Title: General Features of the Thermalization of Particle Detectors and the Unruh Effect.

Speakers: Tales Rick Perche

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Abstract: In this talk we will discuss the notion of thermality for quantum field theories in curved spacetimes, and how it relates to the Unruh effect and Hawking radiation. Then we will argue that particle detectors are physical systems which can act as thermometers, thermalizing to the temperature of the field. We will show that any non-relativistic quantum system undergoing appropriate trajectories can probe the field's temperature, regardless of how they are coupled to the field.

Thermalization of Particle Detectors and the Unruh Effect

T. Rick Perche

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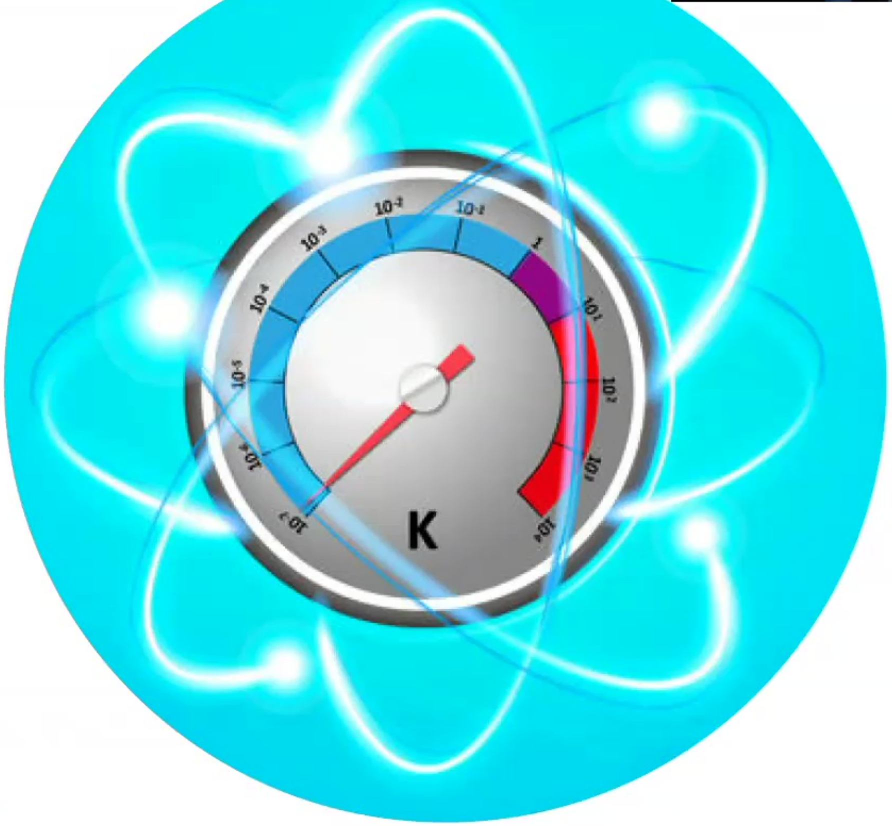
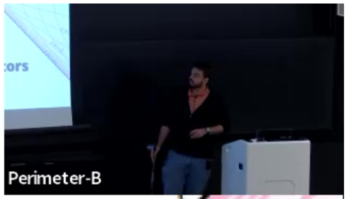
Bourses d'études supérieures du Canada
Vanier
Canada Graduate Scholarships

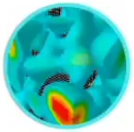


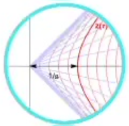
[1] T. Rick Perche, "General features of the thermalization of particle detectors and the unruh effect," Phys. Rev. D 24 104, 065001 (2021).



Outline


Introduction: The Unruh Effect


Thermality in QFT: KMS Condition

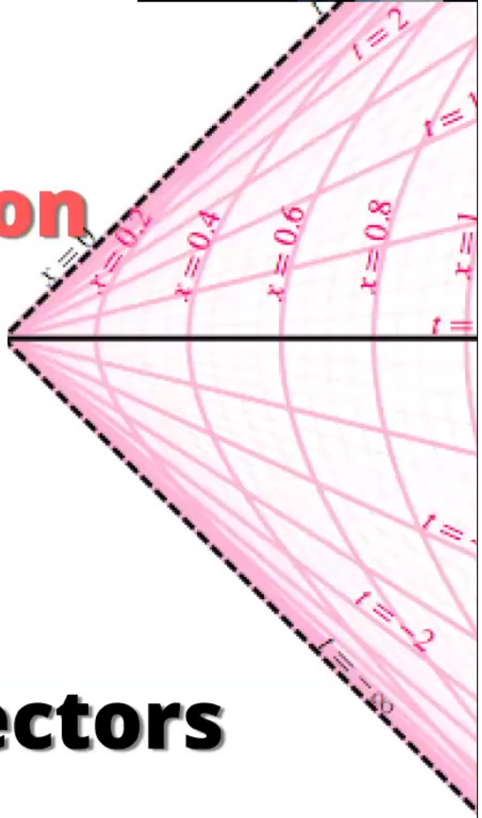

1) Rindler Modes Perspective


2) AQFT Perspective

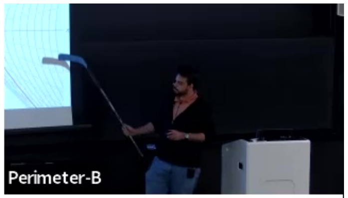

3) Detector Perspective

General Thermalization of Detectors

Summary

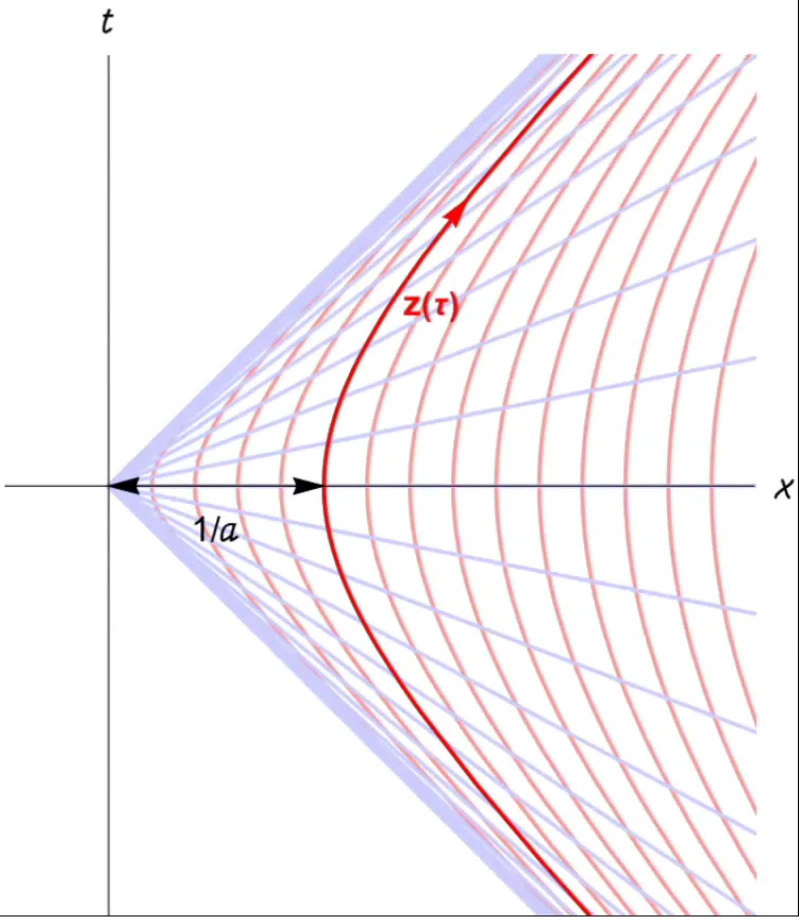


Introduction: The Unruh Effect



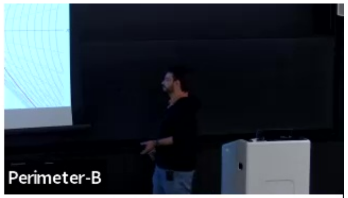
Accelerated observers in the vacuum experience a thermal bath of temperature

$$T_U = \frac{\hbar a}{2\pi c k_B}.$$



[2] W. G. Unruh, "Notes on black-hole evaporation," Phys. Rev. D 14, 870–892 (1976).

Introduction: The Unruh Effect

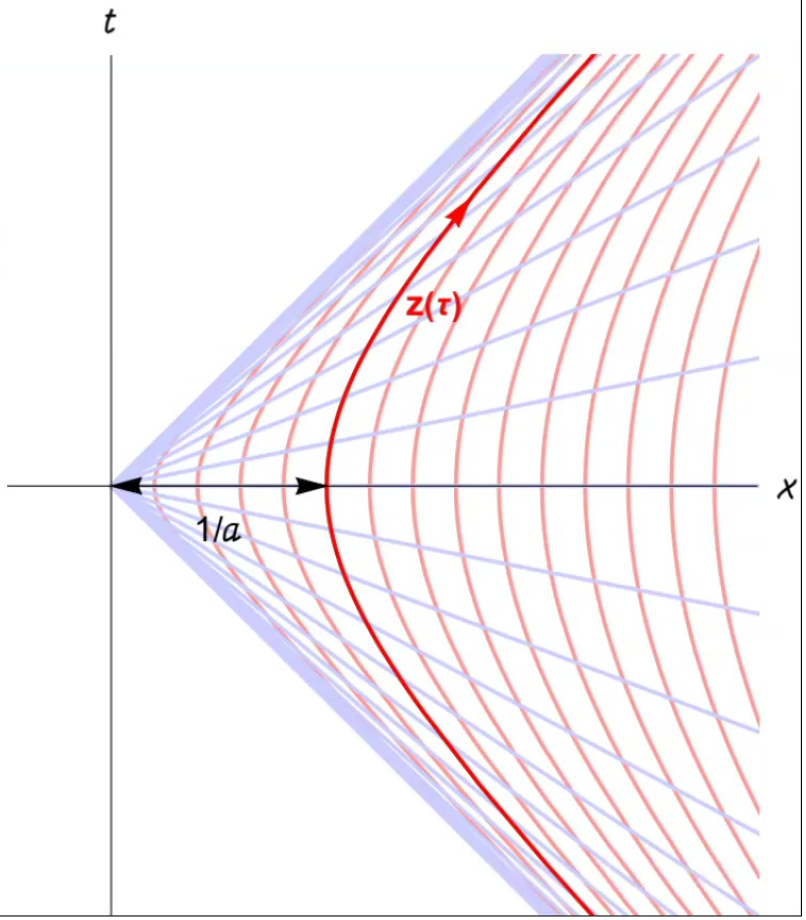


Accelerated observers in the vacuum experience a thermal bath of temperature

HOW?

$$T_U = \frac{\hbar a}{2\pi c k_B}.$$

What does that even mean?



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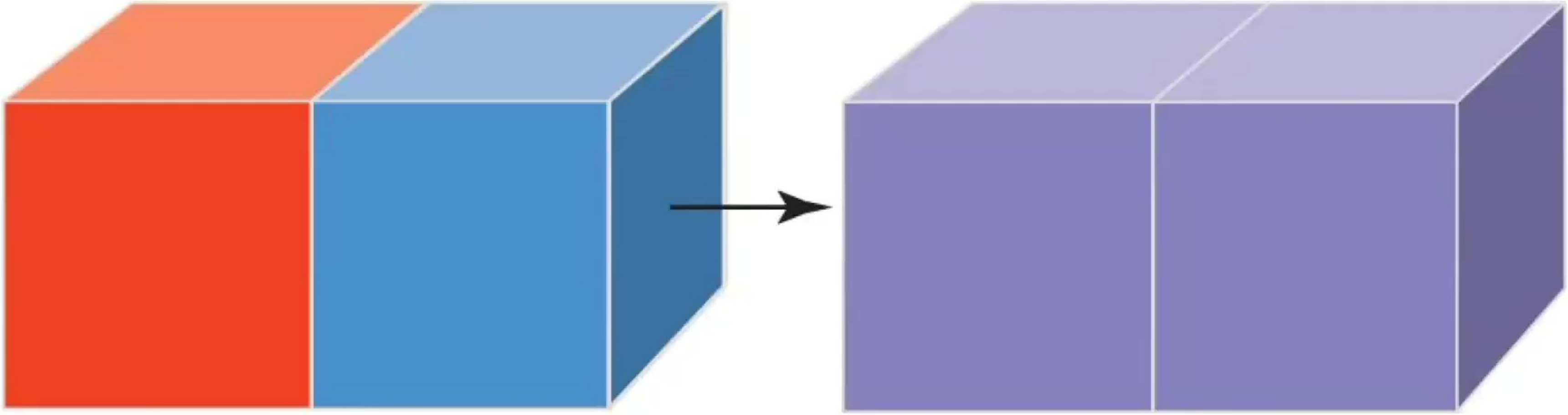
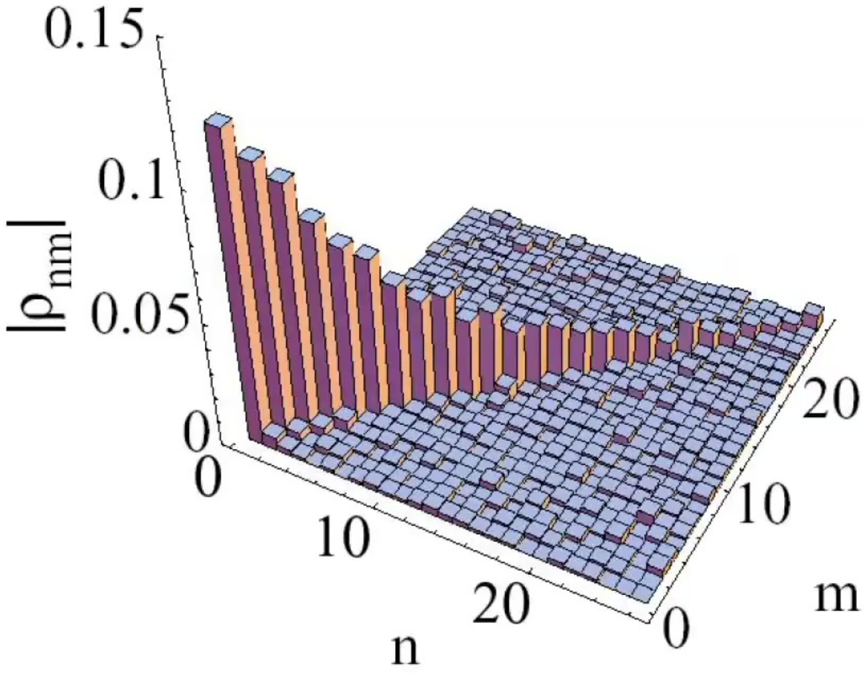
Thermality in QFT: The KMS Conditi



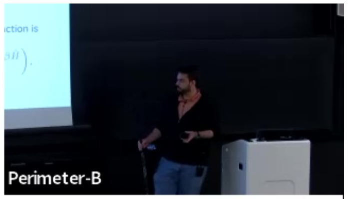
Gibbs States

In a quantum system with a time independent Hamiltonian, a Gibbs state $\hat{\rho}$ with inverse temperature β is defined as:



$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{\mathcal{Z}}$$

Gibbs States



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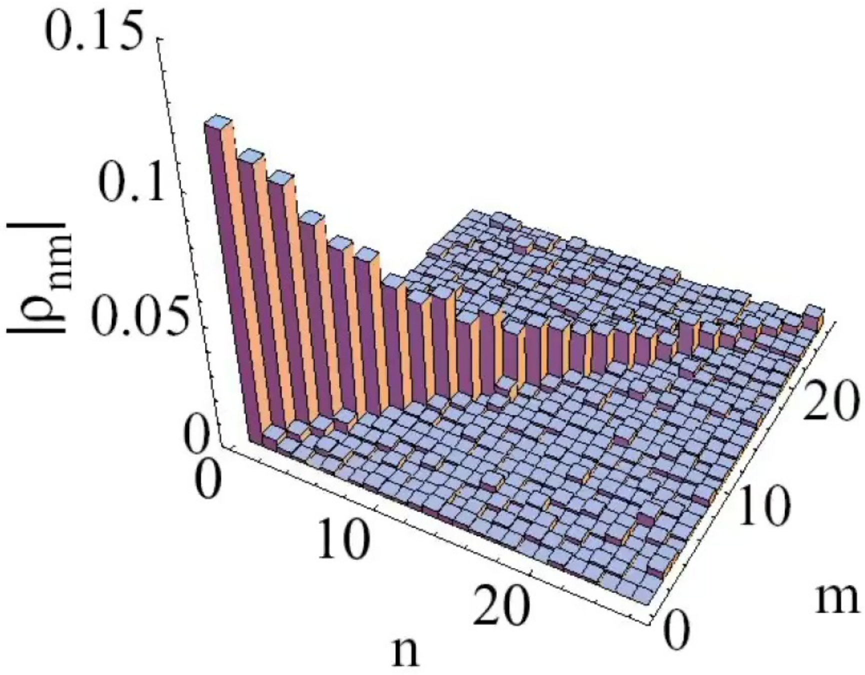
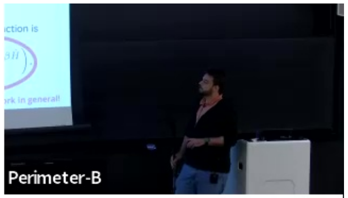


$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{\mathcal{Z}}$$

where the partition function is

$$\mathcal{Z} = \mathrm{tr}\left(e^{-\beta \hat{H}}\right).$$

Gibbs States



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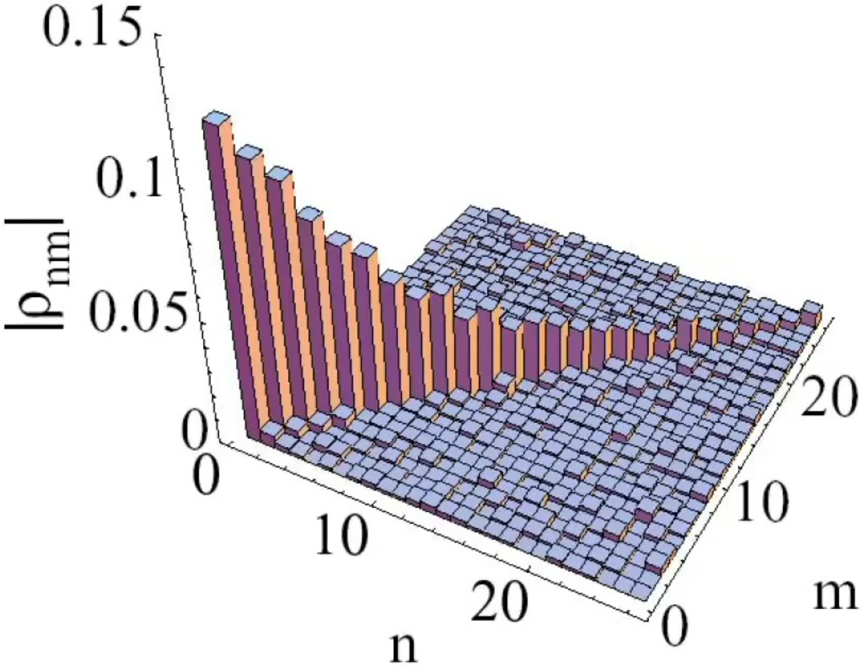


$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{\mathcal{Z}}$$

where the partition function is

$$\mathcal{Z} = \mathrm{tr}\left(e^{-\beta \hat{H}}\right).$$

This doesn't work in general!

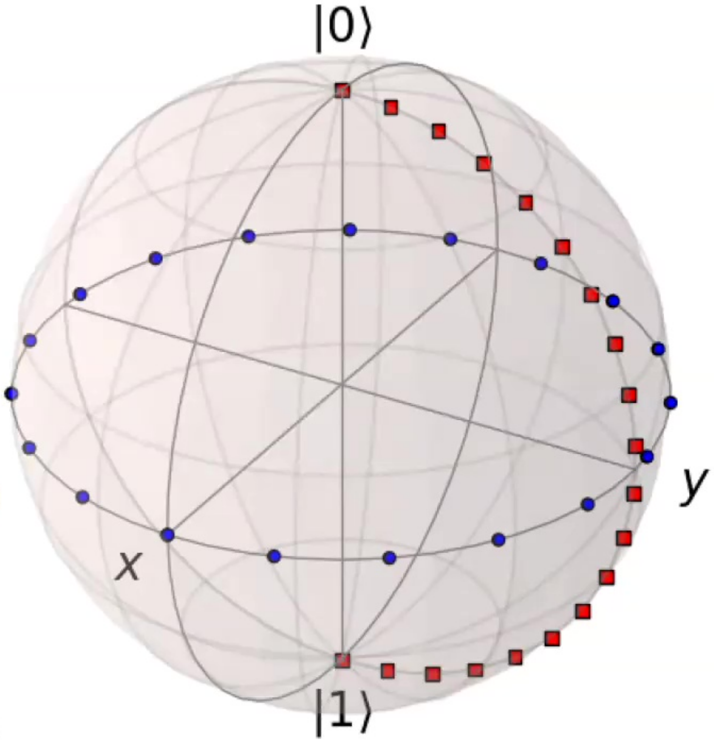
Unitary Time Evolution

The unitary time evolution operator is given by

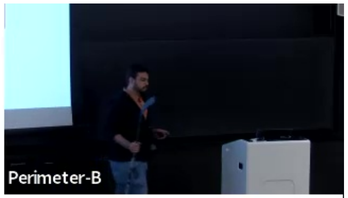
$$\hat{U}(t) = e^{-i\hat{H}t}$$

And in the Heisenberg picture, operators evolve according to:

$$\alpha_t(\hat{A}) = \hat{A}(t) = \hat{U}^\dagger(t)\hat{A}\hat{U}(t)$$



The KMS Condition



Then any Gibbs state can be shown to satisfy the KMS condition

$$\boxed{\left\langle \alpha_t(\hat{A})\hat{B} \right\rangle_{\hat{\rho}} = \left\langle \hat{B}\,\alpha_{t+i\beta}(\hat{A}) \right\rangle_{\hat{\rho}}}$$

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$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{\mathcal{Z}} \qquad \alpha_t(\hat{A}) = e^{i\hat{H}t}\,\hat{A}\,e^{-i\hat{H}t}$$

The KMS Condition

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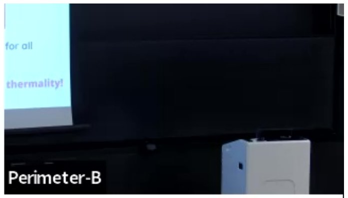
Conversely, any state that satisfies the KMS condition for all operators is a Gibbs state.

The KMS Condition



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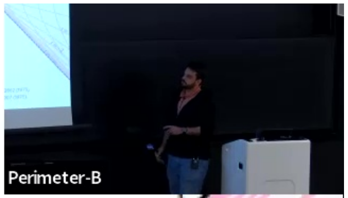
Conversely, any state that satisfies the KMS condition for all operators is a Gibbs state.

The KMS condition defines a more general notion of thermality!

The 3 Approaches



We will discuss 3 ways of seeing the Unruh effect:

1) Rindler Modes Perspective

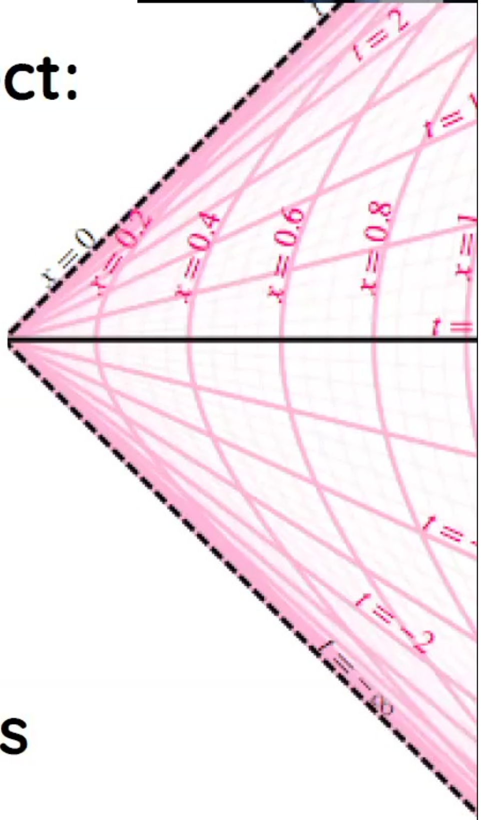
→ Different quantization schemes

2) AQFT Perspective

→ Operator valued distributions

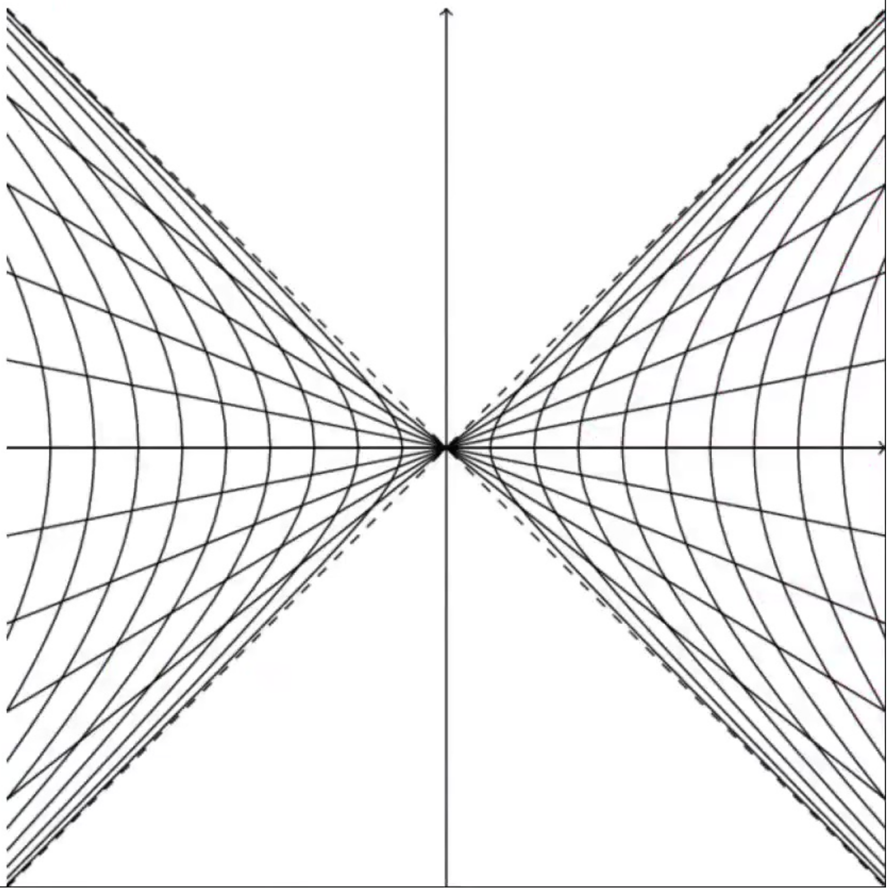
3) Detector Perspective

→ Localized non-relativistic quantum systems



[2] W. G. Unruh, "Notes on black-hole evaporation," Phys. Rev. D 14, 870–892 (1976).
[3] Stephen A. Fulling, "Nonuniqueness of canonical field quantization in riemannian space-time," Phys. Rev. D 7, 2850–2862 (1973).
[4] J. J Bisognano and E. H. Wichmann, "On the Duality Condition for a Hermitian Scalar Field," J. Math. Phys. 16, 985–1007 (1975).
[5] John Earman, "The Unruh effect for philosophers," Stud. Hist. Philos. M P 42, 81–97 (2011).

1) Rindler Modes Perspective



1) Rindler Modes Perspective

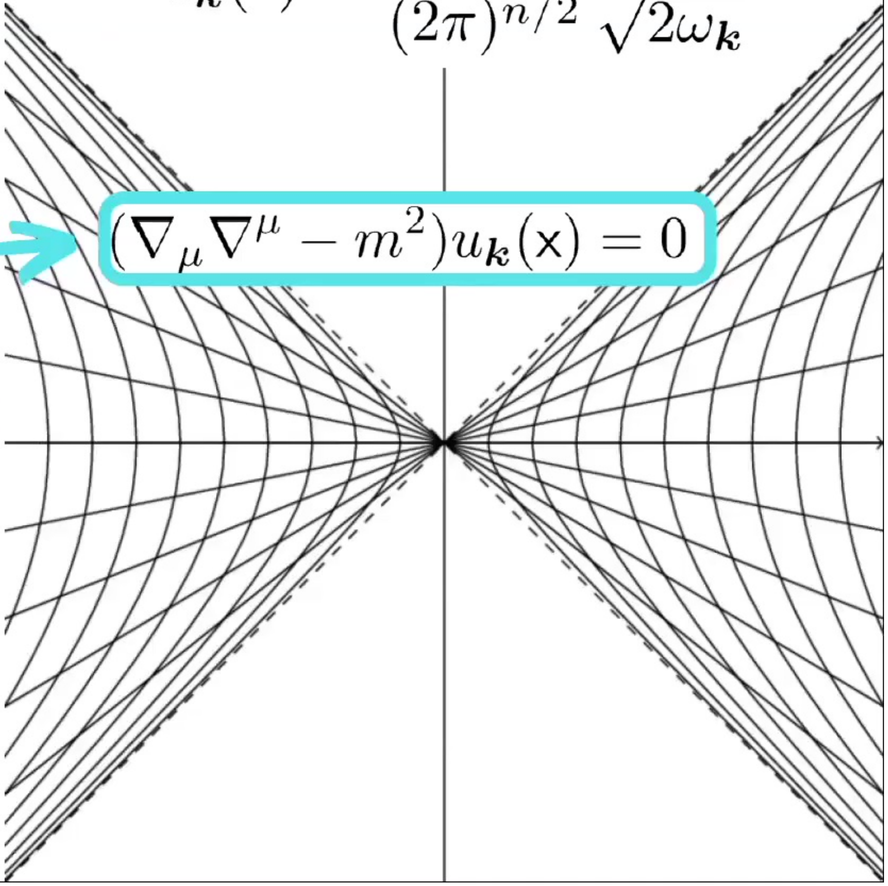
One way of describing a quantum field is in terms of a choice of modes:

$$\hat{\phi}(\mathsf{x}) = \int \mathrm{d}^n \boldsymbol{k} \left(u_{\boldsymbol{k}}(\mathsf{x}) \hat{a}_{\boldsymbol{k}} + u^*_{\boldsymbol{k}}(\mathsf{x}) \hat{a}^\dagger_{\boldsymbol{k}} \right)$$

We then impose commutation relations

$$\left[\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}\right] = \delta^{(n)}(\boldsymbol{k} - \boldsymbol{k}')\mathbb{1},$$

$$u_{\boldsymbol{k}}(\mathsf{x}) = \frac{1}{(2\pi)^{n/2}} \frac{e^{i\mathbf{k}\cdot\mathsf{x}}}{\sqrt{2\omega_{\boldsymbol{k}}}}$$



$$\boxed{(\nabla_\mu \nabla^\mu - m^2) u_{\boldsymbol{k}}(\mathsf{x}) = 0}$$

1) Rindler Modes Perspective

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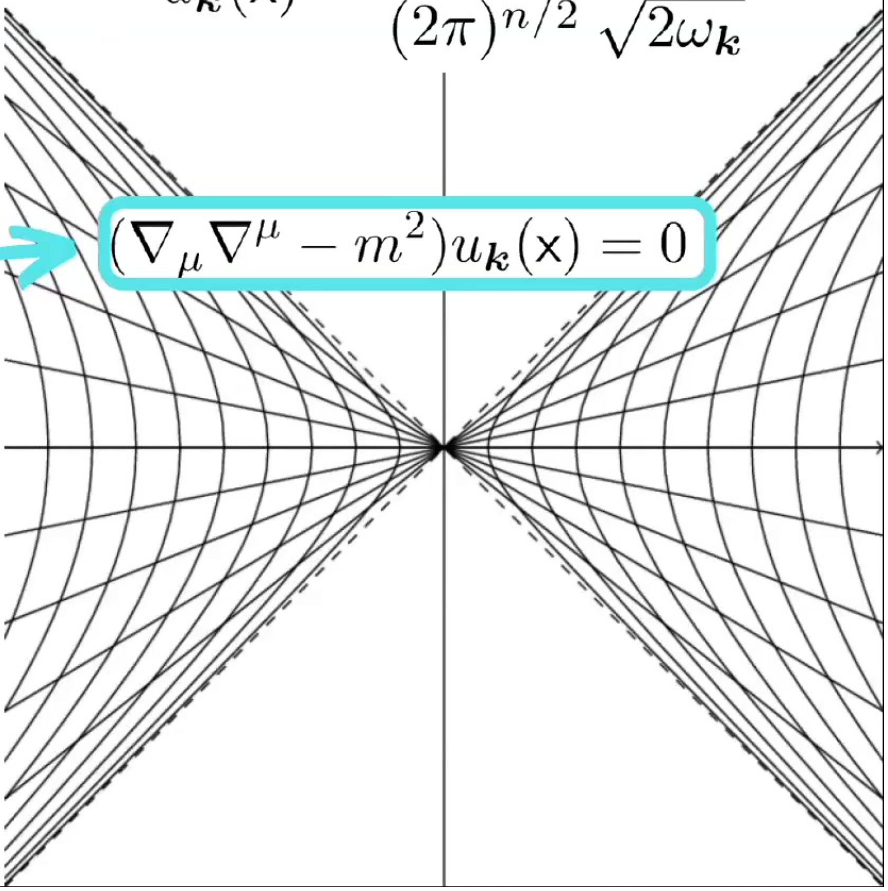


We then impose commutation relations

$$\left[\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}\right] = \delta^{(n)}(\boldsymbol{k} - \boldsymbol{k}')\mathbb{1},$$

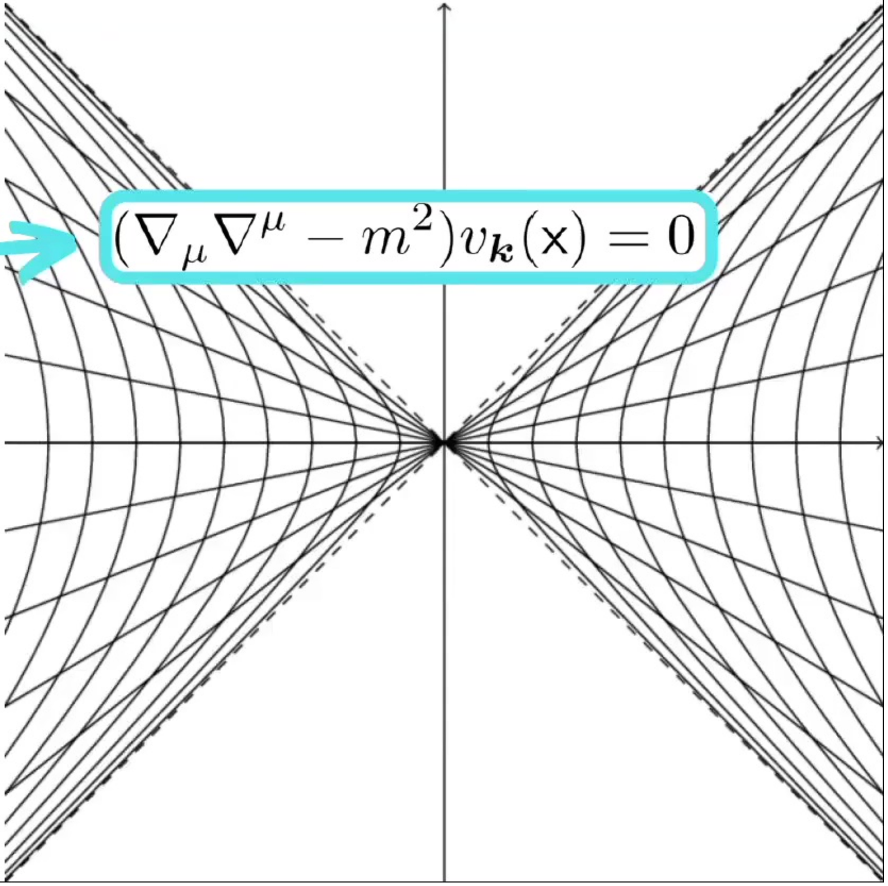
and the vacuum $|0\rangle$ is defined by

$$\hat{a}_{\boldsymbol{k}} |0\rangle = 0$$

1) Mode Decompositions

We could instead pick another mode decomposition

$$\hat{\phi}(\mathsf{x}) = \int \mathrm{d}^n \boldsymbol{k} \left(v_{\boldsymbol{k}}(\mathsf{x}) \hat{b}_{\boldsymbol{k}} + v^*_{\boldsymbol{k}}(\mathsf{x}) \hat{b}^\dagger_{\boldsymbol{k}} \right)$$



$$\boxed{(\nabla_\mu \nabla^\mu - m^2) v_{\boldsymbol{k}}(\mathsf{x}) = 0}$$

and impose commutation relations

$$[\hat{b}_{\boldsymbol{k}}, \hat{b}^\dagger_{\boldsymbol{k}'}] = \delta^{(n)}(\boldsymbol{k} - \boldsymbol{k}')\mathbb{1}$$

and the vacuum $|0'\rangle$ will be defined by

$$\hat{b}_{\boldsymbol{k}} |0'\rangle = 0$$

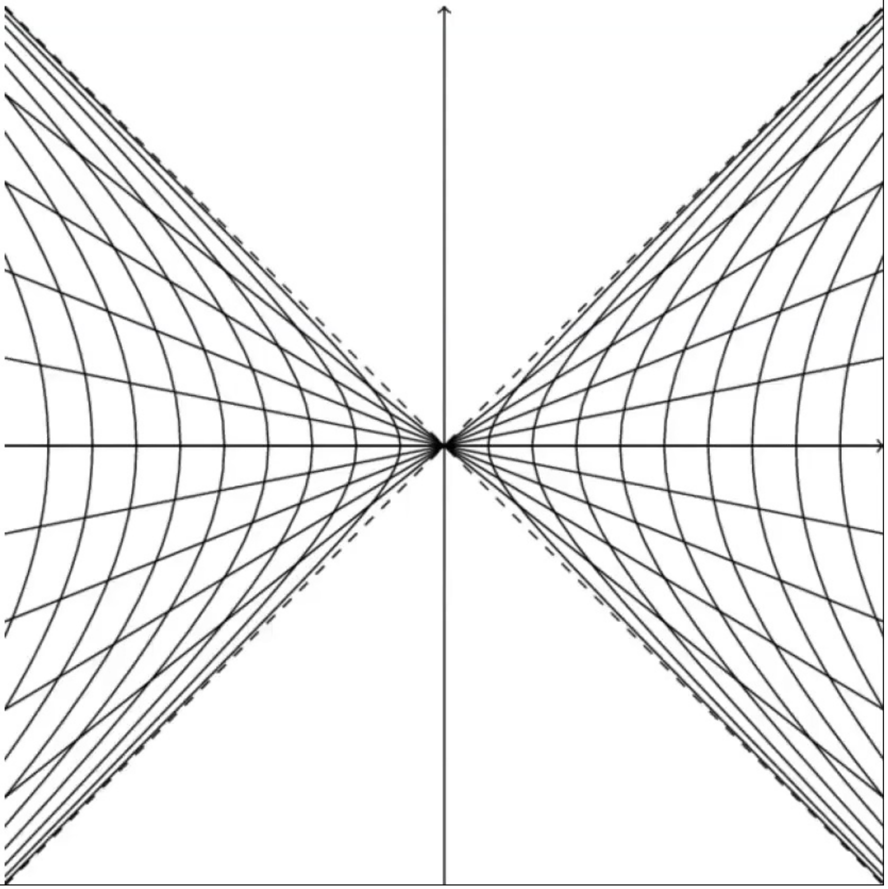
1) Different Vacua

The vacua $|0\rangle$ and $|0'\rangle$ will not be the same in general!

What we call vacuum depends on a choice of modes.

"No excitations" means no excitations with respect to a given basis of modes.

In some scenarios, it is possible to associate modes with physical situations



1) The Minkowski Vacuum

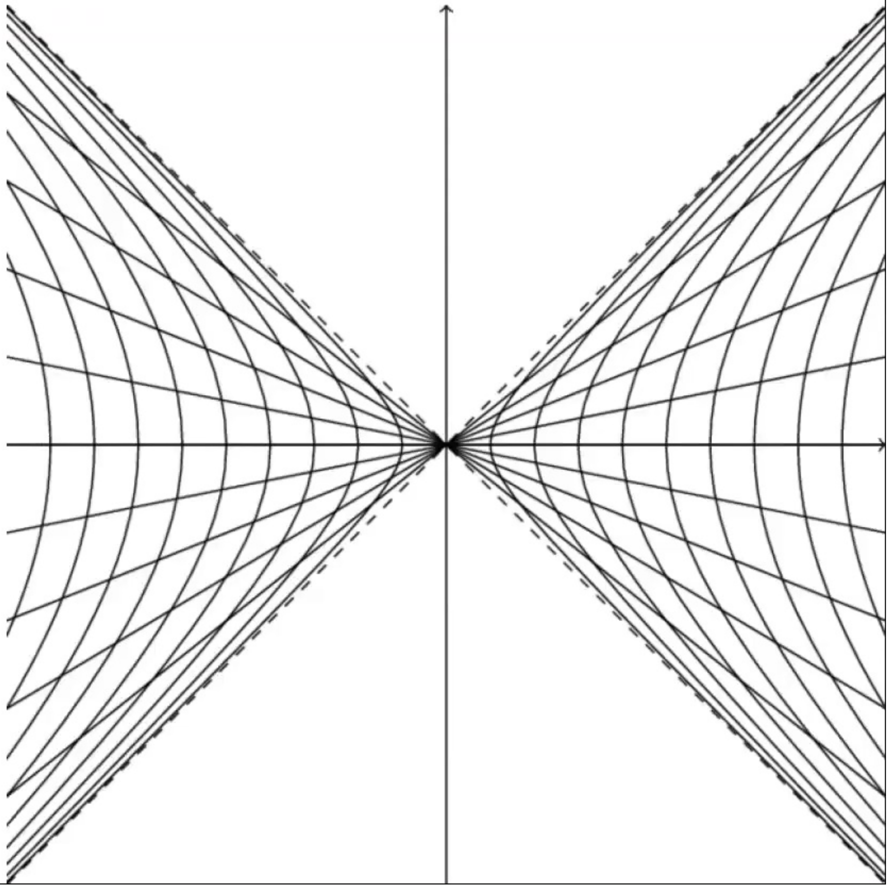
In Minkowski spacetime, the choice of modes

$$u_{\boldsymbol{k}}(\mathsf{x}) = \frac{1}{(2\pi)^{n/2}} \frac{e^{\mathrm{i}\mathsf{k}\cdot\mathsf{x}}}{\sqrt{2\omega_{\boldsymbol{k}}}}$$

yields the Minkowski vacuum $|0_M\rangle$.

The Minkowski vacuum is such that "inertial observers see no particles".

It is also invariant under all symmetries of Minkowski spacetime.



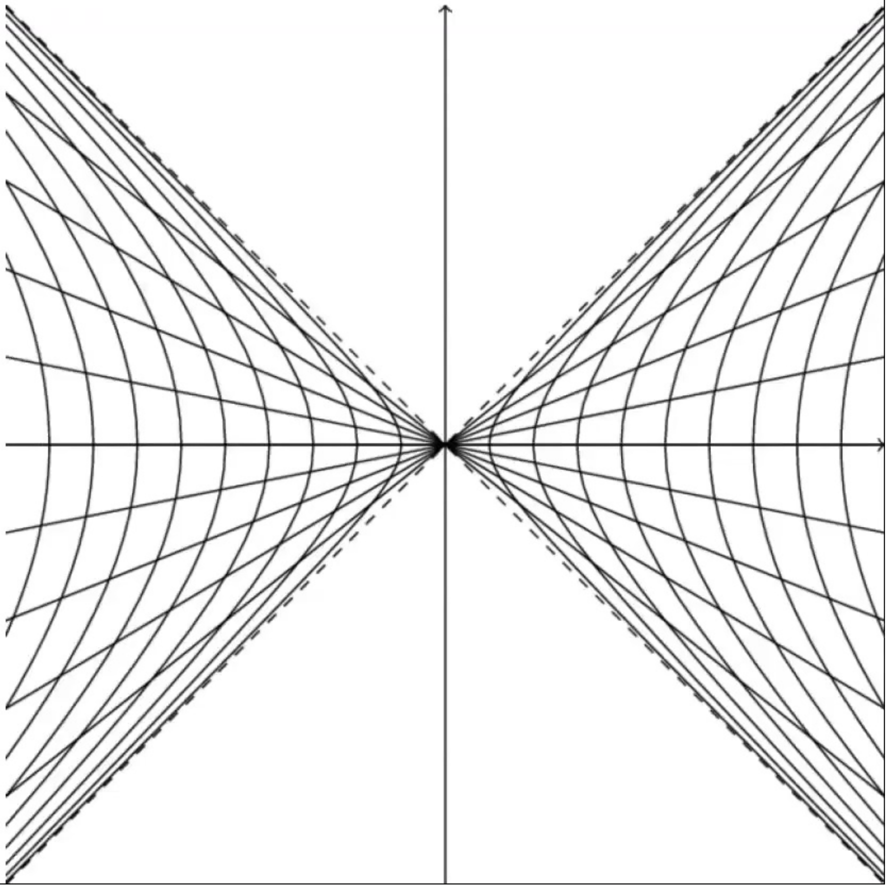
1) The Rindler Vacuum

We can also solve the Klein-Gordon equation in Rindler coordinates

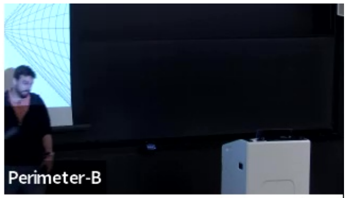
$$t = \frac{1}{a} e^{a\xi} \sinh(a\tau)$$
$$x = \frac{1}{a} e^{a\xi} \cosh(a\tau)$$

which are adapted to a uniformly accelerated observer with acceleration a.

The obtained modes $v_{\boldsymbol{k}}(\mathsf{x})$ are the Rindler modes, and define the Rindler vacuum $|0_R\rangle$.



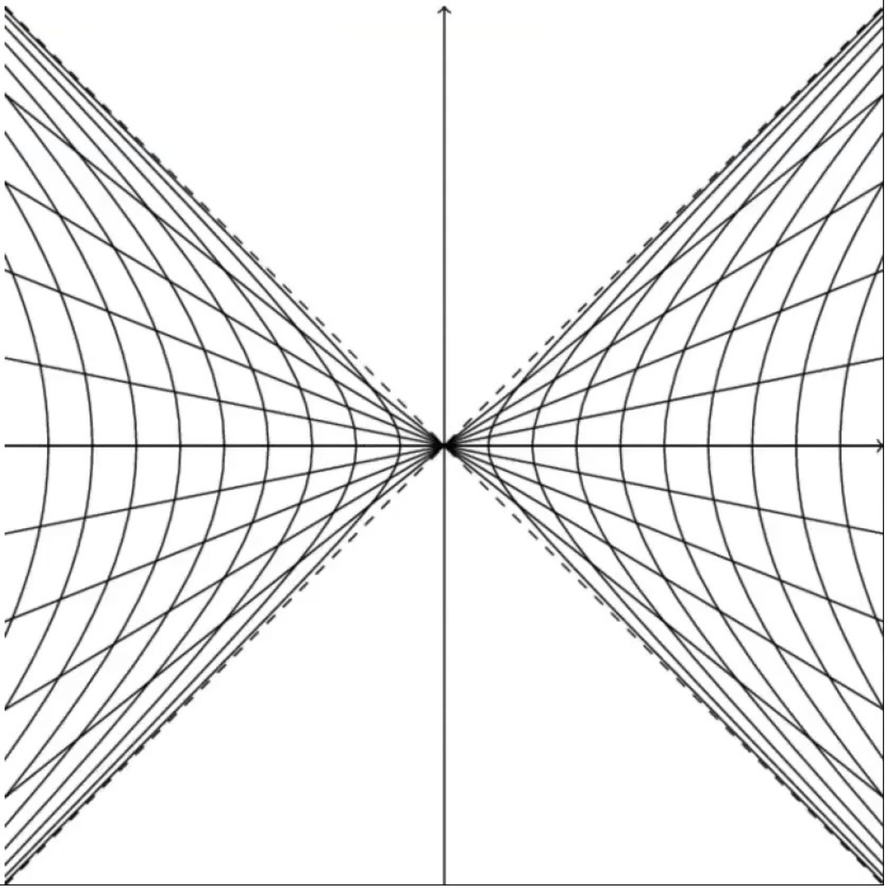
1) The Unruh Effect



The Rindler vacuum $|0_R\rangle$ and the corresponding creation and annihilation operators $\hat{b}^\dagger_{\boldsymbol{k}}, \hat{b}_{\boldsymbol{k}}$ define a notion of "particles" for accelerated observers.

If the field is in the Minkowski vacuum, the particle density operator for an accelerated observer gives

$$\langle \hat{n}_{\boldsymbol{k}} \rangle = \langle 0_M | \, \hat{b}^\dagger_{\boldsymbol{k}} \hat{b}_{\boldsymbol{k}} \, | 0_M \rangle$$



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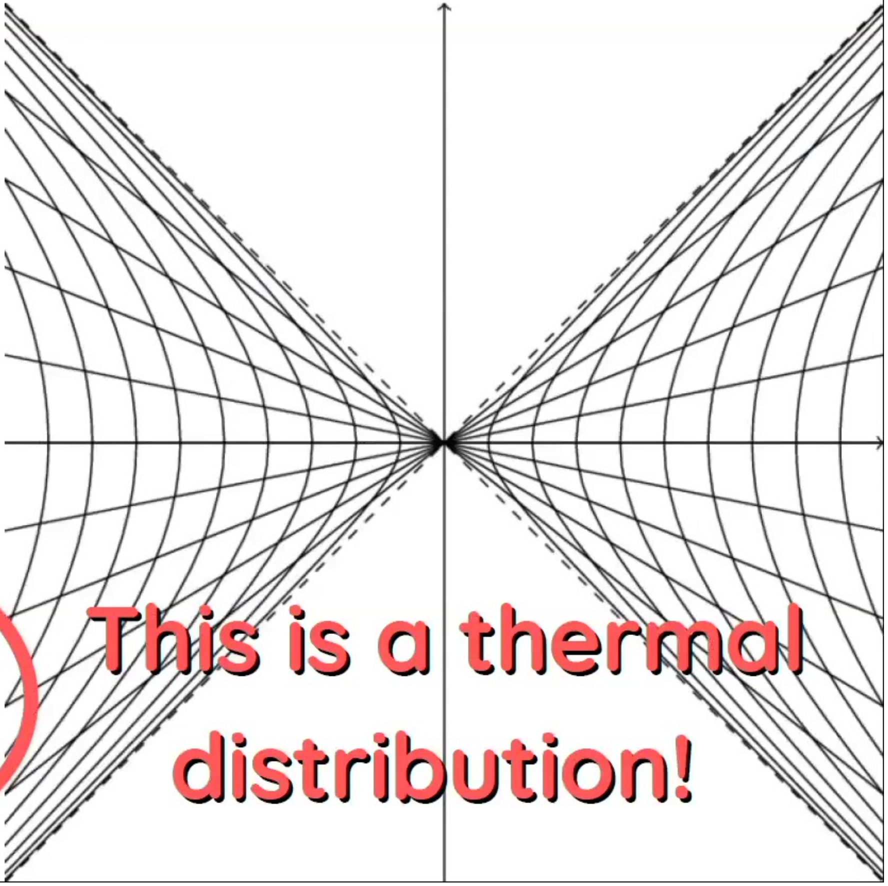
The Rindler vacuum $|0_R\rangle$ and the corresponding creation and annihilation operators $\hat{b}^\dagger_{\boldsymbol{k}}, \hat{b}_{\boldsymbol{k}}$ define a notion of "particles" for accelerated observers.

If the field is in the Minkowski vacuum, the particle density operator for an accelerated observer gives

$$\langle \hat{n}_{\boldsymbol{k}} \rangle = \langle 0_M | \, \hat{b}^\dagger_{\boldsymbol{k}} \hat{b}_{\boldsymbol{k}} \, | 0_M \rangle \boxed{\propto \frac{1}{e^{\frac{2\pi\omega_{\boldsymbol{k}}}{a}} - 1}}$$



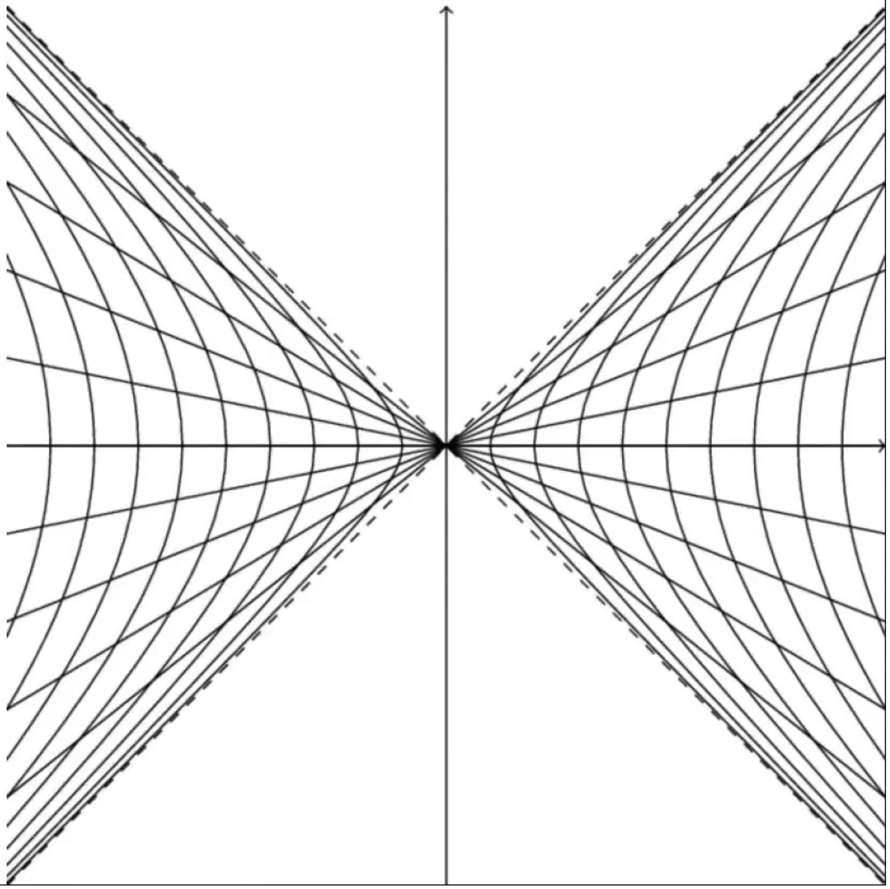
This is a thermal distribution!

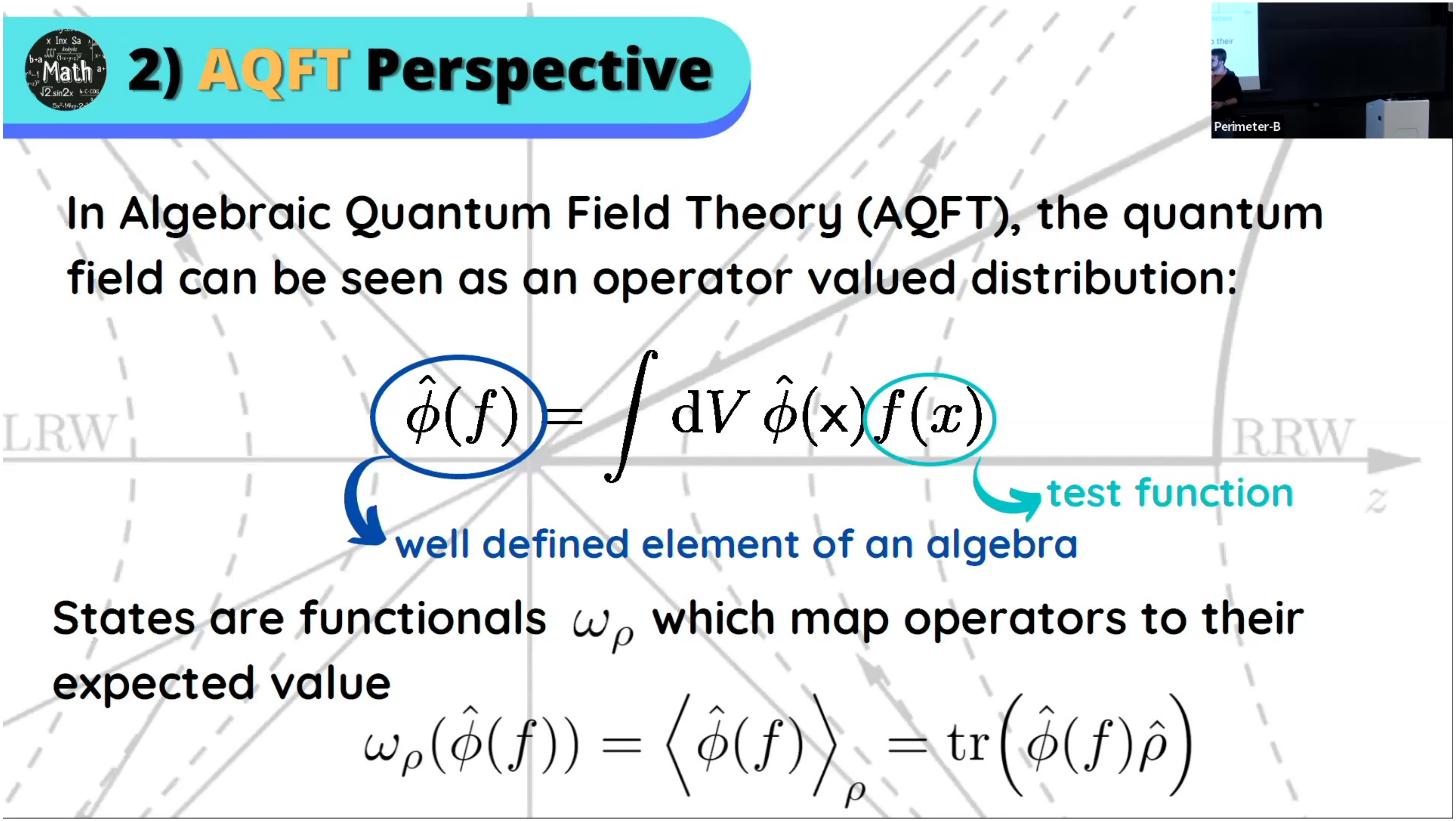
1) Rindler Modes Perspective

Issues with this derivation:

1) Requires modes (which are intrinsically non-local) to talk about temperature for a local observer.

2) The Rindler vacuum is non-physical: it has a divergent energy density in the lightcone.





2) AQFT Perspective

In Algebraic Quantum Field Theory (AQFT), the quantum field can be seen as an operator valued distribution:

$$\boxed{\hat{\phi}(f)} = \int \mathrm{d}V \, \hat{\phi}(\mathrm{x}) \boxed{f(x)}$$

test function

well defined element of an algebra

States are functionals ω_ρ which map operators to their expected value

$$\omega_\rho(\hat{\phi}(f)) = \left\langle \hat{\phi}(f) \right\rangle_\rho = \mathrm{tr}\left(\hat{\phi}(f)\hat{\rho}\right)$$

2) Thermality in AQFT

Thermality is then understood in terms of the KMS condition.

If α_τ is an operation corresponding to a notion of time evolution, then the KMS condition for a state with inverse temperature β reads

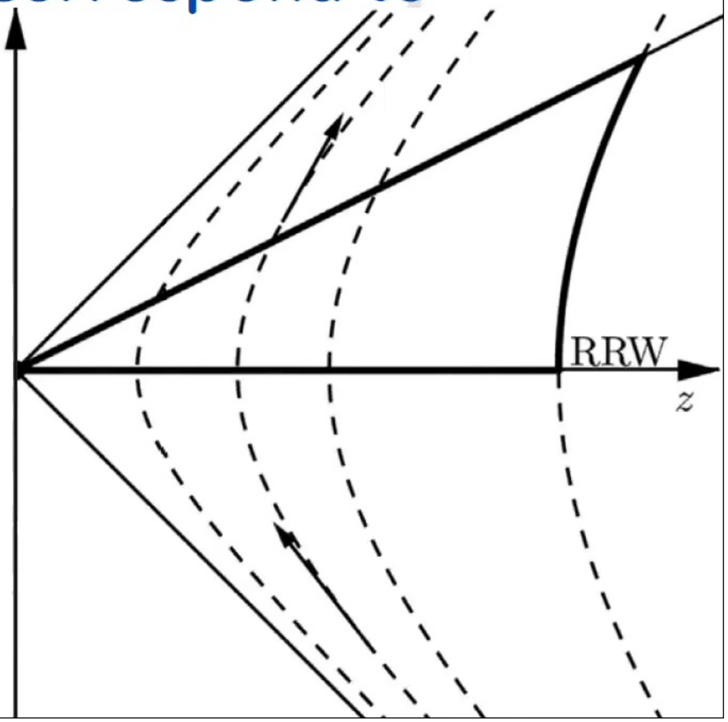
$$\left\langle \alpha_\tau(\hat{A})(f)\hat{B}(g) \right\rangle_\rho = \left\langle \hat{B}(g)\alpha_{\tau+i\beta}(\hat{A})(f) \right\rangle_\rho$$

for all operators $\hat{A}$ and $\hat{B}$ in the algebra of the quantum field theory.

2) The Unruh Effect

The boost generators define a notion of time evolution in Minkowski spacetime, whose orbits correspond to uniformly accelerated observers.

In the AQFT language, this time evolution is implemented by an operation α_τ.



2) The Unruh Effect

It can be shown that the Minkowski vacuum is a KMS state with respect to the time evolution generated by α_τ!

If the time parameter τ is adjusted to match the proper time of a uniformly accelerated observer, we find

$$\beta = \frac{2\pi}{a}$$

[3] J. J Bisognano and E. H. Wichmann, “On the Duality Condition for a Hermitian Scalar Field,” J. Math. Phys. 16, 985-1007 (1975).

2) AQFT Perspective

Issues with the AQFT perspective:

1) It is also non-local: we need to apply the "time evolution" to the field operators and take the expected value of the field state, which is defined everywhere in space. Although here one can talk about local field operators.

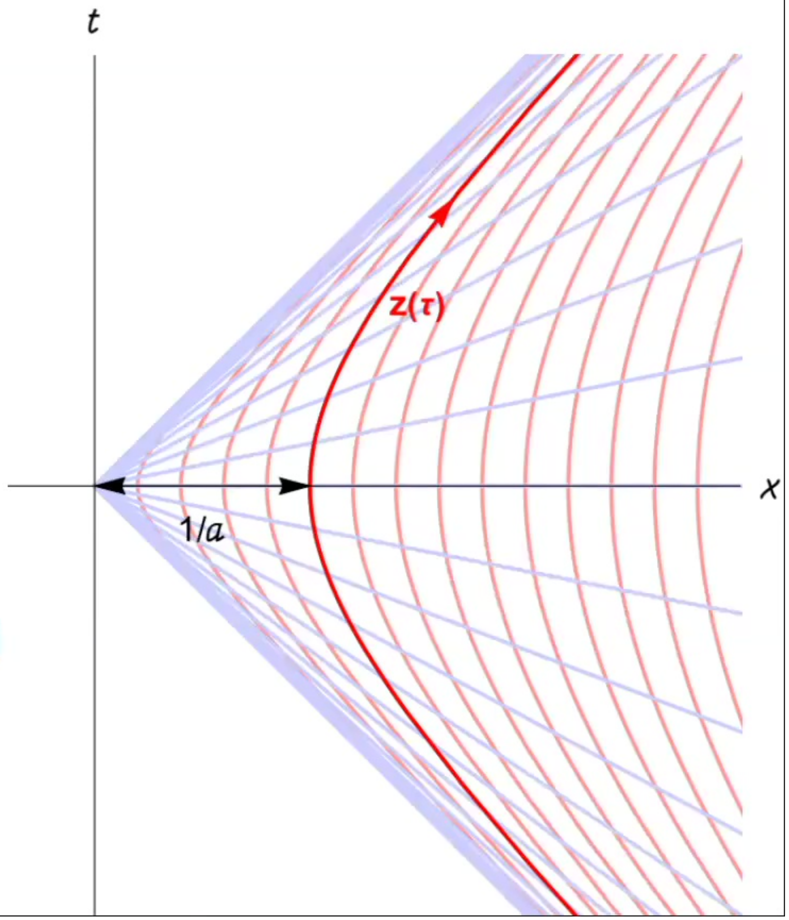
2) There is no physical time evolution associated to α_τ, it is merely a mathematical operation.

3) **Detector** Perspective

A **particle detector** is a

- Localized.
- Non-relativistic quantum system.
- That couples to a quantum field.

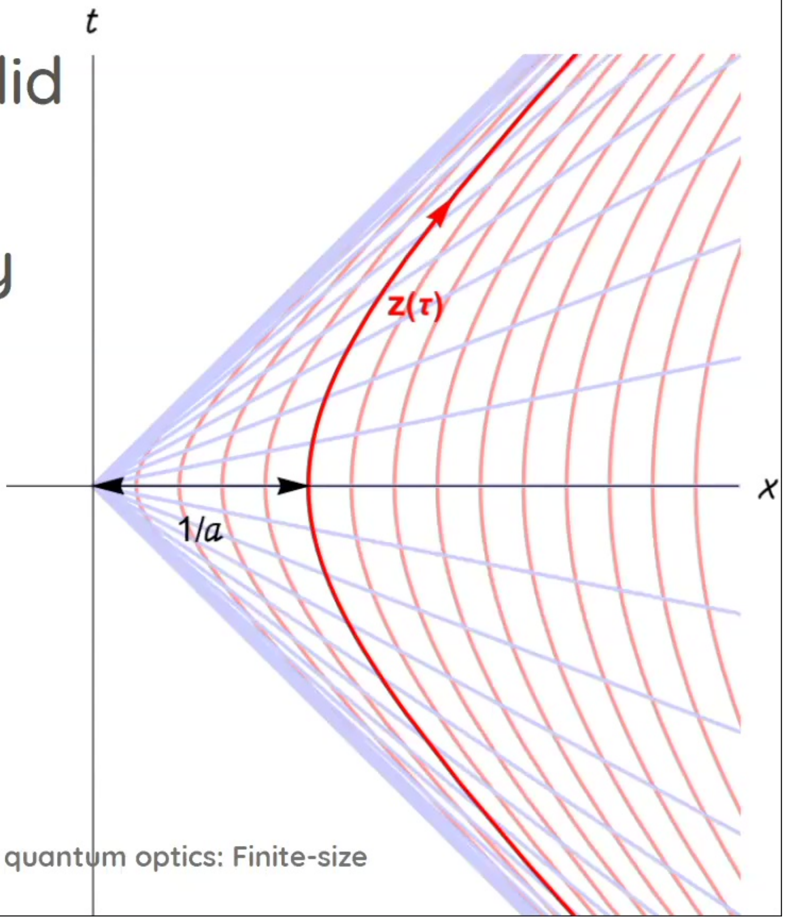
e.g. an atom coupled to the quantum electromagnetic field.



3) **Detector** Perspective

"From the retinas of our eyes to the solid state sensors at the LHC, we never measure a quantum field other than by coupling something to it."

Particle detector models are the theoretical framework for the "something".

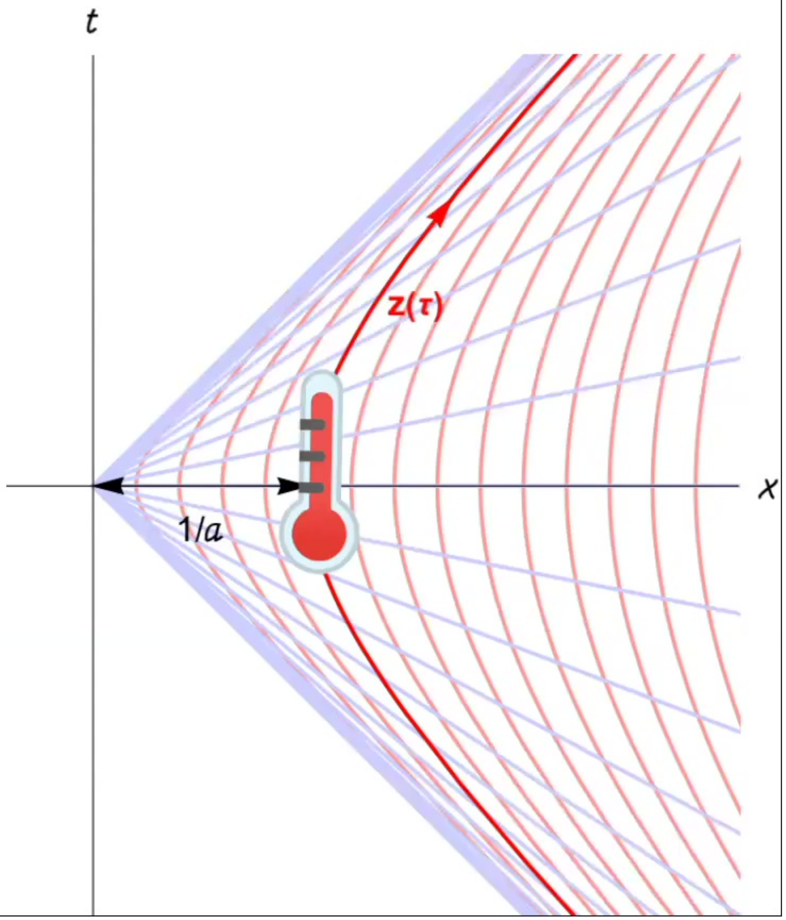


[6] Eduardo Martín-Martínez, T. Rick Perche, and Bruno de S. L. Torres, "General relativistic quantum optics: Finite-size particle detector models in curved spacetimes," Phys. Rev. D 101, 045017 (2020).

3) Detector Perspective

Particle detectors can be used as thermometers for probing the Unruh effect.

It is enough to consider a detector undergoing a uniformly accelerated trajectory, and ask whether it ends in a thermal state.



3) The UDW Model

The simplest particle detector is the two-level UDW model.

- It undergoes a trajectory $\mathsf{z}(\tau)$, parametrized by proper time.
- Its free Hamiltonian is $\hat{H}_{\mathrm{D}} = \Omega\hat{\sigma}^{+}\hat{\sigma}^{-}$.
- The interaction with the field $\hat{\phi}(\mathsf{x})$ is prescribed by the interaction Hamiltonian density:

$$\hat{h}_I(\mathsf{x}) = \underbrace{\lambda}_{\text{coupling constant}}\Lambda(\mathsf{x})\hat{\mu}(\tau)\hat{\phi}(\mathsf{x})$$

[6] Eduardo Martín-Martínez, T. Rick Perche, and Bruno de S. L. Torres - Phys. Rev. D 101, 045017 (2020)

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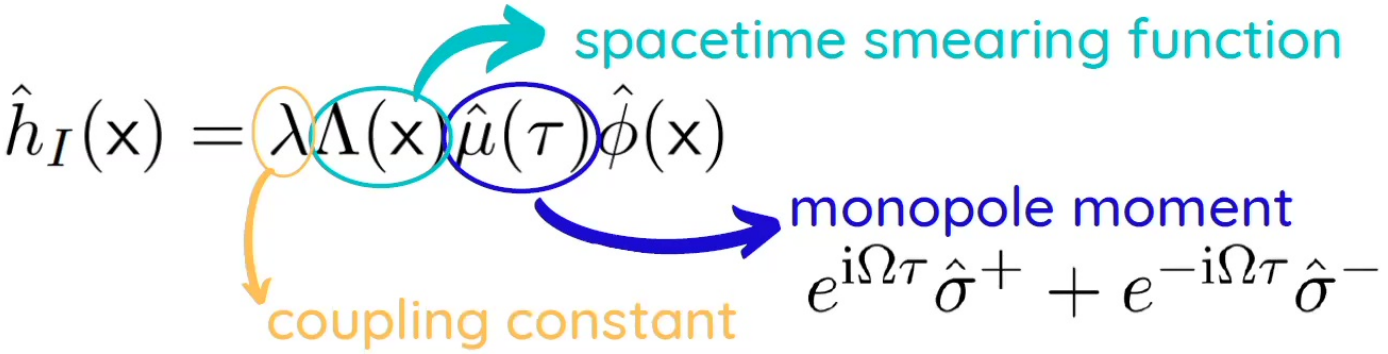


$$\hat{h}_I(\mathsf{x}) = \lambda\Lambda(\mathsf{x})\hat{\mu}(\tau)\hat{\phi}(\mathsf{x})$$

spacetime smearing function

monopole moment

$$e^{\mathrm{i}\Omega\tau}\hat{\sigma}^+ + e^{-\mathrm{i}\Omega\tau}\hat{\sigma}^-$$

coupling constant

[6] Eduardo Martín-Martínez, T. Rick Perche, and Bruno de S. L. Torres - Phys. Rev. D 101, 045017 (2020)

3) The UDW Model

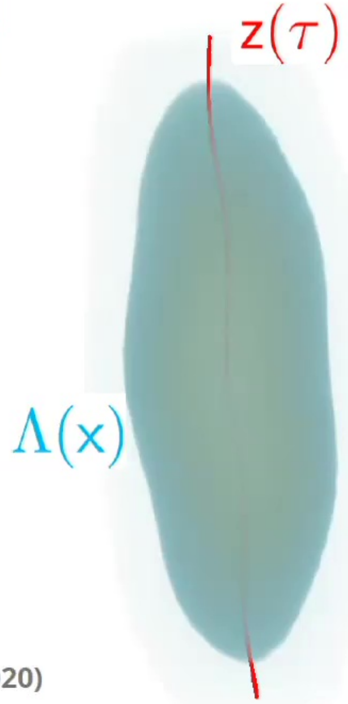
$$\hat{h}_I(\mathrm{x}) = \lambda\Lambda(\mathrm{x})\hat{\mu}(\tau)$$

It is **not** merely a theoretical idealization!

Appropriate choices of $\Lambda(\mathrm{x})$ can mimic physical interactions*, such as

Light-Matter interaction.[7-9]

Interactions of nucleons with neutrinos.[10,11]



[7] Alejandro Pozas-Kerstjens and Eduardo Martín-Martínez - Phys. Rev. D 94, 064074 (2016)
[8] Nicholas Funai, Jorma Louko, and Eduardo Martín-Martínez - Phys. Rev. D 99, 065014 (2019)
[9] Richard Lopp and Eduardo Martín-Martínez - Phys. Rev. A 103, 013703 (2021)
[10] Bruno de S. L. Torres, T. Rick Perche, André G. S. Landulfo, and George E. A. Matsas - Phys. Rev. D 102, 093003 (2020)
[11] T. Rick Perche and Eduardo Martín-Martínez -Phys. Rev. D 104, 105021 (2021)

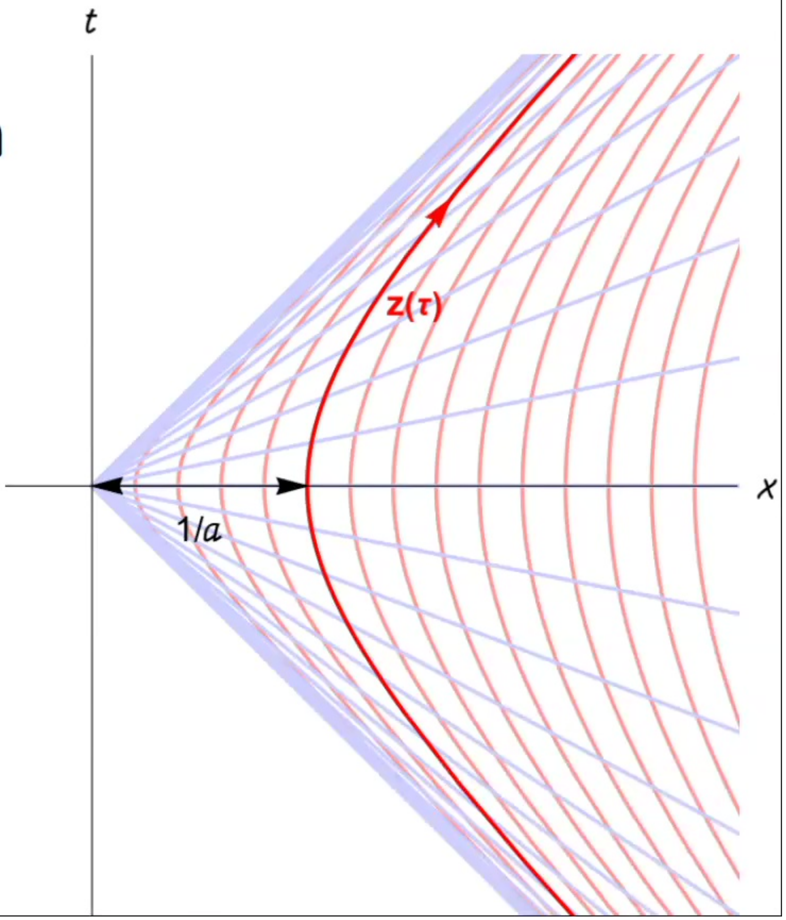
3) The Unruh Effect

Detectors can act as **thermometers**.

The **Excitation-Deexcitation Ratio** can be used to read off the temperature of the detector.

If a detector with energy gap Ω satisfies

$$\frac{p_{g\to e}}{p_{e\to g}} = e^{-\beta\Omega}$$



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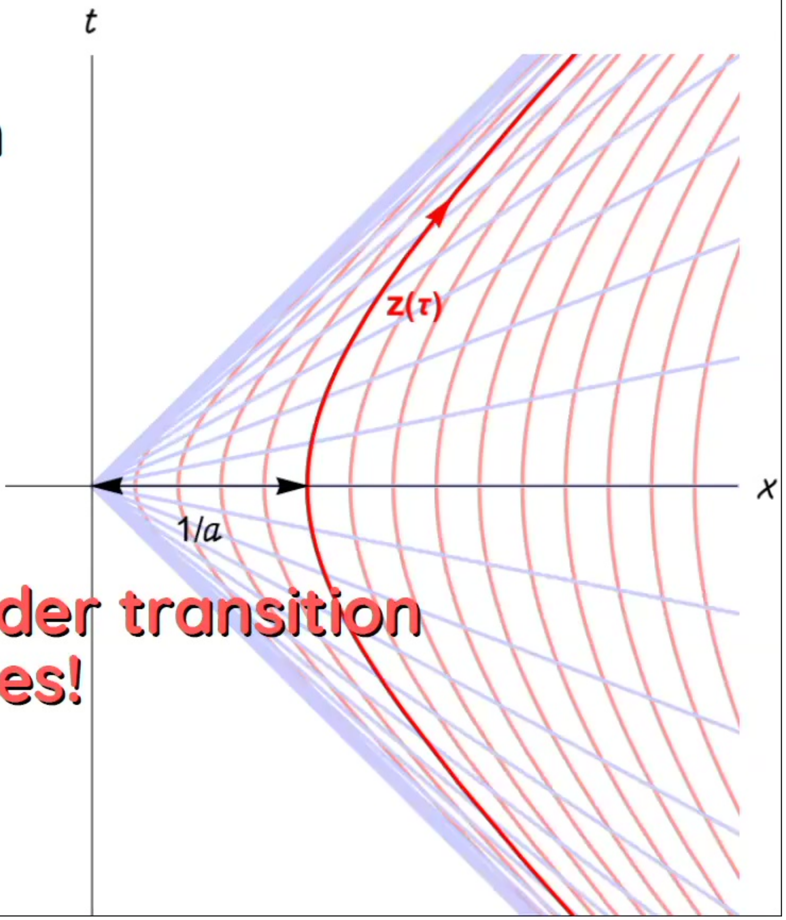

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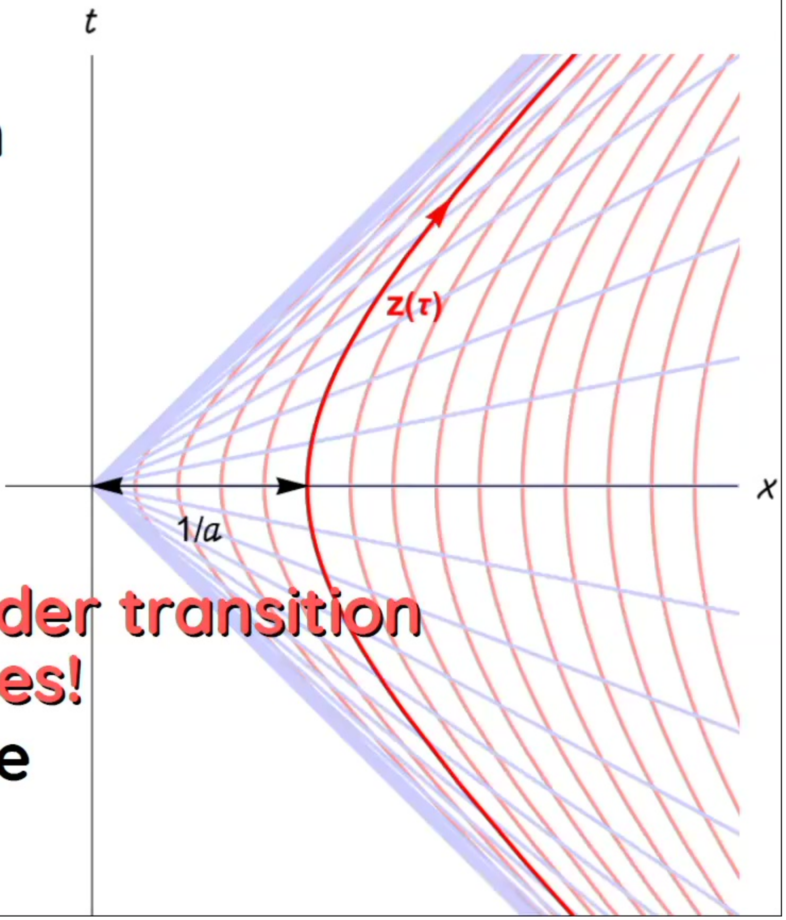
then it is* in a thermal state of inverse temperature β.

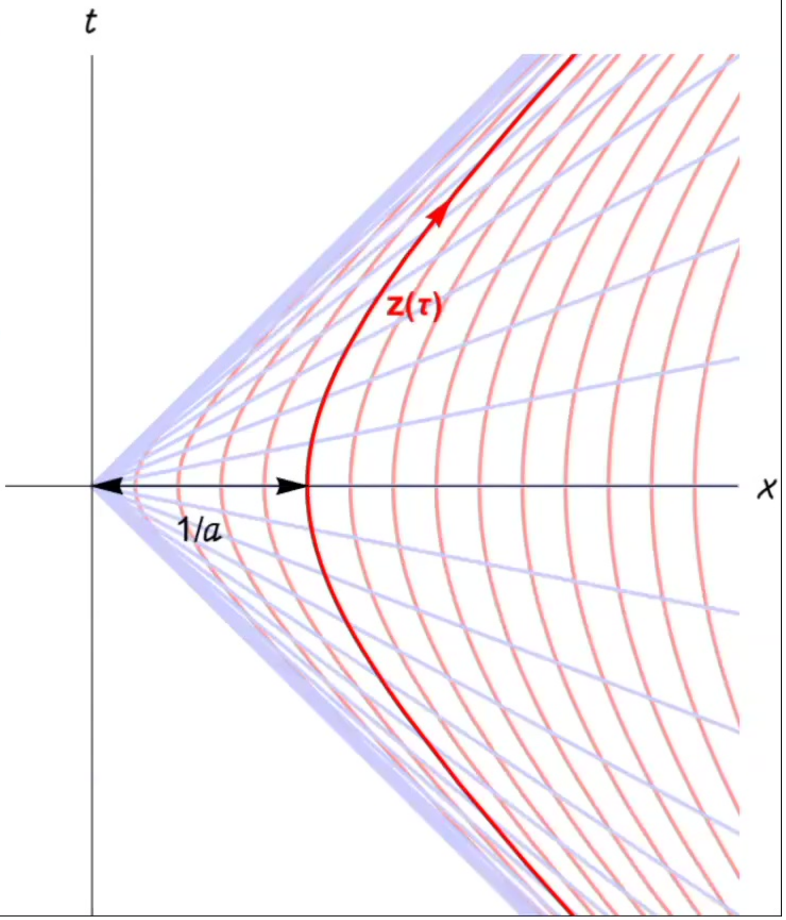


3) The Unruh Effect

We consider a uniformly accelerated UDW detector coupled to the Minkowski vacuum.

The EDR for a UDW detector coupled for long times reads

$$\lim_{T\to\infty}\frac{p_{g\to e}}{p_{e\to g}} = e^{-\frac{2\pi\Omega}{a}}$$

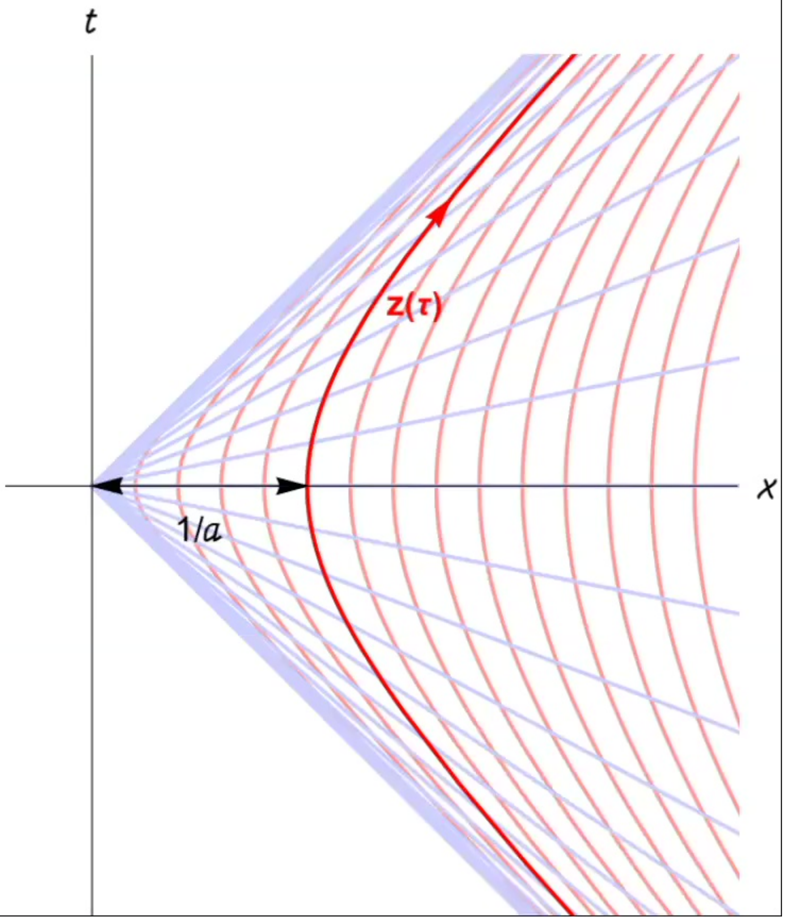


3) The Unruh Effect

Issues with the detector approach:

1) Calculations are perturbative, which makes thermalization harder to study.

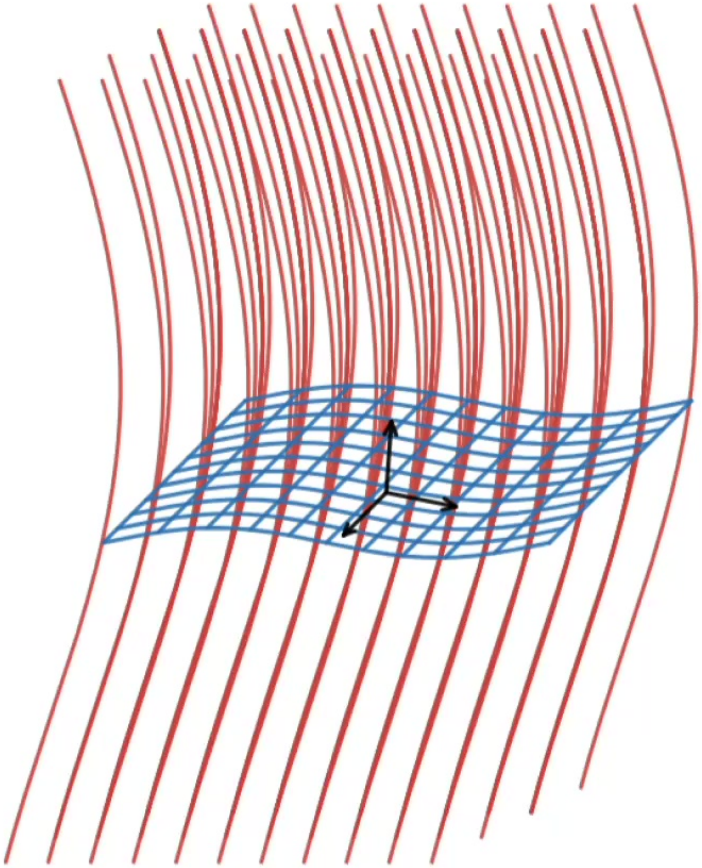
2) Is not entirely done within QFT: it requires an external non-relativistic quantum system.



General Thermalization of Detectors

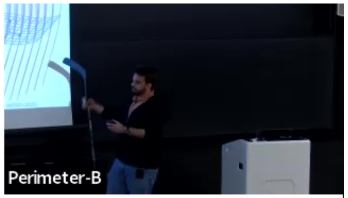
It is possible to use detectors to probe the KMS temperature in more general setups.

If the field state satisfies the KMS condition with respect to the local time evolution around the detector's trajectory



[1] T. Rick Perche, “General features of the thermalization of particle detectors and the Unruh effect,” Phys. Rev. D 104, 065001 (2021).

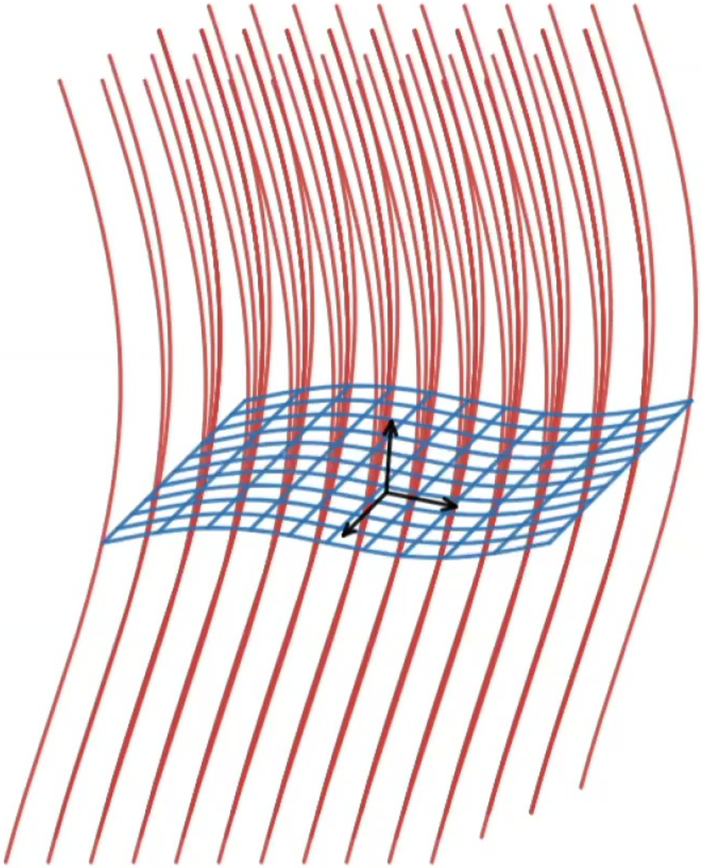
Time Flow Around a Trajectory



Consider a timelike trajectory $z(\tau)$.

We pick a Fermi Normal Coordinates $\xi = (\tau, \boldsymbol{\xi})$ around the trajectory.

The τ parameter corresponds to the trajectory proper time, and extends the time parameter locally around the curve.



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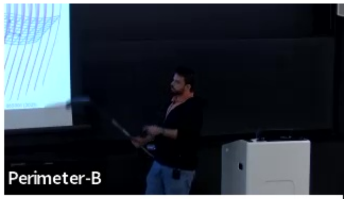
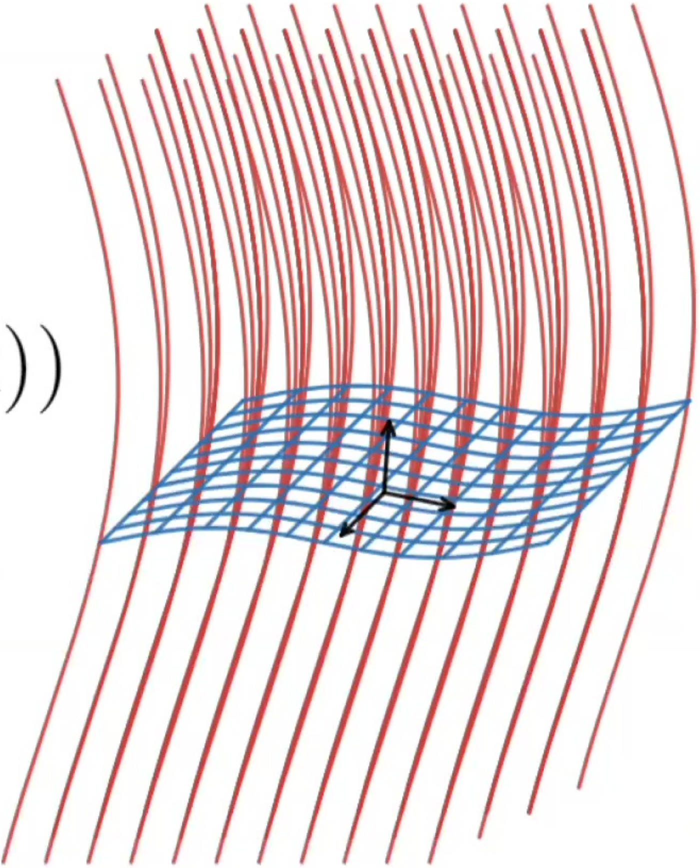
Time Flow Around a Trajectory





The flow Φ_τ associated to τ defines an operation in AQFT:

$$\alpha_\tau(\hat{\phi})(f) := \int \mathrm{d}V \hat{\phi}(\mathsf{x}) f(\Phi_\tau(\mathsf{x}))$$

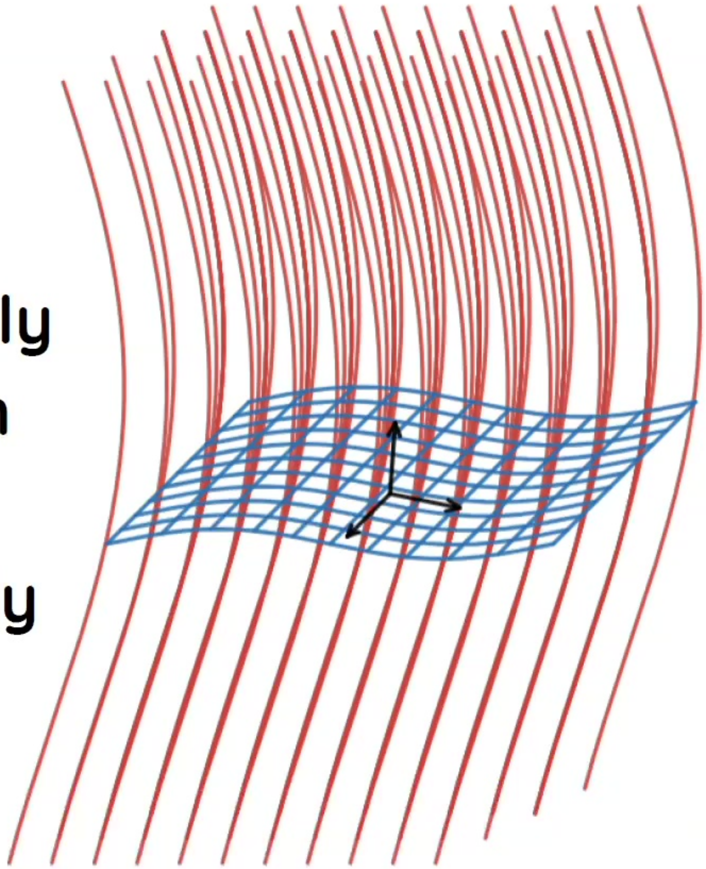
Our assumption is that the field is in a KMS state with respect to the time flow generated by α_τ.

[1] T. Rick Perche, “General features of the thermalization of particle detectors and the Unruh effect,” Phys. Rev. D 104, 065001 (2021).

Thermalization of Detectors

In [5] it was proved that the detector thermalizes to the KMS temperature of the field if:

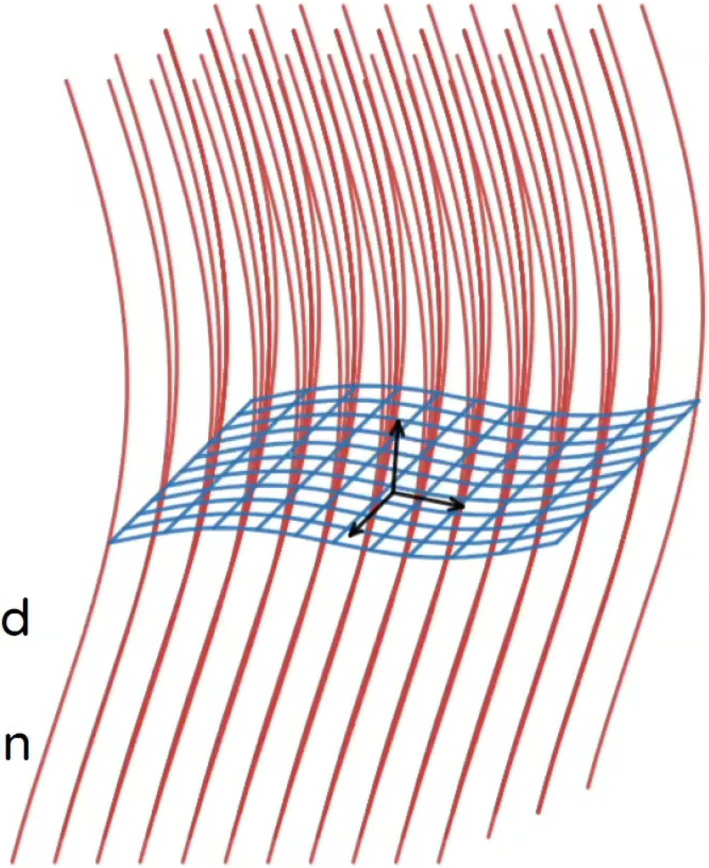
- The detector is rigid and sufficiently small compared to its acceleration and the curvature of spacetime.
- The interaction lasts for sufficiently long times compared to the detector gap.



[1] T. Rick Perche, "General features of the thermalization of particle detectors and the Unruh effect," Phys. Rev. D 104, 065001 (2021).

The Generality of the Result

Moreover, this result is valid
for any* particle detector
that couples to any operator
in any quantum field theory,
and is valid in curved spacetimes.



e.g. an atom coupled with the electromagnetic field
uniformly accelerated in flat spacetimes, or
around a black hole, probing Hawking radiation

[1] T. Rick Perche, “General features of the thermalization of particle detectors and the Unruh effect,” Phys. Rev. D 104, 065001 (2021).

Summary

Thermality in QFT is phrased in terms of the KMS condition

The Minkowski vacuum is a KMS state with respect to uniformly accelerated time translations.

Particle detectors thermalize to the KMS temperature of the field associated with their motion.

→ This is true for any detector coupled to any operator of any quantum field theory in curved spacetimes.

Overall, this can be seen as a generalization of the Unruh effect.

[1] T. Rick Perche, "General features of the thermalization of particle detectors and the Unruh effect," Phys. Rev. D 104, 065001 (2021).

Thank You!

General features of the thermalization of particle detectors and the Unruh effect

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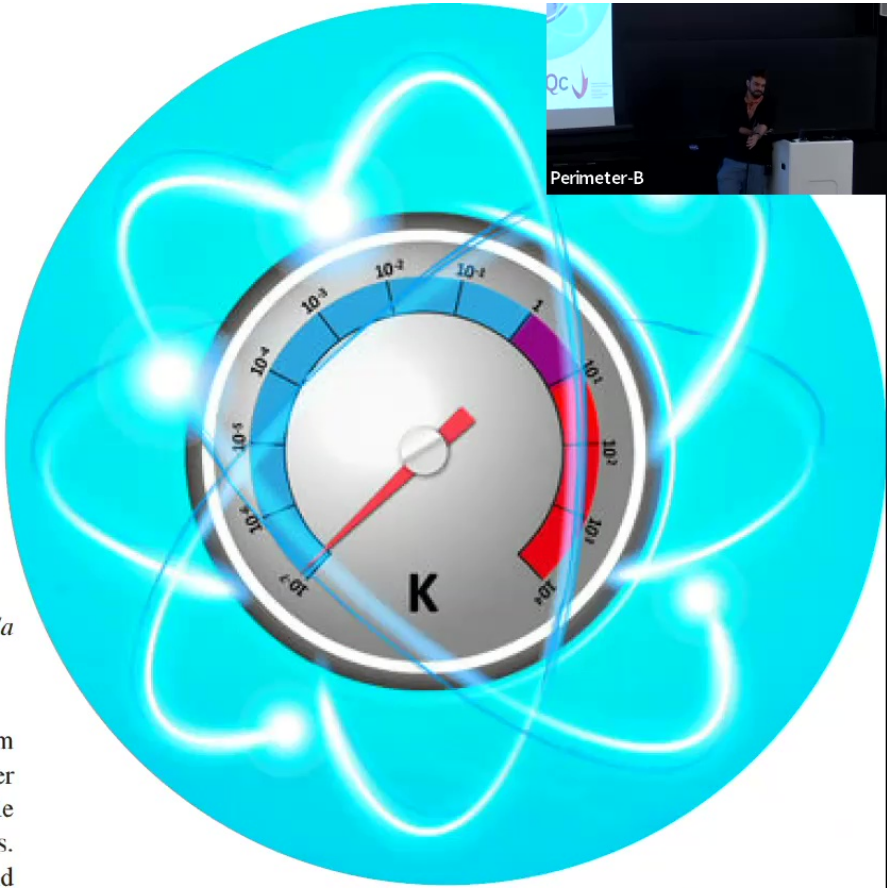
We study the thermalization of smeared particle detectors that couple locally to *any* operator in a quantum field theory in curved spacetimes. We show that if the field state satisfies the Kubo-Martin-Schwinger condition with inverse temperature β with respect to the detector's local notion of time evolution, reasonable assumptions ensure that the probe thermalizes to the temperature $1/\beta$ in the limit of long interaction times. Our method also imposes bounds on the size of the system with respect to its proper acceleration and spacetime curvature in order to accurately probe the Kubo-Martin-Schwinger temperature of the field. We then apply this formalism to a uniformly accelerated detector probing the Minkowski vacuum of any *CPT* symmetric quantum field theory, and show that the detector thermalizes to the Unruh temperature, independently of the operator it couples to. This exemplifies yet again the robustness of the Unruh effect, even when arbitrary smeared detectors are used to probe general operators in a quantum field theory.

DOI: 10.1103/PhysRevD.104.065001



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