Title: Looking for Quantum-Classical Gaps in Causal Structures

Speakers: Marina Maciel Ansanelli

Collection: Young Researchers Conference

Date: June 20, 2022 - 9:10 AM

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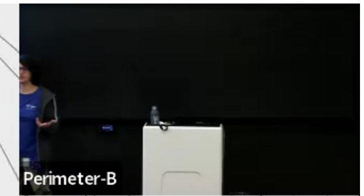
Abstract: A fundamental area in statistical analysis is the study of which causal structures connecting the events of interest can explain the correlations that are observed between them. This is done through the falsification of invalid causal models. Our causal structure might posit the existence of hidden (unobserved) causes between the observed events. For example, if we see a positive correlation between the numbers of shark attacks and ice cream sales, we do not expect to explain it by a direct causal influence between these two things; instead, there should be a hidden common cause (for example, the Summer) that explains the correlation. Physicists also have a vested interest in falsifying causal hypotheses involving hidden variables. Bell's Theorem, for example, highlights the failure of many such classical causal hypotheses to explain the correlations predicted by quantum theory. In the scenario which Bell considered, if instead of treating the unobserved causes of classical random variables we treat them as potentially entangled quantum systems, we can explain a strictly larger set of correlations. Out project explores a simple but difficult question: In what other causal structures this also happens? In other words, for a given causal hypothesis, would the set of correlations it can explain expand if we relax our assumptions regarding posited unobservable systems to allow for shared entanglement? By a series of tricks developed during the PSI Winter School, we found that allowing for quantum causes makes an operational difference in a large number of causal hypotheses involving four observed variables. This work is of general interest as it generalizes Bell's Theorem: it exposes (qualitatively novel?!) advantages afforded by quantum theory over classical models. Bell's Theorem has proven crucially insightful in efforts to provide a causal accounting of quantum theory, and has inspired a plethora of quantum information theoretic protocols; similar dividends may be implicitly suggested by this work.



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01

Motivation: Strong non-classicality

Intrinsically Quantum Properties?

- Superposition
  Double slit experiment
  Interferometer
- Non-Commutativity of Measurements
- Uncertainty Principle
- Entanglement

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- Superposition
  Double slit experiment
  Interferometer
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- Entanglement
- No-Cloning Theorem
- No-Broadcasting
- Monogamy of Pure Entanglement
- Teleportation Protocol

Not

Intrinsically Quantum Properties

All of them have analogues in a classically-simulable model called "Spekkens Toy Theory"

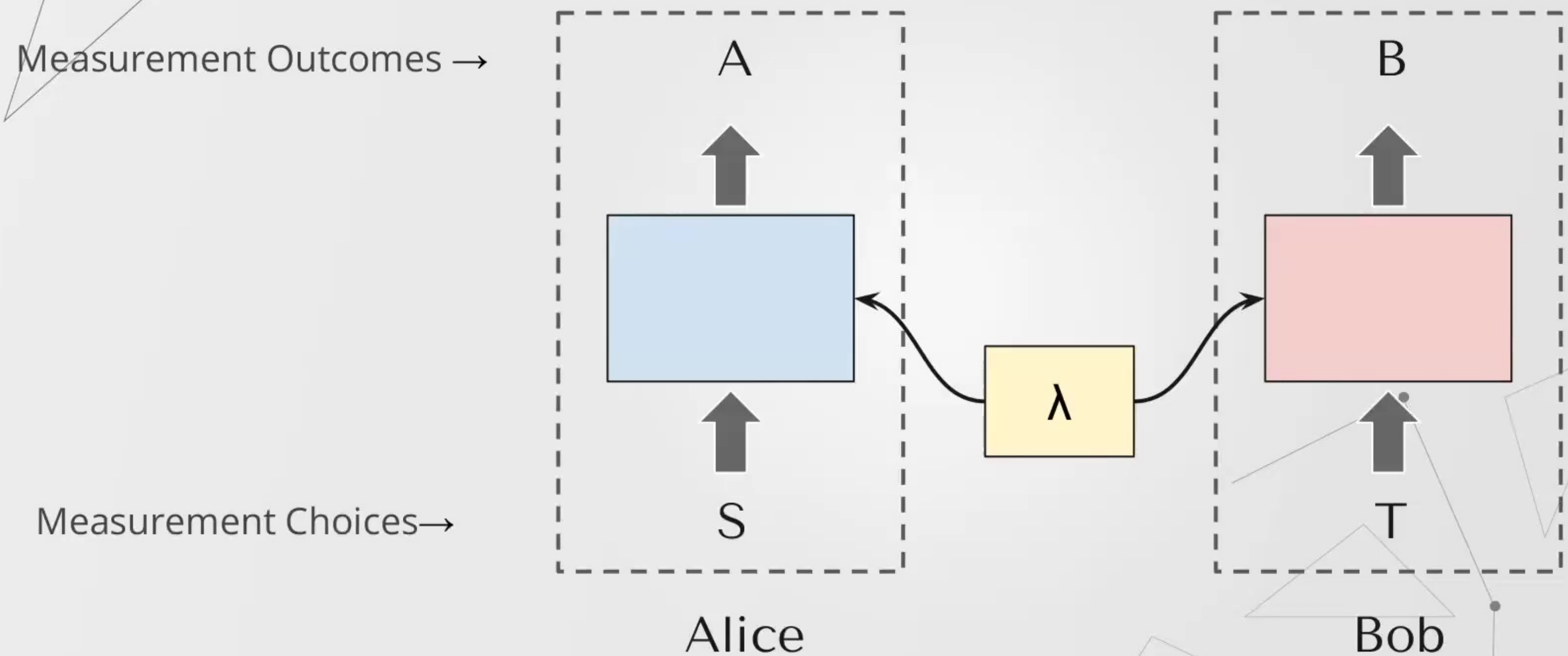
- Superposition
  Double slit experiment
  Interferometer
- Non-Commutativity of Measurements
- Uncertainty Principle
- Entanglement
- No-Cloning Theorem
- No-Broadcasting
- Monogamy of Pure Entanglement
- Teleportation Protocol

How?
Using a restriction in our knowledge about the system

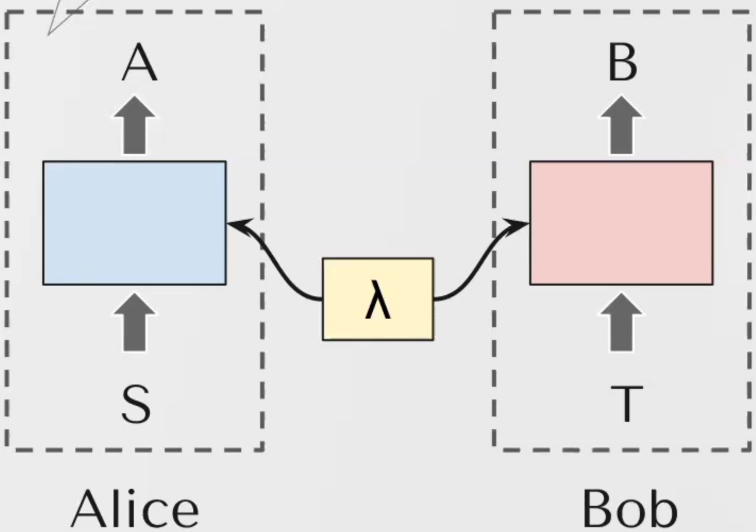
See:

- R.W. Spekkens: "In defense of the epistemic view of quantum states: a toy theory"; arxiv 0401052
- Catani et.al: "Why interference phenomena do not capture the essence of quantum theory"; arxiv 2111.13727

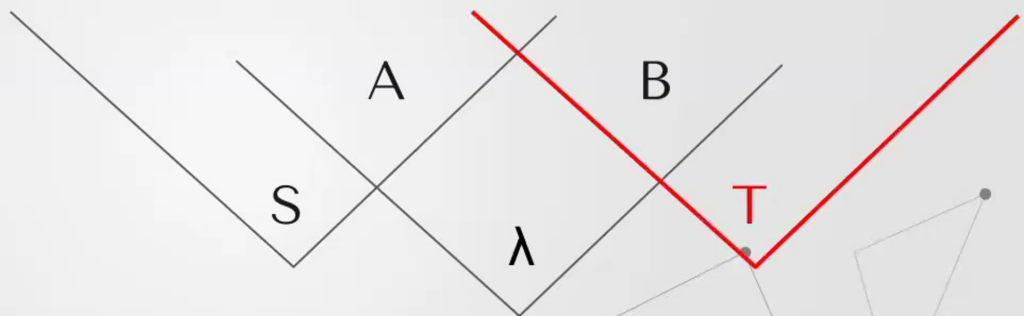
Bell-CHSH Scenario



Bell-CHSH Scenario

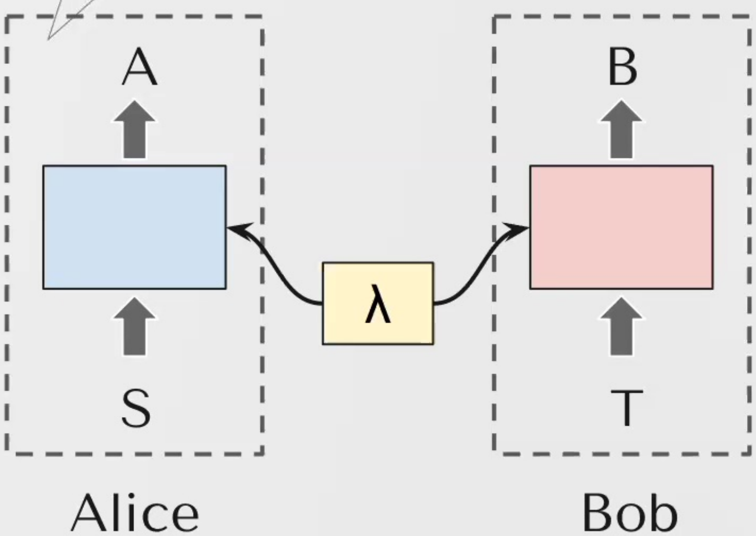


Space-like Separation



$A \perp BT \mid S\lambda$

Bell-CHSH Scenario



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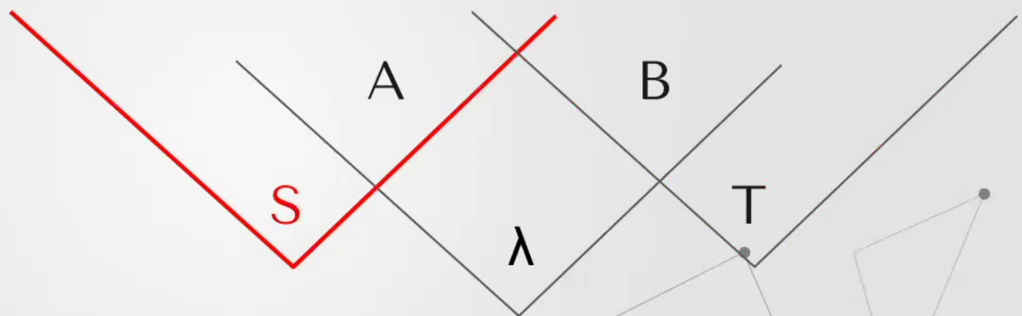
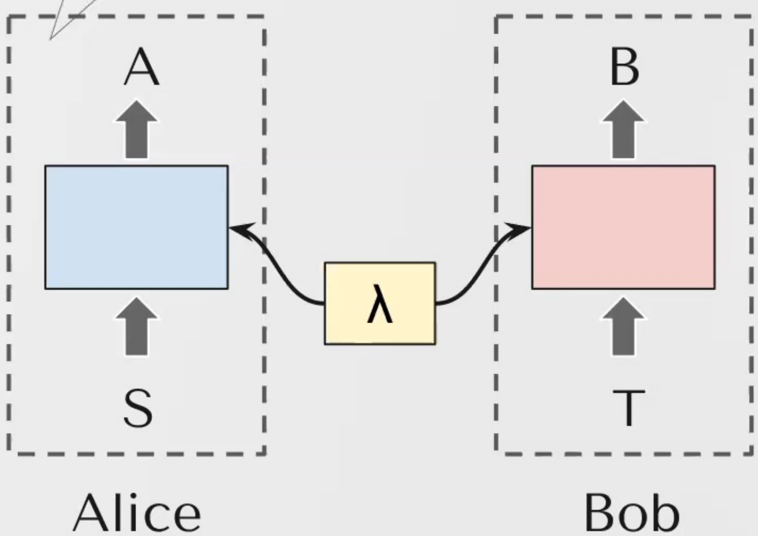


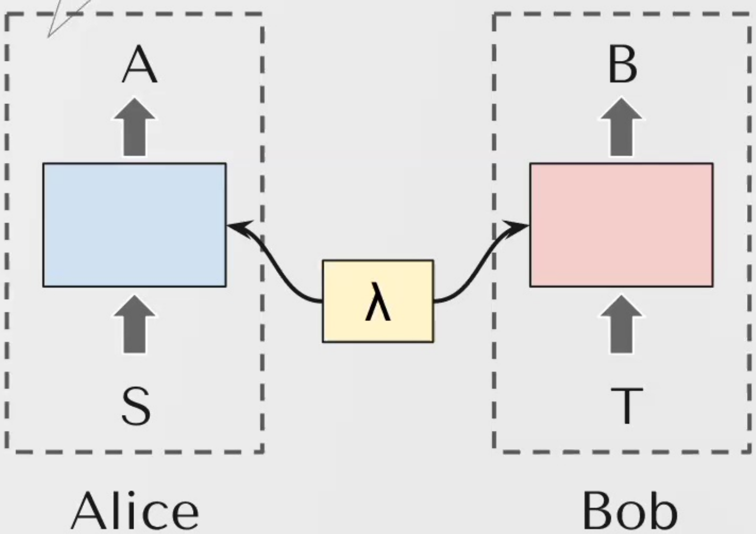
$$A \perp BT \mid S\lambda \text{ and } B \perp AS \mid T\lambda$$

Bell-CHSH Scenario



Alice and Bob are free to choose S and T

Bell-CHSH Scenario

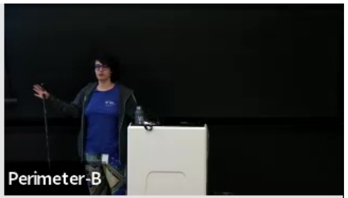


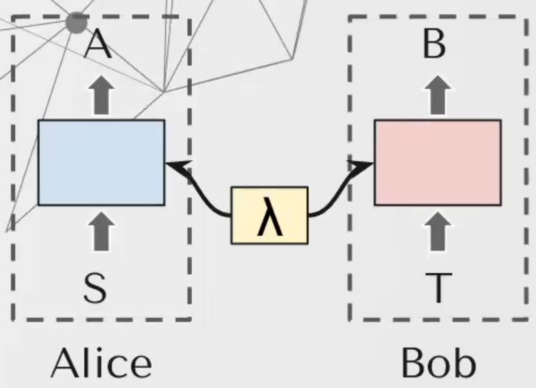
Alice and Bob are free to choose S and T

$$S \perp T\lambda \text{ and } T \perp S\lambda$$

(No Superdeterminism)

Bell-CHSH Scenario





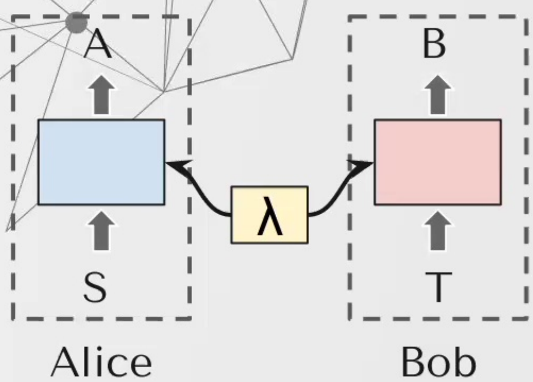
$$A \perp BT \mid S\lambda \text{ and } B \perp AS \mid T\lambda$$

(Local Causality)

$$S \perp T\lambda \text{ and } T \perp S\lambda$$

(No Superdeterminism)

Bell-CHSH Scenario



λ is a classical random variable

$$A \perp BT \mid S\lambda \text{ and } B \perp AS \mid T\lambda$$

(Local Causality)

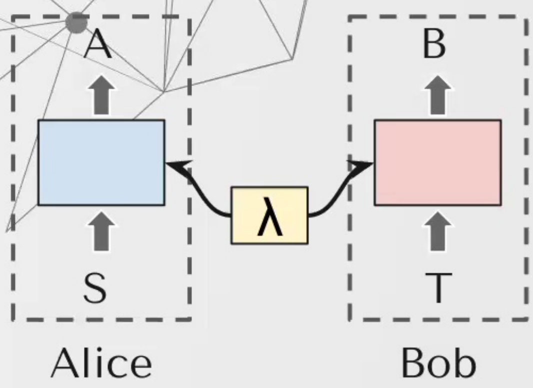
$$S \perp T\lambda \text{ and } T \perp S\lambda$$

(No Superdeterminism)

$$\frac{1}{4}\sum_{a=b} P_{AB|ST}(ab|00) + \frac{1}{4}\sum_{a=b} P_{AB|ST}(ab|01) + \frac{1}{4}\sum_{a=b} P_{AB|ST}(ab|10) + \frac{1}{4}\sum_{a\neq b} P_{AB|ST}(ab|11) \leq 3/4$$

(Bell's Inequality)

Bell-CHSH Scenario



λ is a classical random variable

$$A \perp BT \mid S\lambda \;\text{ and }\; B \perp AS \mid T\lambda$$

(Local Causality)

$$S \perp T\lambda \;\text{ and }\; T \perp S\lambda$$

(No Superdeterminism)

QM and experiments violate this inequality

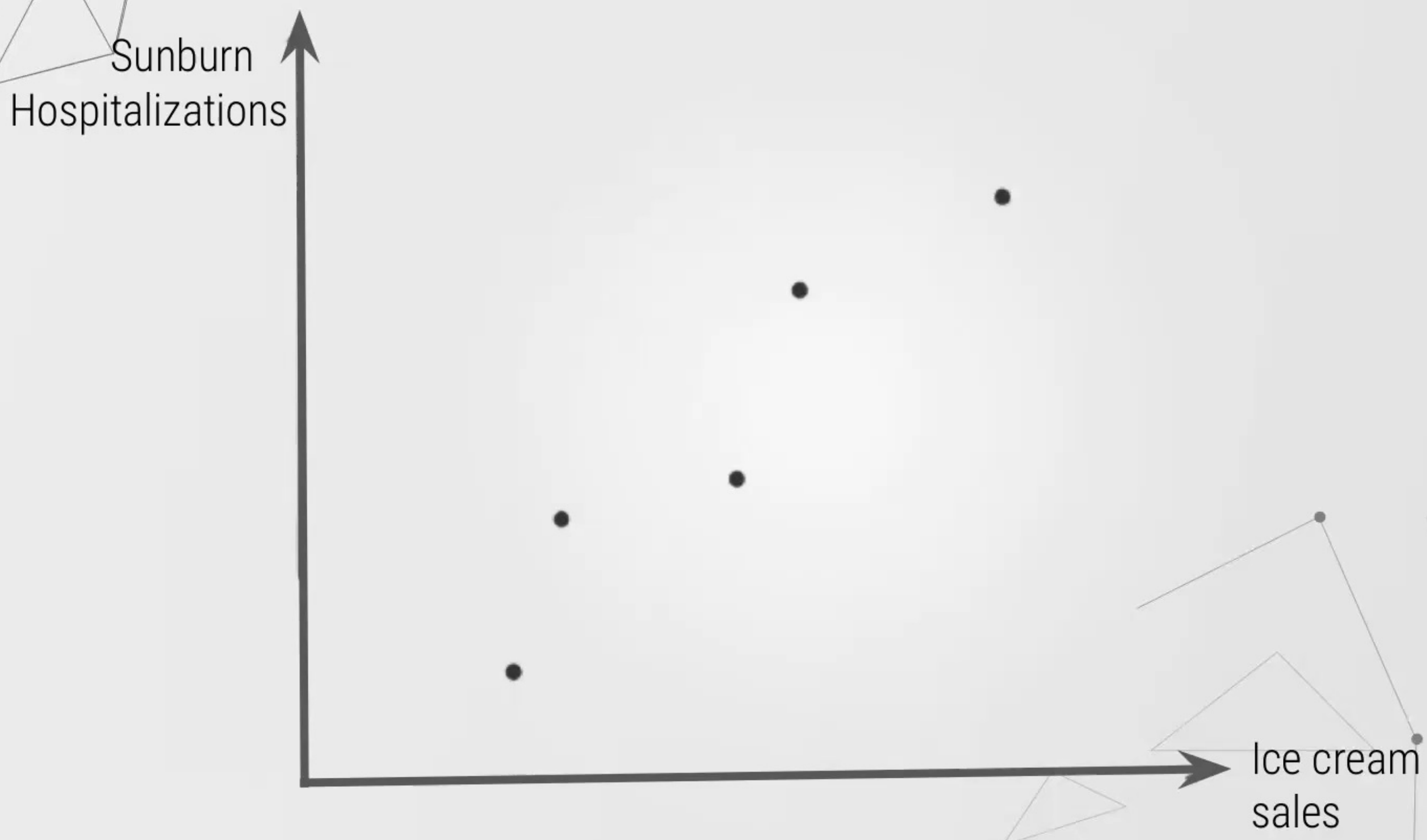
$$\frac{1}{4}\sum_{a=b} P_{AB|ST}(ab|00) + \frac{1}{4}\sum_{a=b} P_{AB|ST}(ab|01) +$$
$$\frac{1}{4}\sum_{a=b} P_{AB|ST}(ab|10) + \frac{1}{4}\sum_{a\neq b} P_{AB|ST}(ab|11) \leq 3/4$$

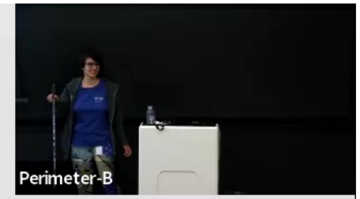
(Bell's Inequality)

02

Causal Inference

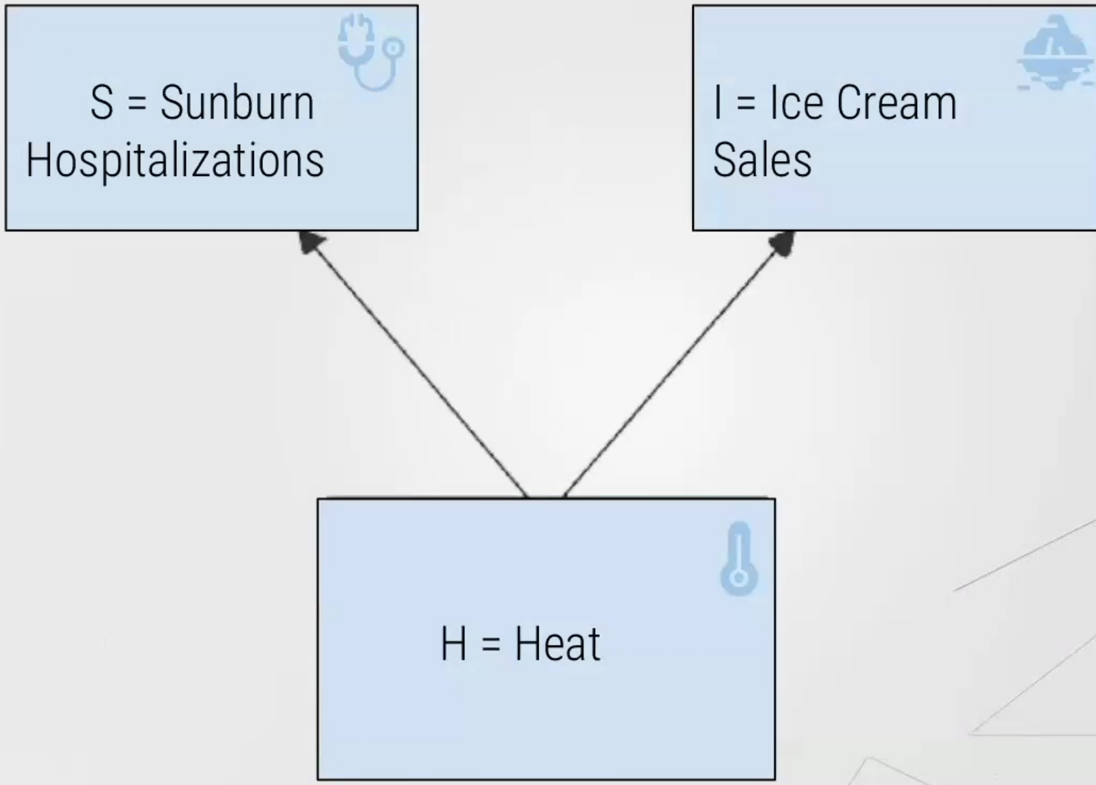
Causal Inference





Causal Inference

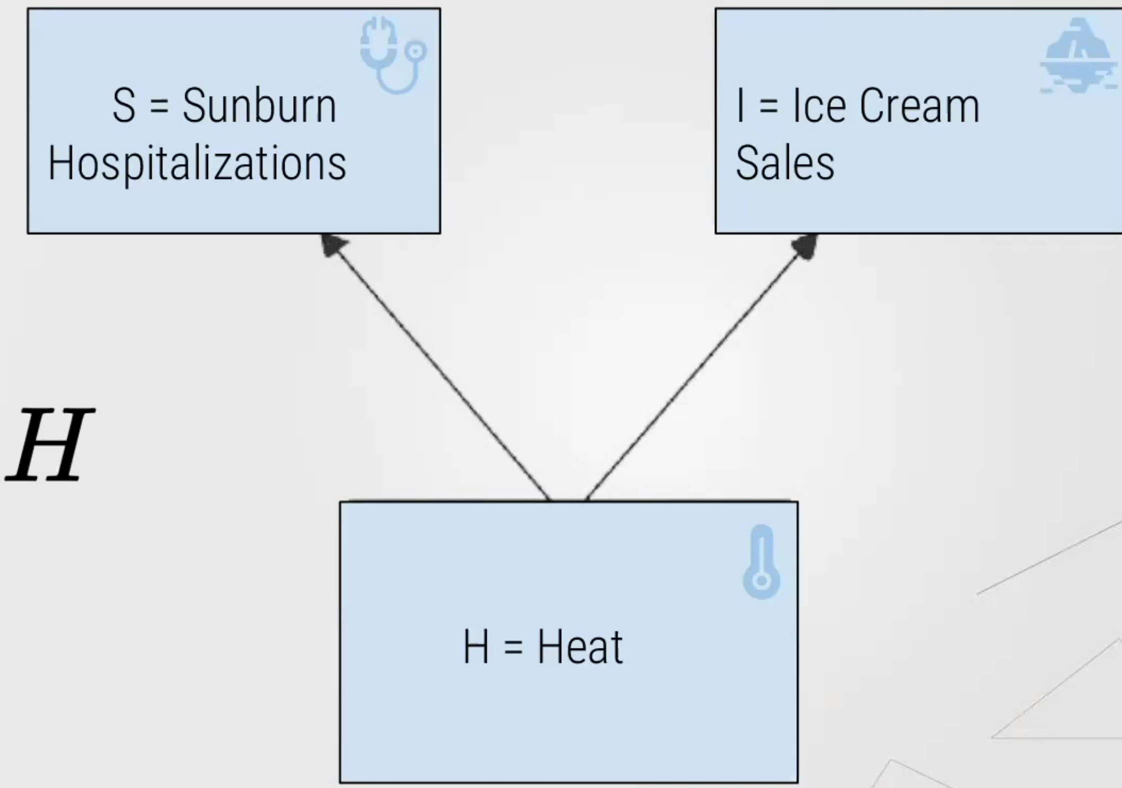
Correlation $\nRightarrow$ Causation



H is a common cause

Causal Inference

Constraints on the Probability Distributions

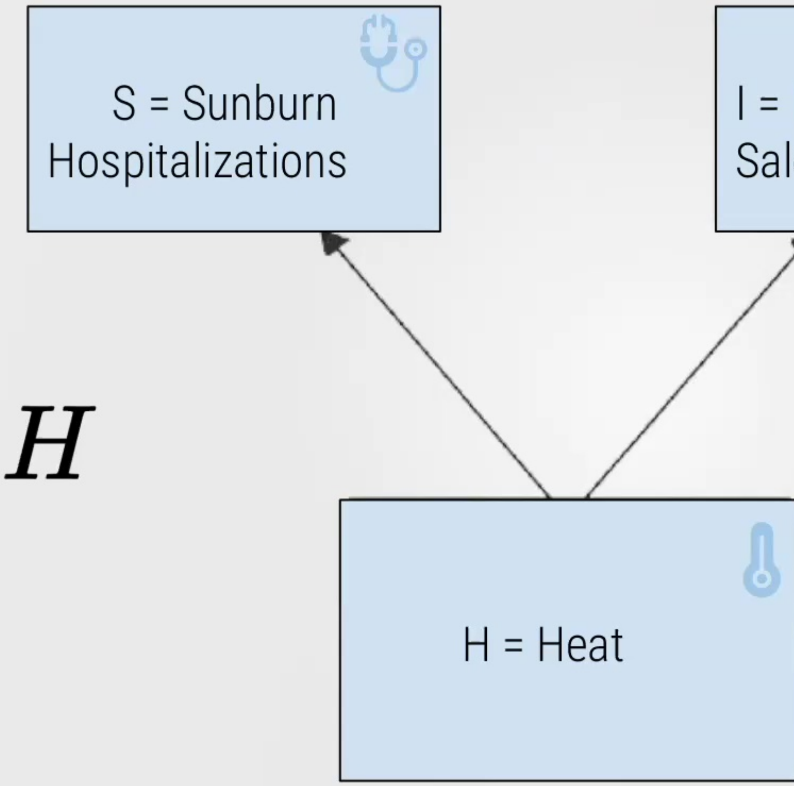


H is a common cause

$$S \perp I \mid H$$

Causal Inference

Constraints on the Probability Distributions



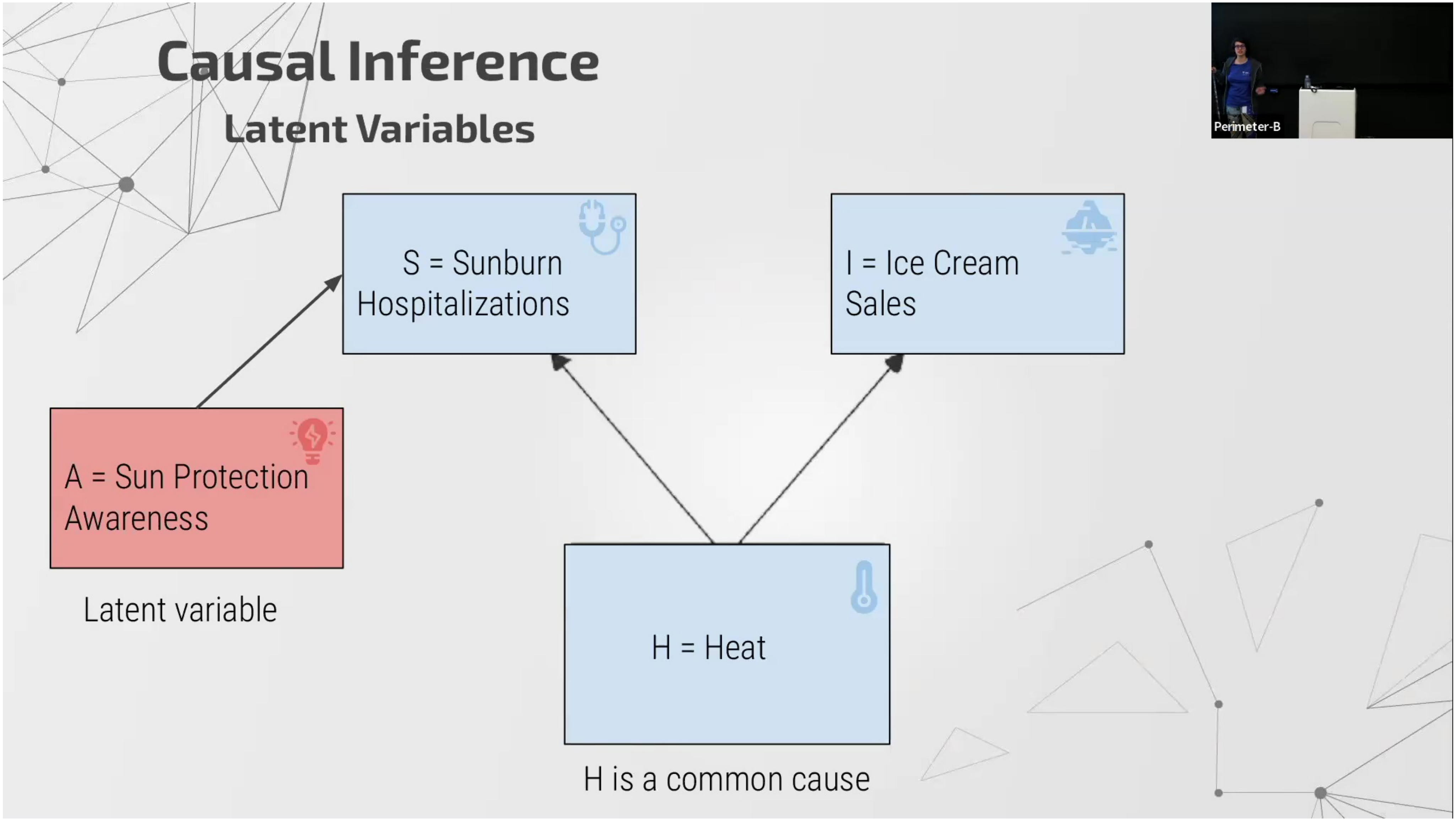
H is a common cause

$$S \perp I \mid H$$

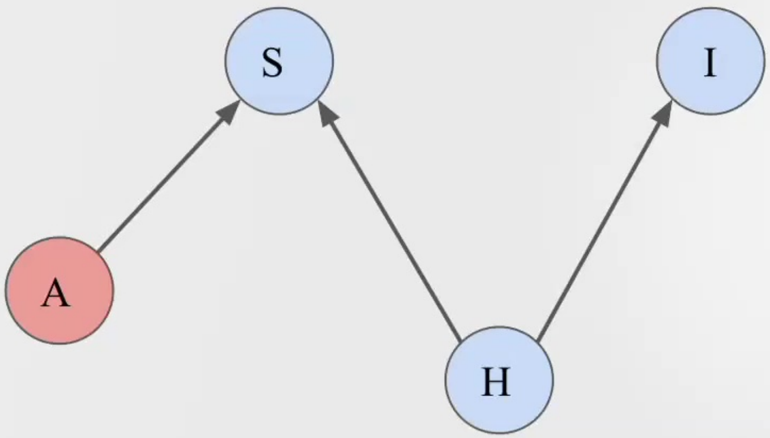
If we could measure H:

$$P(ISH) = P(I|H)P(S|H)P(H)$$

Restriction on the probability distributions that are compatible with this causal structure


Causal Inference
Latent Variables
S = Sunburn Hospitalizations
I = Ice Cream Sales
A = Sun Protection Awareness
Latent variable
H = Heat
H is a common cause
Perimeter-B

Causal Inference



DAG = Directed Acyclic Graph

Applications to Classical Data Analysis

Article | Open Access | Published: 04 March 2021

Masks and distancing during COVID-19: a causal framework for imputing value to public-health interventions

Andres Babino & Marcelo O. Magnasco

Scientific Reports **11**, Article number: 5183 (2021) | Cite this article

ELSEVIER

Journal of Econometrics
Volume 220, Issue 1, January 2021, Pages 23-62

Causal impact of masks, policies, behavior on early covid-19 pandemic in the U.S.

Victor Chernozhukov [a], Hiroyuki Kasahara [b], Paul Schrimpf [b]

Article | Open Access | Published: 11 August 2020

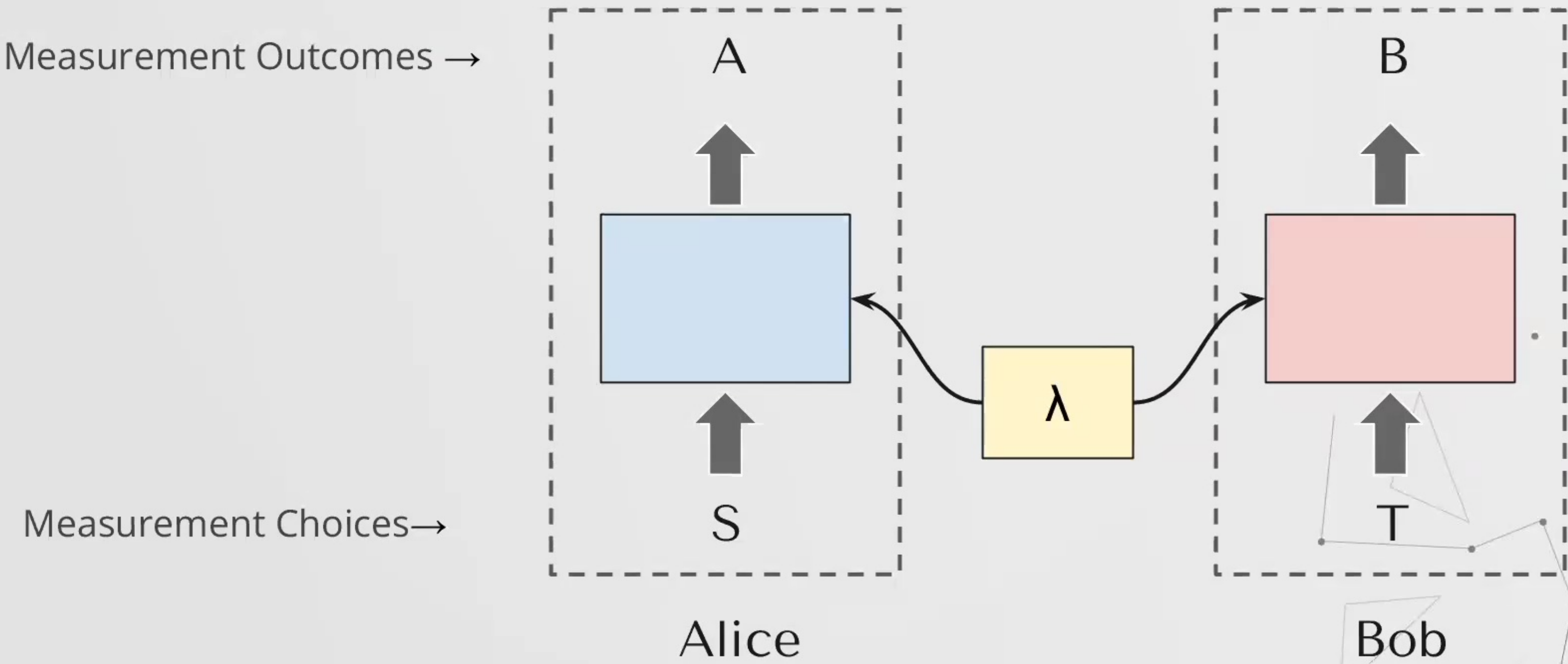
Improving the accuracy of medical diagnosis with causal machine learning

Jonathan G. Richens, Ciarán M. Lee & Saurabh Johri

Nature Communications **11**, Article number: 3923 (2020) | Cite this article

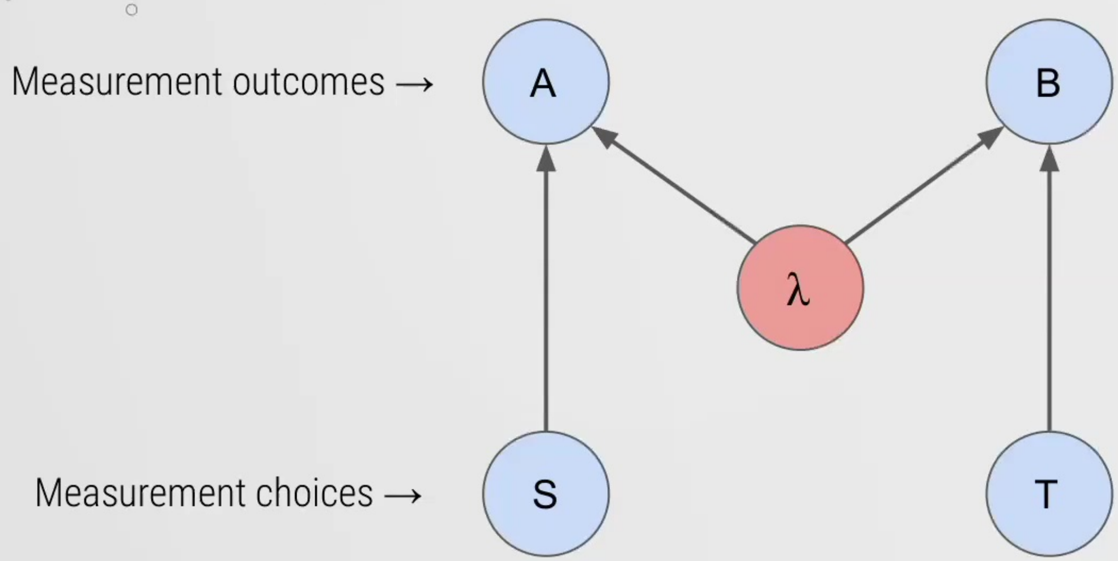
Applications to Quantum Foundations

Bell-CHSH Scenario



Applications to Quantum Foundations
The Bell's Causal Structure

C.J. Wood and R. 1208.4119 (2015)



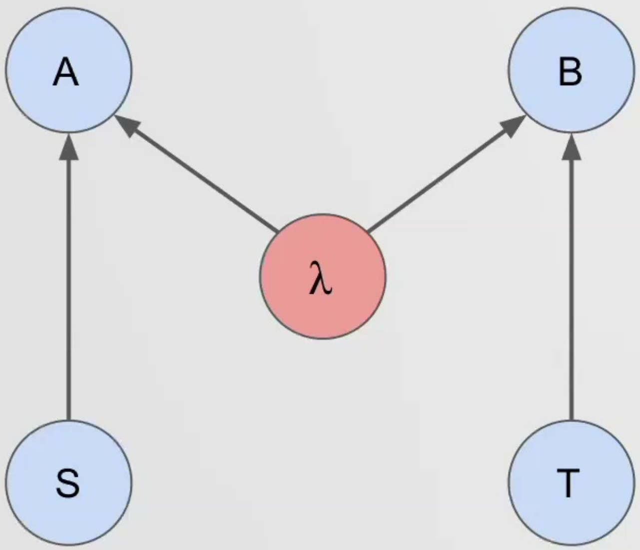
Bell's Causal Structure

Nothing here is quantum, every node is associated with a *classical random variable*

Applications to Quantum Foundations

The Bell's Causal Structure

C.J. Wood and R.
1208.4119 (2015)



Bell's Causal Structure

Constraints:

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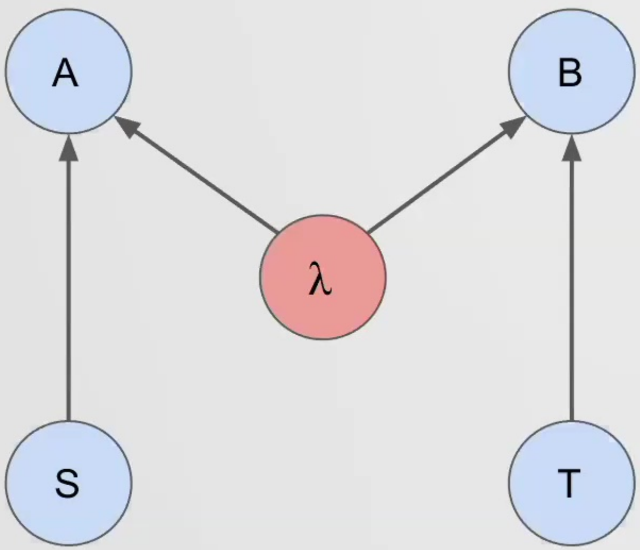
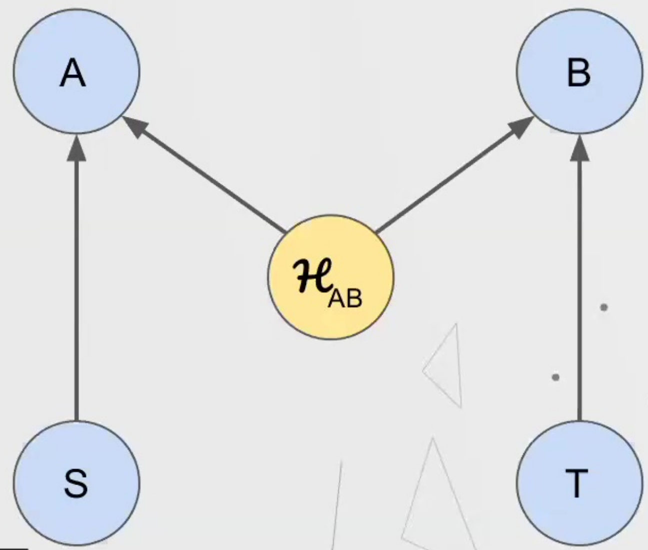


λ is a classical random variable

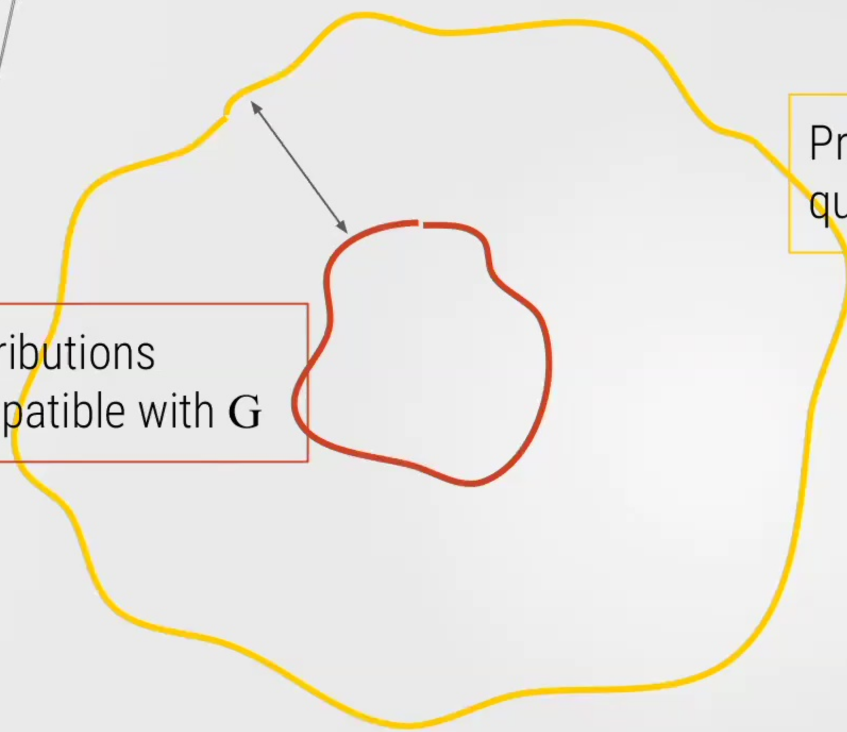
Quantum - Classical gap



$\mathcal{H}_{AC}$ is a quantum system

Applications to Quantum Foundations

QC Gaps



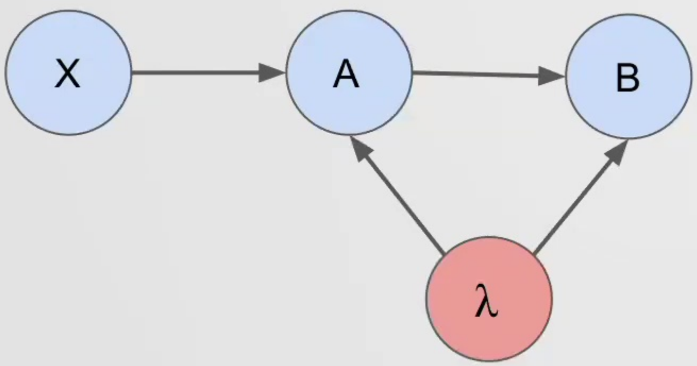
Probability distributions quantumly compatible with $\mathbf{G}$

Probability distributions classically compatible with $\mathbf{G}$

Inequalities are quantumly violated!

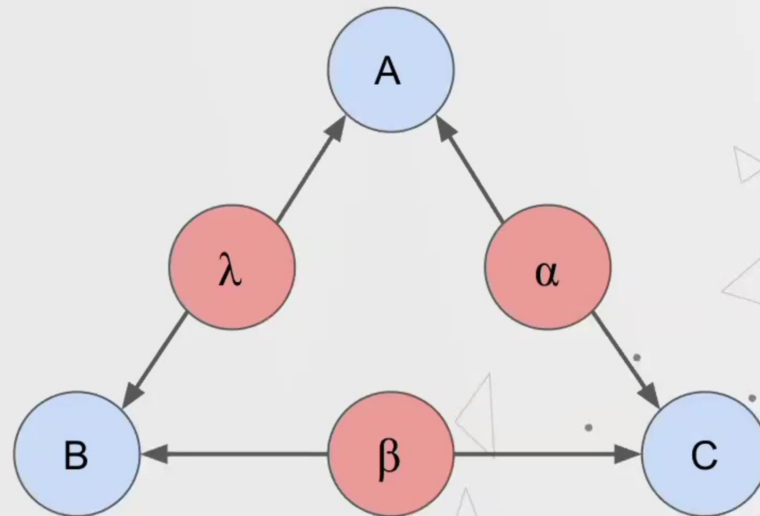
Applications to Quantum Foundations

Other Structures that have QC Gaps



Instrumental Scenario

T. Van Himbeeck et.al.: arxiv 1804.04119 (2019)

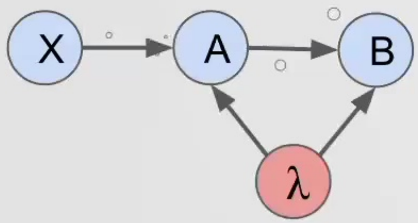


Triangle Scenario

E. Wolfe et.al. : arxiv 1909.10519 (2021)

Applications to Quantum Foundations

Other Structures that have QC Gaps



Instrumental Scenario

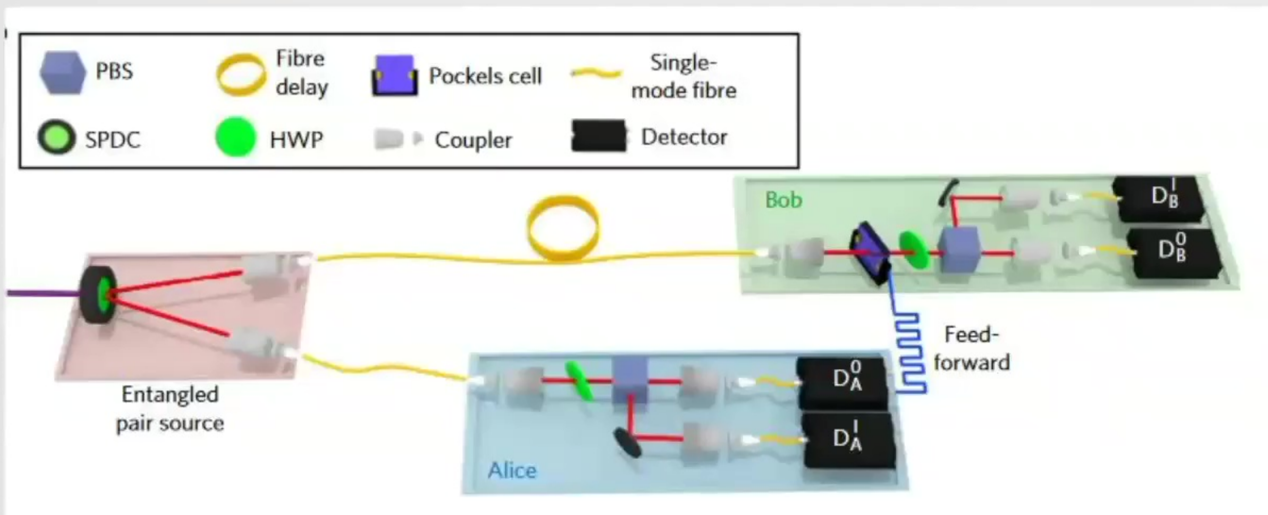

nature physics

ARTICLES

https://doi.org/10.1038/s41567-017-0008-5

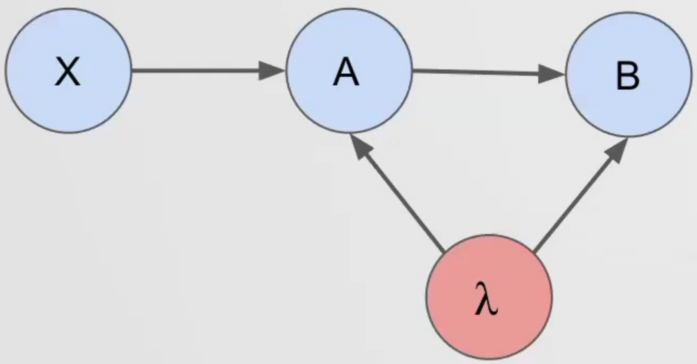
Quantum violation of an instrumental test

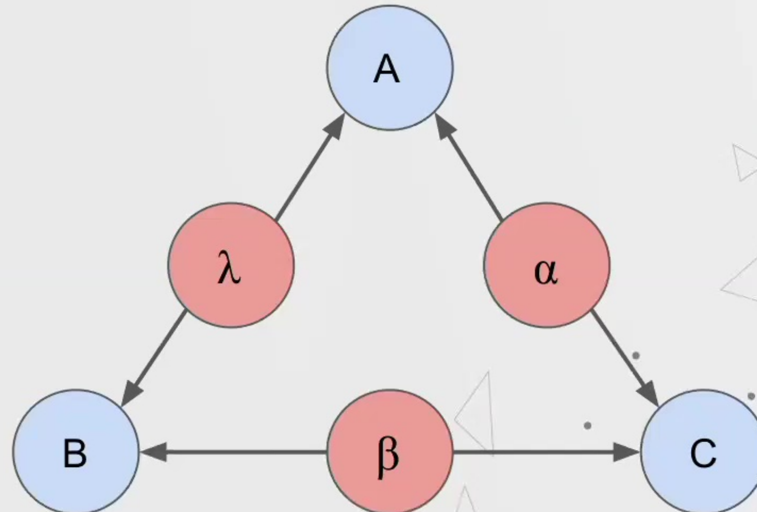
Rafael Chaves[1*], Gonzalo Carvacho[2], Iris Agresti[2], Valerio Di Giulio[2], Leandro Aolita[3], Sandro Giacomini[2] and Fabio Sciarrino[2*]



Applications to Quantum Foundations

Other Structures that have QC Gaps





Instrumental Scenario

T. Van Himbeeck et.al.: arxiv 1804.04119 (2019)

Triangle Scenario

E. Wolfe et.al. : arxiv 1909.10519 (2021)

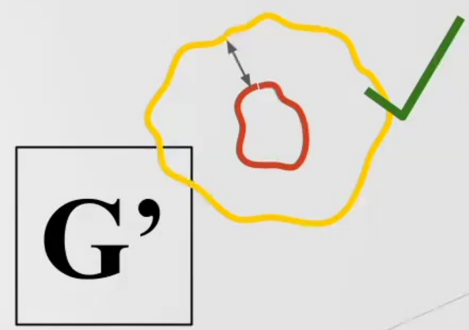
03

Tricks to prove QC Gaps in other causal structures

Results from the Winter School

Tricks to prove QC Gaps

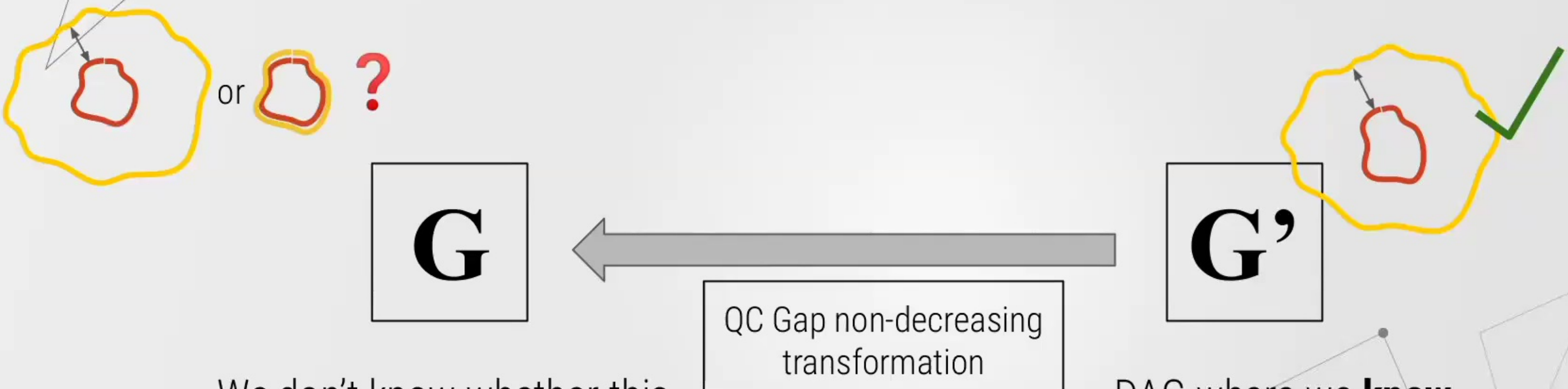
The idea



DAG where we **know** there is a QC Gap

Tricks to prove QC Gaps

The idea

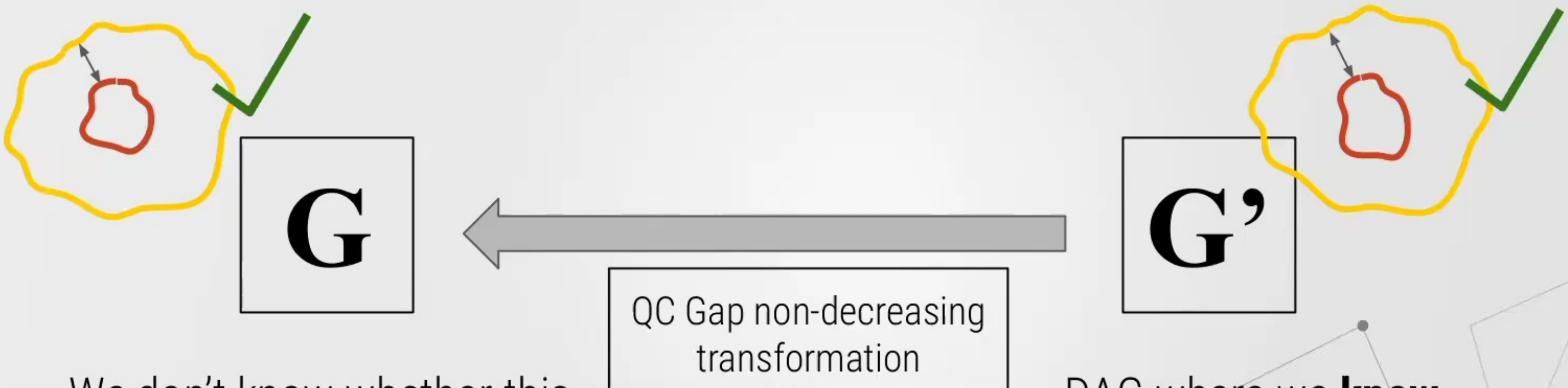


We don’t know whether this DAG has a QC Gap

DAG where we **know** there is a QC Gap

Tricks to prove QC Gaps

The idea



We don’t know whether this DAG has a QC Gap

DAG where we **know** there is a QC Gap

Tricks to prove QC Gaps

The idea

QC Gap non-increasing transformation

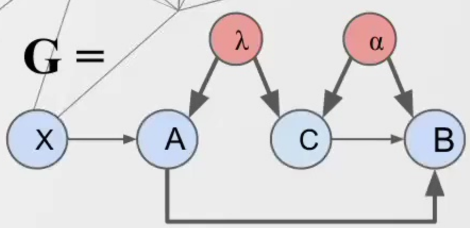
$$\textbf{G} \longrightarrow \textbf{G'}$$

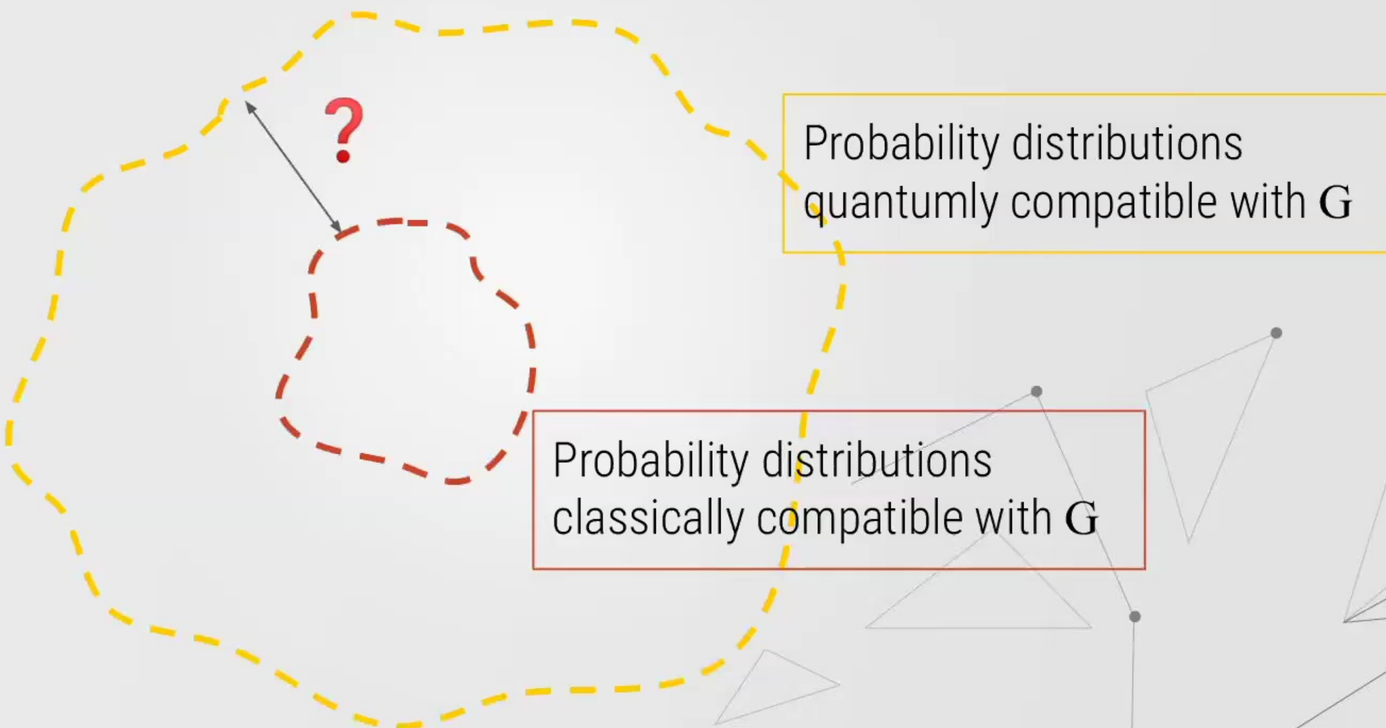
We don't know whether this DAG has a QC Gap

DAG where we **know** there is a QC Gap

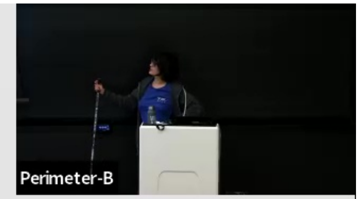
Tricks to prove QC Gaps

Example



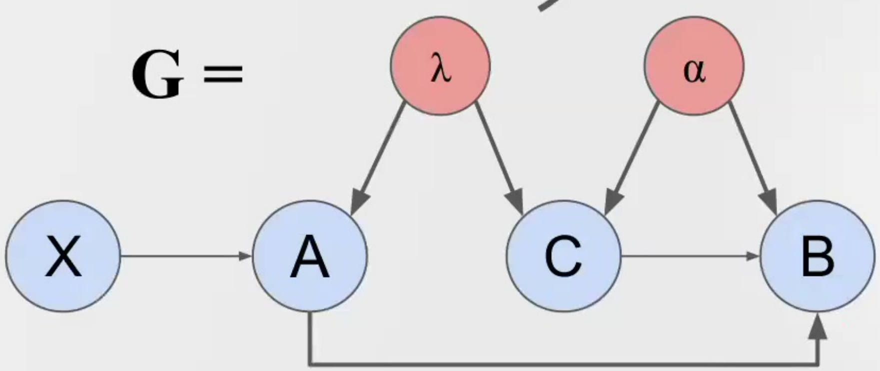


Tricks to prove QC Gaps



Example

G =



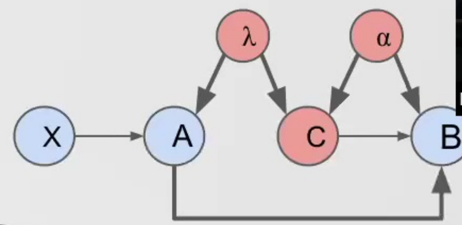
G' =

We don't know whether this DAG has a QC Gap

DAG where we **know** there is a QC Gap

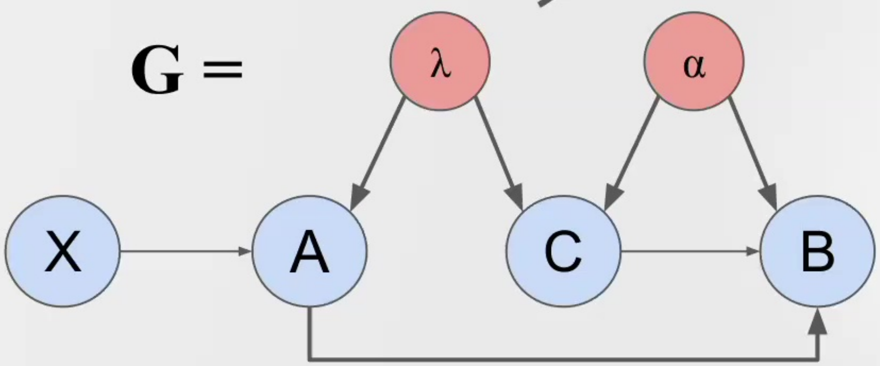
Tricks to prove QC Gaps

Example



Marginalize C

$\mathbf{G} =$



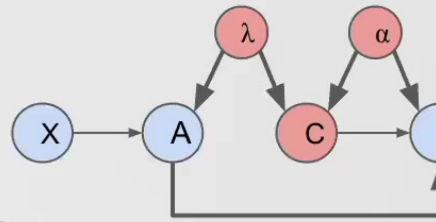
We don’t know whether this DAG has a QC Gap

$\mathbf{G'} =$

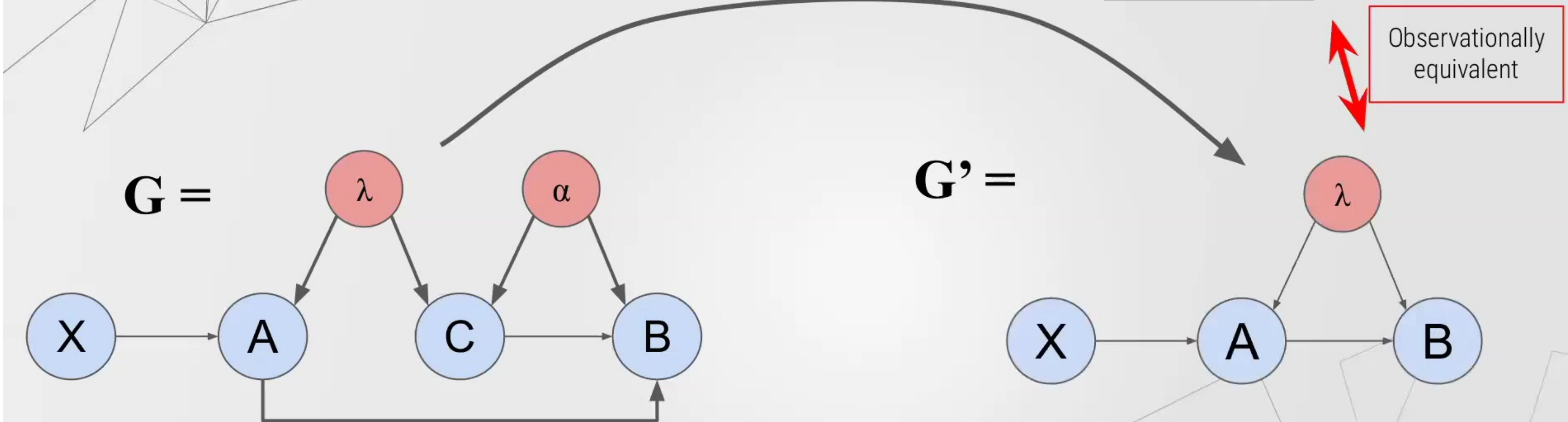
DAG where we **know** there is a QC Gap

Tricks to prove QC Gaps

Example



Marginalize C

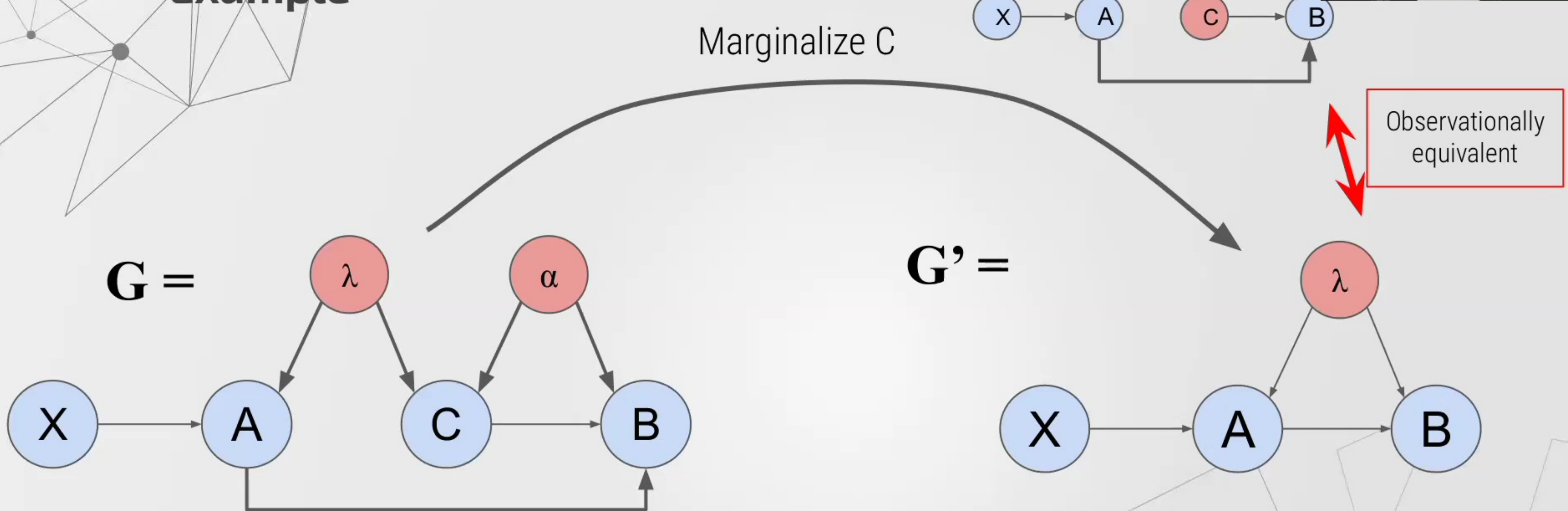


We don't know whether this DAG has a QC Gap

DAG where we **know** there is a QC Gap

Tricks to prove QC Gaps

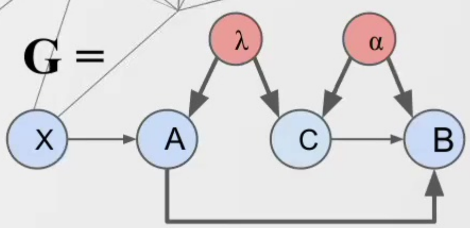
Example

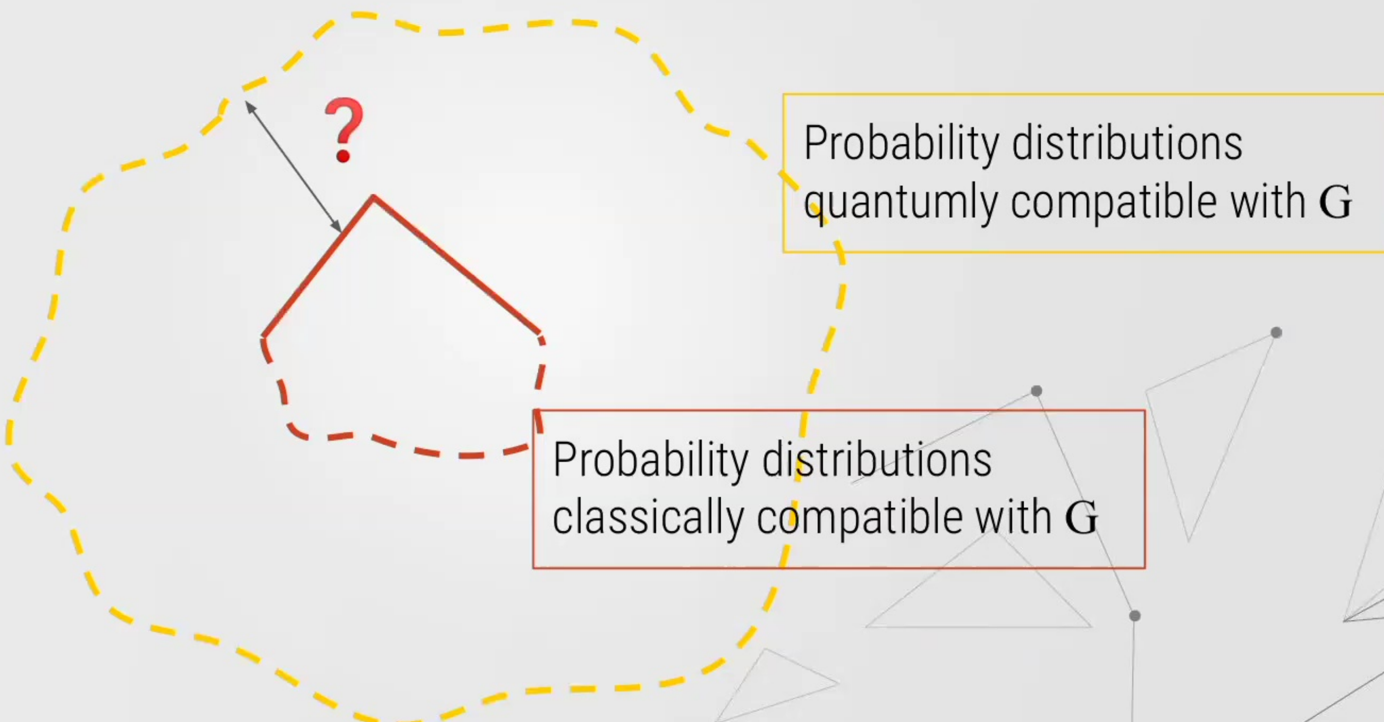


Inequalities involving $\sum_c P^G(x,a,b,c)$ $\longleftrightarrow$ Inequalities involving $P^{G'}(x,a,b)$

Tricks to prove QC Gaps

Example

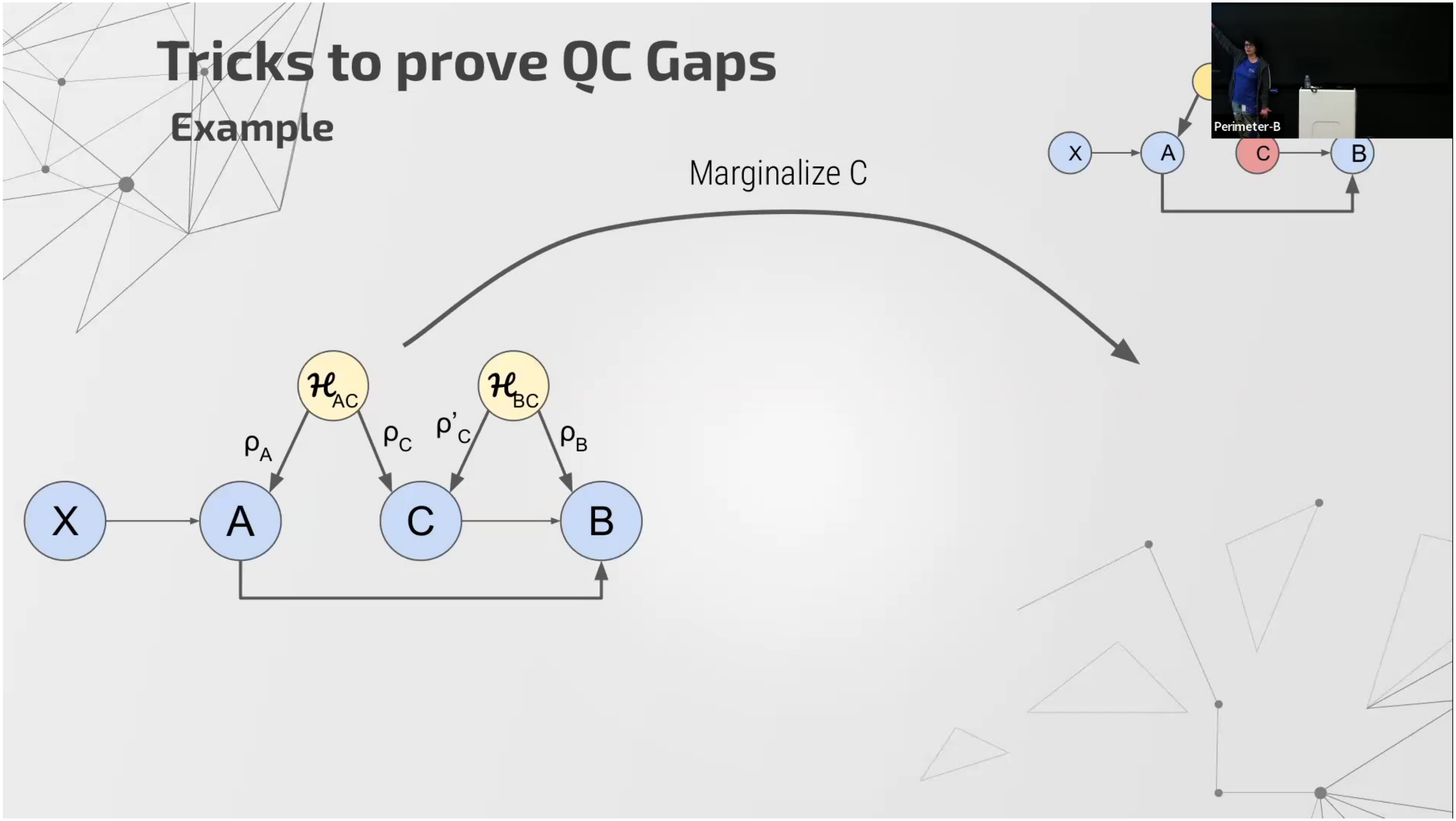
$\mathbf{G} =$ 



Classically, it has some non-trivial inequalities

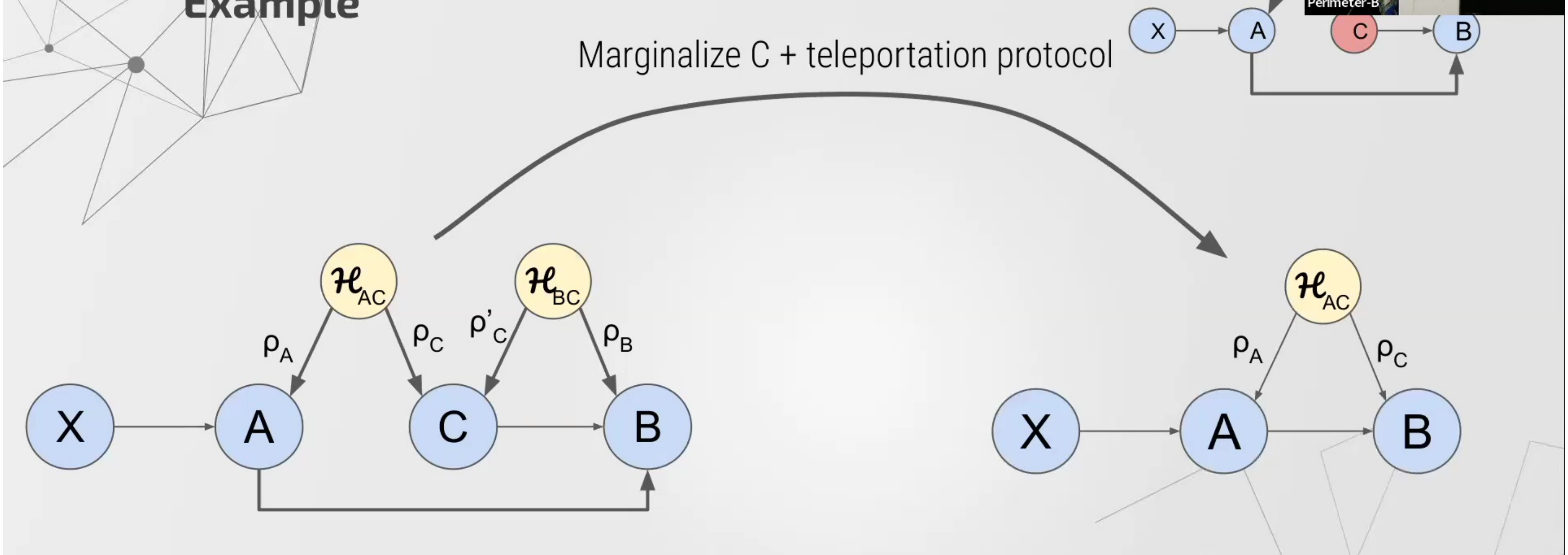
Tricks to prove QC Gaps

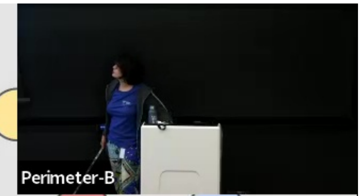
Example



Tricks to prove QC Gaps

Example

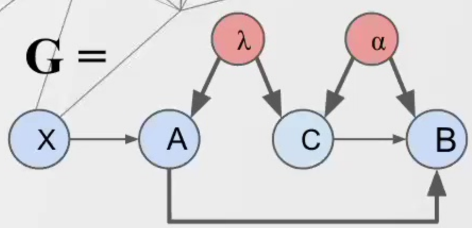


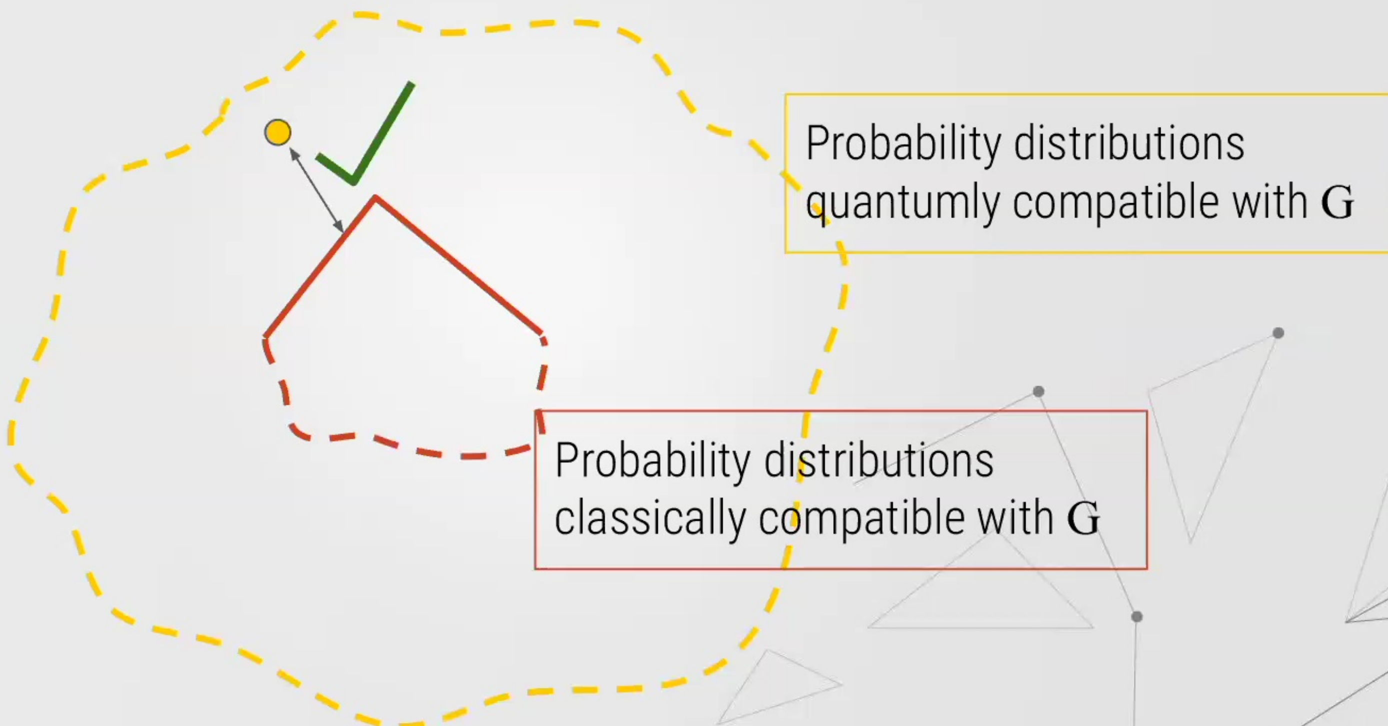


Tricks to prove QC Gaps

Example

$\mathbf{G} =$

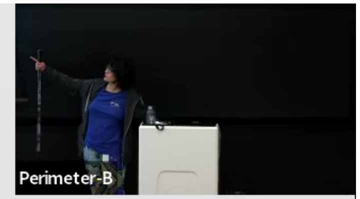


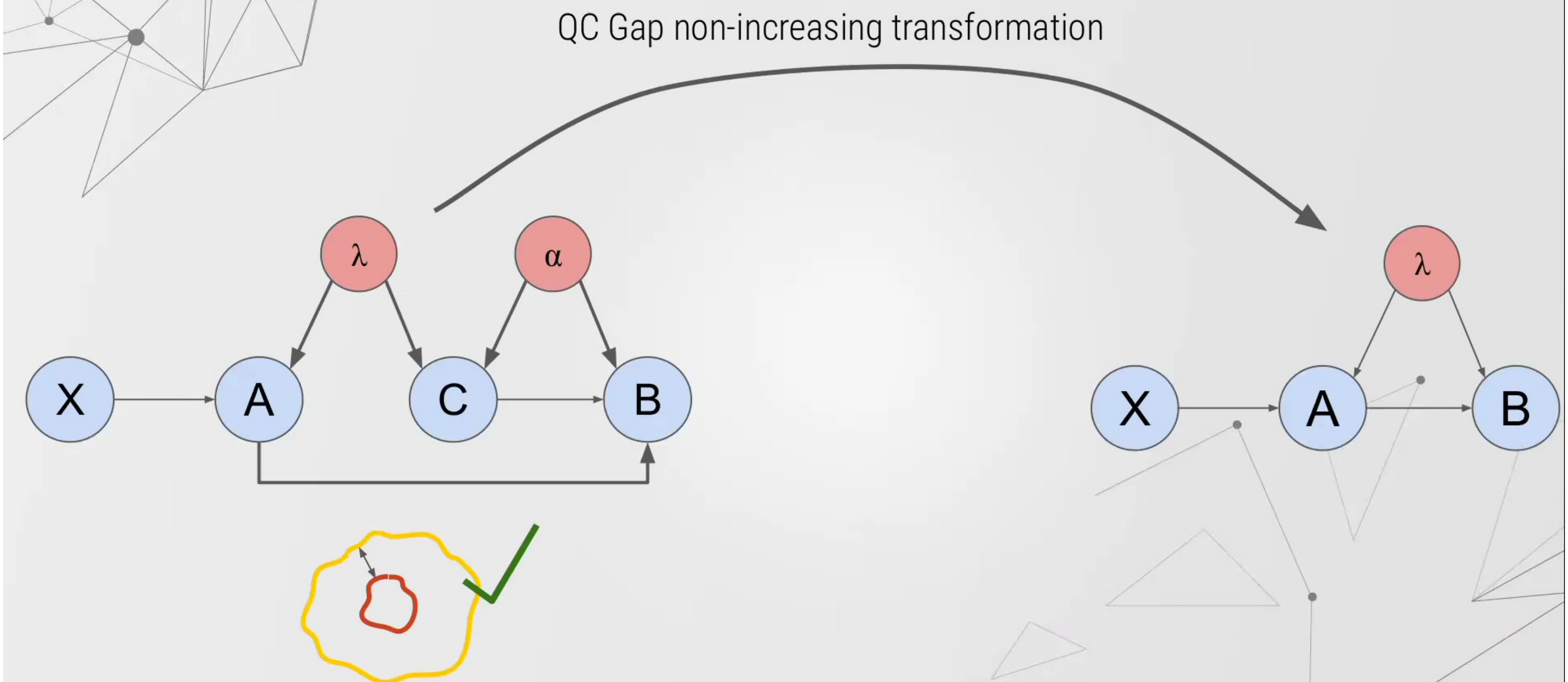


Inequalities are quantumly violated!

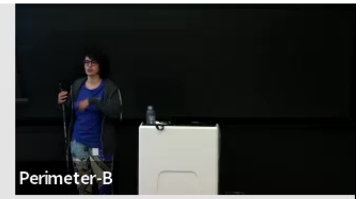
Tricks to prove QC Gaps

Example



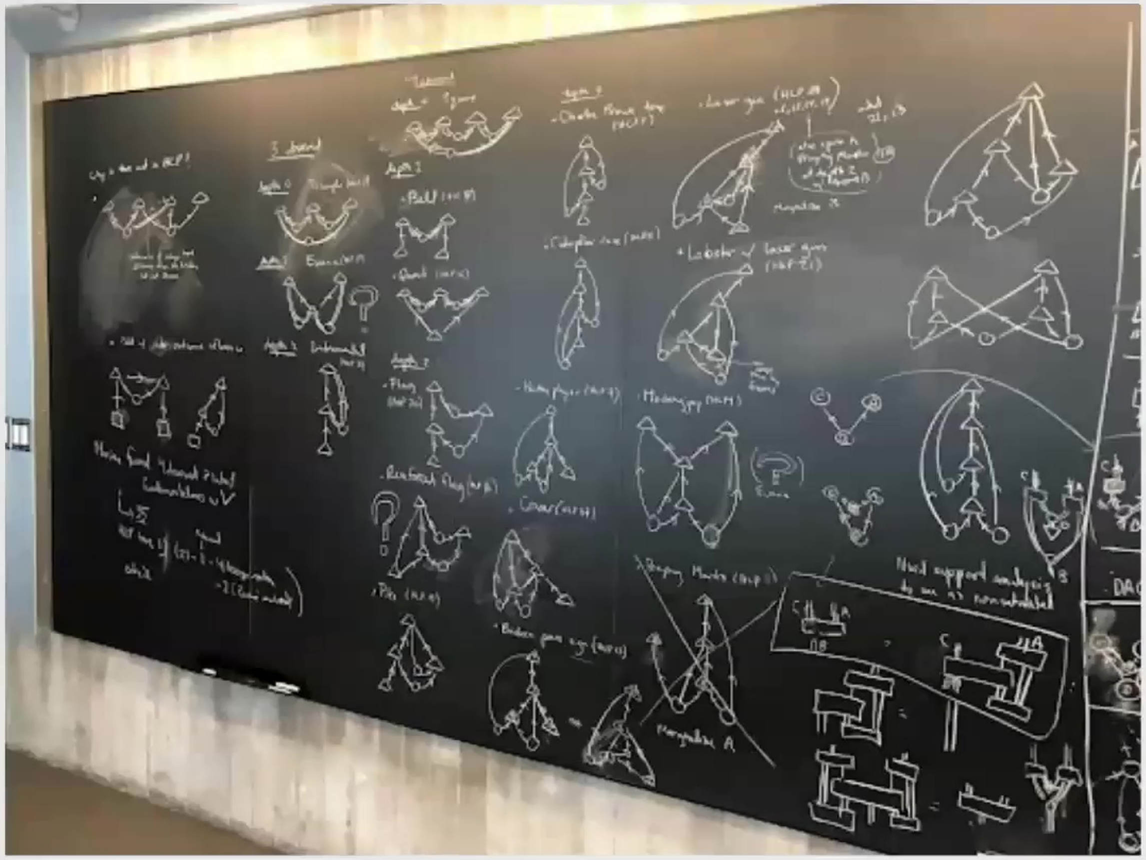


Marginalization is just one trick. There are others!



Tricks to prove QC Gaps

These tricks helped to prove QC Gaps in more DAGs



Tricks to prove QC Gaps

Indeed, many more DAGs

For DAGs with 4 visible variables

Total number of fundamental equivalence classes of causal structures	331
Ones where we proved a QC Gap	284

Code made in Python

Summary

1

Studying the underlying causal relationships between events of interest is important for both Classical Data Analysis and Quantum Foundations.

Bell's theorem can be looked at from this point of view.

2

Finding QC Gaps in more causal structures:

- Potential to find new resources
- Better fundamental understanding of quantum mechanics (new no-go theorems?)

3

Our results: Tricks that prove a QC Gap in G given that we already know that there is a QC gap in another structure G'. Using the tricks, we could prove QC Gaps in a large number of causal structures

THANKS

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