Title: On the continuum limit of spin foams: graviton dynamics and area metric induced corrections

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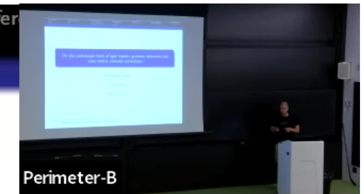
Abstract: The semi-classical limit of spin foams leads to the Area Regge action. It was long thought that this action leads to flatness and does, in particular, not allow for propagating gravitons. I will present the first systematic studies of the continuum limit of the Area Regge action, using different versions of regular hypercubic lattices. These studies have shown that the Area Regge action does in its continuum limit, lead to leading order to general relativity, and thus to propagating gravitons. The higher order corrections depend on the choice of triangulation for the hypercubic lattice. However, there seems to be a preferred choice, for which the Area Regge action is not singular. In this case the correction term approximates the square of the Weyl curvature tensor, and can be interpreted to arise from an area metric dynamics. We therefore conjecture that the continuum limit of spin foams is described by an area metric theory.



On the continuum limit of spin foams: graviton dynamics and area metric induced corrections [1]

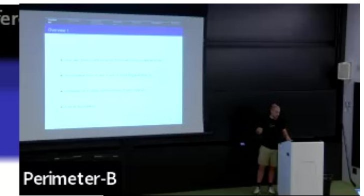
Athanasios Kogios

Perimeter Institute

June 21, 2022

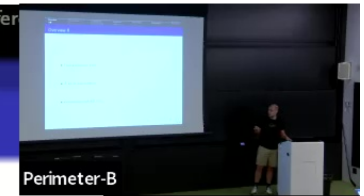
[1]Based on joint work with Bianca Dittrich [1]

Overview I

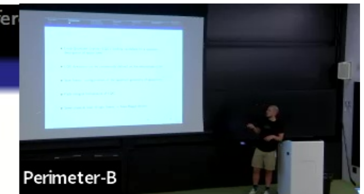


- Solve spin foams (path integral) (Plebanski action or discretization).
- Semi-classical limit of spin foams $\rightarrow$ Area Regge Action [2].
- Implement on a lattice (discretization of path integral).
- A lot of calculations.

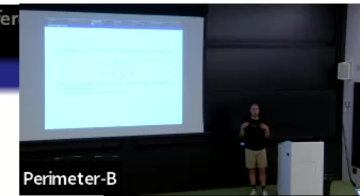
Overview II



- Find continuum limit.
- A lot of calculations.
- Connection with GR (?).



- Loop Quantum Gravity (LQG), leading candidate for a quantum description of space-time.
- LQG dynamics can be consistently defined on the microscopic level.
- Spin foams: configurations of the quantum geometry of space-time.
- Path integral formulation of LQG.
- Semi-classical limit of spin foams $\rightarrow$ Area Regge Action.



Length Regge action is a well established approximation to Einstein-Hilbert action.

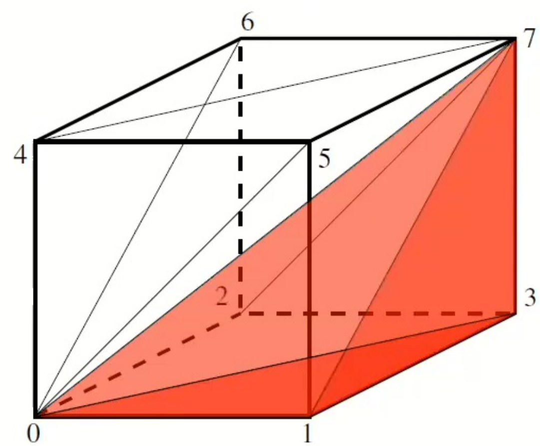
$$S_{LR} = \frac{1}{\kappa} \sum_t A_t(L_e)\epsilon(L_e) \tag{1}$$

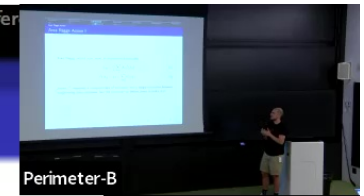
$$\epsilon_t^L(L_e) = 2\pi - \sum_{\sigma \supset t} \theta_t^\sigma(L_e) \tag{2}$$

where θ_t^σ is the dihedral angle between two tetrahedra sharing the triangle t in the four-simplex-simplex σ.
In Length Area Calculus ϵ measures curvature!

Length Regge Action II

- On a triangulated manifold (not regular lattice as eg QCD).
- 10 edges on a four-simplex, when gluing two simplices in a tetrahedron 14 length variables.





Area Regge Action I

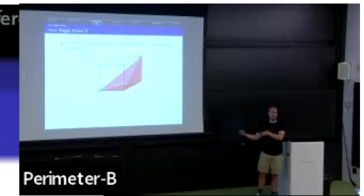
Area Regge action uses areas as fundamental variables.

$$S_{AR} = \frac{1}{\kappa} \sum_t A_t \epsilon^A (A_t) \tag{3}$$

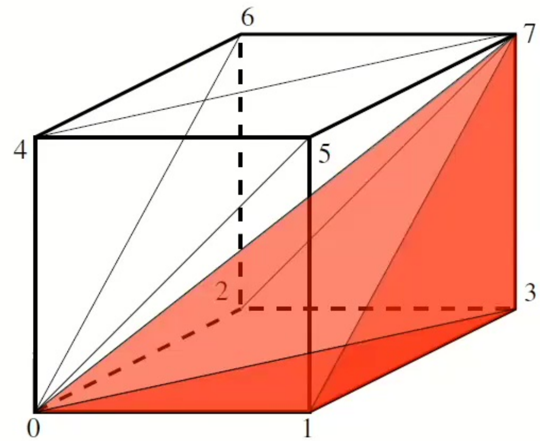
$$\epsilon_t^A (A_t) = 2\pi - \sum_{\sigma \supset t} \theta_t^\sigma (A_t) \tag{4}$$

where ϵ_t^σ measures a conglomerate of curvature and a shape mismatch between neighboring four-simplices, not the curvature as before (keep in mind #1)!

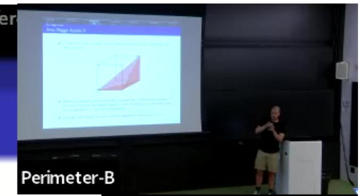
Area Regge Action II



- 10 areas on a four-simplex, when gluing two simplices in a tetrahedron 16 area variables.



- Mismatch between areas and length variables due to differences between the three-dimensional dihedral angles in the tetrahedron, as induced by the area assignments to the two four-simplices.
- Jacobian from length to area variables degenerate (keep in mind #2).



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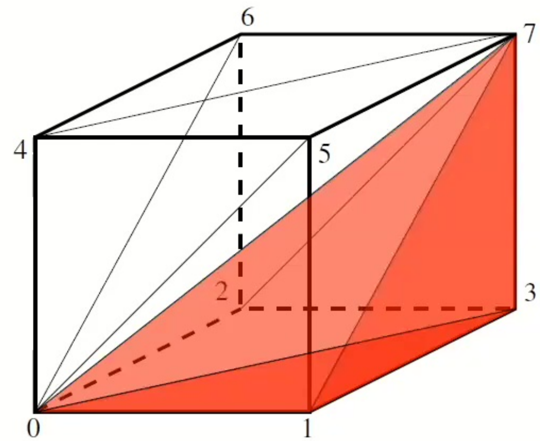

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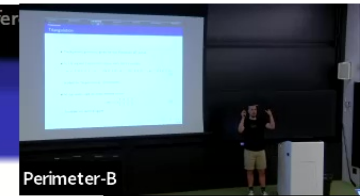


Triangulation

- Background geometry given by flat Euclidean 4D space.
- In [3] regular hypercubic lattice with lattice vectors:

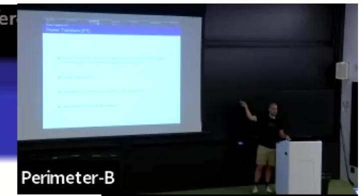
$$e_0 = \lambda(0,0,0,1),\ e_1 = \lambda(0,0,1,0),\ e_2 = \lambda(0,1,0,0) \quad ,\ e_3 = \lambda(1,0,0,0) \tag{5}$$

leaded to "hypercubical" constraints.

- In our work, add an inner central vertex:

$$m_0 = \lambda(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \tag{6}$$

Jacobian no more singular.



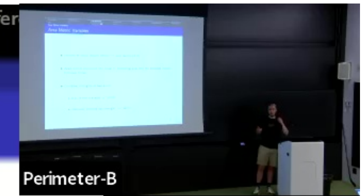
Fourier Transform (FT)

- Split into boundary (not containing the inner vertex) and bulk triangles (containing the inner vertex). Integrate out the latter.

- Fourier Transform.

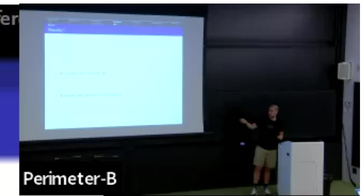
- Expand Fourier coefficients in terms of lattice constant.

- Derivative expansion of the Hessian.



Area Metric Variables

- Instead of usual length metric $\rightarrow$ area metric [4–6].
- Area metric measures the areas of parallelograms and the dihedral angles between these.
- Combine triangles in two ways:
  - sum of two triangles: α^+ sector
  - difference between two triangles: α^- sector.

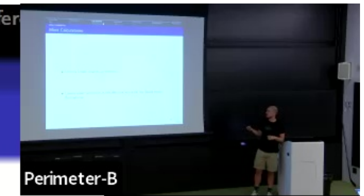


Results I

- Leading order: Recover GR!

- Lowest order correction: Weyl squared!

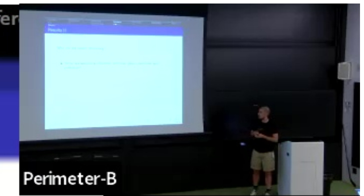
More Calculations



- Identify length degrees of freedom.

- Lowest order correction in the effective action for the length metric fluctuations.

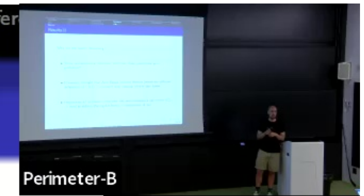
Results II



Why are the results interesting?

- Some speculations in literature, first time these calculations were performed!
- Previously thought that Area Regge imposes flatness (remember different definitions of ϵ #1) $\rightarrow$ incorrect semi-classical limit of spin foams.
- Degeneracy of Jacobian (remember why we introduced a new lattice #2) $\rightarrow$ need to deform the regular lattice $\rightarrow$ suppression of dof.

Results II



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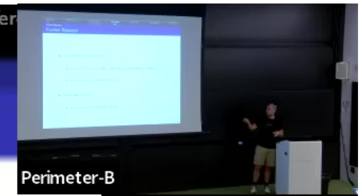
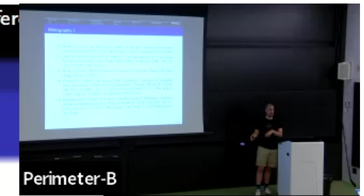
Further Research



- Using already defined lattices:
  - Fourier Transform without integrating out bulk degrees of freedom.
  - Check consistency between lattices.
- Define new lattices.
  - Check consistency between lattices.



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