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Speakers: Brayden Hull

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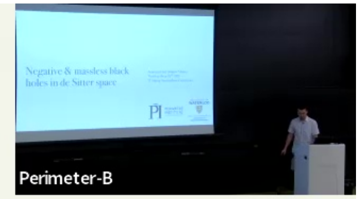
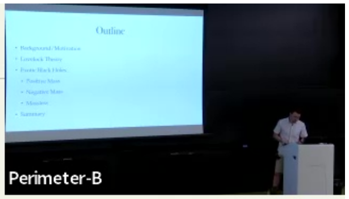


Negative & massless black holes in de Sitter space

Brayden Hull (Robert Mann)
Tuesday, June 21^{st} 2022
PI Young Researchers Conference

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

UNIVERSITY OF WATERLOO
FACULTY OF MATHEMATICS
Department of Applied Mathematics



Outline

- Background / Motivation
- Lovelock Theory
- Exotic Black Holes
  - Positive Mass
  - Negative Mass
  - Massless
- Summary

Black Holes

- Black holes are interesting and exciting objects, although quite strange as well.
- Singularities, event horizons, closed time like curves
- In General Relativity black holes are described by event horizons which possess constant curvature: spherical, hyperbolic.
- Exotic black holes in higher curvature theories do not need to possess constant curvature horizons
- Exotic black holes have novel thermodynamic phenomena

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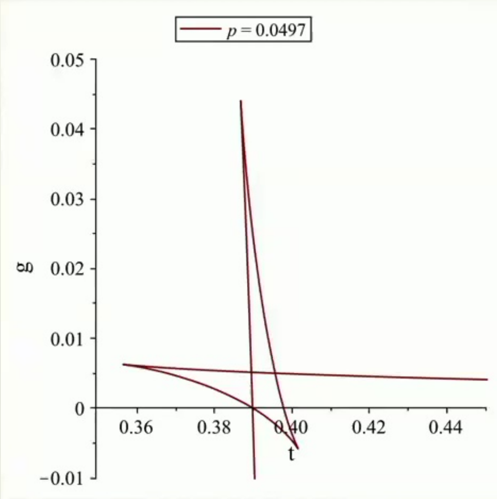
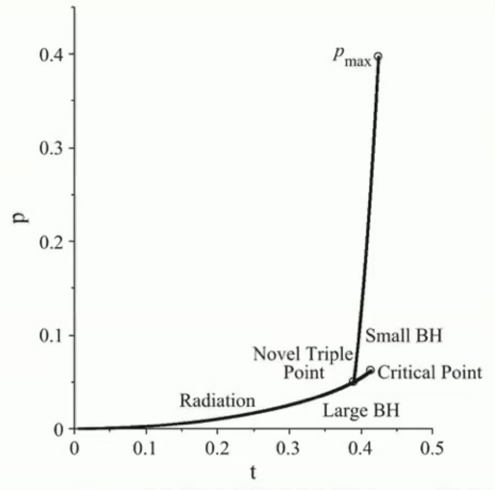
Motivation

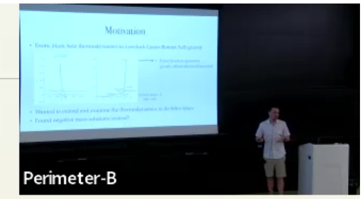
- Exotic black hole thermodynamics in Lovelock Gauss-Bonnet AdS gravity





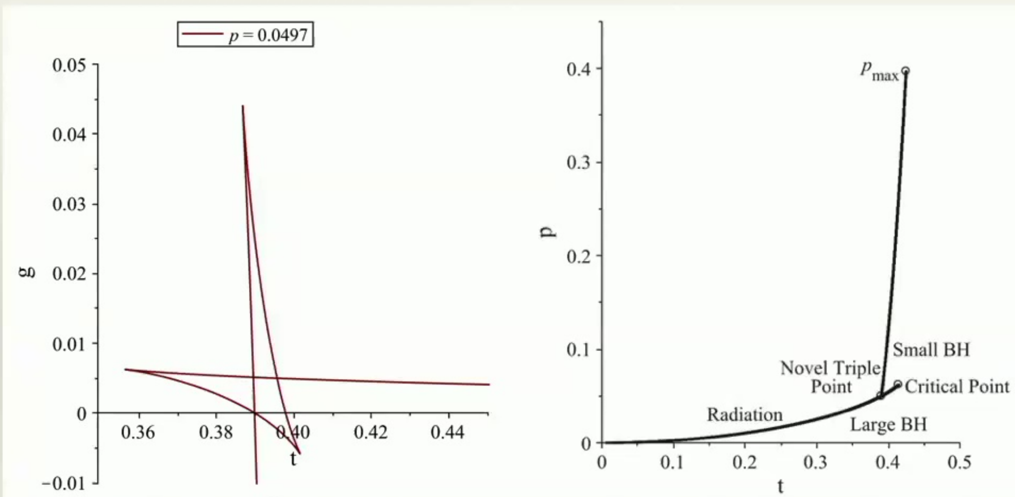
- Event horizon geometry greatly effects thermodynamics!

Hull,B. Mann, R.
PRD 2021.



Motivation

- Exotic black hole thermodynamics in Lovelock Gauss-Bonnet AdS gravity



- Event horizon geometry greatly effects thermodynamics!

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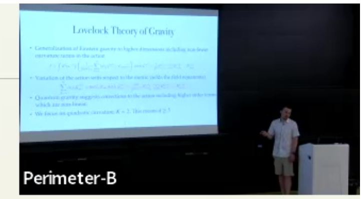
PRD 2021.

- Wanted to extend and examine the thermodynamics in de Sitter space
- Found negative mass solutions instead!



Lovelock Theory of Gravity

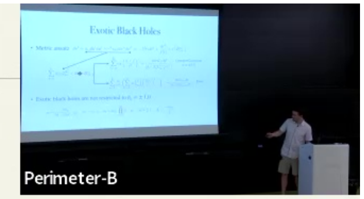
- Generalization of Einstein gravity to higher dimensions including non-linear curvature terms in the action

$$S = \int d^d x \sqrt{-g} \left(\frac{1}{16\pi G_N} \sum_{k=0}^{K} (\hat{\alpha})_k \mathcal{L}^{(k)} + \mathcal{L}_{\text{matter}} \right) \text{ with } \mathcal{L}^{(k)} = \frac{1}{2^k} \delta^{a_1 b_1 \ldots a_k b_k}_{c_1 d_1 \ldots c_k d_k} R^{c_1 d_1}_{a_1 b_1} \ldots R^{c_k d_k}_{a_k b_k}$$

- Variation of the action with respect to the metric yields the field equation(s)

$$\sum_{k=0}^{K} \hat{\alpha}_{(k)} \mathcal{G}^{(k)}_{ab} = 8\pi G_n T_{ab} \text{ with } \mathcal{G}^{(k)}_{ab} = \frac{g_{ab}}{2^{(k+1)}} \delta^{a_1 b_1 \ldots a_k b_k}_{c_1 d_1 \ldots c_k d_k} R^{c_1 d_1}_{a_1 b_1} \ldots R^{c_k d_k}_{a_k b_k}$$

- Quantum gravity suggests corrections to the action including higher order terms which are non-linear.
- We focus on quadratic curvature, $K = 2$. This means $d \geq 5$



Exotic Black Holes

- Metric ansatz $ds^2 = g_{ij}dy^i dy^j + r^2\gamma_{\alpha\beta}dx^\alpha dx^\beta \Rightarrow -f(r)dt^2 + \dfrac{dr^2}{f(r)} + r^2 d\Sigma^2_{d-2}$

$$\sum_{k=0}^{K} \hat{\alpha}_{(k)}\mathcal{G}^{(k)}_{ab} = 0 \Rightarrow d\Sigma^2_{d-2}$$

$$\sum_{k=0}^{K} \alpha_k \left(\frac{\kappa - f}{r^2}\right)^k = \frac{16\pi G_N M}{(d-2)\Sigma^{(\kappa)}_{d-2} r^{d-1}}$$

Constant Curvature $\kappa = \pm 1{,}0$

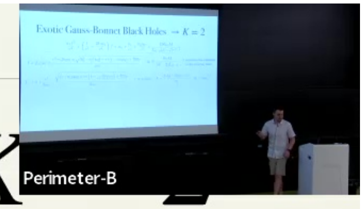
$$\sum_{n=0}^{K} \frac{b_n}{r^{2n}} \left(\sum_{k=n}^{K} \alpha_k \binom{k}{n} \left(\frac{f(r)}{r^2}\right)^{k-n}\right) = \frac{16\pi G_N M}{(d-2)\Sigma_{d-2} r^{d-1}}$$

Exotic

- Exotic black holes are not restricted to $b_n = \pm 1{,}0$

$$\alpha_0 = \frac{\hat{\alpha}_{(0)}}{(d-1)(d-2)}, \quad \alpha_1 = \hat{\alpha}_{(1)}, \quad \alpha_k = \hat{\alpha}_{(k)} \prod_{n=3}^{2k}(d-n) \quad \text{for } k \geq 2 \quad \Lambda = \frac{-\hat{\alpha}_{(0)}}{2}$$

Exotic Gauss-Bonnet Black Holes $\rightarrow K$



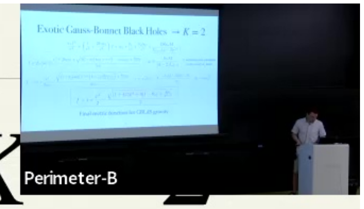
$$\frac{\alpha_2 f^2}{r^4} + \left(\frac{1}{r^2} + \frac{2b_1\alpha_2}{r^4}\right) f + \alpha_0 + \frac{b_1}{r^2} + \frac{b_2\alpha_2}{r^4} = \frac{16 G_n M}{\Sigma_{d-2}(d-2)r^{d-1}}$$

$$f = f_{\pm}(\mathrm{m}) \equiv \frac{r^2 + 2b_1\alpha_2 \pm \sqrt{(b_1^2 - b_2)\,4\alpha_2^2 + r^4\,(1 - 4\alpha_2\alpha_0) + \frac{8\mathrm{m}\alpha_2}{r^{d-5}}}}{2\alpha_2} \qquad \mathrm{m} \equiv \frac{8\pi M}{(d-2)\Sigma_{d-2}}$$

$\pm$ separates the solutions in the small α_2 limit

$$f_- = 1 + \frac{r^2}{2\alpha_2} - \frac{\sqrt{(1-b_2)4\alpha_2^2 + r^4\left(1 + \frac{2\Lambda\alpha_2}{(d-2)(d-1)}\right) + \frac{8\mathrm{m}\alpha_2}{r^{d-5}}}}{2\alpha_2} \qquad r = x\sqrt{\alpha_2}, \quad \Lambda = \frac{2z(d-2)(d-1)}{\alpha_2}, \quad \mathrm{m} = m\alpha_2^{\frac{d-3}{2}}$$

Exotic Gauss-Bonnet Black Holes $\rightarrow K$



$$\frac{\alpha_2 f^2}{r^4} + \left(\frac{1}{r^2} + \frac{2b_1\alpha_2}{r^4}\right) f + \alpha_0 + \frac{b_1}{r^2} + \frac{b_2\alpha_2}{r^4} = \frac{16 G_n M}{\Sigma_{d-2}(d-2)r^{d-1}}$$

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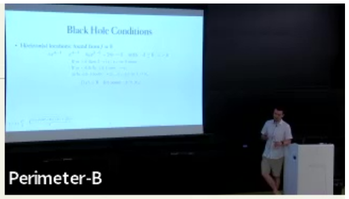
$$\boxed{f = 1 + \frac{x^2}{2} - \frac{\sqrt{(1+4z)x^4 + 4(1-b_2) + \frac{8m}{x^{d-5}}}}{2}}$$

Final metric function for GBLdS gravity

Black Hole Conditions



- Horizon(s) locations: found from $f = 0$

$$zx^{d-1} - x^{d-3} - b_2 x^{d-5} + 2m = 0 \quad \text{with} \quad d \geq 6, \;\; z > 0$$

If $m > 0$ then $2 \rightarrow (x_c, x_+)$ or 0 roots

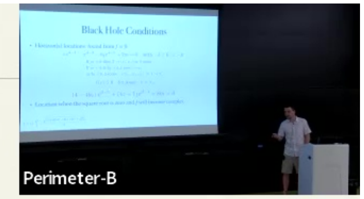
If $m < 0$ & $b_2 > 0, 1$ root $\rightarrow x_c$

or $b_2 < 0, 3$ roots $\rightarrow (x_-, x_+, x_c)$ or $1 \rightarrow x_c$

$$f(x) > 0 \quad \text{for some} \quad x > x_+$$

$$f = 1 + \frac{x^2}{2} - \frac{\sqrt{(1+4z)x^4 + 4(1-b_2) + \frac{8m}{x^{d-5}}}}{2}$$

Black Hole Conditions



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$$\text{If } m > 0 \text{ then } 2 \to (x_c, x_+) \text{ or } 0 \text{ roots}$$

$$\text{If } m < 0 \ \& \ b_2 > 0, 1 \text{ root } \to x_c$$

$$\text{or } b_2 < 0, 3 \text{ roots } \to (x_-, x_+, x_c) \text{ or } 1 \to x_c$$

$$f(x) > 0 \quad \text{for some} \quad x > x_+$$

$$(4 - 4b_2)\, x^{d-5} + (4z+1)x^{d-1} + 8m = 0$$

- Location when the square root is zero and f will become complex

$$f = 1 + \frac{x^2}{2} - \frac{\sqrt{(1+4z)x^4 + 4(1-b_2) + \frac{8m}{x^{d-5}}}}{2}$$

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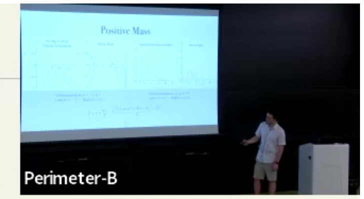
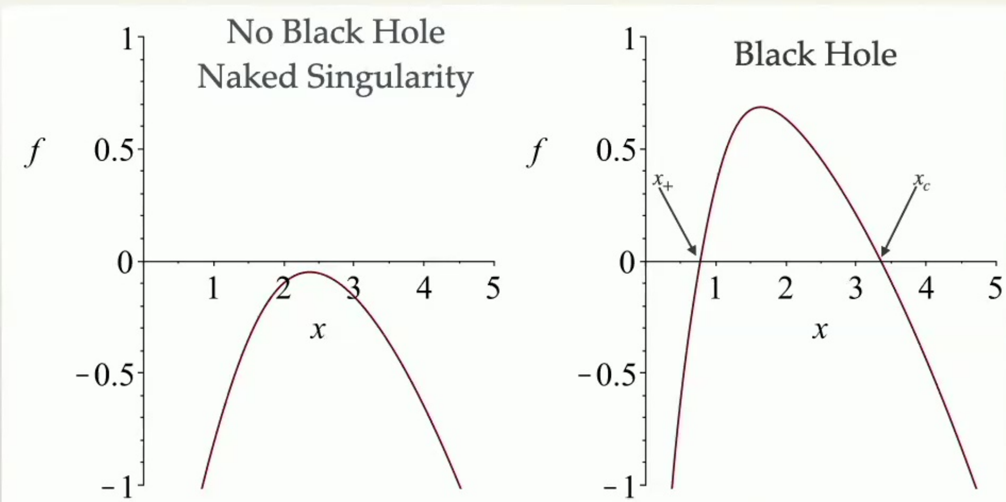
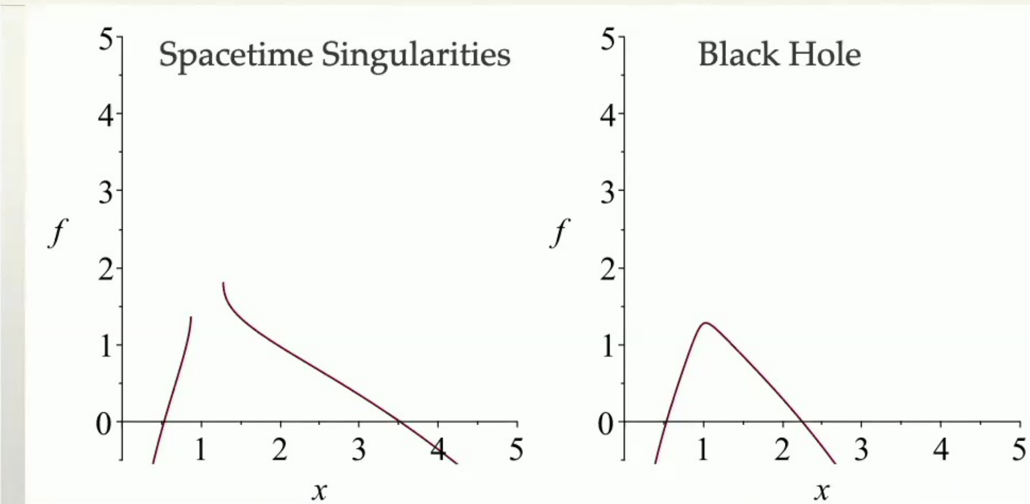


Positive Mass

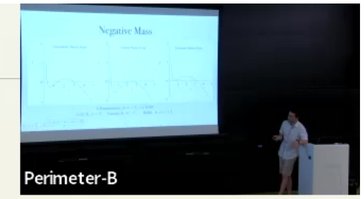




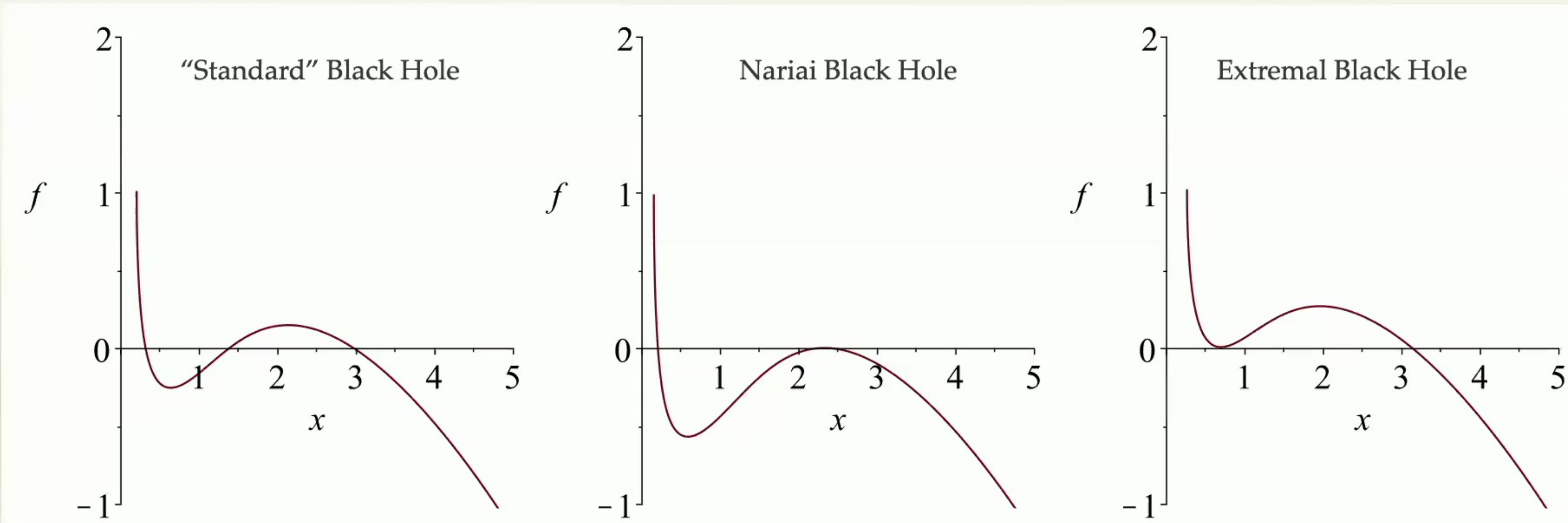
6 Dimensions $m = 1,\ z = 0.1.$
Left: $b_2 = -2$ Right: $b_2 = 2$

6 Dimensions $m = 1,\ b_2 = 3.5.$
Left: $z = 0.1$ Right: $z = 0.3$

$$f = 1 + \frac{x^2}{2} - \frac{\sqrt{(1+4z)x^4 + 4(1-b_2) + \frac{8m}{x^{d-5}}}}{2}$$



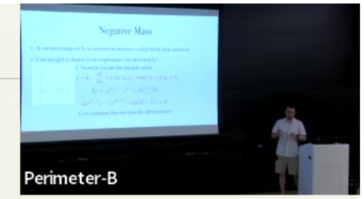
Negative Mass



6 Dimensions, $m = -3$, $z = 0.09$.

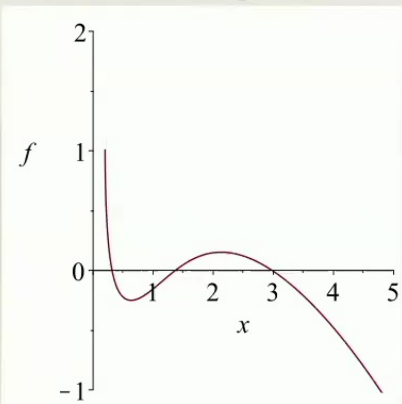
Left: $b_2 = -2$ Center: $b_2 = -3$ Right: $b_2 = -1.3$

$$f = 1 + \frac{x^2}{2} - \frac{\sqrt{(1+4z)x^4 + 4(1-b_2) + \frac{8m}{x^{d-5}}}}{2}$$



Negative Mass

- A certain range of b_2 is needed to ensure a valid black hole solution
- Can we get a closed form expression for allowed b_2?



- Need to locate the double roots

$$f = 0, \quad \frac{\partial f}{\partial x} = 0 \text{ for } b_2, x \quad \text{with } \{x > 0, b_2 < 0\}$$

$$b_2 = x^4 z - x^2 + 2x^{5-d} m$$

$$2x^{d-1} z - x^{d-3} - m(d-5) = 0$$

Can examine this for specific dimensions

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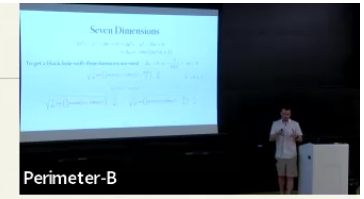


Seven Dimensions

$$2x^6 z - x^4 - 2m = 0 \to 2y^3 z - y^2 - 2m = 0$$

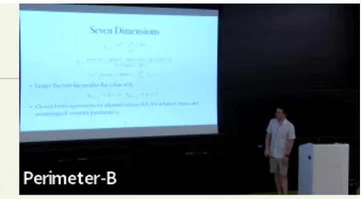
$$\Rightarrow \Delta_3 \equiv -8m\left(54z^2 m + 1\right)$$

To get a black hole with three horizons we need $\Delta_3 > 0 \Rightarrow -\dfrac{1}{54z^2} < m < 0$

$$\sqrt{\frac{1}{3z}\cos\left(\frac{1}{3}\arccos(1+108mz^2) - \frac{2\pi k}{3}\right) - \frac{1}{6z}} \qquad k = 0, 1, 2$$

$$x_0 > x_1, \quad x_2 \propto i$$

$$\sqrt{\frac{1}{3z}\cos\left(\frac{1}{3}\arccos(1+108mz^2)\right) - \frac{1}{6z}} \qquad \sqrt{\frac{1}{3z}\cos\left(\frac{1}{3}\arccos(1+108mz^2) - \frac{2\pi}{3}\right) - \frac{1}{6z}}$$



Seven Dimensions

$$b_2 = \frac{zx^6 - x^4 + 2m}{x^2}$$

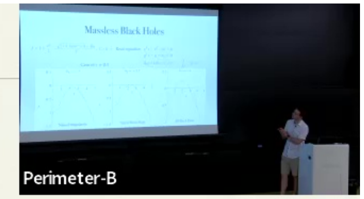
$$b_2 = \frac{8\cos(A)^3 + 432mz^2 - 36\cos(A)^2 + 30\cos(A) - 7}{72z^2\cos(A) - 36z}$$

$$A = \frac{1}{3}\arccos(1 + 108mz^2) - \frac{2\pi k}{3} \quad k = 0, 1$$

- Larger the root the smaller the value of b_2

$$b_{2_{min}} \rightarrow k = 0,\ b_{2_{max}} \rightarrow k = 1$$

- Closed form expressions for allowed values of b_2 for arbitrary mass and cosmological constant parameter z



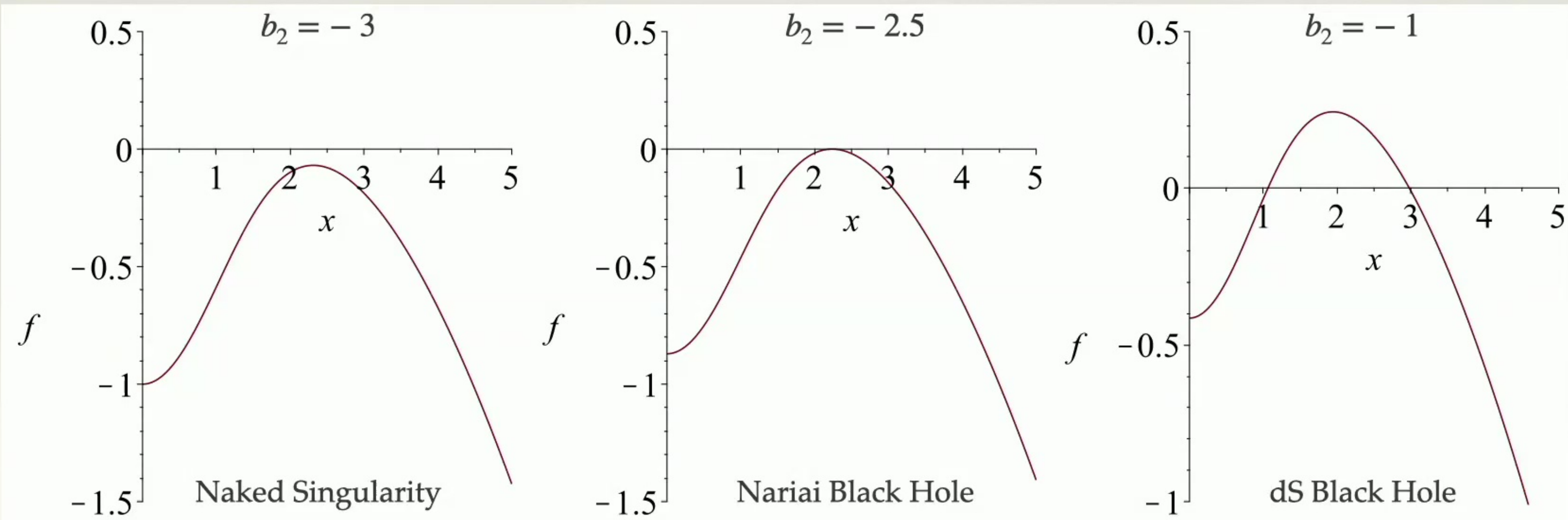
Massless Black Holes

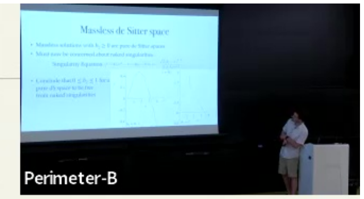
$$f = 1 + \frac{x^2}{2} - \frac{\sqrt{(1+4z)x^4 + 4 - 4b_2}}{2}, \; f = 0 \rightarrow$$ Root equation $$x^4 z - x^2 - b_2 = 0$$

$$y^2 z - y - b_2 = 0$$

$$\Delta_2 \equiv 4zb_2 + 1 \;\geq 0 \qquad -\frac{1}{4z} < b_2 < 0$$

Case of $z = 0.1$



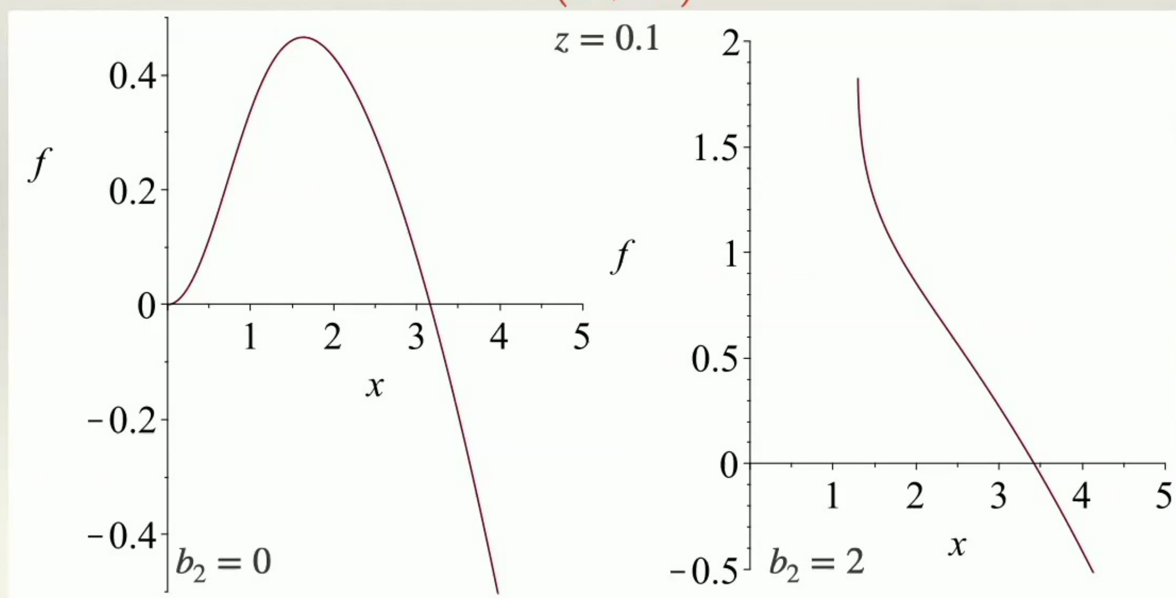


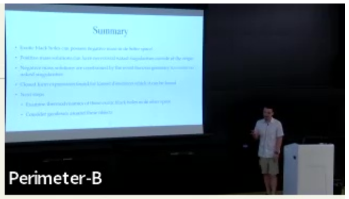
Massless de Sitter space

- Massless solutions with $b_2 \geq 0$ are pure de Sitter spaces
- Must now be concerned about naked singularities

Singularity Equation $(1+4z)x^4 + 4 - 4b_2 = 0 \Rightarrow x = \dfrac{\sqrt{2}\,(b_2-1)^{1/4}}{(1+4z)^{1/4}}$

- Conclude that $0 \leq b_2 \leq 1$ for a pure dS space to be free from naked singularities





Summary

- Exotic black holes can possess negative mass in de Sitter space!
- Positive mass solutions can have non-trivial naked singularities outside of the origin
- Negative mass solutions are constrained by the event horizon geometry to ensure no naked singularities
- Closed form expressions found for lowest dimension which it can be found
- Next steps
    - Examine thermodynamics of these exotic black holes in de sitter space
    - Consider geodesics around these objects

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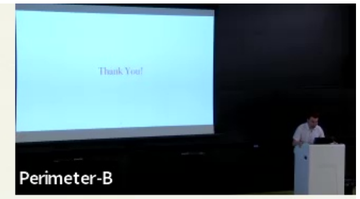

Thank You!

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