

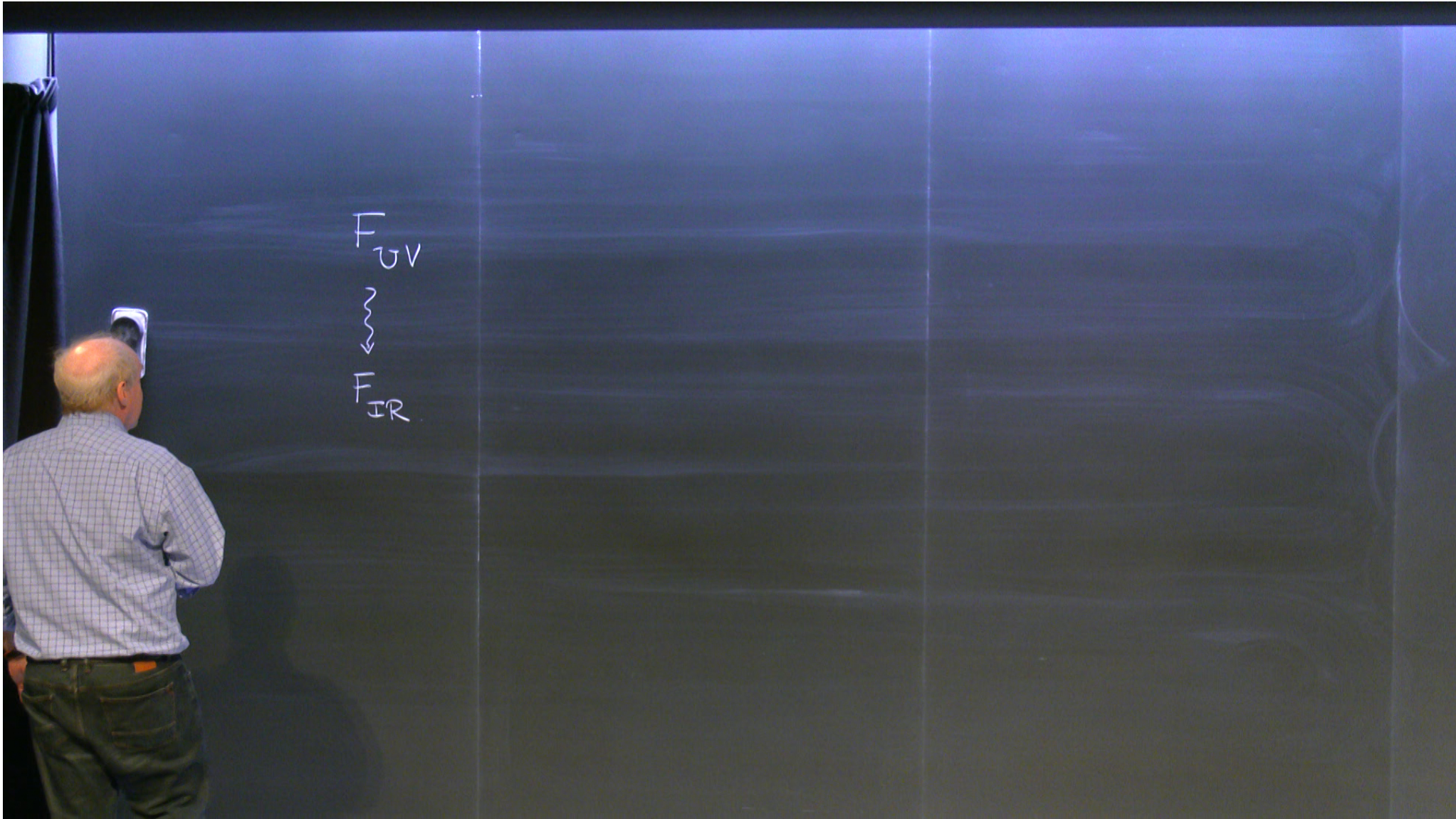
Title: Finite symmetry in QFT

Speakers: Dan Freed

Collection: Global Categorical Symmetries

Date: June 17, 2022 - 9:00 AM

URL: <https://pirsa.org/22060038>



$$(\sigma, \rho) \in F_{UV}$$



$$(\sigma, \rho) \in F_{IR} \text{ ~topological field}$$

Q. $\exists \tilde{\lambda}$ left (σ, ρ) -module

s.t. $\rho \otimes_{\sigma} \tilde{\lambda} = \lambda$ invertible

If ρ regular: $A \otimes_A \tilde{\lambda}(\rho) \cong \tilde{\lambda}(\rho) = \lambda(\rho)$

$$(\sigma, p) \in F_{UV}$$



$$(\sigma, p) \in F_{IR}^{\text{topological field theory}}$$

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If p regular: $A \otimes_A \hat{\lambda}(p) \simeq \hat{\lambda}(p) = \lambda(p)$

Choi-Cordova-Hsien-Lam-Shao
 Kaidi-Ohmori-Zhu



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Abelian Duality.

Def'n: A, A' abelian gps. A
bimorphism

$$b: A \times A' \rightarrow \mathbb{C}^\times$$

is nondegenerate if

$$b(a, a') = 1 \text{ for all } a' \in A' \Rightarrow a = 1$$

$$a \in A \Rightarrow a' = 1$$

$$(\sigma, \rho) \in F_{UV}$$



$$(\sigma, \rho) \in F_{\mathbb{R}}^{\text{topological}}$$

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Kaidi-Ohmari-Zhang.

Abelian Duality.

Def'n: A, A' abelian gps. A bicharacterism

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$$a \in A \Rightarrow a' = 1$$

$$A \cong (A')^\vee \quad A' \cong A^\vee$$

Pontrjagin duality.

Ex: $\ell \in \mathbb{Z}^{>0}$ $\mathbb{Z}/\ell\mathbb{Z}$, μ_ℓ ℓ^{th} roots of unity

$$b: \mu_\ell \times \mathbb{Z}/\ell\mathbb{Z} \rightarrow \mathbb{C}^\times$$

$$\lambda, k \mapsto \lambda^k$$

$$b(a, a') = 1 \text{ for all } a' \in A' \Rightarrow a = 1$$

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Pontrjagin duality.

Ex: $l \in \mathbb{Z}^{>0}$ $\mathbb{Z}/l\mathbb{Z}$, μ_l l^{th} roots of unity

$$b: \mu_l \times \mathbb{Z}/l\mathbb{Z} \rightarrow \mathbb{C}^\times$$

$$\lambda, k \mapsto \lambda^k$$

Fourier transform: $\text{Fun}(A) \xrightarrow{\cong} \text{Fun}(A')$

Spectra $\longleftrightarrow$ Infinite loop spaces.

$$T \longleftrightarrow \Omega^\infty T \sim \mathcal{X}_T$$

FHT: $B^0 A$ is $\mathcal{X}_{\Sigma BHA}$

$$B^2 A \rightarrow \mathcal{X} \rightarrow BG.$$

$$B^3 \mathbb{Z}/2 \rightarrow \mathcal{X} \xrightarrow{\text{not}} B^2 \mathbb{Z}/2$$

(S_1^2)

$$b(a, a') = 1 \text{ for all } a \in A' \Rightarrow a = 1$$

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(S_1^2)

$$T \times T' \rightarrow IC^\times$$

Ex: A finite abelian

$$HA \times HA' \rightarrow HC^\times \rightarrow IC^\times$$

T π -finite spectrum
 X_T ω -loop space

Defn: Fix n ($m = n+1$)
 $\sigma = \sigma_T^{(n+1)} = \sigma_T^{(n+1)}$
 ρ assoc. to $* \rightarrow X_T$

Dual data $(\sigma^\vee, \rho^\vee)$

Invertible domain will
 $\delta: \sigma \rightarrow \sigma^\vee$
 $\delta^\vee: \sigma^\vee \rightarrow \sigma$

$$\sigma^\vee = \sigma_T^{(n+1)}$$

$$b(e, e') = 1 \text{ for all } e, e' \in A' \Rightarrow a = 1$$

$$a \in A \Rightarrow a' = 1$$

$$A \xrightarrow{\sim} (A')^\vee \quad A' \xrightarrow{\sim} A^\vee$$

Pontrjagin dual

$$\text{Ex: } l \in \mathbb{Z}^{>0}$$

$$\mu_l \xrightarrow{\text{ } l^{\text{th}} \text{ roots of unity}} \mathbb{C}^\times$$

T π -finite spectrum

X_T ω -loop space

Defn: Fix n ($m = n+1$)

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ρ assoc. to $* \rightarrow X_T$

Dual data $(\sigma^\vee, \rho^\vee)$

Invertible domain will

$$\delta: \sigma \rightarrow \sigma^\vee$$

$$\delta^\vee: \sigma^\vee \rightarrow \sigma$$

$$\sigma^\vee = \sigma_T^{(n+1)} / \Sigma^n T^\vee, \quad \rho^\vee \text{ basepoint}$$

$$(\ast_T \times \ast_{\Sigma^n T^\vee}, \mu)$$

$$(\ast_T, 0)$$

$$(\ast_{\Sigma^n T^\vee}, 0)$$

$$\mu \text{ represents } T \times \Sigma^n T^\vee \rightarrow \Sigma^n \mathbb{I} \mathbb{C}^\times$$

$$b(a, a') = 1 \text{ for all } a \in A' \Rightarrow a = 1$$

$$a \in A \Rightarrow a' = 1$$

$$A \xrightarrow{\sim} (A')^\vee \quad A' \xrightarrow{\sim} A^\vee$$

Pontrjagin duality.

$$\text{Ex: } l \in \mathbb{Z}^{>0} \quad \mathbb{Z}/l\mathbb{Z}, \quad \mu_l \text{ } l^{\text{th}} \text{ roots of unity}$$

$$b: \mu_l \times \mathbb{Z}/l\mathbb{Z} \rightarrow \mathbb{C}^\times$$

$$\lambda, k \mapsto \lambda^k$$

T π -finite spectrum
 X_T ω -loop space

Defn. Fix n ($m = n+1$)
 $\sigma^{(n+1)} = \sigma^{(n+1)}_{\Sigma^n T}$
 $\mu \rightarrow * \rightarrow X_T$
 (σ^{ν}, p^{ν})
 $\sigma^{\nu}: \sigma \rightarrow \sigma^{\nu}$
 $\sigma^{\nu}: \sigma^{\nu} \rightarrow \sigma$

$$\sigma^{\nu} = \sigma^{(n+1)}_{\Sigma^n T^{\nu}}, \quad p^{\nu} \text{ basepoint}$$

$$(\Sigma_T^{\nu} \times \Sigma_{\Sigma^n T^{\nu}}^{\nu}, \mu)$$

$$(\Sigma_T^{\nu}, 0)$$

$$(\Sigma_{\Sigma^n T^{\nu}}^{\nu}, 0)$$

μ represents $T \times \Sigma^n T^{\nu} \rightarrow \Sigma^n \mathbb{I} \mathbb{Q}^{\times}$
 $\circ \varepsilon, \varepsilon^{\nu}$ augmentations $(\text{id}_{\Sigma_T^{\nu}}, \text{id}_{\Sigma_{\Sigma^n T^{\nu}}^{\nu}})$

T π -finite spectrum
 X_T ω -loop space

Defn: Fix n ($m = n+1$)
 $\sigma = \sigma_T^{(n+1)} = \sigma_T^{(n)}$
 ρ assoc. to $*$ $\rightarrow X_T$

Dual data $(\sigma^\vee, \rho^\vee)$

Invertible domain with
 $\delta: \sigma \rightarrow \sigma^\vee$
 $\delta^\vee: \sigma^\vee \rightarrow \sigma$

$$\sigma^\vee = \sigma_T^{(n+1)} / \Sigma^n T^\vee, \quad \rho^\vee \text{ basepoint}$$

$$\begin{aligned} & (X_T, X_{\Sigma^n T^\vee}, \mu) \\ & (X_T, 0) \quad (X_{\Sigma^n T^\vee}, 0) \\ & \mu \text{ represents } T^\vee \rightarrow \Sigma^n T^\vee \\ & * \varepsilon, \varepsilon^\vee \text{ as } (id_{X_T}, id_{X_{\Sigma^n T^\vee}}) \end{aligned}$$

Lemma: $\exists \cong$ of right σ -modules
 $\rho^\vee \otimes_{\sigma^\vee} \delta \xrightarrow{\cong} \varepsilon$

$$\begin{array}{ccc} \sigma^\vee & \xrightarrow{\delta} & \sigma \\ \rho^\vee & & \varepsilon \end{array} \xrightarrow{\cong}$$

Pf: $*$ $\rightarrow X_{\Sigma^n T^\vee} \rightarrow X_T$
 $(X_{\Sigma^n T^\vee}, X_T, \mu)$

T π -finite spectrum
 X_T ∞ -loop space

Defn: Fix n ($m = n+1$)
 $\sigma = \sigma_T^{(n+1)} = \sigma_T^{(n)}$
 ρ assoc. to $* \rightarrow X_T$

Dual data $(\sigma^\vee, \rho^\vee)$

Invertible domain with
 $\delta: \sigma \rightarrow \sigma^\vee$
 $\delta^\vee: \sigma^\vee \rightarrow \sigma$

$$\sigma^\vee = \sigma_T^{(n+1)} / \Sigma^n T^\vee, \quad \rho^\vee \text{ basepoint}$$

$$(*_T \times *_T, \mu)$$

$$(*_T, 0)$$

$$(*_{\Sigma^n T}, 0)$$

μ represents

$$\Sigma^n T^\vee \rightarrow \Sigma^n I \mathbb{Q}^*$$

$$* \xrightarrow{\varepsilon, \varepsilon^\vee} \Sigma^n T^\vee \xrightarrow{(id_{X_T}, id_{X_{\Sigma^n T}})} \Sigma^n I \mathbb{Q}^*$$

Lemma: $\exists \cong$ of right σ -modules
 $\rho^\vee \otimes_{\sigma^\vee} \delta \xrightarrow{\cong} \varepsilon$

$$\sigma^\vee \times \sigma \xrightarrow{\cong} \sigma$$

Pf: $* \xleftarrow{\varepsilon^\vee} \Sigma^n T^\vee \xrightarrow{(\varepsilon_T, \varepsilon_T)} (*_T \times *_T, \mu) \xrightarrow{\delta} X_T$

T π -finite spectrum
 X_T ω -loop space

Defn: Fix n ($m = n+1$)
 $\sigma = \sigma_T^{(n+1)} = \sigma_T^{(n)}$
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Dual data $(\sigma^\vee, \rho^\vee)$

Invertible domain with
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$$(\ast_T \times \ast_{\Sigma^n T^\vee}, \mu)$$

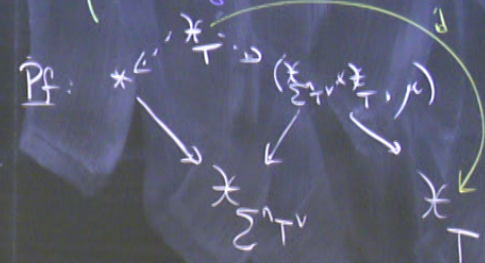
$$(\ast_T, 0)$$

$$(\ast_{\Sigma^n T}, 0)$$

μ represents $T \times \Sigma^n T^\vee \rightarrow \Sigma^n \mathbb{I} \mathbb{Q}^\times$
 $\ast \varepsilon, \varepsilon^\vee$ augmentations $(\text{id}_{X_T}, \text{id}_{X_{\Sigma^n T^\vee}})$

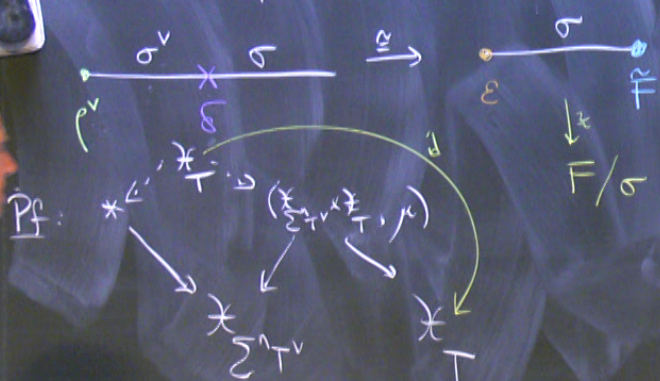
Lemma: $\exists \cong$ of right σ -modules
 $\rho^\vee \otimes_{\sigma^\vee} \delta \xrightarrow{\cong} \varepsilon$

$$\sigma^\vee \times \sigma \xrightarrow{\cong} \sigma$$



Lemma: $\exists$ $n \notin$ right σ -modules

$$p^\nu \otimes_{\sigma^\nu} \delta \xrightarrow{\sim} E$$



$$T \times T' \rightarrow IC^*$$

Ex: A finite abelian

$$HA \rightleftharpoons HA^{\vee} \rightarrow HC^{\times} \rightarrow IC^{\times}$$

Lemma: $\exists$ of right σ -modules
 $\rho^\vee \otimes_{\sigma^\vee} \delta \xrightarrow{\cong} \varepsilon$



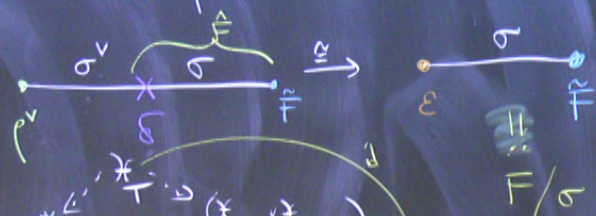
$$T \rtimes T' \rightarrow IC^*$$

Ex: A finite abelian

$$HA \rtimes HA^\vee \rightarrow HC^* \rightarrow IC^*$$

Lemma: $\exists$ of right σ -modules

$$\rho^v \otimes_{\sigma^v} \delta \xrightarrow{\cong} \varepsilon$$



Pf:

$$\begin{aligned} & \ast \xrightarrow{\delta} \ast_{\sum T^v} \xrightarrow{\delta} \ast_{\sum T^v} \xrightarrow{\delta} \ast_{\sum T^v} \\ & \ast \xrightarrow{\delta} \ast_{\sum T^v} \xrightarrow{\delta} \ast_{\sum T^v} \xrightarrow{\delta} \ast_{\sum T^v} \end{aligned}$$

Cor: If $(\sigma, \rho) \in F$, and F^{dual} is its transform under abelian duality, then $F^{\text{dual}} \cong F/\sigma$ and $(\sigma^v, \rho^v) \in F^{\text{dual}}$

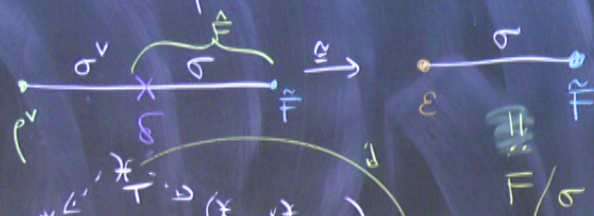
$$T \times T' \rightarrow IC^x$$

Ex: A finite abelian

$$HA \times HA^v \rightarrow HC^x \rightarrow IC^x$$

Lemma: $\exists$ of right σ -modules

$$\rho^v \otimes_{\sigma^v} \delta \xrightarrow{\cong} \varepsilon$$



Pf: $\ast \xrightarrow{\delta} \ast_{\sum T^v} \xrightarrow{\delta} \ast_{\sum T^v} \xrightarrow{\delta} \ast_{\sum T^v}$

Cor: If $(\sigma, \rho) \in F$, and F^{dual} is its transform under abelian duality, then $F^{\text{dual}} \approx F/\sigma$ and $(\sigma^v, \rho^v) \in F^{\text{dual}}$

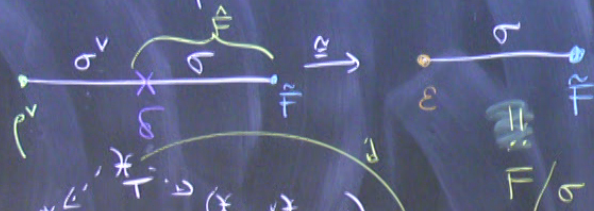
Ex: $n=2, m=3.$

$$T = \sum H A$$

$$T^v = \sum H A^v$$

Lemma: $\exists$ of right σ -modules

$$\rho^v \otimes_{\sigma^v} \delta \xrightarrow{\cong} \varepsilon$$



Pf: $\begin{matrix} * \xleftarrow{\delta} \sum_{T^v} * \xrightarrow{\mu} * \\ \downarrow \quad \downarrow \quad \downarrow \\ * \xleftarrow{\delta} \sum_{T^v} * \xrightarrow{\mu} * \end{matrix}$

Cor: If $(\sigma, \rho) \in F$, and F^{dual} is its transform under abelian duality, then $F^{\text{dual}} \approx F/\sigma$ and $(\sigma^v, \rho^v) \in F^{\text{dual}}$

Ex: $n=2, m=3.$

$$T = \sum H A$$

$$T^v = \sum H A^v$$

$$\sigma(pt) = \text{Vect}[A]$$

$$\sigma^v(pt) = \text{Vect}[A^v] \approx \text{Rep}(A) \quad \left. \begin{matrix} \text{Morita} \\ \cong \end{matrix} \right\}$$

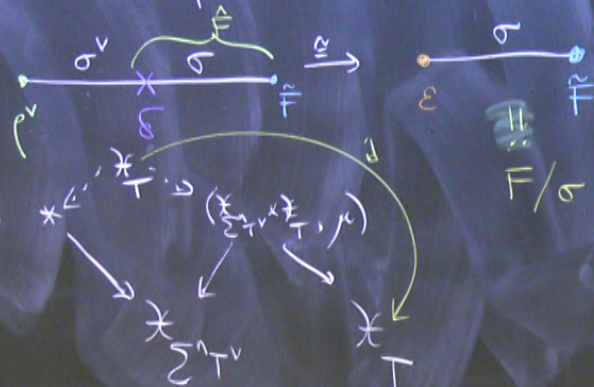
$$\sigma(Y) = \text{Fun}(H'(Y; A))$$

$$\sigma^v(Y) = \text{Fun}(H'(Y; A^v)) \quad \left. \begin{matrix} \text{Pontrjagin} \\ \text{dual} \end{matrix} \right\}$$

Y^2
closed
oriented

Lemma: $\exists$ of right σ -modules

$$\rho^v \otimes_{\sigma^v} \delta \xrightarrow{\cong} \varepsilon$$



Pf:

Cor: If $(\sigma, \rho) \in F$, and F^{dual} is its transform under abelian duality, then $F^{\text{dual}} \approx F/\sigma$ and $(\sigma^v, \rho^v) \in F^{\text{dual}}$

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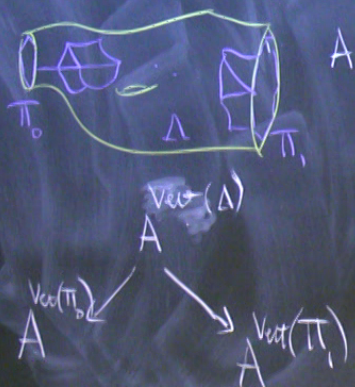
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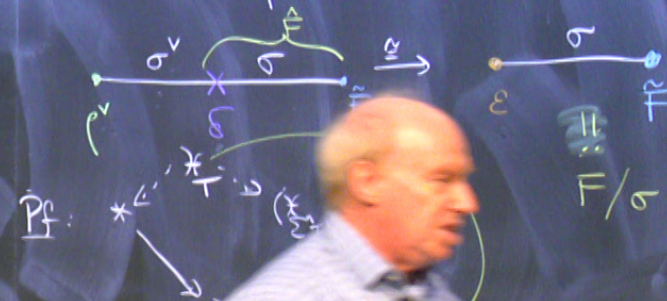
Y^2
closed
oriented

2d Ising model



Lemma: $\exists$ right σ -modules

$$\rho^v \otimes_{\sigma^v} \delta \xrightarrow{\sim} \varepsilon$$

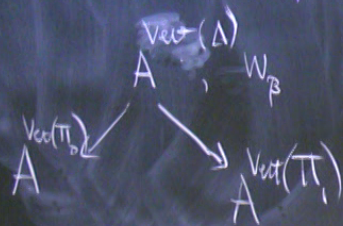


2d Ising model



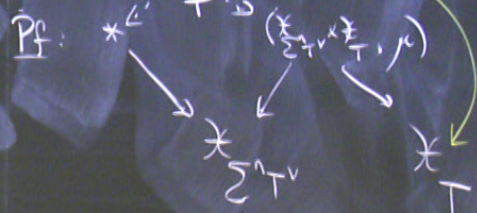
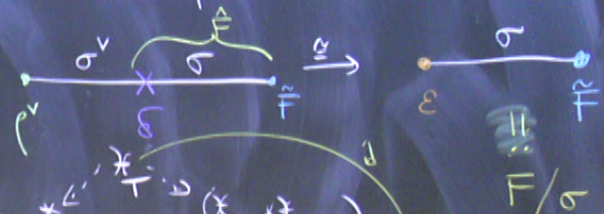
$$A = \mu_2 = \{1, 3\}$$

$$\beta \in \mathbb{R}^{>0}$$



Lemma: $\exists$ right σ -modules

$$\rho^v \otimes_{\sigma^v} \delta \xrightarrow{\cong} \varepsilon$$

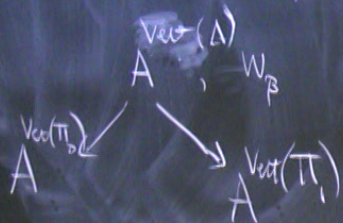


2d Ising model



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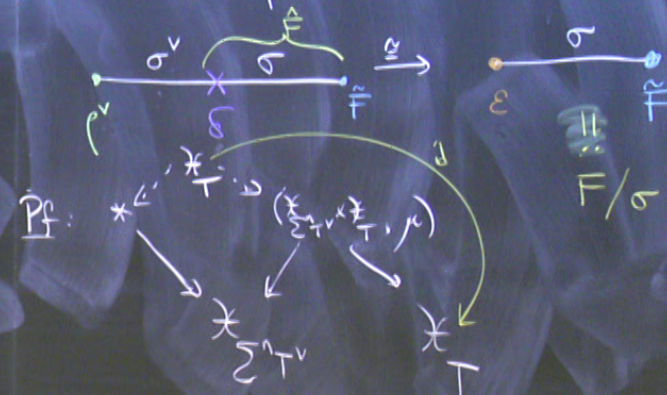
$$\beta \in \mathbb{R}^{>0}$$



$$(\sigma, \rho) \hookrightarrow F_\beta \quad \sigma = \sigma_{BA}^3$$

$$(\sigma, \rho) \hookrightarrow F_\beta^{\text{dual}} \xrightarrow[\cong]{\phi_\beta} F_{\beta^\vee}$$

Lemma: $\exists$ right σ -modules

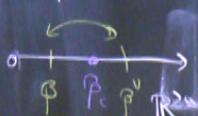
$$\rho^\vee \otimes_{\sigma^\vee} \delta \xrightarrow{\cong} \varepsilon$$


2d Ising model



$$A = \mu_2 = \{ \pm 1 \}$$

$$\beta \in \mathbb{R}^{>0}$$



$$A^{\text{Vec}(\Delta)} = W_\beta$$

$$A^{\text{Vec}(\pi_0)} \rightarrow A^{\text{Vec}(\pi_1)}$$

$$(\sigma, \rho) \hookrightarrow F_\beta \quad \sigma = \sigma_{BA}^3$$

$$(\sigma, \rho) \hookrightarrow F_\beta^{\text{dual}} \xrightarrow[\cong]{\phi_\beta} F_{\beta'}$$

$$F_{\beta_c}^{\text{dual}} \xrightarrow[\cong]{\phi_{\beta_c} = \phi} F_{\beta_c}$$

2d Ising model



$$A = \mu_2 = \{1, 3\}$$



$$(\sigma, \rho) \hookrightarrow F_\beta \quad \sigma = \sigma_{BA}^3$$

$$(\sigma, \rho) \hookrightarrow F_\beta^{\text{dual}} \xrightarrow[\cong]{\phi_\beta} F_{\beta'}$$

$$F_{\beta_c}/\sigma \cong F_{\beta_c}^{\text{dual}} \xrightarrow[\cong]{\phi_{\beta_c} = \phi} F_{\beta_c}$$

$$(\sigma, \rho) \subset F_\beta \quad \sigma = \sigma_{BA}^3$$

$$(\sigma', \rho') \subset F_\beta^{\text{dual}} \xrightarrow[\cong]{\phi_\rho} F_{\beta'}^{\text{dual}}$$

$$F_{\beta_c}/\sigma \xrightarrow[\cong]{\phi_{\beta_c}^{\text{dual}}} F_{\beta_c}^{\text{dual}}$$



$$\delta' \circ \delta : F \rightarrow F$$

quotient defect
problem 1.

$$\Delta = \phi \circ \delta : F \rightarrow F$$

$$\Delta^2 = \delta' \circ \delta$$

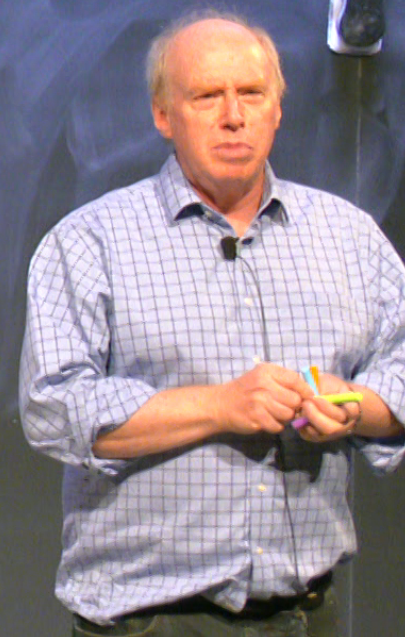
Cor: If $(\sigma, \rho) \subset F$, and F^{dual} is its transform under abelian duality, then $F^{\text{dual}} \subset F/\sigma$ and $(\sigma', \rho') \subset F^{\text{dual}}$

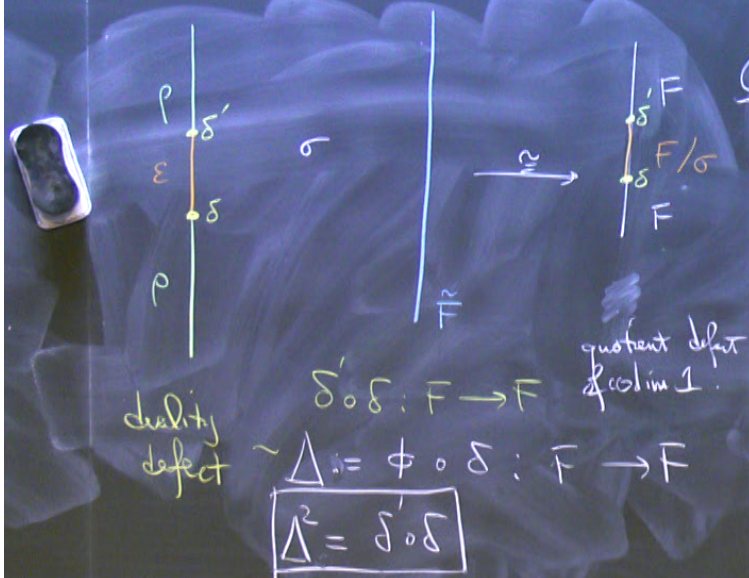
Y^2 closed
oriented

$$\begin{aligned}
 (\sigma, \rho) &\hookrightarrow F_\rho & \sigma &= \sigma_{BA}^3 \\
 (\sigma', \rho) &\hookrightarrow F_\rho^{\text{dual}} \xrightarrow[\cong]{\phi_\rho} F_{\rho'} \\
 F_{\rho_c}/\sigma &\xrightarrow[\cong]{\text{dual}} F_{\rho_c} \xrightarrow[\cong]{\phi_{\rho_c} = \phi} F_{\rho_c}
 \end{aligned}$$



$\delta' \circ \delta : F \rightarrow F$ quotient defect of colim 1.
 duality defect $\sim \Delta := \phi \circ \delta : F \rightarrow F$
 $\Delta^2 = \delta' \circ \delta$





Q: Is there a left $(\sigma, \rho) = (\sigma^\vee, \rho^\vee)$ -module $\tilde{\lambda}$ s.t.

- 1) Δ is implemented
- 2) $\rho \otimes_{\sigma} \tilde{\lambda} =: \lambda$ invertible

(?)

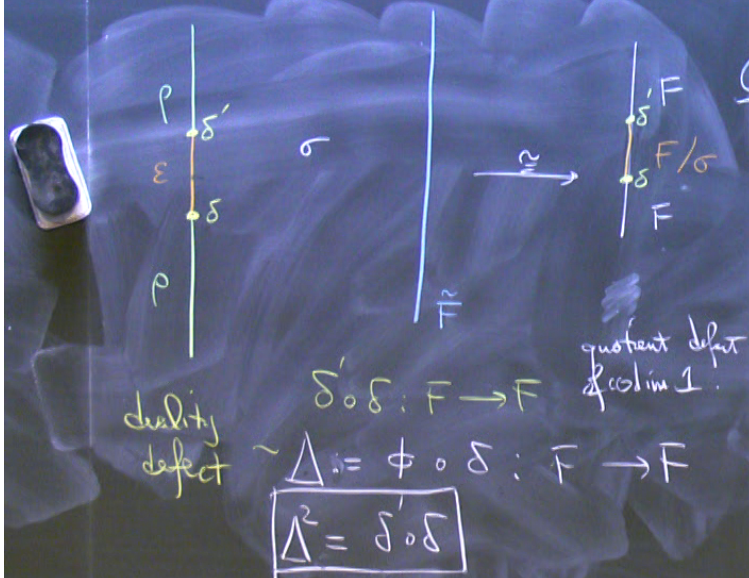
Ex: $n=2, m=3.$

$$T = \sum H A$$

$$T^\vee = \sum H A^\vee$$

$$\left. \begin{aligned} \sigma(pt) &= \text{Vect}[A] \\ \sigma^\vee(pt) &= \text{Vect}[A^\vee] \approx \text{Rep}(A) \end{aligned} \right\} \text{Morita } \cong$$

$$\left. \begin{aligned} \sigma(Y) &= \text{Fun}(H'(Y; A)) \\ \sigma^\vee(Y) &= \text{Fun}(H'(Y; A^\vee)) \end{aligned} \right\} \text{Pontryagin dual}$$



Q: Is there a left $(\sigma, \rho) = (\sigma^\vee, \rho^\vee)$ -module $\tilde{\lambda}$ st.

- 1) Δ is implemented
- 2) $\rho \otimes_{\sigma} \tilde{\lambda} =: \lambda$ invertible

(?)

Ex: $n=2, m=3$.

$$T = \sum H A$$

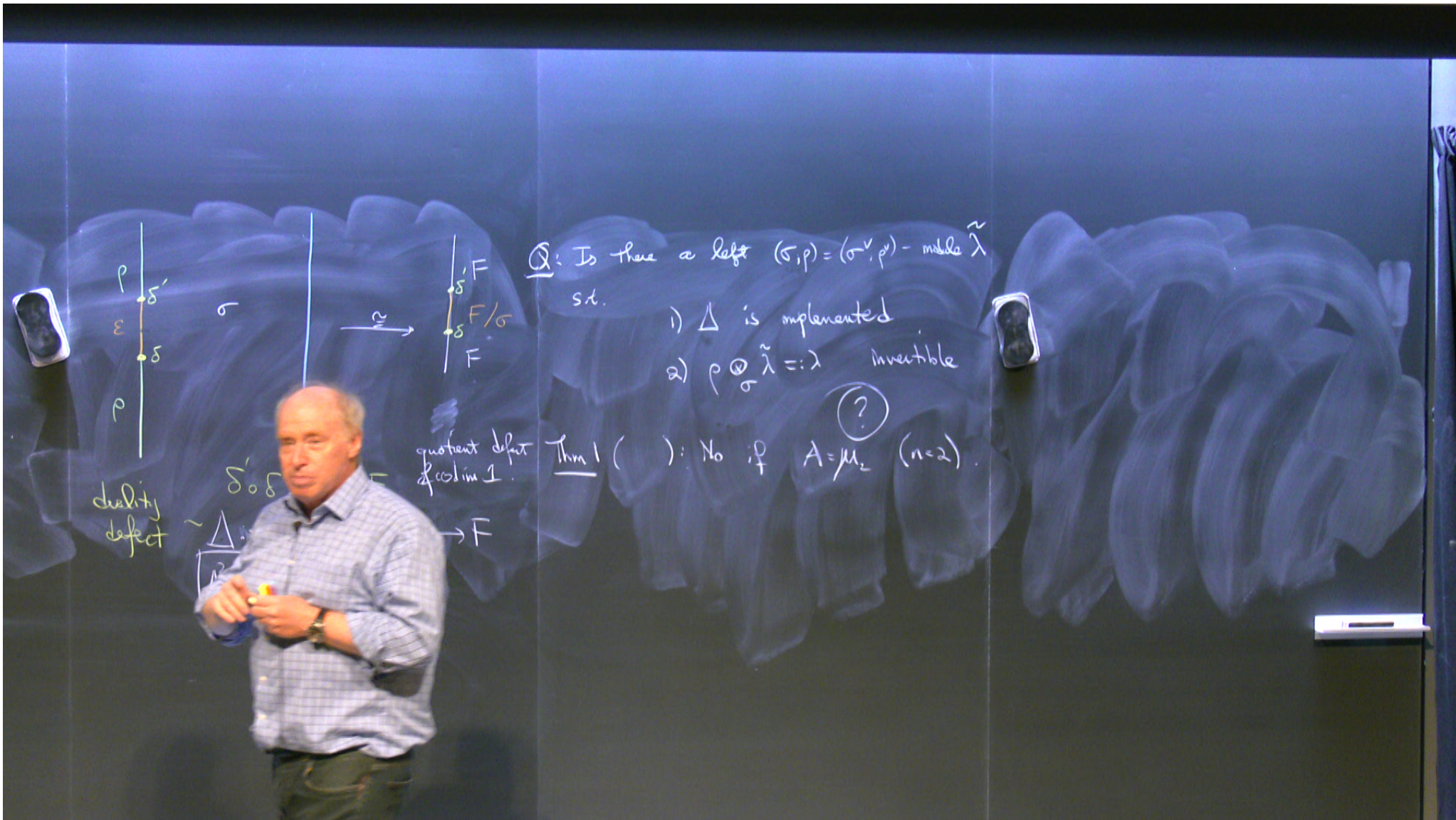
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$$\sigma(Y) = \text{Fun}(H(Y; A))$$

$$\sigma^\vee(Y) = \text{Fun}(H(Y; A^\vee)) \quad \leftarrow \text{Pontryagin dual}$$



$\delta' \circ \delta : F \rightarrow F$
 $\Delta = \phi \circ \delta : F \rightarrow F$
 $\Delta^2 = \delta' \circ \delta$

quotion defect
of codim 1

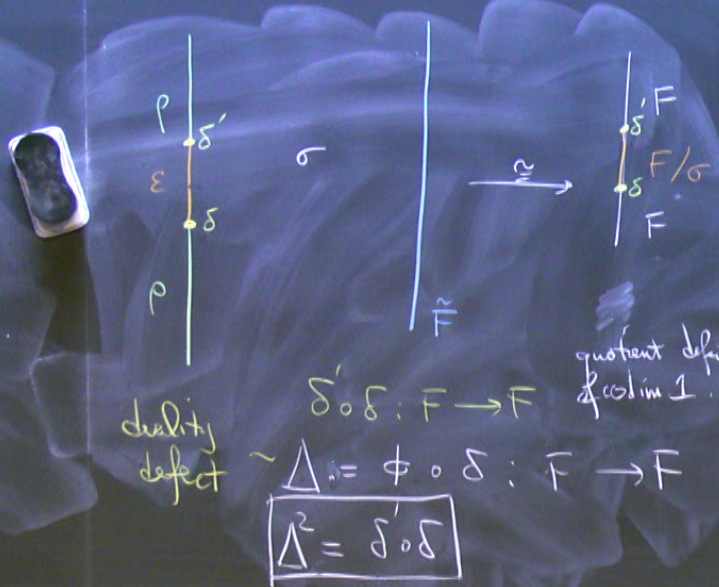
Q: Is there a left $(\sigma, \rho) = (\sigma^\vee, \rho^\vee)$ -module $\tilde{\lambda}$ s.t.

1) Δ is implemented

2) $\rho \otimes_{\sigma} \tilde{\lambda} =: \lambda$ invertible

Thm 1 (): No if $A = \mu_n$ ($n < 2$)

Proof 1: $\lambda(\rho)$ invertible Vect-module

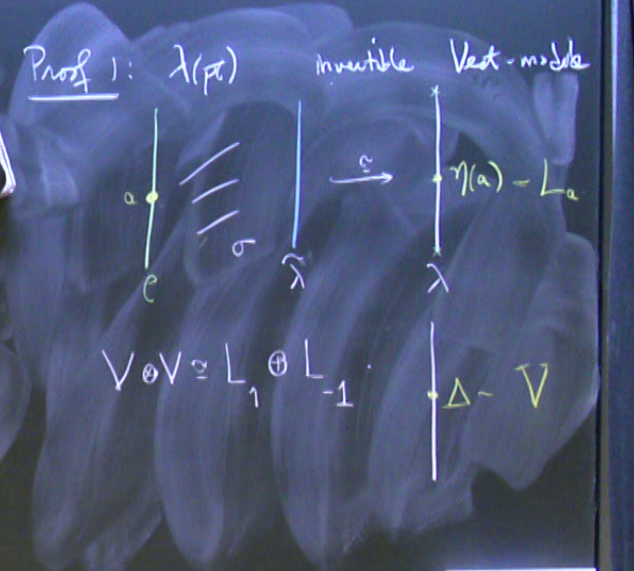


Q: Is there a left $(\sigma, \rho) = (\sigma^\vee, \rho^\vee)$ -module $\tilde{\lambda}$ s.t.

1) Δ is implemented

2) $\rho \otimes_{\sigma} \tilde{\lambda} =: \lambda$ invertible

Thm 1 (): No if $A = \mu_2$ ($n=2$)



2nd Proof: $1, g, \Delta$
 $g^2 = 1$
 $\Delta^2 = 1 + g$

No fiber functor.

3rd Proof:

$$\begin{aligned} (\sigma, \rho) &\hookrightarrow F_\rho & \sigma &= \sigma_{BA}^3 \\ (\sigma', \rho) &\hookrightarrow F_\rho^{\text{dual}} & \xrightarrow[\cong]{\phi_\rho} & F_{\rho'} \\ F_{\rho_c}/\sigma &= F_{\rho_c}^{\text{dual}} & \xrightarrow[\cong]{\phi_{\rho_c} \circ \phi} & F_{\rho_c} \end{aligned}$$

$\begin{array}{c} \rho \\ \delta' \\ \varepsilon \\ \delta \\ \rho \end{array}$
 $\xrightarrow{\sigma}$
 $\begin{array}{c} \delta' F \\ \delta F/\sigma \\ F \end{array}$

$\delta' \circ \delta: F \rightarrow F$
 $\sim \Delta := \phi \circ \delta: F \rightarrow F$
 $\boxed{\Delta^2 = \delta' \circ \delta}$

duality defect
 quotient defect
 Thm

2nd Proof: $1, g, \Delta$

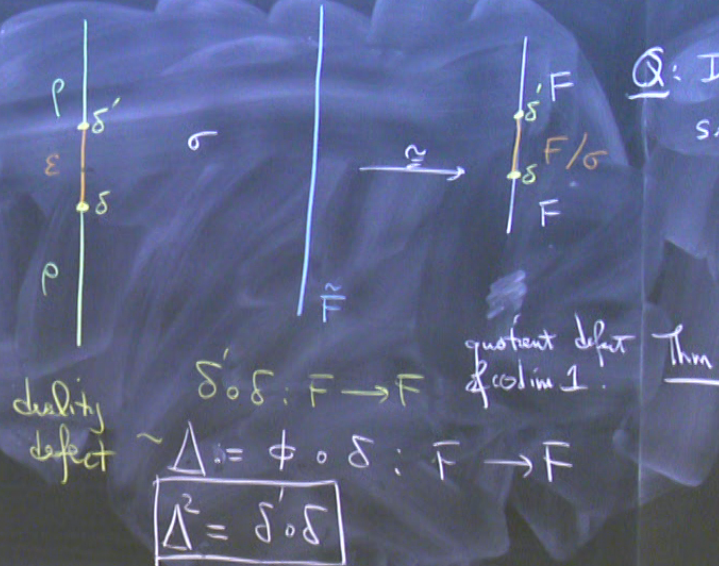
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3rd Prof: Id σ -model into A
has no $\sqrt{\quad}$.

$n=4$ Example



2nd Proof: $1, g, \Delta$

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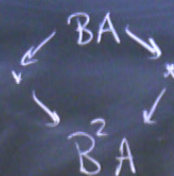
No fiber functor.

3rd Proof: Id σ -model into A
has no $\sqrt{\quad}$.

$n=4$ Example

$$T = \sum^2 HA$$

$$T^\vee = \sum^2 HA^\vee$$



$$\chi_T = B^2A$$

$n=4$ Example

$$T = \sum^2 HA$$

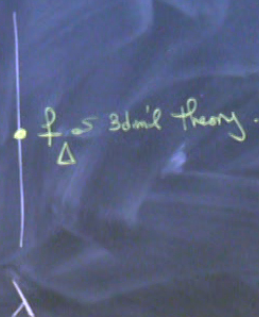
$$T^V = \sum^2 A^V$$

$$\chi_T = B^2 A$$

$\sigma =$

Thm 2: No $\tilde{\lambda}$ for $A = \mu_2$.

$$\begin{aligned} \varphi_{+ \Delta}^{\otimes 2} &= A\text{-gauge thry} \\ &= \sigma_{BA}^{(3)} \end{aligned}$$



Q: Is there a left $(\sigma, \rho) = (\sigma^V, \rho^V)$ -module $\tilde{\lambda}$ s.t.

- 1) Δ is implemented
- 2) $\rho \otimes_{\sigma} \tilde{\lambda} =: \lambda$ invertible

(?)

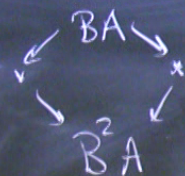
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$n=4$ Example

$$T = \sum^2 HA$$

$$T^V = \sum^2 HA^V$$

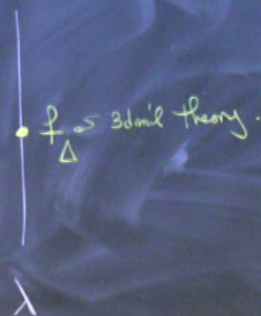


$$X_T = B^2A$$

$$\sigma = \sigma_{BA}^{(n)}[\Delta] / -$$

Thm 2: No $\tilde{\lambda}$ for $A = \mu_2$.

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Q: Is there a left $(\sigma, \rho) = (\sigma^V, \rho^V)$ -module $\tilde{\lambda}$ s.t.

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im 1 (): No φ $A = \mu_2$ (n=2) ?

