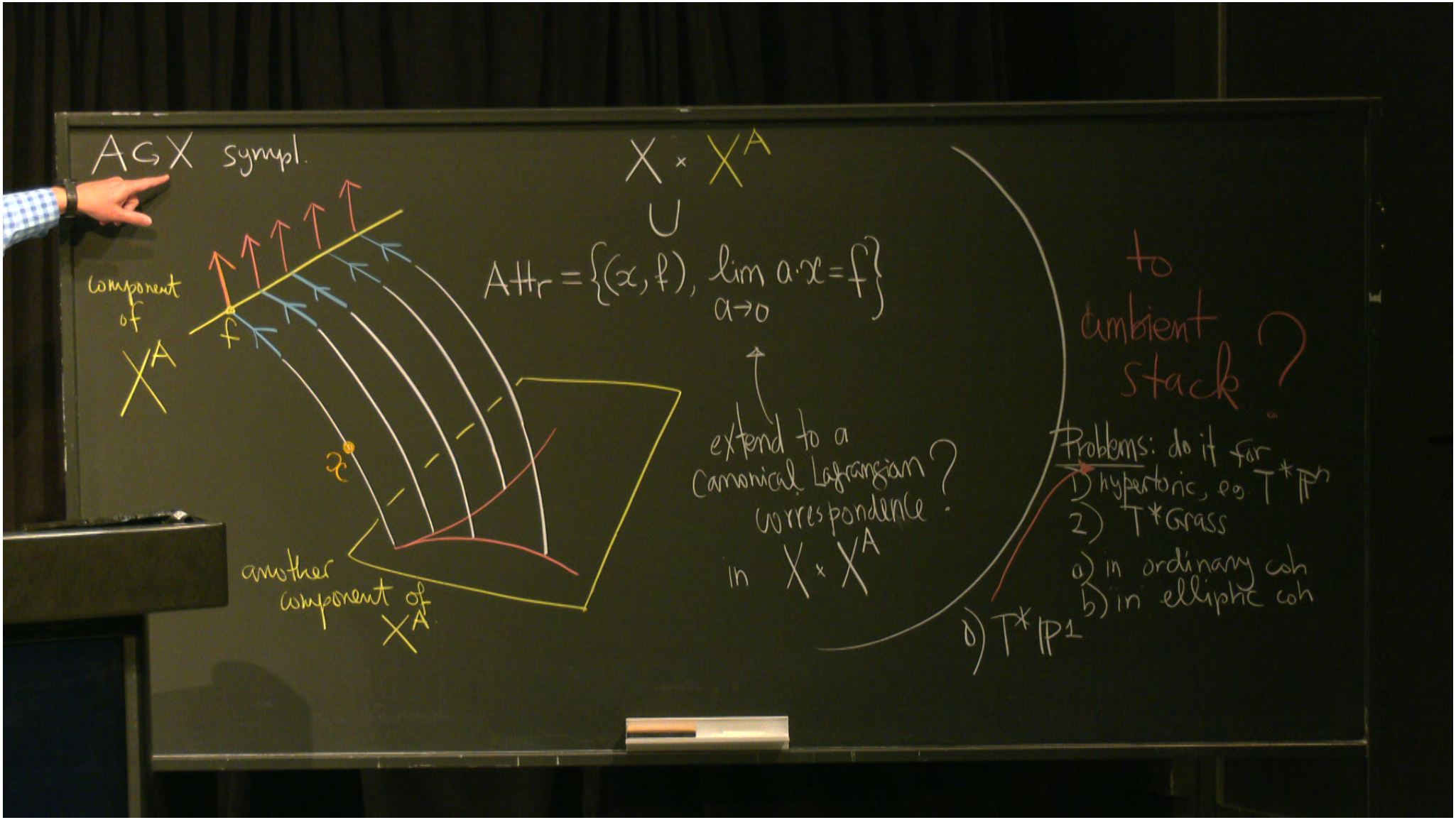
Title: 3d Gauge Theory and Elliptic Stable Envelopes I

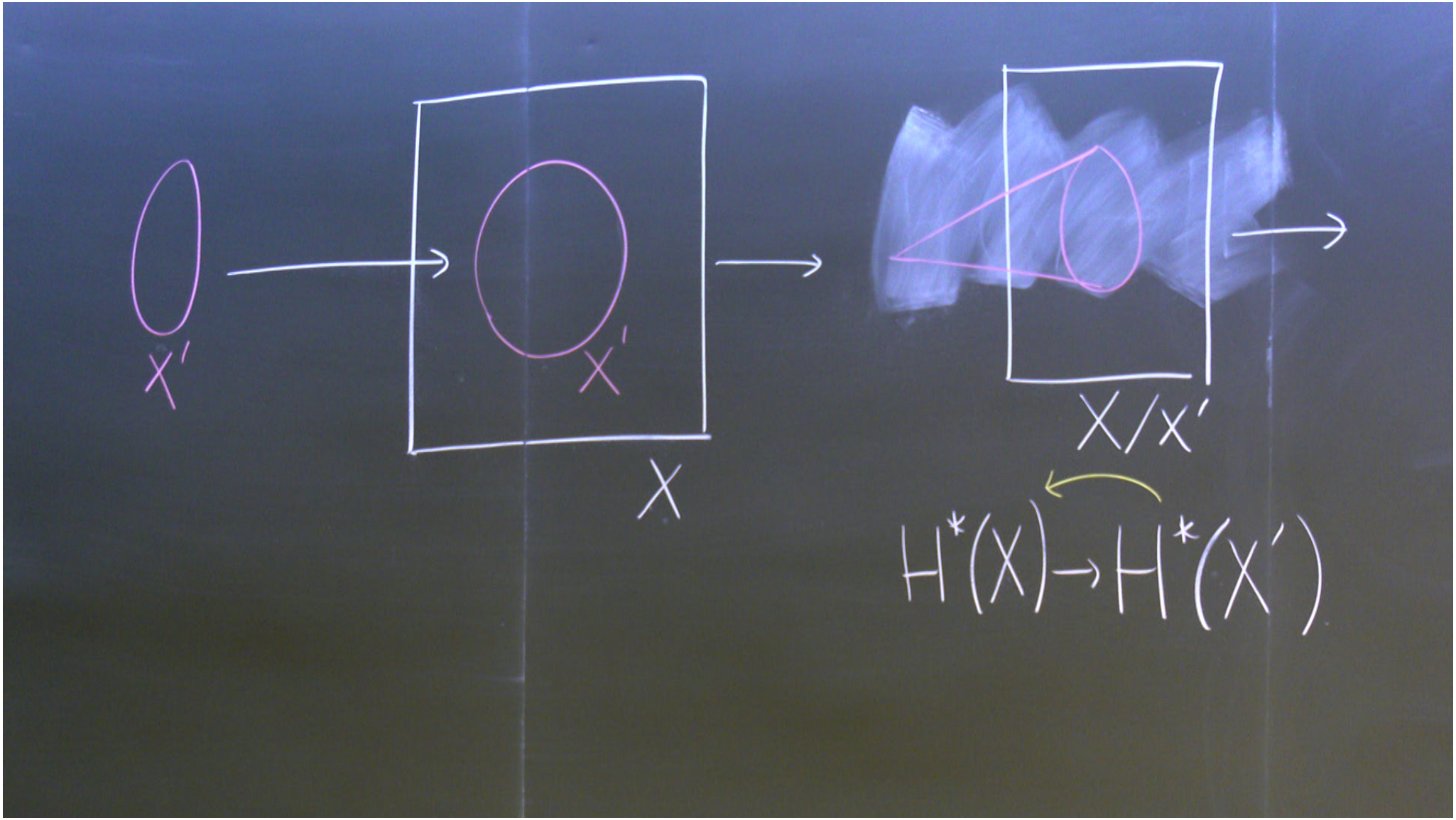
Speakers: Andrei Okounkov

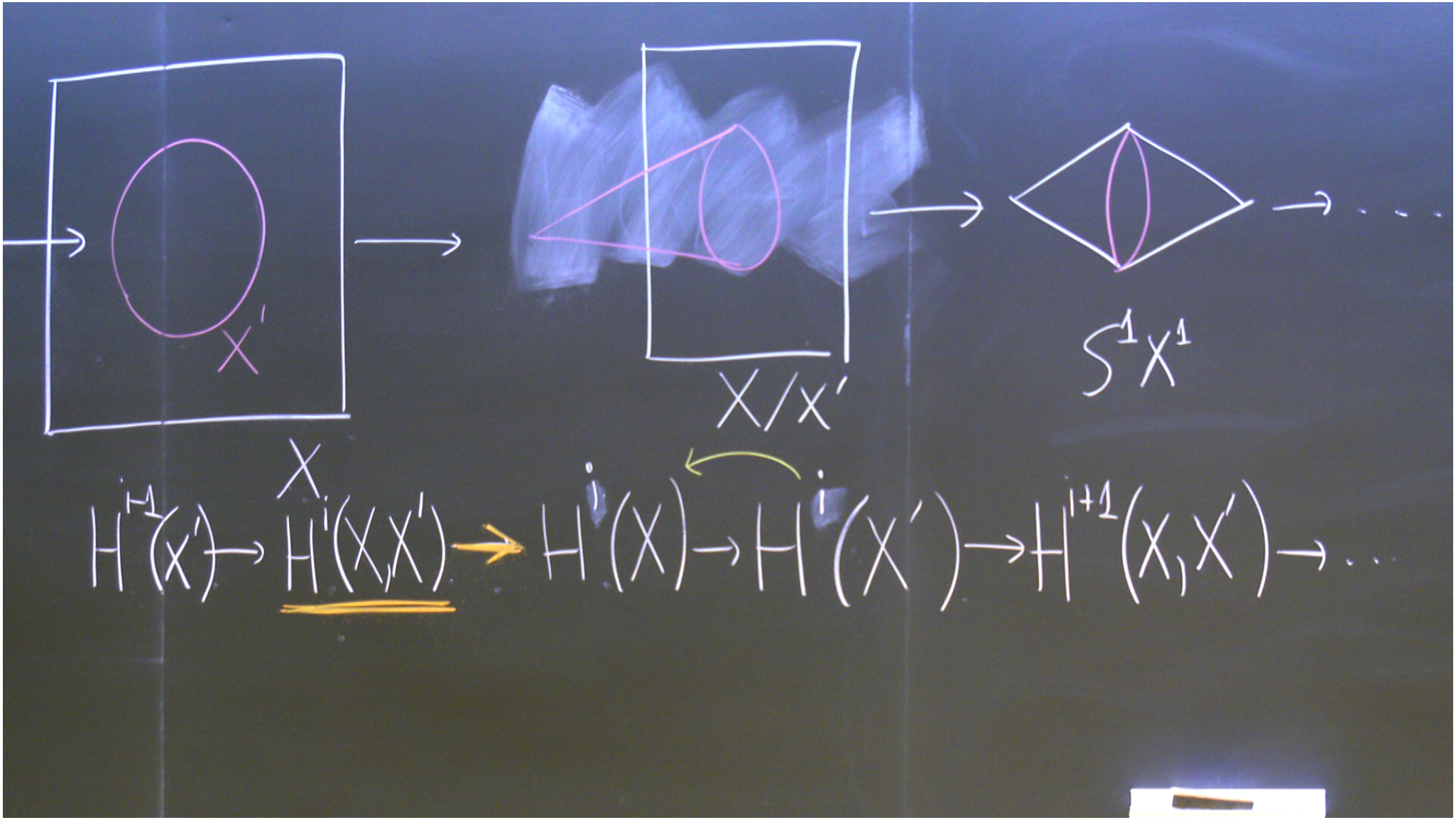
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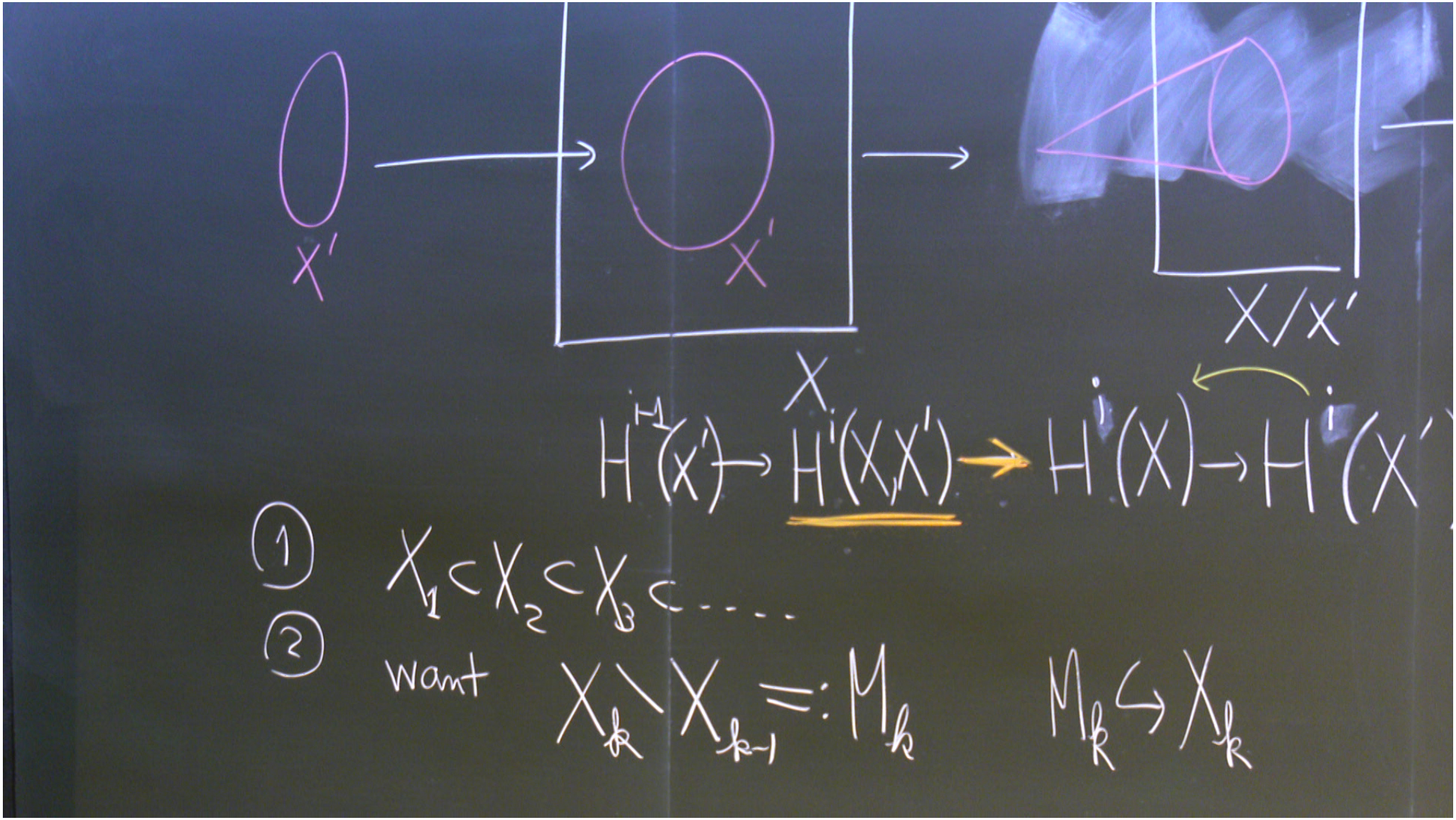
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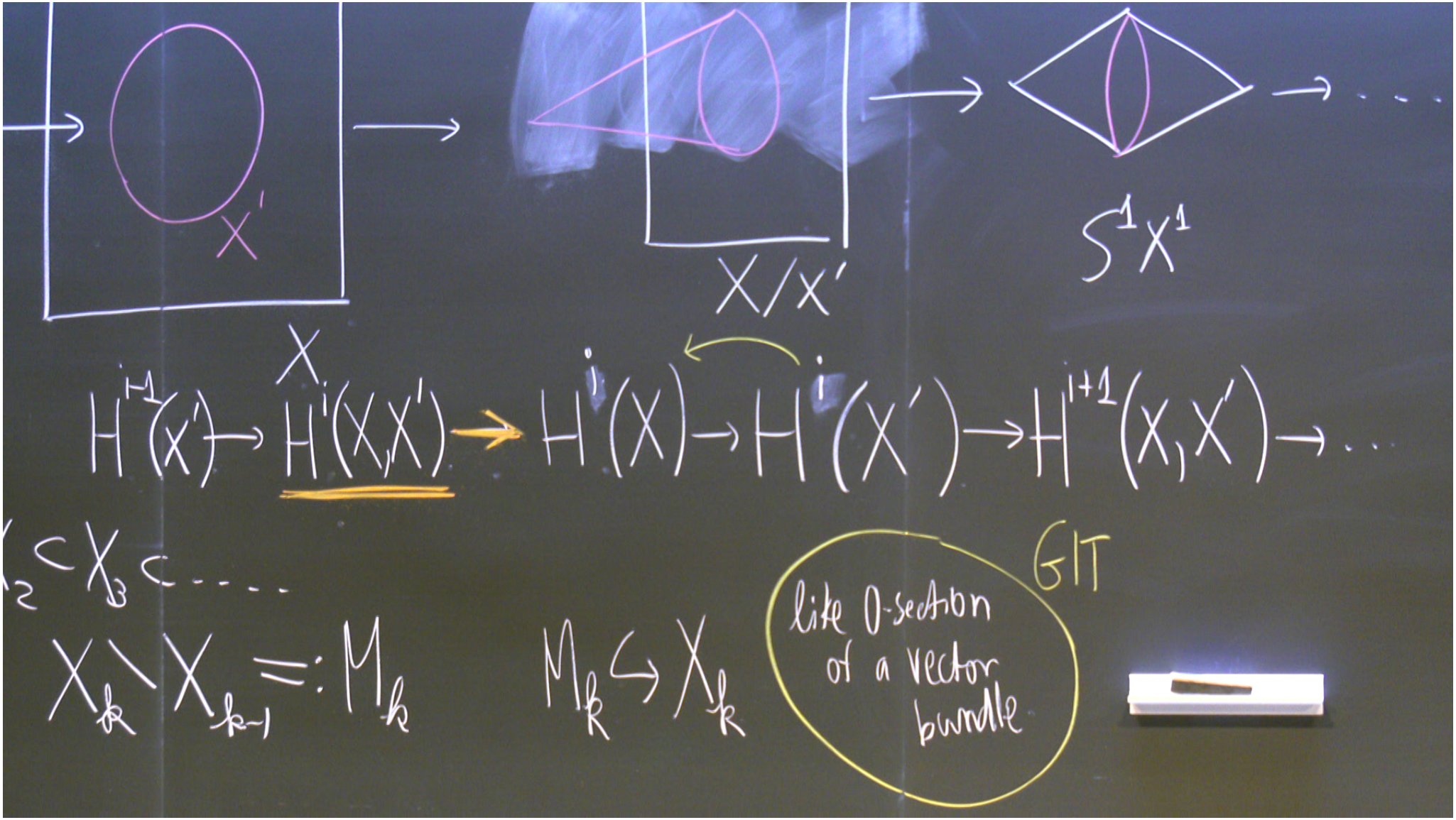
URL: https://pirsa.org/22060064


A ⊂ X sympl.
component of X^A
f
another component of X^A
$X \times X^A$
$\cup$
$Attr = \{(x, f), \lim_{a \to 0} a \cdot x = f\}$
extend to a canonical Lagrangian correspondence in $X \times X^A$?
to ambient stack?
Problems: do it for
1) hypertoric, eg $T^*\mathbb{P}^n$
2) T^*Grass
a) in ordinary coh
b) in elliptic coh
0) $T^*\mathbb{P}^1$

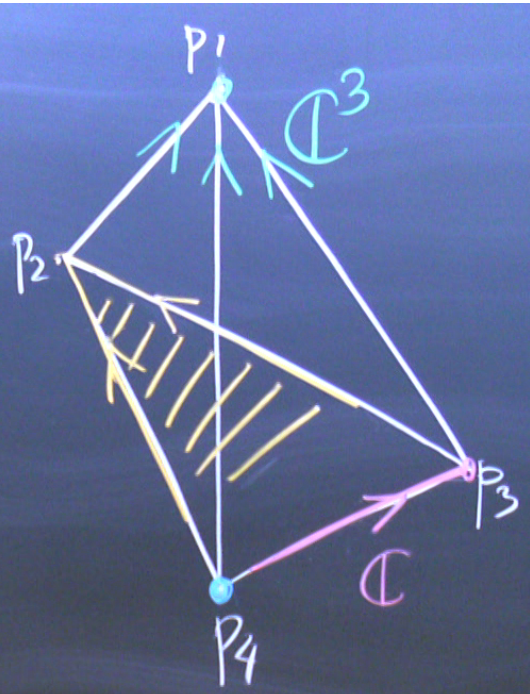

X'
X'
X
X/X'
H*(X)→H*(X')

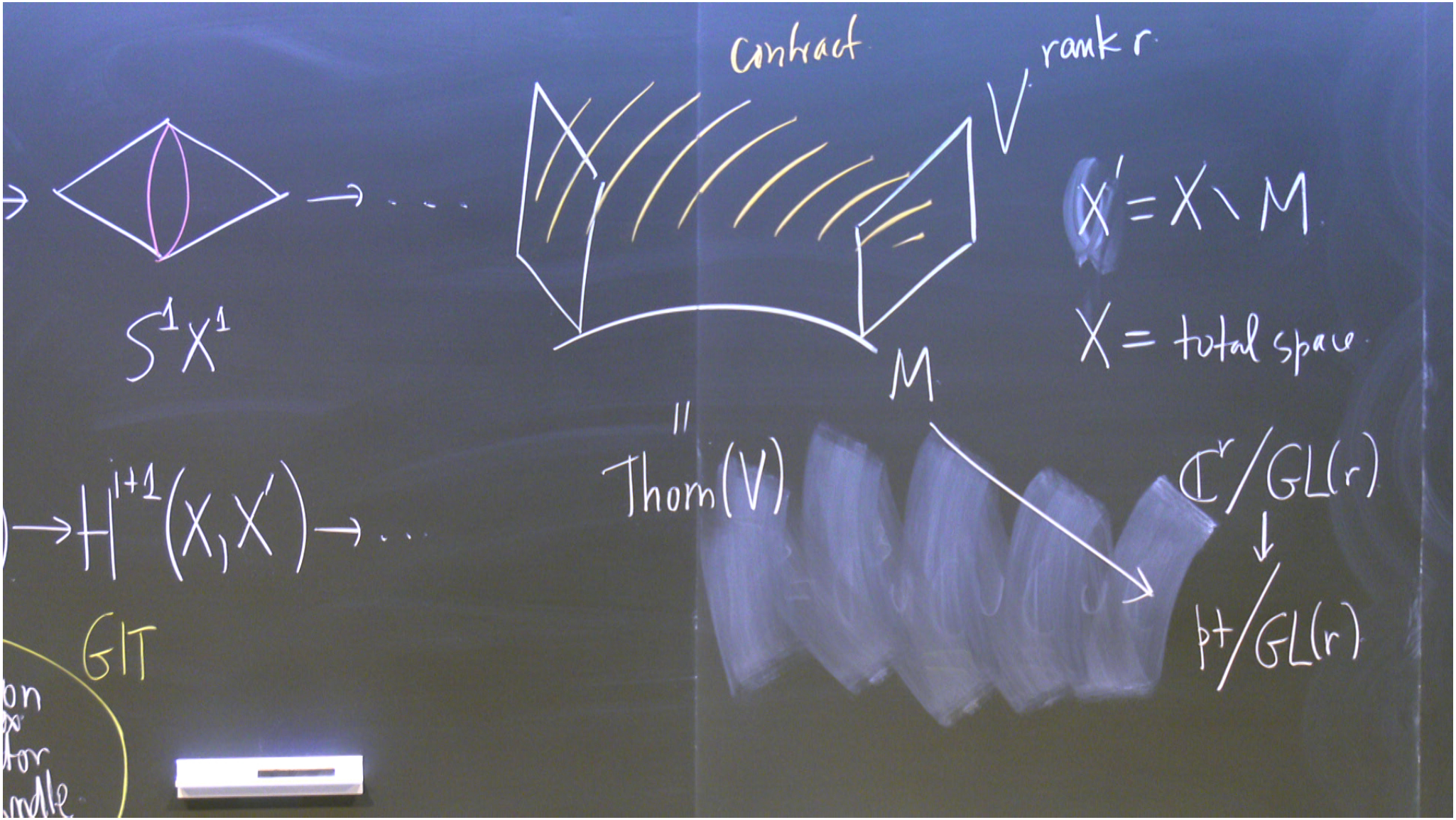

X'
X
X/X'
S¹X¹
H^{i-1}(X')→ H^i(X,X') → H^i(X) → H^i(X') → H^{i+1}(X,X') → ...

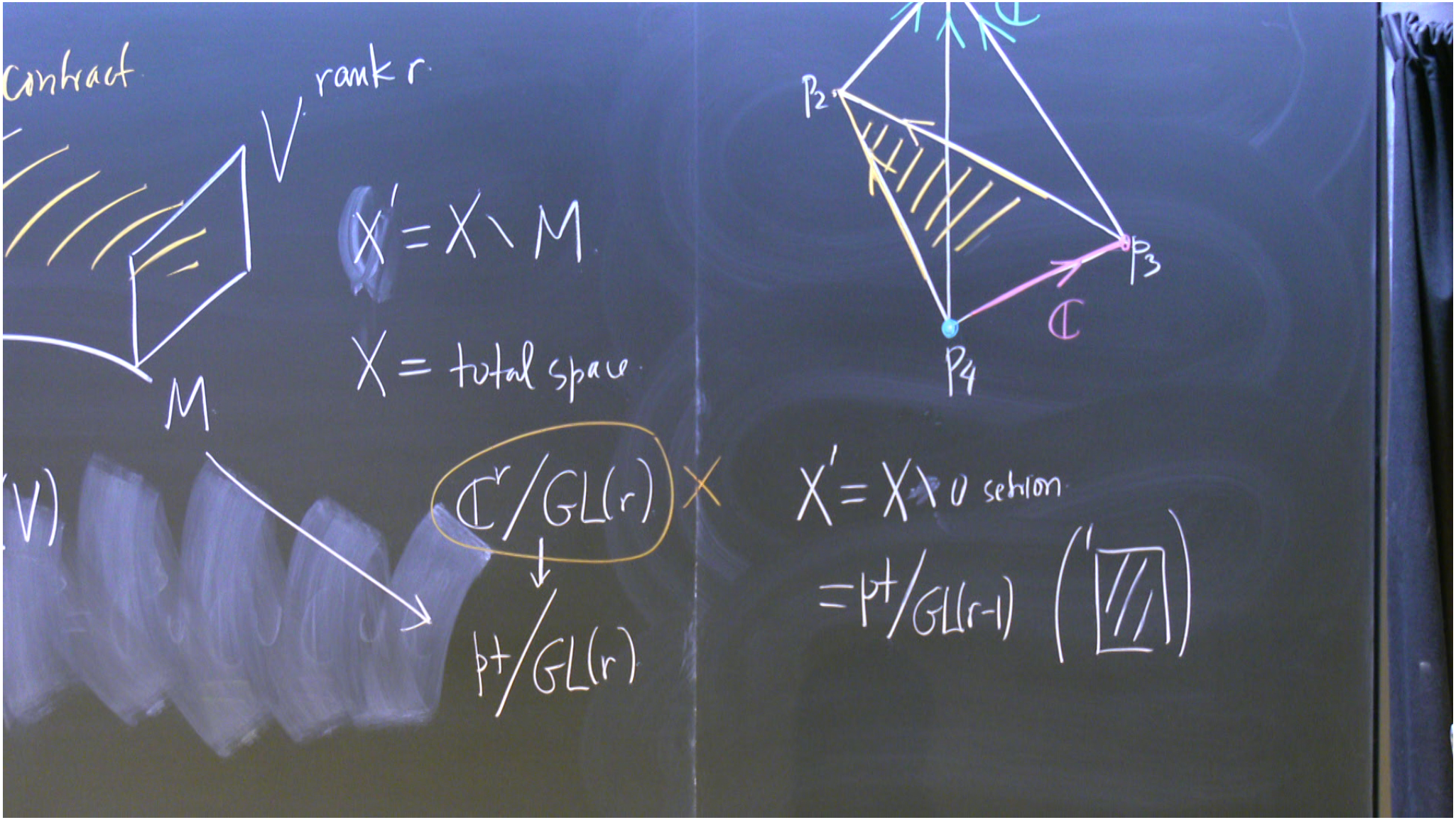

X'
X'
X
X/X'
$H^{i-1}(X') \to H^i(X,X') \to H^i(X) \to H^i(X')$
① $X_1 \subset X_2 \subset X_3 \subset \cdots$
② want $X_k \setminus X_{k-1} =: M_k$
$M_k \hookrightarrow X_k$

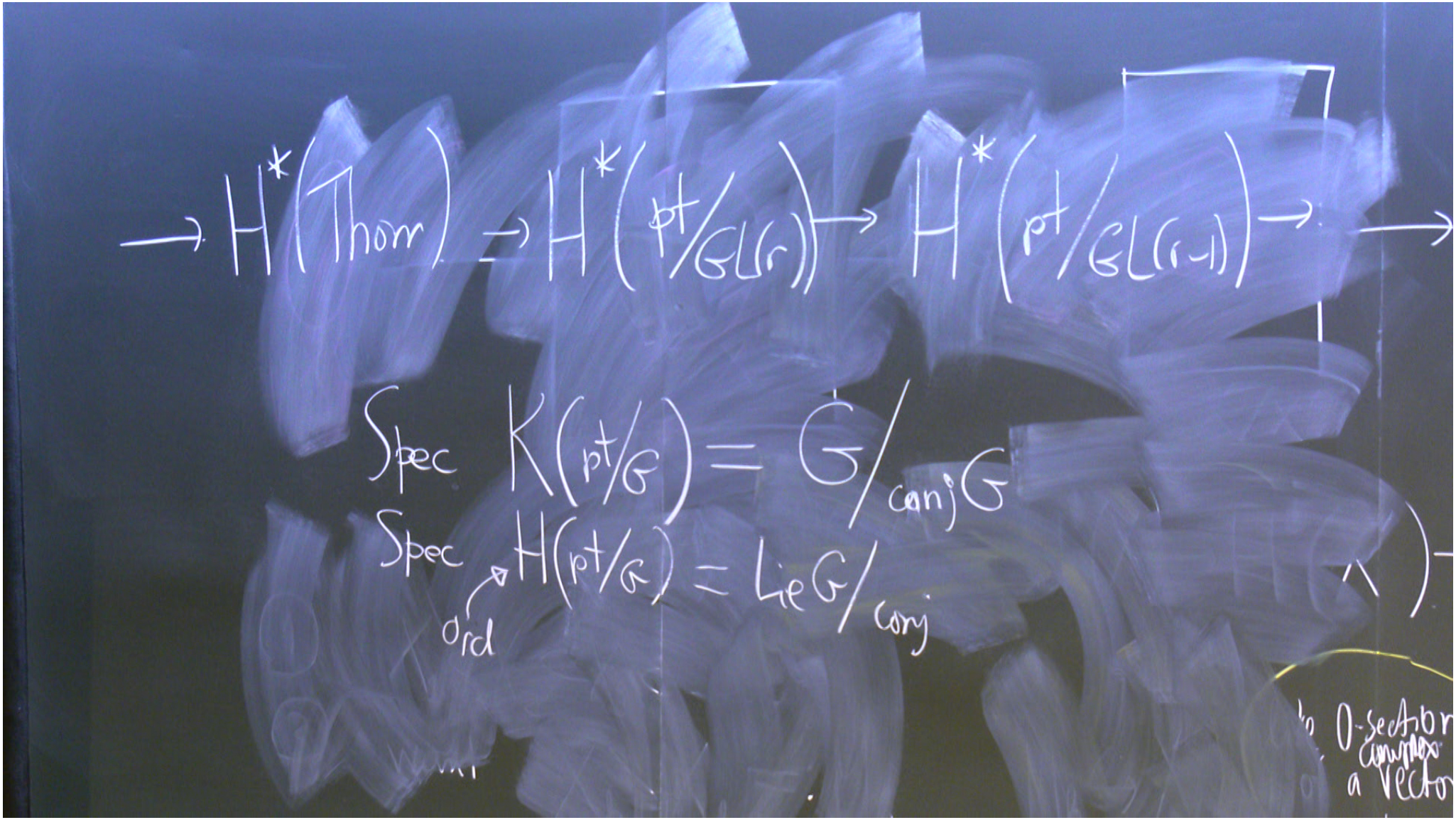

X'
X/X'
S^1X^1
$H^{i-1}(X') \to H^i(X,X') \to H^i(X) \to H^i(X') \to H^{i+1}(X,X') \to \dots$
$X_2 \subset X_3 \subset \dots$
$X_k \setminus X_{k-1} =: M_k$
$M_k \hookrightarrow X_k$
like 0-section of a vector bundle
GIT

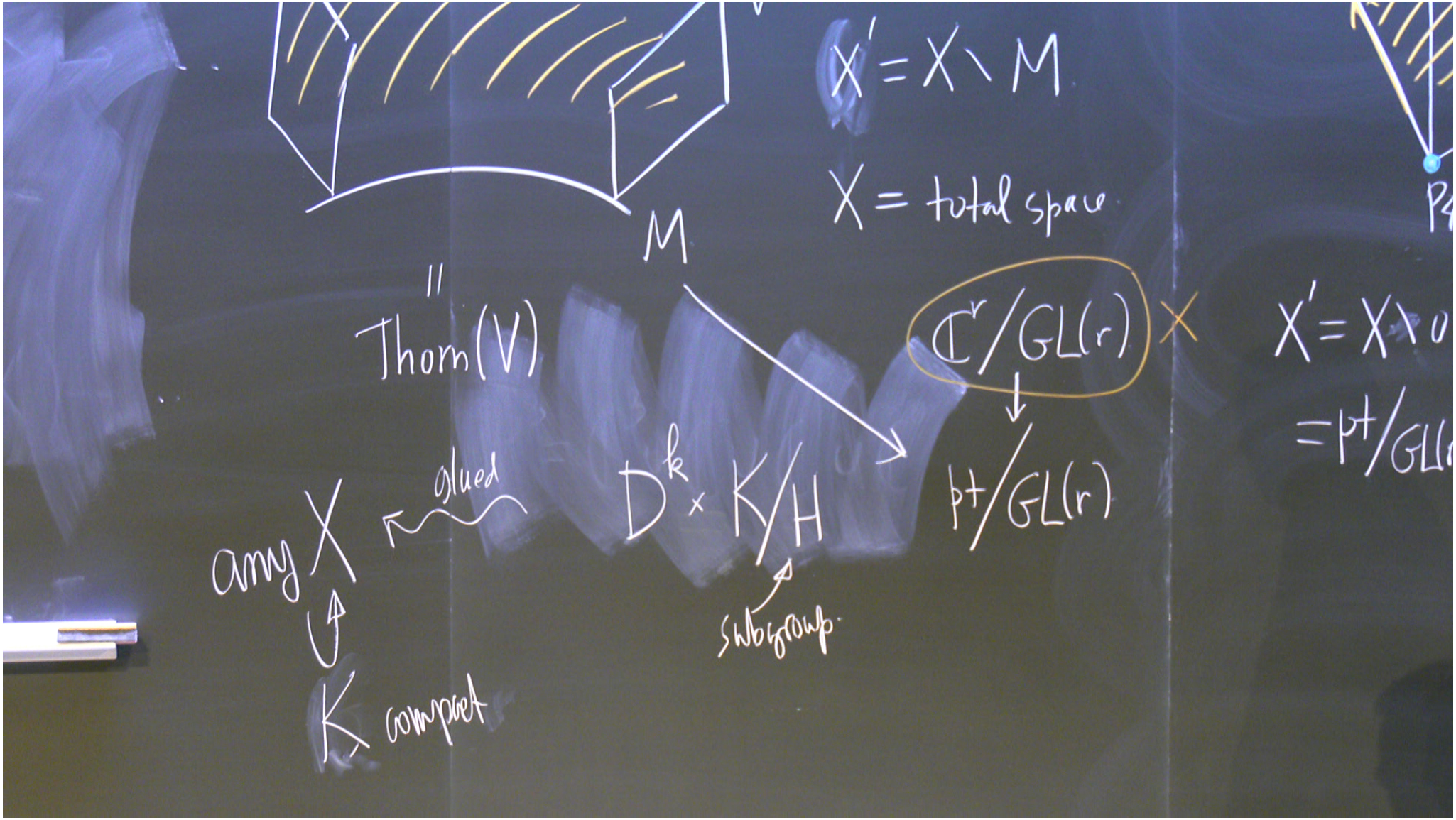
$$\mathbb{P}^3 = \mathbb{C}^3 \cup \mathbb{C}^2 \cup \mathbb{C}^1 \cup \mathbb{C}^0$$

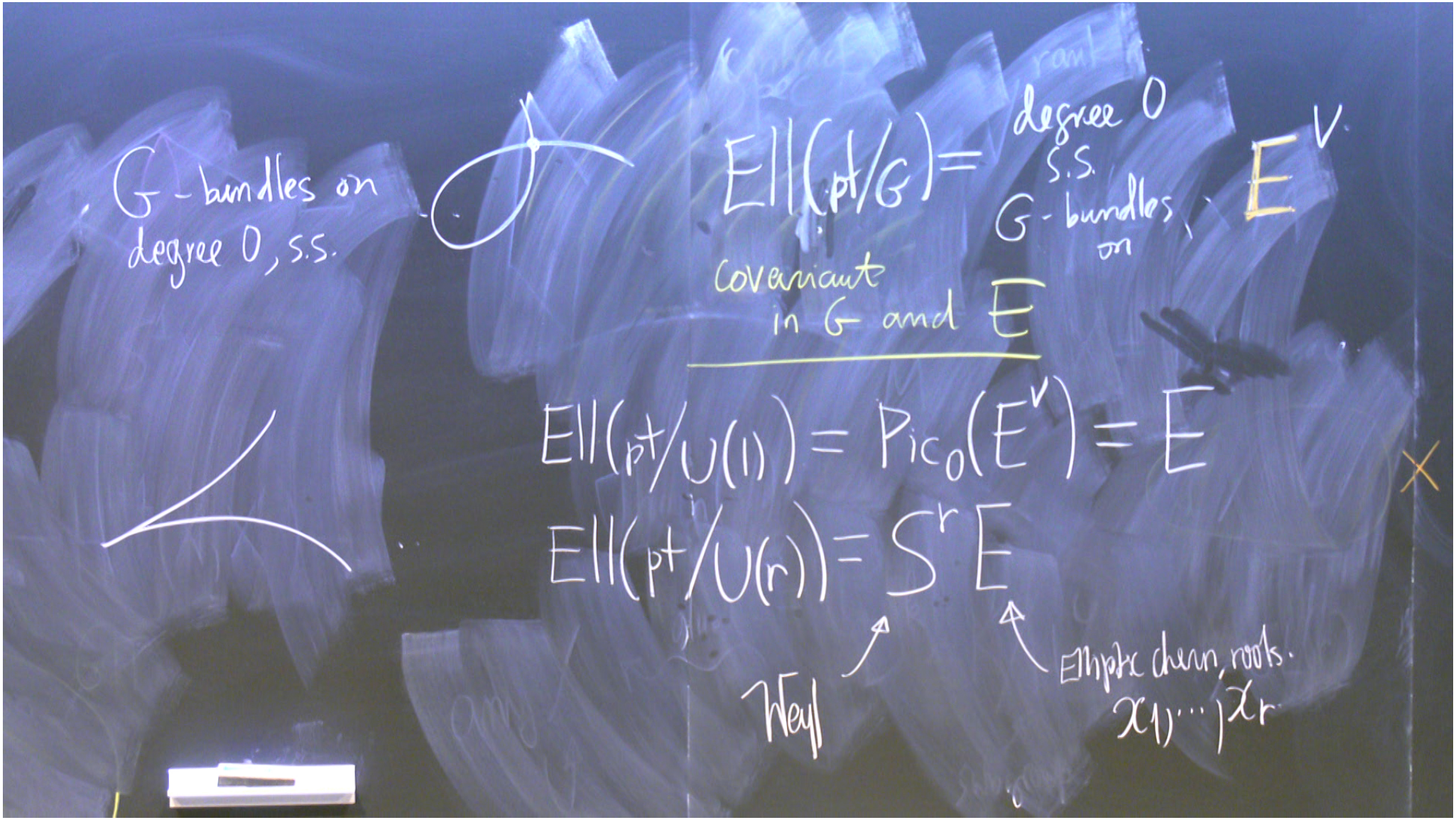


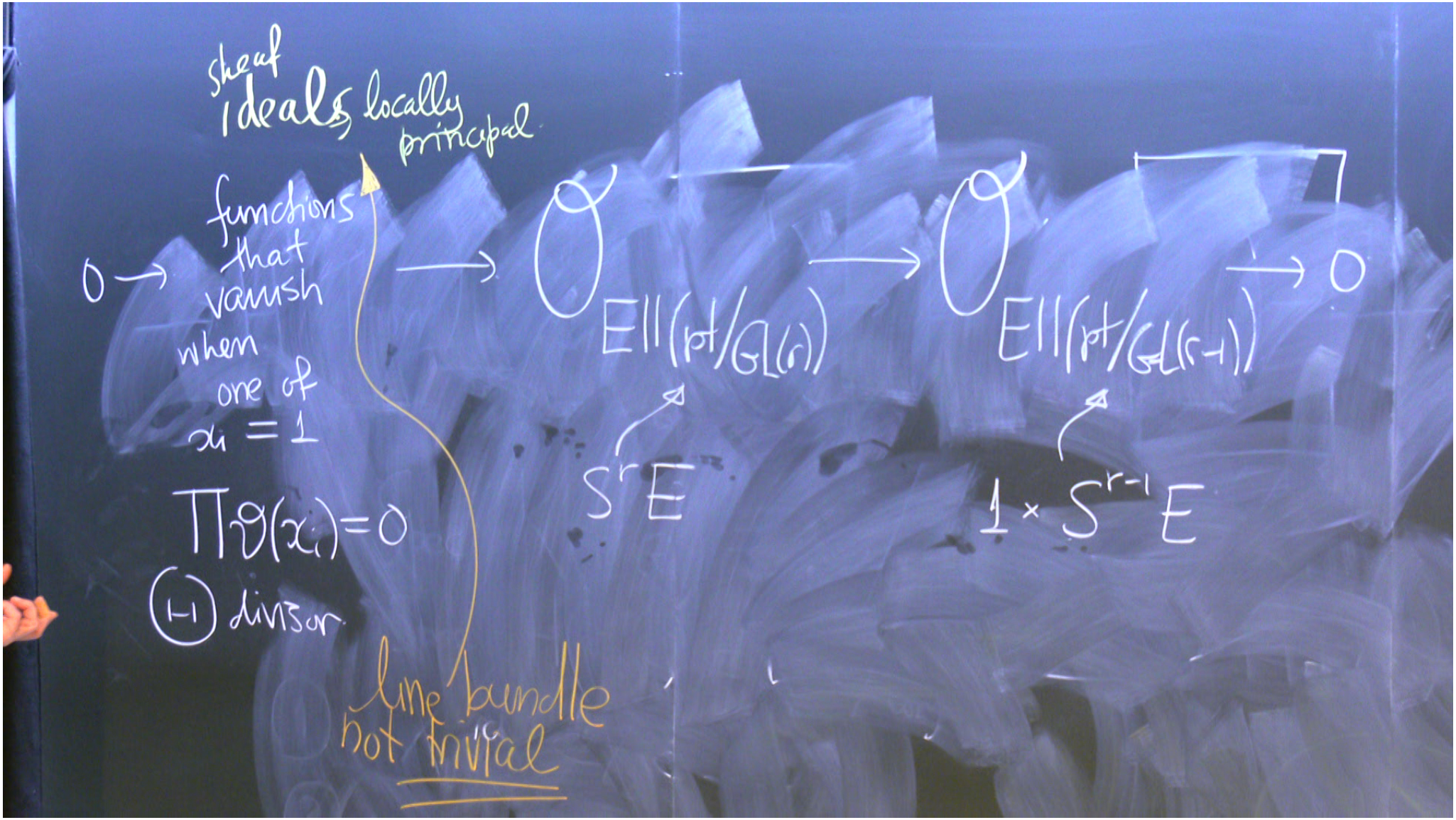

contract
rank r
V
$X' = X \setminus M$
X = total space
$S^1 X^1$
M
$\| $
Thom(V)
$\mathbb{C}^r/GL(r)$
$pt/GL(r)$
$\to H^{i+1}(X, X') \to \ldots$
GIT

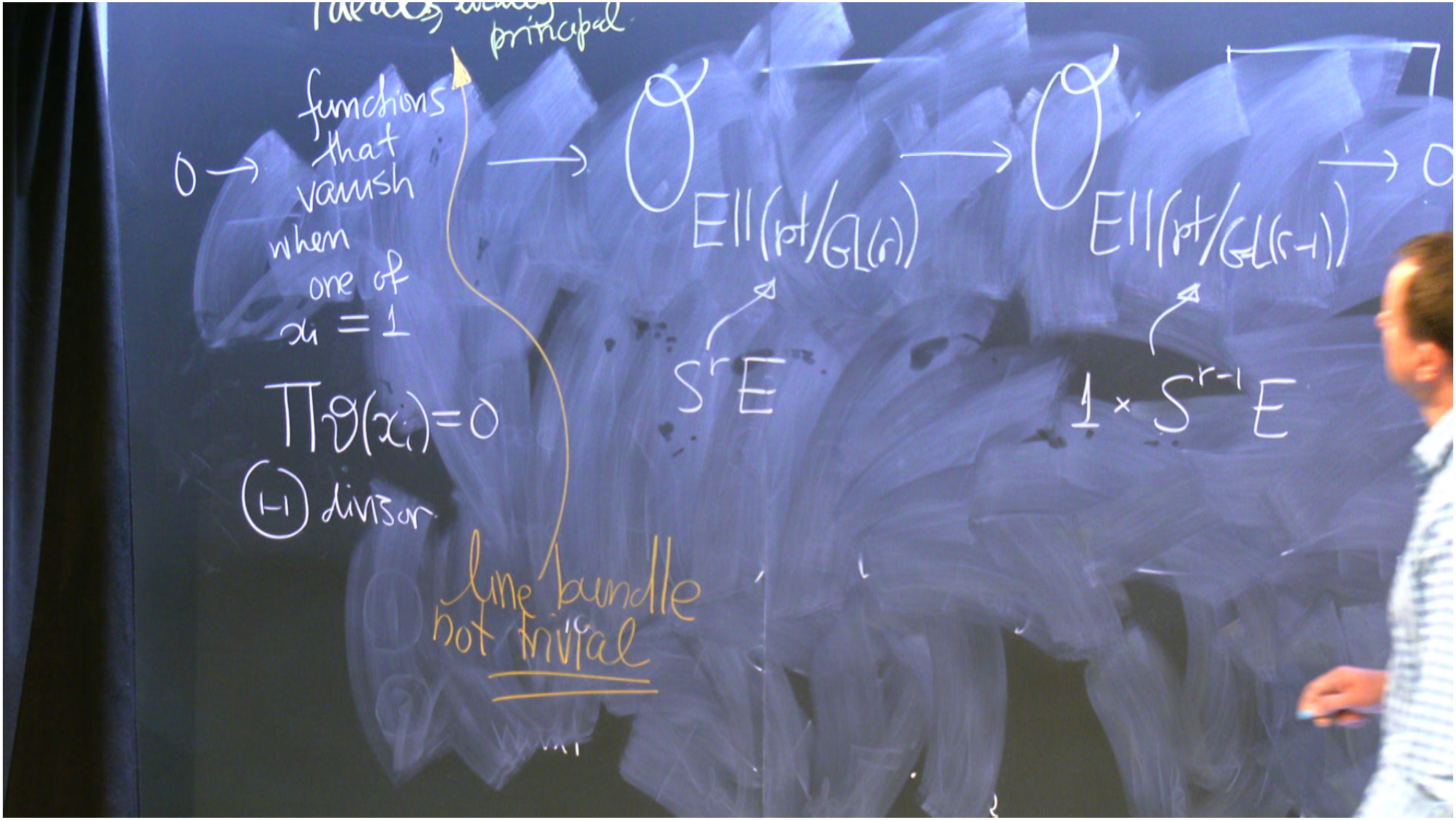

contract
rank r
V
M
$X' = X \setminus M$
X = total space
$\mathbb{C}^r/GL(r)$
$pt/GL(r)$
$X' = X \setminus 0$ section
$= pt/GL(r-1)$
p_2
p_3
p_4
$\mathbb{C}$

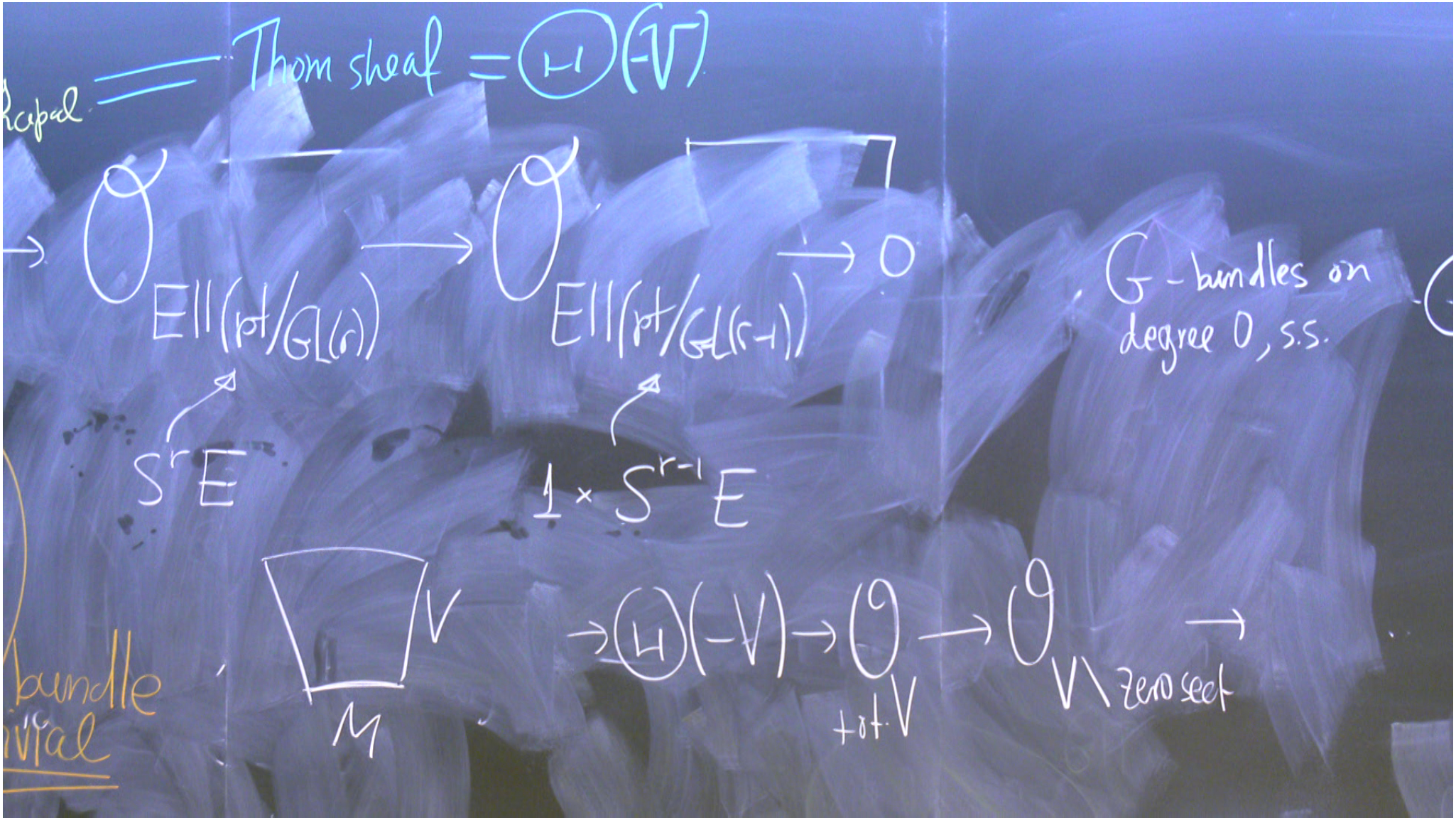

$\to H^*(\mathrm{Thom}) \to H^*(pt/GL(r)) \to H^*(pt/GL(r-1)) \to$
$\mathrm{Spec}\ K(pt/G) = G/_{\mathrm{conj}G}$
$\mathrm{Spec}\ H(pt/G) = \mathrm{Lie}G/_{\mathrm{Conj}}$
ord
0-section
complex
a vecto

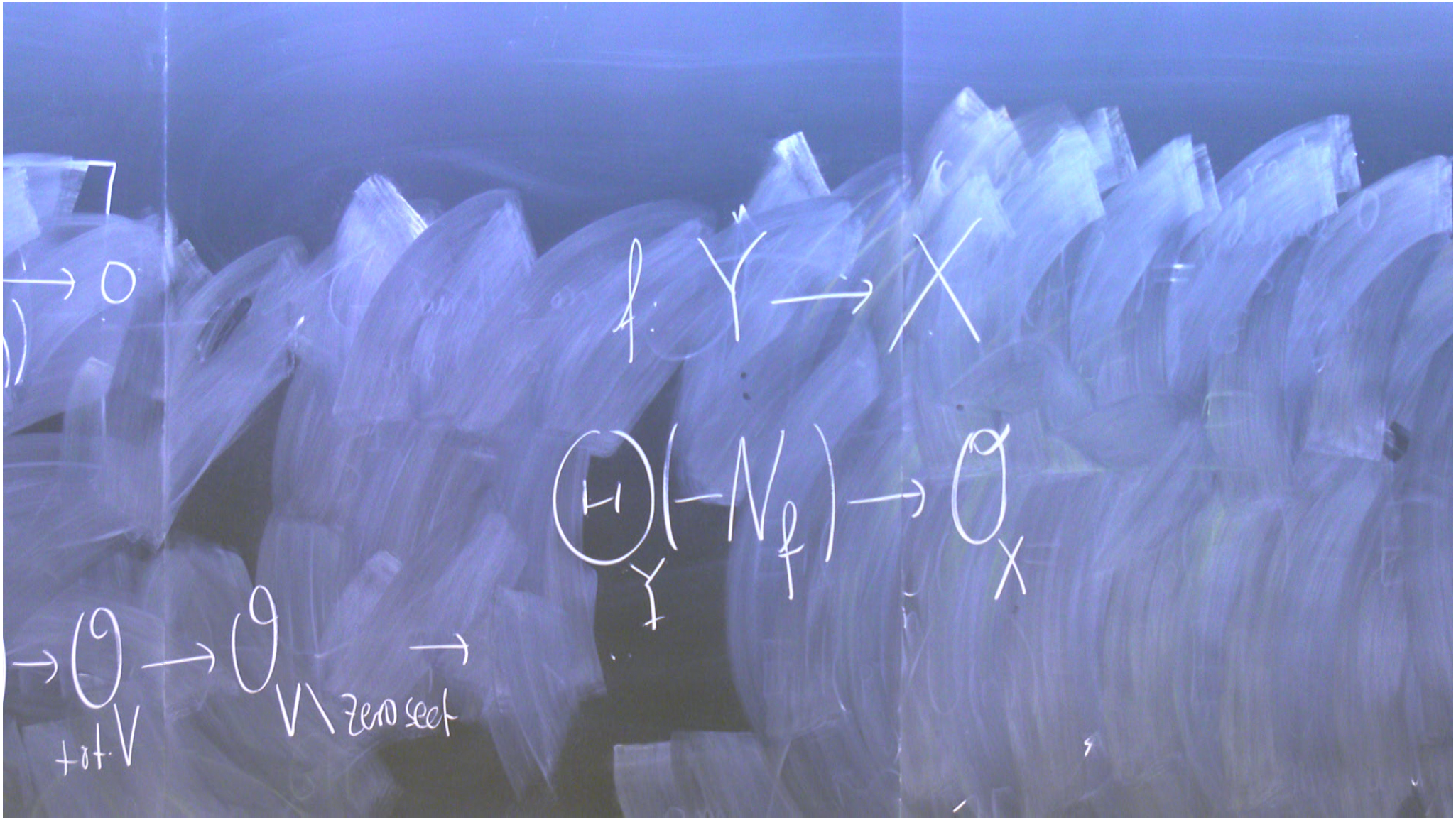

$X' = X \setminus M$
X = total space
M
$\parallel$
Thom(V)
$\mathbb{C}^r/GL(r)$ $\times$
$X' = X \setminus$
$= pt/GL$
any X
glued
$D^k \times K/H$
$pt/GL(r)$
K compact
subgroup

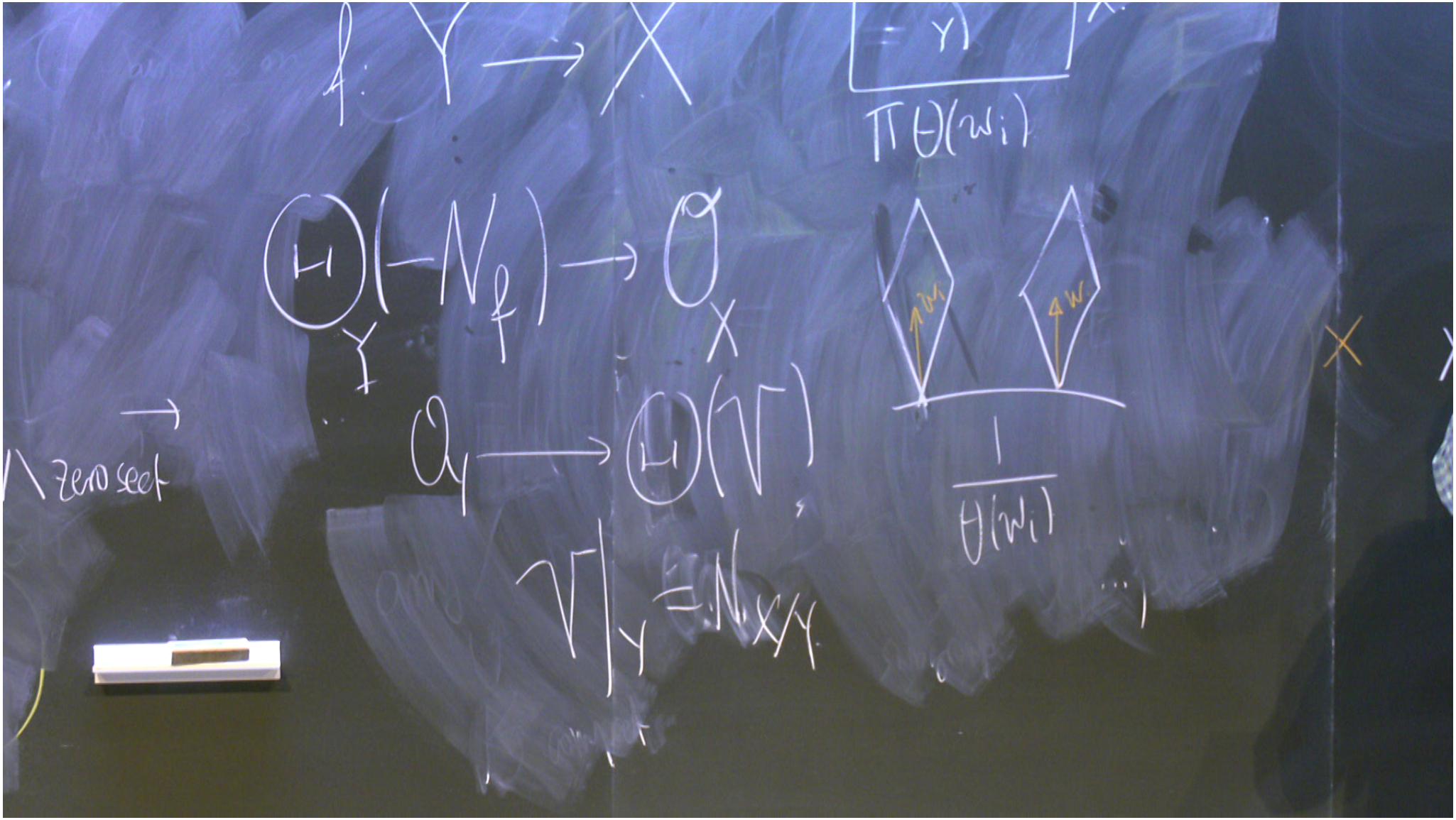

G - bundles on degree 0, s.s.
Ell(pt/G) = degree 0 s.s. G-bundles on E^V
covariant in G and E
Ell(pt/U(1)) = Pic_0(E^V) = E
Ell(pt/U(r)) = S^r E
Weyl
Elliptic chern roots. x_1 ... x_r

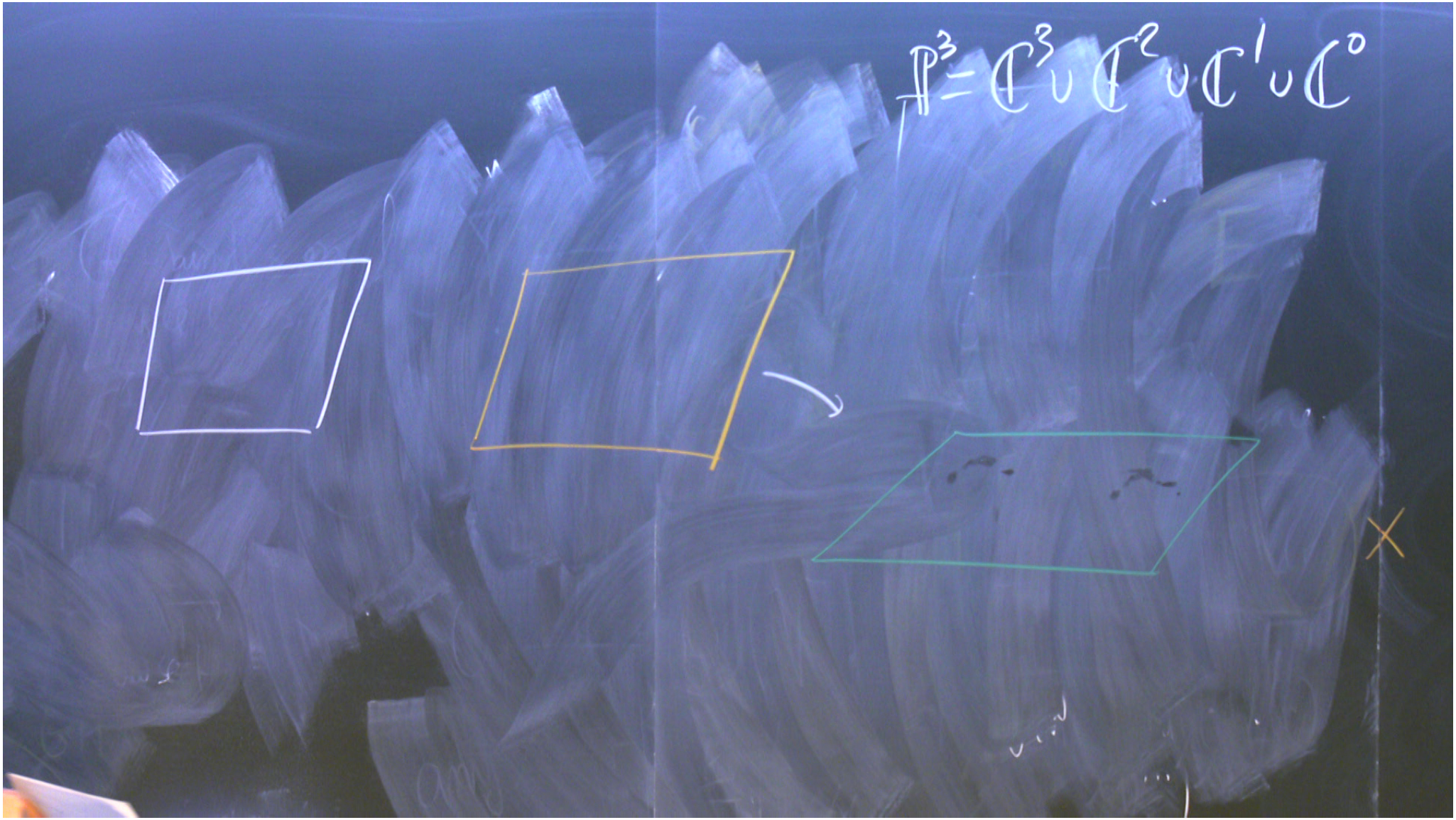

sheaf
ideals
locally
principal
functions
that
vanish
when
one of
$x_i = 1$
$\prod \vartheta(x_i) = 0$
(1-1) divisor
line bundle
not trivial
$0 \to$
$\to 0 \to$
$Ell(pt/GL(r))$
$Ell(pt/GL(r-1))$
$\to 0$
$S^r E$
$1 \times S^{r-1} E$


principal
functions that vanish when one of $x_i = 1$
$\prod \vartheta(x_i) = 0$
(-1) divisor
line bundle not trivial
$0 \to$
$\to$
$\to$
$\to$
$\mathbb{P}(\mathcal{O} \oplus S^r E)$
$\mathbb{P}(\mathcal{O} \oplus S^{r-1} E)$
$S^r E$
$1 \times S^{r-1} E$

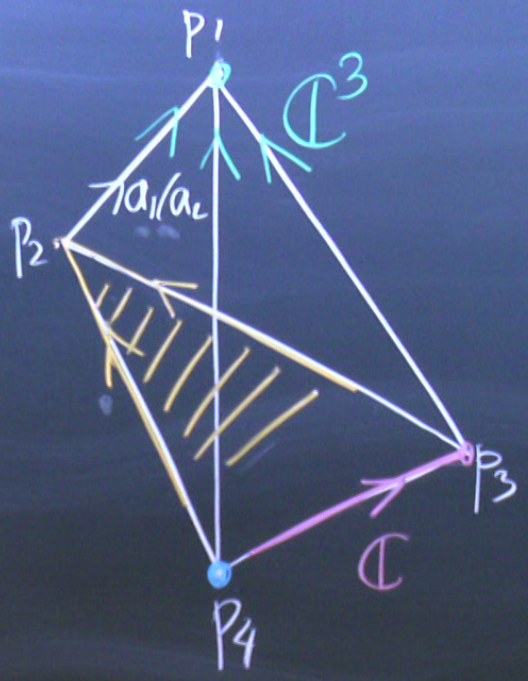

Thom sheaf = $\mathcal{O}(-V)$
$\to \mathcal{O} \to \mathcal{O} \to 0$
$Ell(pt/GL(r))$
$Ell(pt/GL(r-1))$
$S^r E$
$1 \times S^{r-1} E$
G - bundles on degree 0, s.s.
V
M
$\to \mathcal{O}(-V) \to \mathcal{O} \to \mathcal{O}$
tot V
V \ zero sect
bundle
ivial


$f: Y \to X$
$\mathcal{O}_Y(-N_f) \to \mathcal{O}_X$
$\to \mathcal{O} \to \mathcal{O}$
$V \setminus \text{zero set}$
$\to$
$\to 0$
$+ \text{zero set}$

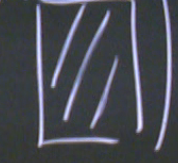

$f: Y \to X$
$\Pi \Theta(w_i)$
$\mathcal{O}(-N_f) \to \mathcal{O}$
Y
X
w_1
w_2
$\mathcal{O}_Y \to \mathcal{O}(\Gamma)$
$\frac{1}{\Theta(w_i)}$
zero sect
$\Gamma|_Y = N_{X/Y}$

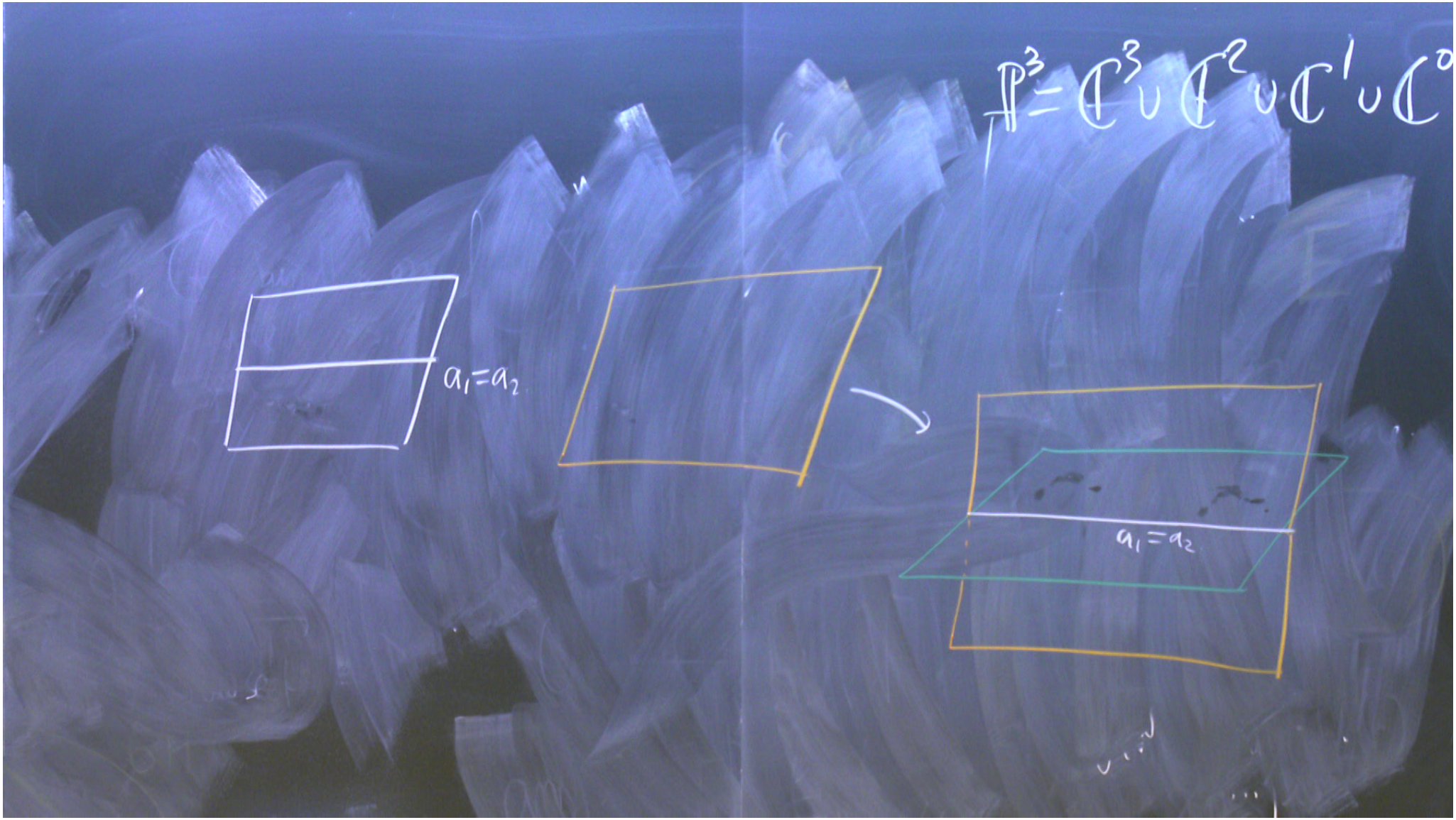

$\mathbb{P}^3 = \mathbb{C}^3 \cup \mathbb{C}^2 \cup \mathbb{C}^1 \cup \mathbb{C}^0$

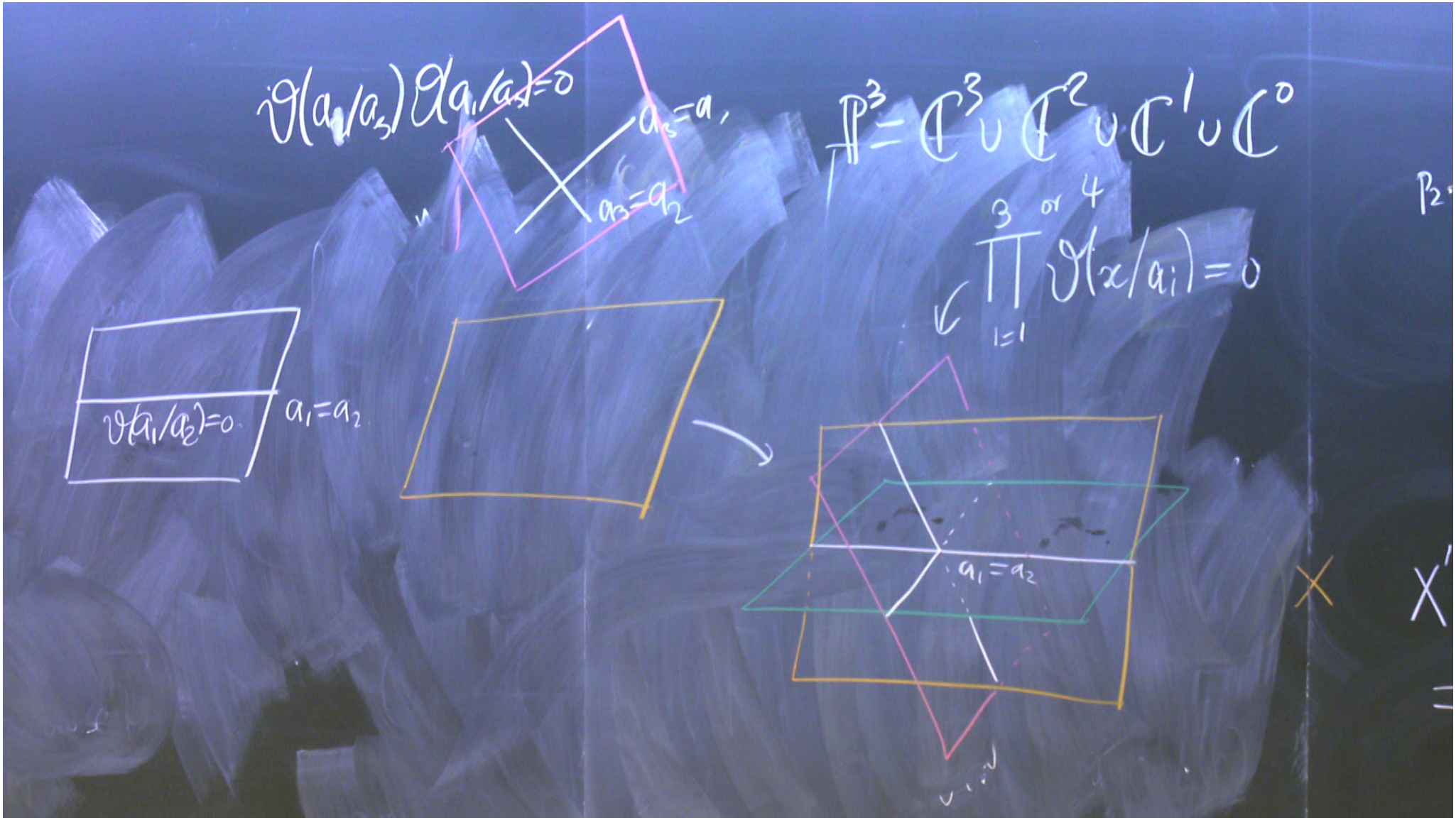
$\mathbb{P}^3 = \mathbb{C}^3 \cup \mathbb{C}^2 \cup \mathbb{C}^1 \cup \mathbb{C}^0$

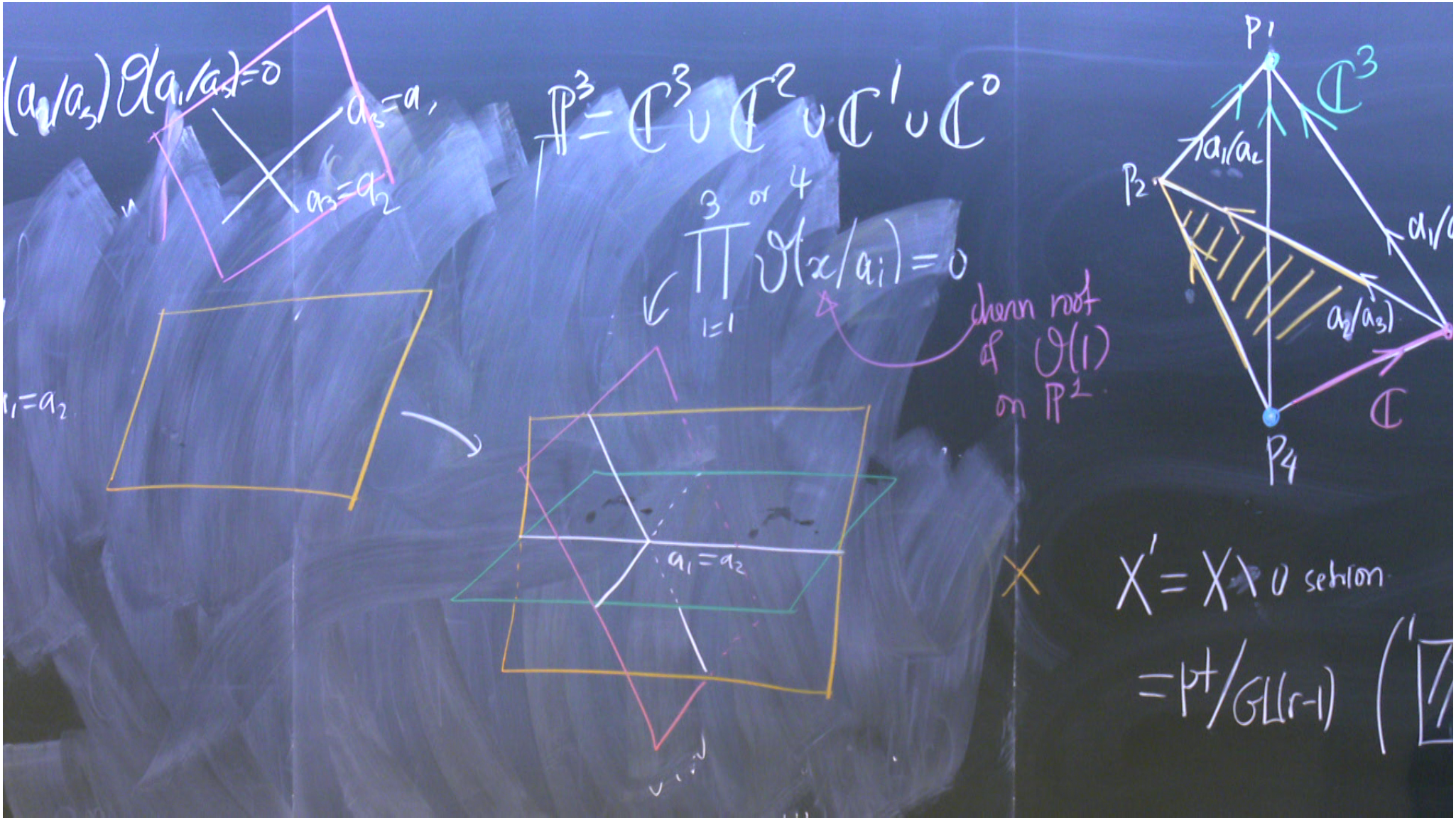


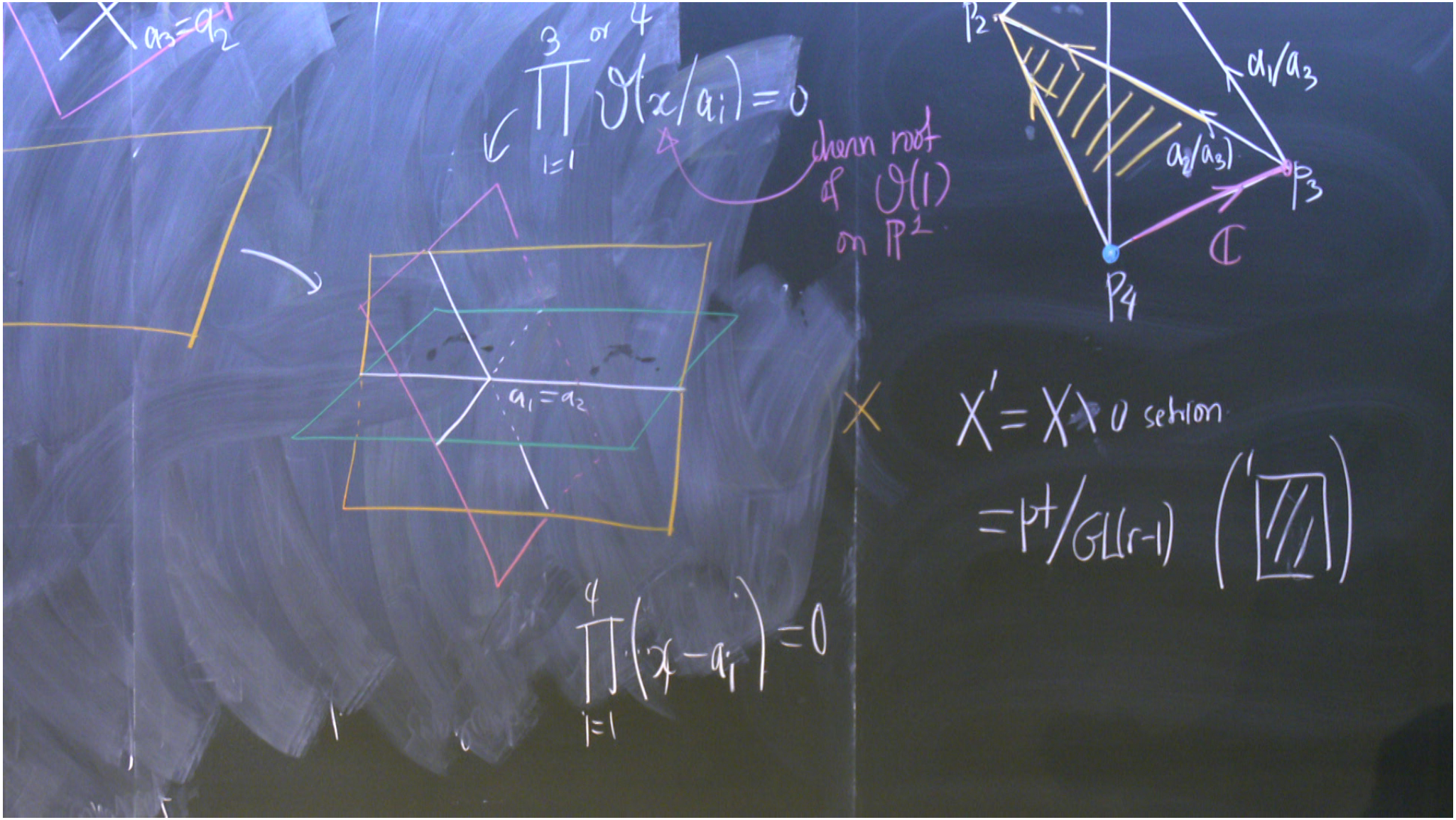
$X' = X \setminus$ U section

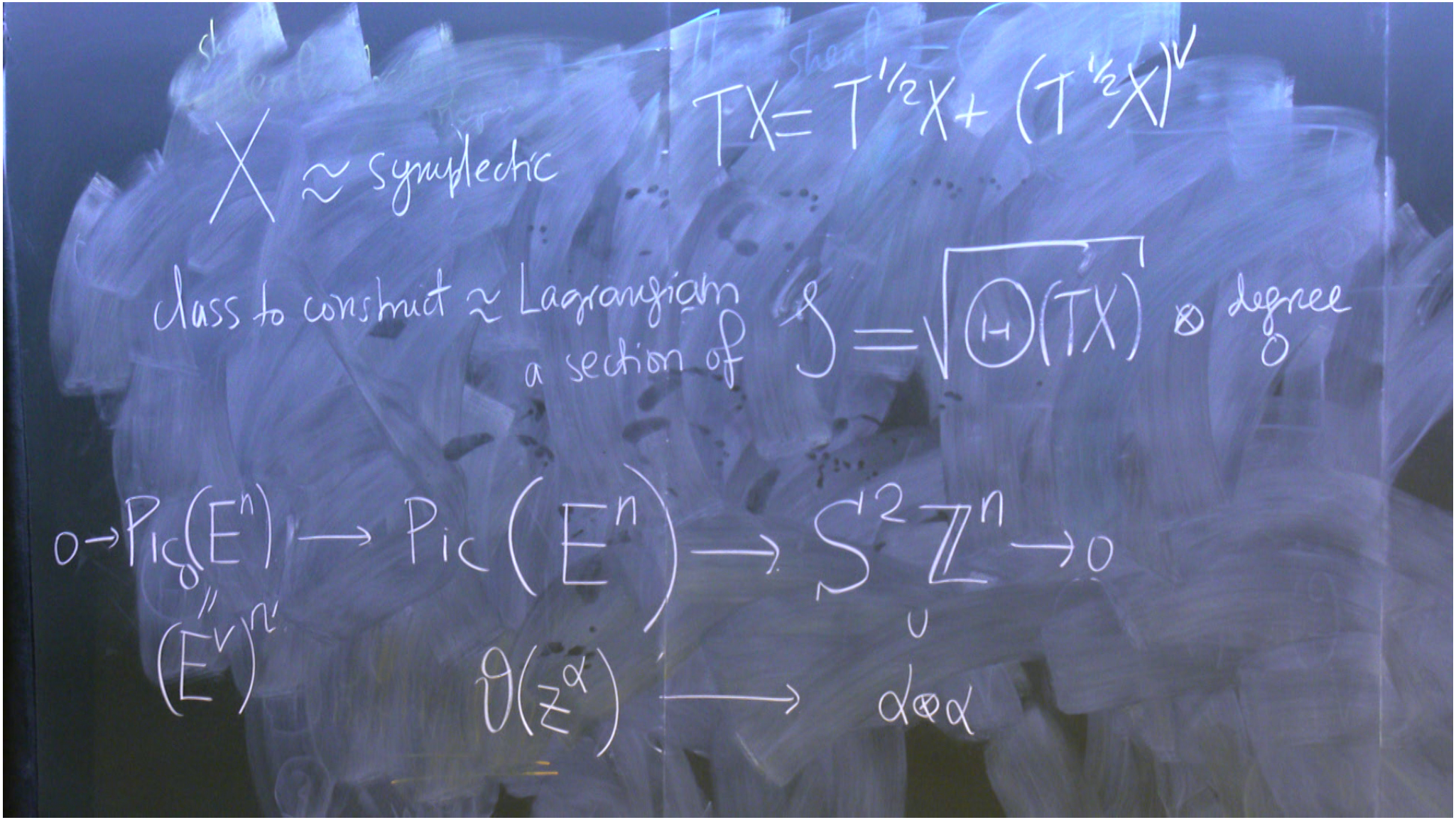
$= pt/GL(r-1)$ ()

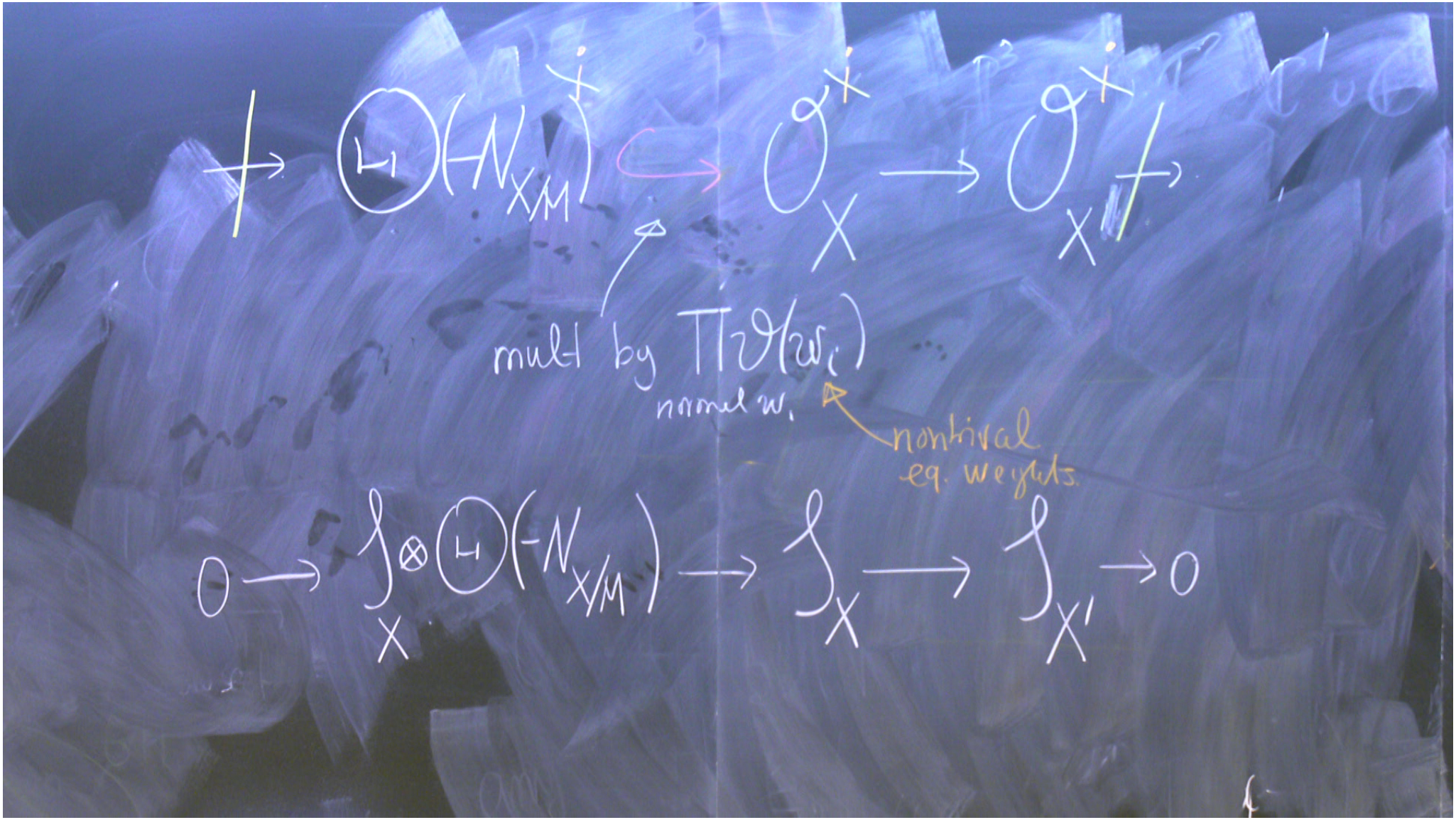

$\mathbb{P}^3 = \mathbb{C}^3 \cup \mathbb{C}^2 \cup \mathbb{C}^1 \cup \mathbb{C}^0$
$a_1 = a_2$
$a_1 = a_2$


$\vartheta(a_1/a_3)\vartheta(a_1/a_2)=0$
$a_3=a_1$
$a_3=a_2$
$\mathbb{P}^3=\mathbb{C}^3\cup\mathbb{C}^2\cup\mathbb{C}^1\cup\mathbb{C}^0$
$\prod_{i=1}^{3 \text{ or } 4}\vartheta(x/a_i)=0$
$\vartheta(a_1/a_2)=0$
$a_1=a_2$
$a_1=a_2$
X
X'


$\mathbb{P}^3 = \mathbb{C}^3 \cup \mathbb{C}^2 \cup \mathbb{C}^1 \cup \mathbb{C}^0$
$\prod_{i=1}^{3 \text{ or } 4} \vartheta(x/a_i) = 0$
chern root of $\mathcal{O}(1)$ on $\mathbb{P}^2$
$a_3 = a_1$
$a_3 = a_2$
$a_1 = a_2$
$a_1 = a_2$
P1
P2
P4
$\mathbb{C}^3$
$\mathbb{C}$
$X' = X \setminus 0$ section
$= pt/GL(r-1)$


$a_3 = a_2$
$\prod_{i=1}^{3 \text{ or } 4} \vartheta(x/a_i) = 0$
chern root of $\mathcal{O}(1)$ on $\mathbb{P}^1$
$a_1 = a_2$
$\prod_{i=1}^{4} (x - a_i) = 0$
P_2
a_1/a_3
a_2/a_3
P_3
P_4
$\mathbb{C}$
$X' = X \setminus 0$ section
$= pt/GL(r-1)$


$X \approx$ symplectic
$TX = T^{1/2}X + (T^{1/2}X)^\vee$
class to construct $\approx$ Lagrangian
a section of $\mathcal{S} = \sqrt{\Theta(TX)} \otimes$ degree 0
$0 \to \mathrm{Pic}_0(E^n) \to \mathrm{Pic}(E^n) \to S^2\mathbb{Z}^n \to 0$
$(E^\vee)^{n'}$
$\vartheta(z^\alpha) \longrightarrow \alpha \otimes \alpha$


mult by $\prod \vartheta(zw_i)$
normal w_i
nontrivial eq. weights