Title: Viewing asymptotic symmetries through conformal mappings

Speakers: Hong Zhe Chen

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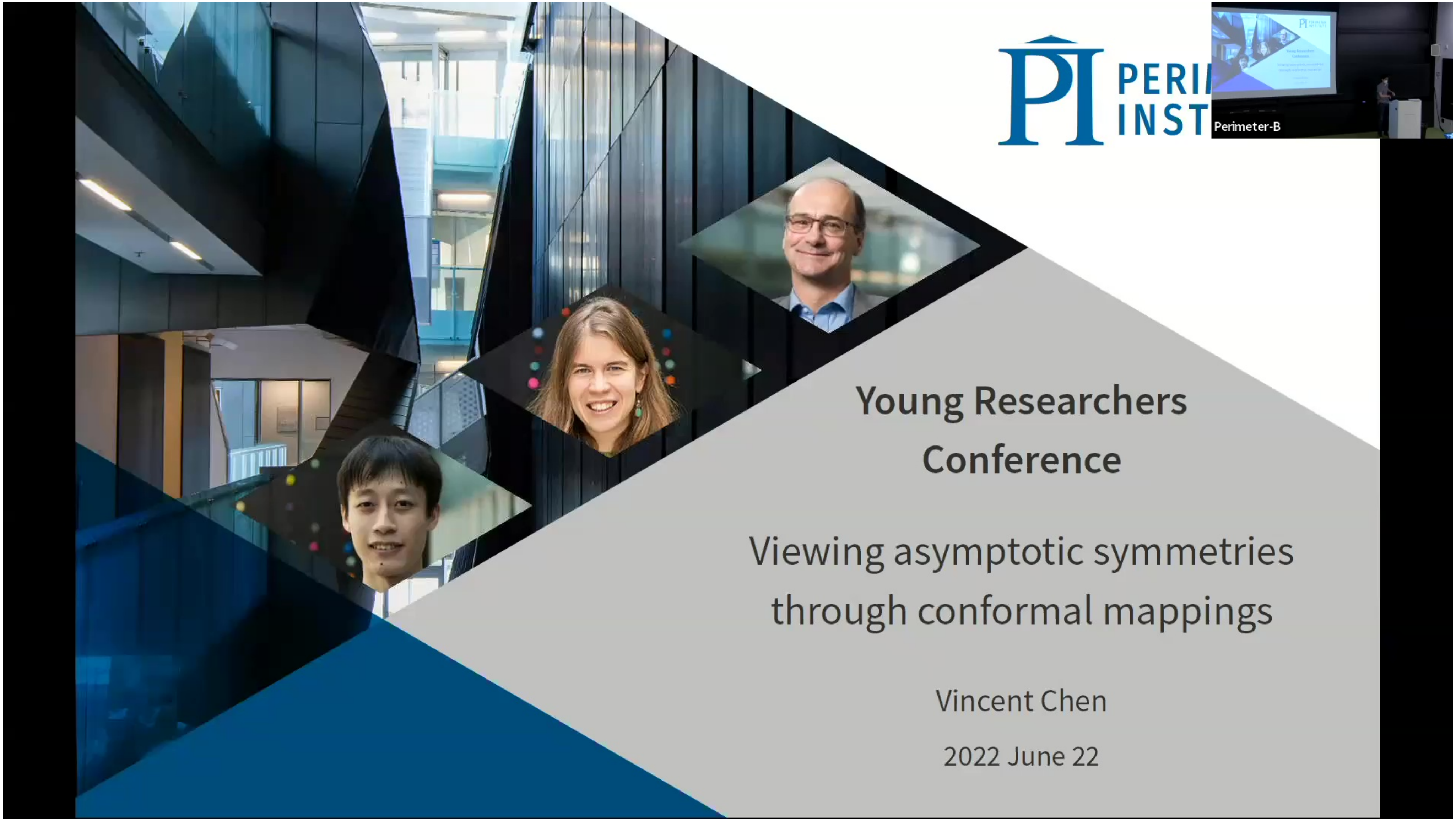
Abstract: Over the past decade, many infrared phenomena of gauge theories, such as soft theorems and memory effects, have been shown to be manifestations of asymptotic symmetries which persist to the spacetime boundary. In this talk, I will discuss ongoing work, in collaboration with Robert Myers and Ana-Maria Raclariu, which recasts the asymptotic symmetries of gauge theories in Minkowski spacetime through conformal mappings. Through a mapping to the Einstein static universe, I will describe how conservation of asymptotic charge can be viewed as a smoothness constraint for image sources passing through spacelike infinity. Additionally, I will sketch how asymptotic charge flux through a subregion of null infinity is mapped to edge modes. This will then allow us to quantify fluctuations in asymptotic charge flux by relation to edge mode entropies, which have been well-studied in literature. Altogether, the general theme of my talk will be how new insights can be obtained by conformally mapping asymptotic structures of gauge theories to various settings.


Perimeter-B
Young Researchers Conference
Viewing asymptotic symmetries through conformal mappings
Vincent Chen
2022 June 22

Overview

1. Introduction
2. The Einstein universe and image charges
3. Edge mode entropy at $\mathscr{I}^+$
4. Summary

Introduction

The infrared triangle

[Strominger:2017zoo]

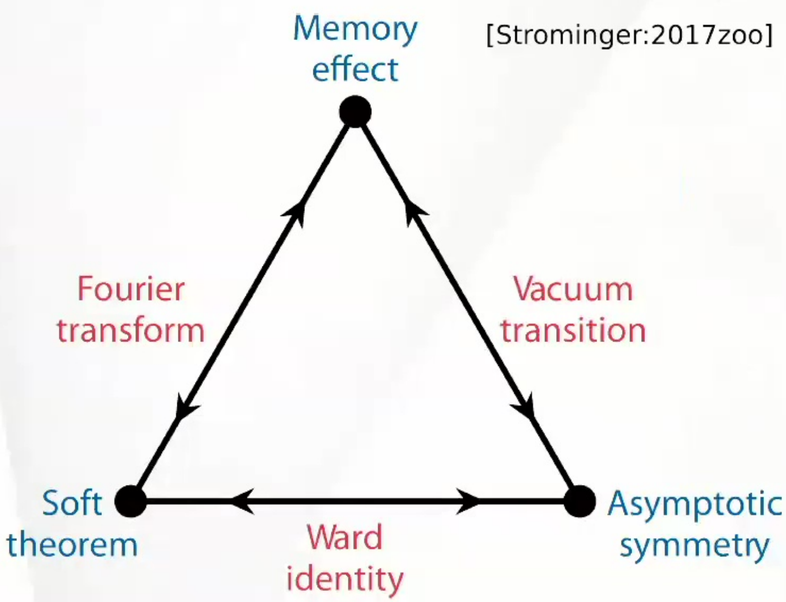


Figure 1: The IR triangle for gauge theories

Many infrared (IR) phenomena *e.g.*

- Memory effect
- Soft theorem

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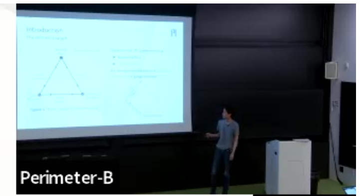

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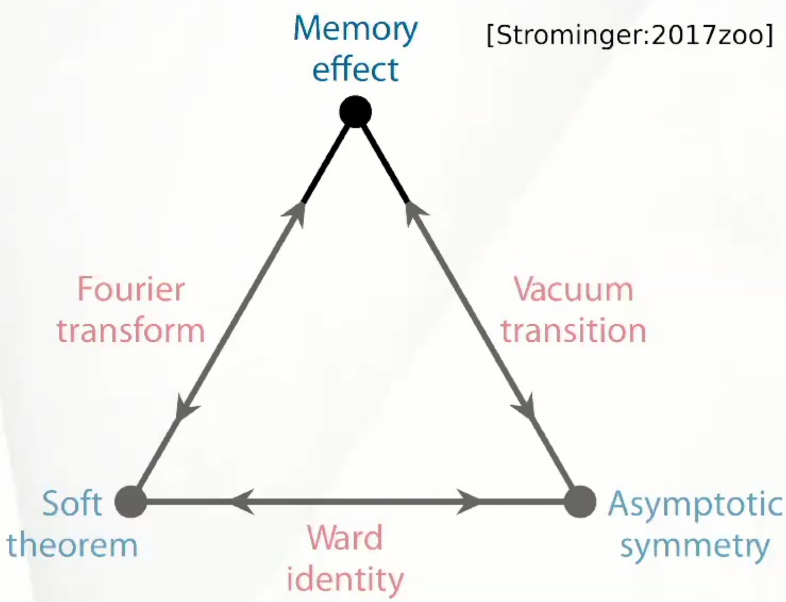

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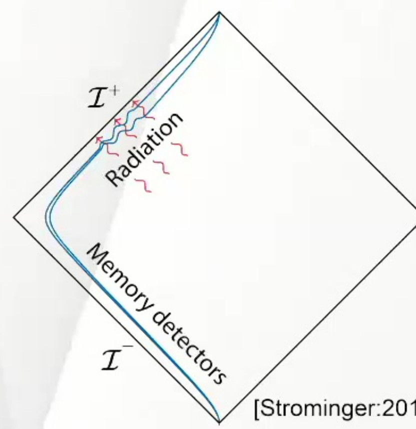

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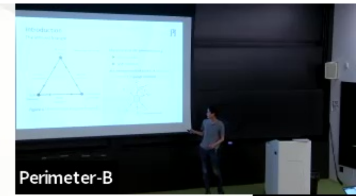

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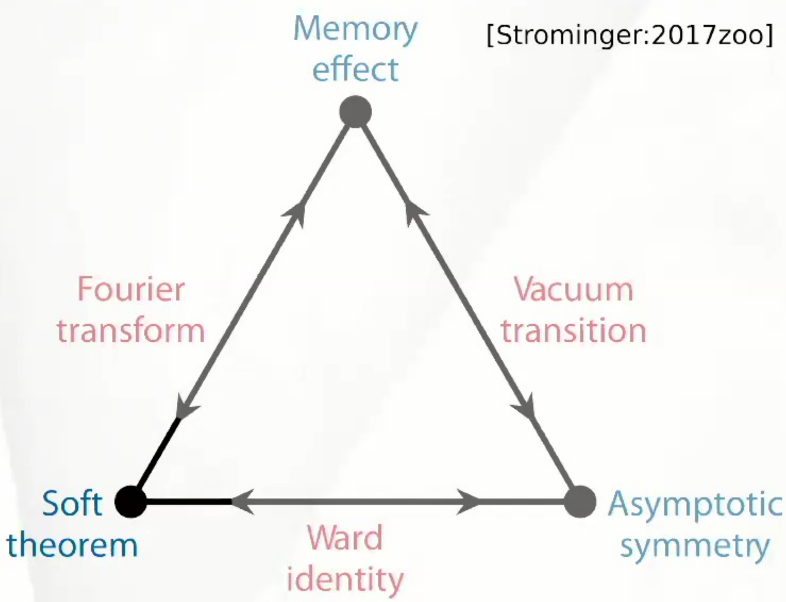

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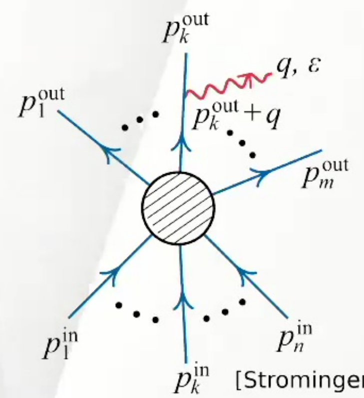

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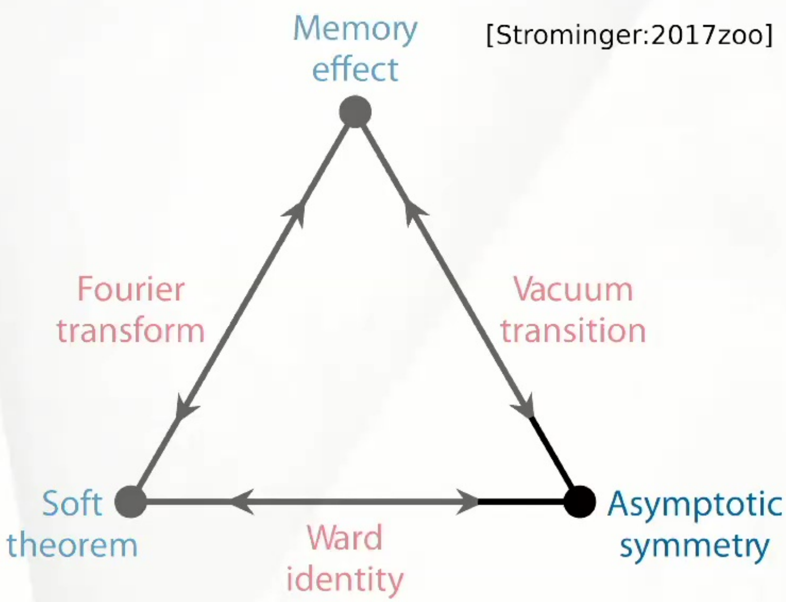

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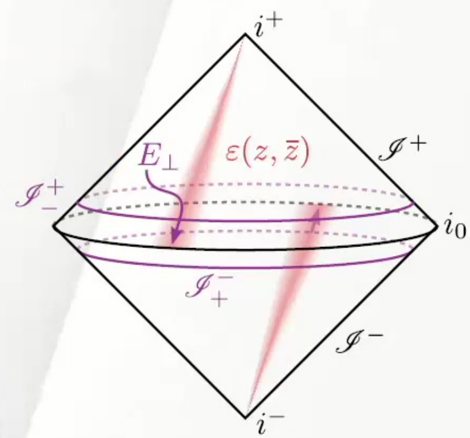

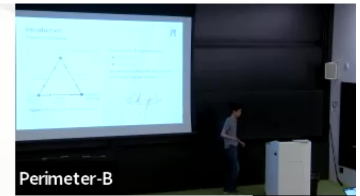

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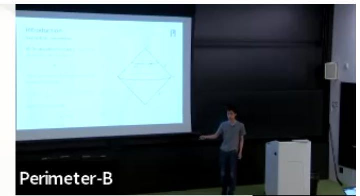


Introduction

Asymptotic symmetries

At the spacetime boundary, asymptotic symmetries do not vanish:

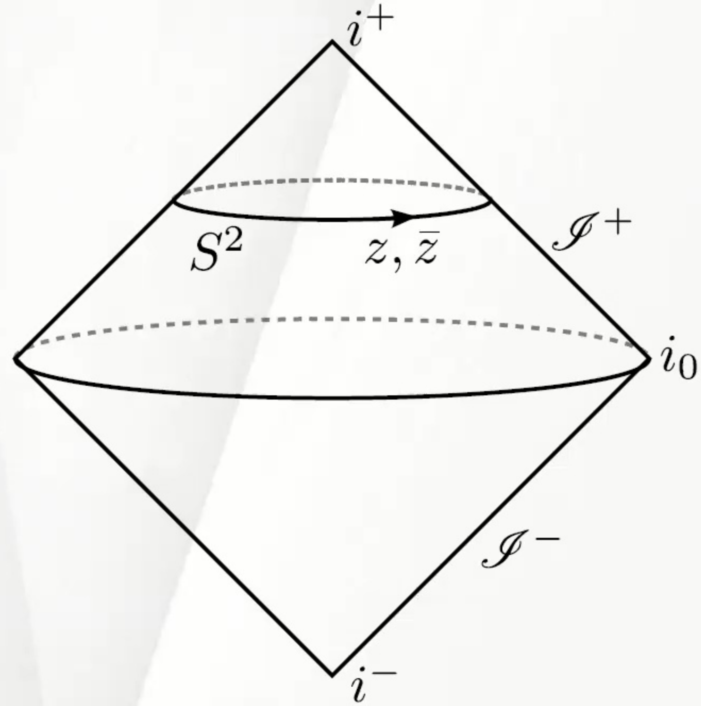
$$\delta \mathbf{A} \xrightarrow[r\to\infty]{} d\varepsilon(z,\bar{z}) \tag{1}$$

These symmetries have corresponding asymptotic charges:

$$Q_\varepsilon^{\pm} = \int_{\mathscr{I}^{\pm}_{\mp}} \varepsilon\, E_\perp = Q_\varepsilon^{\mathrm{soft}\pm} + Q_\varepsilon^{\mathrm{hard}\pm} \tag{2}$$

which are conserved:

$$Q_\varepsilon^{+} = Q_\varepsilon^{-} \tag{3}$$



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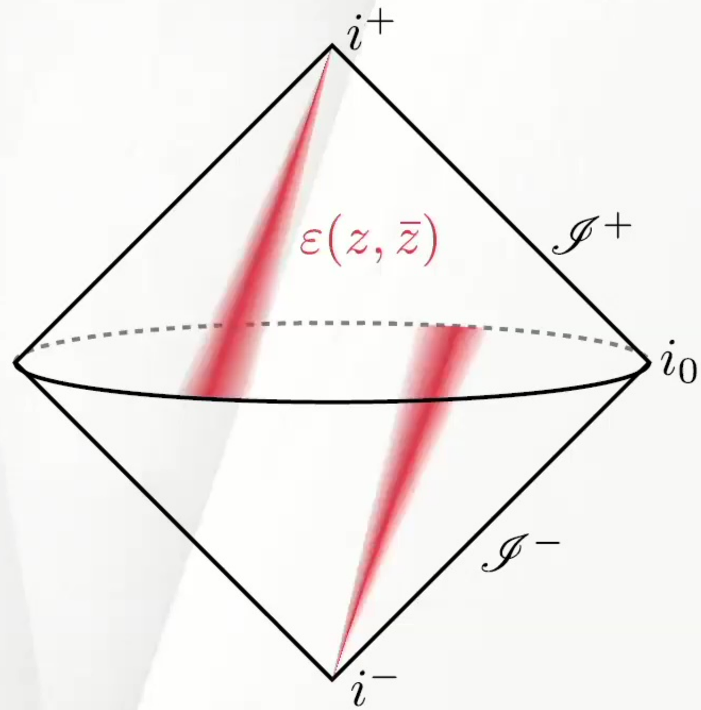

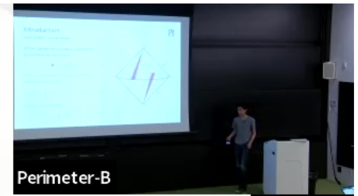

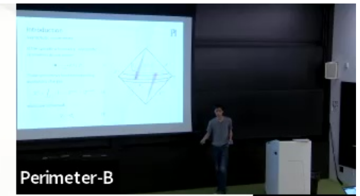

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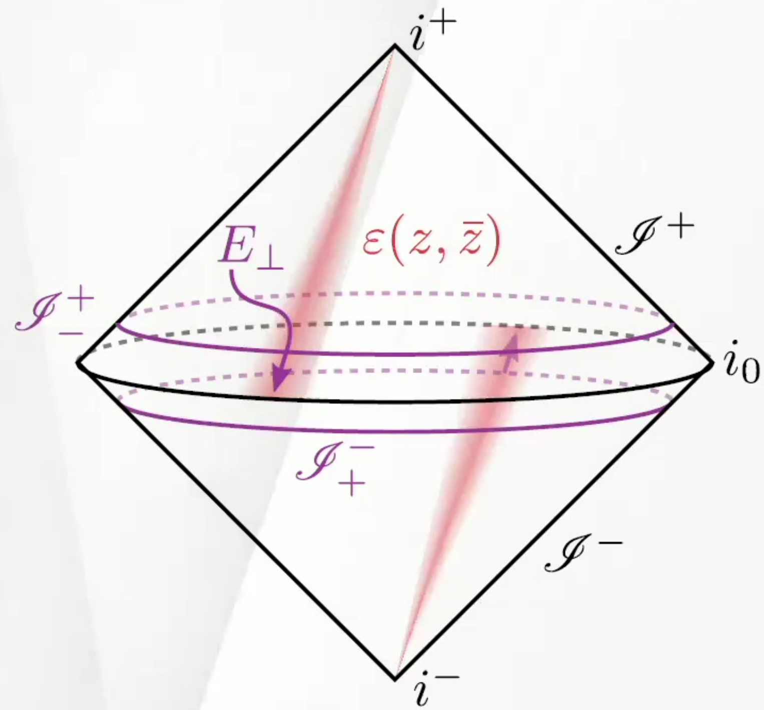

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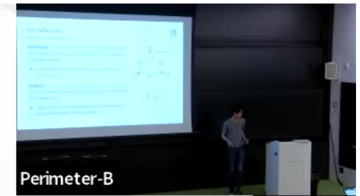
Introduction

Motivation and method



Motivation

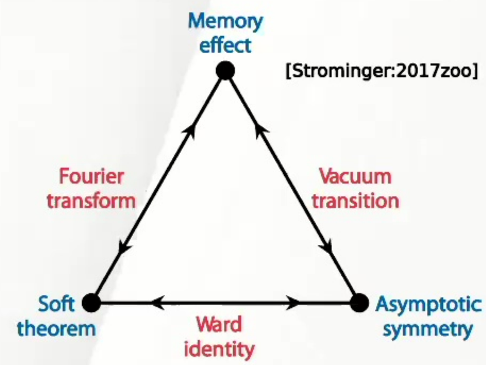
Asymptotic symmetries are related to many phenomena in gauge theories.

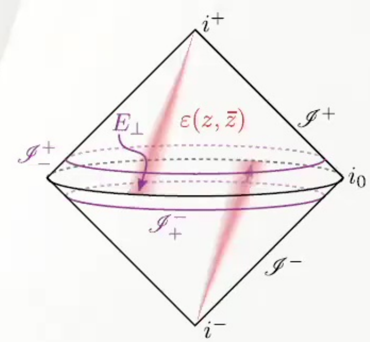
- want to better understand these symmetries and their charges

Method

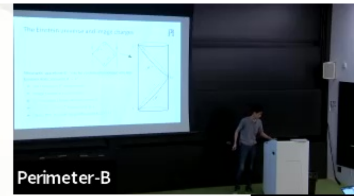
Points at infinity can be conformally mapped to points at finite separation.

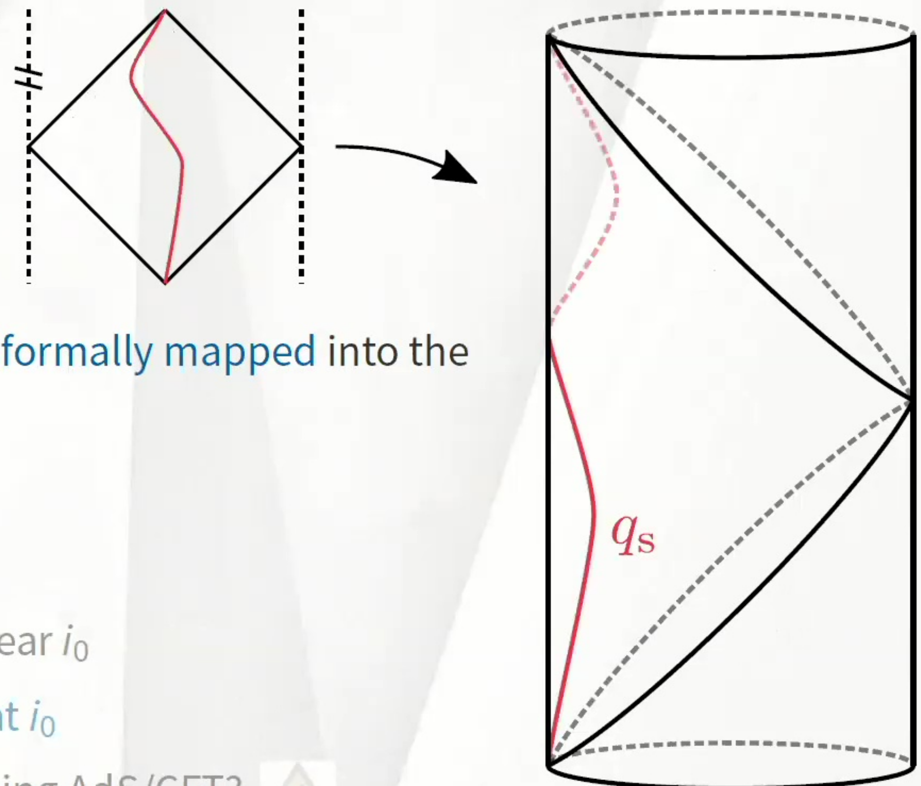
- large-distance concepts mapped to more familiar or better studied short-distance ideas





The Einstein universe and image charges





Minkowski spacetime $\mathbb{R}^{1,3}$ can be conformally mapped into the Einstein static universe $\mathbb{R} \times S^3$.

- net charge on S^3 must vanish
- image charge(s) run through i_0
- $Q_\varepsilon^\pm \leftrightarrow$ image charge velocities near i_0
- $Q_\varepsilon^+ = Q_\varepsilon^- \iff \mathrm{C}^1$ trajectories at i_0
- Check this in super Yang-Mills using AdS/CFT?

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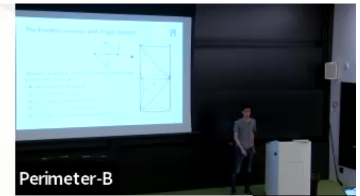

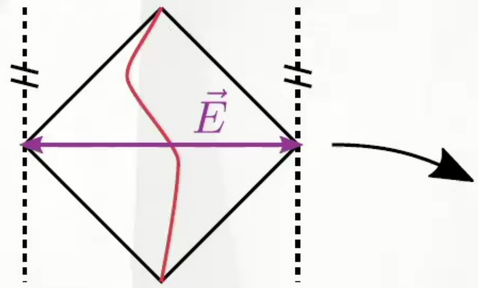

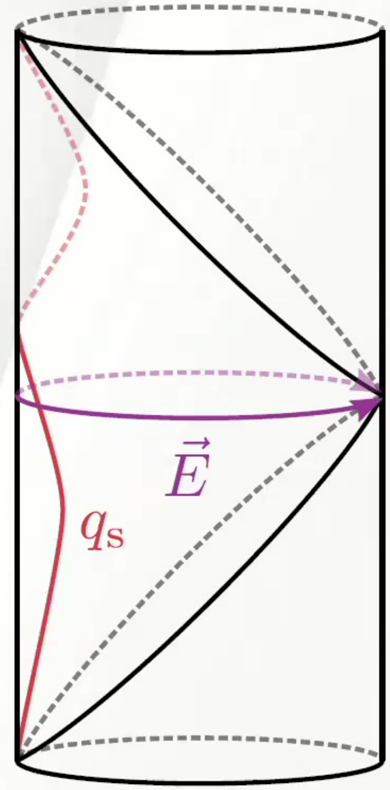

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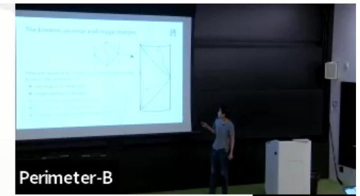

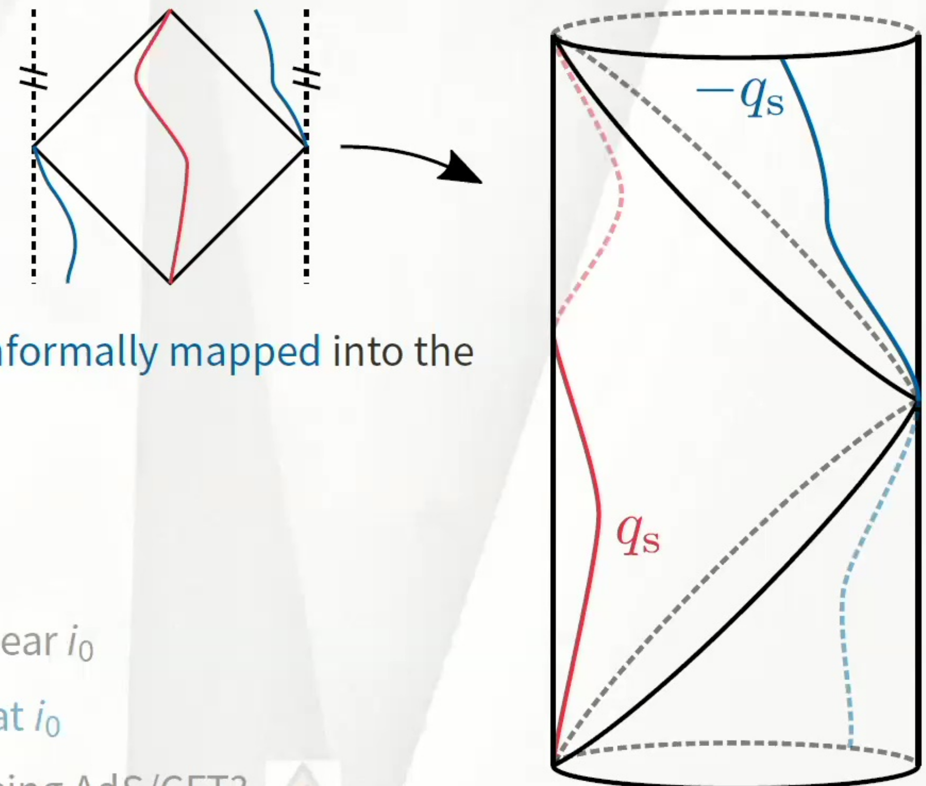

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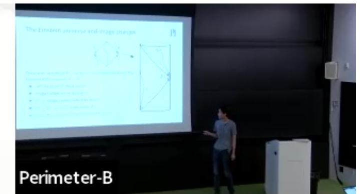

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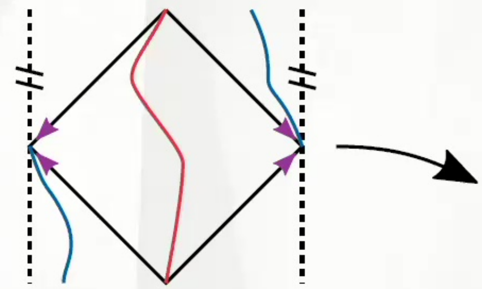

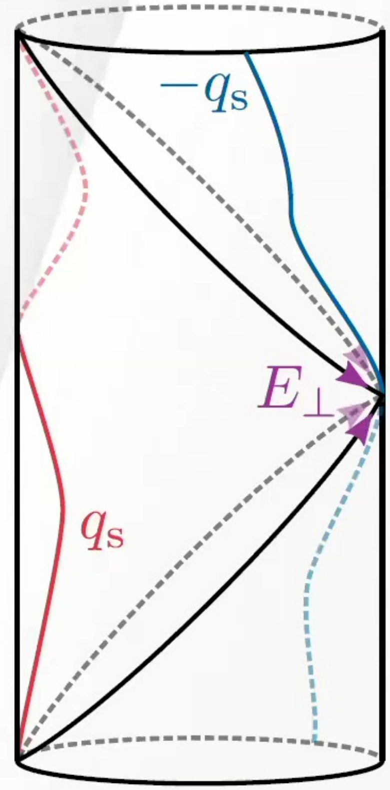

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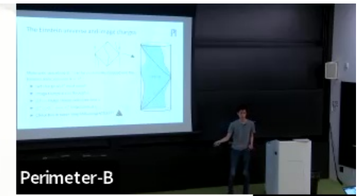

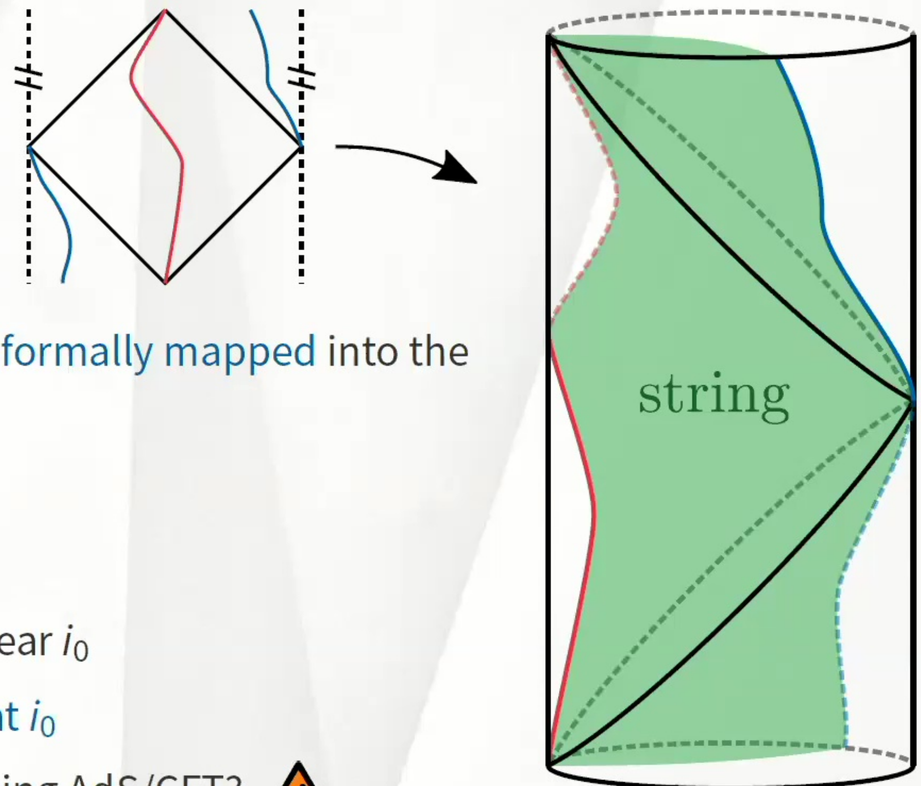

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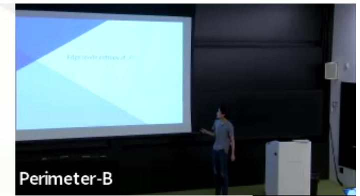

Edge mode entropy at $\mathscr{I}^+$

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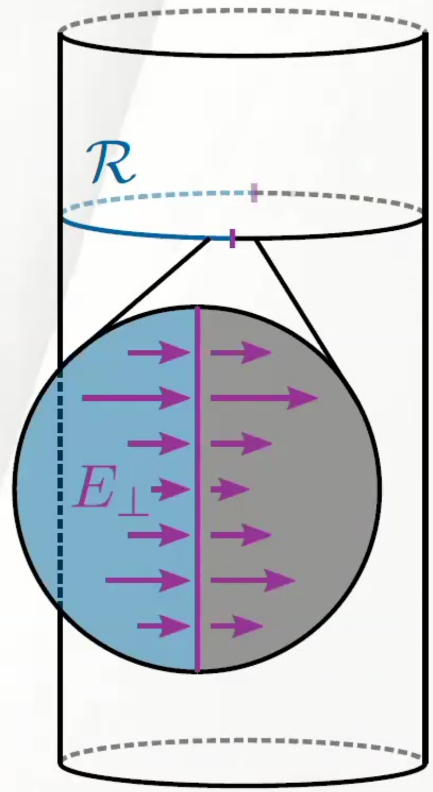
Defining edge modes

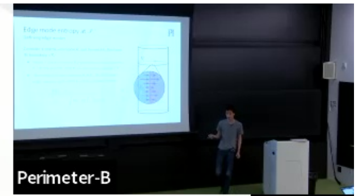
Consider a spatial subregion $\mathcal{R}$ and the electric field near its boundary $\partial\mathcal{R}$:

- Gauss's law constrains the perpendicular component $E_\perp$ of the electric field to be continuous across $\partial\mathcal{R}$.
- [Donnelly:2015hxa] showed that $E_\perp$ fluctuations, *i.e.* edge modes, contribute to entanglement entropy:

$$S[\mathcal{R}] = S_\perp + \cdots \tag{4}$$

$$S_\perp = -\int [dE_\perp]\, p(E_\perp) \log p(E_\perp) \tag{5}$$





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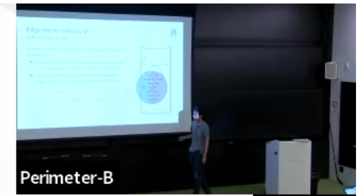

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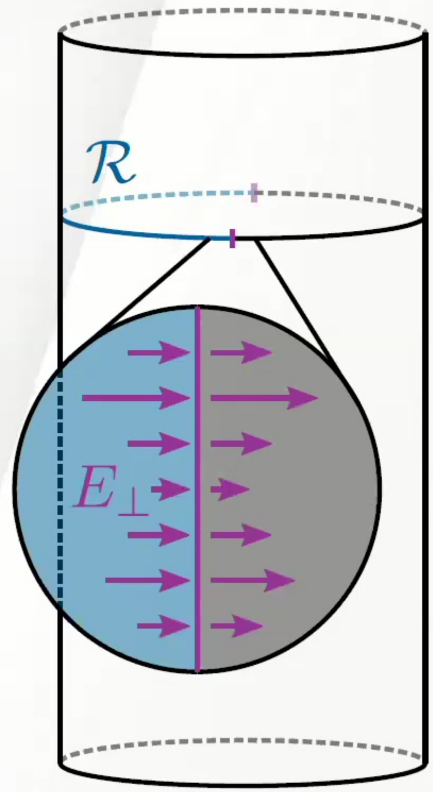
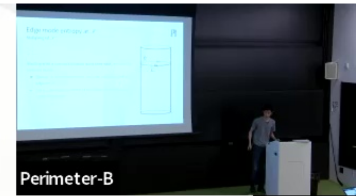


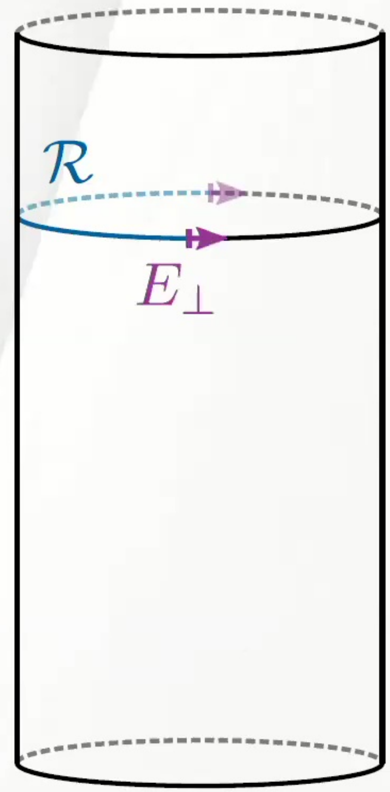
Edge mode entropy at $\mathscr{I}^+$

Mapping to $\mathscr{I}^+$



Starting with a constant Einstein static time slice, let us do an extreme boost:

- Balls $\mathcal{R}$ in the Einstein static universe conformally map to caps on $\mathscr{I}^+$.
- What is the interpretation of the edge mode entropy $S_\perp$ under this map?



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Edge mode entropy at $\mathscr{I}^+$

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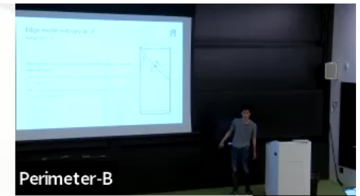

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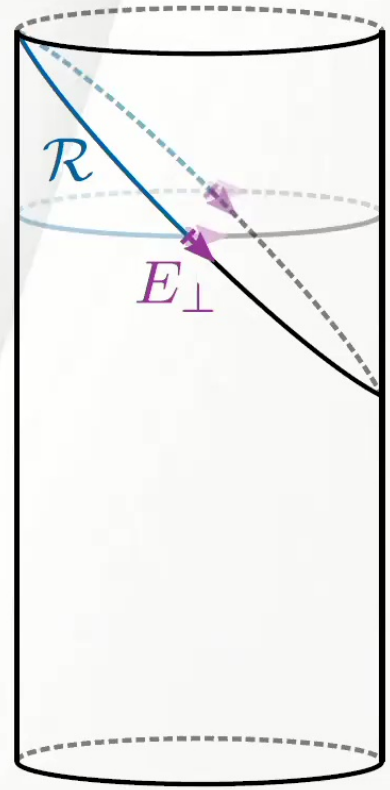

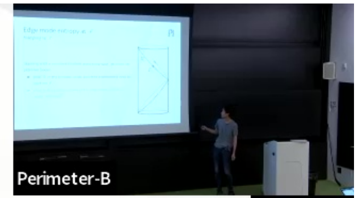

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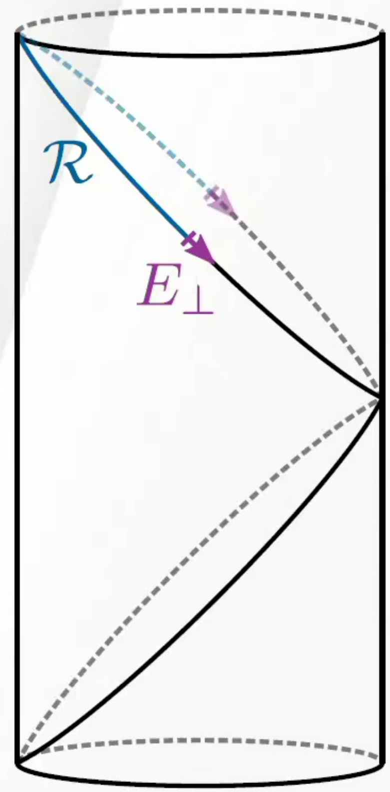

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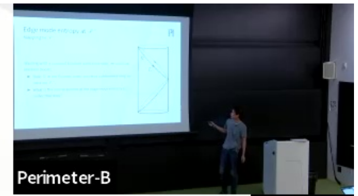

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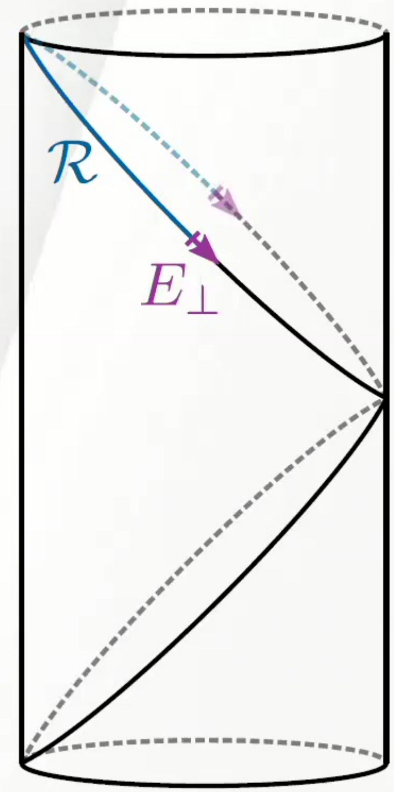

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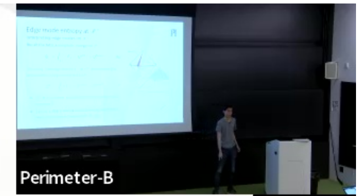


Edge mode entropy at $\mathscr{I}^+$

Interpreting edge modes on $\mathscr{I}^+$

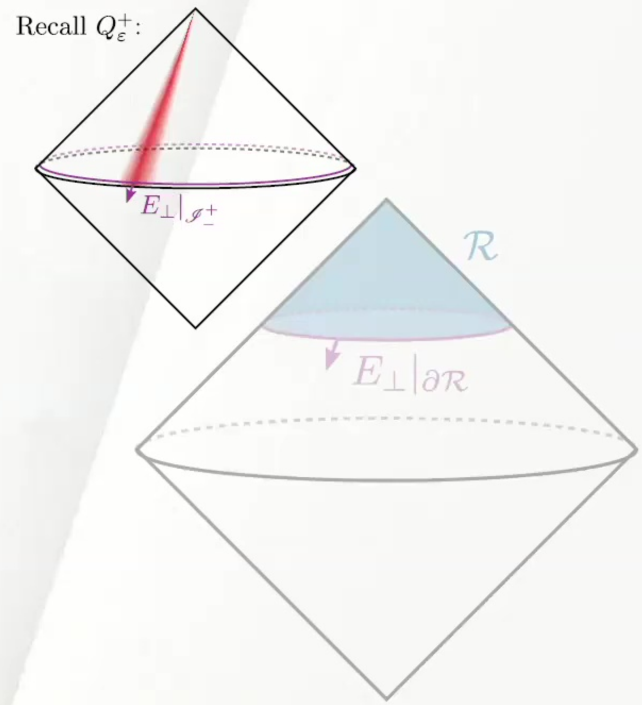
Recall the total asymptotic charge on $\mathscr{I}^+$:

$$Q_\varepsilon^+ = \int_{\mathscr{I}^+_-} \varepsilon\, E_\perp = Q_\varepsilon^{\mathrm{soft}+} + Q_\varepsilon^{\mathrm{hard}+} \tag{6}$$

Similarly, the edge modes $E_\perp$ at $\partial\mathcal{R}$ determines the asymptotic charge restricted to $\mathcal{R}$:

$$Q_\varepsilon^{\mathcal{R}} = \int_{\mathcal{R}} \varepsilon\, E_\perp = Q_\varepsilon^{\mathrm{soft}\mathcal{R}} + Q_\varepsilon^{\mathrm{hard}\mathcal{R}} \tag{7}$$

- $Q_\varepsilon^{\mathcal{R}}$ fluctuations are quantified by the edge mode entropy $S_\perp$.
- This is a step towards understanding asymptotic symmetries for subregions of $\mathscr{I}$.



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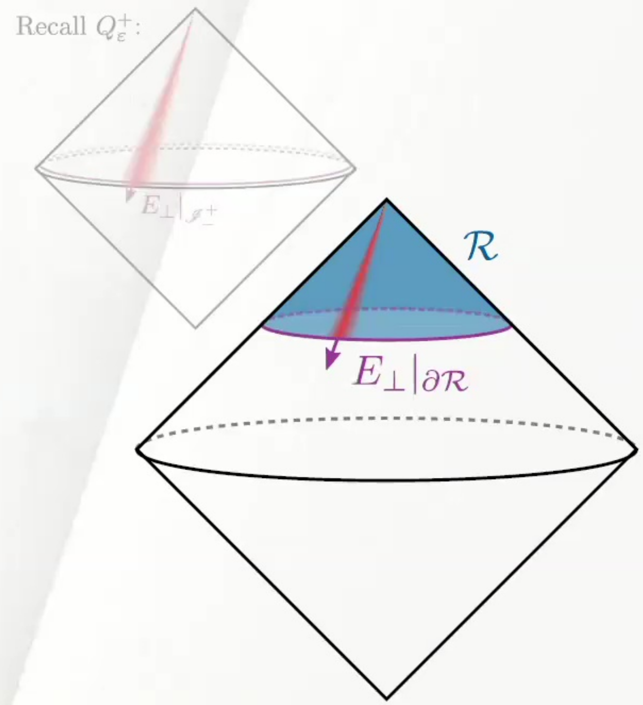
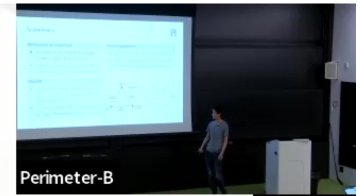




Summary

Motivation and method

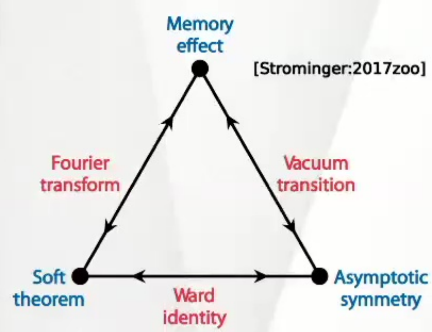
- Asymptotic symmetries & charges relate to many IR phenomena.
- These conformally map to familiar or better-studied short-distance ideas.

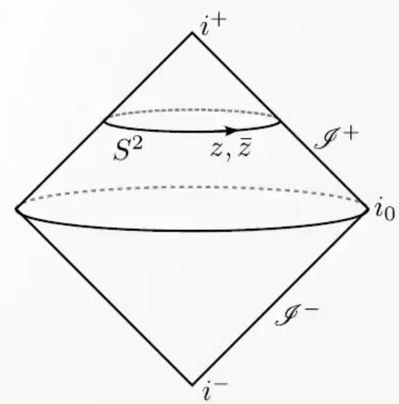
Results

- asymptotic charge conservation $Q^+_\varepsilon = Q^-_\varepsilon \iff C^1$ image charge trajectories at i_0
- $Q^{\mathcal{R}}_\varepsilon$ fluctuations in $\mathcal{R} \subset \mathscr{I}^+$ described by edge mode entropies $S_\perp$

Future questions

- Can $Q^+_\varepsilon = Q^-_\varepsilon$ be tested with AdS/CFT?
- How are spacetime and $\mathscr{I}$ subregions described in celestial holography?





Edge mode entropy at $\mathscr{I}^+$

Interpreting edge modes on $\mathscr{I}^+$

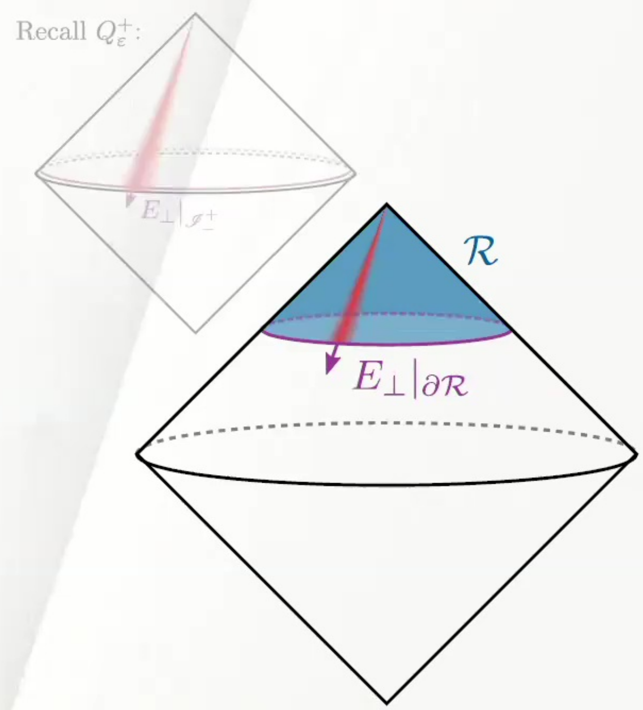
Recall the total asymptotic charge on $\mathscr{I}^+$:

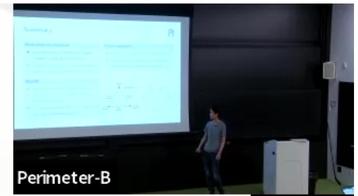
$$Q_\varepsilon^+ = \int_{\mathscr{I}_-^+} \varepsilon\, E_\perp = Q_\varepsilon^{\text{soft}+} + Q_\varepsilon^{\text{hard}+} \qquad (6)$$

Similarly, the edge modes $E_\perp$ at $\partial\mathcal{R}$ determines the asymptotic charge restricted to $\mathcal{R}$:

$$Q_\varepsilon^{\mathcal{R}} = \int_{\mathcal{R}} \varepsilon\, E_\perp = Q_\varepsilon^{\text{soft}\mathcal{R}} + Q_\varepsilon^{\text{hard}\mathcal{R}} \qquad (7)$$

- $Q_\varepsilon^{\mathcal{R}}$ fluctuations are quantified by the edge mode entropy $S_\perp$.
- This is a step towards understanding asymptotic symmetries for subregions of $\mathscr{I}$. ⚠





Summary

Motivation and method

- Asymptotic symmetries & charges relate to many IR phenomena.
- These conformally map to familiar or better-studied short-distance ideas.

Results

- asymptotic charge conservation $Q^+_\varepsilon = Q^-_\varepsilon \iff \mathrm{C}^1$ image charge trajectories at i_0
- $Q^{\mathcal{R}}_\varepsilon$ fluctuations in $\mathcal{R} \subset \mathscr{I}^+$ described by edge mode entropies $S_\perp$

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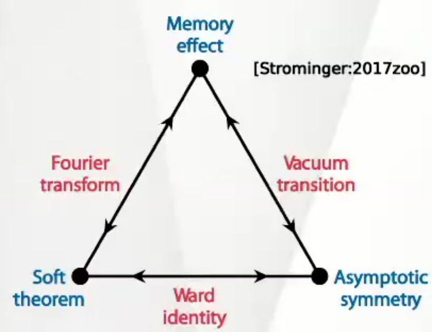

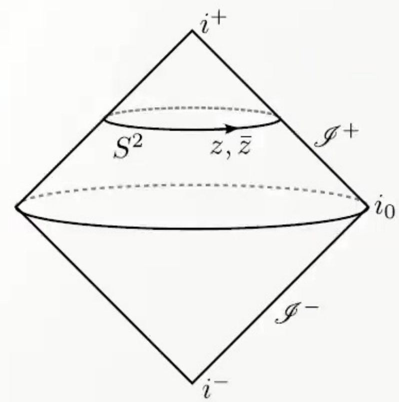

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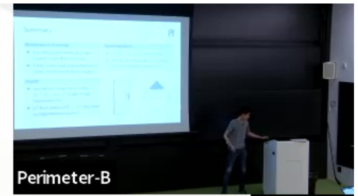

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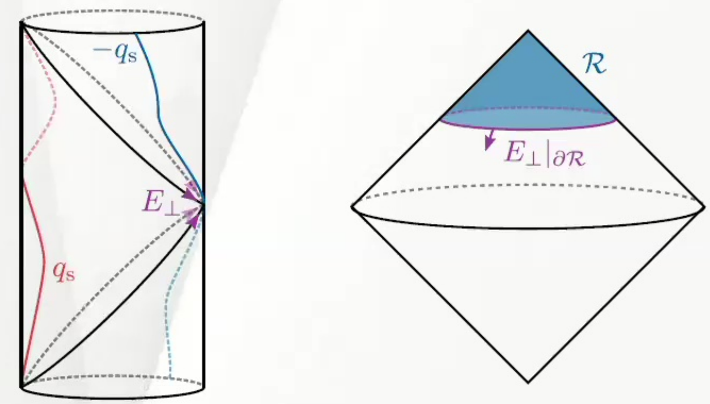

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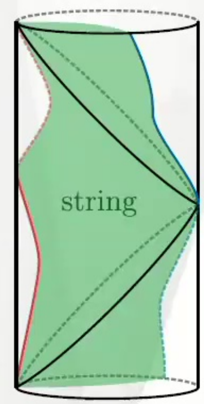

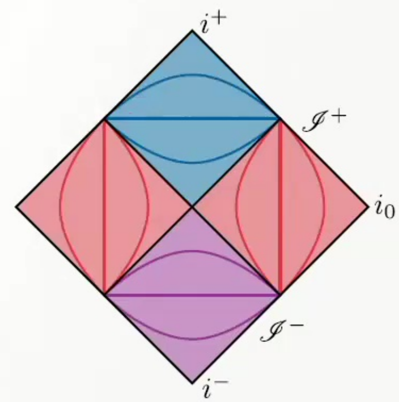

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AdS/CFT for Wilson lines

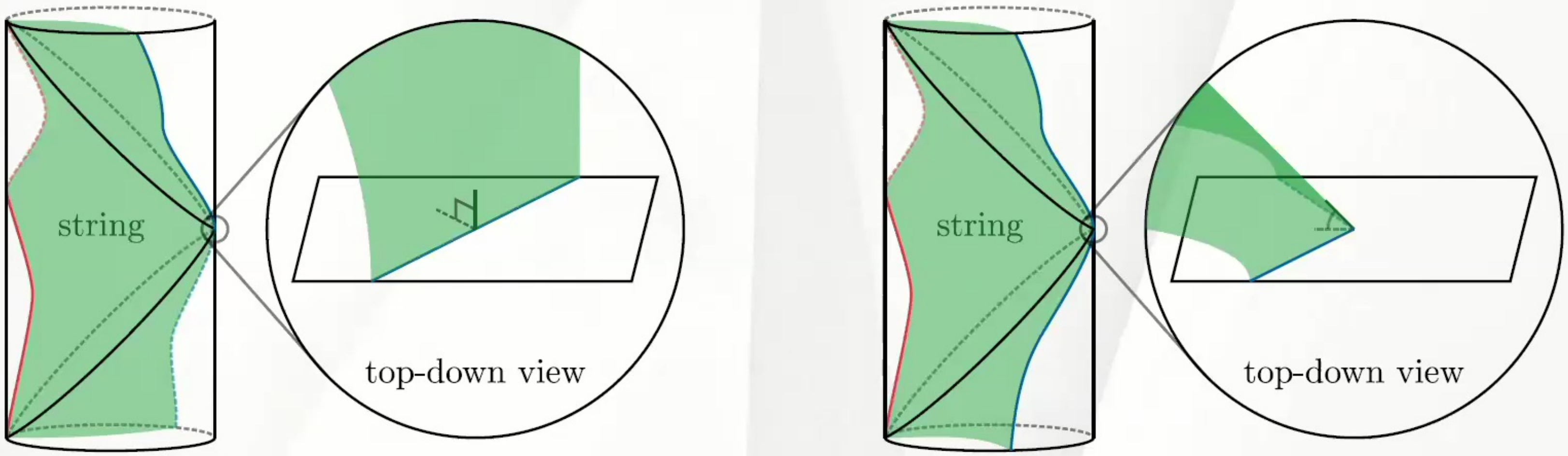


Figure 2: String duals of smooth and cusped Wilson lines.

Celestial holography

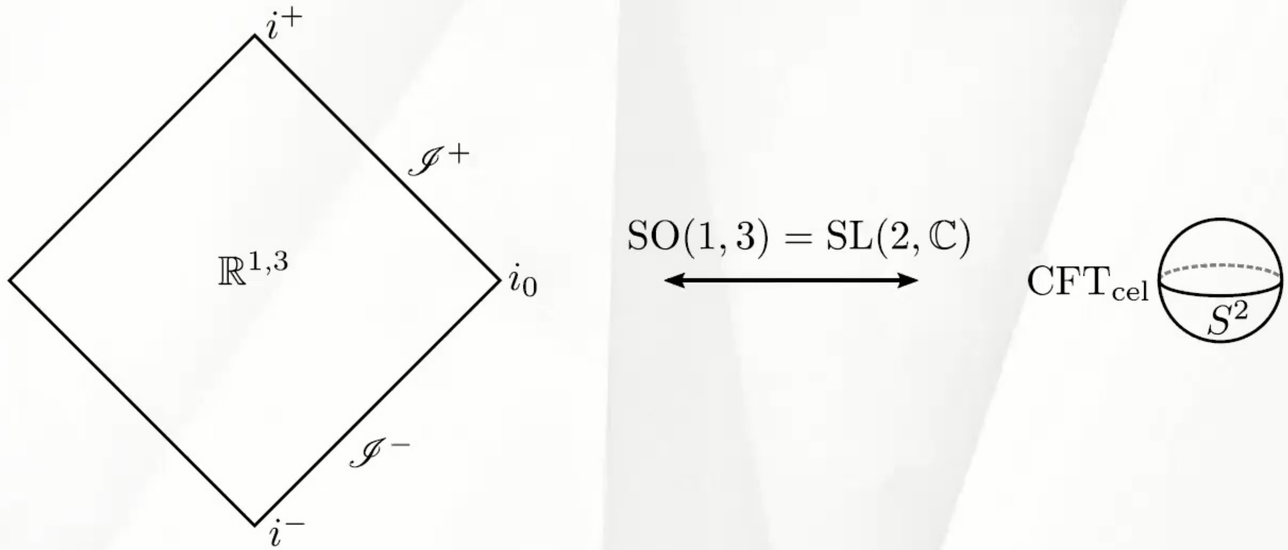
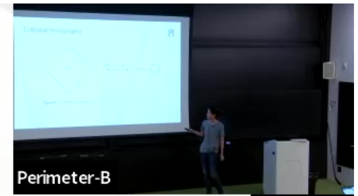
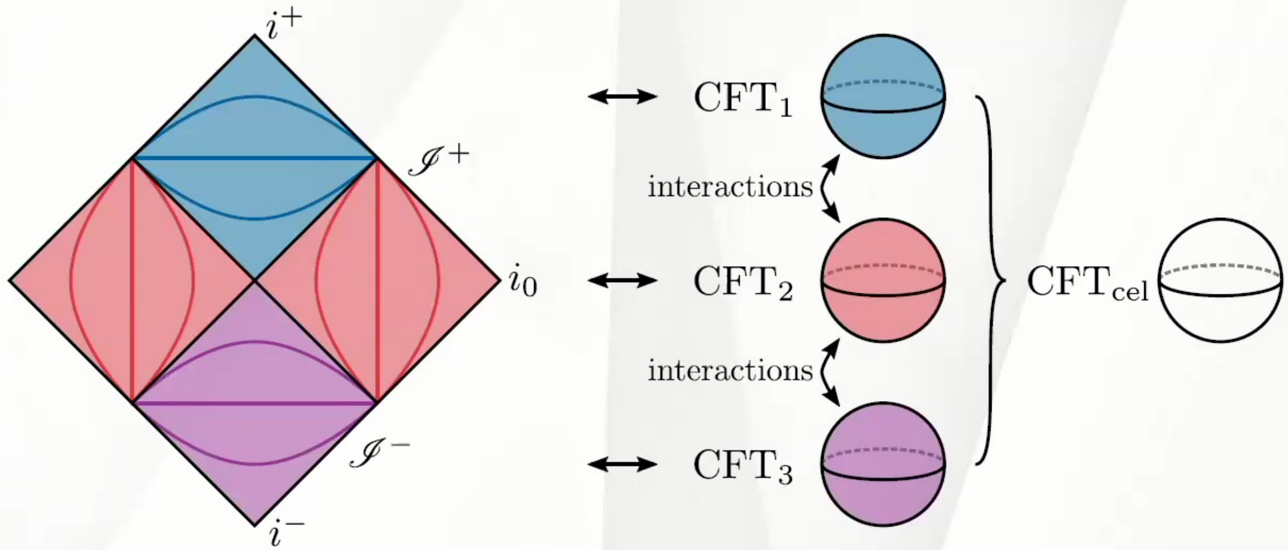


Figure 3: Celestial holography and a decomposition of the celestial CFT.



Celestial holography



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