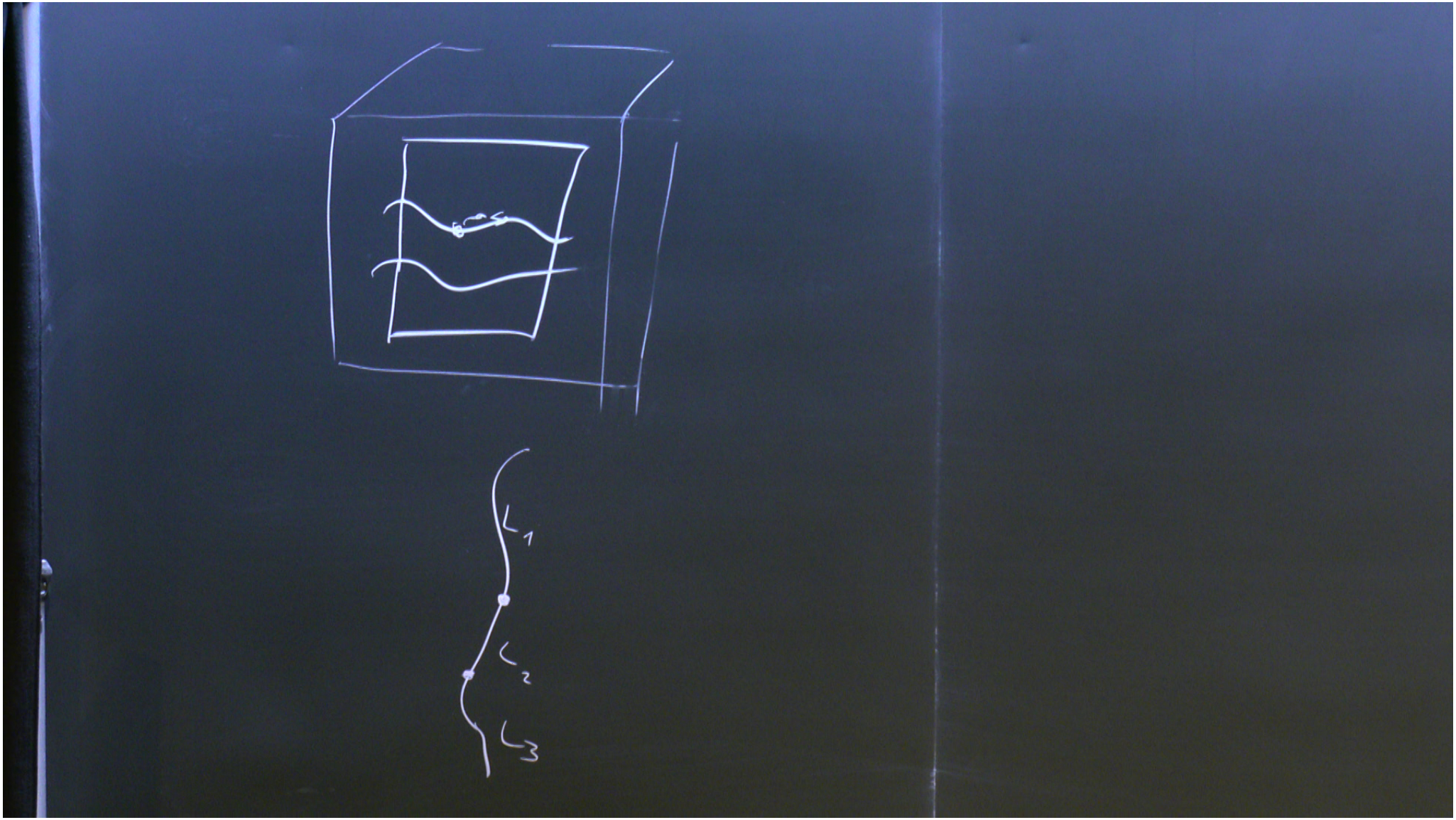
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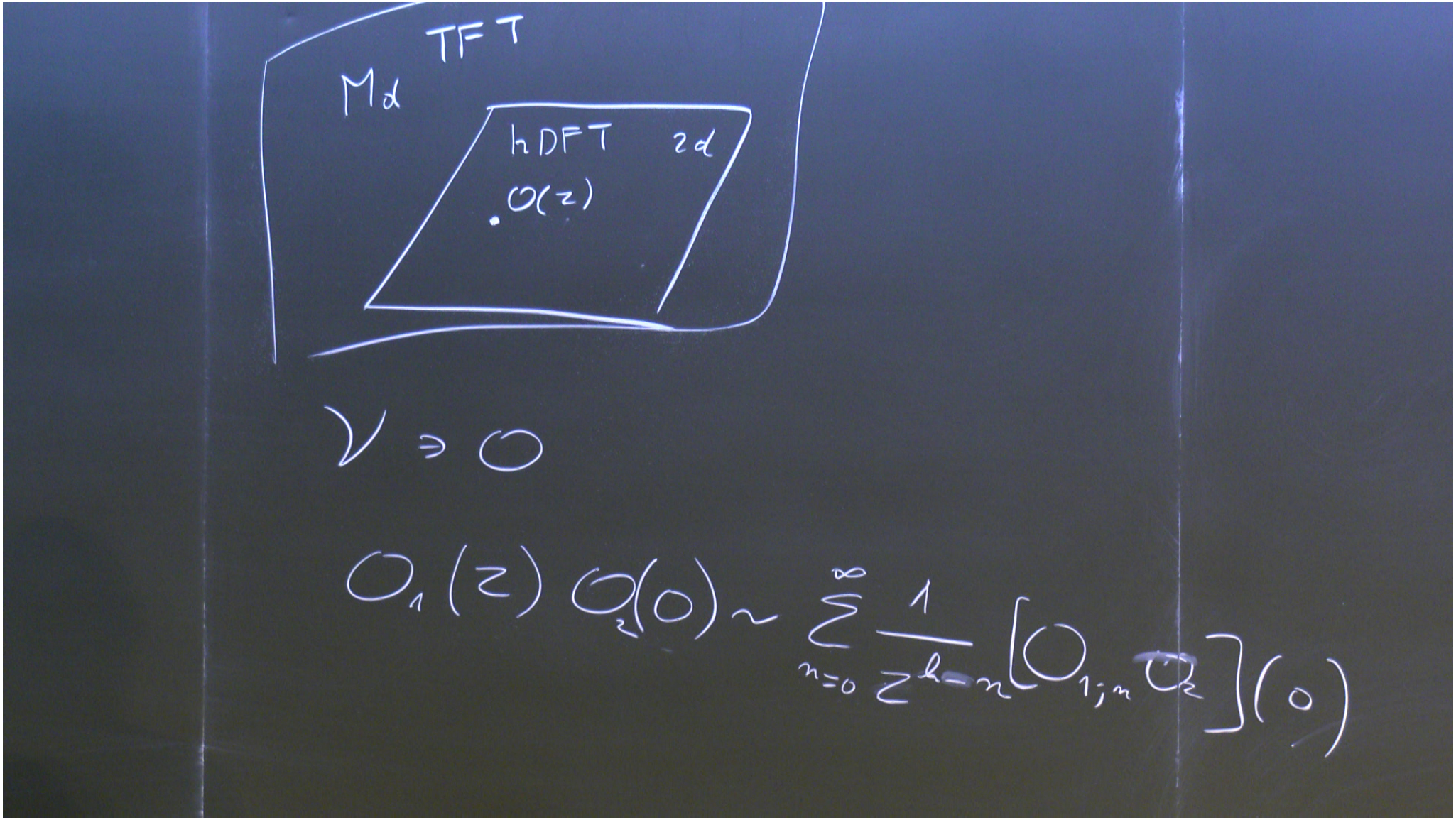
Speakers: Davide Gaiotto

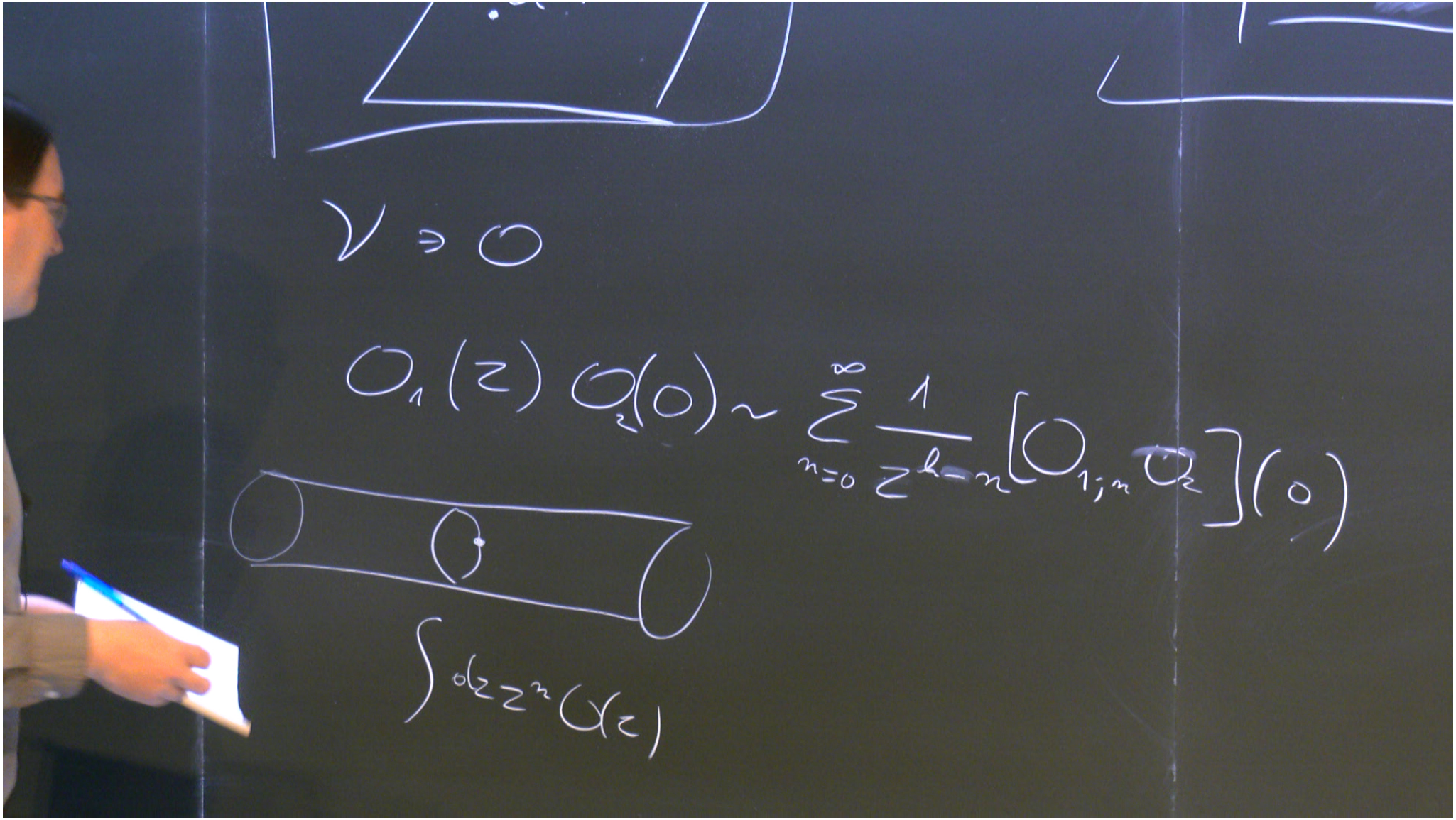
Collection: QFT for Mathematicians 2022

Date: June 21, 2022 - 3:30 PM

URL: https://pirsa.org/22060068

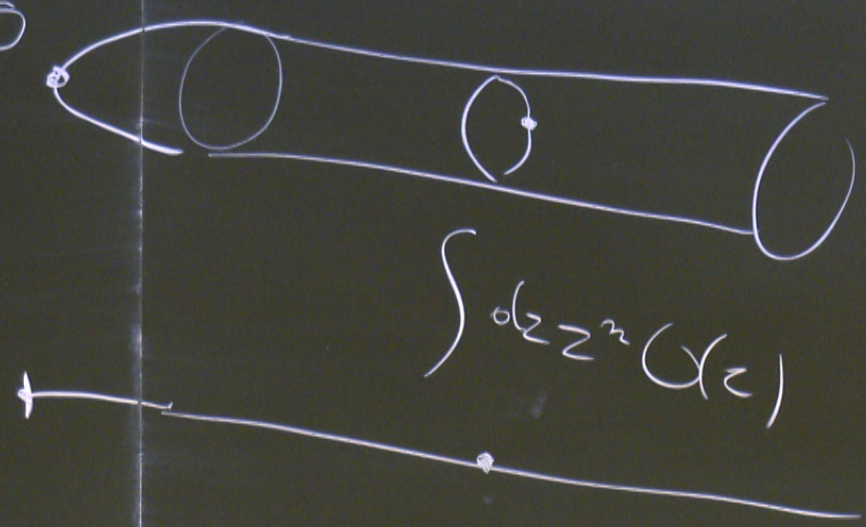

L1
L2
L3


TFT
Md
hDFT
2d
O(z)
V ⇒ O
$O_1(z)\,O_2(0) \sim \sum_{n=0}^{\infty} \frac{1}{z^{h-n}} [O_{1;n}\,O_2](0)$


$\nu > 0$
$O_1(z)\,O_2(0) \sim \sum_{n=0}^{\infty} \frac{1}{z^{h-n}} [O_{1;n}\,O_2](0)$
$\int_0 dz\, z^n \alpha(z)$

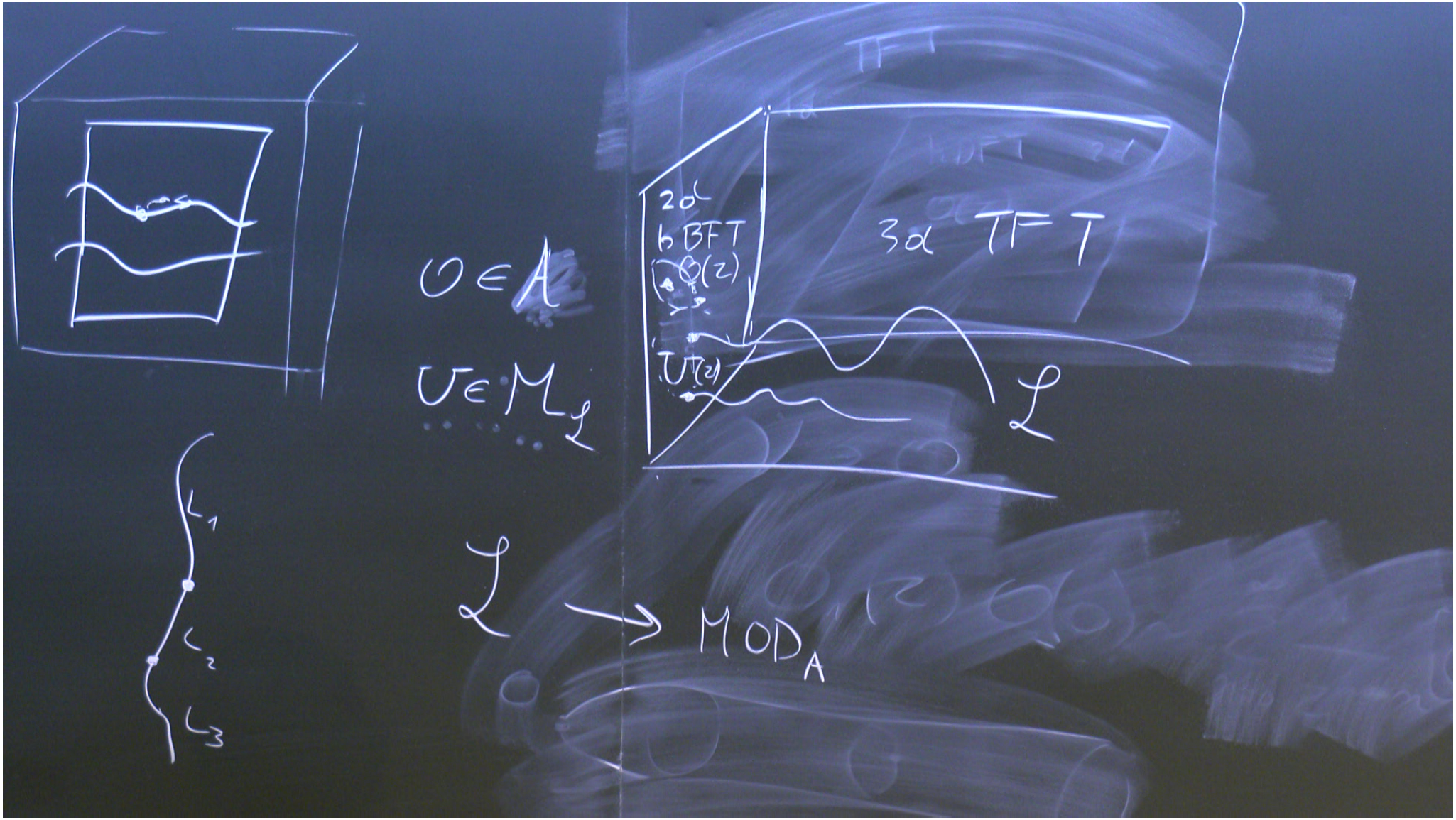
$V \Rightarrow 0$

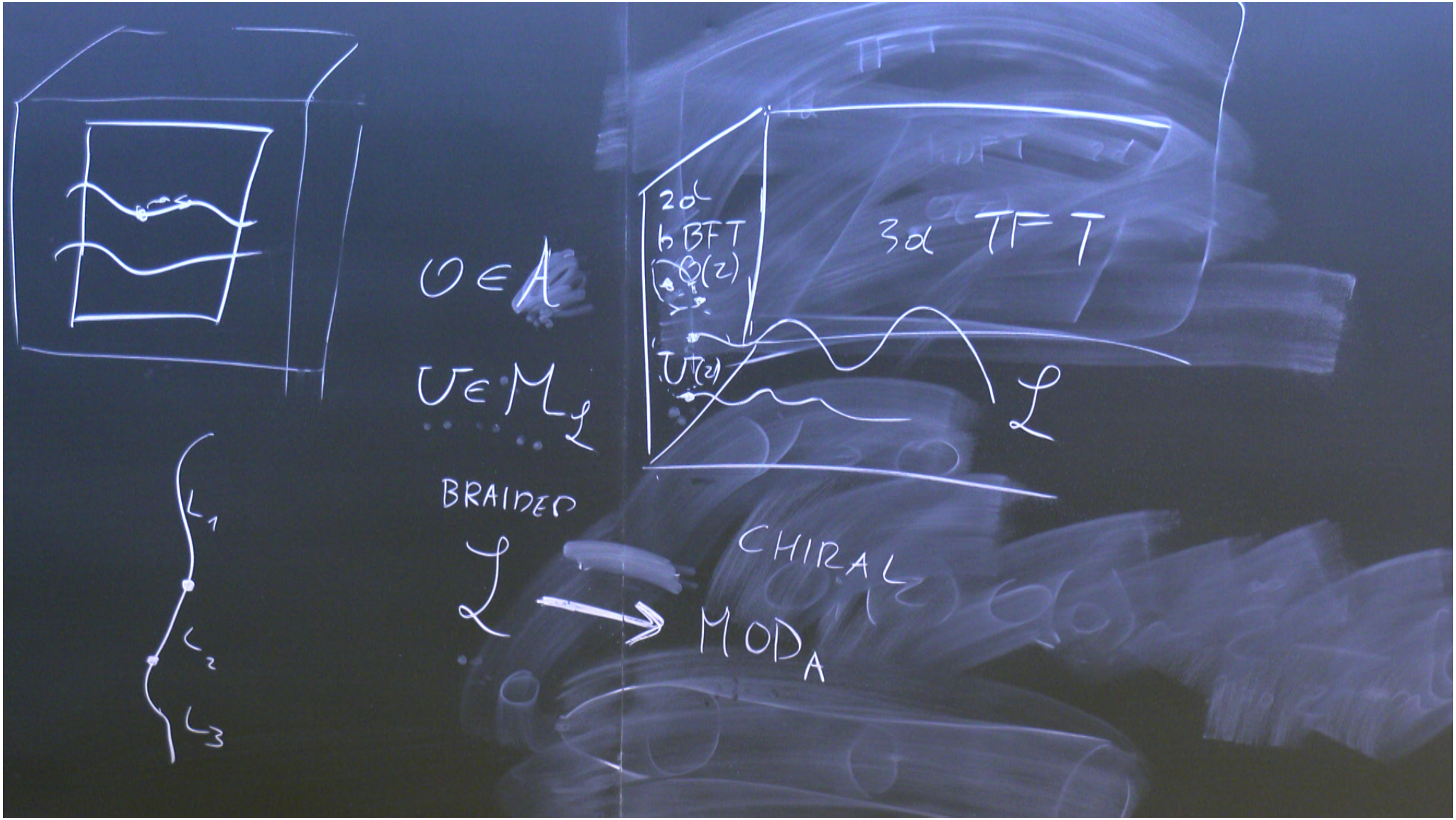
$$\mathcal{O}_1(z)\,\mathcal{O}_2(0) \sim \sum_{n=0}^{\infty} \frac{1}{z^{h-n}} \left[\mathcal{O}_{1;n}\,\mathcal{O}_2\right](0)$$

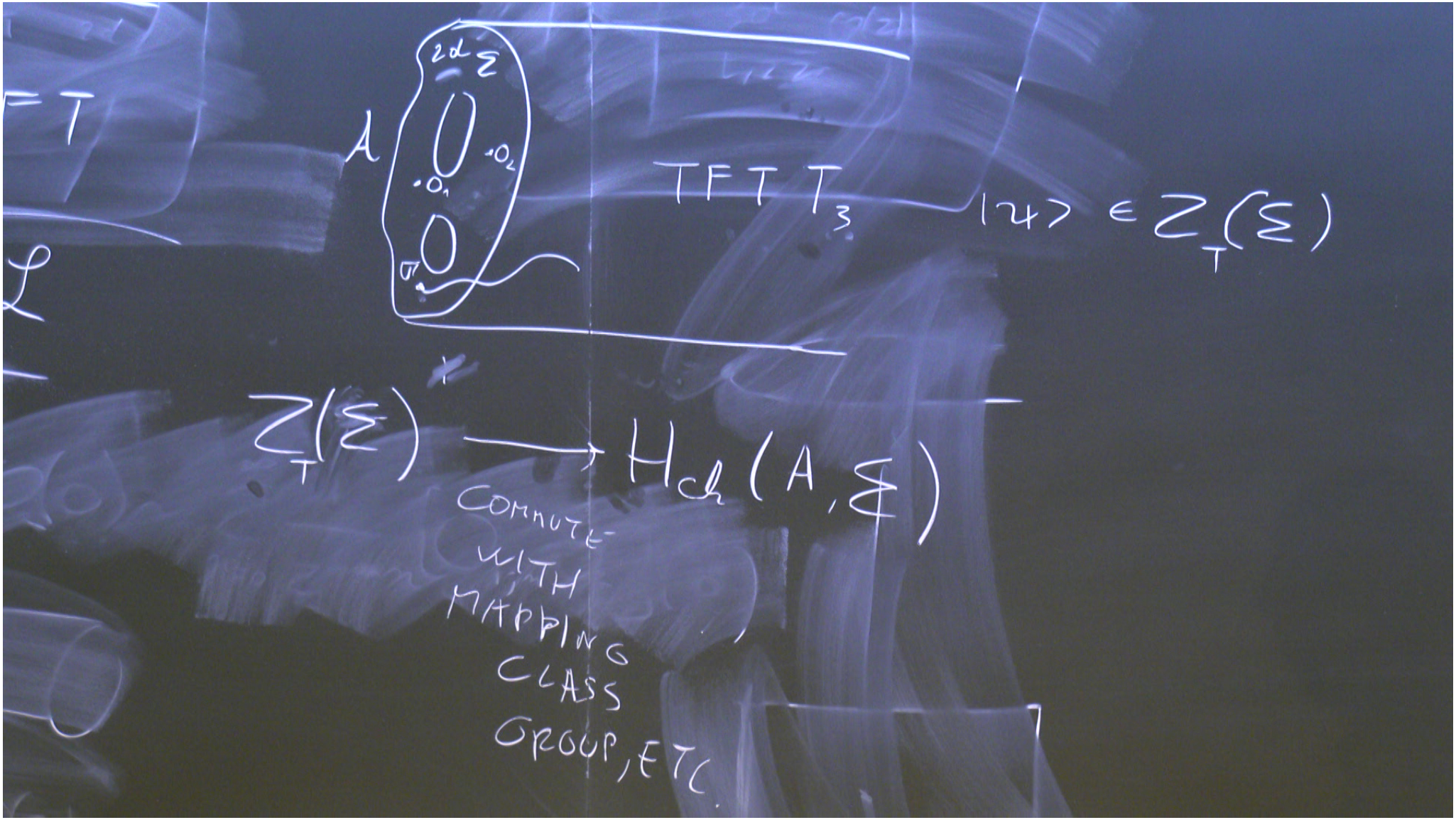


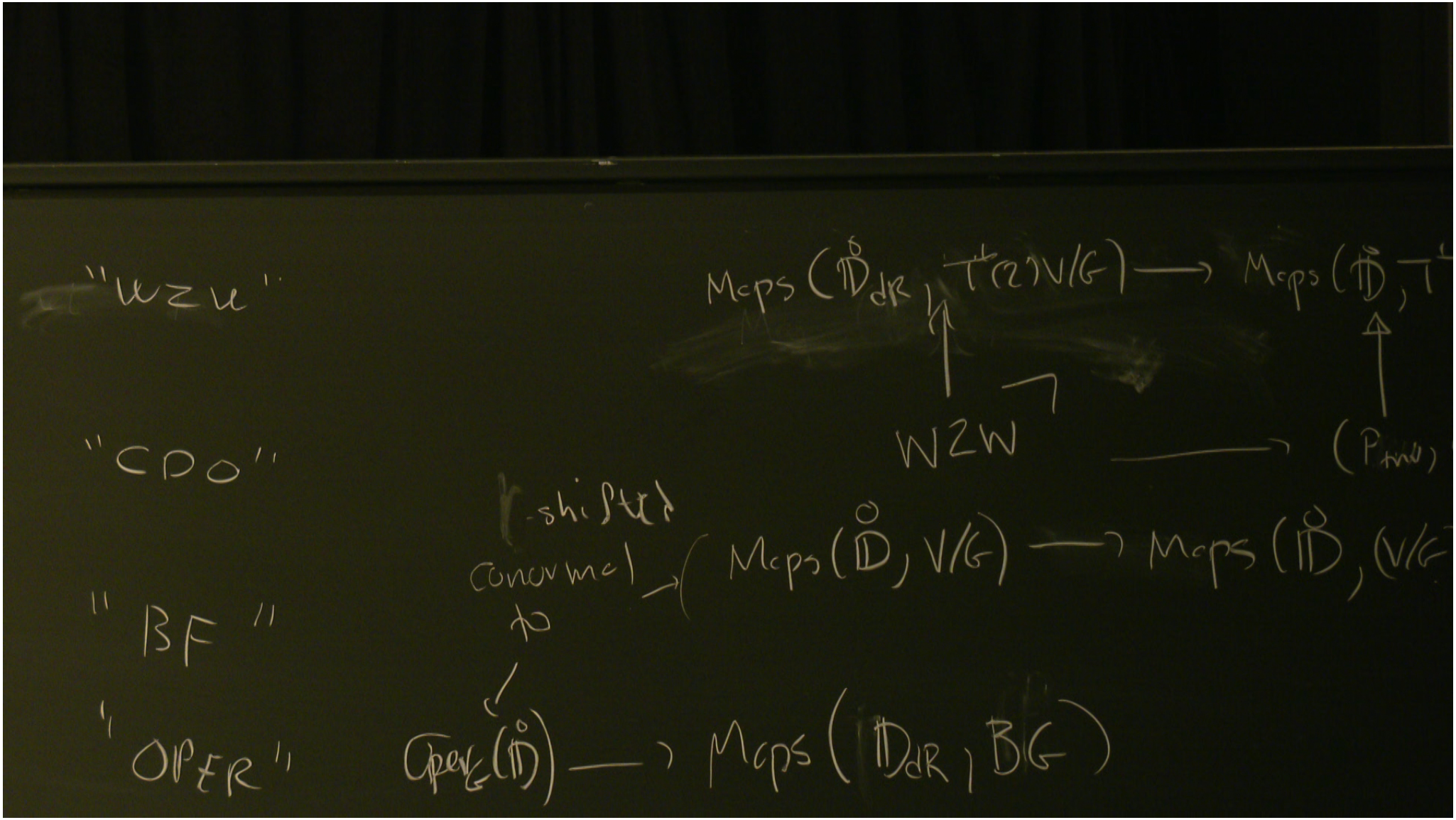
$$\sum_{=0}^{\infty} \frac{1}{z^{h-n}} [O_{1;n} O_2](0)$$

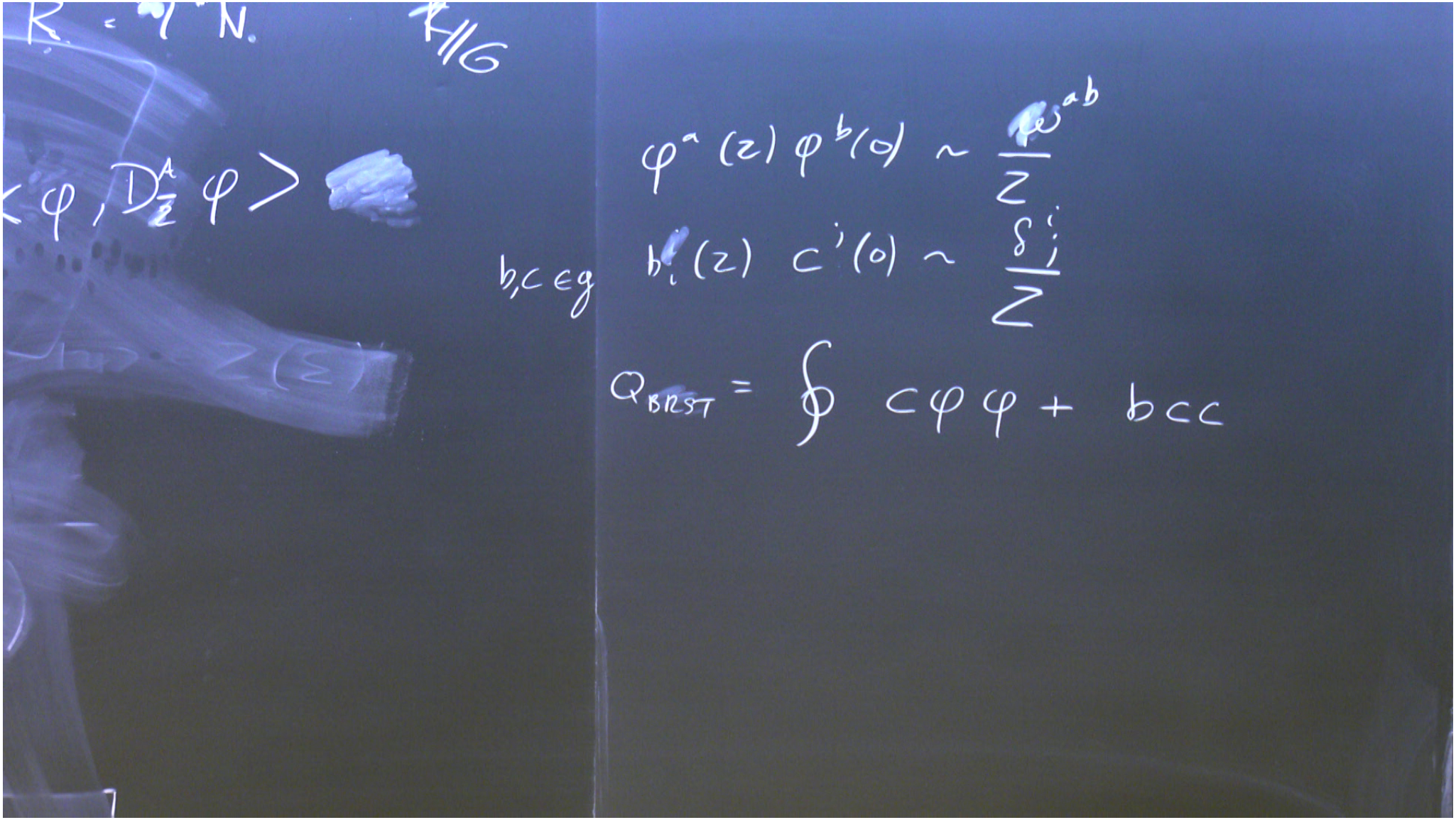


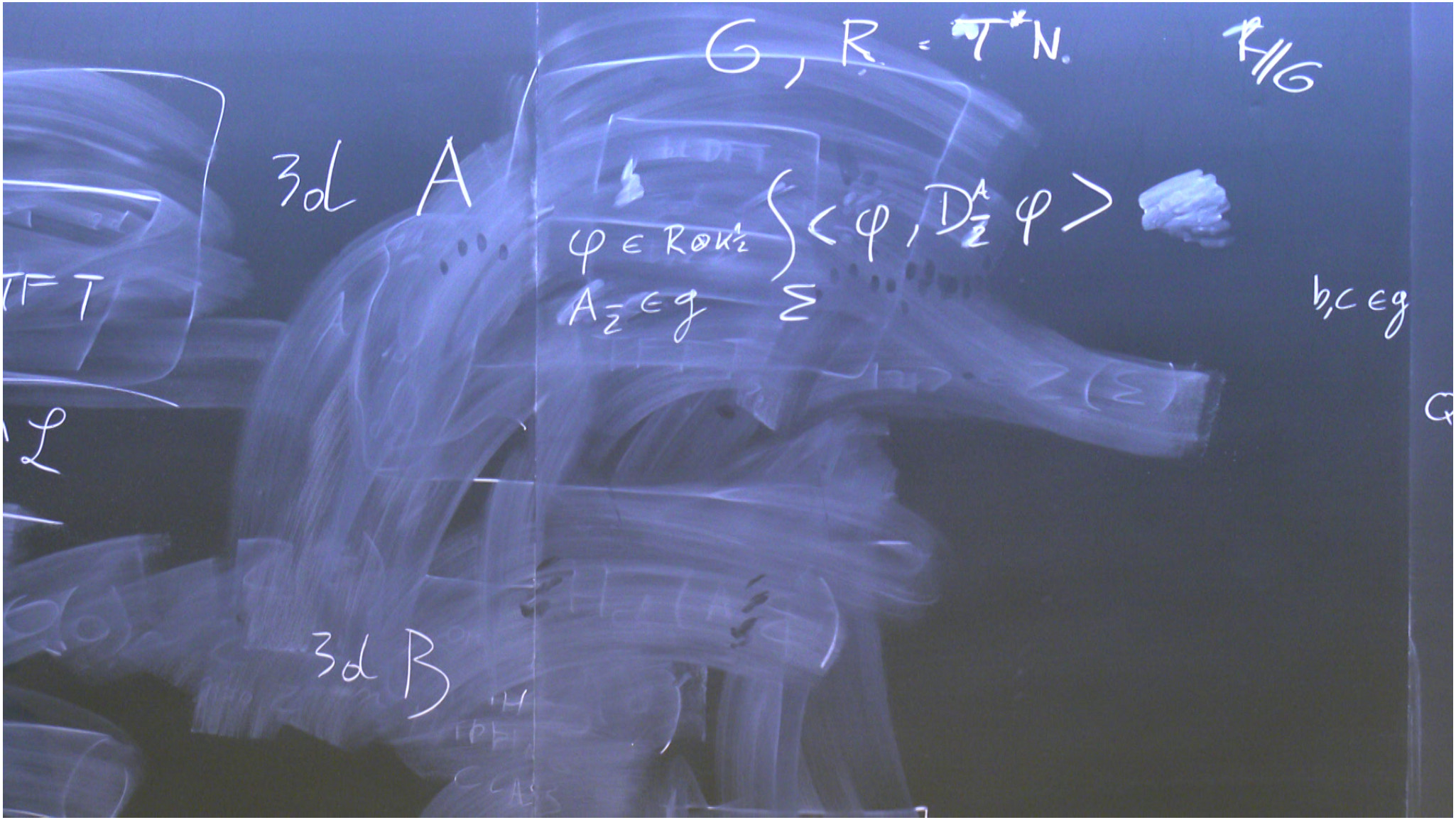

TFT
2d
bBFT
3d TFT
$\mathcal{L}$
$\mathcal{L} \to MOD_A$

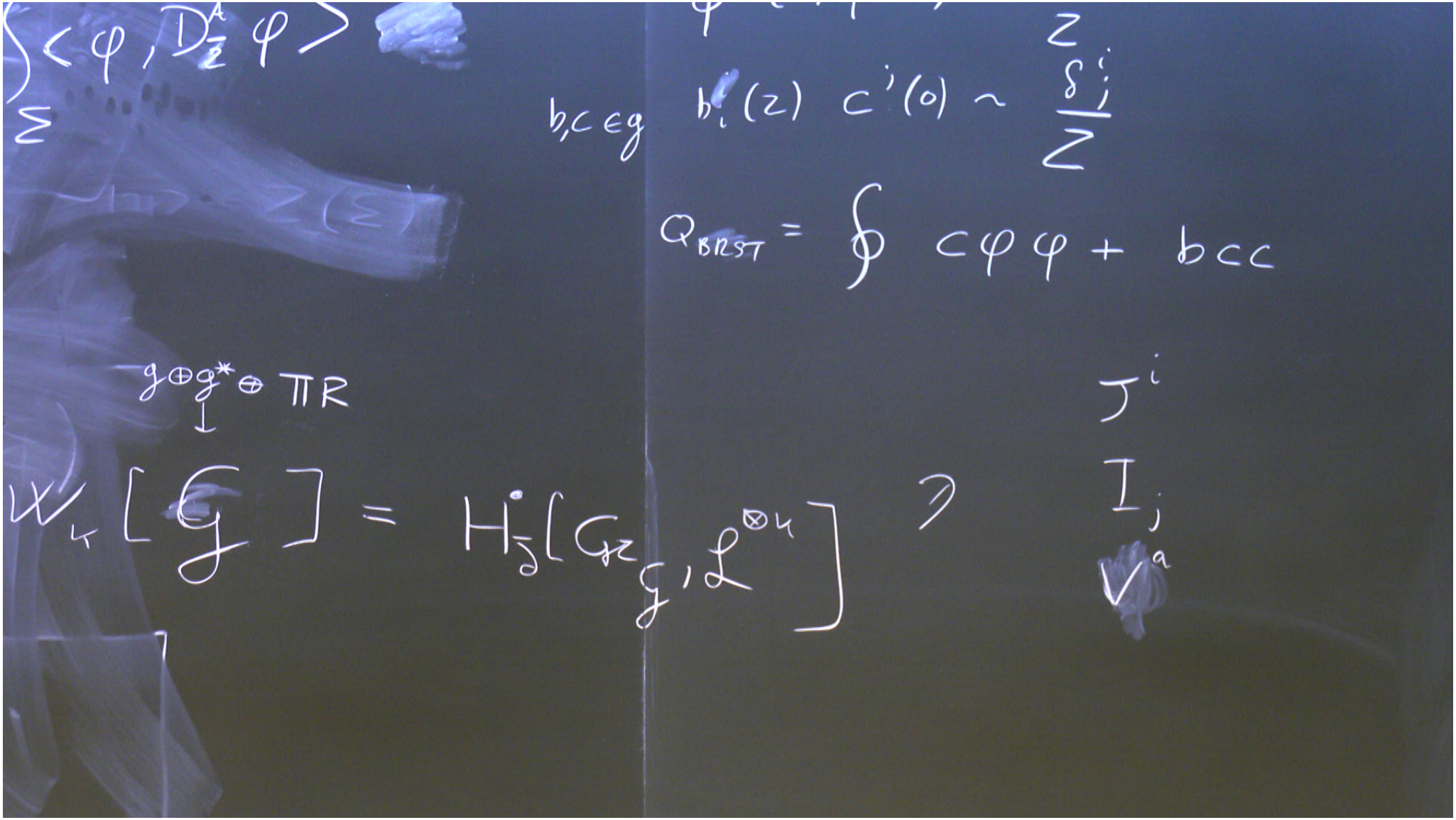

TFT
2d
bBFT
3d TFT
$\mathcal{O} \in \mathcal{A}$
$U \in M_{\mathcal{L}}$
BRAIDED
$\mathcal{L}$
CHIRAL
$\mathcal{L}$
MOD_A
L_1
L_2
L_3

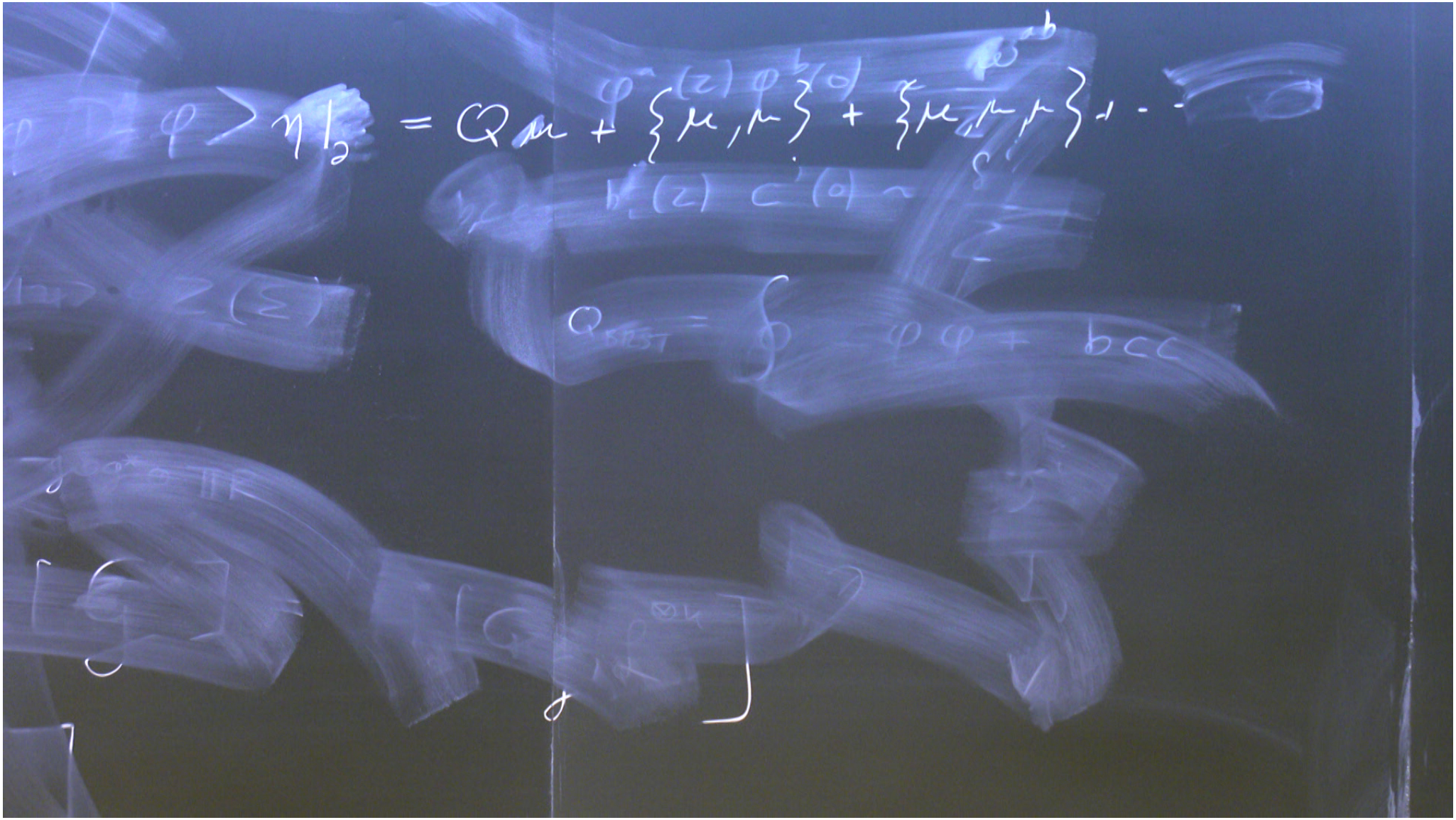

$\mathcal{A}$
TFT T_3
$|\psi\rangle \in Z_T(\Sigma)$
$Z_T(\Sigma)$
$H_{ch}(A, \Sigma)$
COMMUTE WITH MAPPING CLASS GROUP, ETC.

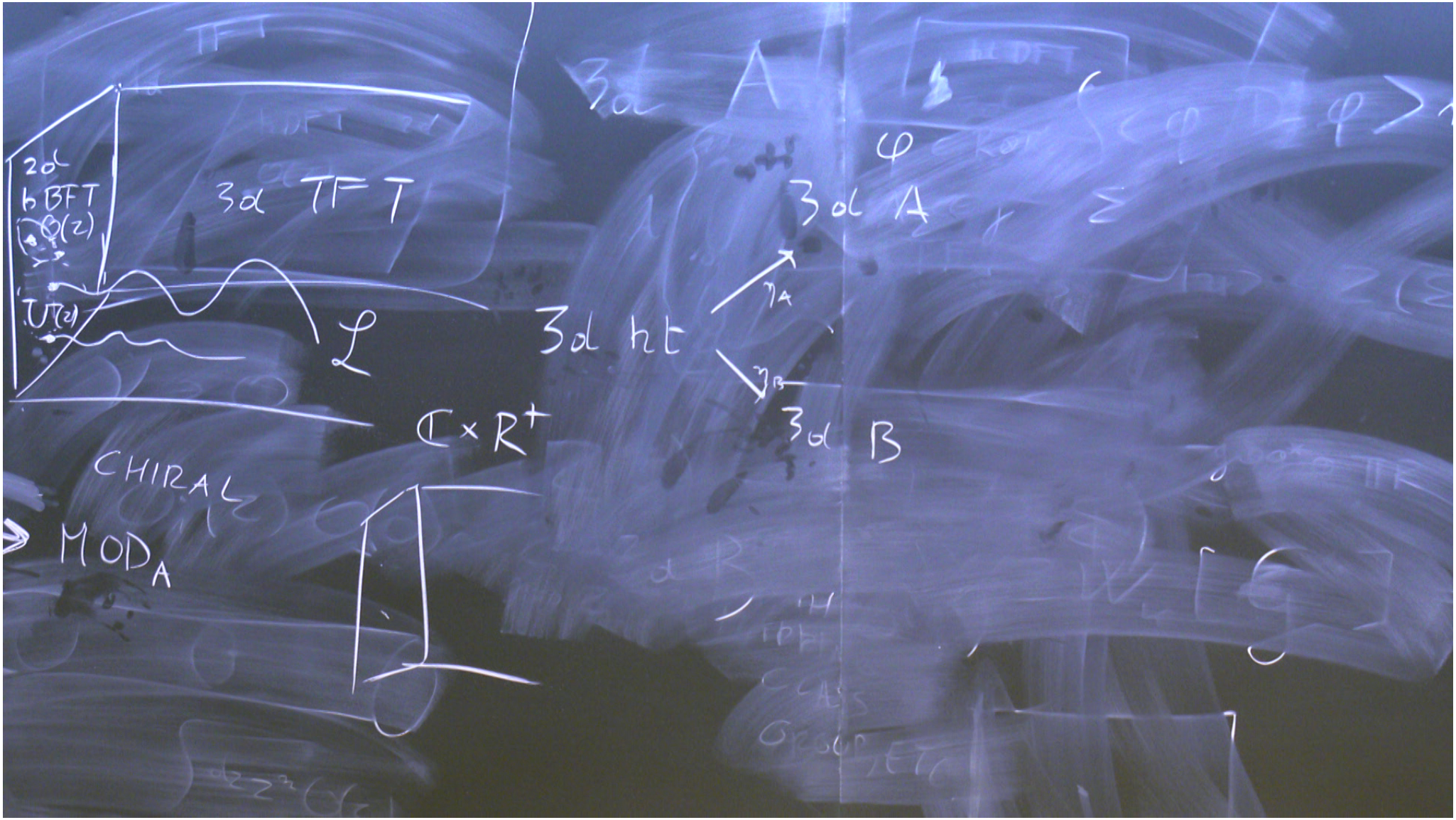

"WZW"
"CDO"
"BF"
"OPER"
Maps(D_dR, T(2)V/G)
Maps(D, T
W2W
(P_trivG)
shifted
conormal
to
Maps(D, V/G)
Maps(D, (V/G
Gper_G(D)
Maps(D_dR, BG)


$\varphi^a(z)\,\varphi^b(0) \sim \frac{\omega^{ab}}{z}$
$b,c \in g$
$b_i(z)\,c^j(0) \sim \frac{\delta_i^j}{z}$
$Q_{BRST} = \oint c\varphi\varphi + bcc$

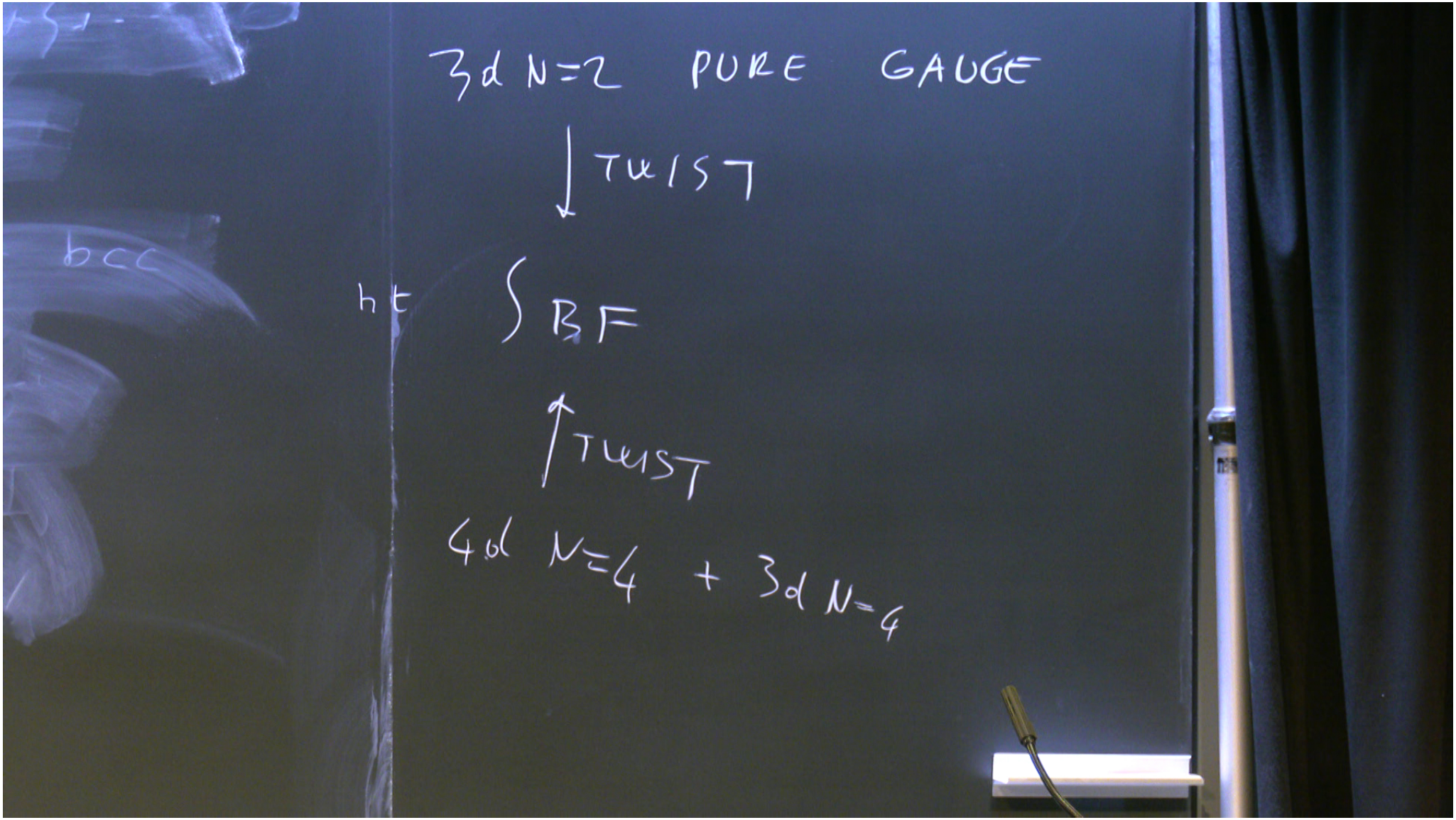

G, R = T*N.
R//G
3d A
$\varphi \in \mathrm{Ker}\, D_{\frac{1}{2}}$
$A_{\frac{1}{2}} \in g$
$\int \langle \varphi, D^{A}_{\frac{1}{2}} \varphi \rangle$
Σ
b, c ∈ g
3d B

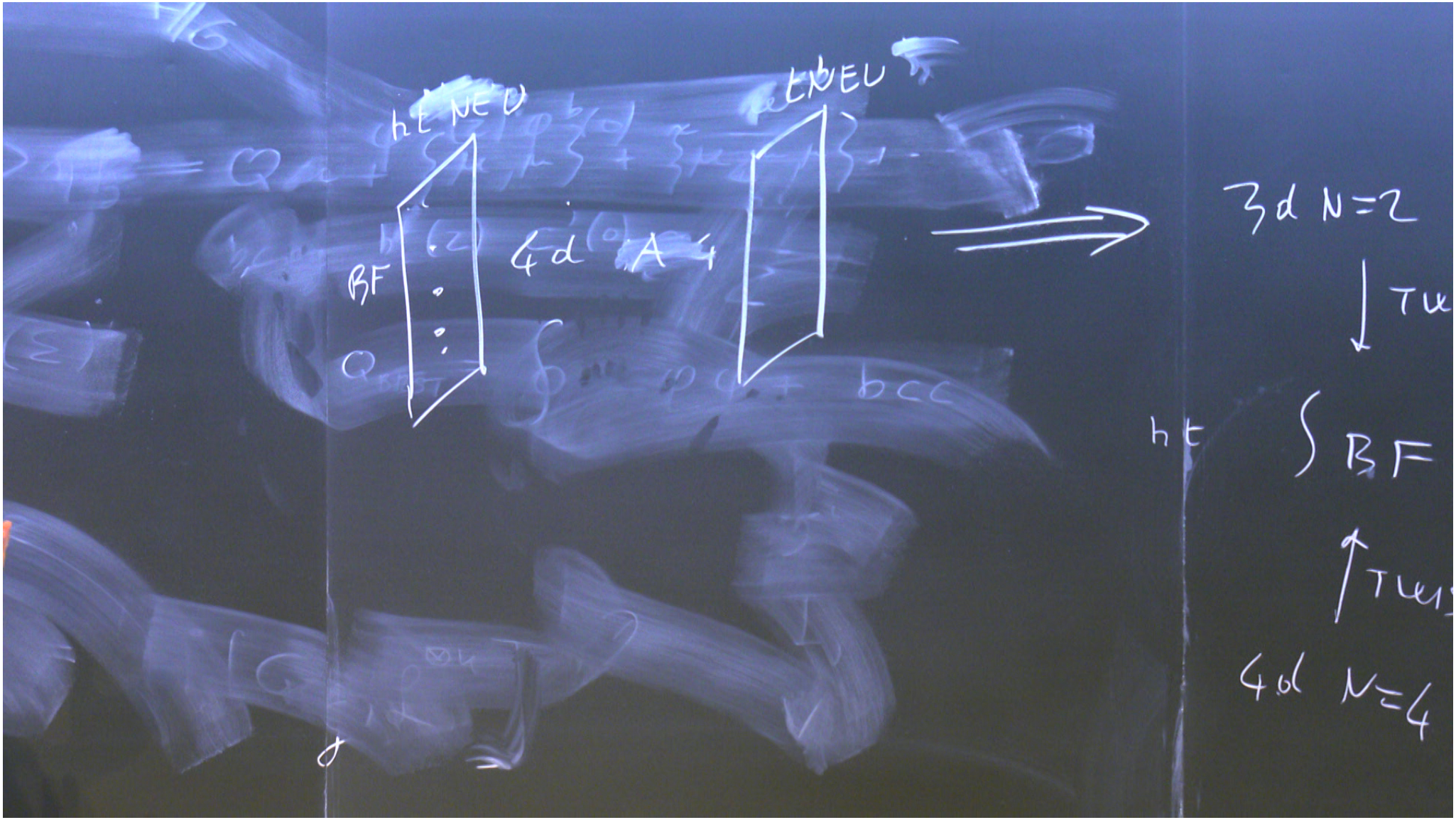

$\int \langle \varphi, D_z^A \varphi \rangle$
Σ
$b, c \in g$
$b_i(z)\ c^j(0) \sim \frac{\delta_i^j}{z}$
$Q_{BRST} = \oint c\varphi\varphi + bcc$
$g \oplus g^* \oplus TR$
$W_4[G] = H_3[G_{\mathbb{Z}}, \mathcal{L}^{\otimes 4}]$
$\supset$
J^i
I_j
V^a



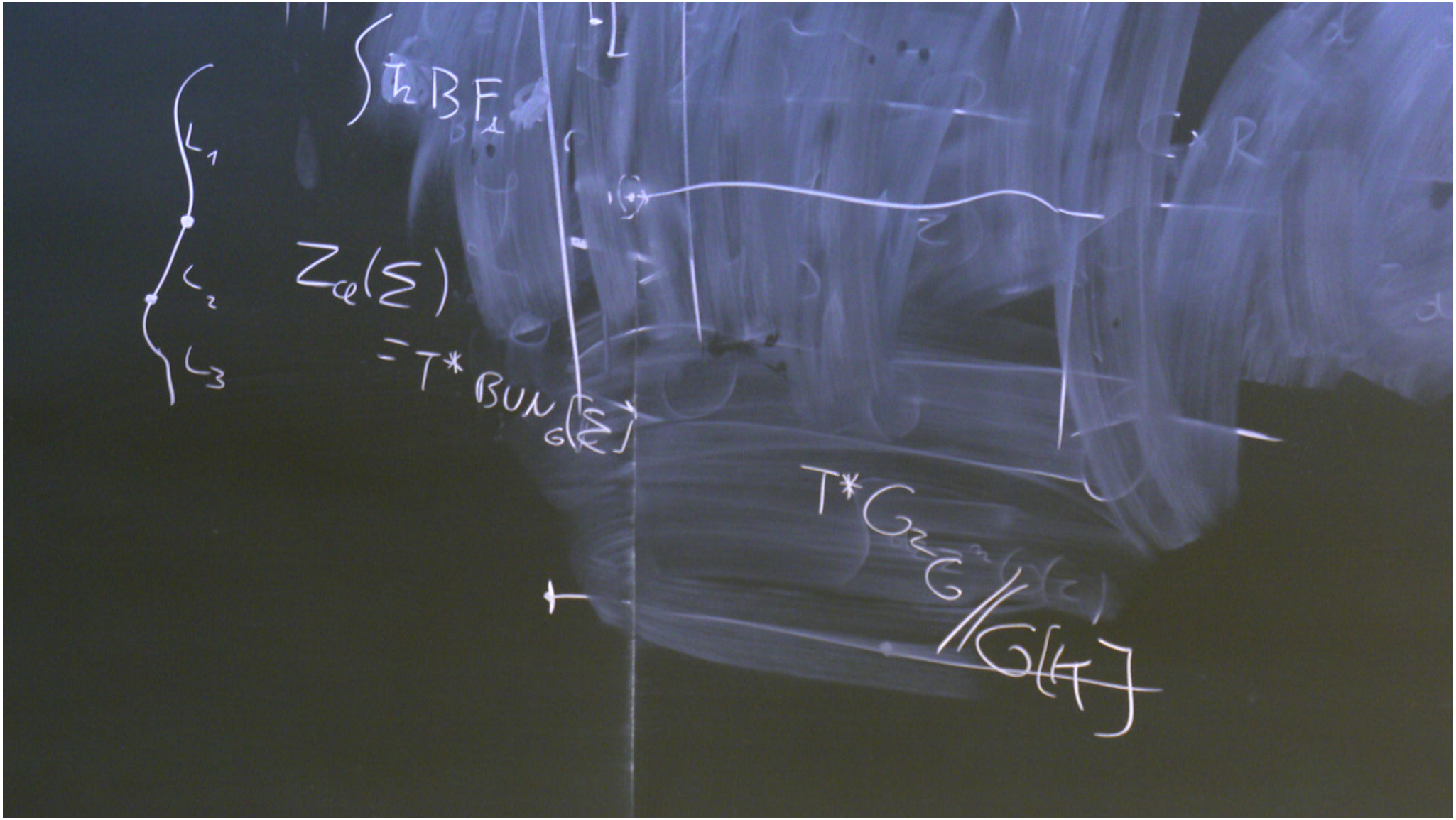

TFT
3d A
2d
bBFT
3d TFT
3d A
3d bt
3d B
CHIRAL
MOD A

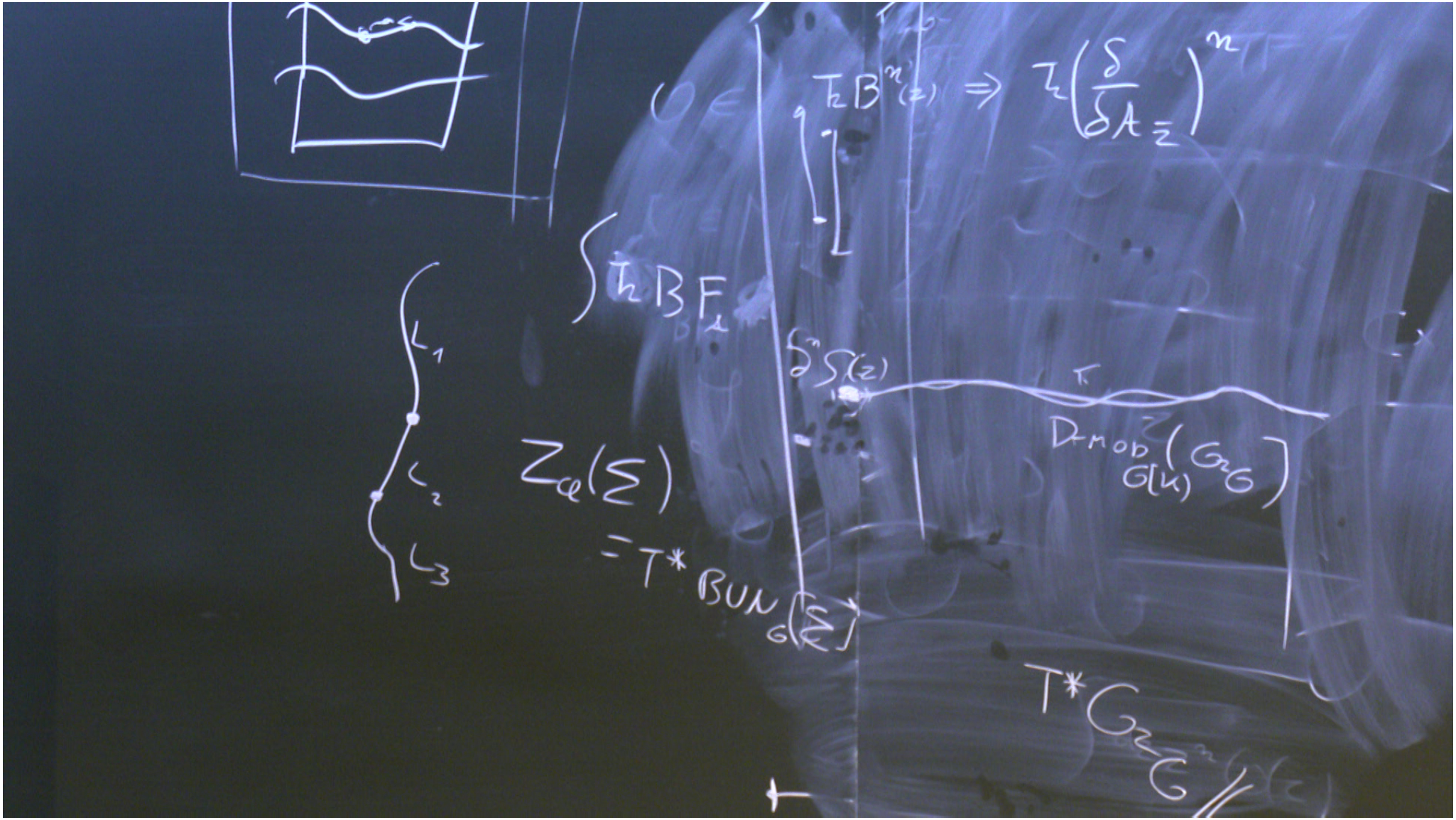

3d
4d A TWIST
BF

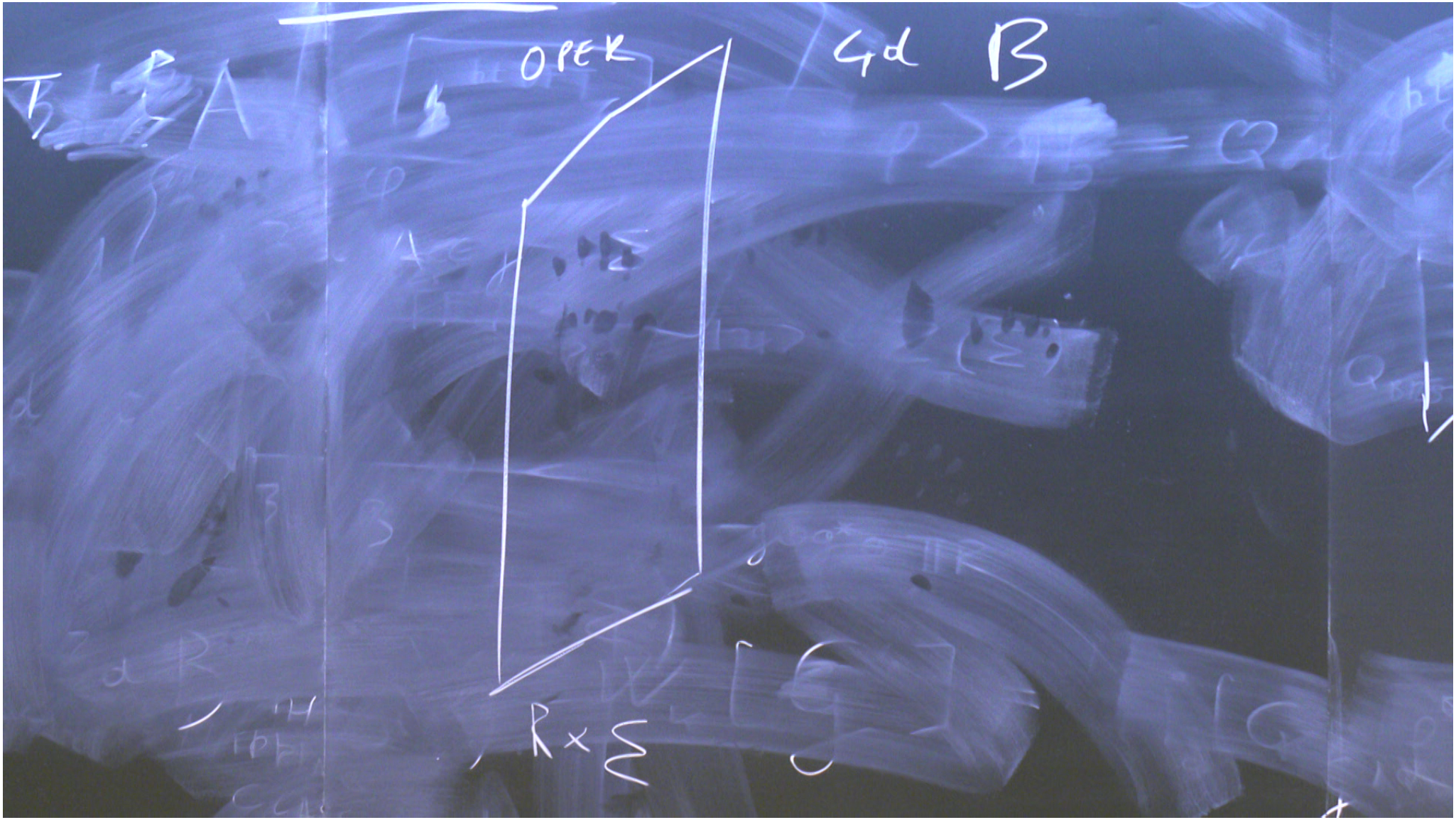

3d N=2 PURE GAUGE
TWIST
BCC
ht
BF
TWIST
4d N=4 + 3d N=4

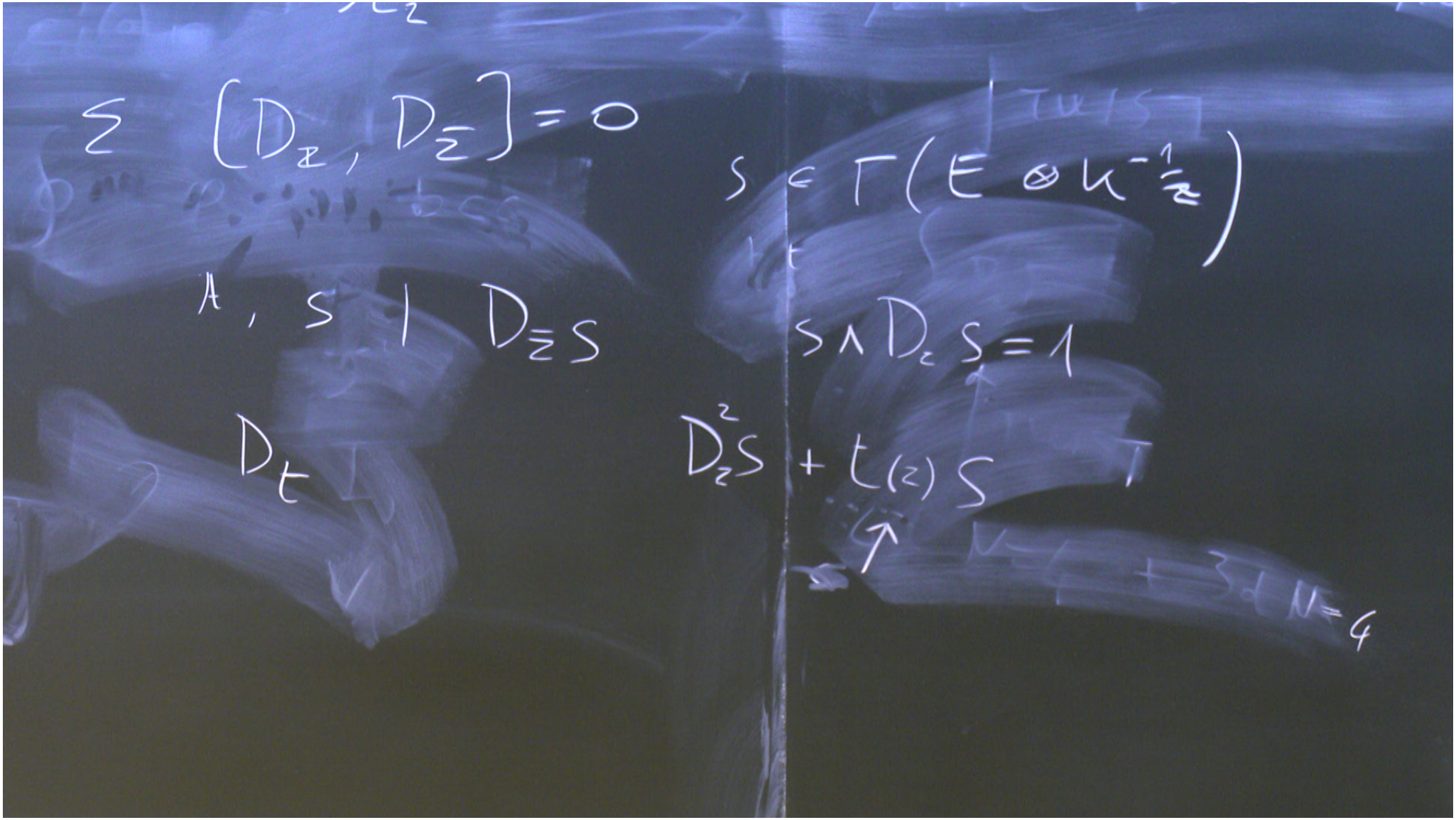

BF
4d
3d N=2
4d N=4

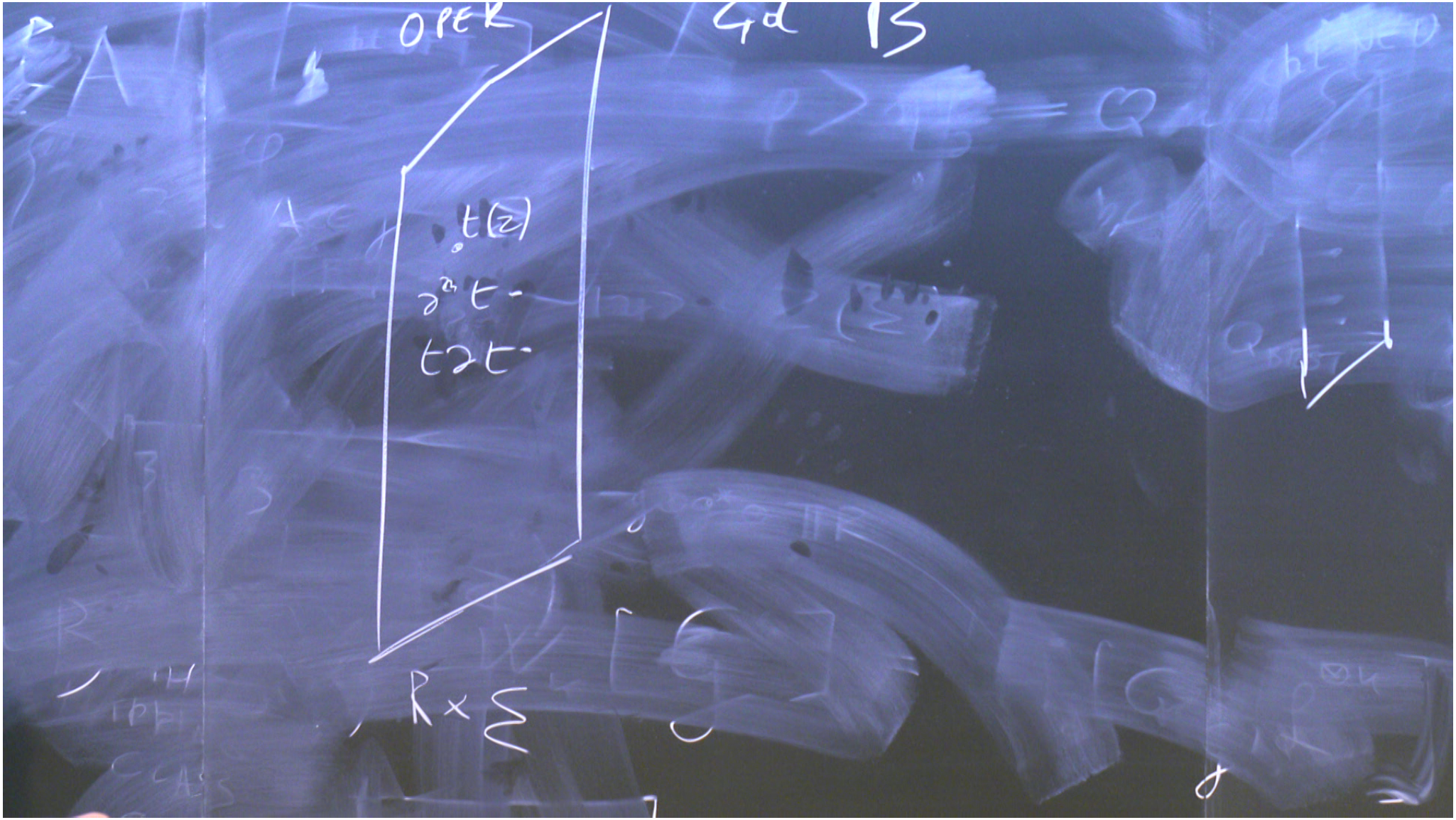

4d A
$\int_\Sigma \hbar BF$
L_1
L_2
L_3
$Z_Q(\Sigma)$
$= T^* BUN_G(\Sigma)$
$\hbar B^n(z) \Rightarrow \hbar \left(\frac{\delta}{\delta A_z}\right)^n$
$\mathbb{C} \times R$

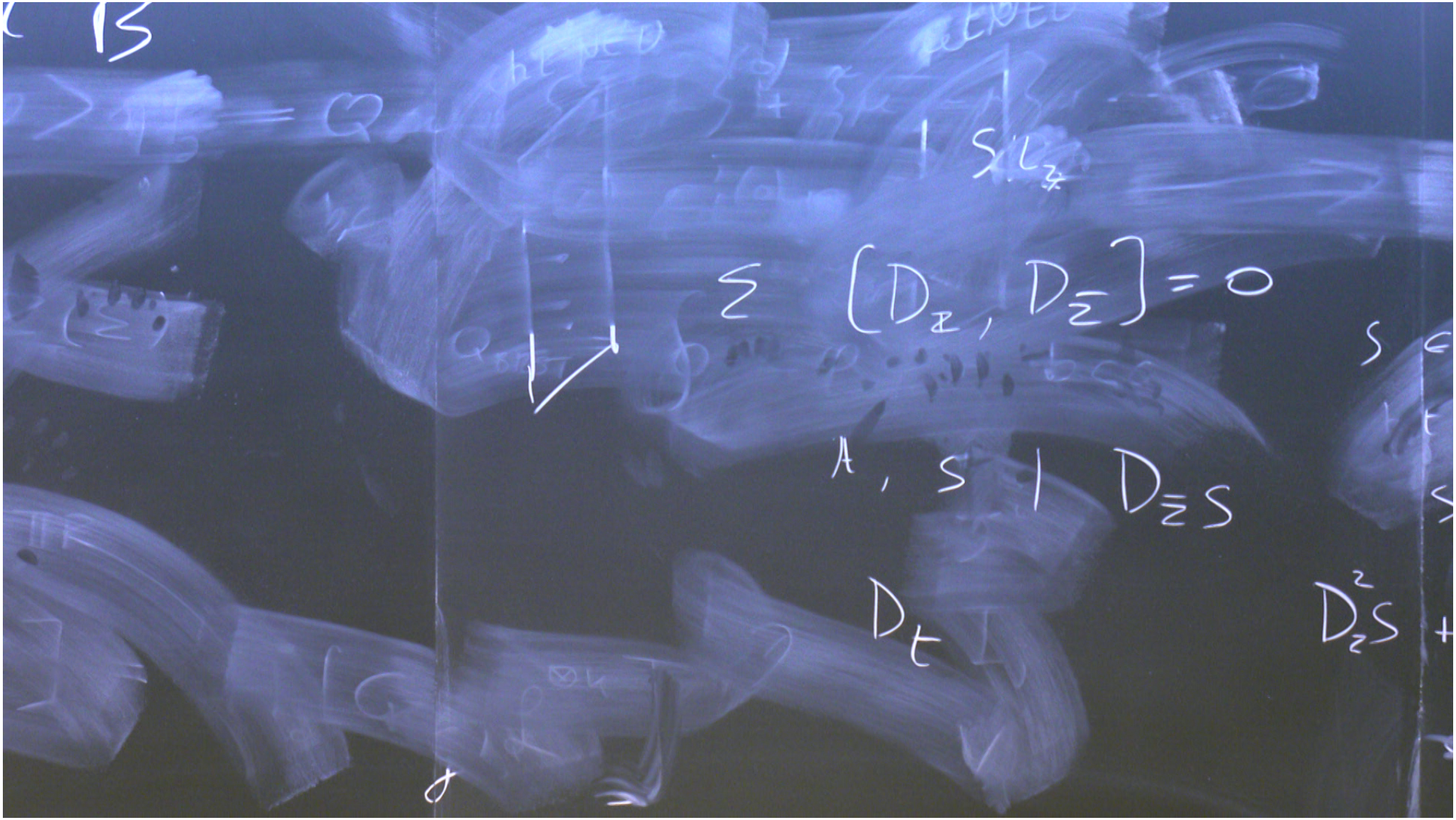

$\int_\Sigma \mathrm{Tr} B F_A$
L_1
L_2
L_3
$Z_{cl}(\Sigma)$
$= T^* \mathrm{BUN}_G(\Sigma)$
$T^* G_{rG}$
$/\!/ G[t]$

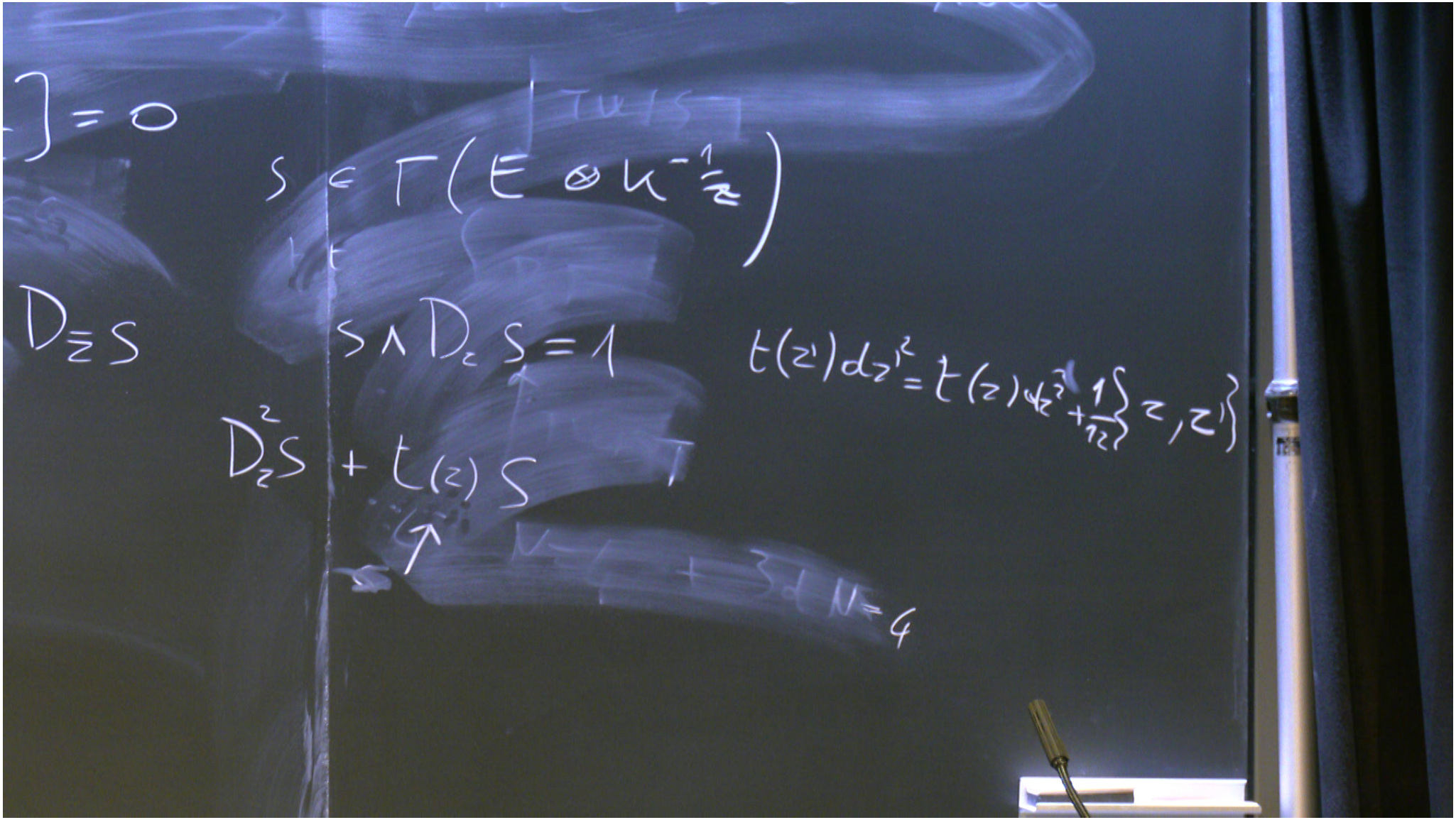

$\int_{\Sigma} \mathrm{Tr} B F$
L_1
L_2
L_3
$Z_Q(\Sigma)$
$= T^* \mathrm{BUN}_G(\Sigma)$
$\mathrm{Tr} B^n(z) \Rightarrow \mathrm{Tr}\left(\frac{\delta}{\delta A_z}\right)^n$
$\delta^n S(z)$
$D\text{-mod}_{G(\mathcal{K})}(Gr_G)$
$T^* Gr_G$

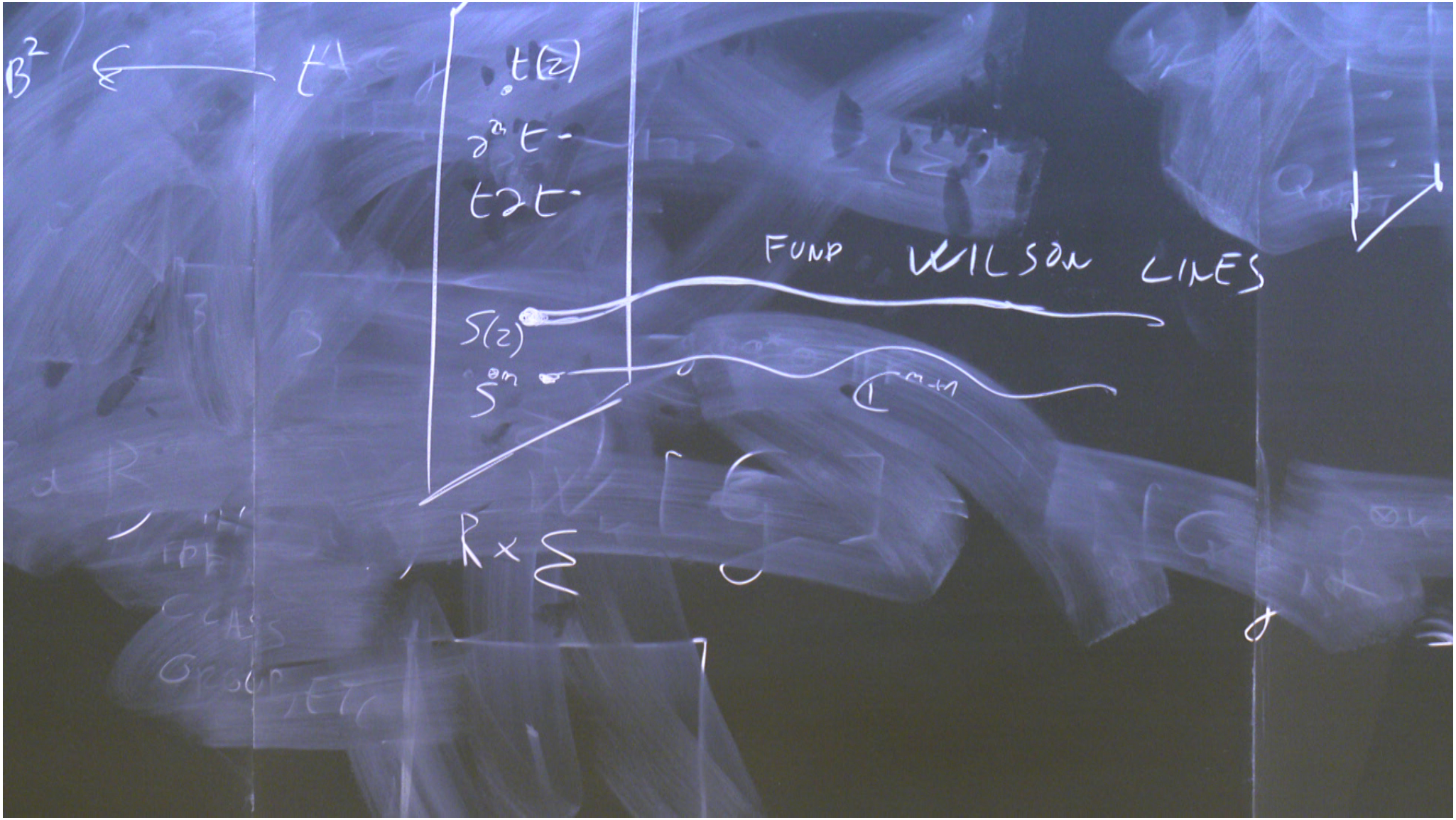

OPER
R×Σ

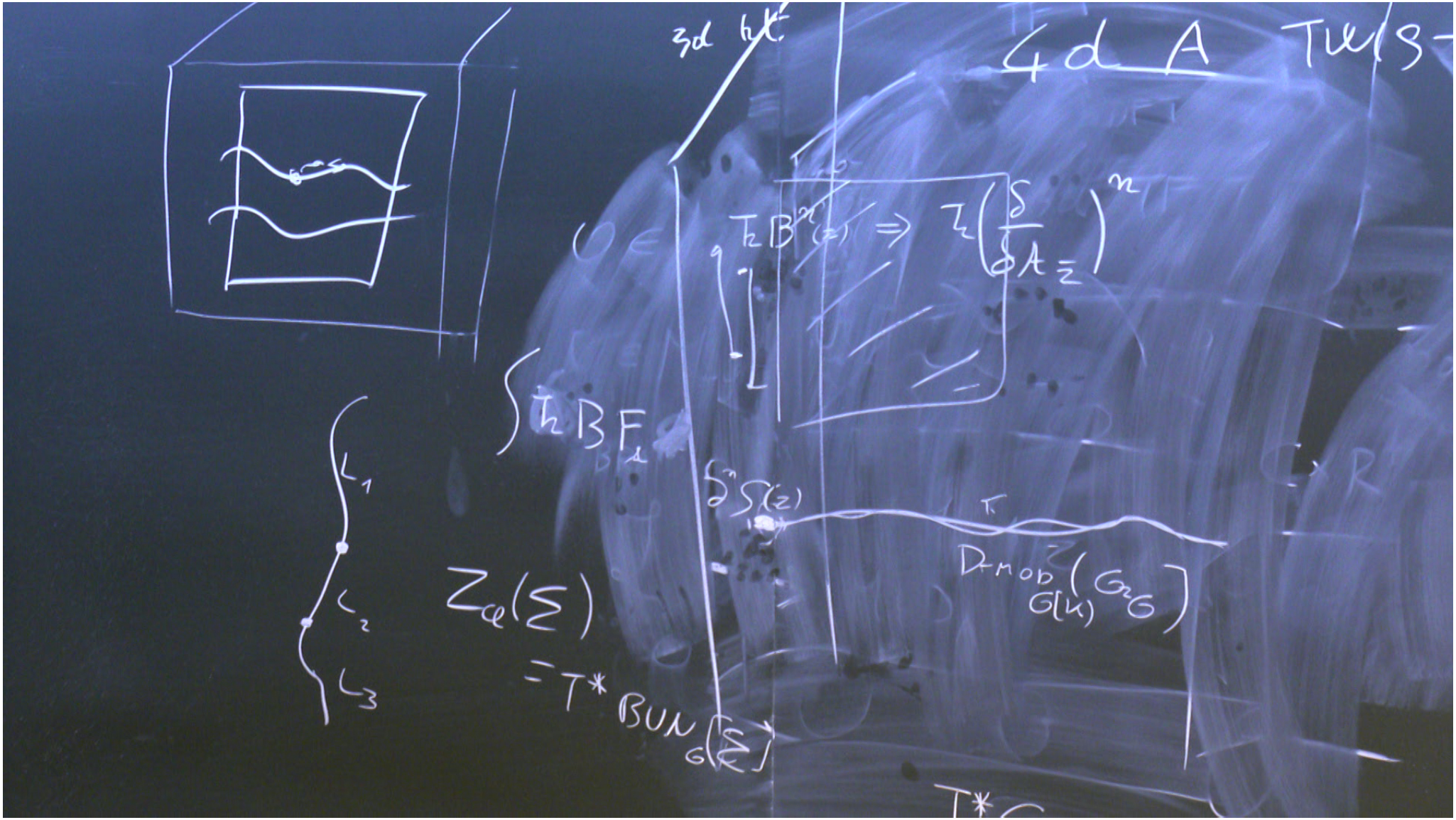

$\Sigma \quad [D_z, D_{\bar{z}}] = 0$
$s \in \Gamma(E \otimes K^{-\frac{1}{2}})$
$A, s \mid D_{\bar{z}} s$
$s \wedge D_z s = 1$
D_t
$D_z^2 s + t(z) s$
$N = 4$

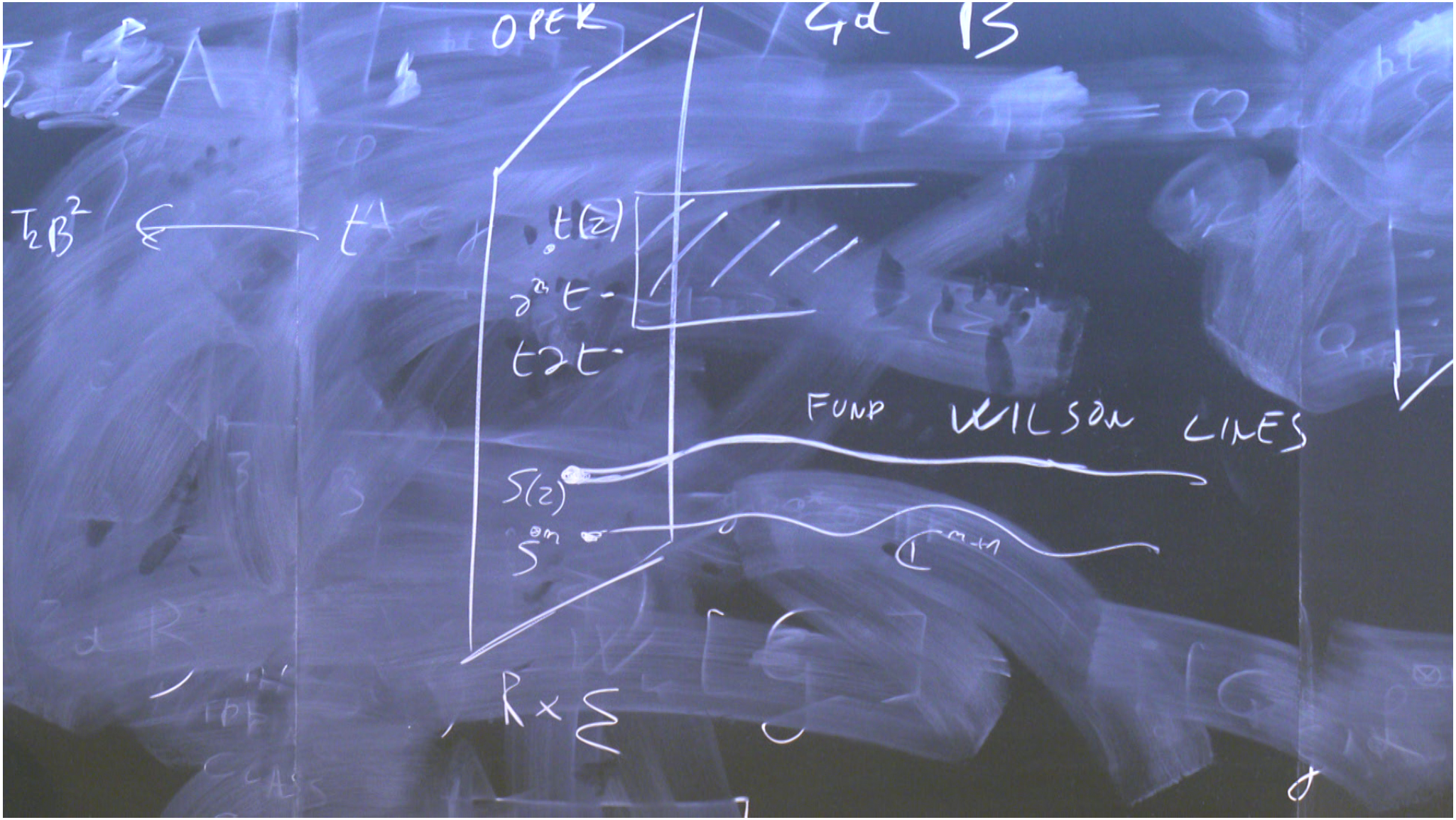

OPER
t(z)
$\partial^n t -$
$t \partial t -$
$R \times \Sigma$

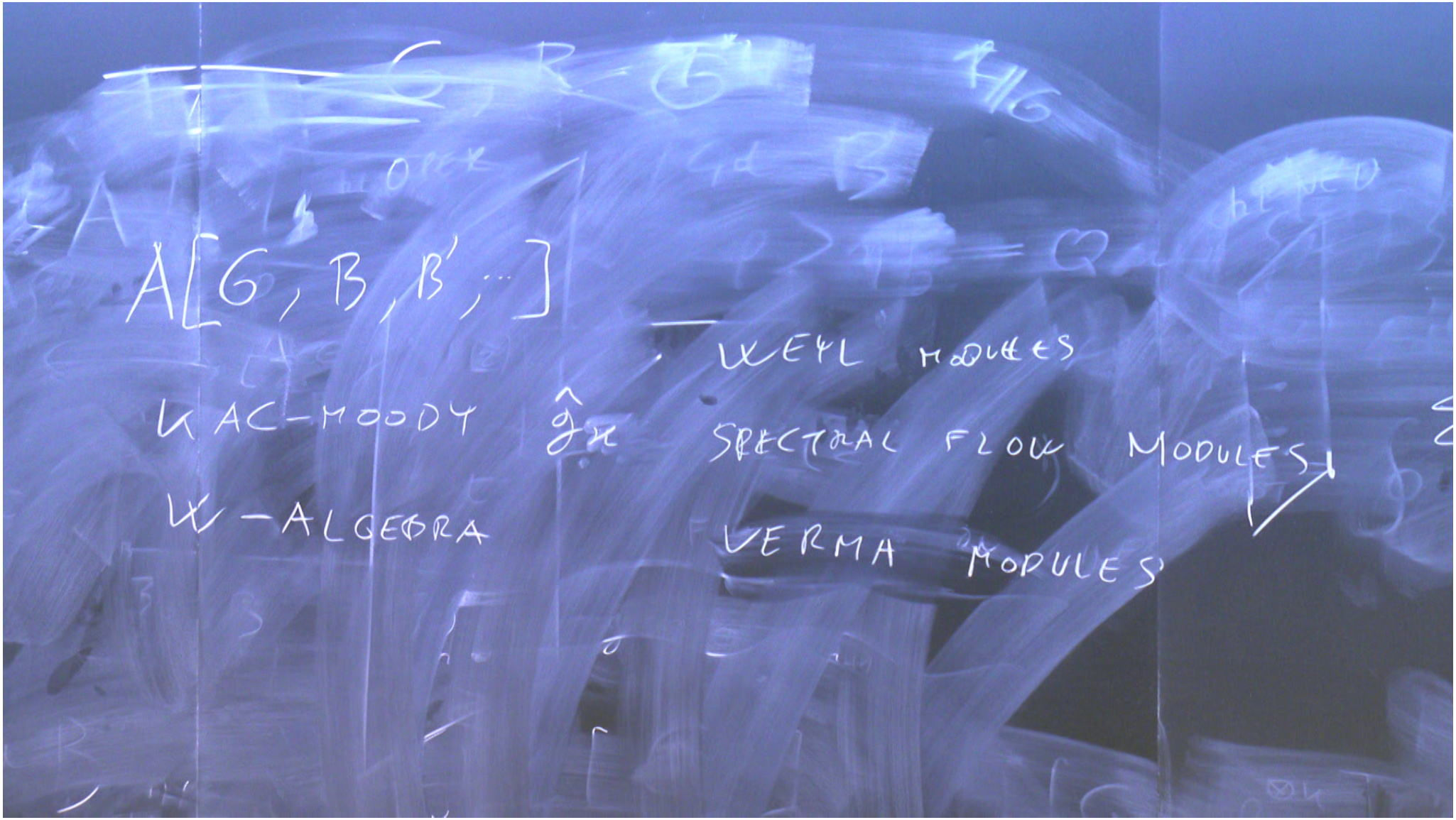

$[D_z, D_{\bar{z}}] = 0$
$A, S \mid D_{\bar{z}} S$
D_t
$D_z^2 S$

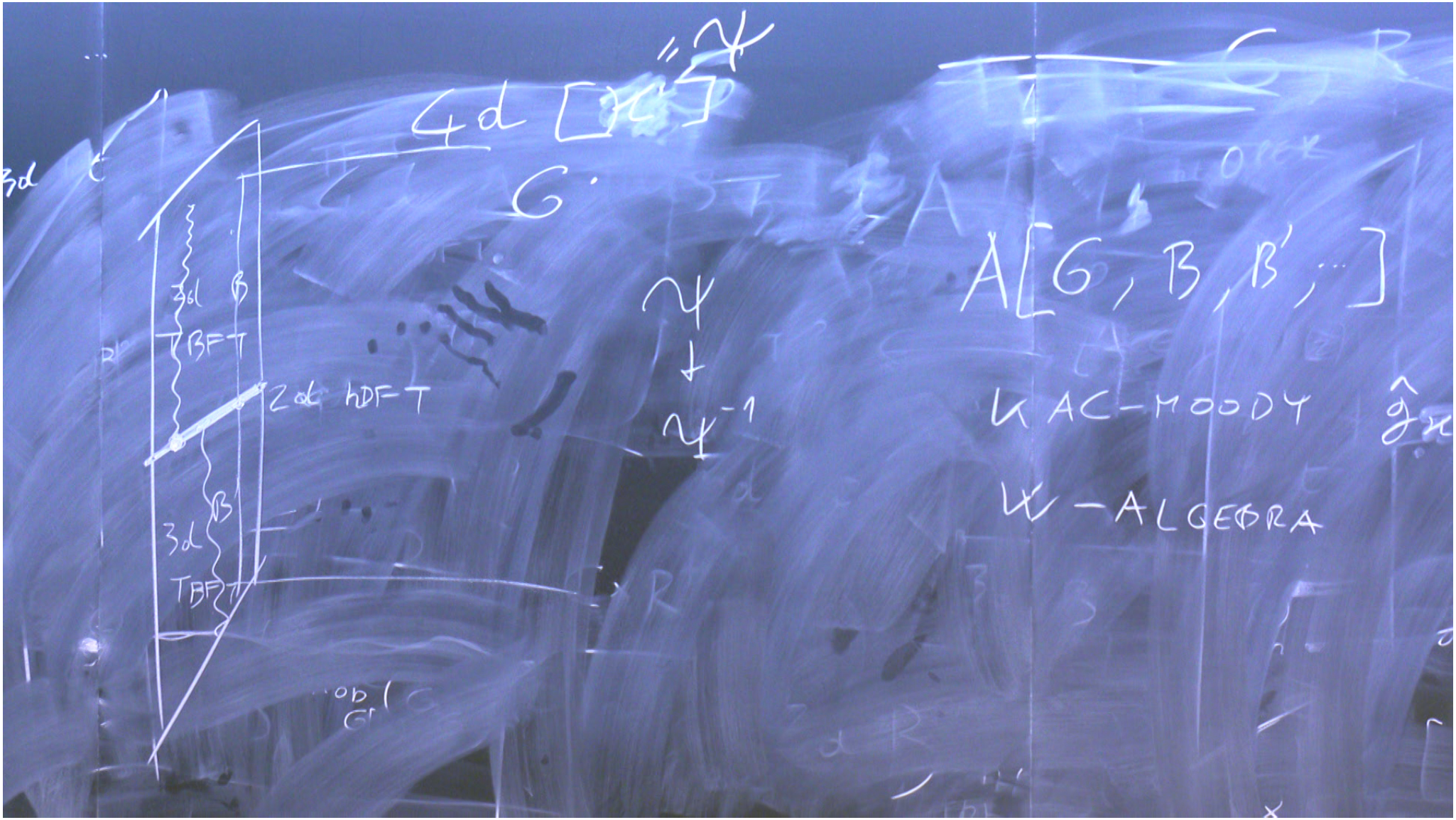

$s \in \Gamma(E \otimes K^{-\frac{1}{2}})$
$D_z s$
$s \wedge D_z s = 1$
$t(z)dz^2 = t(z)dz^2 + \frac{1}{2}\{z, z'\}$
$D_z^2 s + t(z) s$
$N = 4$

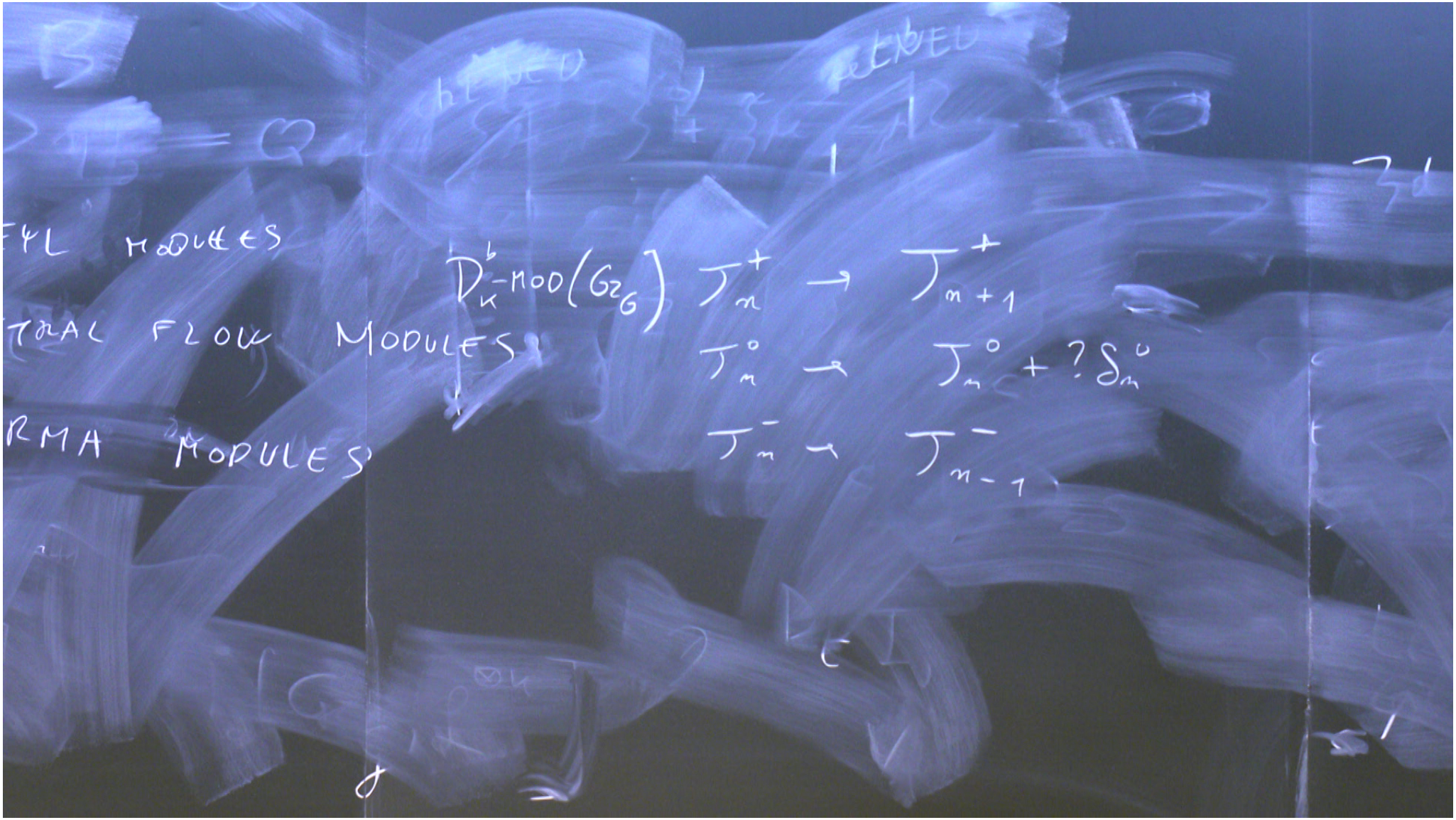

B^2
$t(z)$
$\partial^a t -$
$t \partial t -$
FUND WILSON LINES
$S(z)$
S^{2n}
$R \times \Sigma$


3d hyt
4d A Tw/s
$\int_{\Sigma} \mathrm{Tr} B F_A$
L_1
L_2
L_3
$Z_Q(\Sigma)$
$= T^* \mathrm{BUN}_G(\Sigma)$
$\partial^n S(z)$
$D\text{-mod}\left(\frac{G_2}{G(\mathcal{K})}\right)$
T^*G

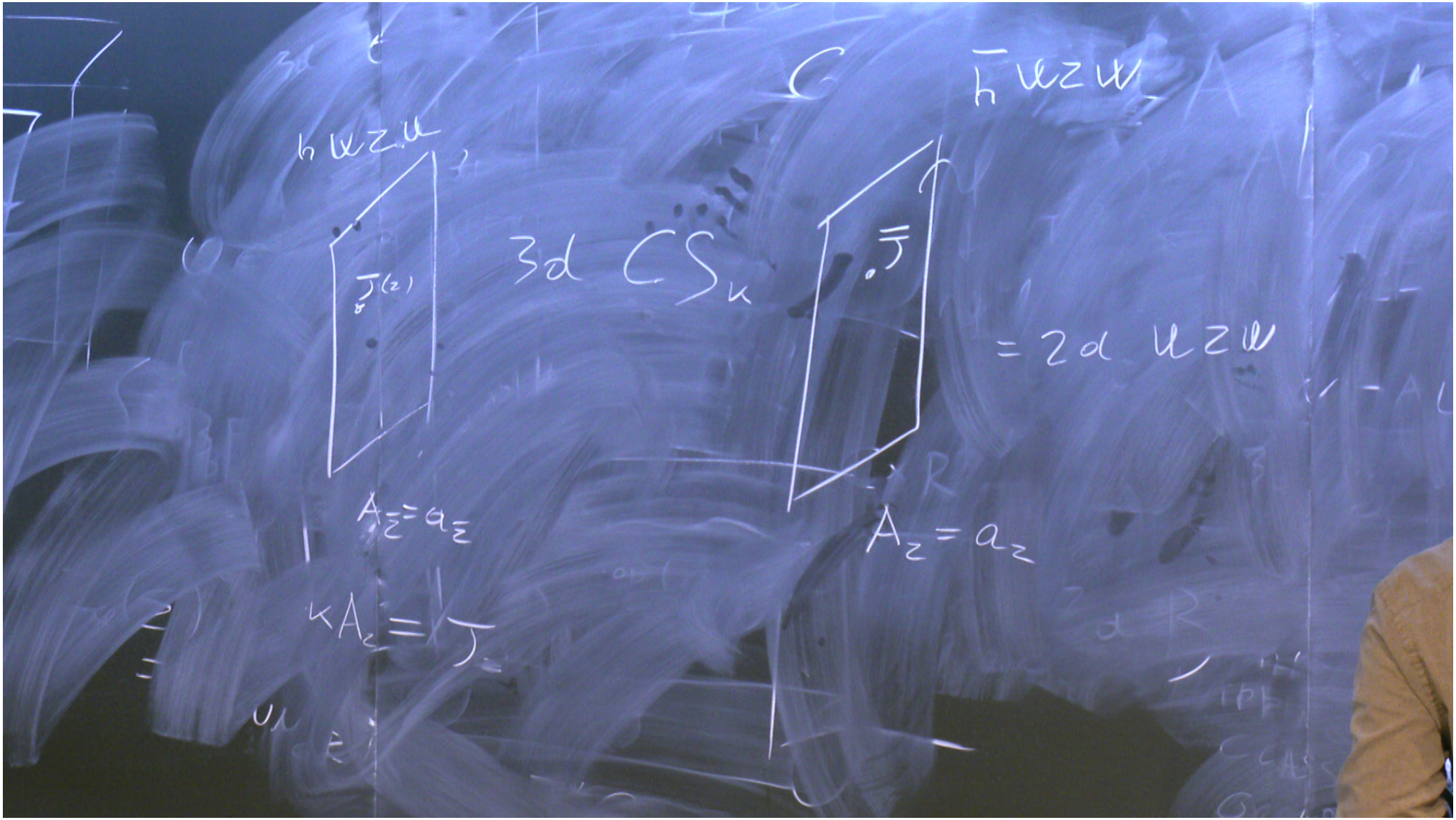

OPER
FUND
WILSON
LINES
R × Σ

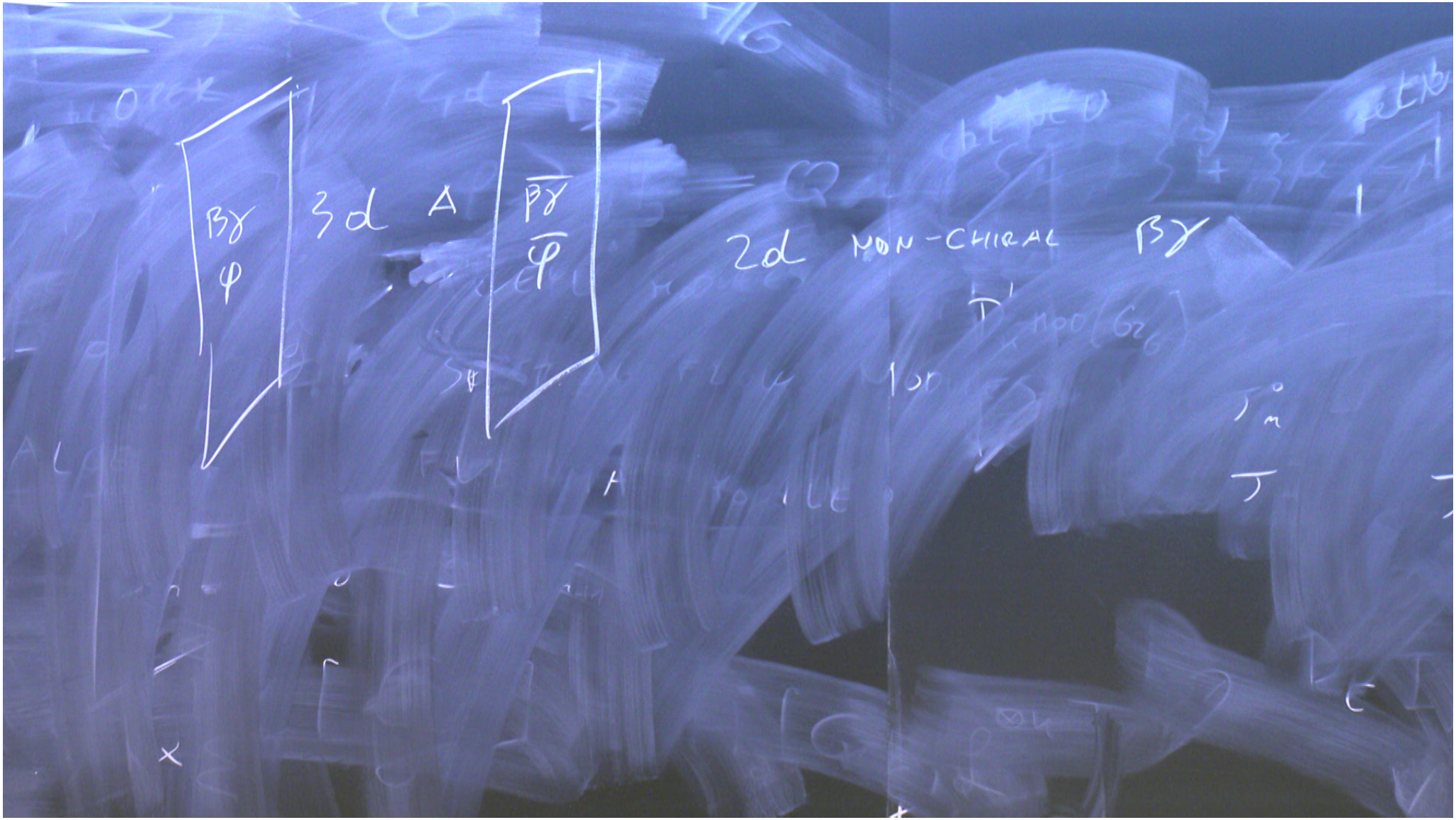

A[G, B, B', ...]
KAC-MOODY
W-ALGEBRA
WEYL MODULES
SPECTRAL FLOW MODULES
VERMA MODULES

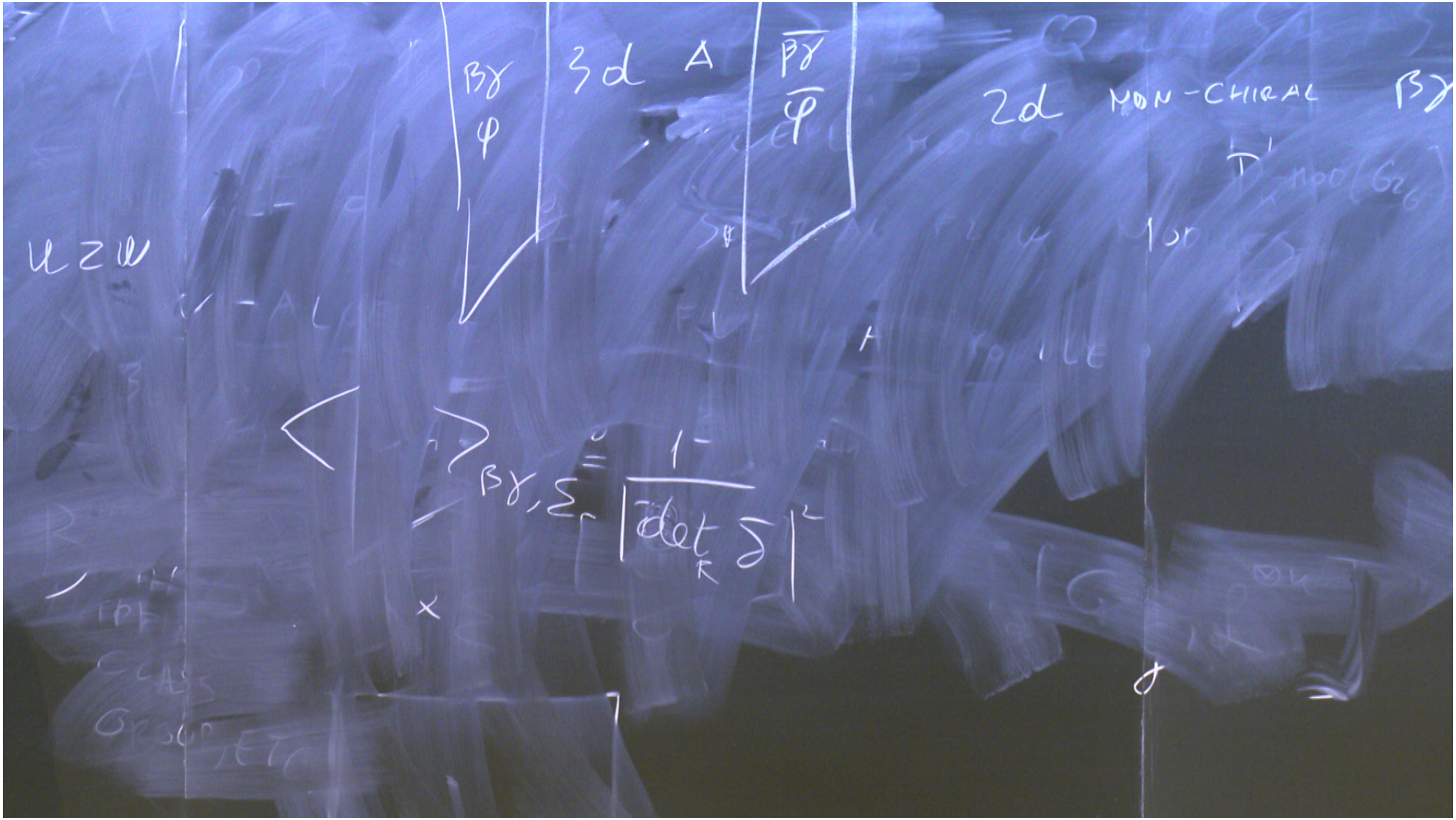

4d
G
3d
TBFT
2d hDF-T
3d
TBFT
A[G, B, B', ...]
KAC-MOODY
W-ALGEBRA

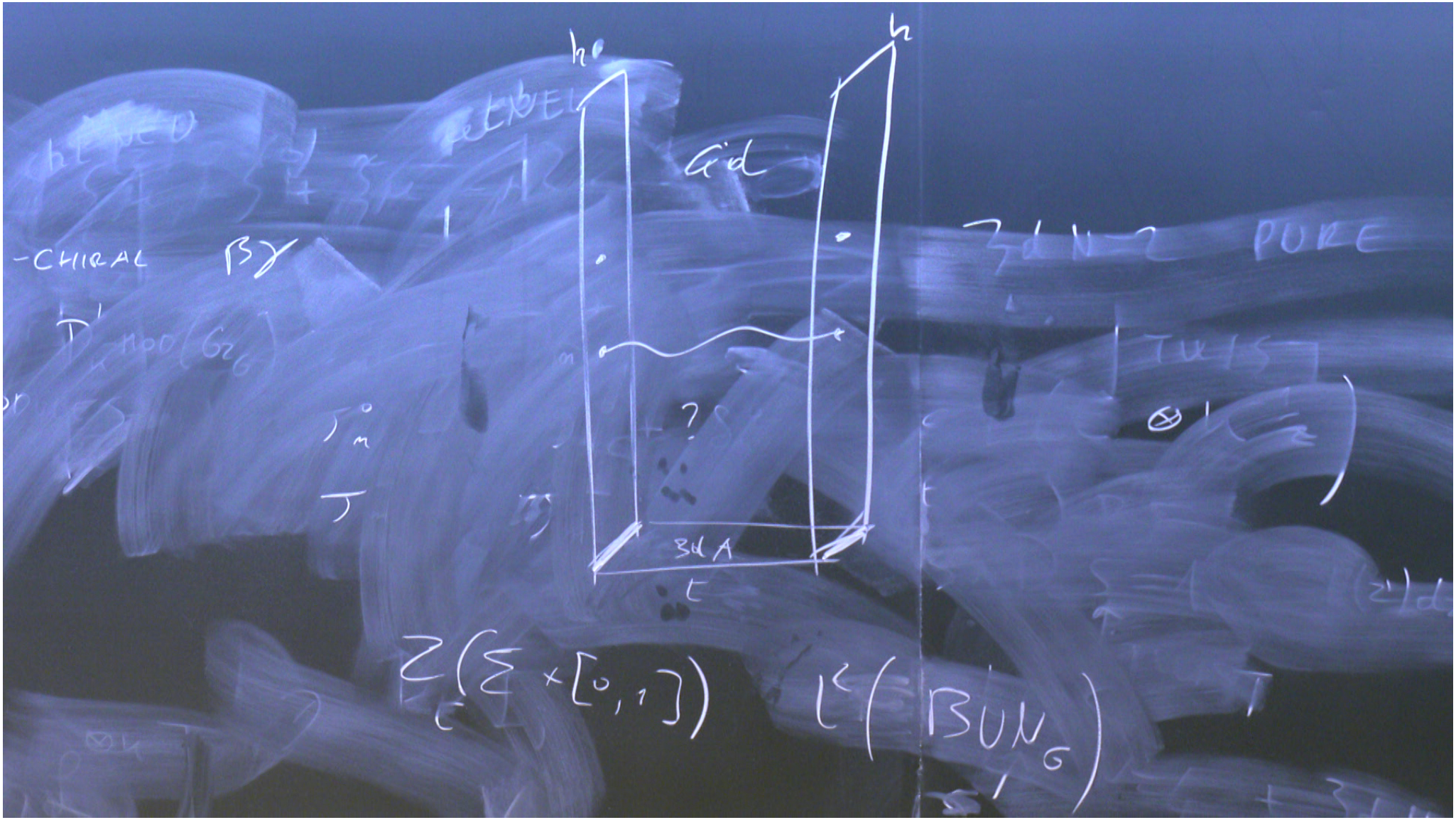

FYL MODULES
TORAL FLOW MODULES
RMA MODULES
$D^b_K\text{-mod}(G_{2_G})$
$\mathcal{J}^+_n \to \mathcal{J}^+_{n+1}$
$\mathcal{J}^0_n \to \mathcal{J}^0_n + ?\,\delta^0_n$
$\mathcal{J}^-_n \to \mathcal{J}^-_{n-1}$


3d
3d B
TBFT
2d hDFT
B 2d hDFT
3d
TBFT
ALG, B,
AC-MOD

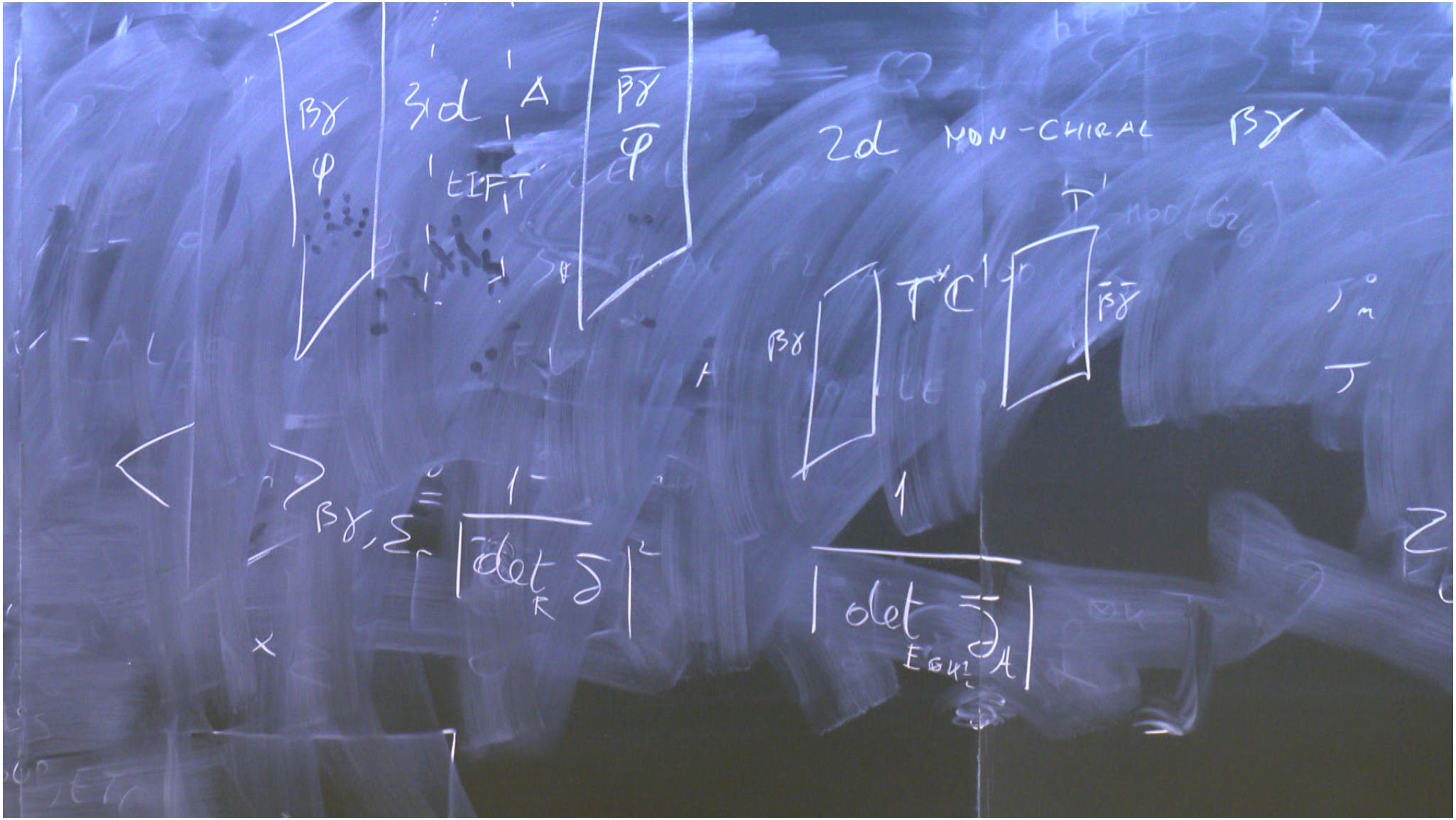

3d CS_k
= 2d
A_z = a_z
A_z = a_z

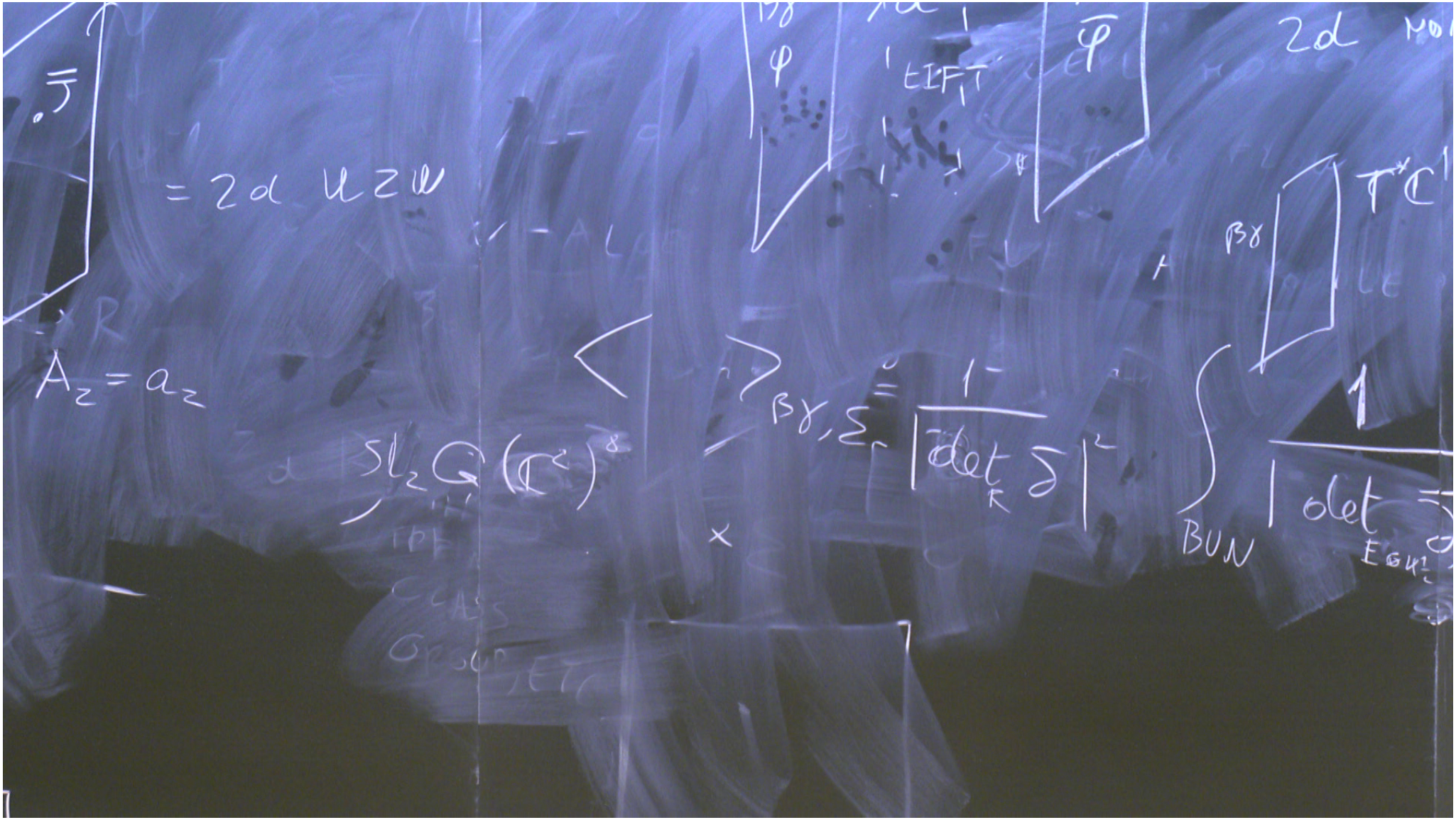

3d
2d NON-CHIRAL

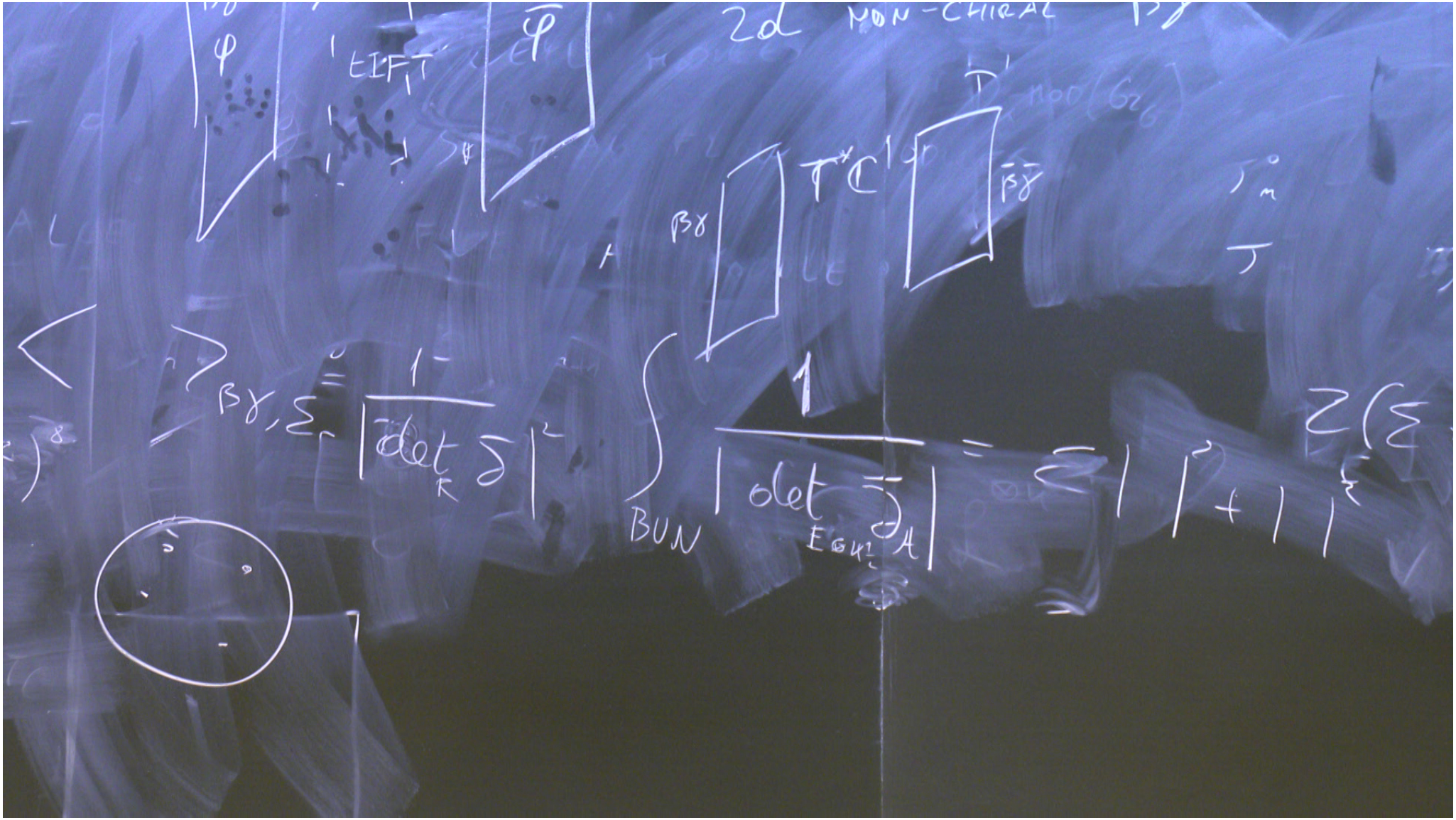

3d A
2d NON-CHIRAL

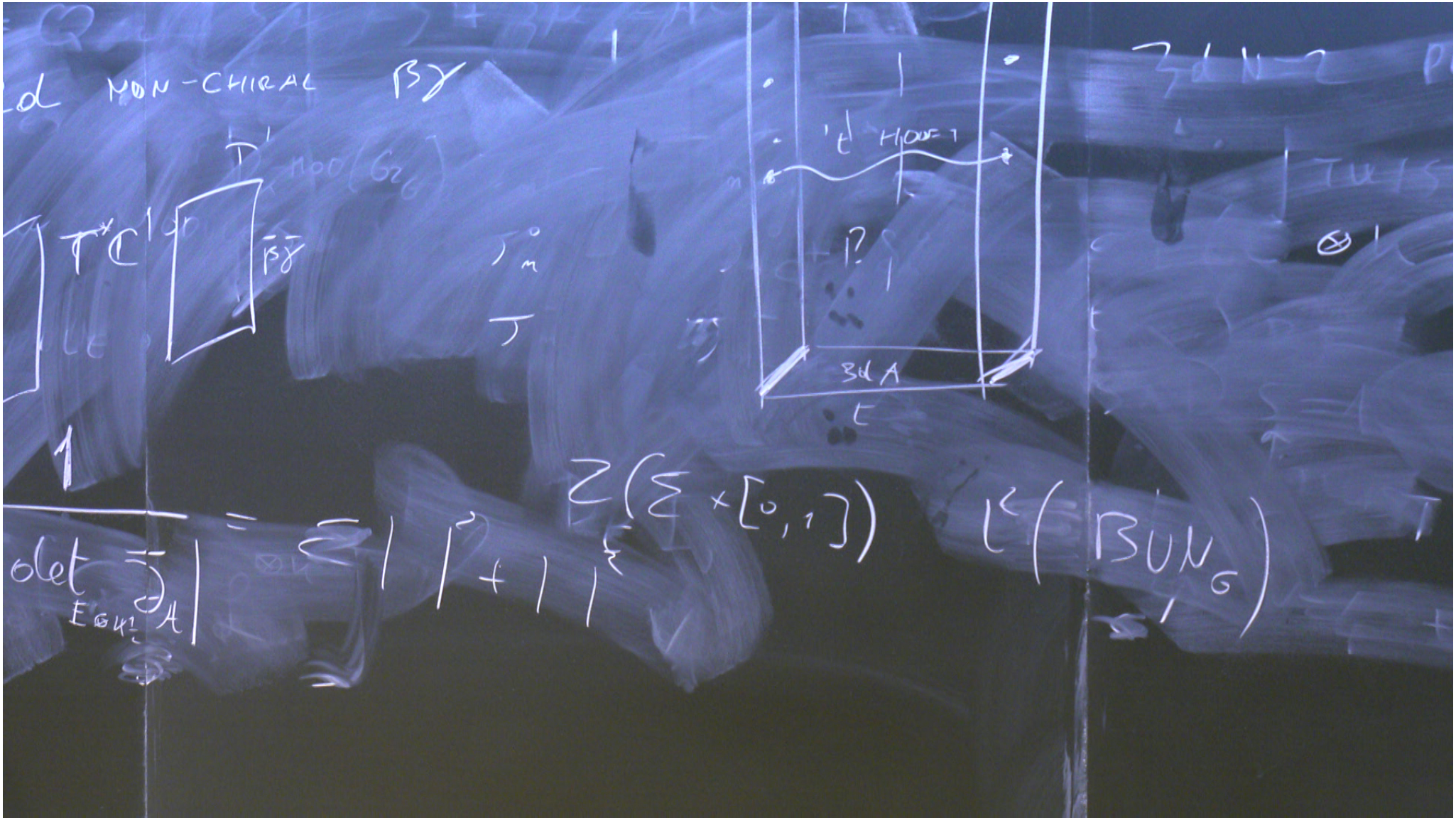

h
h
Grid
?
3d N=2 PURE
-CHIRAL
BY
TWIST
$Z_G(\Sigma \times [0,1])$
$L^c(BUN_G)$


3d
A
LIFT
2d
NON-CHIRAL


2d
LIFT
2d NON-CHIRAL


$A_z = a_z$
$SL_2 G(\mathbb{C})$
$\det_R \partial$
BUN
$T^*\mathbb{C}$


2d NON-CHIRAL
LIFT
BUN
EGM


NON-CHIRAL
BY
HOLE
BUA