

Title: Improving 3D Codes under Biased Noise

Speakers: Eric Huang

Collection: Young Researchers Conference

Date: June 20, 2022 - 10:40 AM

URL: <https://pirsa.org/22060045>

Abstract: Quantum error correction is necessary for scalable quantum computation. Topological quantum error correcting codes have exceptional properties that make them ideal for future experiments. Massive gains can be achieved if the code is optimized for the noise, which we demonstrate for some 3D codes under biased noise.

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Improving 3D Codes for Biased Noise

Supervisor: Arthur Pesah, Michael Vasmer, Arpit Dua, Christopher Chubb

Eric Huang

Page 1 of 70 Layer Layer 1 215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Motivation

The diagram illustrates the progression of quantum computing technology. It begins at a point labeled 'START', marked with a red pushpin. A path leads from 'START' through a box labeled 'NISQ' (Noisy Intermediate-Scale Quantum). This path then leads to a box labeled 'ERROR CORRECT', which is connected to a box labeled 'SCALE'. From 'SCALE', the path continues through a box labeled 'QEC' (Quantum Error Correction) and finally reaches a box labeled 'Quantum Computing'. A large, wavy barrier labeled 'NOISE' is shown on the right side of the path, representing the challenge of maintaining quantum coherence. The 'Quantum Computing' area is depicted with colorful, overlapping arcs, suggesting a complex or multi-faceted future state. The diagram is presented in a hand-drawn style on a white background within a presentation software interface.

Page 2 of 70 Layer Layer 1

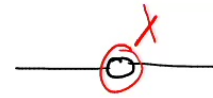
215%

Qubits and Pauli Operators

$$\chi|\psi\rangle = \beta|0\rangle + \alpha|1\rangle$$

Qubit

Flip



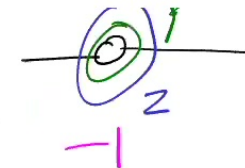
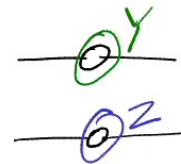
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Phase

$$Z|\psi\rangle = \alpha|0\rangle - \beta|1\rangle$$



$$YZ = -ZY$$

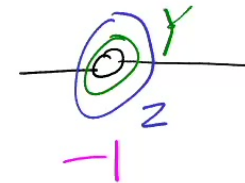
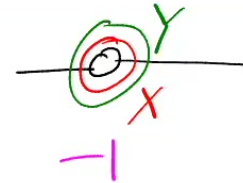
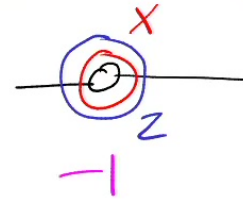
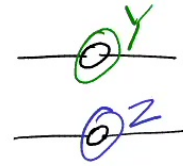
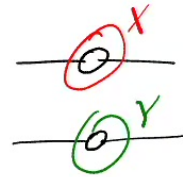
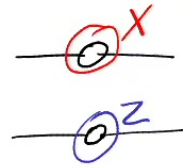


Commutate or Anti-Commutate

$$XZ = -ZX$$

$$XY = - YX$$

$$YZ = -ZY$$



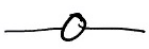
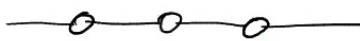
File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Encode 1-qubit state in subspace of 3-qubit states

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ Encode $|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$

$|0\rangle \mapsto |000\rangle = |0\rangle \otimes |0\rangle \otimes |0\rangle$

$|1\rangle \mapsto |111\rangle = |1\rangle \otimes |1\rangle \otimes |1\rangle$

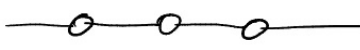
Page 5 of 70 Layer Layer 1 215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

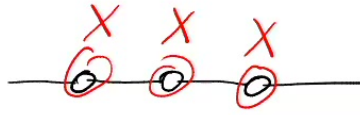
Perimeter-B

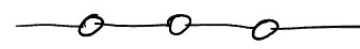
Logical operations still work

$$|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$$


$\bar{X}$

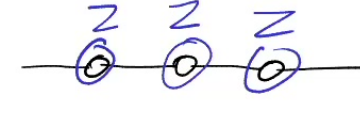
 $\rightarrow$

$$|\psi\rangle = \beta|000\rangle + \alpha|111\rangle$$


$$|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$$


$\bar{Z}$

 $\rightarrow$

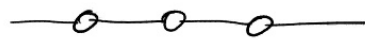
$$|\psi\rangle = \alpha|000\rangle - \beta|111\rangle$$


Page 6 of 70 Layer Layer 1

215%

Suppose 1 qubit gets flipped

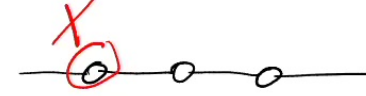
$$|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$$



Noise

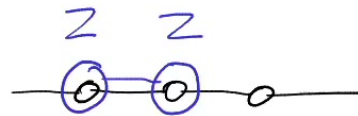


Error $X \otimes I \otimes I$

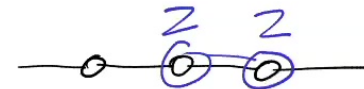


Measure "Stabilisers"

$Z \otimes Z \otimes I$

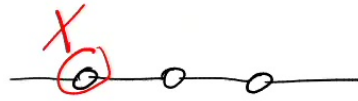


$I \otimes Z \otimes Z$

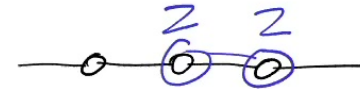
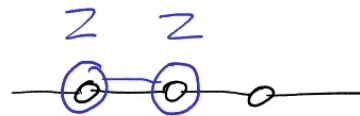


Measure stabilisers to get "syndromes"

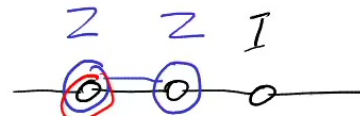
Error



Measure Stabilisers



Commutation



$$\begin{matrix} X & I & I \\ (-1) & (+1) & (+1) \end{matrix}$$

$$\begin{matrix} I & Z & Z \\ (+1) & (+1) & (+1) \end{matrix}$$

$$ZX = -XZ$$

$$XI = +IX$$

Syndromes

$$-1$$

$$+1$$

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

What is the error?

Page 9 of 70 Layer Layer 1

215%

Perimeter-B

Pirsa: 22060045

Page 10/69

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Error

Measure Stabilisers

Commutation

Syndromes

+1 -1 +1

+1 -1 -1

Page 10 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Stabilisers

Z Z

Z Z

Syndromes

$+|$

$+|$

$-|$

$-|$

$+|$

$-|$

Correction

X

X

X

Page 11 of 70 Layer Layer 1

215%

Result

FAIL!!! $|000\rangle \rightarrow |111\rangle$

Logical error!

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Code and Decoder

Stabilisers define Code

Corrections define Decoder

+ |

+ |

- |

- |

+ |

- |

+ |

- |

Page 13

- +

of 70

Layer Layer 1

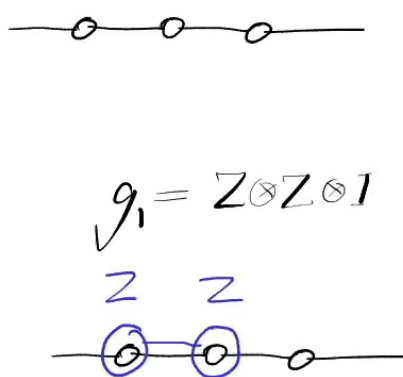
215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Stabilisers "Stabilise" the codespace

$$|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$$
$$g_k |\psi\rangle = |\psi\rangle$$

$$g_1 = Z \otimes Z \otimes I$$

$$g_2 = I \otimes Z \otimes Z$$

Page 14 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Actually, there are many valid corrections

$g \in \mathcal{G}$ stabiliser $|\psi\rangle \in \text{codespace}$ i.e. $g|\psi\rangle = |\psi\rangle$

E error (after measurement) C correction i.e. $CE|\psi\rangle = |\psi\rangle$

$gCE|\psi\rangle = g|\psi\rangle = |\psi\rangle$ 😊

i.e. gC also a valid correction $\Rightarrow 2^k$ valid corrections

Page 16 of 70 Layer Layer 1 215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Noise models

Bit Flip

$$\rho \rightarrow (1-p)\rho + pX\rho X$$

Diagram illustrating the Bit Flip noise model:

- Initial state: ρ (represented by a circle with a dot on a horizontal line).
- Transition with probability p : $\rho \rightarrow X\rho X$ (represented by a circle with a dot crossed out with a red 'X').
- Transition with probability $1-p$: $\rho \rightarrow \rho$ (represented by the original circle with a dot).

Page 18 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

IID Noise

$$\Pr\left[-\overset{X}{\textcircled{0}}-0-0\right] = \Pr\left[-0-\overset{X}{\textcircled{0}}-0\right] = \Pr\left[-0-0-\overset{X}{\textcircled{0}}\right]$$

et. cetera...

Page 19 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Phase flip

$$\rho \mapsto (1-p)\rho + pZ\rho Z$$

Bit flip code cannot detect this

Page 20 of 70 Layer Layer 1

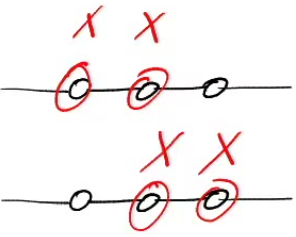
215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Can use these stabilisers instead



But then it can't correct bit flips...

Page 21 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Make Text

Page 22 of 70 Layer Layer 1

215%

Perimeter-B

More types of noise

Depolarising

$$\rho \mapsto (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$$

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Loading...

$1-p_x-p_y-p_z$

→

→

Page 23 of 70 Layer Layer 1 215%

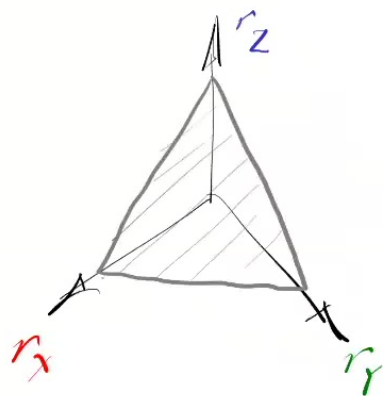
Biased Noise!

The diagram shows a horizontal line on the left with a point labeled 0 . Three arrows originate from this point and point to the right. The top arrow is labeled P_X , the middle arrow is labeled P_Y , and the bottom arrow is labeled P_Z . A fourth arrow originates from the same point on the left and points downwards and to the right, labeled $1 - P_X - P_Y - P_Z$. On the right side, there are four horizontal lines. The top line has a point labeled X circled in red. The second line has a point labeled Y circled in green. The third line has a point labeled Z circled in blue. The bottom line has a point labeled 0 .

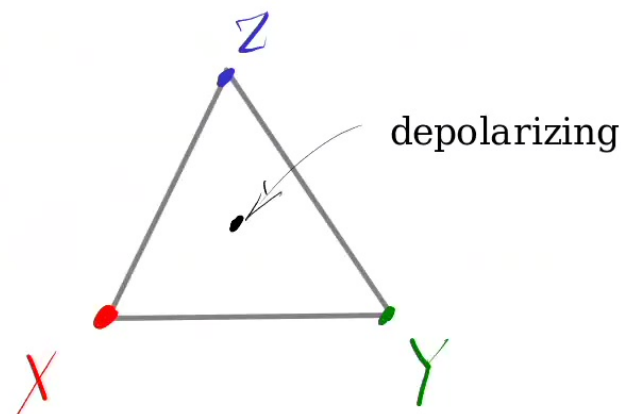
General Pauli Channel (r_X, r_Y, r_Z)

$$\rho \mapsto (1-p)\rho + p(r_X X \rho X + r_Y Y \rho Y + r_Z Z \rho Z)$$

$$r_X + r_Y + r_Z = 1$$



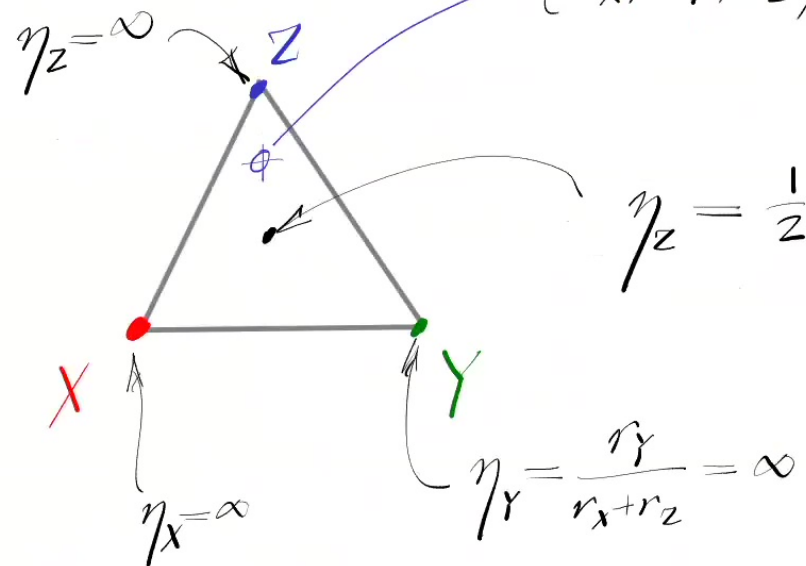
project



Channel

Bias Ratio

$$\eta_z \in [0, \infty)$$



File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

The diagram illustrates the relationship between three components and a final outcome:

- Code**: Contains the handwritten text $z21$ and 122 .
- Noise Model**: Contains the handwritten text r_x , r_y , and r_z .
- Decoder**: Contains the handwritten text $\neg + \rightarrow x11$ and $+ - \rightarrow 1xx$.
- Probability of Success**: A central box at the bottom.

Arrows indicate the flow of information from the Code, Noise Model, and Decoder boxes to the Probability of Success box.

Page 26 of 70 Layer Layer 1

215%

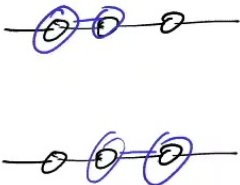
File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

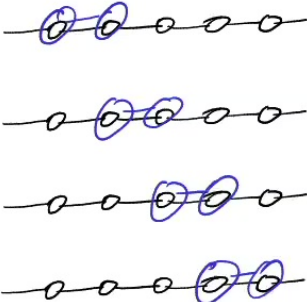
Perimeter-B

Family of Codes

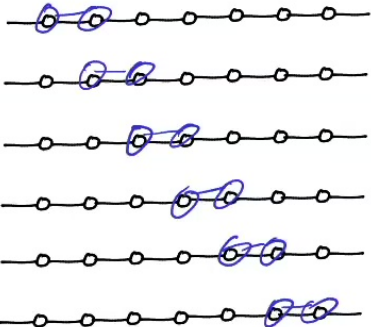
$n=3$



$n=5$



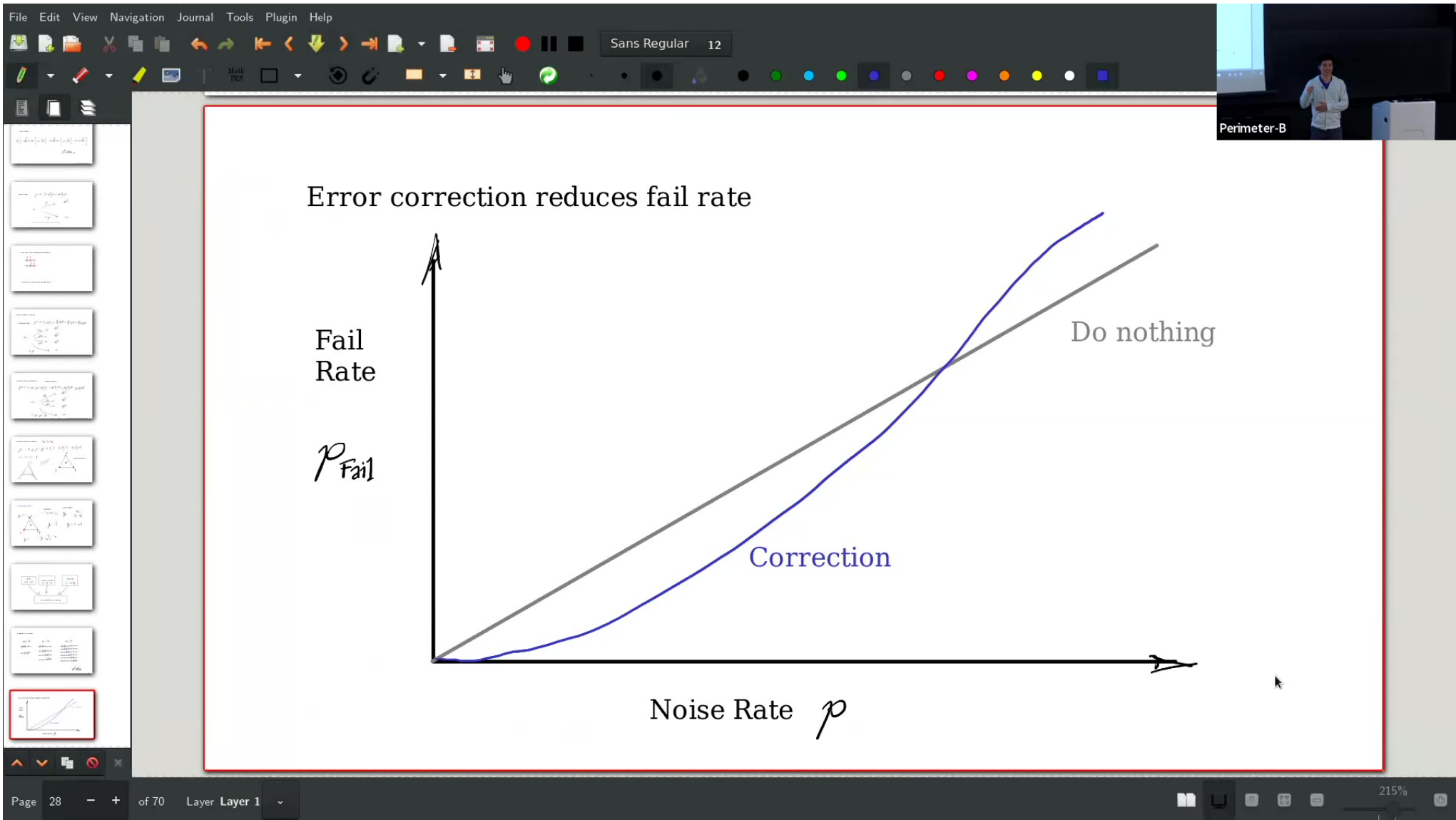
$n=7$

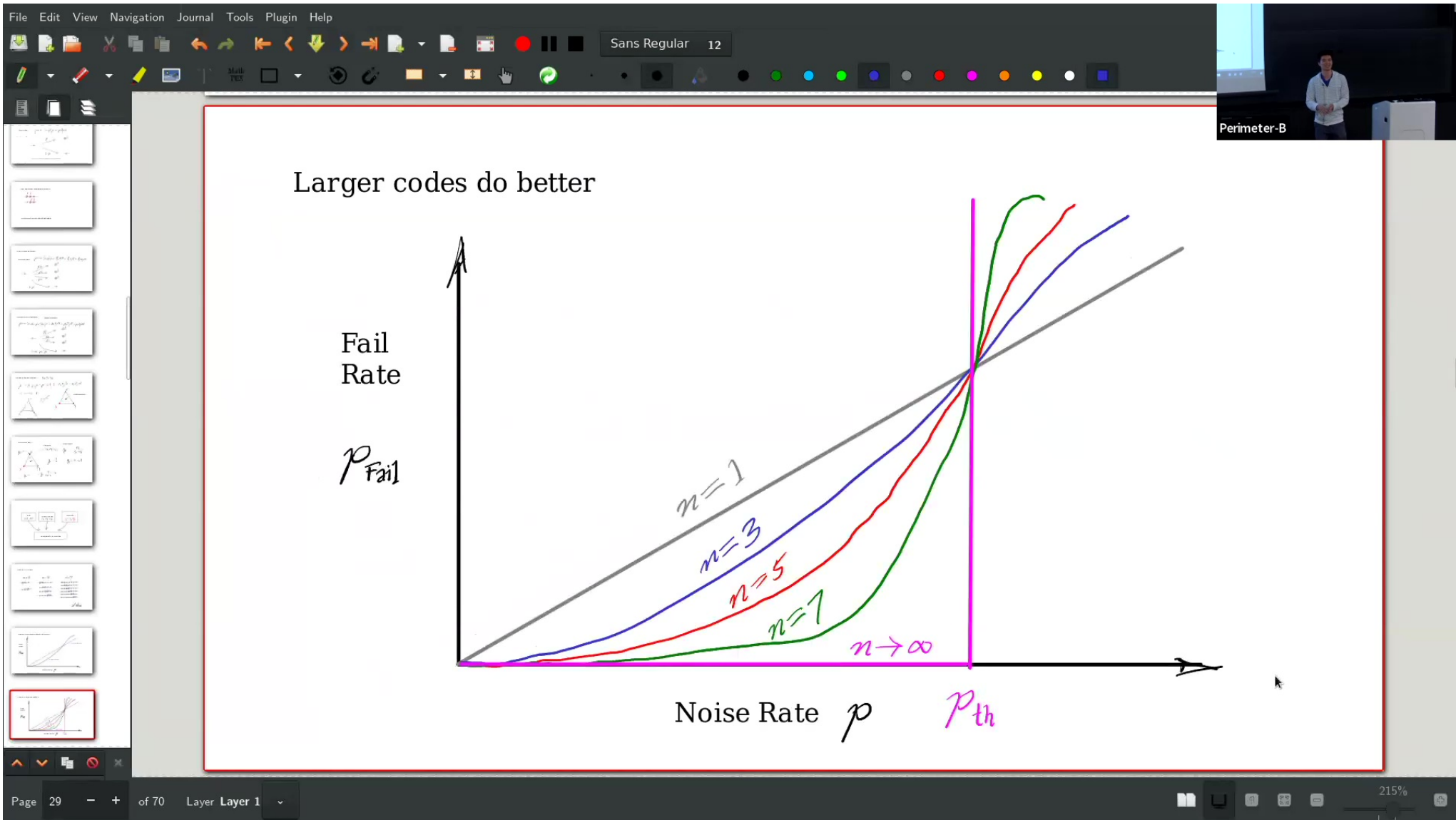


et cetera

Page 27 of 70 Layer Layer 1

215%





File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

The diagram illustrates the relationship between four components in a quantum error correction system:

- Code Family**: Contains several small diagrams of quantum codes, including surface codes and LDPC codes.
- Noise Model**: Contains the parameters r_x , r_y , and r_z .
- Decoder**: Contains the mapping rules for decoding, such as $- \rightarrow X$ and $+ \rightarrow I$, and the threshold p_{th} .
- Noise Threshold p_{th}** : The central concept that all three components influence.

Arrows indicate that the Code Family, Noise Model, and Decoder all contribute to determining the Noise Threshold p_{th} .

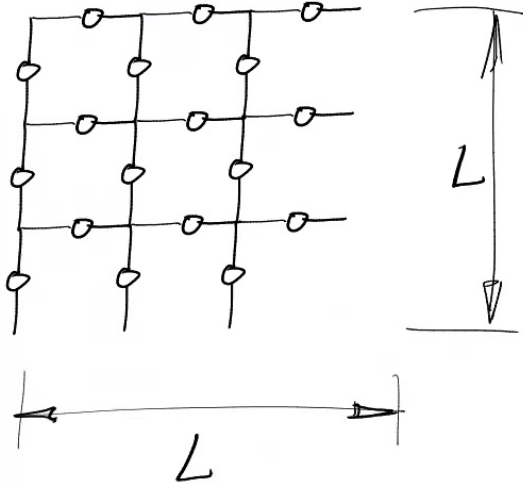
Page 31 of 70 Layer Layer 1

215%

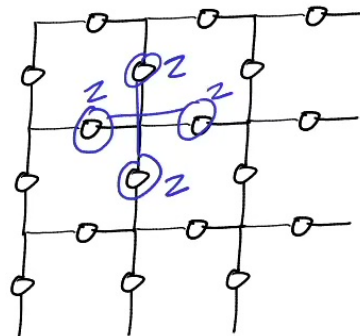
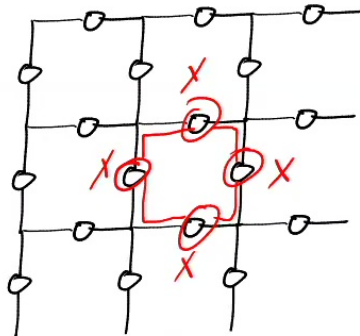
File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Toric Code Stabilisers



$n = 2L^2$



Perimeter-B

Page 32 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Multitool

Page 33 of 70 Layer Layer 1

215%

Perimeter-B

Stabilisers Commute

$+1$

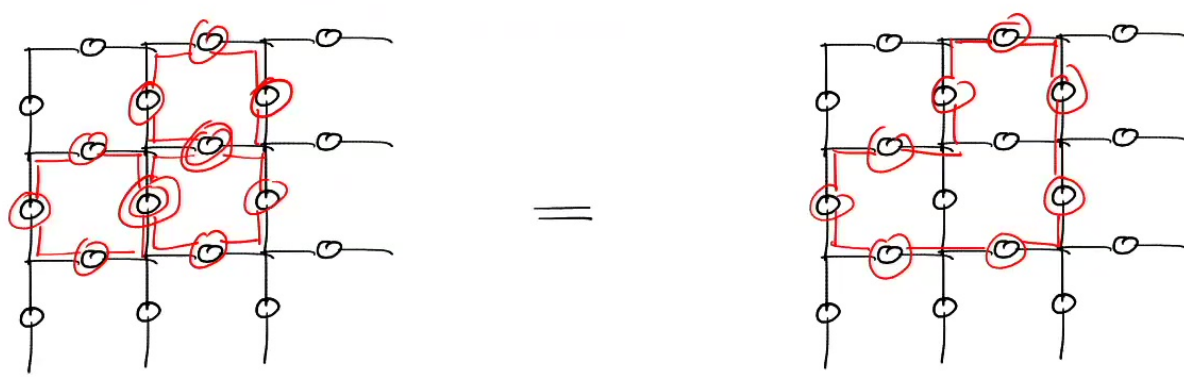
$(-1) \times (-1) = +1$

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Stabiliser group is contractible loops



Page 35 of 70 Layer Layer 1

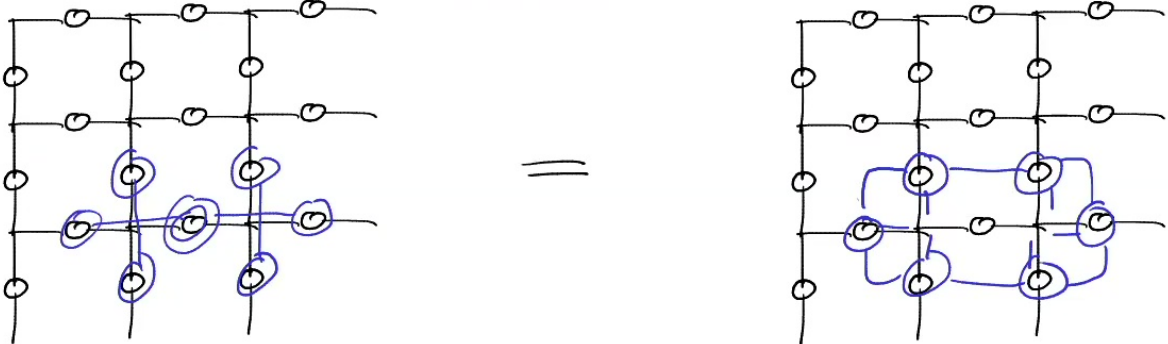
215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Stabiliser group is contractible loops



Page 36 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help
Sans Regular 12

=

Only $L^2 - 1$ independent generators like

Page 38 of 70 Layer Layer 1
215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

$\bar{X}_1$

$\bar{Z}_1$

$\bar{X}_1 \bar{Z}_1 = -\bar{Z}_1 \bar{X}_1$

Page 40 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Number of Qubits

Number of Stabiliser Generators

Number of Encoded Logical Qubits

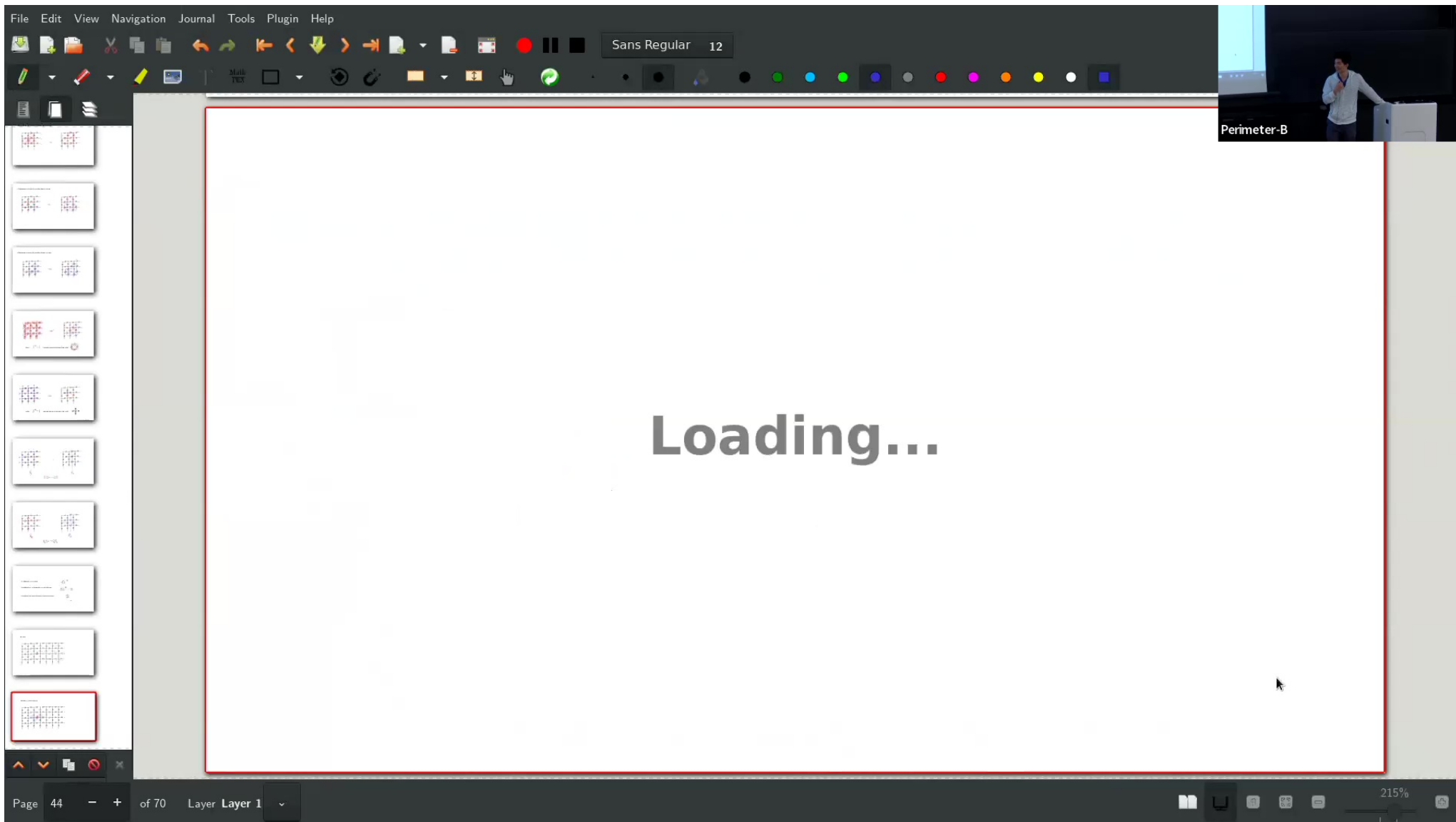
$2L^2$

$2L^2 - 2$

2

Page 42 of 70 Layer Layer 1

215%



File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Strings of Errors

Perimeter-B

Page 46 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Correct by Pair-Matching

Minimum-Weight Pair Matching

Page 47 of 70 Layer Layer 1

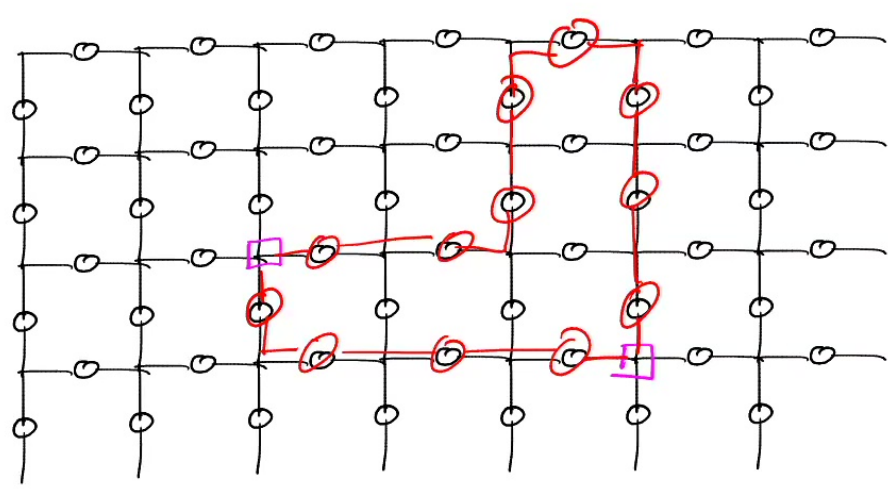
215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Correct by Pair-Matching



Closed loops: stabiliser!

Page 48 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Uncorrectable Errors

The diagram illustrates a quantum circuit with 5 qubits and 6 gates. The left side shows the circuit with a vertical line of errors (circles) on the fourth qubit, highlighted by a pink square. An arrow labeled "decode" points to the right side, which shows the same circuit with the errors on the fourth qubit highlighted by a red circle.

Page 49 of 70 Layer Layer 1

215%

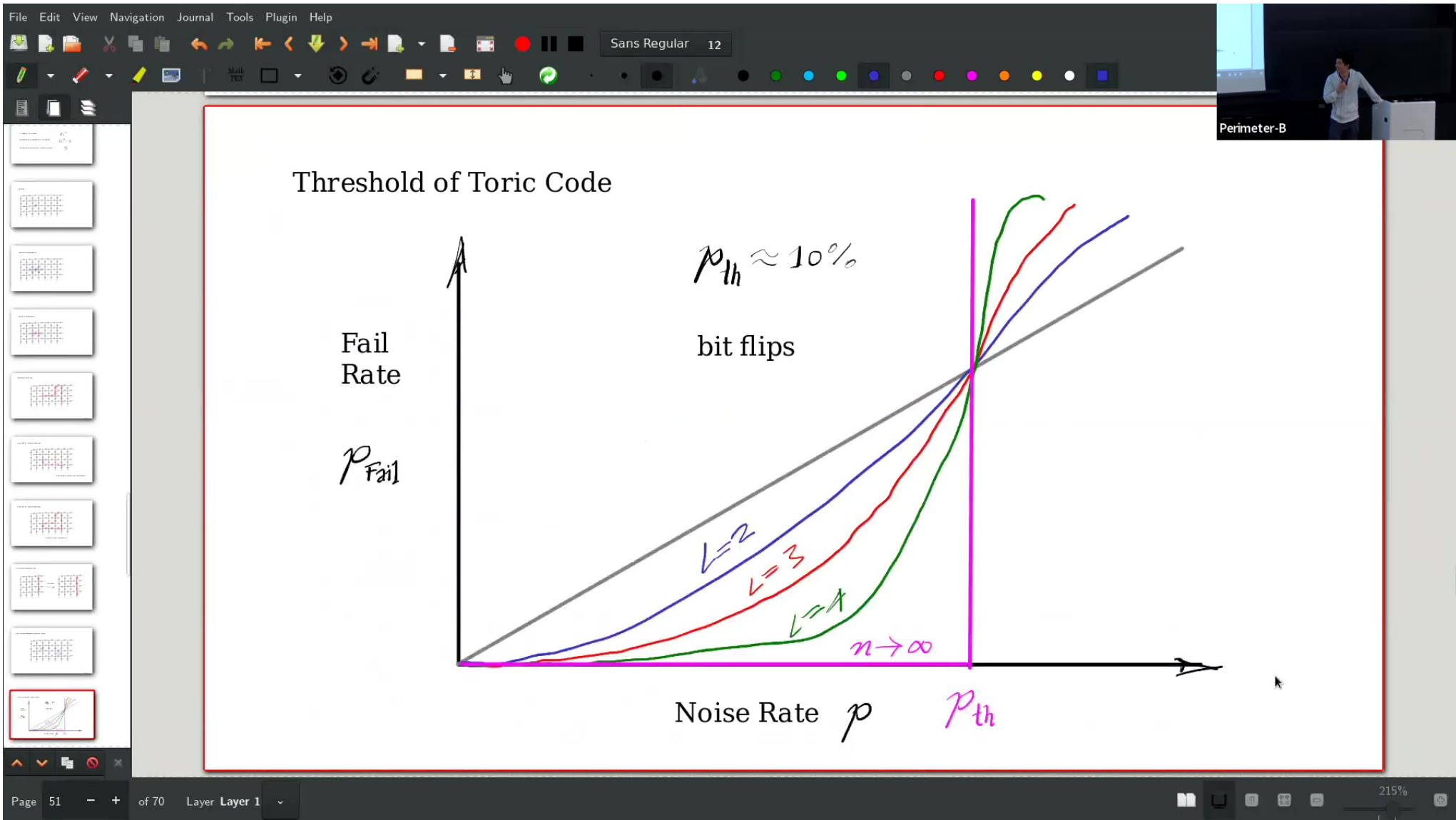
File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Can correct both X and Z errors

Page 50 of 70 Layer Layer 1 215%



File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Thresholds of Toric Code

0.11

0.11

0.11

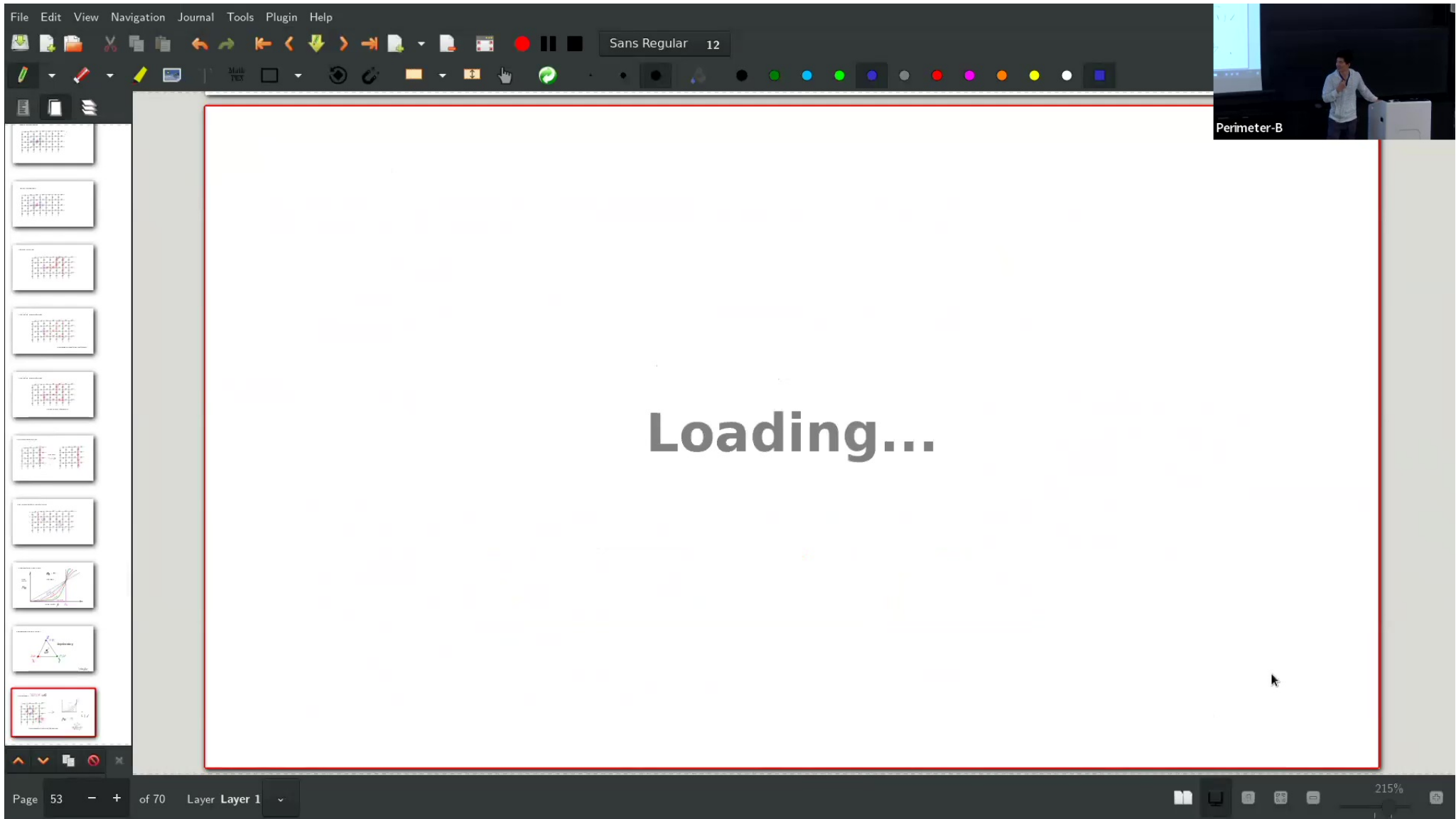
0.50

depolarizing

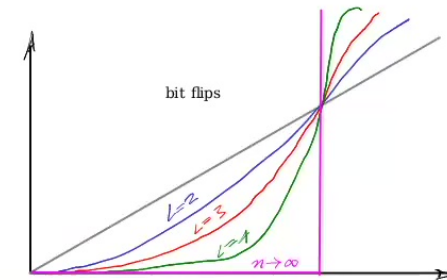
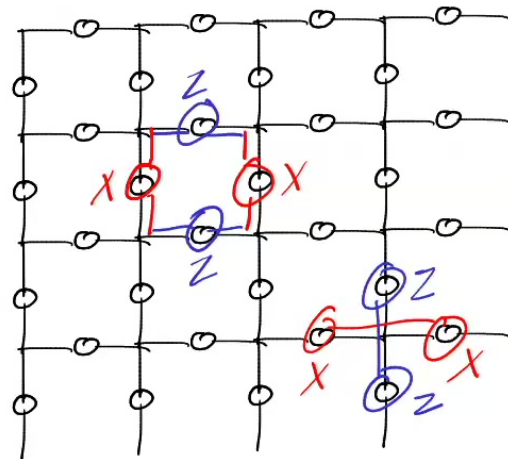
*vbwPen

Page 52 of 70 Layer Layer 1

215%



Cool Hack: "XZZX" code


$$p_{th} \approx 50\%$$


Can optimize code for threshold!

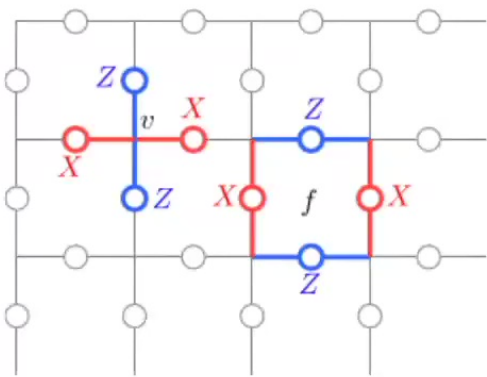


File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

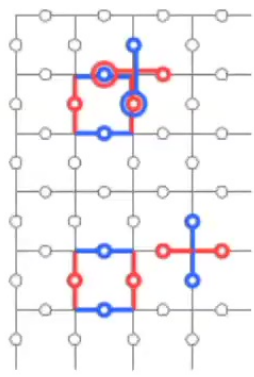
Perimeter-B

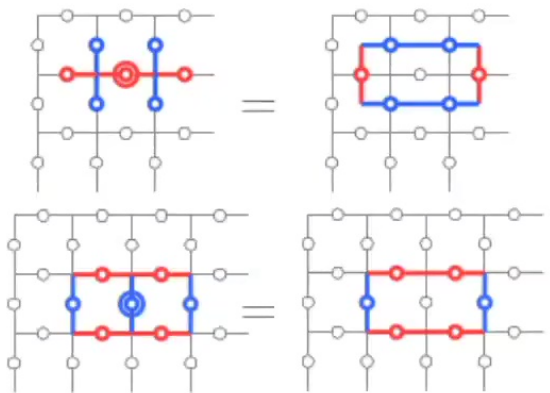
Same Properties as 3D Toric Code



Generators

All commute





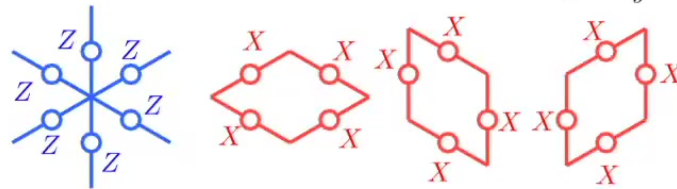
Products contractible loops

Page 54 of 70 Layer Layer 1

215%

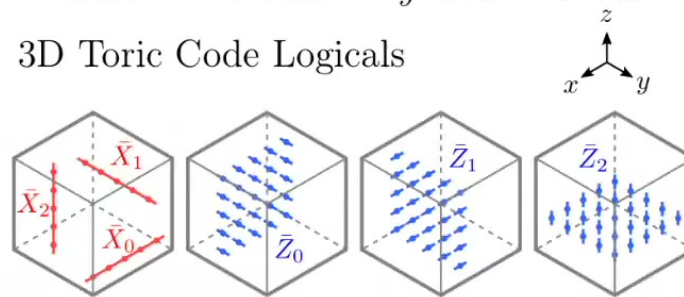
3D Toric Code

3D Toric Code Stabilizers



Vertex	x -face	y -face	z -face
--------	-----------	-----------	-----------

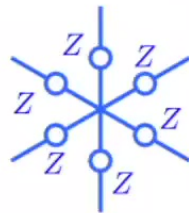
3D Toric Code Logicals



Strings

Minimum Weight Perfect Matching in 3D

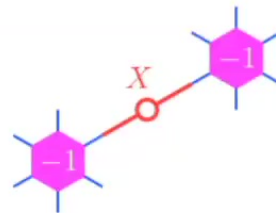
Point Sector Excitations



Stabilizer

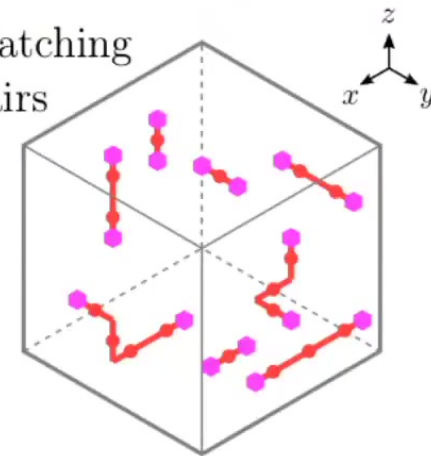


Error



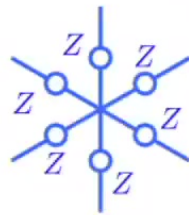
Syndrome pair

Matching pairs



Minimum Weight Perfect Matching in 3D

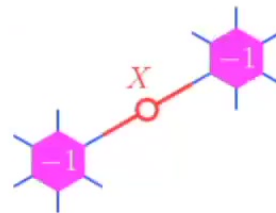
Point Sector Excitations



Stabilizer

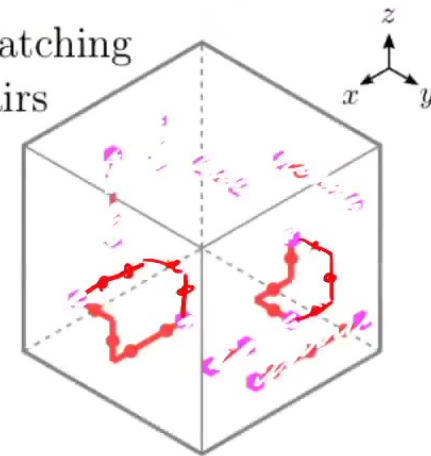


Error



Syndrome pair

Matching pairs



File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Loop Sector Excitations

Stabilizers

Error

Syndromes

Loops

The diagram illustrates the relationship between various concepts in quantum error correction. On the left, a red hexagonal lattice of nodes is labeled "Stabilizers". Above it, a red hexagonal lattice of nodes is labeled "Loop Sector Excitations". In the center, a blue line with a circle and a "Z" is labeled "Error". To the right of the error, a pink diamond shape with a blue line and a "Z" is labeled "Syndromes". Further right, a blue shape is labeled "Loops". On the far right, a 3D cube is shown with a coordinate system (x, y, z) and several blue lines with circles and "Z"s, representing excitations or errors within the cube.

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Diagram illustrating a sequence of transformations on a hexagonal prism structure, showing the progression from a simple prism to a complex, multi-faceted 3D shape.

Page 59 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Question

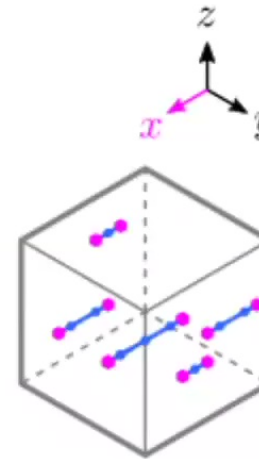
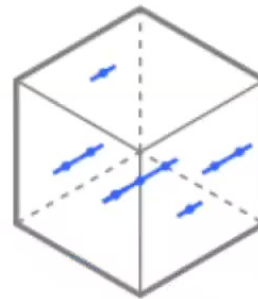
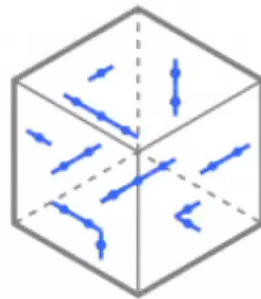
Can we do the same hack to increase the threshold for 3D codes with biased noise?

Yes!

Page 60 of 70 Layer Layer 1 215%

Pure Z Noise Dephasing

Dephasing point sector

Stabilizers 

all noise

detected noise

syndrome

$$[\text{star}, \text{arrow}] = [\text{star}, \text{dot}] = 0$$

pairs on lines

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Page 64 of 70 Layer Layer 1

215%

File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

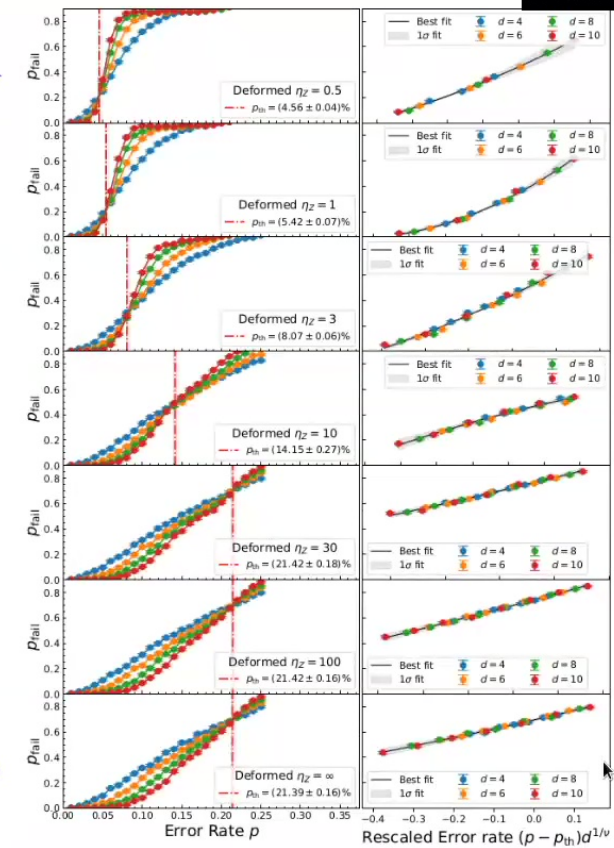
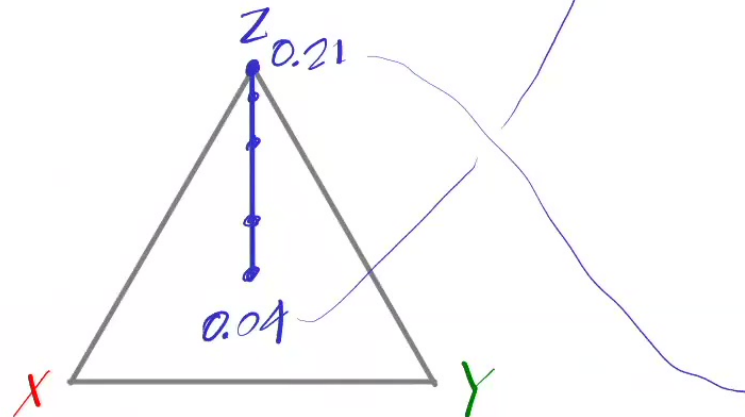
The main visualization area displays two 3D models of a grid-based structure. The left model shows a perspective view of a grid with blue spheres and orange rectangles. The right model shows a similar view from a different angle. The grid is composed of light gray squares, and the blue spheres are connected by lines. The orange rectangles are placed on the grid, some overlapping the spheres. The background is white, and the entire scene is enclosed in a red border.

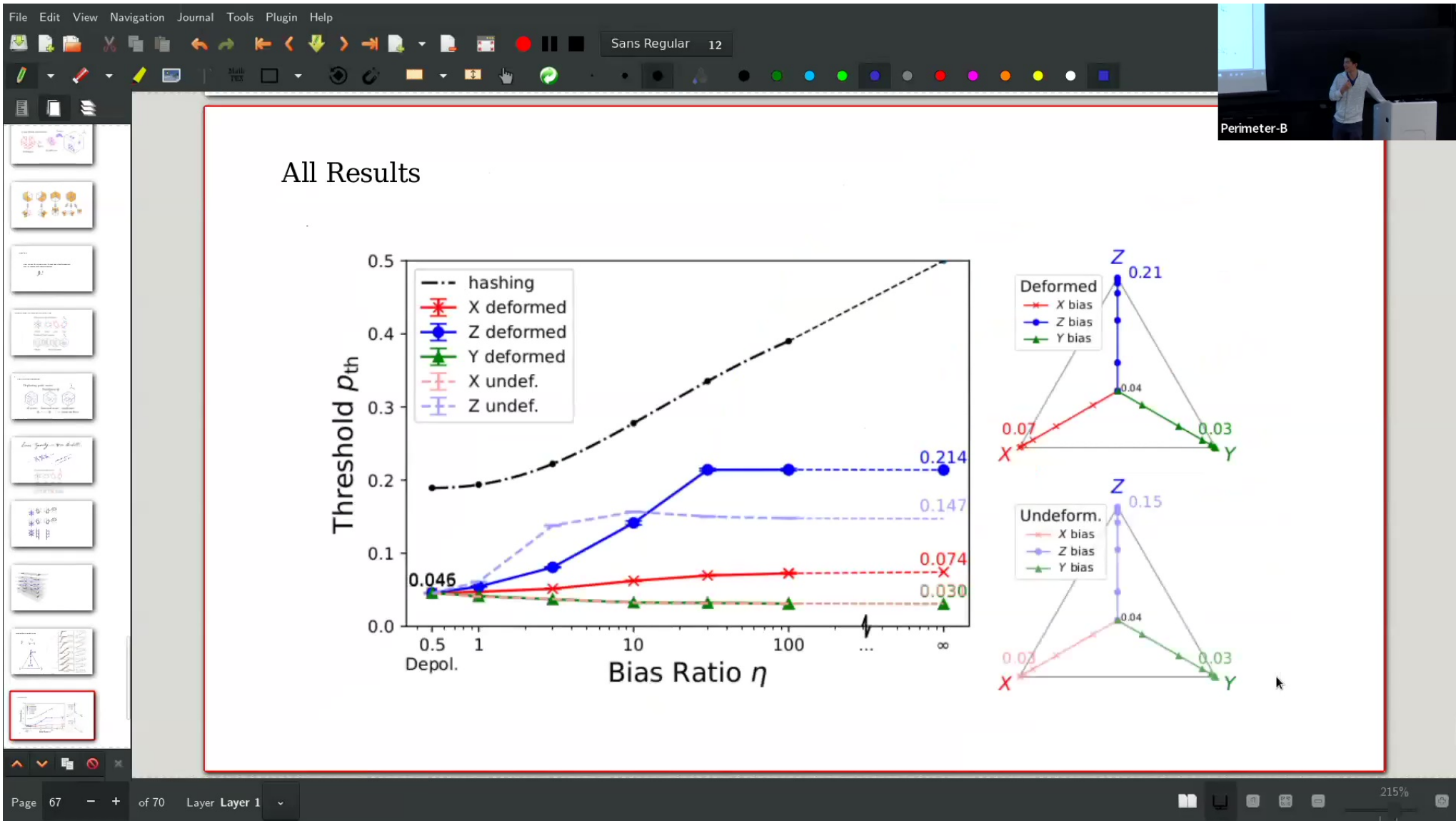
Page 65 of 70 Layer Layer 1

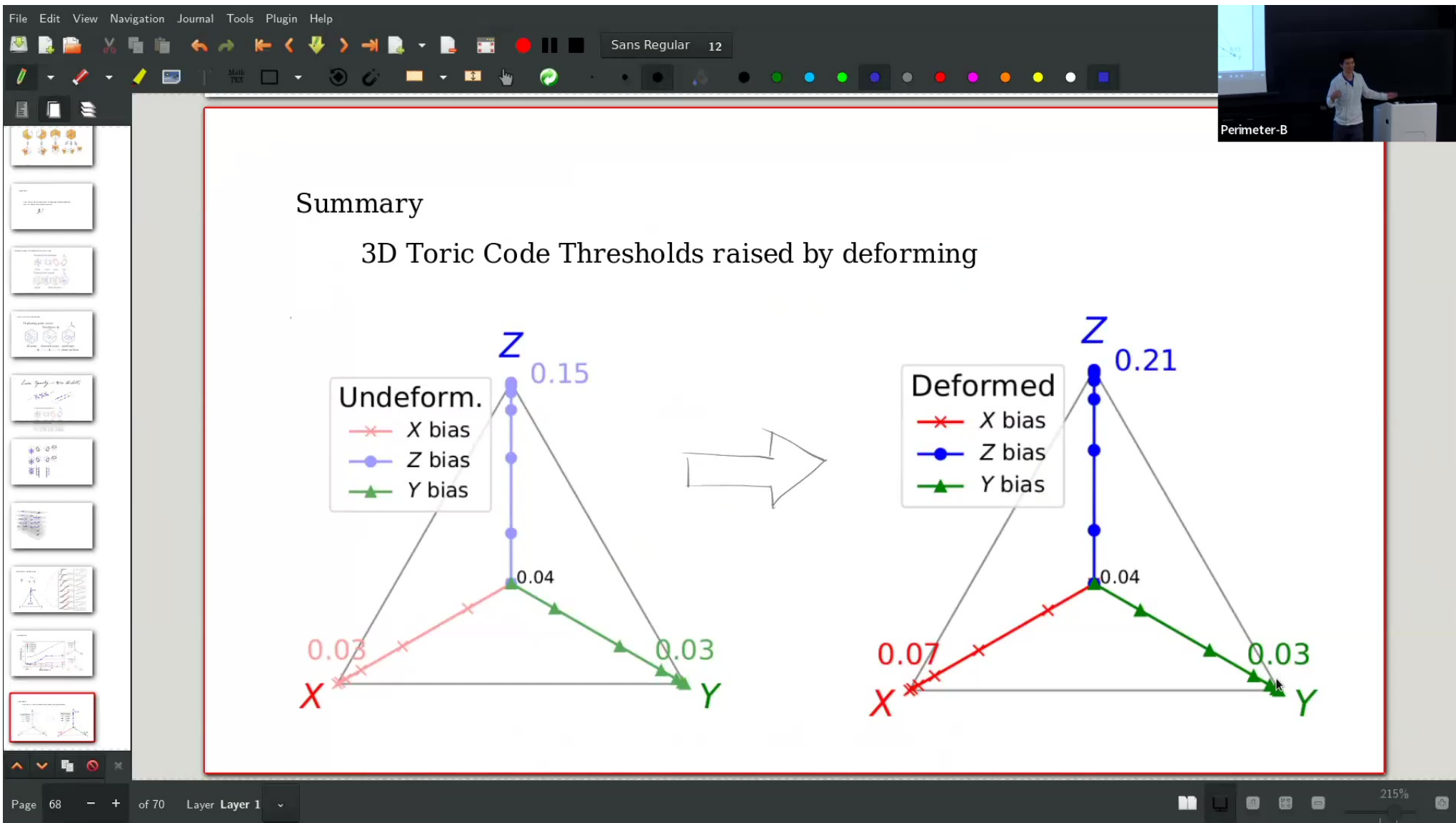
215%

Repeat for Finite Bias

$$\eta = \frac{r_z}{r_x + r_y}$$







File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Future Directions

- Better Decoders -- Tensor network, BP+OSD, etc.
- Optimal Threshold -- Statistical Mechanics mapping
- Noisy measurements -- Fault-tolerant thresholds
- Other Deformations: -- e.g. Rhombic Code, Fracton code

Page 69 of 70 Layer Layer 1 215%

File Edit View Navigation Journal Tools Plugin Help

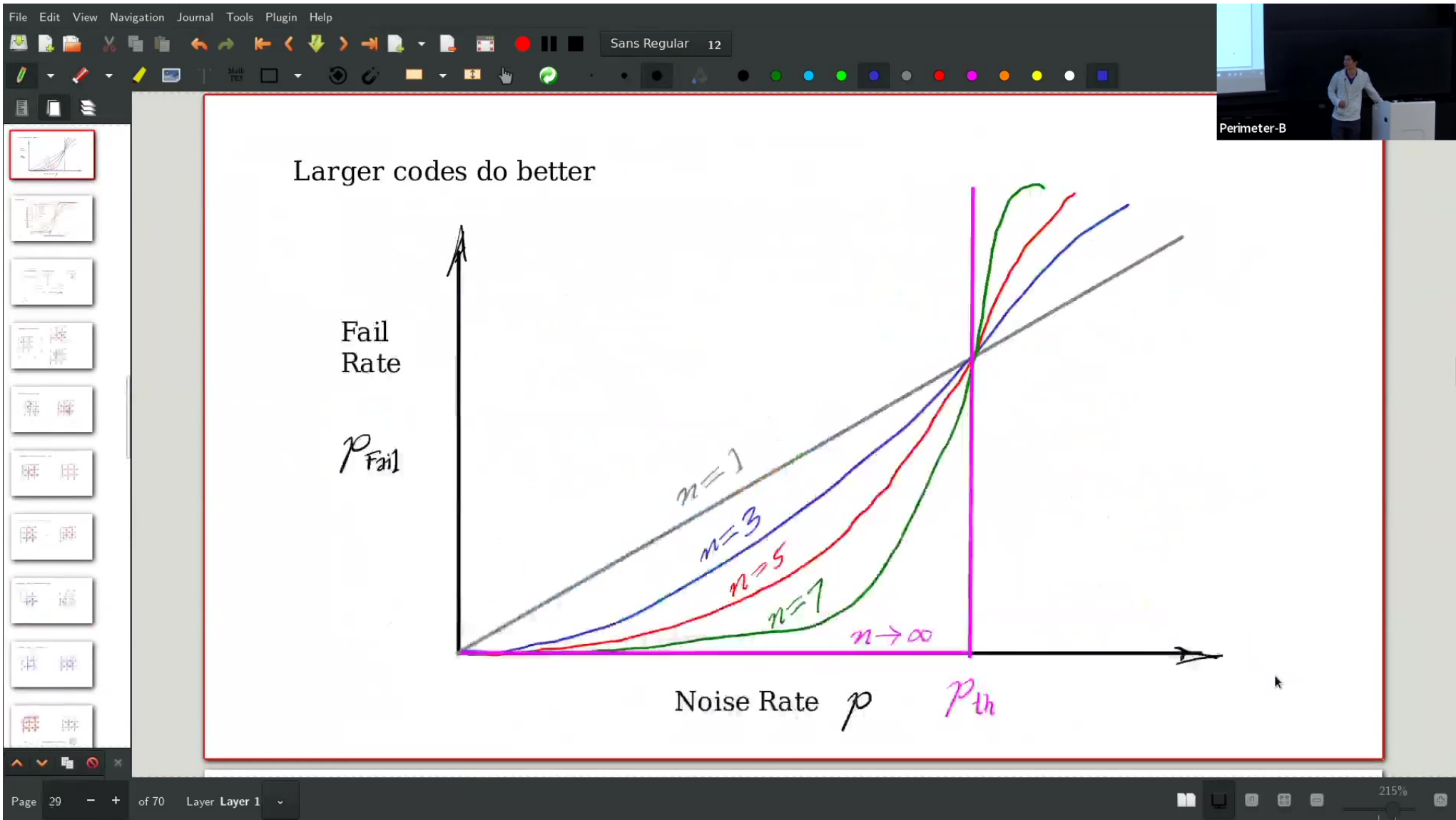
Sans Regular 12

Perimeter-B

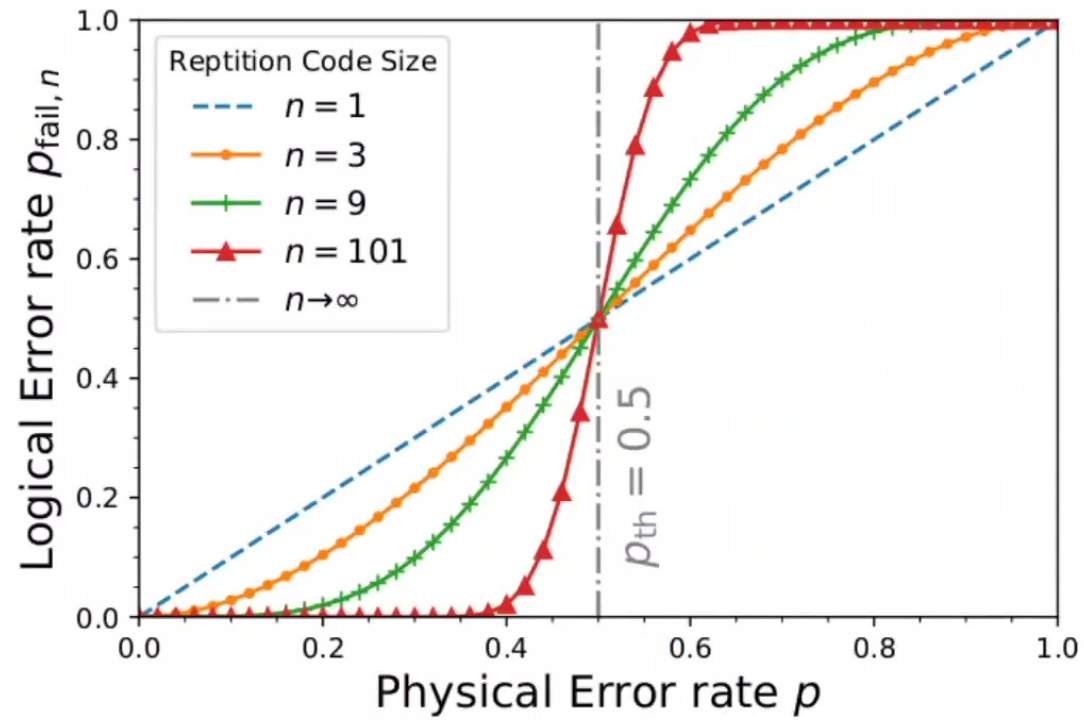
The image shows a hand-drawn diagram on a white background, consisting of two identical 4x4 grids of nodes. Each node is a small circle. The nodes are connected by horizontal and vertical lines. On each horizontal line, there are three arrows pointing to the right. On each vertical line, there are three arrows pointing downwards. The diagram is drawn with black ink.

Page 70 of 70 Layer Layer 1

215%



Threshold

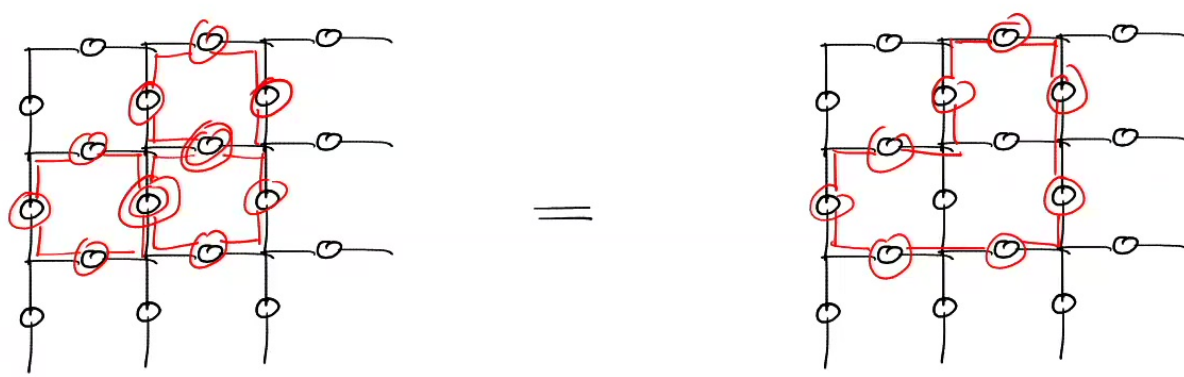


File Edit View Navigation Journal Tools Plugin Help

Sans Regular 12

Perimeter-B

Stabiliser group is contractible loops



Page 35 of 70 Layer Layer 1

215%