Title: Black Hole Thermalization from Microstate Spectral Statistics

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Collection: Young Researchers Conference

Date: June 22, 2022 - 10:55 AM

URL: https://pirsa.org/22060058

Abstract: The detection of gravitational waves has opened up new observational windows into the physics of black holes and has the potential to shed light on how imposing unitary evolution modifies the near horizon dynamics. In this talk, I will present how recent developments in holography have provided a way of understanding the physics of black hole thermalization in terms of the spacing statistics of black hole microstates. Based on this, I will suggest that the issue of measuring the quantum aspects of black holes from their ringdown depends on the spectral statistics of their microstates. I will then suggest that certain microstate statistics lend to the possibility of deviations in the ringdown behaviour of black holes in the form of "echoes" which might be interpreted as being due to Planck-scale microstructure near the horizon.


# Black Hole Thermalization from Microstate Spectral Statistics

arXiv: 2110.03188 (Main Content)
arXiv: 1906.02653

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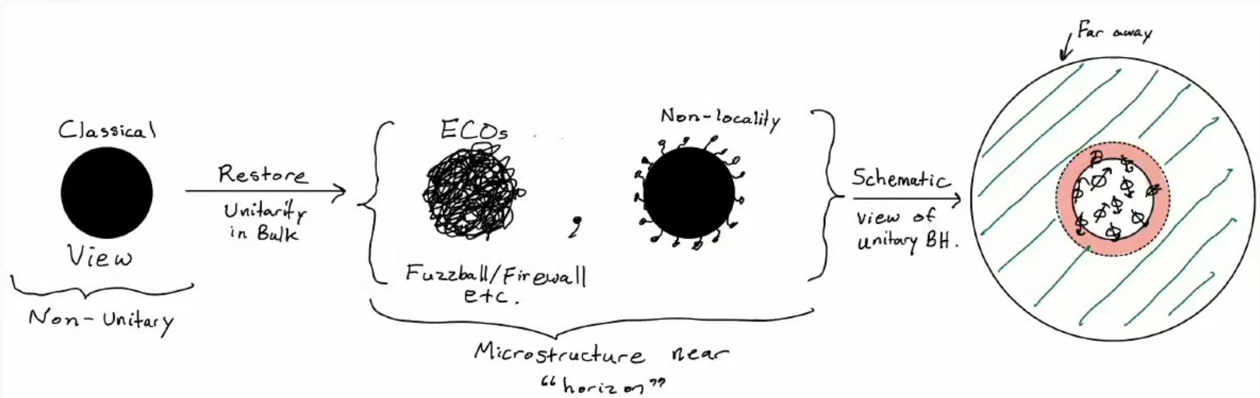
# BH Unitarity and the need for Microstructure

- AdS/CFT formulates quantum gravity in AdS in terms of unitary CFT.
- $\Rightarrow$ BH evolution is unitary from CFT perspective.




- Unitarity $\rightarrow$ Microstructure.

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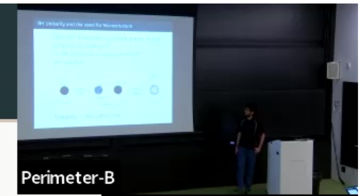
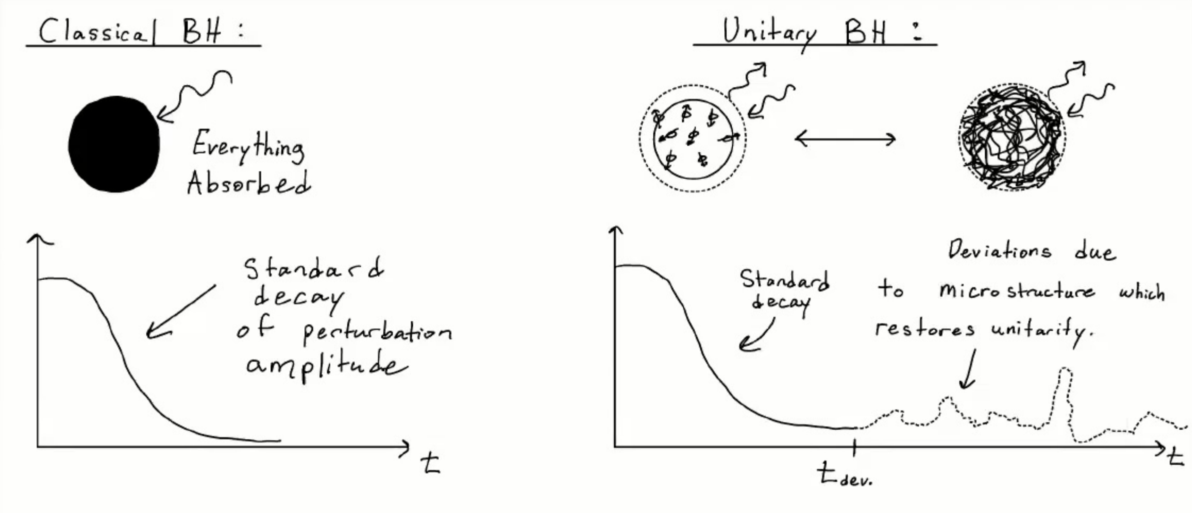


# Ringdown of Classical vs Unitary BH




- How and when do the deviations manifest?
- How is this related to details of the unitary description?

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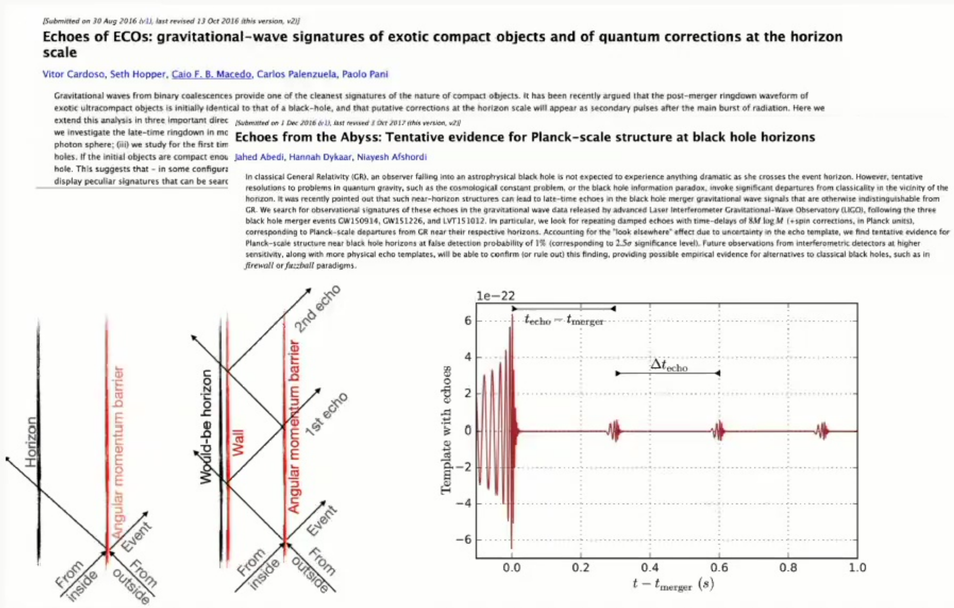
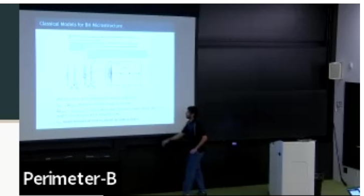
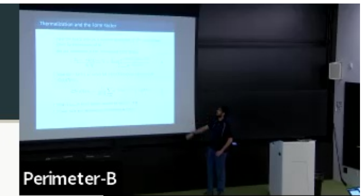
# Classical Models for BH Microstructure




- Wall has some semi-reflective boundary conditions.
- $t_{dev} \sim \Delta t_{echo}$ related to vicinity of wall to horizon.
- $\Delta t_{echo} \sim t_{scrambling} \simeq \beta \ln(S)$ when wall is placed proper radial Planck length from horizon (KS & Afshordi 2019).
- $t_{dev}$ **rough measure of how localized microstructure is.**

# Thermalization and the Form Factor




- View the black hole as a thermal ensmeble of $e^{S_{BH}}$ microstates given by eigenstates of $\mathcal{H}$.
- We are interested in the normalized form factor:

$$\frac{\mathcal{Z}(\beta+it)\mathcal{Z}(\beta-it)}{\mathcal{Z}(\beta)^2} = \frac{\sum_{n,m} e^{-\beta(E_m+E_n)} e^{i(E_n-E_m)t}}{\sum_{n,m} e^{-\beta(E_m+E_n)}} \tag{1}$$

- View form factor as proxy for 2-point function calculation. $\langle \mathcal{O}(t)\mathcal{O}(0)\rangle_\beta$:

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle_\beta = \frac{1}{\mathcal{Z}(\beta)} \sum_{n,m} e^{-\beta E_n} e^{i(E_n-E_m)t} |\langle n|\mathcal{O}|m\rangle|^2 \tag{2}$$

- How is $t_{dev}$ in form factor related to details of $\mathcal{H}$?
- Do we also see behaviour resembling echoes?

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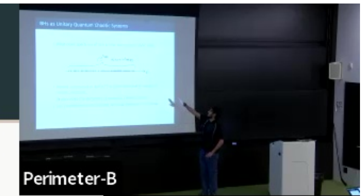
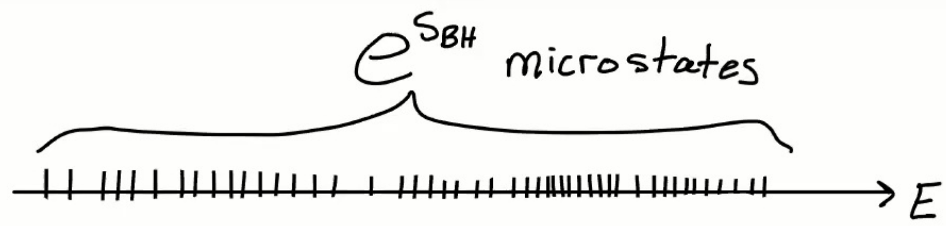



# BHs as Unitary Quantum Chaotic Systems

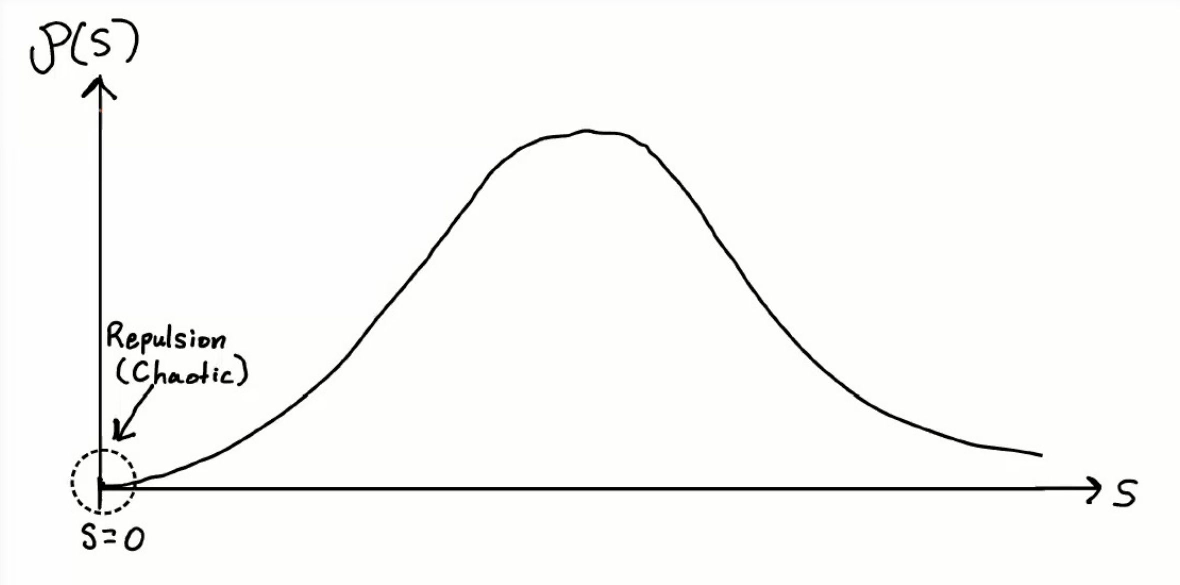
- What does spectrum of black hole microstates "look" like?




- Recent progresses in AdS/CFT suggest BHs dual to quantum chaotic systems.
- $\mathcal{H}$ describes the dynamics of quantum chaotic system.
- Has consequences on spacing statistics between microstates.

# Microstate Spacing Statistics and Quantum Chaos


- Generally quantum chaotic systems exhibit eigenvalue "repulsion."
- Repulsion quantified by nearest neighbor spacing (NNS) distributions.

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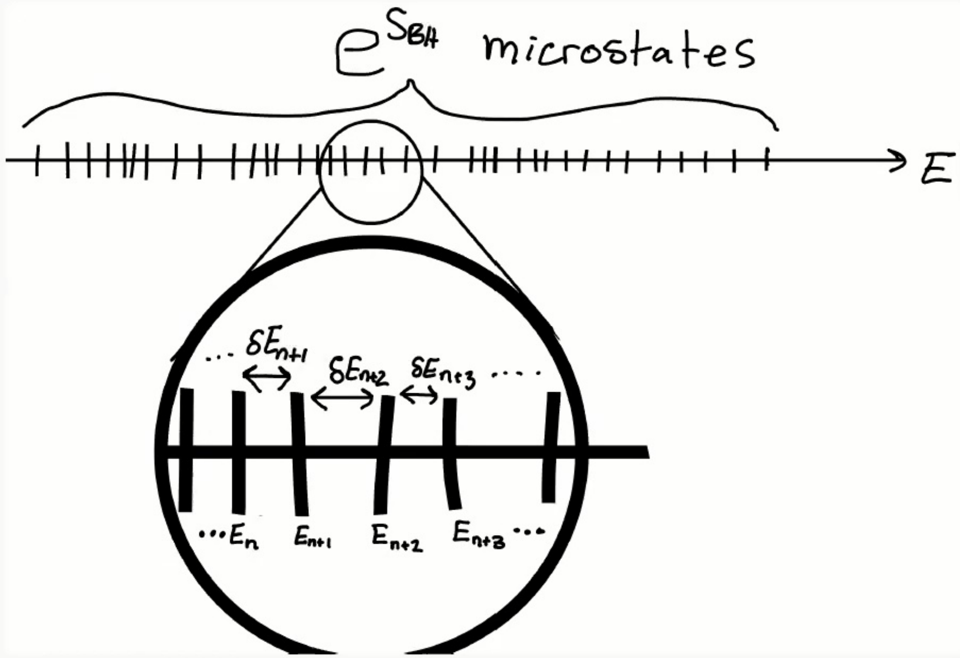
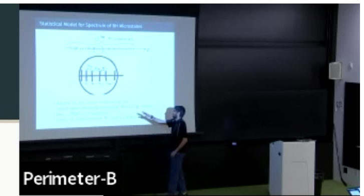
## Statistical Model for Spectrum of BH Microstates




- Assume $\delta E_k$ are random variables that are independent-identically-distributed: $\mathcal{P}(\delta E)$ (**i.i.d.** model) .
- $\lim_{\delta E \to 0} \mathcal{P}(\delta E) = 0$ (repulsion).
- Using i.i.d. model we have: $\mathcal{P} \mapsto \langle \mathcal{Z}(\beta + it)\mathcal{Z}(\beta - it) \rangle$.

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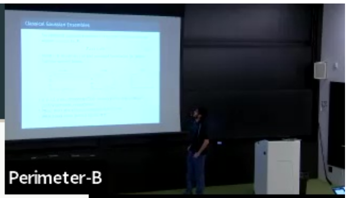
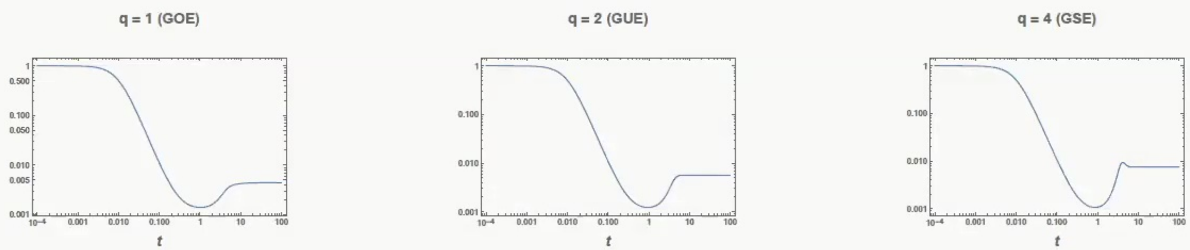
## Classical Gaussian Ensembles




- For standard Gaussian ensembles of random matrices, we have Wigner surmise for $\mathcal{P}$:

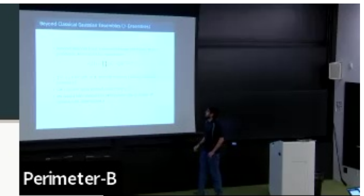
$$\mathcal{P}_q(s) \sim s^q e^{-s^2} \tag{3}$$

- Using i.i.d. model we can plot averaged form factor for Wigner surmise spacing below:




- $q = 1, 2, 4$ for orthogonal (GOE), unitary (GUE), and symplectic (GSE) ensembles, respectively.
- These don't give echoes in spectral form factor.
- What about more general values of $q$?

## Beyond Classical Gaussian Ensembles ($\beta$-Ensembles)




- Random matrices from $\beta$-ensembles have the following joint probability distribution for eigenvalues:

$$P_q(\lambda) \sim \prod_{i<j} |\lambda_i - \lambda_j|^q e^{-\sum_i \lambda_i^2/2} \tag{4}$$

- $q = 1, 2, 4$ for GOE, GUE, and GSE (recover Classical Gaussian ensembles).
- Can consider more general values of $q > 0$.
- We expect NNS statistics to exhibit even higher degree of repulsion for large values $q$.

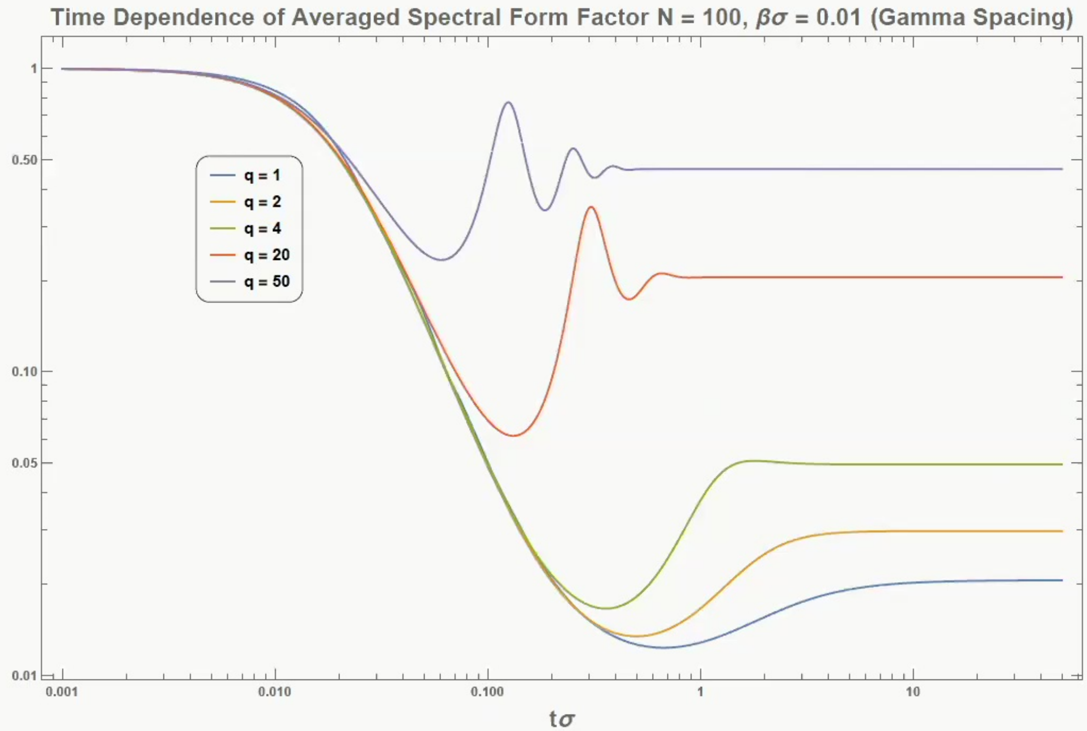
# i.i.d. Model with Gamma Distribution


- Consider i.i.d. model with Gamma distribution:

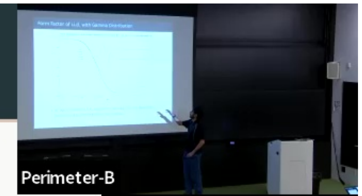
$$\mathcal{P}_q(x) = \Theta(x)\frac{x^q e^{-x/\sigma}}{\Gamma(1+q)\sigma^{1+q}} \tag{5}$$

- $\sigma$ determines average spacing between nearest neighbor pairs.
- $q$ measures degree of repulsion.
- Use i.i.d. model to study how form factor behaviour changes with $q$.

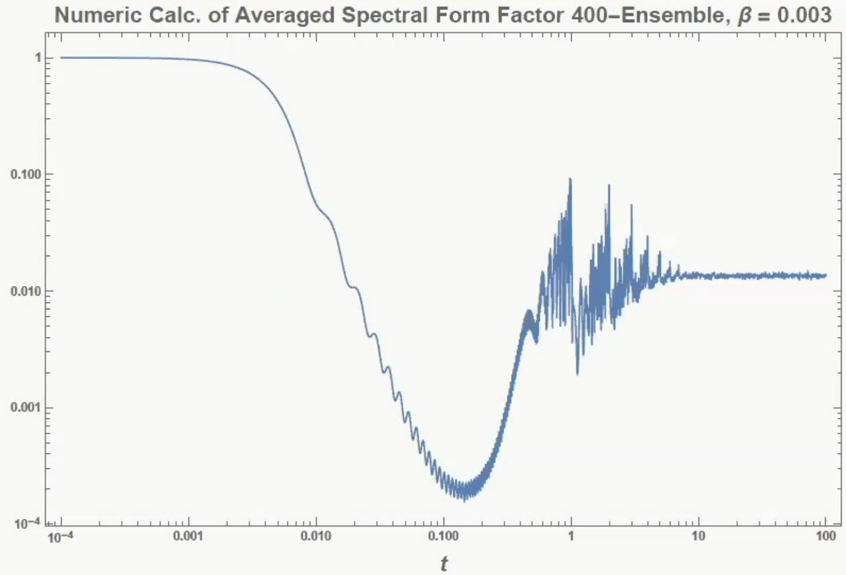
# Form Factor of i.i.d. with Gamma Distribution

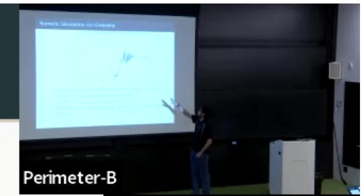



- As you increase q (i.e. repulsion) you start to see late-time oscillations (echoes) before the plateau.



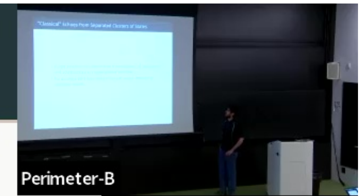
# Numeric Calculation 400-Ensemble




- Numerical average over 100 samples ($q = 400$ at finite temp.)
- Echoes manifest on time scales of the order $t \sim \langle \delta E \rangle^{-1} \sim t_{\mathrm{Heisenberg}} \gg t_{scrambling}$.
- Makes these echoes different from the ones found in classical models.

## "Classical" Echoes from Separated Clusters of States




- To get deviations on earlier time scales violate i.i.d. assumption and add structure to coarse grained spectrum.
- For example, let's now instead consider simple interacting oscillator model.

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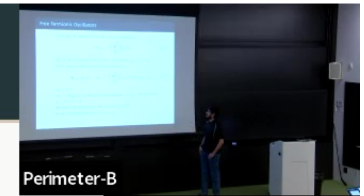
# Free Fermionic Oscillators




- Consider $N$ identical free fermionic oscillators:

$$\mathcal{H}_{FHO} = \frac{\omega_0}{2} \sum_{k=1}^{N} [b_k^\dagger, b_k] \tag{6}$$

- $b_k^\dagger, b_k$ are creation-annihilation operators $\{b_k, b_l^\dagger\} = \delta_{kl}$.
- Fock states diagonalize the Hamiltonian:

$$\mathcal{H}_{FHO} \left| n_1, n_2, .., n_N \right\rangle = \frac{\omega_0}{2} \sum_{k=1}^{N} (2n_k - 1) \left| n_1, n_2, .., n_N \right\rangle \tag{7}$$

- $n_k = 0, 1$
- $N + 1$ distinct clusters of states at $E(p) = \omega_0(p - N/2)$ where $p = 0, 1, 2, .., N$.
- Number of microstates for value of $p$ is $\binom{N}{p}$.
- Form factor is periodic $\tau = 2\pi/\omega_0$

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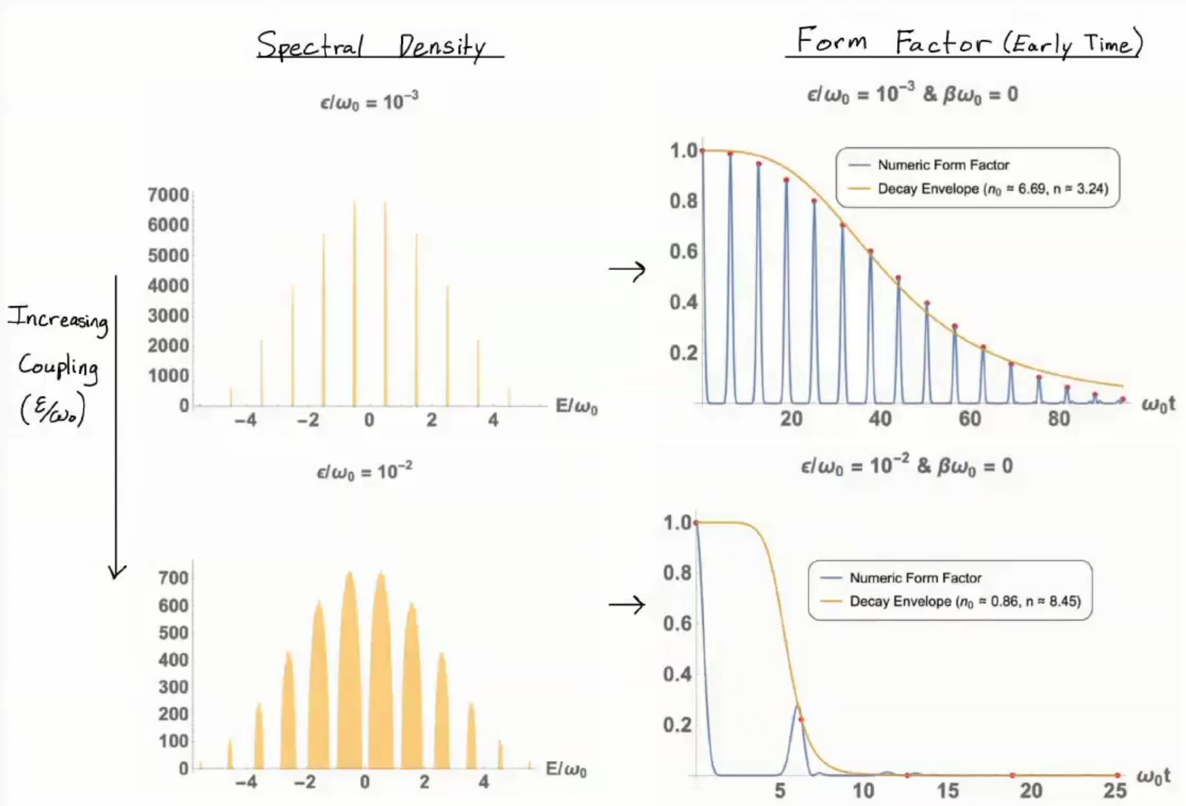
# Spectral Densities and Form Factors (varying coupling)

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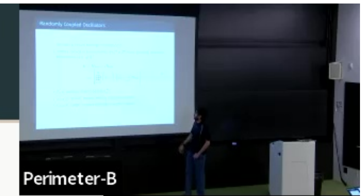
## Randomly Coupled Oscillators

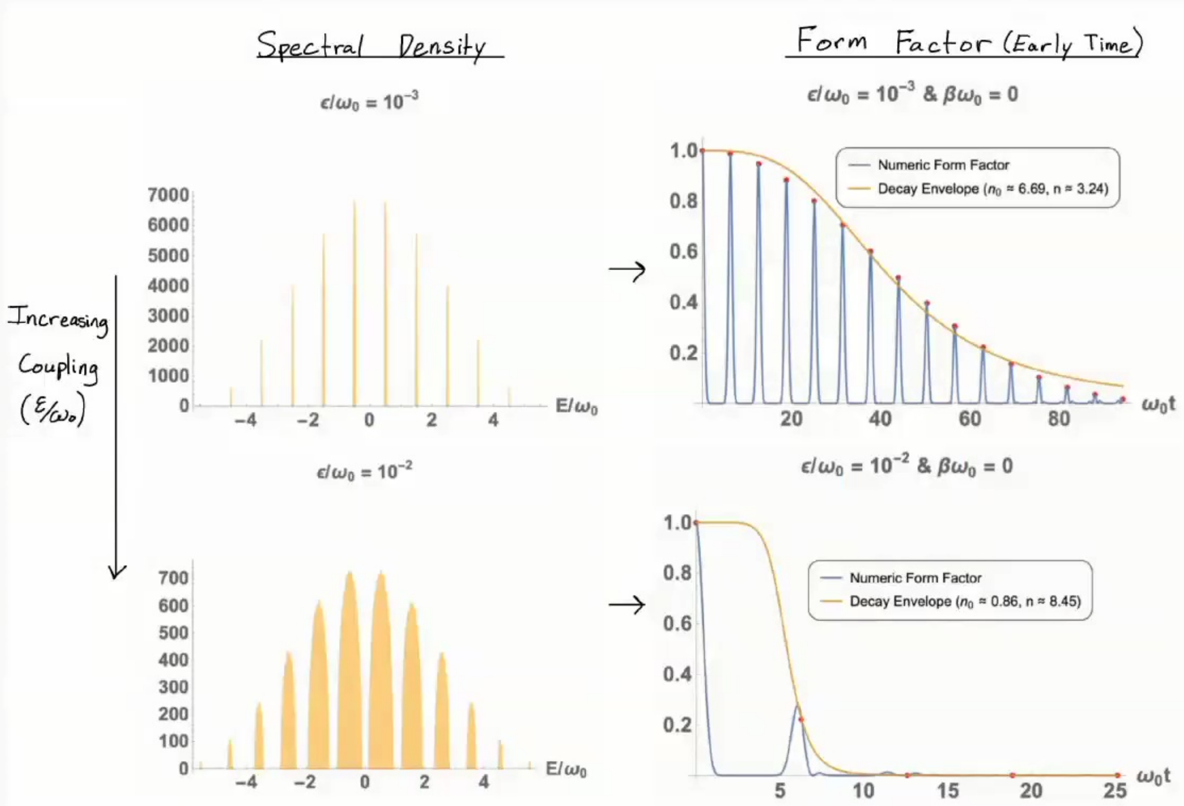



- Introduce chaos through interactions.
- Simple choice is random matrix ($2^N \times 2^N$) from classical Gaussian ensembles (e.g. GUE):

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_{FHO} + \epsilon \mathcal{H}_{GUE} \\ &= \omega_0 \left[ \bigoplus_{p=0}^{N} \left( p - \frac{N}{2} \right) \mathcal{I}_p + \frac{\epsilon}{\omega_0} \mathcal{H}_{GUE} \right]\end{aligned} \tag{8}$$

- $\mathcal{I}_p$ is identity matrix of size $\binom{N}{p}$.
- $\epsilon/\omega_0$ is “small” means weakly coupled regime.
- $\epsilon/\omega_0$ is “large” means strongly coupled regime.

# Spectral Densities and Form Factors (varying coupling)

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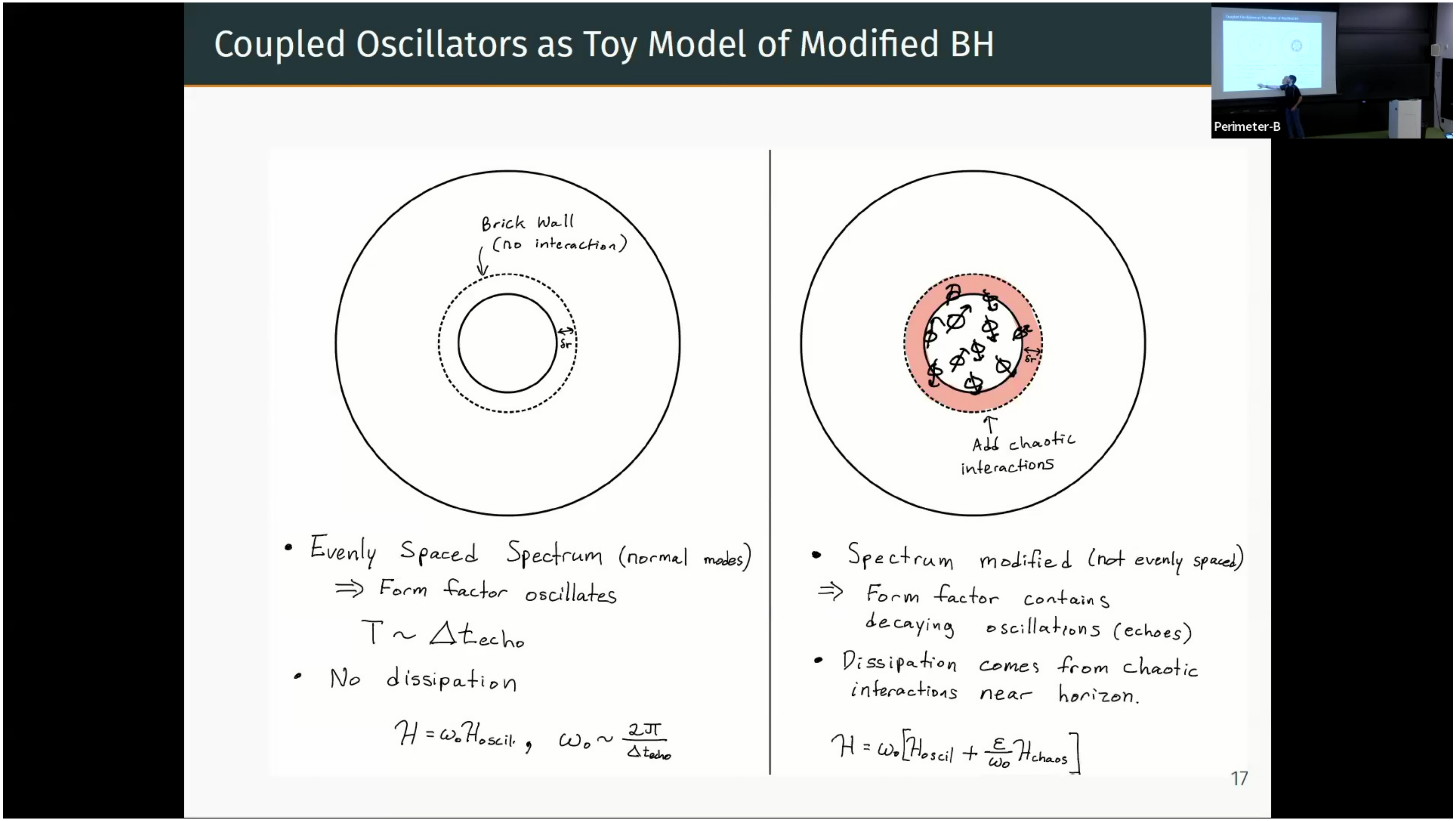
Perimeter-B



Coupled Oscillators as Toy Model of Modified BH
Perimeter-B
Brick Wall
(no interaction)
δr
Add chaotic interactions
δr
• Evenly Spaced Spectrum (normal modes)
⇒ Form factor oscillates
T ~ Δt_echo
• No dissipation
H = ω₀ H_oscil , ω₀ ~ 2π / Δt_echo
• Spectrum modified (not evenly spaced)
⇒ Form factor contains decaying oscillations (echoes)
• Dissipation comes from chaotic interactions near horizon.
H = ω₀ [H_oscil + ε/ω₀ H_chaos]
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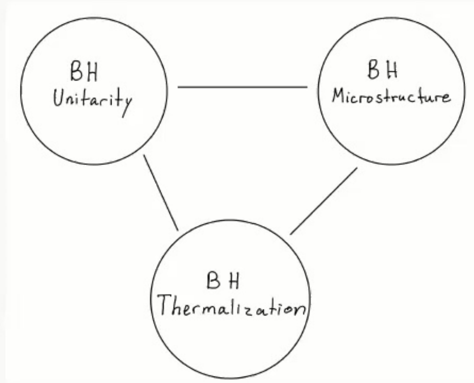
# Summary




- $t_{dev}$ related to vicinity of microstructure to "horizon."
- For microstructure localized within proper Planck length of horizon, expect $t_{dev} \gtrsim \beta \ln(S) \sim t_{scrambling}$ .
- Deviations in the form of echoes occur for systems with:
  - Enhanced eigenvalue repulsion ($\beta$-ensembles).
  - Regularly spaced cluster of states (coupled oscillator example).

**BH microstate statistics play a central role in understanding BH microstructure and thermalization in the bulk.**

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