Title: Asymptotic charges in Maxwell theory.

Speakers: Nicolas Cresto

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Asymptotic Charges in Maxwell's Theory

Nicolas Cresto

Supervised by Laurent Freidel

Young PI researchers conference

June 22, 2022

- Introduction
- Set up
- Maxwell's Equations
- Asymptotic Expansion
- Asymptotic Charges
- Hypergeometric Function
- Multipoles
- Future

INTRODUCTION

- This is still very much a work in progress.
- Therefore, I'd like to emphasize the way the research is done, on top of the results themselves.
- The **notation** does matter.

Motivations & Context

- Local Holography Program.
- Asymptotic symmetries.
- Infinite dimensional symmetry group.
- Reconstruction of bulk data thanks to boundary data (on the Celestial Sphere).

Metric & Newman-Penrose Tetrad

- Retarded coordinates $u = t - r$ and r with stereographic coordinates on the sphere $z, \bar{z}$.

$$ds^2 = -du^2 - 2dudr + 2r^2 q_{z\bar{z}} dz d\bar{z}, \quad q_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2} \equiv P(z)^{-2}$$

- Newman-Penrose tetrad $(k, m, \overline{m}, \ell)$:

$$k = \partial_u - \frac{1}{2}\partial_r, \quad \ell = \partial_r, \quad m = \frac{\widehat{m}}{r} = \frac{P(z)}{r}\partial_z, \quad \overline{m} = \frac{P(z)}{r}\partial_{\bar{z}}.$$

$$g^{\mu\nu} = -(\ell^\mu k^\nu + k^\mu \ell^\nu) + m^\mu \overline{m}^\nu + \overline{m}^\mu m^\nu.$$

DERIVATIVE ON THE SPHERE

- There is an isomorphism between STF tensors on the sphere and **two scalars** of definite helicity:

$$\mathcal{O}_{\langle A_1 \dots A_s \rangle} = \mathcal{O}_s \widehat{\bar{\mathbf{m}}}_{A_1} \dots \widehat{\bar{\mathbf{m}}}_{A_s} + \mathcal{O}_{-s} \widehat{\mathbf{m}}_{A_1} \dots \widehat{\mathbf{m}}_{A_s}.$$

- Intrinsic derivatives on the sphere

$$D\mathcal{O}_s \equiv \mathcal{O}_{\widehat{m} \dots \widehat{m} | \widehat{m}} \doteq \widehat{m}^{A_1} \dots \widehat{m}^{A_s} \widehat{m}^{A} D_A \mathcal{O}_{A_1 \dots A_s}$$
$$D\mathcal{O}_s = P^{-1+s} \partial_z \left(P^{-s} \mathcal{O}_s \right),$$
$$\bar{D}\mathcal{O}_s = P^{-1-s} \partial_{\bar{z}} \left(P^{s} \mathcal{O}_s \right).$$

- They satisfy the commutation relation

$$\boxed{[\bar{D}, D]\mathcal{O}_s = s\mathcal{O}_s.}$$

Derivative on the sphere

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Spin Spherical Harmonics

- A function on the sphere can in general be decomposed on the basis of spherical harmonics $Y_{l,m}$. The advantage of working with scalar quantities of definite spin is that any operator $\mathcal{O}_s$ can be expanded in the basis of so-called spin spherical harmonics $Y^s_{l,m}$, with $l \geqslant |s|$.

$$\bar{D}D\,Y^s_{l,m} = -\frac{1}{2}(l-s)(l+s+1)Y^s_{l,m},$$

$$DY^s_{l,m} = \sqrt{\frac{(l-s)(l+s+1)}{2}}Y^{s+1}_{l,m},$$

$$\bar{D}Y^s_{l,m} = -\sqrt{\frac{(l+s)(l-s+1)}{2}}Y^{s-1}_{l,m}.$$

Maxwell's theory

- Dual tensor $\tilde{F}_{\mu\nu} = (\star F)_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\varepsilon_{\mu\nu\alpha\beta}g^{\alpha\rho}g^{\beta\lambda}F_{\rho\lambda}$.
- Sefl-dual tensor $F^{+}_{\mu\nu} \equiv \frac{1}{2}\big(F_{\mu\nu} - i\tilde{F}_{\mu\nu}\big)$.
- Newman-Penrose scalars

$$
\begin{aligned}
\phi_0 &\equiv F_{\ell m} = F^{+}_{\ell m} \\
\phi_1 &\equiv \frac{1}{2}\big(F_{\ell k} + F_{\overline{m}m}\big) = F^{+}_{\ell k} = F^{+}_{\overline{m}m} \\
\phi_2 &\equiv F_{\overline{m}k} = F^{+}_{\overline{m}k}
\end{aligned}
$$

Maxwell à la Newman-Penrose

- Maxwell's equations and the Bianchi identities take the form

$$\nabla^\mu F^+_{\mu\nu} = 0.$$

$$\boxed{\begin{aligned} (2 + r\partial_r)\phi_1 &= \bar{D}\phi_0, \\ \left(\frac{1}{2} + \frac{r\partial_r}{2} - r\partial_u\right)\phi_0 &= -D\phi_1, \\ (1 + r\partial_r)\phi_2 &= \bar{D}\phi_1, \\ \left(1 + \frac{r\partial_r}{2} - r\partial_u\right)\phi_1 &= -D\phi_2. \end{aligned}}$$

RADIAL EXPANSION

- We get a close equation for ϕ_0:

$$\boxed{\left(1+\frac{r\partial_r(3+r\partial_r)}{2}-r\partial_u(3+r\partial_r)\right)\phi_0+D\bar{D}\phi_0=0} \quad (1)$$

- We posit the following ansatz:

$$\phi_0=\frac{1}{r^3}\sum_{n=0}\frac{\phi_0^{(n)}}{r^n},\quad \phi_1=\frac{1}{r^2}\sum_{n=0}\frac{\phi_1^{(n)}}{r^n},\quad \phi_2=\frac{1}{r}\sum_{n=0}\frac{\phi_2^{(n)}}{r^n}.$$

- Plugging in (1) and commuting the derivatives on the sphere,

$$(n+1)\partial_u\phi_0^{(n+1)}+\frac{n(n+3)}{2}\phi_0^{(n)}+\bar{D}D\phi_0^{(n)}=0. \quad (2)$$

The magical ansatz

- We introduce a new set of objects Q_n (of spin n) defined via

$$\boxed{\phi_0^{(n)} \equiv \phi_{0,G}^{(n)} + \phi_{0,L}^{(n)} = \phi_{0,G}^{(n)} + \frac{(-1)^n}{n!}\bar{D}^n Q_{n+1}}.$$

- Since $Q_{n+1} \in \bigoplus_{l=n+1}^{\infty} V_{n+1}^l$, $\bar{D}^n Q_{n+1} \in \bigoplus_{l=n+1}^{\infty} V_1^l$.

  On the other hand, $\phi_0^{(n)} \in \bigoplus_{l=1}^{\infty} V_1^l$.

  Therefore we call the missing part $\phi_{0,G}^{(n)} \in \bigoplus_{l=1}^{n} V_1^l$.

Charges

- $G^{(n)} \equiv \left[\phi^{(n)}_{0,G}\right]_{l=n}$ are **conserved charges**:

$$\boxed{\partial_u G^{(l)} = 0, \qquad G^{(l)} \in V^l_1}.$$

Charges

- $G^{(n)} \equiv \left[\phi_{0,G}^{(n)}\right]_{l=n}$ are **conserved charges**:

$$\boxed{\partial_u G^{(l)} = 0, \qquad G^{(l)} \in V_1^l}.$$

- Eq. (2) splits into two equations, one for the global part of $\phi_0^{(n)}$ and another for the local part:

$$\partial_u \phi_{0,G}^{(n+1)} = -\frac{1}{n+1}\left(\frac{n(n+3)}{2} + \bar{D}D\right)\phi_{0,G}^{(n)},$$

$$\cancel{\bar{D}^n}\partial_u Q_{n+1} = \cancel{\bar{D}^n} D Q_n.$$

Q_n are made of higher spin **charges** that suggest a higher spin **symmetry**.

Charges on the Celestial Sphere

$$\boxed{\phi_{0,G}^{(n)} = \sum_{l=1}^{n} \alpha_l^{(n)} G^{(l)} u^{n-l}}$$

- About the local part, assume that $Q_{-1} = 0$ (**no radiation**), then Q_n is a polynomial in u. Denoting $q_p \equiv q_p(z, \bar{z})$ the successive constants of integration, we have

$$Q_n = \sum_{p=0}^{n} D^{n-p} q_p \frac{u^{n-p}}{(n-p)!}.$$

- Notice that q_p has helicity p and represents the **conserved charge** associated to Q_p.

Hypergeometric resummation

- Global part

$$\phi_{1-s,G} = \frac{1}{r^{2+s}} \sum_{n=1}^{\infty} \frac{\phi_{1-s,G}^{(n)}}{r^n} = \sum_{l=1}^{\infty} \frac{G_s^{(l)}}{r^{l+3}} F_{l,s}\left(-\frac{u}{2r}\right)$$

- Local part

$$\phi_{1-s,L} = \delta_{0,s}\frac{q_0}{r^2} + \sum_{l=1}^{\infty}\sum_{p=1}^{l} \frac{(-1)^{p-s}}{(p-s)!} \frac{\bar{D}^{p-s} q_p^{(l)}}{r^{p+2}} F_{l,s}^{(p)}(\tau)$$

Relation to Multipole Moments

- A generic scalar potential ϕ has an expansion in multipole

$$\phi \sim \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \partial_L \left(\frac{\hat{M}_L(u)}{r} \right) = \sum_{l=0}^{\infty} \sum_{k=0}^{l} \frac{\#}{r^{k+1}} \partial_u^{l-k} M^{(l)}$$

- In our case

$$\phi_{1,L} = \sum_{l=0}^{\infty} \sum_{k=0}^{l} \frac{\#}{r^{k+2}} \partial_u^{l-k} \mathcal{E}_l^{(l)}$$

$$\mathcal{E}_l^{(l)} = \sum_{p=0}^{l} \frac{(-2)^p l!}{(l+p)!} u^{l-p} \bar{D}^p q_p^{(l)}$$

- The asymptotic charge $q_p^{(l)}$ is related to the multipole (l).

What comes next

- Matching condition between $\mathcal{I}^+$ and $\mathcal{I}^-$.
- Multipole expansion valid in general? Inclusion of radiation...

WHAT COMES NEXT

- Matching condition between $\mathcal{I}^+$ and $\mathcal{I}^-$.
- Multipole expansion valid in general? Inclusion of radiation...
- Lots of parallels with gravity.
- The coefficients of the NP scalars expanded in $1/r$ (and in particular Ψ_0 which encodes outgoing radiations) are identified with charges generating asymptotic symmetries and soft higher-spin symmetries found by OPE methods in the celestial CFT framework.
- Extension to (A)dS.
- Extension to Yang-Mill.