Title: Measurement-Protected Order in Monitored Quantum Circuits with Continuous Symmetry

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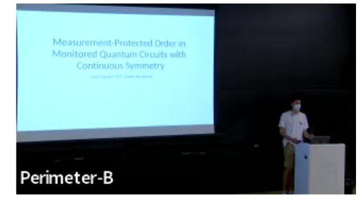
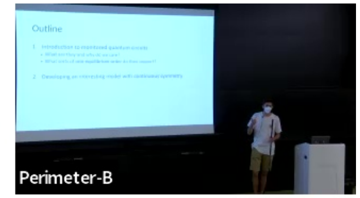
Abstract: Monitored quantum circuits, composed of local unitary operators and projective measurements, have recently emerged as a rich setting for studying non-equilibrium quantum dynamics. In such systems, sufficient densities of measurements can protect a highly-monitored steady state phase with area law entanglement. Furthermore, it has been shown that such area law phases can host a measurement-protected Ising ferromagnetic order. However, it is not yet known whether such measurement-protected order is a generic phenomenon or whether it relies on the discrete Ising symmetry. To begin answering this question, we introduce a circuit model with continuous symmetry where ferromagnetic order arises in the steady state. Notably, our model requires feedback based on measurement results in order to generate this ferromagnetic order



Measurement-Protected Order in Monitored Quantum Circuits with Continuous Symmetry

Jake Hauser (UC Santa Barbara)

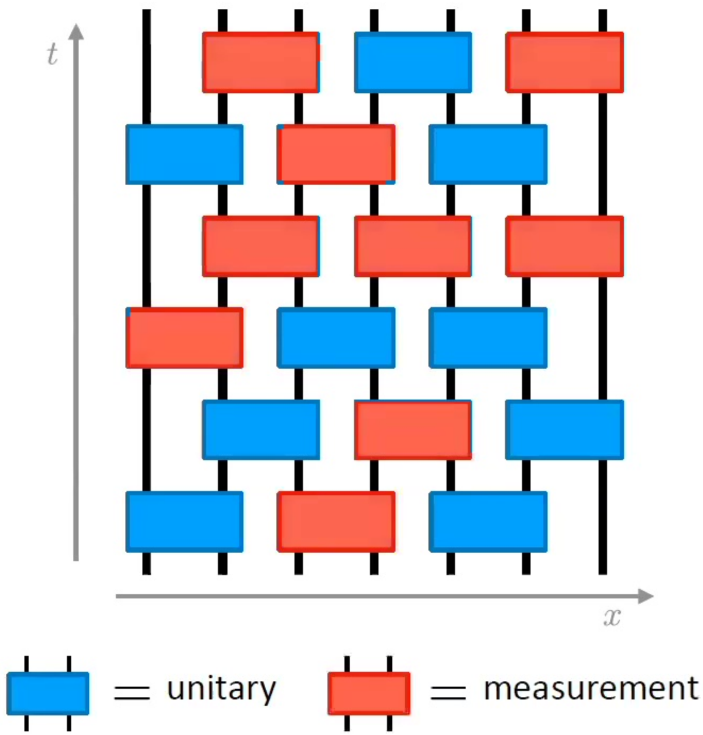
Outline



1. Introduction to **monitored quantum circuits**
   - What are they and why do we care?
   - What sorts of **non-equilibrium order** do they support?

2. Developing an interesting model with **continuous symmetry**

Monitored quantum circuits

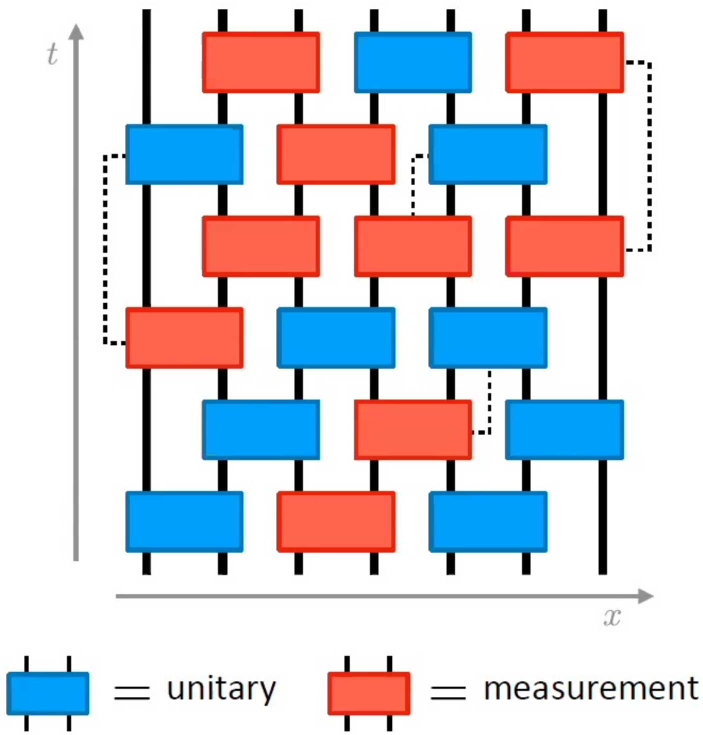


- A minimal model for quantum evolution:
  - **unitary**
  - **local**
- More generally, we allow **measurements**

$$M = \sum_{i=1}^{n} m_i P_i \qquad |\psi\rangle \to \frac{P_i|\psi\rangle}{\sqrt{\langle\psi|P_i|\psi\rangle}}$$

with probability $\langle\psi|P_i|\psi\rangle$

Monitored quantum circuits



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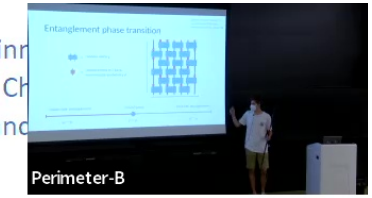
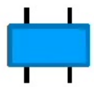
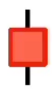
- Classical **feedback** from measurements is also allowed

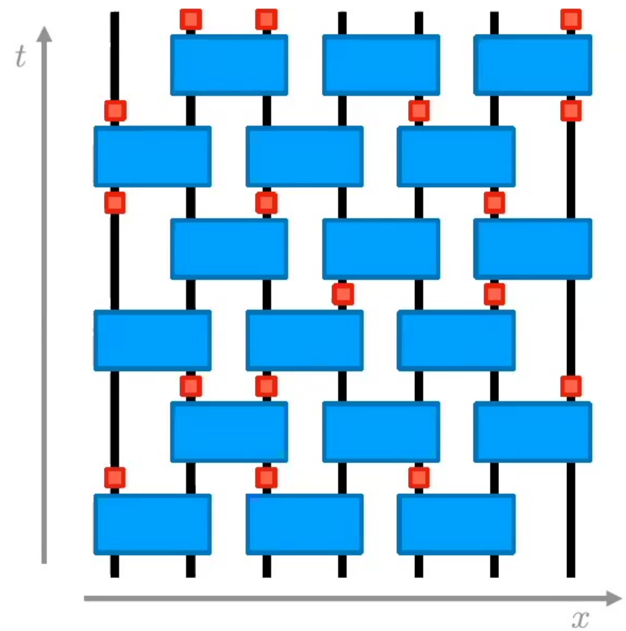
Skinn
Li, Ch
Nanc



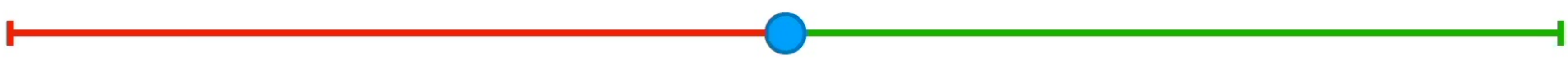
Entanglement phase transition

 = random unitary

 = measurement in Z basis, inserted with probability p

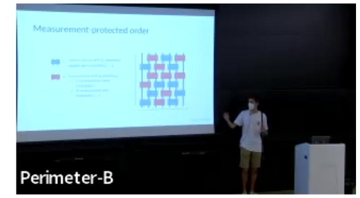


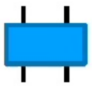
volume law entanglement | critical point | area law entanglement

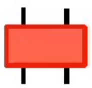


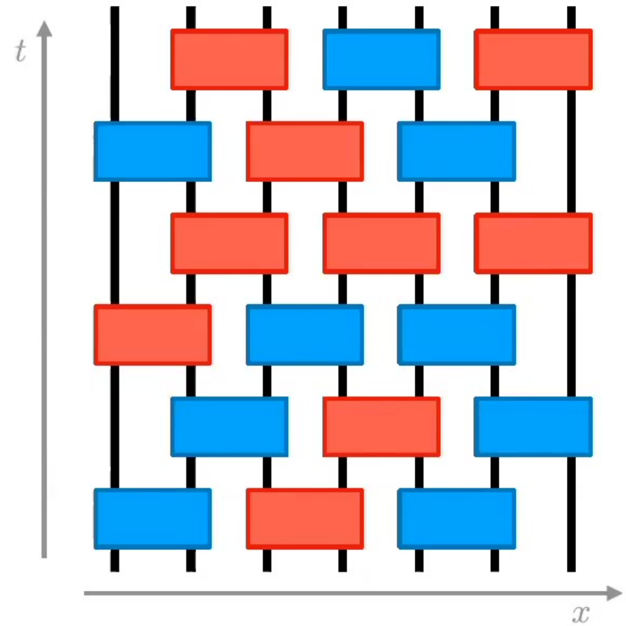
$p < p_c$ | $p = p_c$ | $p > p_c$

Measurement-protected order



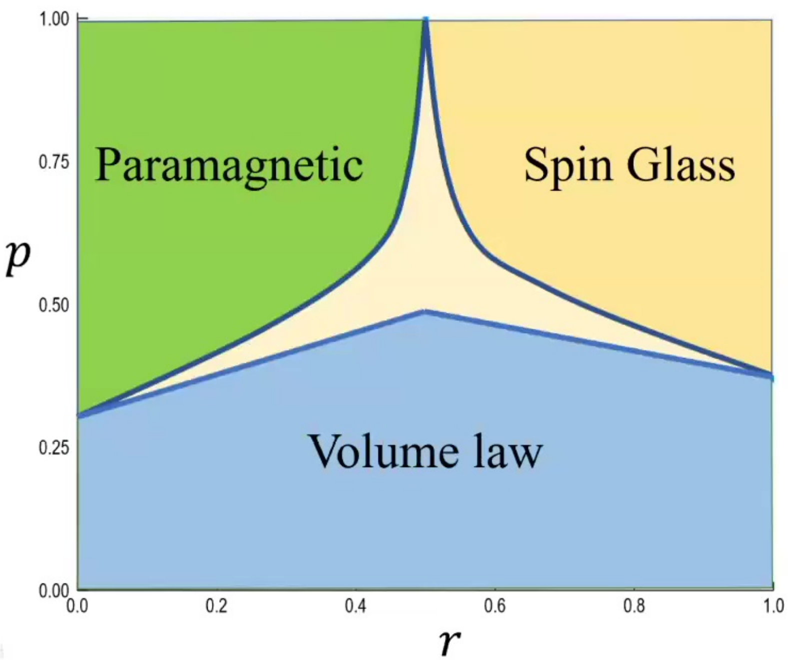
 = random unitary with **Z_2 symmetry** applied with probability *1 – p*

 = measurement with probability *p*:
- *ZZ* measurement with probability *r*
- *XI* measurement with probability *1 – r*



Sang & Hsieh 2021

Measurement-protected order



Paramagnetic:

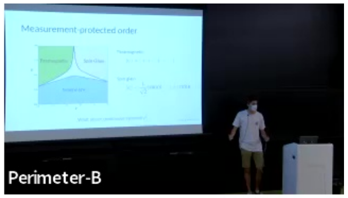
$$|\psi\rangle = |++-+-\cdots\rangle$$

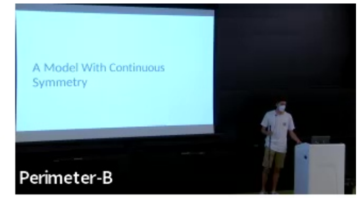
Spin glass:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00101\cdots\rangle + |11010\cdots\rangle)$$

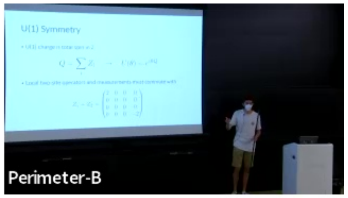
What about continuous symmetry?

Sang & Hsieh 2021





A Model With Continuous Symmetry



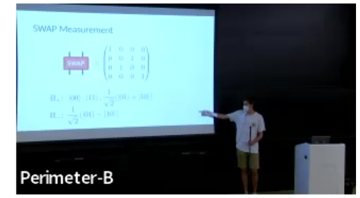
U(1) Symmetry

- U(1) charge is total spin in Z:

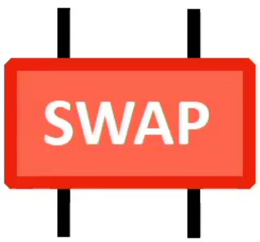
$$Q = \sum_i Z_i \quad \rightarrow \quad U(\theta) = e^{i\theta Q}$$

- Local two-site operators and measurements must commute with

$$Z_1 + Z_2 = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$



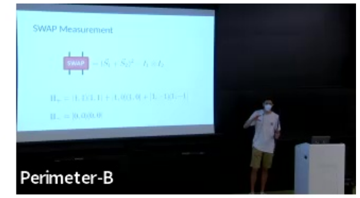
SWAP Measurement



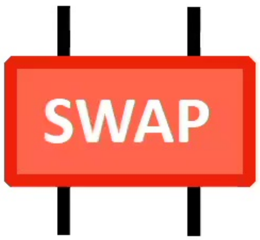
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Pi_+ : \;\; |00\rangle \, , |11\rangle \, , \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$\Pi_- : \;\; \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



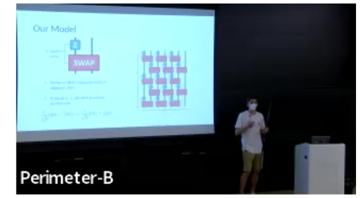
SWAP Measurement



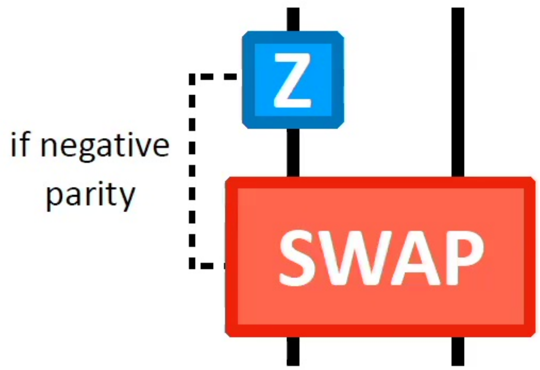
$$= (\vec{S}_1 + \vec{S}_2)^2 - I_1 \otimes I_2$$

$$\Pi_+ = |1,1\rangle\langle 1,1| + |1,0\rangle\langle 1,0| + |1,-1\rangle\langle 1,-1|$$

$$\Pi_- = |0,0\rangle\langle 0,0|$$

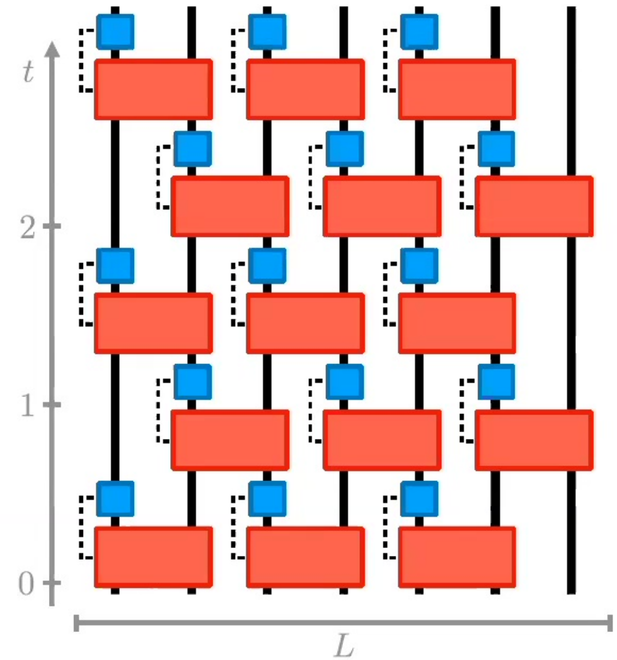


Our Model

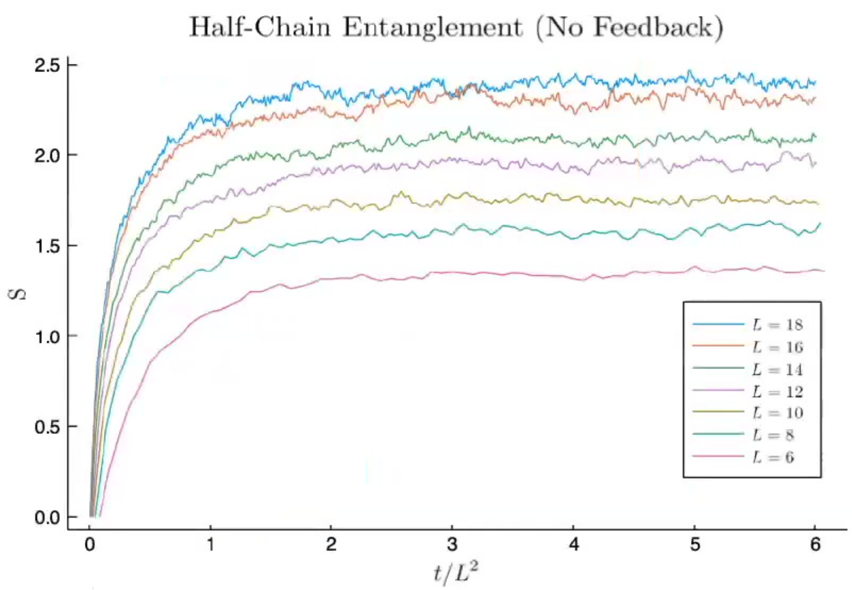


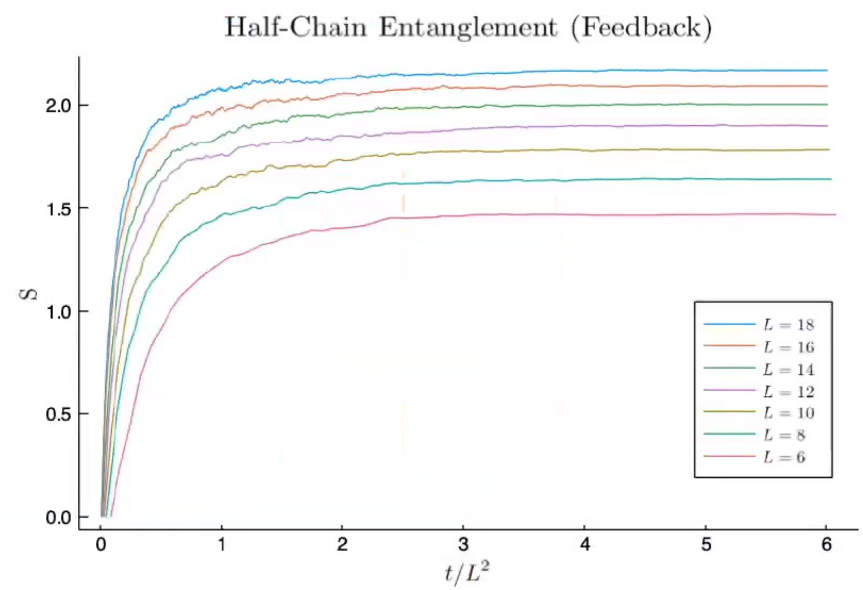
1. Perform SWAP measurement on adjacent sites
2. If result is -1, act with Z unitary on first site

$$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \rightarrow \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

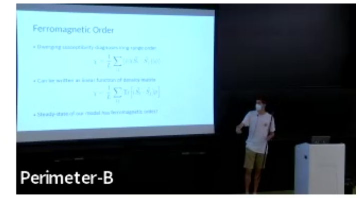


Reaching a steady state





Ferromagnetic Order



- Diverging susceptibility diagnoses long-range order

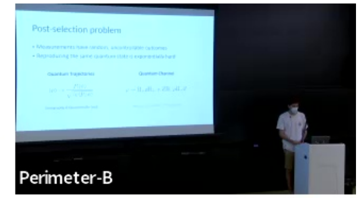
$$\chi = \frac{1}{L} \sum_{ij} \langle \psi | (\vec{S}_i \cdot \vec{S}_j) | \psi \rangle$$

- Can be written as linear function of **density matrix**

$$\chi = \frac{1}{L} \sum_{ij} \mathrm{Tr}\left[(\vec{S}_i \cdot \vec{S}_j) \rho \right]$$

- Steady-state of our model has ferromagnetic order!

Post-selection problem



- Measurements have random, uncontrollable outcomes
- Reproducing the same quantum state is exponentially hard

Quantum Trajectories

$$|\psi\rangle \to \frac{P_i|\psi\rangle}{\sqrt{\langle\psi|P_i|\psi\rangle}}$$

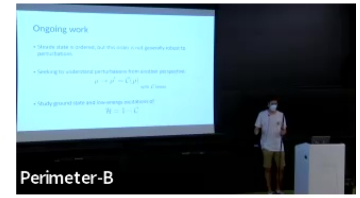
Tomography is exponentially hard

Quantum Channel

$$\rho \to \Pi_+\rho\Pi_+ + Z\Pi_-\rho\Pi_-Z$$

Natural observable of the system

Ongoing work



- Steady state is ordered, but this order is not generally robust to perturbations

- Seeking to understand perturbations from another perspective:

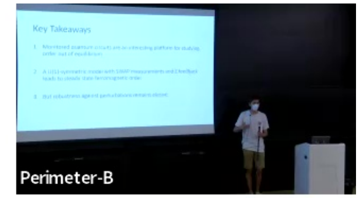
$$\rho \to \rho' = \mathcal{C}(\rho)$$

with $\mathcal{C}$ linear

- Study ground state and low-energy excitations of

$$\mathcal{H} = 1 - \mathcal{C}$$

Key Takeaways

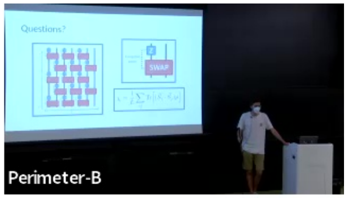


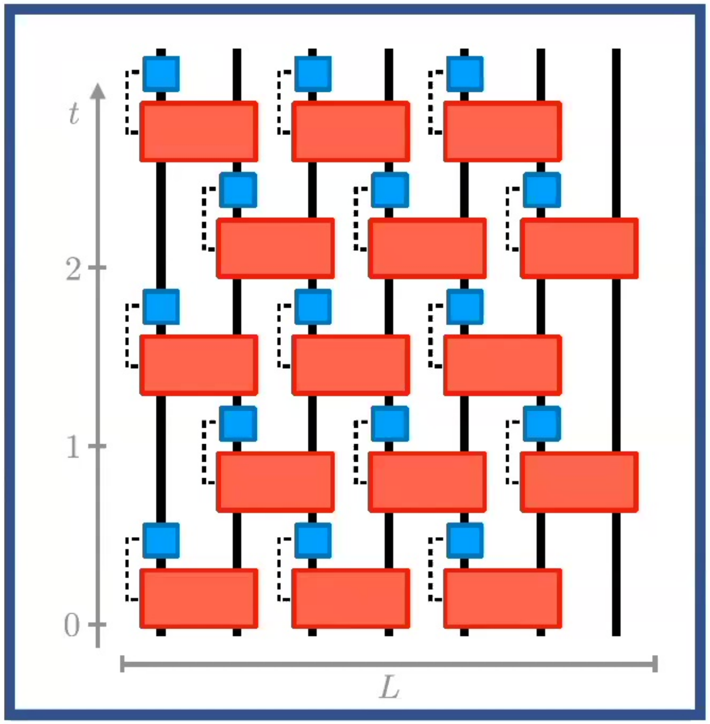
1. Monitored quantum circuits are an interesting platform for studying order out of equilibrium

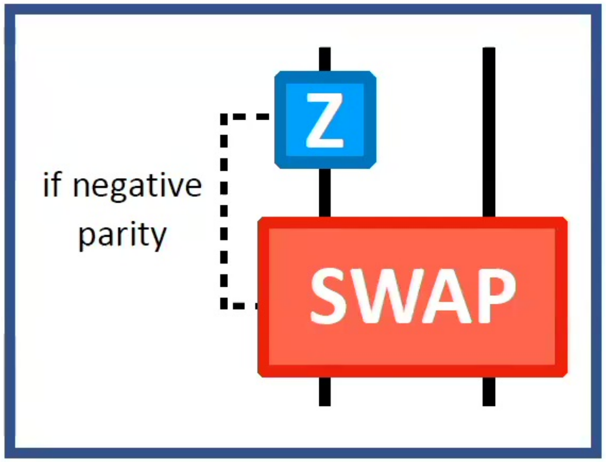
2. A U(1)-symmetric model with SWAP measurements and Z feedback leads to steady state ferromagnetic order

3. But robustness against perturbations remains elusive

Questions?







$$\chi = \frac{1}{L} \sum_{ij} \mathrm{Tr}\left[(\vec{S}_i \cdot \vec{S}_j) \rho \right]$$