Title: Reflecting scalar fields in numerical relativity

Speakers: Conner Dailey

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Abstract: Black hole echoes have been considered as new probes to standard gravitational waveforms. Here, I consider reflections of scalar waves around a black hole as a model of black hole echoes arising from scalar fields. This problem is difficult due to the need for a proper understanding of the characteristic fields that propagate in numerical relativity. Using the "Einstein-Christoffel" system, I model the characteristic fields and the boundary conditions in such a way as to properly reflect scalar waves at a boundary using the full power of Einstein's equations.

Reflecting Scalar Fields in Numerical Relativity

The full IBVP in Spherical Symmetry

Conner Dailey

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June 21, 2022

Reflecting Scalar Fields in GR

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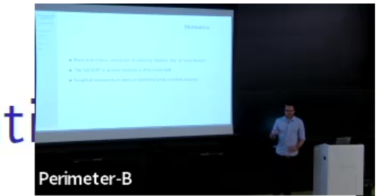
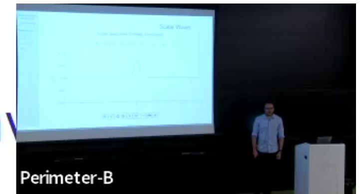
Motivati



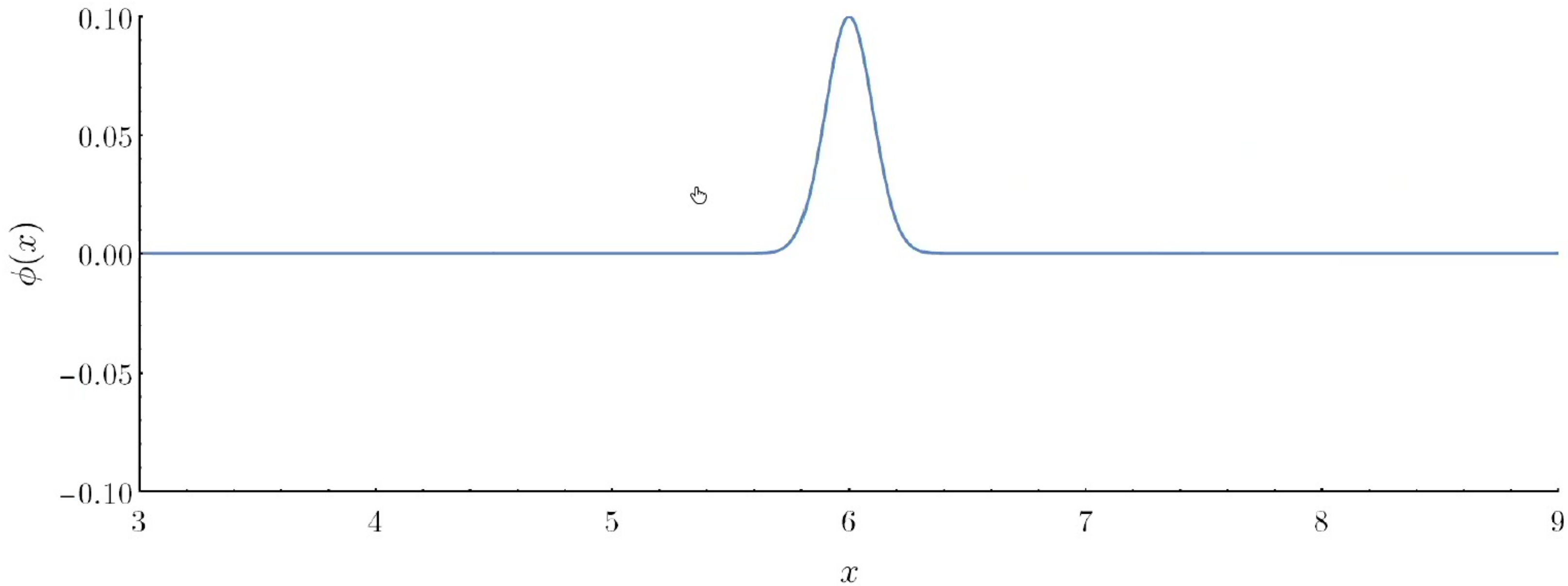
- Black hole echoes, indications of reflecting behavior near an event horizon
- The full IBVP in general relativity is often overlooked
- Simplified simulations in spherical symmetry using a modern language

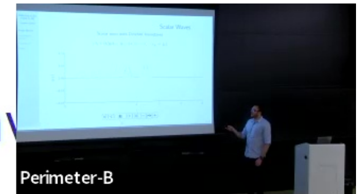
Scalar Wa



Scalar wave with Dirichlet boundaries

$$(\partial_t + \partial_x)(\partial_t - \partial_x)\phi(x,t) = 0\,, \quad c_{\pm} = \pm 1$$





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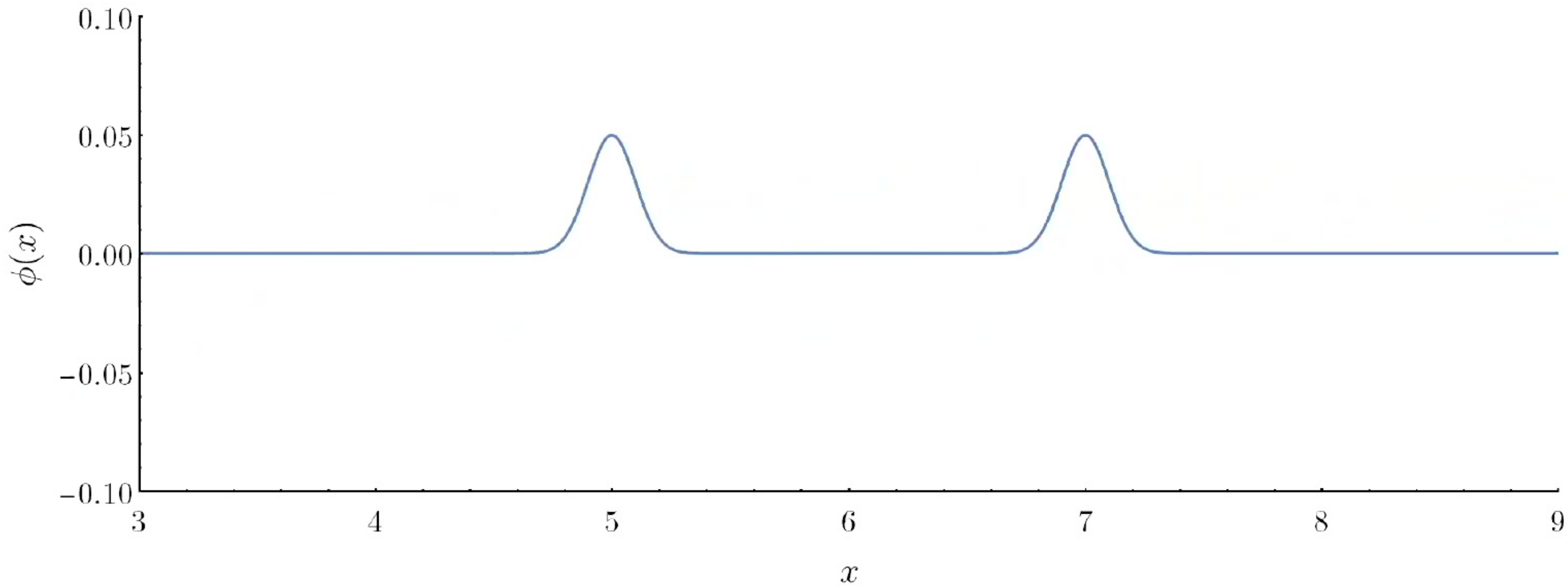

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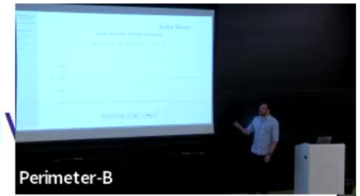

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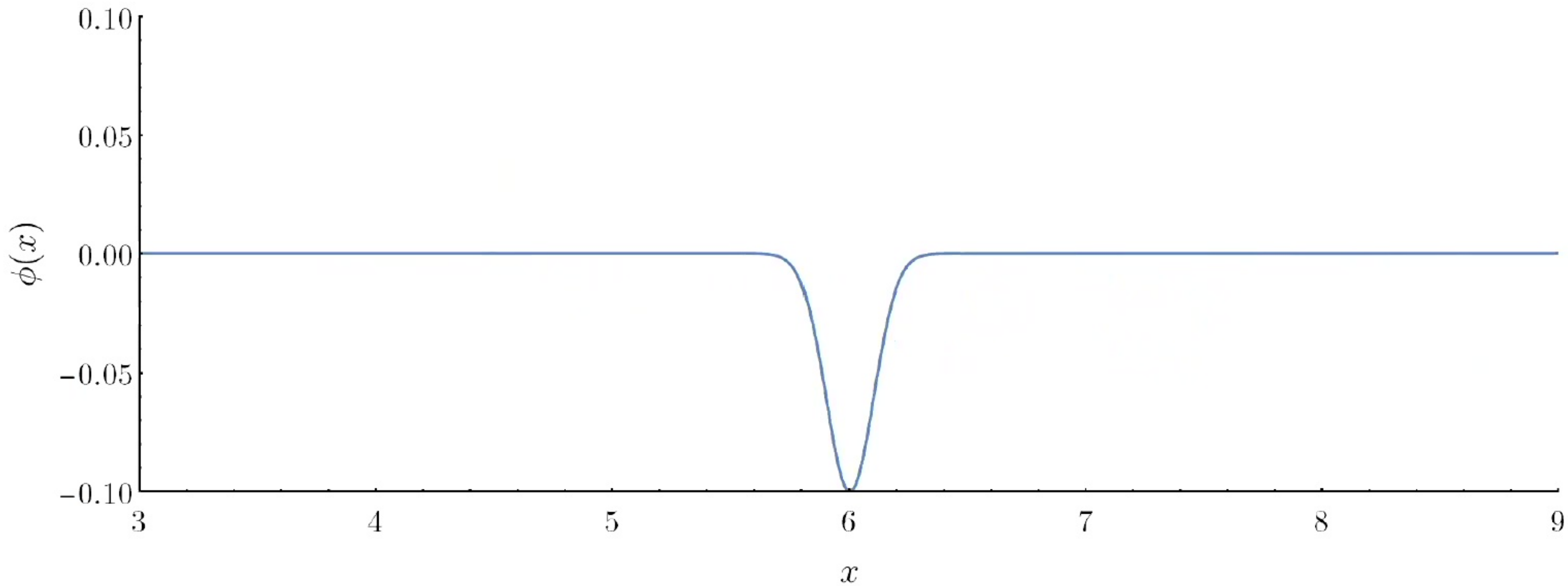
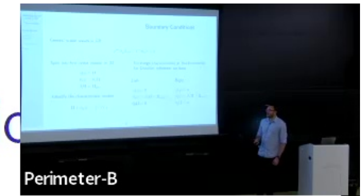


Boundary Conditic



General scalar waves in GR

$$g^{\mu\nu}\partial_\mu\partial_\nu\phi - \Gamma^\mu\partial_\mu\phi = 0$$

Split into first order system in 1D

$$\partial_t\phi = \Pi$$
$$\partial_t\psi = \partial_r\Pi$$
$$\partial_t\Pi = \Pi_{\text{rhs}}$$

Identify the characteristic modes

$$\Pi + c_\mp\psi\,, \quad v = c_\pm$$

Exchange characteristics at the boundaries, for Dirichlet reflection we have

Left	Right
$\partial_t\phi = 0$	$\partial_t\phi = 0$
$\partial_t\psi = \partial_r\Pi + \Pi_{\text{rhs}}/c_+$	$\partial_t\psi = \partial_r\Pi + \Pi_{\text{rhs}}/c_-$
$\partial_t\Pi = 0$	$\partial_t\Pi = 0$

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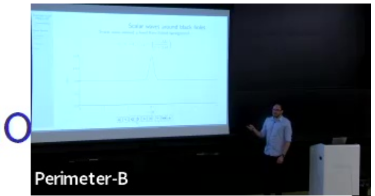
Scalar Waves

Coupling to Gravity

Constraints

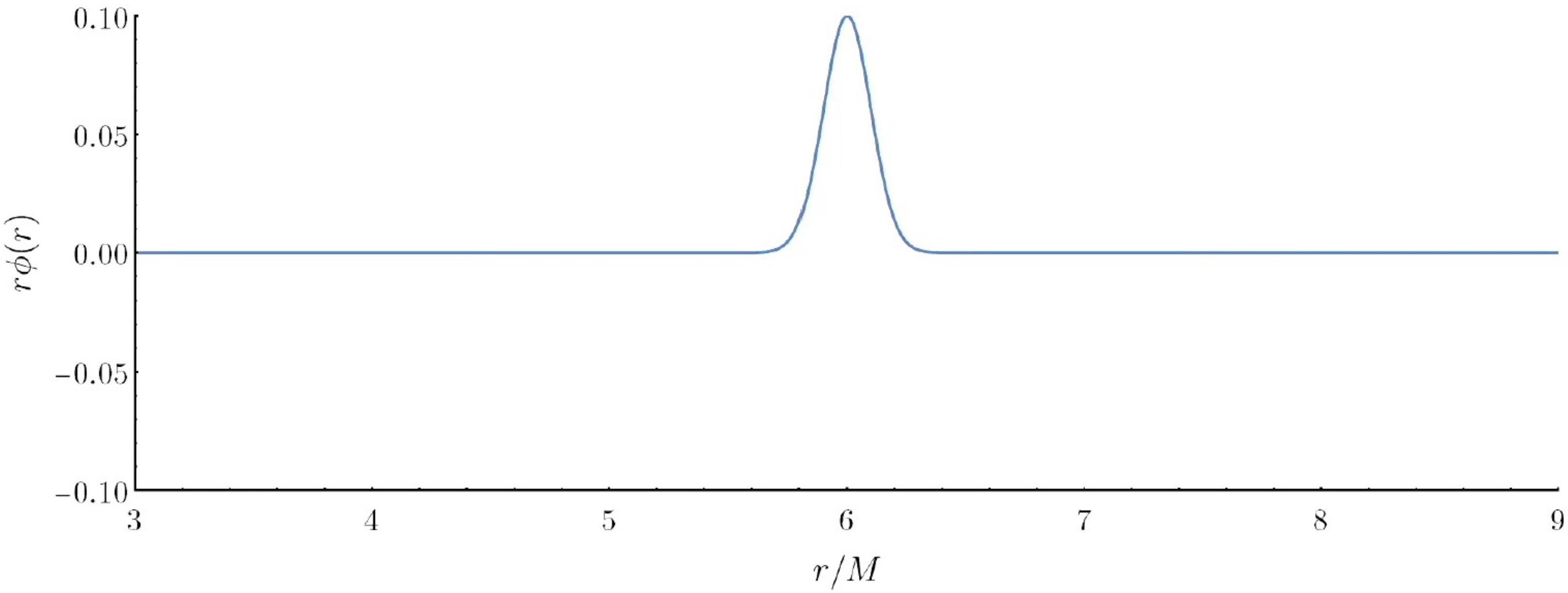
Results



Scalar waves around black ho

Scalar wave around a fixed Kerr-Schild background

$$c_- = -1\,, \quad c_+ = \left(\frac{r-2M}{r+2M}\right)$$



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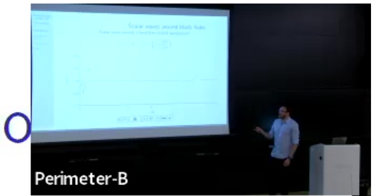

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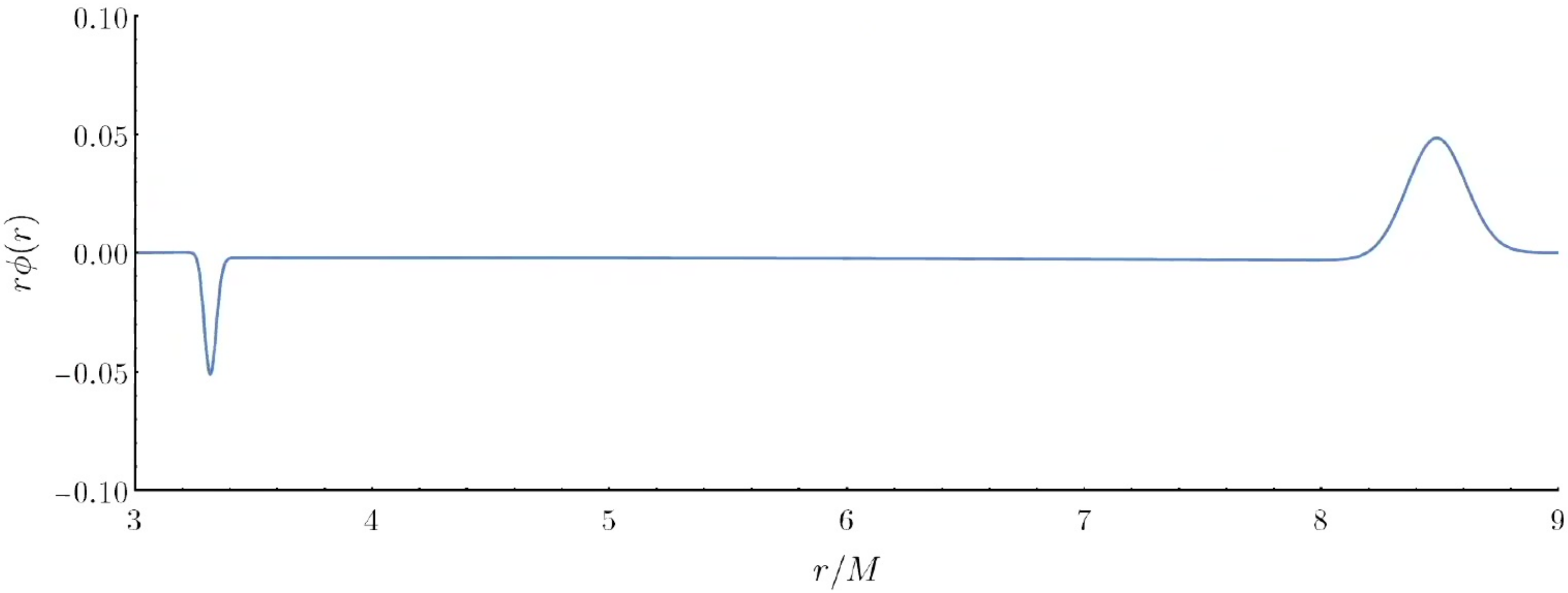

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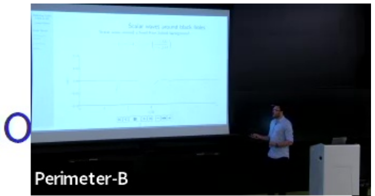

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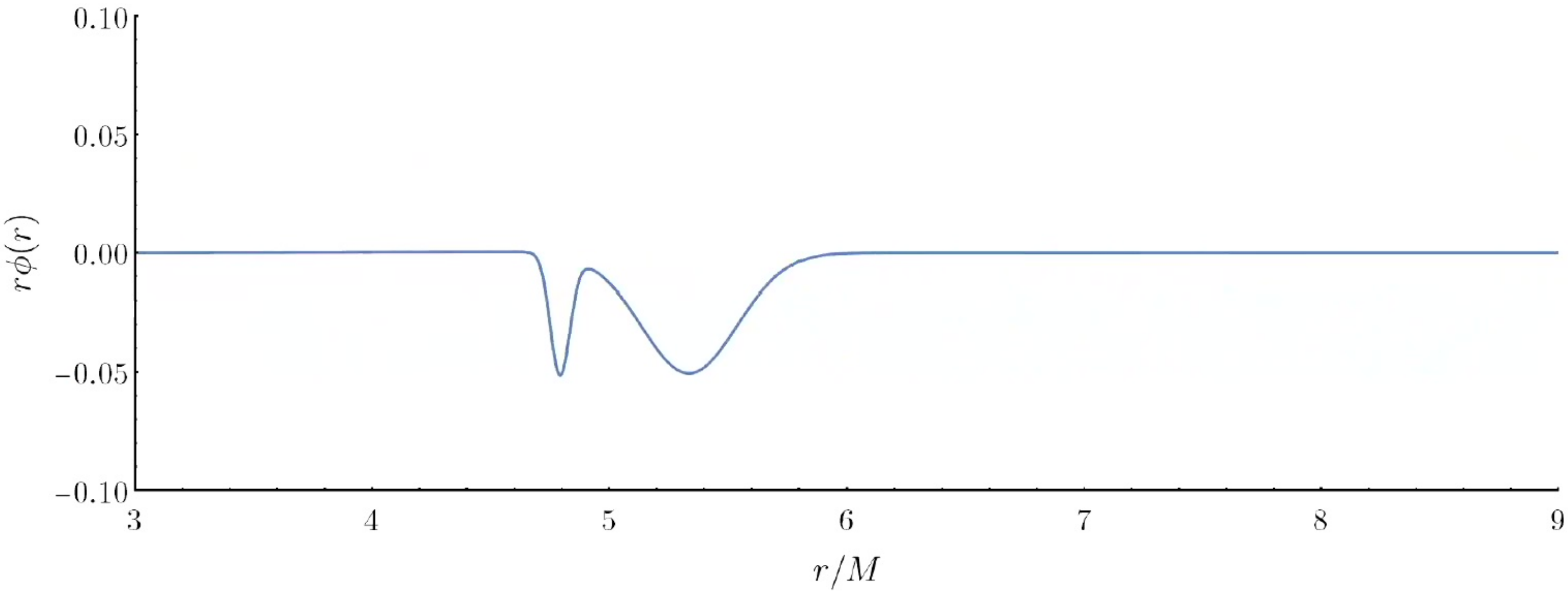
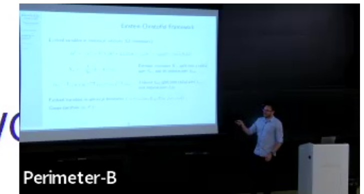


Numerical Evolution in

- Vector of evolved variables $\vec{u}$
- We need a system of equations $\partial_t \vec{u} = \cdots$
- If the system is higher order in time, you can add more variables
- Lots of freedom of choice here, we just want a system that is stable numerically and correctly solves Einstein's equations
- For a proper IBVP, we would like physical and well-defined characteristic speeds

Einstein-Christoffel Framew



Evolved variables in numerical relativity (EC framework)

$$ds^2 = -(\alpha^2 - \beta^r \beta_r)dt^2 + 2\beta_r dt dr + \gamma_{rr} dr^2 + \gamma_{\theta\theta}(d\theta^2 + \sin^2(\theta) d\phi^2)$$

$$K_{ij} = -\frac{1}{2\alpha}(\partial_t - \pounds_\beta)\gamma_{ij}$$

Extrinsic curvature K_{ij} split into a radial part K_{rr} and an angular part $K_{\theta\theta}$

$$f_{kij} = \Gamma_{(ij)k} + \gamma_{ki}\gamma^{lm}\Gamma_{[lj]m} + \gamma_{kj}\gamma^{lm}\Gamma_{[li]m}$$

3-tensor f_{kij} split into radial part f_{rrr} and angular part $f_{r\theta\theta}$

Evolved Variables in spherical symmetry $\vec{u} = (\gamma_{rr}, \gamma_{\theta\theta}, K_{rr}, K_{\theta\theta}, f_{rrr}, f_{r\theta\theta})$

Gauge variables (α, β^r)

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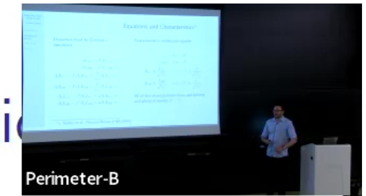
Equations and Characteristi



Dynamics fixed by Einstein's equations

$$\partial_t \gamma_{rr} - \beta^r \partial_r \gamma_{rr} = \cdots \,,$$
$$\partial_t \gamma_{\theta\theta} - \beta^r \partial_r \gamma_{\theta\theta} = \cdots \,,$$
$$\partial_t K_{rr} - \beta^r \partial_r K_{rr} + \frac{\alpha}{\gamma_{rr}} \partial_r f_{rrr} = \cdots \,,$$
$$\partial_t K_{\theta\theta} - \beta^r \partial_r K_{\theta\theta} + \frac{\alpha}{\gamma_{rr}} \partial_r f_{r\theta\theta} = \cdots \,,$$
$$\partial_t f_{rrr} - \beta^r \partial_r f_{rrr} + \alpha \partial_r K_{rr} = \cdots \,,$$
$$\partial_t f_{r\theta\theta} - \beta^r \partial_r f_{r\theta\theta} + \alpha \partial_r K_{\theta\theta} = \cdots \,.$$

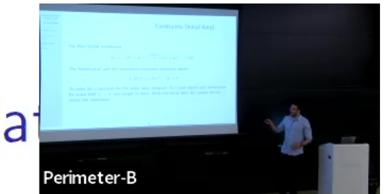
Characteristics modes and speeds

$$\gamma_{rr} \,, \quad v = -\beta^r$$
$$\gamma_{\theta\theta} \,, \quad v = -\beta^r$$
$$K_{rr} \pm \frac{f_{rrr}}{\sqrt{\gamma_{rr}}} \,, \quad v = -\beta^r \pm \frac{\alpha}{\sqrt{\gamma_{rr}}}$$
$$K_{\theta\theta} \pm \frac{f_{r\theta\theta}}{\sqrt{\gamma_{rr}}} \,, \quad v = -\beta^r \pm \frac{\alpha}{\sqrt{\gamma_{rr}}}$$

All of the characteristics have well-defined and physical speeds ($v \leq 1$)

[1] L. Kidder *et al.*, *Physical Review D* **62** (2000).

Constraints (Initial da



Use Kerr-Schild coordinates

$$ds^2 = -dt^2 + dr^2 + \frac{2M(r)}{r}(dt + dr)^2 + r^2 d\Omega^2$$

The Hamiltonian and the momentum constraint equations require

$$\partial_r M(r) = \rho\, 4\pi r^2\,, \quad S_r = 0$$

To solve the constraints for the initial data, integrate $M(r)$ and require zero momentum for scalar field $S_r = 0$, very simple to solve. With this initial data, the system initially solves the constraints.

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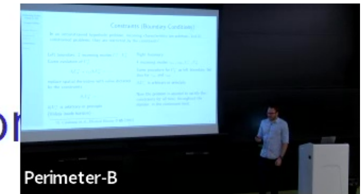
Constraints (Boundary Conditio



In an unconstrained hyperbolic problem, incoming characteristics are arbitrary, but in constrained problems, they are restricted by the constraints[2].

Left boundary: 2 incoming modes U_r^+, U_θ^+

Given evolution of U_θ^+

$$\partial_t U_\theta^+ + c_+ \partial_r U_\theta^+ = \cdots$$

replace spatial derivative with value dictated by the constraints

$$\partial_r U_\theta^+ = \cdots$$

$\partial_t U_r^+$ is arbitrary in principle

(Unless inside horizon)

Right boundary:

4 incoming modes γ_{rr}, $\gamma_{\theta\theta}$, U_r^-, U_θ^-

Same procedure for U_θ^- as left boundary, but also for γ_{rr} and $\gamma_{\theta\theta}$

$\partial_t U_r^-$ is arbitrary in principle

Now the problem is assured to satisfy the constraints for all time, throughout the domain, in the continuum limit.

[2] G. Calabrese *et al.*, *Physical Review D* **65** (2002).

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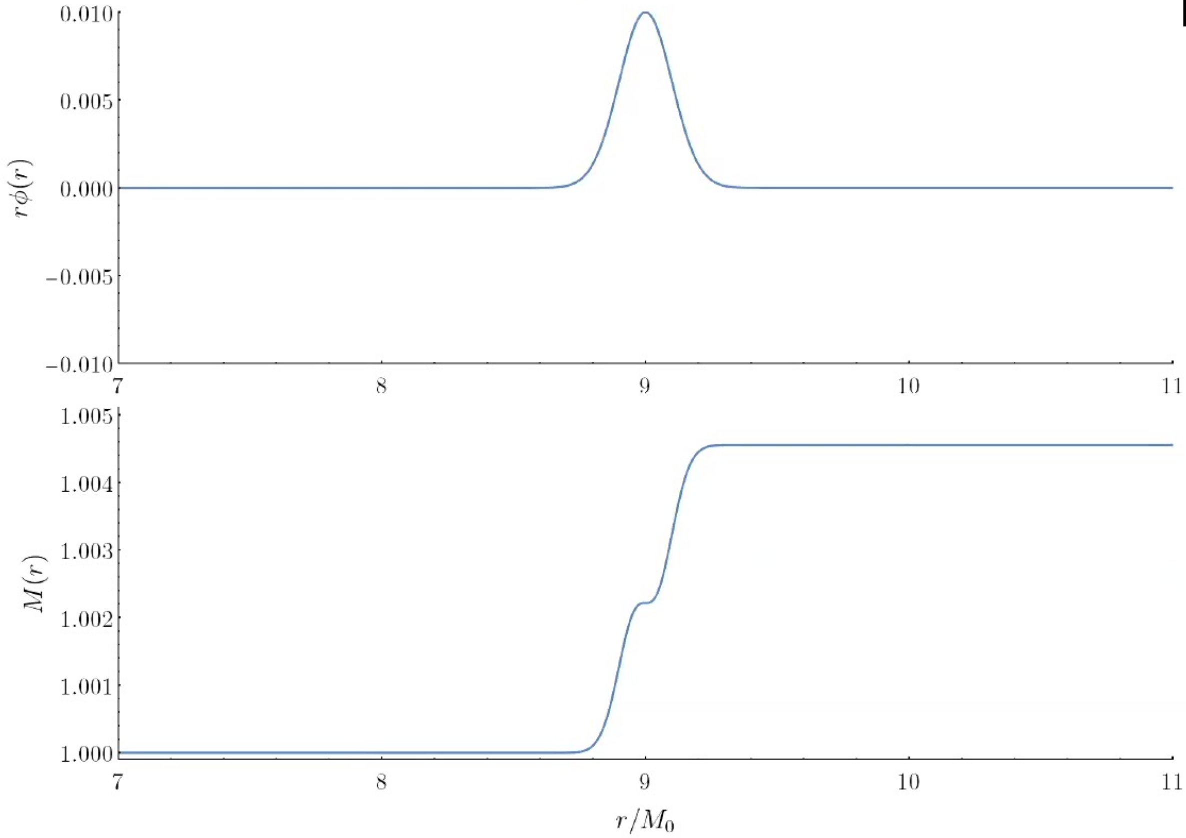
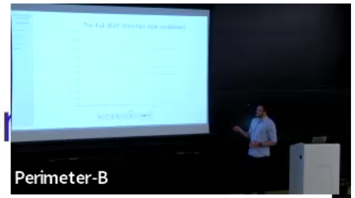
The Full IBVP (Dirichlet style condition





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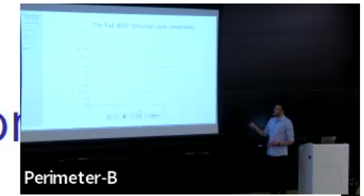

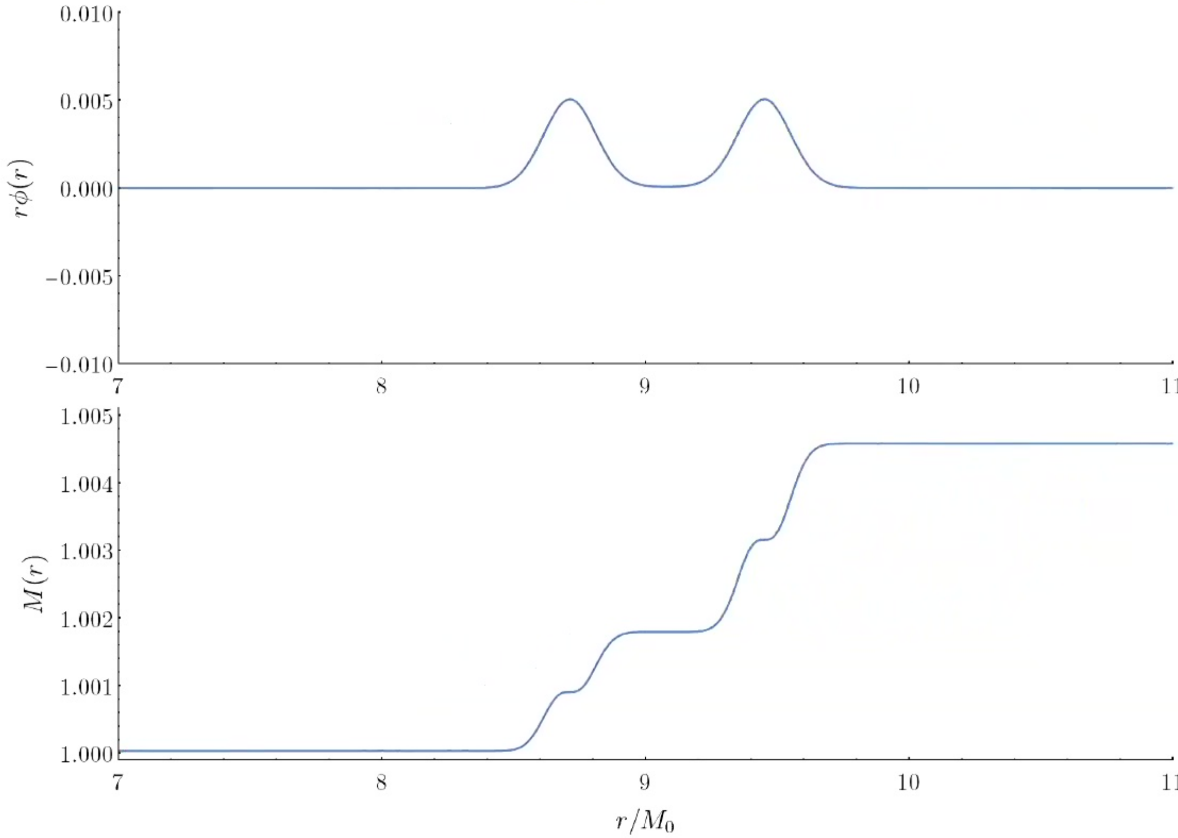

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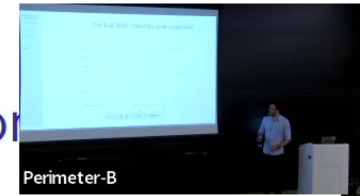
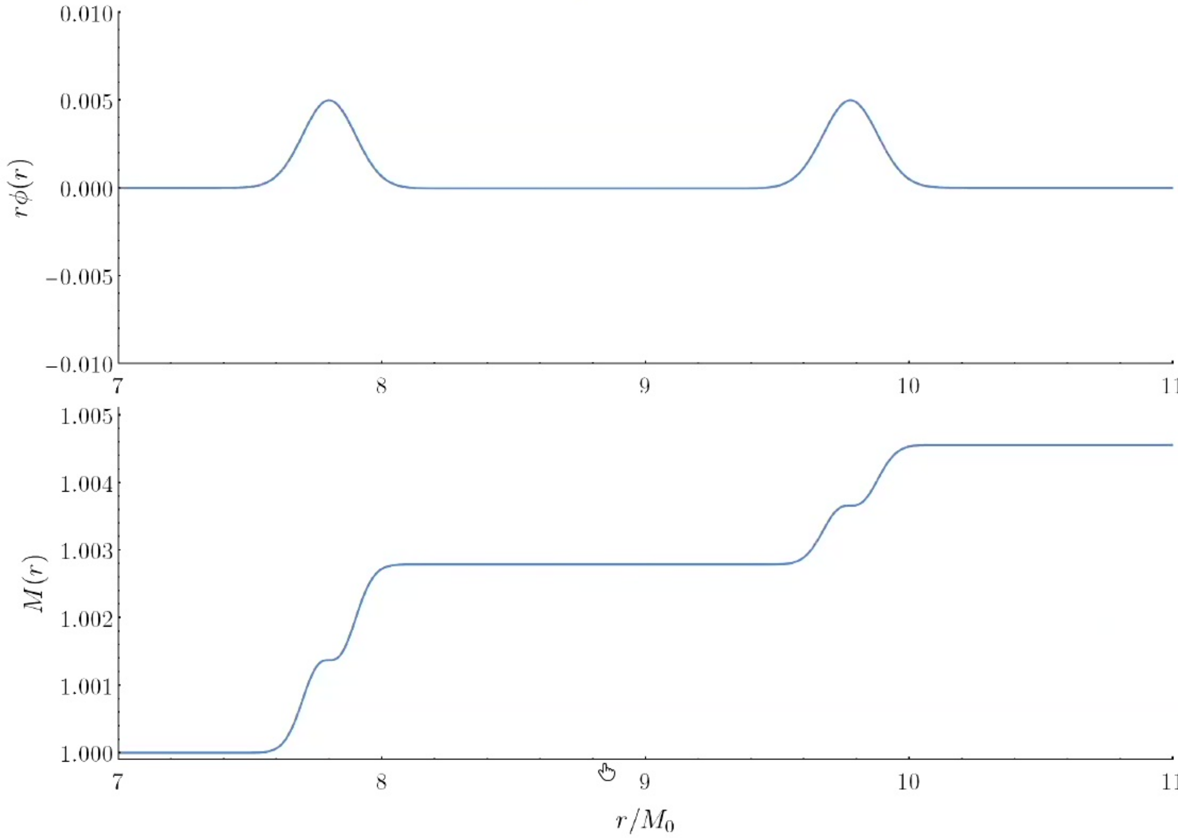
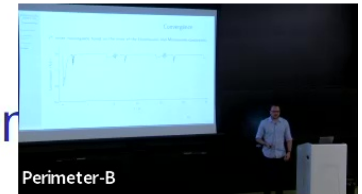
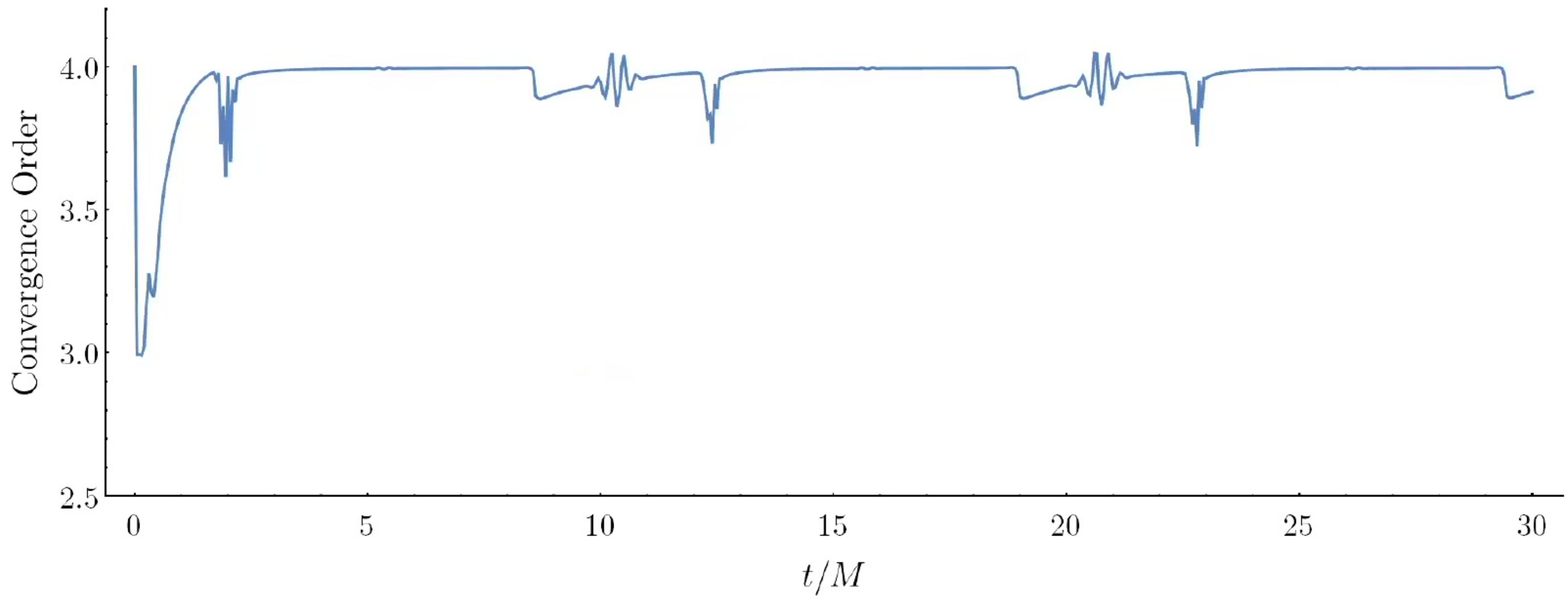




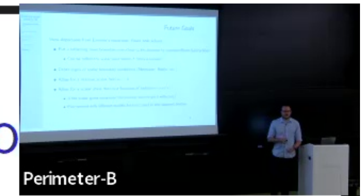
Converger



4^{th} order convergence based on the norm of the Hamiltonian and Momentum constraints



Future Go



Show departures from Einstein's equations: Black hole echoes

- Put a reflecting inner boundary very close to the horizon to simulate Black hole echoes
  - Can we reflect the scalar wave before it forms a horizon?
- Other types of scalar boundary conditions (Neumann, Robin, etc.)
- Allow for a massive scalar field $m > 0$
- Allow for a scalar mass that is a function of radius $m \rightarrow m(r)$
  - If the scalar gains mass near the horizon, can we get a reflection?
  - Play around with different models for $m(r)$, tied to the apparent horizon

The Full IBVP (Dirichlet style conditio