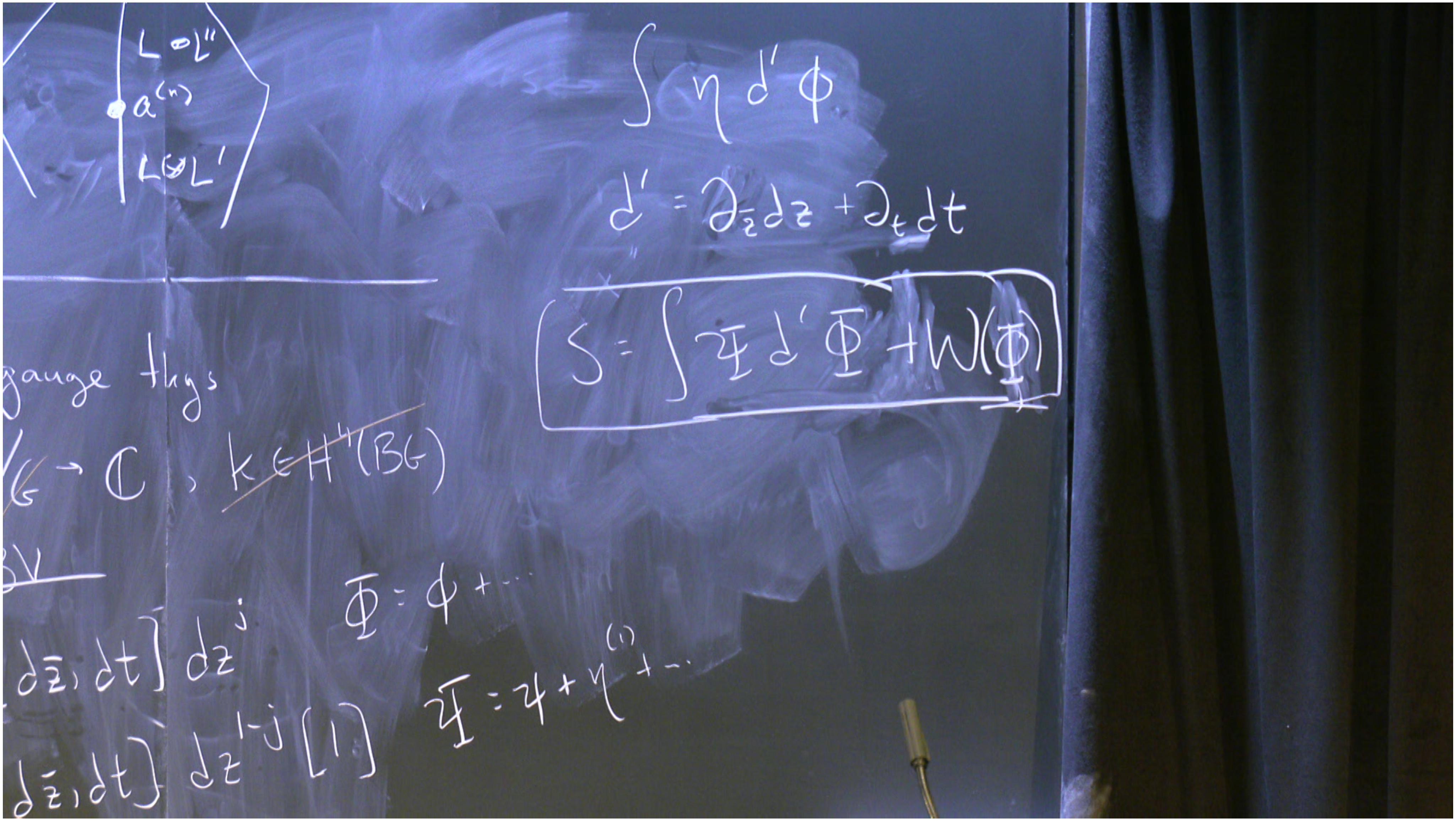
data: $\underset{\text{red}}{G}$, $\underset{\text{cx}}{V}$, $W : V/G \to \mathbb{C}$, $\cancel{k \in H^4(BG)}$

<u>twisted BV</u>

V: chiral multiplets valued in V

$$\Phi \in V[d\bar{z}, dt]\, dz^{j}$$

$$\Psi \in V^{*}[d\bar{z}, dt]\, dz^{1-j}[1]$$


$L \otimes L''$
$a^{(n)}$
$L \otimes L'$
$\int \eta \, d' \phi$
$d' = \partial_{\bar z} d\bar z + \partial_t dt$
$S = \int \Psi \, d' \Phi + W(\Phi)$
gauge thys
$\mathbb{C}$, $k \in H^4(BG)$
BV
$\Phi = \phi + \cdots$
$\Psi = \psi + \eta^{(1)} + \cdots$

BV rules

Fields is -1 shifted symp^c

$\mathcal{O}_{\text{Fields}}$ has a $+1$ Poisson bracket

$$\{\Phi(x), \Phi(x')\}_{BV} = \delta^{(3)}(x-x')$$

$$Q = \{S, -\}$$

$$Q^2 = 0 \iff \{S, S\} = 0$$