Title: Asymptotic charges in Maxwell theory.

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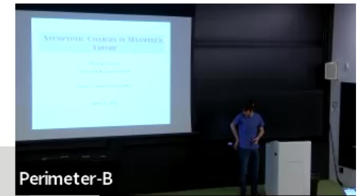


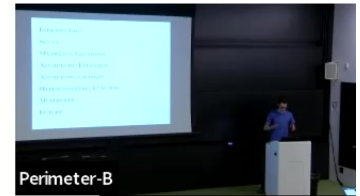
Asymptotic Charges in Maxwell's Theory

Nicolas Cresto

Supervised by Laurent Freidel

Young PI researchers conference

June 22, 2022



Introduction

Set up

Maxwell's Equations

Asymptotic Expansion

Asymptotic Charges

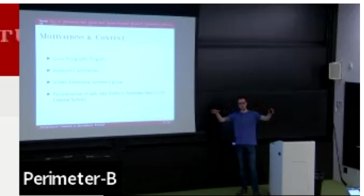
Hypergeometric Function

Multipoles

Future

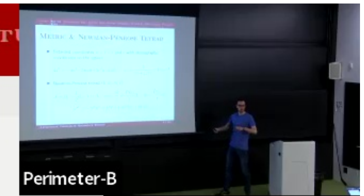
Introduction

- This is still very much a work in progress.
- Therefore, I'd like to emphasize the way the research is done, on top of the results themselves.
- The **notation** does matter.



Motivations & Context

- Local Holography Program.
- Asymptotic symmetries.
- Infinite dimensional symmetry group.
- Reconstruction of bulk data thanks to boundary data (on the Celestial Sphere).



METRIC & NEWMAN-PENROSE TETRAD

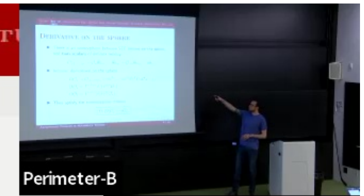
- Retarded coordinates $u = t - r$ and r with stereographic coordinates on the sphere $z, \bar{z}$.

$$ds^2 = -du^2 - 2du dr + 2r^2 q_{z\bar{z}} dz d\bar{z}, \quad q_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2} \equiv P(z)^{-2}$$

- Newman-Penrose tetrad $(k, m, \overline{m}, \ell)$:

$$k = \partial_u - \frac{1}{2}\partial_r, \quad \ell = \partial_r, \quad m = \frac{\widehat{m}}{r} = \frac{P(z)}{r}\partial_z, \quad \overline{m} = \frac{P(z)}{r}\partial_{\bar{z}}.$$

$$g^{\mu\nu} = -(\ell^\mu k^\nu + k^\mu \ell^\nu) + m^\mu \overline{m}^\nu + \overline{m}^\mu m^\nu.$$



Derivative on the sphere

- There is an isomorphism between STF tensors on the sphere and **two scalars** of definite helicity:

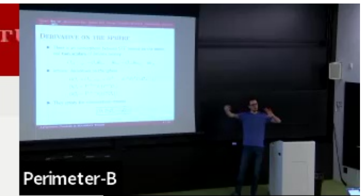
$$\mathcal{O}_{\langle A_1 \dots A_s\rangle} = \mathcal{O}_s \widehat{\bar{\mathbf{m}}}_{A_1} \dots \widehat{\bar{\mathbf{m}}}_{A_s} + \mathcal{O}_{-s} \widehat{\mathbf{m}}_{A_1} \dots \widehat{\mathbf{m}}_{A_s}.$$

- Intrinsic derivatives on the sphere

$$D\mathcal{O}_s \equiv \mathcal{O}_{\widehat{m}\dots\widehat{m}|\widehat{m}} \doteq \widehat{m}^{A_1} \dots \widehat{m}^{A_s} \widehat{m}^{A} D_A \mathcal{O}_{A_1 \dots A_s}$$
$$D\mathcal{O}_s = P^{-1+s} \partial_z \left(P^{-s} \mathcal{O}_s\right),$$
$$\bar{D}\mathcal{O}_s = P^{-1-s} \partial_{\bar{z}} \left(P^{s} \mathcal{O}_s\right).$$

- They satisfy the commutation relation

$$\boxed{[\bar{D}, D]\mathcal{O}_s = s\mathcal{O}_s.}$$



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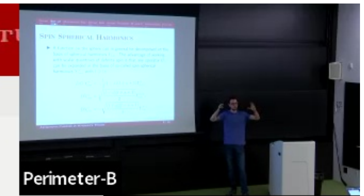


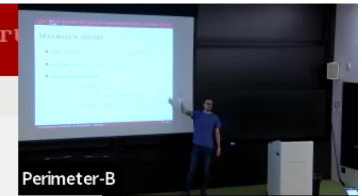
Spin Spherical Harmonics

- A function on the sphere can in general be decomposed on the basis of spherical harmonics $Y_{l,m}$. The advantage of working with scalar quantities of definite spin is that any operator $\mathcal{O}_s$ can be expanded in the basis of so-called spin spherical harmonics $Y^s_{l,m}$, with $l \geqslant |s|$.

$$\bar{D}D\,Y^s_{l,m} = -\frac{1}{2}(l-s)(l+s+1)Y^s_{l,m},$$

$$DY^s_{l,m} = \sqrt{\frac{(l-s)(l+s+1)}{2}}Y^{s+1}_{l,m},$$

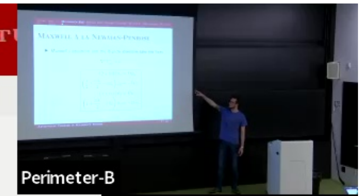
$$\bar{D}Y^s_{l,m} = -\sqrt{\frac{(l+s)(l-s+1)}{2}}Y^{s-1}_{l,m}.$$



MAXWELL'S THEORY

- Dual tensor $\tilde{F}_{\mu\nu} = (\star F)_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\varepsilon_{\mu\nu\alpha\beta}g^{\alpha\rho}g^{\beta\lambda}F_{\rho\lambda}$.
- Sefl-dual tensor $F^{+}_{\mu\nu} \equiv \frac{1}{2}\left(F_{\mu\nu} - i\tilde{F}_{\mu\nu}\right)$.
- Newman-Penrose scalars

$$\phi_0 \equiv F_{\ell m} = F^{+}_{\ell m}$$
$$\phi_1 \equiv \frac{1}{2}\left(F_{\ell k} + F_{\overline{m}m}\right) = F^{+}_{\ell k} = F^{+}_{\overline{m}m}$$
$$\phi_2 \equiv F_{\overline{m}k} = F^{+}_{\overline{m}k}$$

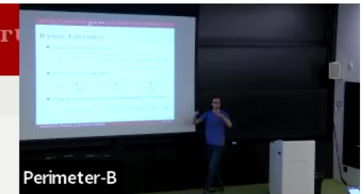


Maxwell à la Newman-Penrose

- Maxwell's equations and the Bianchi identities take the form

$$\nabla^\mu F^+_{\mu\nu} = 0.$$

$$\boxed{\begin{aligned} (2 + r\partial_r)\phi_1 &= \bar{D}\phi_0, \\ \left(\frac{1}{2} + \frac{r\partial_r}{2} - r\partial_u\right)\phi_0 &= -D\phi_1, \\ (1 + r\partial_r)\phi_2 &= \bar{D}\phi_1, \\ \left(1 + \frac{r\partial_r}{2} - r\partial_u\right)\phi_1 &= -D\phi_2. \end{aligned}}$$



Radial Expansion

- We get a close equation for ϕ_0:

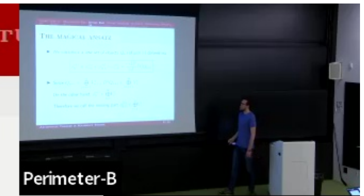
$$\boxed{\left(1 + \frac{r\partial_r(3 + r\partial_r)}{2} - r\partial_u(3 + r\partial_r)\right)\phi_0 + D\bar{D}\phi_0 = 0} \qquad (1)$$

- We posit the following ansatz:

$$\phi_0 = \frac{1}{r^3}\sum_{n=0}\frac{\phi_0^{(n)}}{r^n}, \quad \phi_1 = \frac{1}{r^2}\sum_{n=0}\frac{\phi_1^{(n)}}{r^n}, \quad \phi_2 = \frac{1}{r}\sum_{n=0}\frac{\phi_2^{(n)}}{r^n}.$$

- Plugging in (1) and commuting the derivatives on the sphere,

$$(n+1)\partial_u\phi_0^{(n+1)} + \frac{n(n+3)}{2}\phi_0^{(n)} + \bar{D}D\phi_0^{(n)} = 0. \qquad (2)$$



The magical ansatz

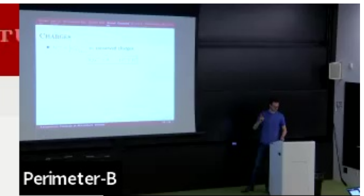
- We introduce a new set of objects Q_n (of spin n) defined via

$$\boxed{\phi_0^{(n)} \equiv \phi_{0,G}^{(n)} + \phi_{0,L}^{(n)} = \phi_{0,G}^{(n)} + \frac{(-1)^n}{n!}\bar{D}^n Q_{n+1}}.$$

- Since $Q_{n+1} \in \bigoplus_{l=n+1}^{\infty} V_{n+1}^l$, $\bar{D}^n Q_{n+1} \in \bigoplus_{l=n+1}^{\infty} V_1^l$.

  On the other hand, $\phi_0^{(n)} \in \bigoplus_{l=1}^{\infty} V_1^l$.

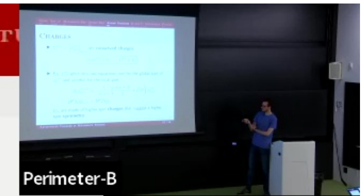
  Therefore we call the missing part $\phi_{0,G}^{(n)} \in \bigoplus_{l=1}^{n} V_1^l$.



Charges

- $G^{(n)} \equiv \left[\phi_{0,G}^{(n)}\right]_{l=n}$ are **conserved charges**:

$$\boxed{\partial_u G^{(l)} = 0, \qquad G^{(l)} \in V_1^l}.$$



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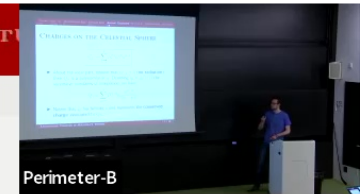
$$\boxed{\partial_u G^{(l)} = 0, \qquad G^{(l)} \in V_1^l}.$$

- Eq. (2) splits into two equations, one for the global part of $\phi_0^{(n)}$ and another for the local part:

$$\partial_u \phi_{0,G}^{(n+1)} = -\frac{1}{n+1}\left(\frac{n(n+3)}{2} + \bar{D}D\right)\phi_{0,G}^{(n)},$$

$$\cancel{\bar{D}^n}\partial_u Q_{n+1} = \cancel{\bar{D}^n} D Q_n.$$

Q_n are made of higher spin **charges** that suggest a higher spin **symmetry**.



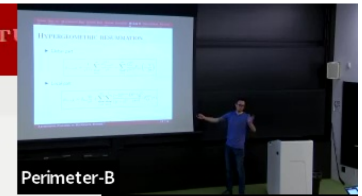
Charges on the Celestial Sphere

$$\boxed{\phi_{0,G}^{(n)} = \sum_{l=1}^{n} \alpha_l^{(n)} G^{(l)} u^{n-l}}$$

- About the local part, assume that $Q_{-1} = 0$ (**no radiation**), then Q_n is a polynomial in u. Denoting $q_p \equiv q_p(z, \bar{z})$ the successive constants of integration, we have

$$Q_n = \sum_{p=0}^{n} D^{n-p} q_p \frac{u^{n-p}}{(n-p)!}.$$

- Notice that q_p has helicity p and represents the **conserved charge** associated to Q_p.



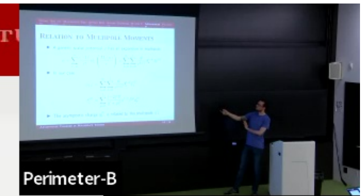
Hypergeometric resummation

- Global part

$$\phi_{1-s,G} = \frac{1}{r^{2+s}} \sum_{n=1}^{\infty} \frac{\phi_{1-s,G}^{(n)}}{r^n} = \sum_{l=1}^{\infty} \frac{G_s^{(l)}}{r^{l+3}} F_{l,s}\left(-\frac{u}{2r}\right)$$

- Local part

$$\phi_{1-s,L} = \delta_{0,s}\frac{q_0}{r^2} + \sum_{l=1}^{\infty}\sum_{p=1}^{l} \frac{(-1)^{p-s}}{(p-s)!} \frac{\bar{D}^{p-s} q_p^{(l)}}{r^{p+2}} F_{l,s}^{(p)}(\tau)$$



Relation to Multipole Moments

- A generic scalar potential ϕ has an expansion in multipole

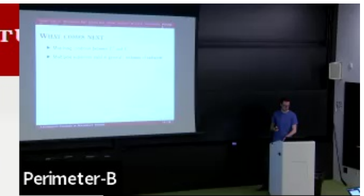
$$\phi \sim \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \partial_L \left(\frac{\hat{M}_L(u)}{r} \right) = \sum_{l=0}^{\infty} \sum_{k=0}^{l} \frac{\#}{r^{k+1}} \partial_u^{l-k} M^{(l)}$$

- In our case

$$\phi_{1,L} = \sum_{l=0}^{\infty} \sum_{k=0}^{l} \frac{\#}{r^{k+2}} \partial_u^{l-k} \mathcal{E}_l^{(l)}$$

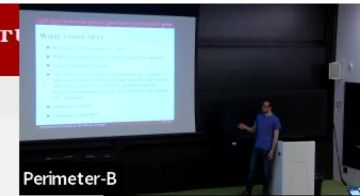
$$\mathcal{E}_l^{(l)} = \sum_{p=0}^{l} \frac{(-2)^p l!}{(l+p)!} u^{l-p} \bar{D}^p q_p^{(l)}$$

- The asymptotic charge $q_p^{(l)}$ is related to the multipole (l).



What comes next

- Matching condition between $\mathcal{I}^+$ and $\mathcal{I}^-$.
- Multipole expansion valid in general? Inclusion of radiation...



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- Matching condition between $\mathcal{I}^+$ and $\mathcal{I}^-$.
- Multipole expansion valid in general? Inclusion of radiation...
- Lots of parallels with gravity.
- The coefficients of the NP scalars expanded in $1/r$ (and in particular Ψ_0 which encodes outgoing radiations) are identified with charges generating asymptotic symmetries and soft higher-spin symmetries found by OPE methods in the celestial CFT framework.
- Extension to (A)dS.
- Extension to Yang-Mill.