Title: Quantum Field Theory, Separation of Scales, and Beyond

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Abstract: We will review the role of Quantum Field Theory (QFT) in modern physics. We will highlight how QFT uses a reductionist perspective as a powerful quantitative tool relating phenomena at different length and energy scales. We will then discuss various examples motivated by string theory and lattice models that challenge this separation of scales and seem outside the standard framework of QFT. These lattice models include theories of fractons and other exotic systems.

Zoom Link: https://pitp.zoom.us/j/97212376441?pwd=bkFIZ0pIVG1xV3JQYlpnNE5Ebllydz09

Quantum Field Theory, Separation of Scales, and Beyond
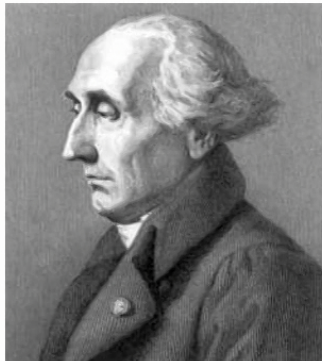
Nathan Seiberg
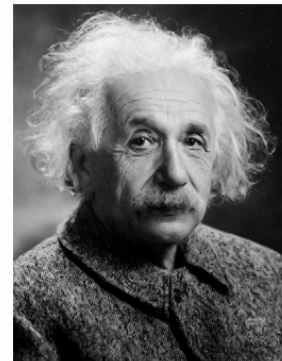
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Classical Physics

Classical Mechanics

Time evolution of a finite number of particles

Ordinary differential equations

Classical Field Theory

Time evolution of an infinite number (continuum) of degrees of freedom, e.g., electromagnetic field, velocity of fluid, metric

Quantum Physics

Quantum Mechanics

Time evolution of a finite number of quantum particles

Operators in a Hilbert space

Functional integral

Quantum Field Theory

Time evolution of an infinite number (continuum) of quantum degrees of freedom, e.g., electromagnetic field

A lot is known. Still very exciting progress.

My personal view: a new intellectual structure is needed – QFT

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Quantum Field Theory is Everywhere

- Particle physics: the language of the Standard Model
  - Enormous success, e.g., the electron magnetic dipole moment is theoretically   *1.001 159 652 18 ...*
    experimentally   *1.001 159 652 180...*
- Condensed matter
  - Description of the long-distance properties of materials: phases and the transitions between them
- Cosmology
  - Early Universe, inflation
- ...

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Quantum Field Theory is Everywhere

- String theory/quantum gravity
  - On the string world-sheet
  - In the low-energy approximation
  - The whole theory (gauge/gravity duality)
- Applications in mathematics especially in geometry and topology

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Separation of Scales

In physics (and in fact in all sciences, including social sciences) we use different effective descriptions at different scales.

- Simplification. Independence of the details at other scales

- Nature is kind to us – reductionism

- Examples

  - Thermodynamics

  - Hydrodynamics

  - Epidemiology

  - Effective field theory and the renormalization group…

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Separation of Scales in QFT

Different field theories describe the phenomena at different length scales (or energy scales).

Different Theories at Short and Long Distances

- We formulate a problem at short distances and the answer is the long-distance behavior

UV theory at short distances

- Possible phases are the possible long-distance behavior

- The IR theory is independent of most of the UV details – universality. Efficient, effective description.

- Even if the UV theory is on a lattice, the IR theory is in the continuum.

IR theory at long distances

  - Approximating a sum by an integral
  - More about that, below.

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The Phases of QFT

Consider the theory in large but finite volume. The spectrum of the Hamiltonian can be

- Gapped/massive, e.g., free massive particles
  - Unique ground state
  - Several states become degenerate as the volume goes to infinity. Topological FT

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- Gapless/massless
  - Free massless particles, e.g., photons
  - Interacting massless particles, notion of particle is ill-defined. Conformal FT

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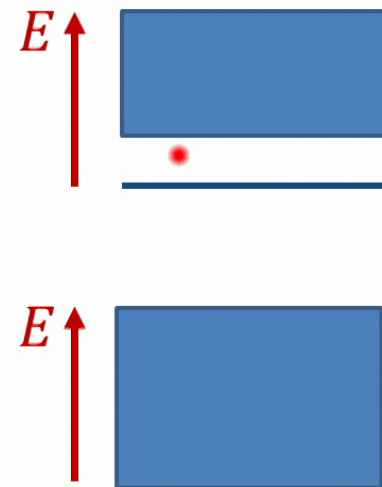
Different Theories at Short and Long Distances

- Continuum QFT classifies and organizes the possible phases and the transitions between them.

- Interesting new phases teach us about QFT.

- The IR theory is expected to be scale invariant
  - Conformal field theories (CFT)
  - ⊃ Topological field theories (TQFT)
  - ⊃ Invertible TQFT

- Interesting flow from the UV to the IR
  - Given the question in the UV, determine the answer in the IR

UV theory at short distances

**IR theory at long distances**

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Concrete example: lattice models

Approximate space (or spacetime) by a lattice, (e.g., a cubic lattice).

Place degrees of freedom at the vertices, links, etc.

Postulate short-range interactions (e.g., coupling between nearest neighbors).

Typically, the low-energy limit of such a system is described by a continuum QFT – the lattice becomes a continuum.

- In condensed matter physics, the lattice is physical.
- In high-energy physics, it is a tool helping us to define the continuum theory (and to perform numerical calculations).

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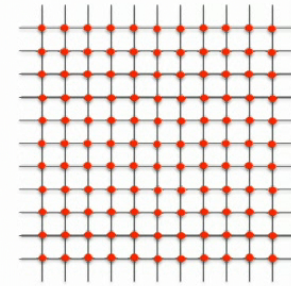
UV/IR mixing – counterexamples to the separation of scales dogma

UV/IR mixing – no separation of scales – long-distance/low-energy phenomena reflect high-energy physics.

- Common in gravity:
  - High energy in a small volume leads to a large black hole, hiding the short-distance physics
  - Dualities relate small $\leftrightarrow$ large
  - Many questions and confusions in quantum gravity circle around this issue
- String theory with vanishing Newton constant is a non-gravitational theory. Typically, it is a QFT. But certain peculiar examples exhibit UV/IR mixing...

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UV/IR mixing – counterexamples to the separation of scales dogma

- Examples based on limits of string theory
  - Little string theory
    - No local operators
    - T-duality
  - Field theory on a non-commutative space
    - dipole with high momentum is large in space along another direction
    - comparing with the same theory on a commutative space, fewer UV divergences and instead new IR divergences

Exotic lattice models (including models of fractons) also lead to UV/IR mixing...

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Exotic lattice models

Many exotic models of various kinds (soon we will review them)

- Lifshitz theory [many references]
- XY-plaquette model [Paramekanti, Balents, Fisher; …]
- Fracton models [Chamon; Haah; Vijay, Haah, and Fu; …]
- Many others

They do not have a standard continuum limit.

This challenge is related to their peculiar properties…

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Exotic models – peculiar properties

- Unusual symmetries (position dependent global symmetries)
- Excitations with restricted mobility (particles restricted to a point, line, etc.)
- Large ground state degeneracy, e.g., $2^{k(L^x,L^y,L^z)}$, where L^i are the number of lattice sites in direction i. The limit $L^i \to \infty$ is not well-defined and often infinite.
- UV/IR mixing (long-distance/low-energy phenomena are sensitive to short-distance details)

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Exotic models – questions

Are these peculiarities related?

(Not all examples have all these peculiarities.)

How should we think of such theories?

How should we organize them?

- Classification

- More examples

- Continuum field theory description?

The X-cube Model [Vijay, Haah, and Fu]

- $\mathbb{Z}_2$ spin on every spatial link (qubit on every link)
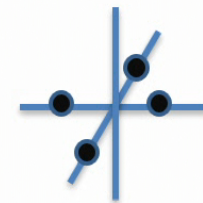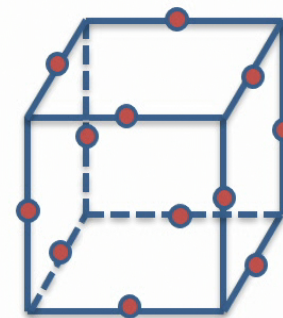- The Hamiltonian has two kinds of terms

$$H = -\sum_c B_c - \sum_s \left(A_s^x + A_s^y + A_s^z\right)$$

- Cube interaction

$$B_c = \prod_{\text{links around } c} \sigma^1$$

- Site interactions

$$A_s^z = \prod_{\text{links at } s \perp \text{ to } z} \sigma^3$$

- Innocent-looking local Hamiltonian

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The X-cube Model [Vijay, Haah, and Fu]

- On a lattice with $L^x \times L^y \times L^z$ sites with periodic boundary conditions, huge ground state degeneracy

$$2^{2(L^x + L^y + L^z) - 3}$$

  - Depends on the number of lattice sites
  - Entropy $2(L^x + L^y + L^z) - 3$ not proportional to the volume (sub-extensive)
  - Infinite in the continuum limit: lattice spacing $a \to 0$, $L^i \to \infty$ with fixed size $\ell^i = aL^i$

- The ground states reflect short distance physics – high momentum modes (of lattice scale) have zero energy – UV/IR mixing.

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The X-cube Model [Vijay, Haah, and Fu]

- Gap in the spectrum.
- Small deformations of the Hamiltonian do not change the low-energy physics.
- Localized excitations with restricted mobility:
  - Fractons are fixed at a point
  - Lineons fixed to move on a line (x, y, or z)
  - Planon fixed to move on a plane (xy, yz, or xz)

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Haah Code [Haah]

Two qubits at every site.

Two kinds of terms in the Hamiltonian
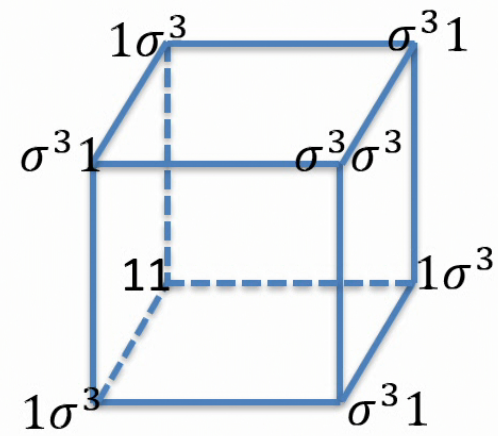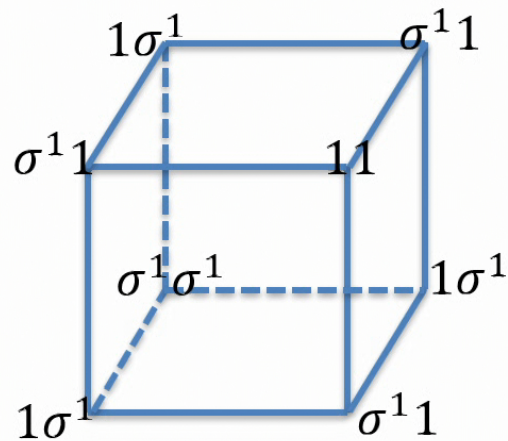
$1\sigma^1$ means the action of $1 \otimes \sigma^1$ on the two qubits.

Haah Code [Haah]

Excitations with restricted mobility.

The ground state degeneracy is $2^{k(L^x, L^y, L^z)}$ with $k(L^x, L^y, L^z)$

- a complicated number-theoretic function of L^x, L^y, L^z
- complicated even in the special case $L = L^x = L^y = L^z$
- not monotonic in L
- bounded by $\sim L$
- for some sequence of $L \to \infty$, $k(L) \to$ finite

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Haah Code [Haah]

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The limit $L^i \to \infty$ is ambiguous. Is there a continuum limit?

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Gapless Models [Pretko]

Motivated by earlier models of symmetric tensor gauge theories in the continuum and the lattice [Xu; …]

$$A_0 \rightarrow A_0 + \partial_0 \alpha$$
$$A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \alpha$$

Gauge invariant electric and magnetic fields

$$E_{ij} = \partial_0 A_{ij} - \partial_i \partial_j A_0$$
$$B_{[ij]k} = \partial_i A_{jk} - \partial_j A_{ik}$$

Lagrangian $\qquad\qquad \mathcal{L} = E^2 - B^2$

Gapless/massless "photon"

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Gapless Models [Pretko]

$$A_0 \rightarrow A_0 + \partial_0 \alpha$$
$$A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \alpha$$

Gauss law with matter $\qquad \sum_{ij} \partial_i \partial_j E_{ij} = \rho$

Conserved charge $\qquad \int d^3 x \, \rho$

Conserved dipole charge $\quad \int d^3 x \, x^i \rho$

Restricted mobility because of the conservations – fractons

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Questions

- More examples, perhaps with more exotic phenomena?
- What is the underlying reason for the bizarre behavior?
- What are the possible theories/phases?
  - Organization/classification
  - Is there a sensible continuum quantum field theory for the long-distance behavior?

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Continuum Lagrangians

The naïve continuum limit of these models leads to unusual Lagrangians.

Schematically,

- $\mathcal{L} = (\partial_t \phi)^2 - \sum_{ij} (\partial_i \partial_j \phi)^2$ [Henley; ...; Chen, Huang; ...; Radičević; ...; NS, Shao; ...]
  - Lifshitz theory: sum over all ij
  - XY-plaquette model [Paramekanti, Balents, Fisher; ...]: sum only over $i \neq j$

- $\mathcal{L} = E^2 - B^2$ [Xu; Xu, Wu; Pretko; ...]

$$E_{ij} = \partial_t A_{ij} - \partial_i \partial_j A_t \quad , \qquad B_{ijk} = \partial_i A_{jk} - \partial_j A_{ik}$$
$$A_t \to A_t + \partial_t \alpha \quad , \qquad A_{ij} \to A_{ij} + \partial_i \partial_j \alpha$$

  - sum over all ij [Xu; Pretko; ...]
  - sum only over $i \neq j$ [Xu, Wu; Ma, Hermele, Chen...]

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Continuum Lagrangians

- Chern-Simons-type Lagrangians of exotic gauge theories lead to gapped theories including the X-cube model [Slagle, Kim; NS, Shao; …]
- Many others

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Gorantla, Lam, NS, Shao

The questions we studied
[Gorantla, Lam, NS, Shao]

- What do continuum field theories based on such Lagrangians mean?
  - Need to study discontinuous field configurations
  - Understand the global structure – fluxes, operators, defects.
- What is the relation between these continuum theories and the underlying lattice models?

Many results about many models…

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Some of our conclusions
[Gorantla, Lam, NS, Shao]

- Typically, the continuum theories have more symmetries than the underlying lattice models.
  - New lattice models exhibit these symmetries. They are closer to the continuum theories.
- Starting with a lattice, we can study two limits:
  - Continuum limit: $a \to 0, L \to \infty$ with fixed length $\ell = aL$ – leading to continuum field theory in finite volume.
  - Thermodynamic limit: $L \to \infty$ with fixed a (and hence, $\ell \to \infty$)

  They do not commute!
- Can also study other limits, leading to different low-energy/continuum theories

UV/IR mixing

Summary

- Quantum field theory is everywhere
  - It appears to be the language of physics
- A fundamental property of quantum field theory is the phenomenon of separation of scales – different effective theories at different scales – reductionism
  - Most of the details of the short-distance UV theory do not affect its long-distance IR description.
- Certain string theory constructions and some lattice models lead to peculiar systems outside the framework of standard QFT – UV/IR mixing

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Summary

- The exotic systems, including fractons, exhibit interesting properties
  - UV/IR mixing:
    - Large ground state degeneracy. Sometimes there is no well defined limit as $L \to \infty$
    - Observables change at the lattice scale – discontinuous in the continuum limit.
  - Excitations with restricted mobility

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Summary

- The exotic systems, including fractons, exhibit interesting properties
  - UV/IR mixing:
    - Large ground state degeneracy. Sometimes there is no well defined limit as $L \to \infty$
    - Observables change at the lattice scale – discontinuous in the continuum limit.
  - Excitations with restricted mobility
- These seem incompatible with the framework of continuum QFT.

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Summary

- We analyzed nonstandard continuum QFTs for these systems

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Thank you
Stay healthy

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