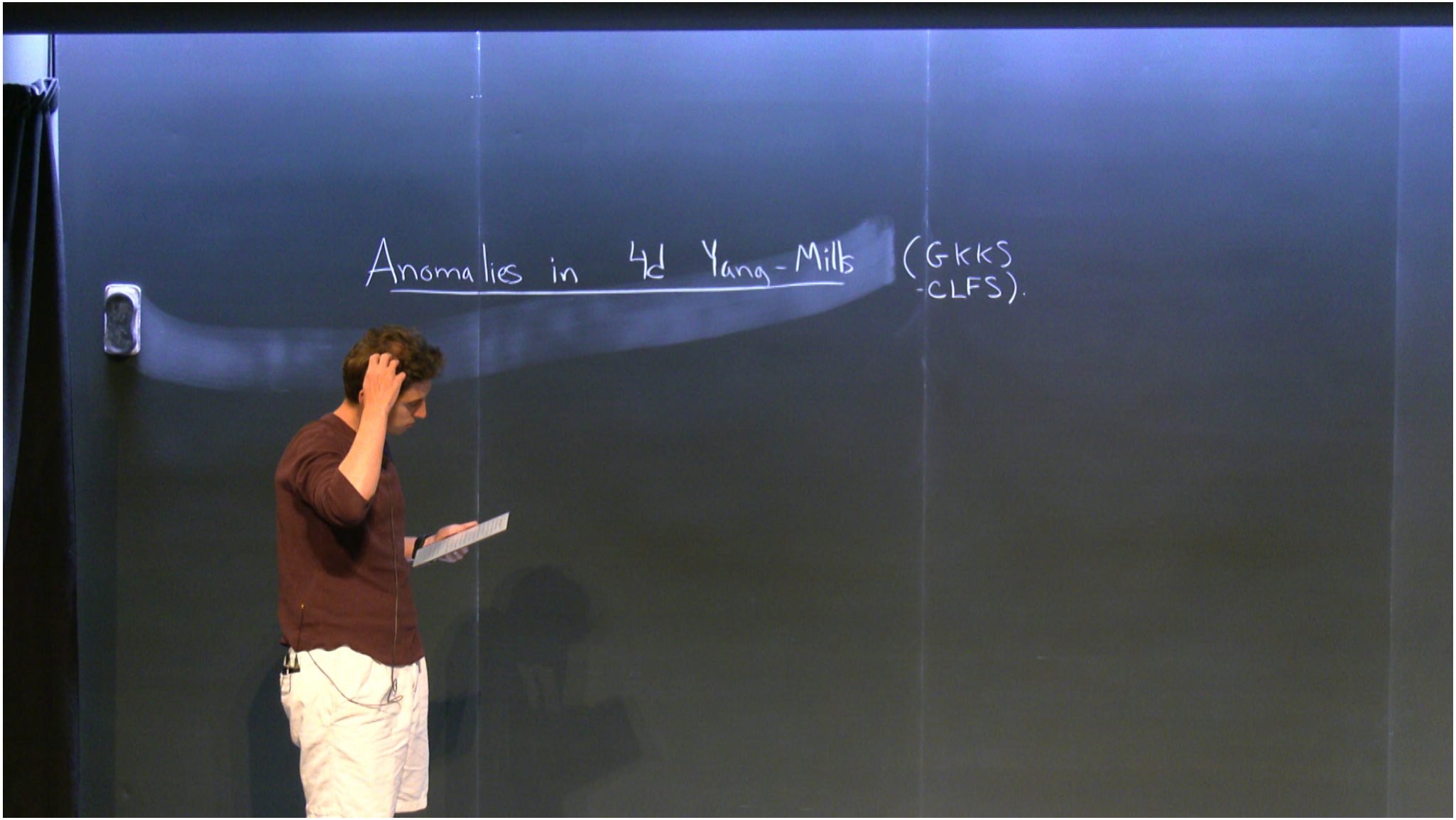
Title: Introduction to Anomalies in Quantum Field Theory

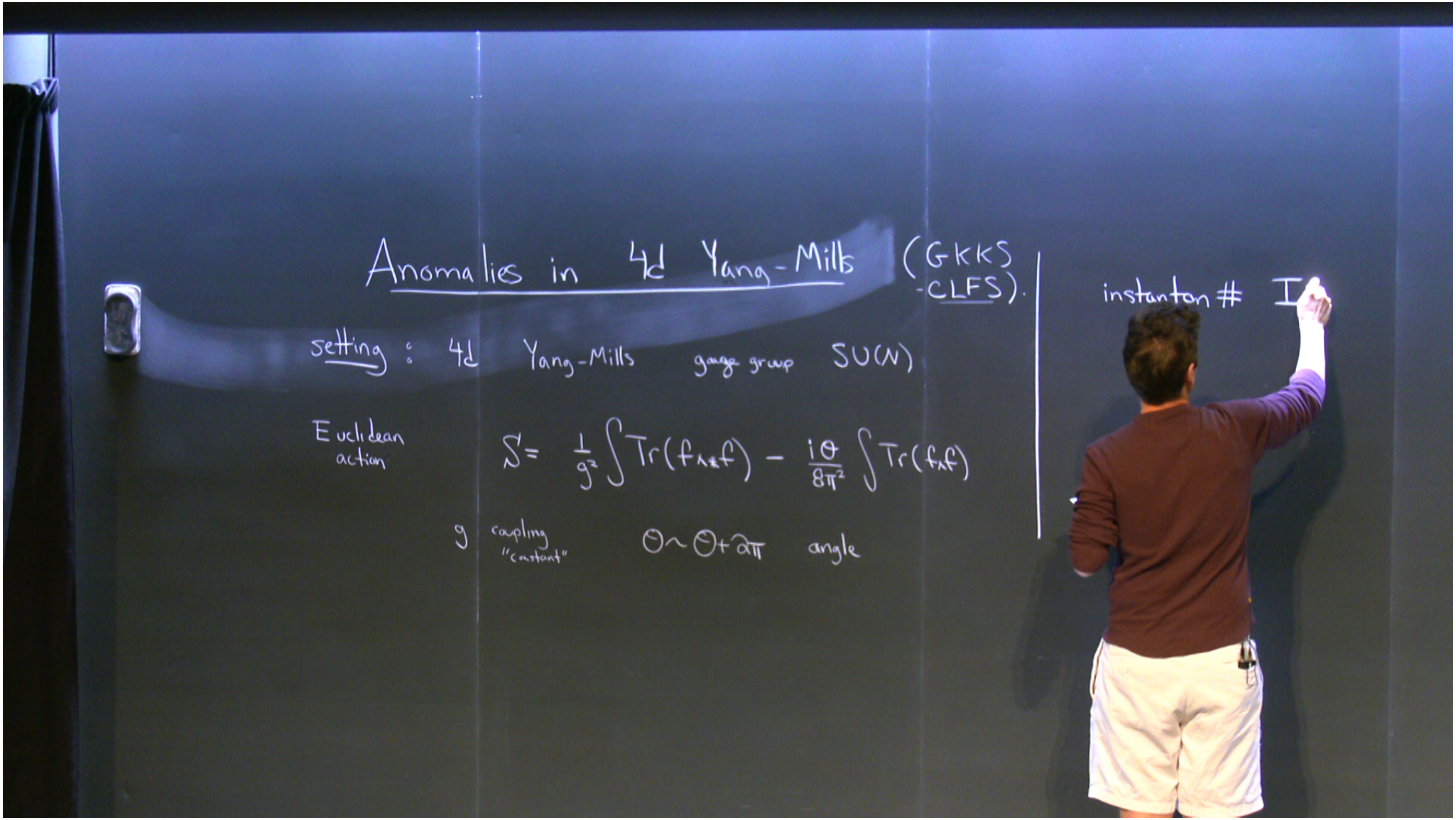
Speakers:

Collection: Global Categorical Symmetries

Date: June 17, 2022 - 1:30 PM

URL: https://pirsa.org/22060037


Anomalies in 4d Yang-Mills (GKKS
-CLFS).


Anomalies in 4d Yang-Mills (GKKS -CLFS).
setting : 4d Yang-Mills gauge group SU(N)
Euclidean action
$S = \frac{1}{g^2}\int \mathrm{Tr}(f_{\wedge *} f) - \frac{i\theta}{8\pi^2}\int \mathrm{Tr}(f_{\wedge} f)$
g coupling "constant"
$\theta \sim \theta + 2\pi$ angle
instanton # I

...alies in 4d Yang-Mills (GKKS-CLFS).

4d Yang-Mills gauge group $SU(N)$

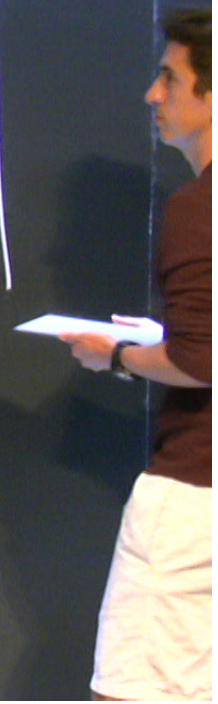
$$S = \frac{1}{g^2}\int \mathrm{Tr}(f\wedge *f) - \frac{i\theta}{8\pi^2}\int \mathrm{Tr}(f\wedge f)$$

g coupling "constant" $\quad \theta \sim \theta + 2\pi$ angle

instanton # $I = \frac{1}{8\pi^2}\int_X \mathrm{Tr}(f\wedge f) = c_2 \in \mathbb{Z}$.

partition function defined by functional integral

$$Z = \sum_{I\in\mathbb{Z}} \int_{\text{space of connections, fixed bundle topology}} Da \; \exp\left(-\frac{1}{g^2}\int \mathrm{Tr}(f\wedge f) + i\theta I\right)$$



(GKKS -CLFS).

(N)

($f_A f$)

instanton # $I = \frac{1}{8\pi^2}\int_X \mathrm{Tr}(f_A f) = c_2 \in \mathbb{Z}$.

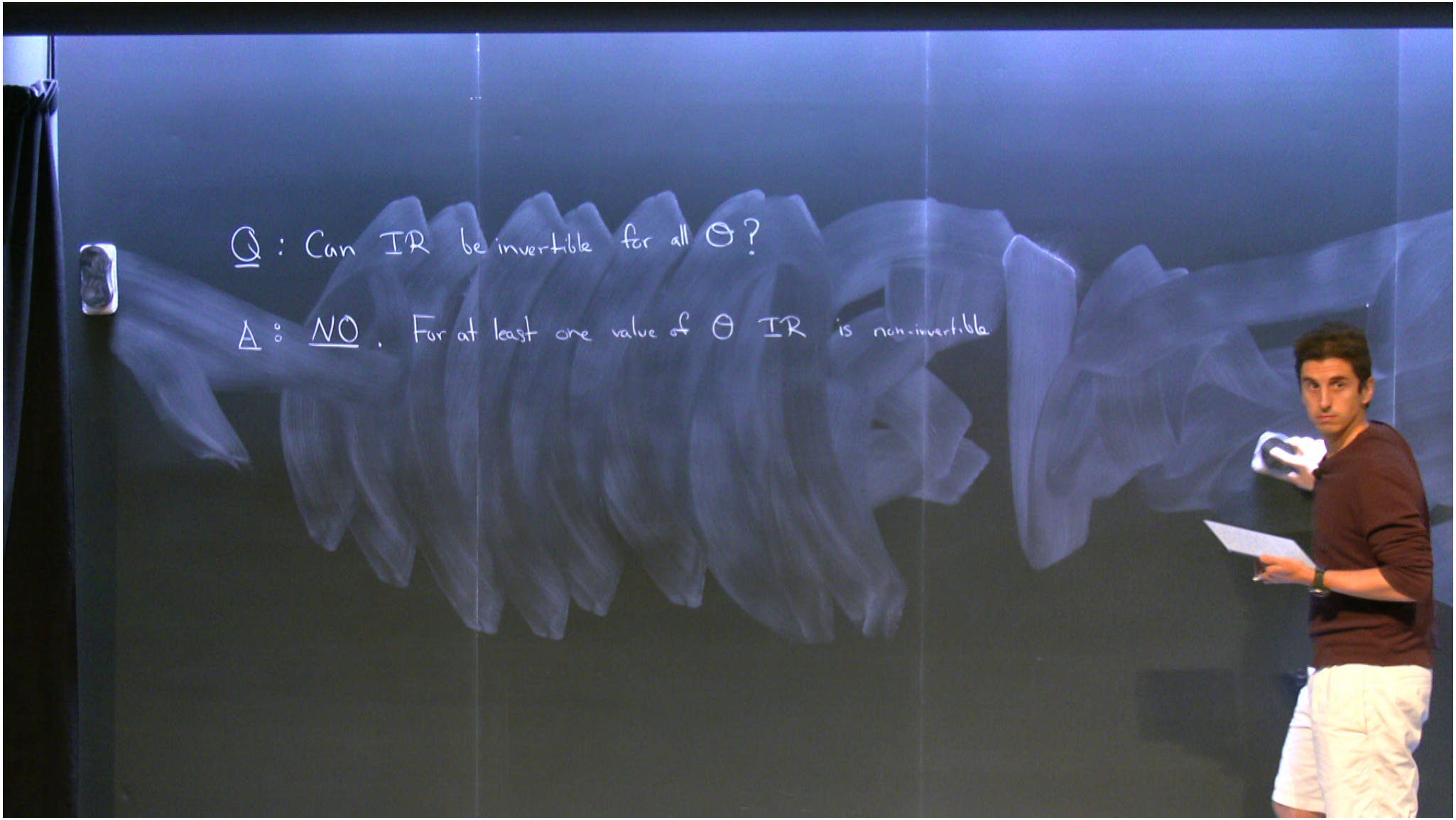
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θ controls how different bundle topologies contribute to sum.

Defines an RG flow. What happens in IR?

Lore: at $\theta = 0$, IR trivially gapped = invertible


Q : Can IR be invertible for all Θ?
A : NO. For at least one value of Θ IR is non-invertible

be invertible for all θ?

at least one value of θ IR is non-invertible

Symmetries

center of $SU(N) \cong \mathbb{Z}_N$ $\Rightarrow$ one-form symmetry $\mathbb{Z}_N^{(1)}$

Charged objects = Wilson lines (detect confinement)

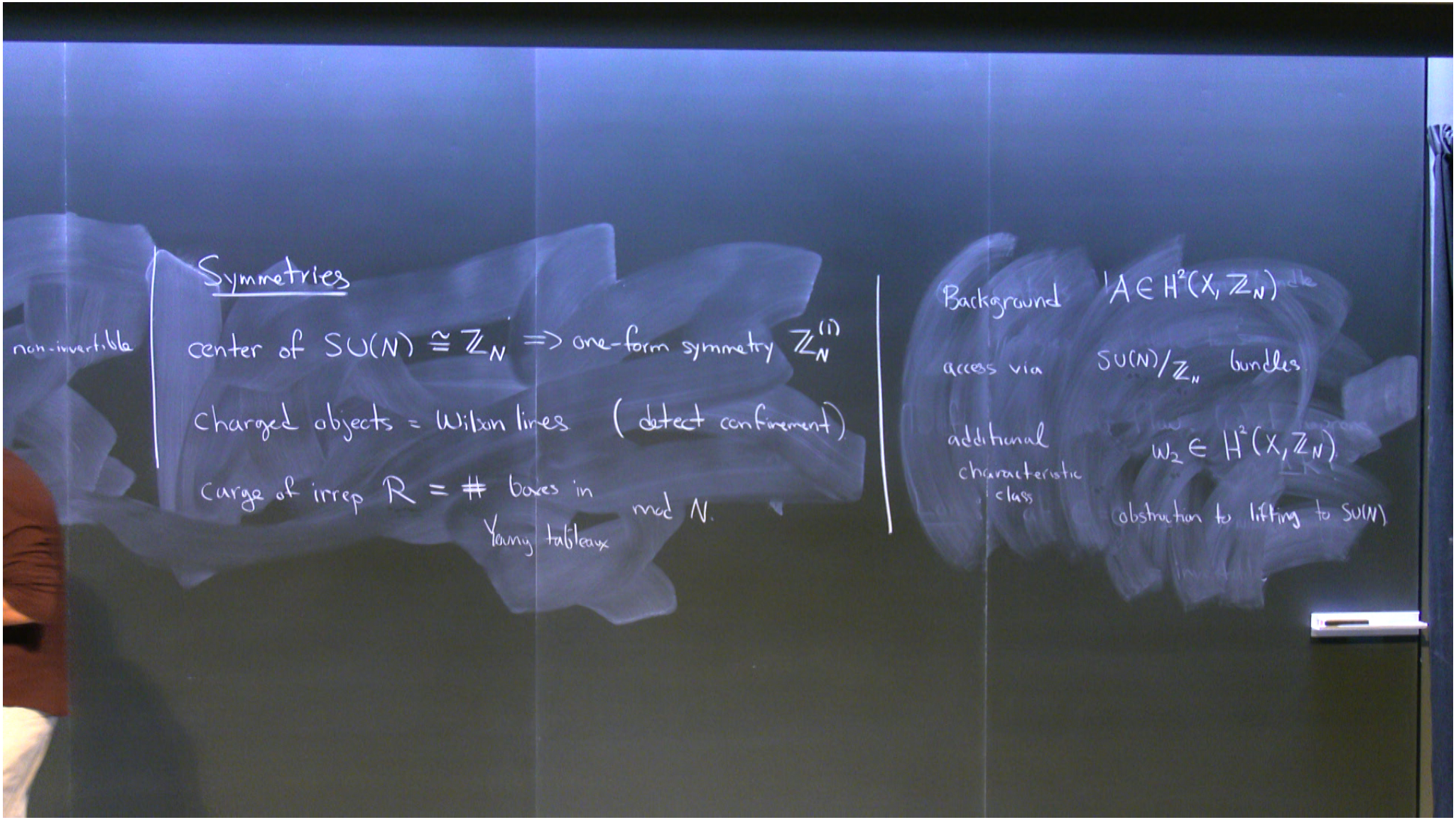
Charge of irrep R = # boxes in Young tableaux mod N.

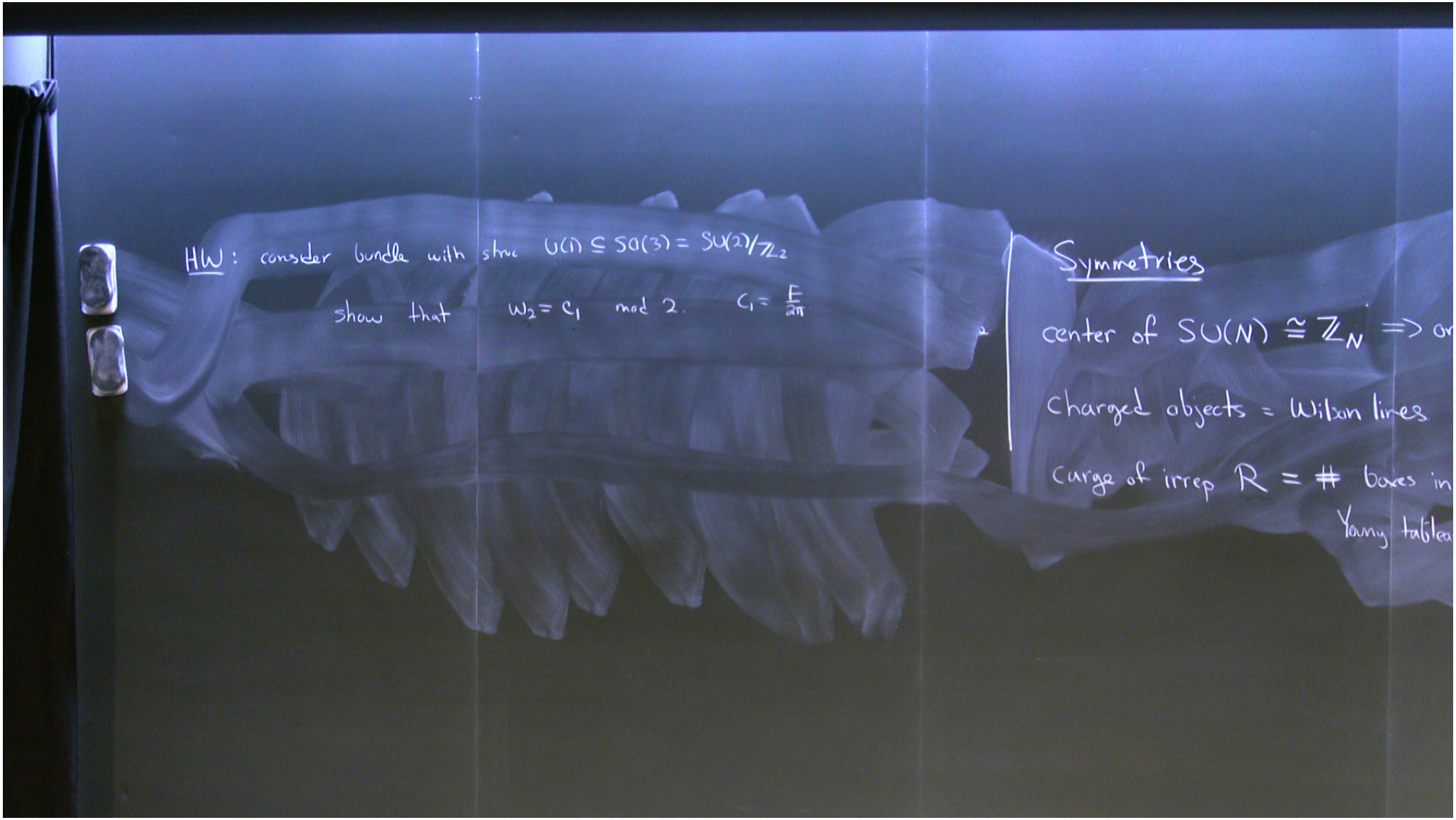
θ con

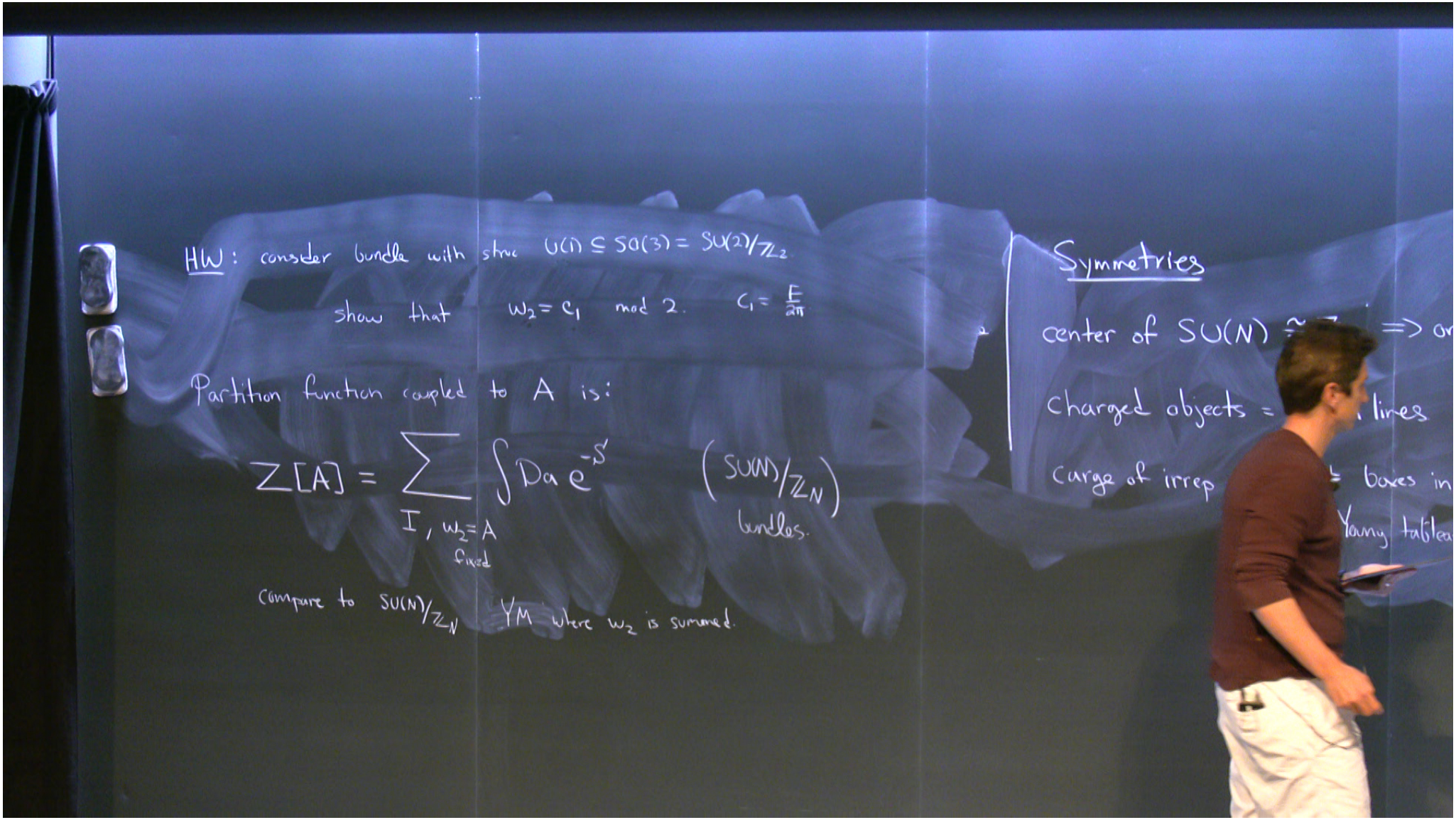
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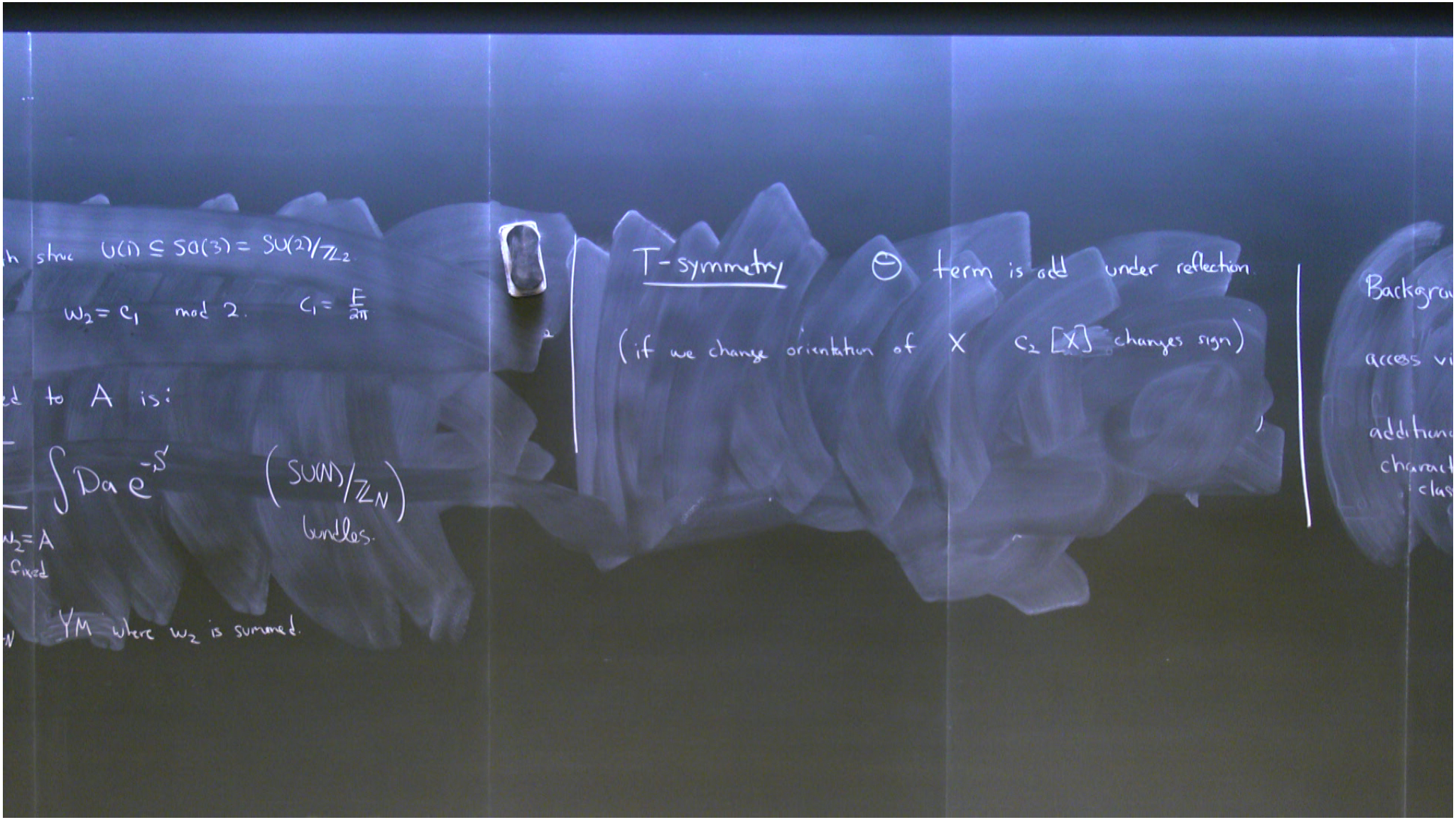
Defines

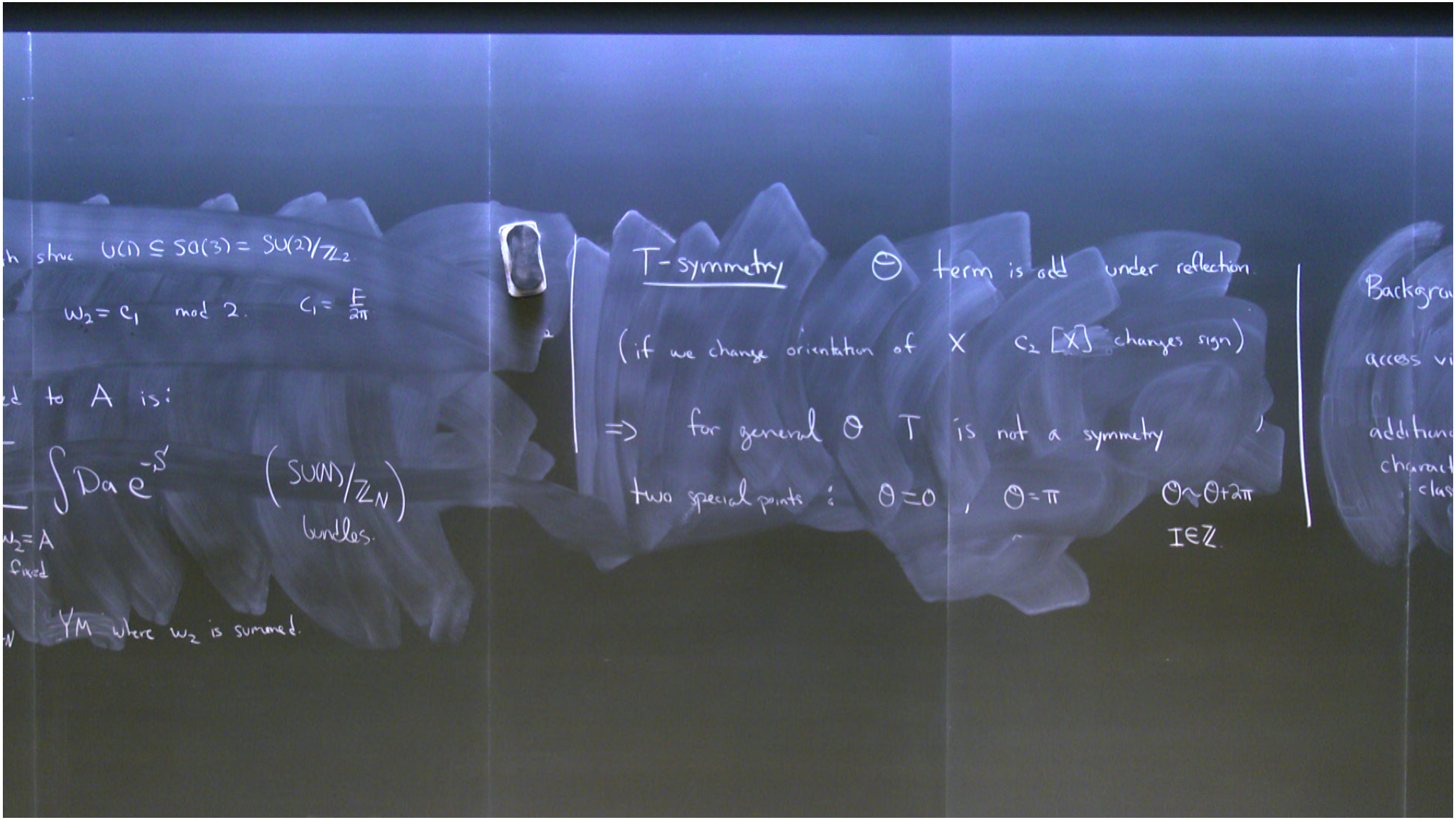
Lore

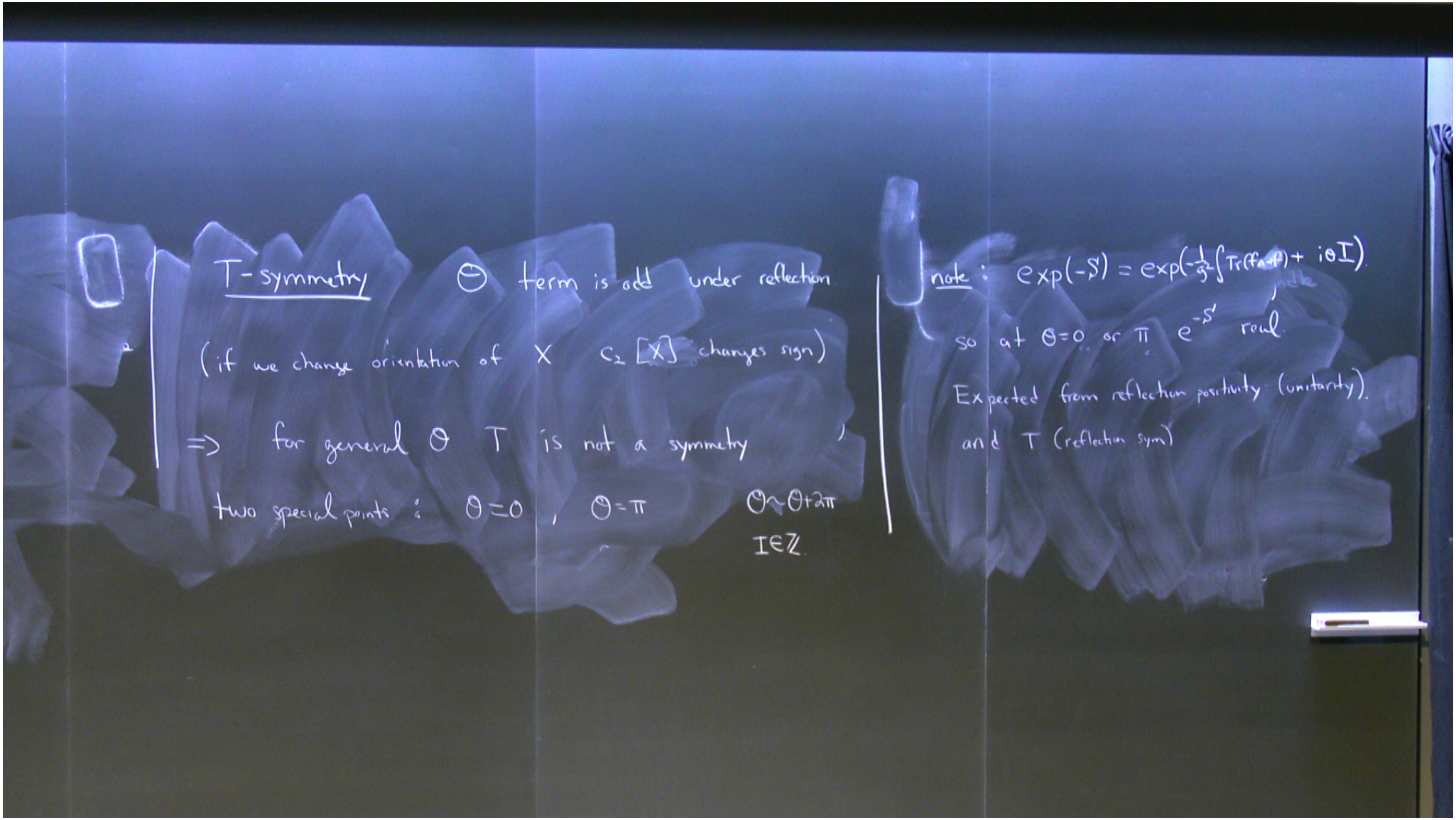

Symmetries
non-invertible
center of $SU(N) \cong \mathbb{Z}_N$ => one-form symmetry $\mathbb{Z}_N^{(1)}$
charged objects = Wilson lines (detect confinement)
charge of irrep R = # boxes in Young tableaux mod N.
Background $A \in H^2(X, \mathbb{Z}_N)$
access via $SU(N)/\mathbb{Z}_N$ bundles
additional characteristic class $w_2 \in H^2(X, \mathbb{Z}_N)$
obstruction to lifting to SU(N)

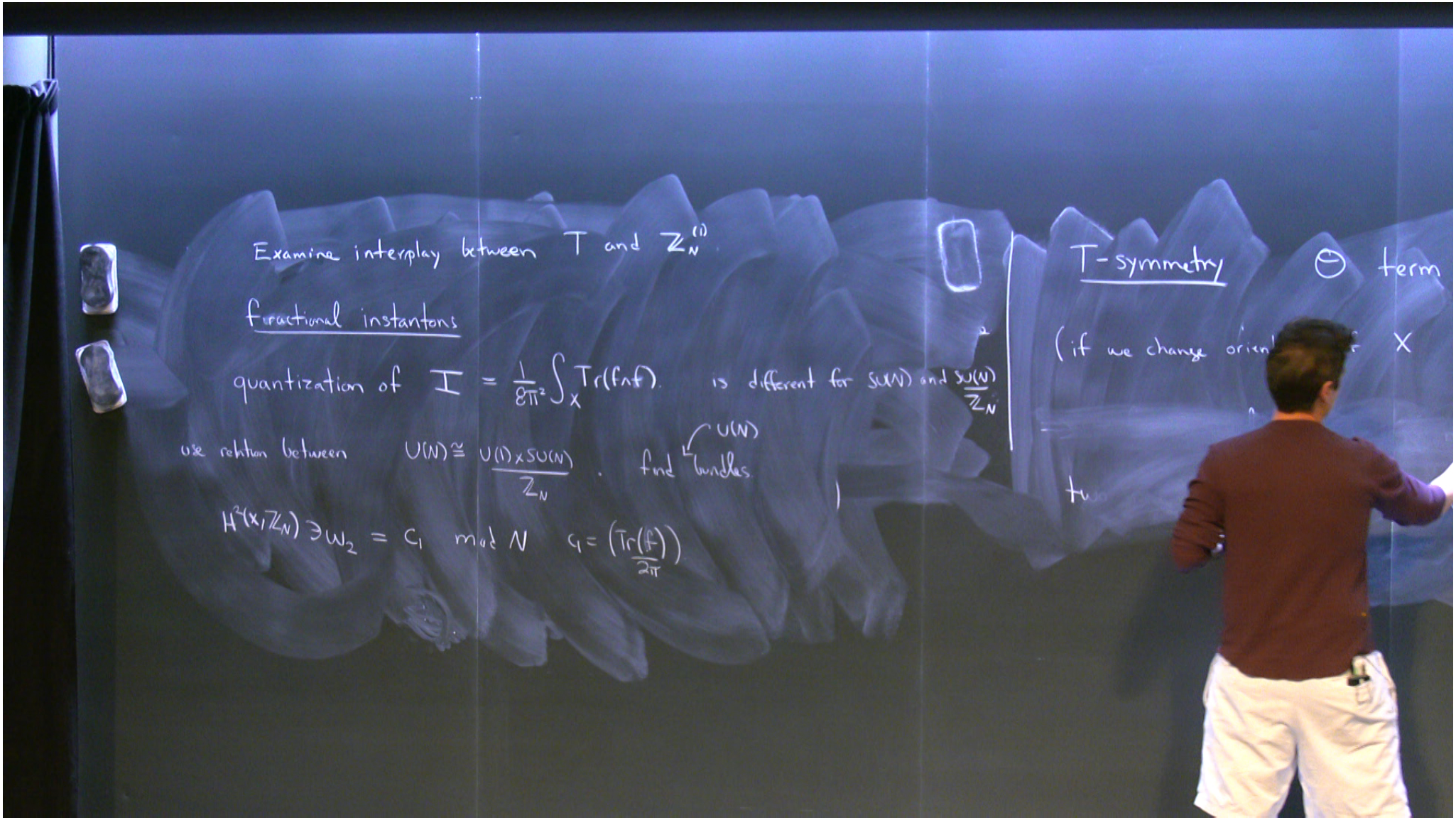

HW: consider bundle with struc $U(1) \subseteq SO(3) = SU(2)/\mathbb{Z}_2$
show that $w_2 = c_1 \mod 2$. $c_1 = \frac{F}{2\pi}$
Symmetries
center of $SU(N) \cong \mathbb{Z}_N$ =>
charged objects = Wilson lines
charge of irrep R = # boxes in
Young tableau

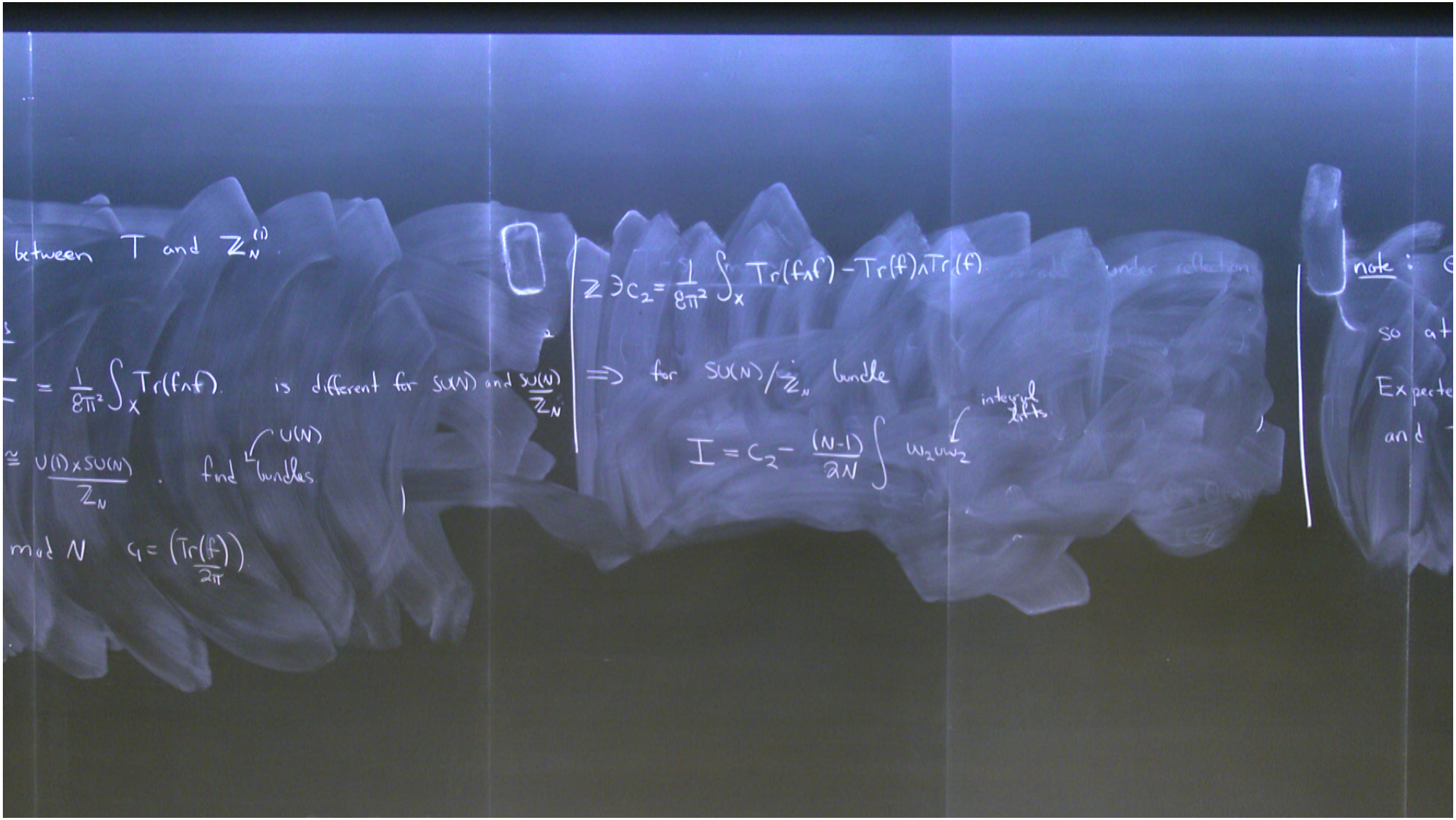

HW: consider bundle with struc $U(1) \subseteq SO(3) = SU(2)/\mathbb{Z}_2$
show that $w_2 = c_1 \mod 2$. $c_1 = \frac{F}{2\pi}$
Partition function coupled to A is:
$Z[A] = \sum_{I,\ w_2 = A \text{ fixed}} \int Da\, e^{-S}$ $\left(\frac{SU(N)}{\mathbb{Z}_N}\right.$ bundles.
Compare to $SU(N)/\mathbb{Z}_N$ YM where w_2 is summed.
Symmetries
center of SU(N)
Charged objects =
lines
Charge of irrep
boxes in
Young tableau

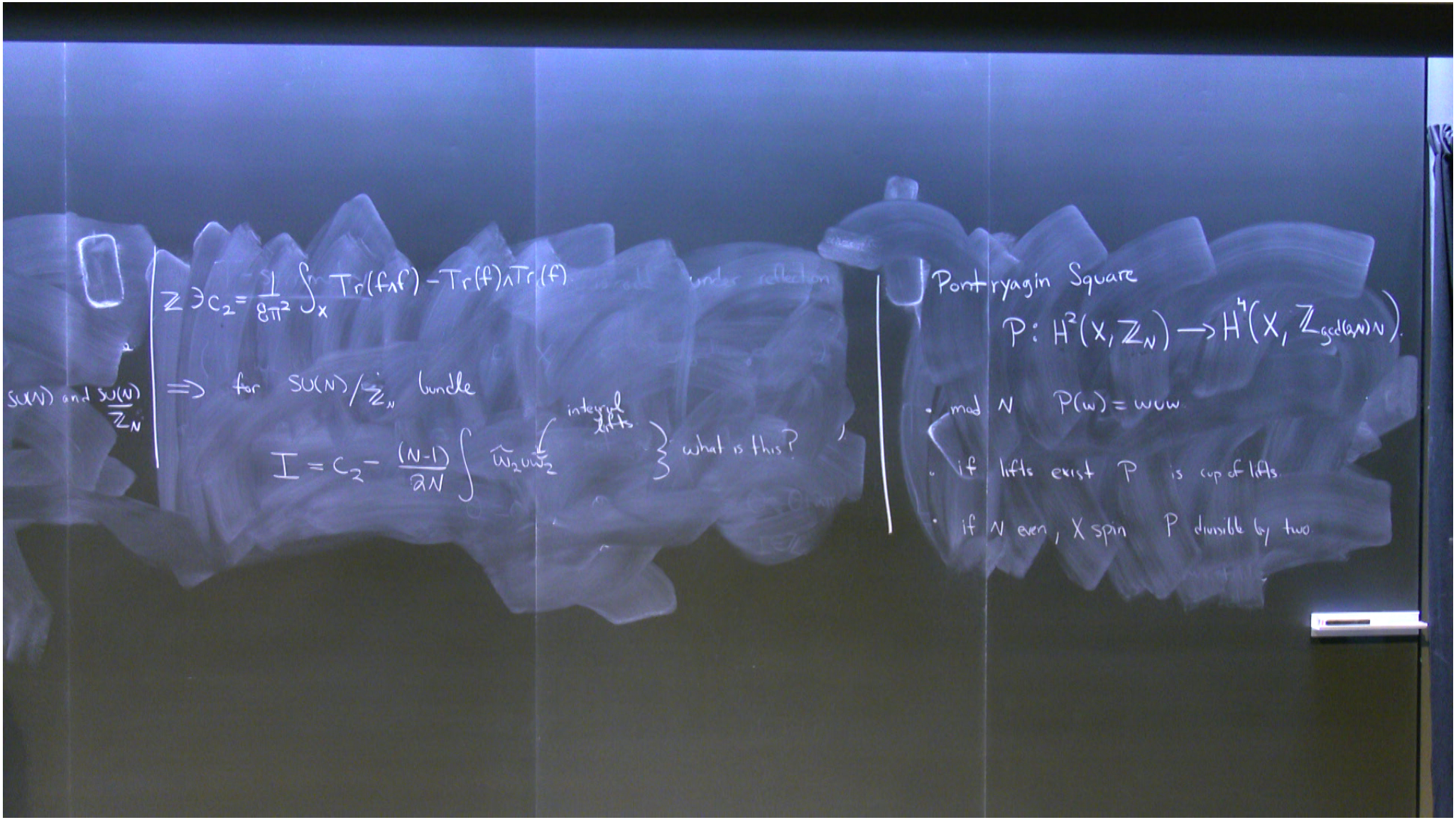

$U(1) \subseteq SO(3) = SU(2)/\mathbb{Z}_2$
$w_2 = c_1 \mod 2. \quad c_1 = \frac{F}{2\pi}$
to A is:
$\int Da\, e^{-S}$
$(SU(N)/\mathbb{Z}_N$ bundles.
$w_2 = A$ fixed
YM where w_2 is summed.
T-symmetry
$\ominus$ term is odd under reflection
(if we change orientation of X $c_2[X]$ changes sign)
Backgrou
access vi
additiona
charact
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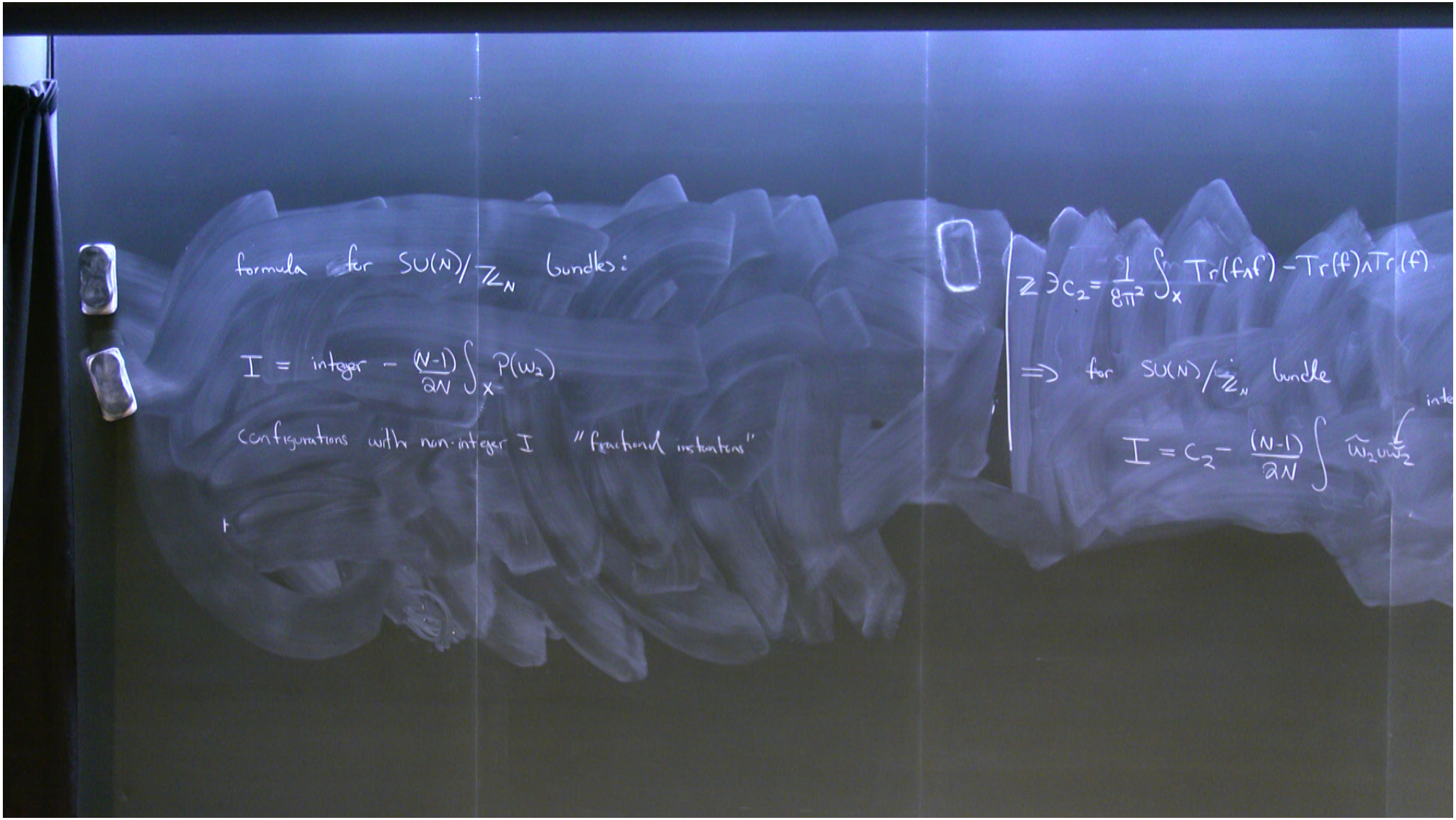

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T-symmetry θ term is odd under reflection
(if we change orientation of X $c_2[X]$ changes sign)
$\Rightarrow$ for general θ T is not a symmetry
two special points: $\theta = 0$, $\theta = \pi$
$\theta \sim \theta + 2\pi$
$I \in \mathbb{Z}$
Backgrou
access vi
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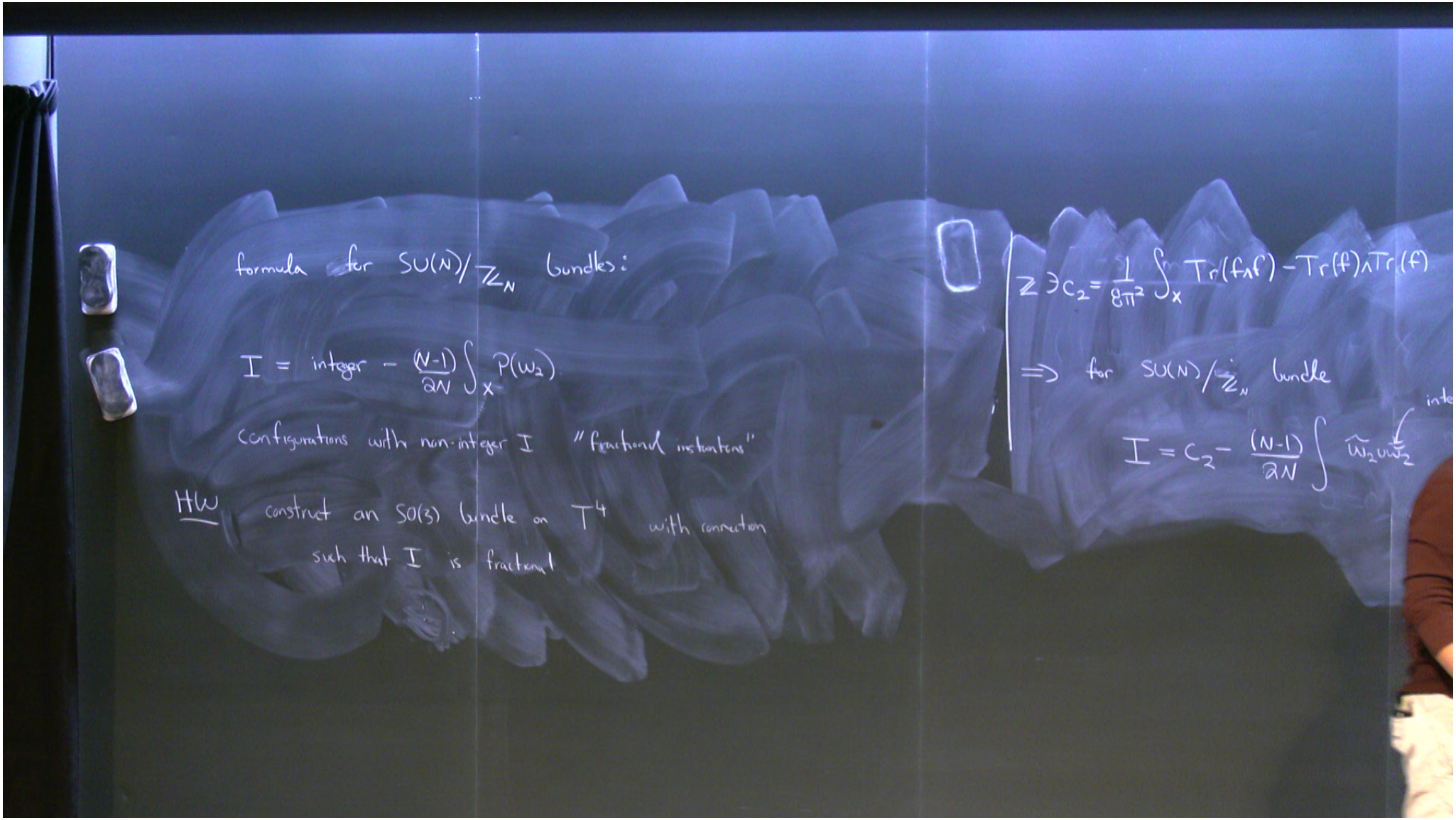

T-symmetry
θ term is odd under reflection
(if we change orientation of X $c_2[X]$ changes sign)
⇒ for general θ T is not a symmetry
two special points: θ=0, θ=π
θ~θ+2π
I∈ℤ
note: $\exp(-S) = \exp(-\frac{1}{g^2}\int \mathrm{Tr}(f_{\mu\nu}f^{\mu\nu}) + i\theta I)$
so at θ=0 or π e^{-S} real
Expected from reflection positivity (unitarity)
and T (reflection sym)

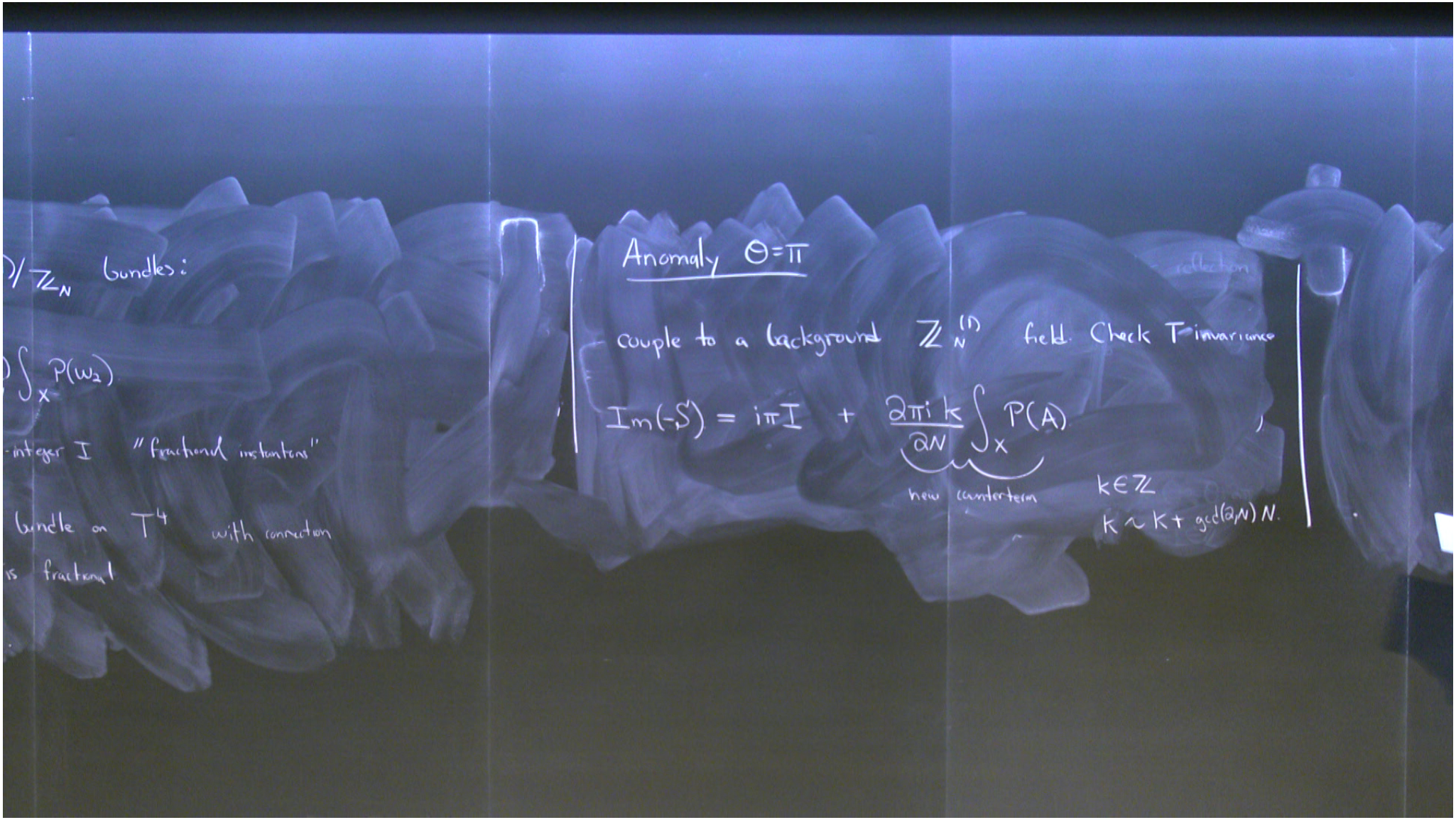

Examine interplay between T and $\mathbb{Z}_N^{(1)}$
fractional instantons
quantization of $I = \frac{1}{8\pi^2}\int_X \mathrm{Tr}(f\wedge f)$ is different for SU(N) and $\frac{SU(N)}{\mathbb{Z}_N}$
use relation between $U(N) \cong \frac{U(1)\times SU(N)}{\mathbb{Z}_N}$. find U(N) bundles
$H^2(X,\mathbb{Z}_N) \ni w_2 = c_1 \mod N \quad c_1 = \left(\frac{\mathrm{Tr}(f)}{2\pi}\right)$
T-symmetry
θ term
(if we change orien
X
tw

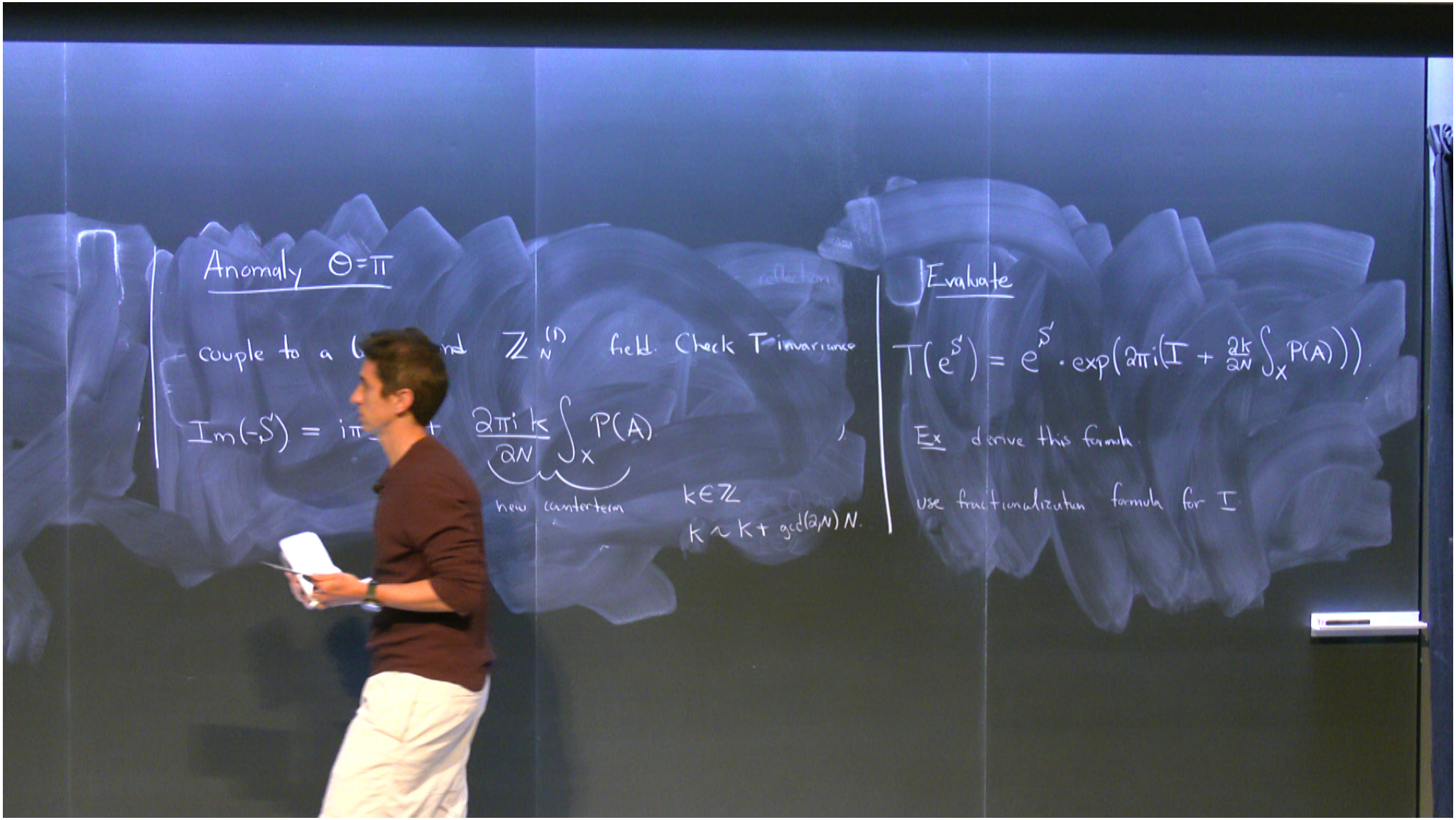

between T and $\mathbb{Z}_N^{(1)}$
$= \frac{1}{8\pi^2}\int_X \mathrm{Tr}(f \wedge f)$ is different for SU(N) and $\frac{SU(N)}{\mathbb{Z}_N}$
$\cong \frac{U(1)\times SU(N)}{\mathbb{Z}_N}$. find U(N) bundles
mod N $c_1 = \left(\frac{\mathrm{Tr}(f)}{2\pi}\right)$
$\mathbb{Z} \ni c_2 = \frac{1}{8\pi^2}\int_X \mathrm{Tr}(f \wedge f) - \mathrm{Tr}(f)\wedge\mathrm{Tr}(f)$
$\Rightarrow$ for $SU(N)/\mathbb{Z}_N$ bundle
$I = c_2 - \frac{(N-1)}{2N}\int w_2 \cup w_2$ integral lifts
note :
so at
Expecte
and

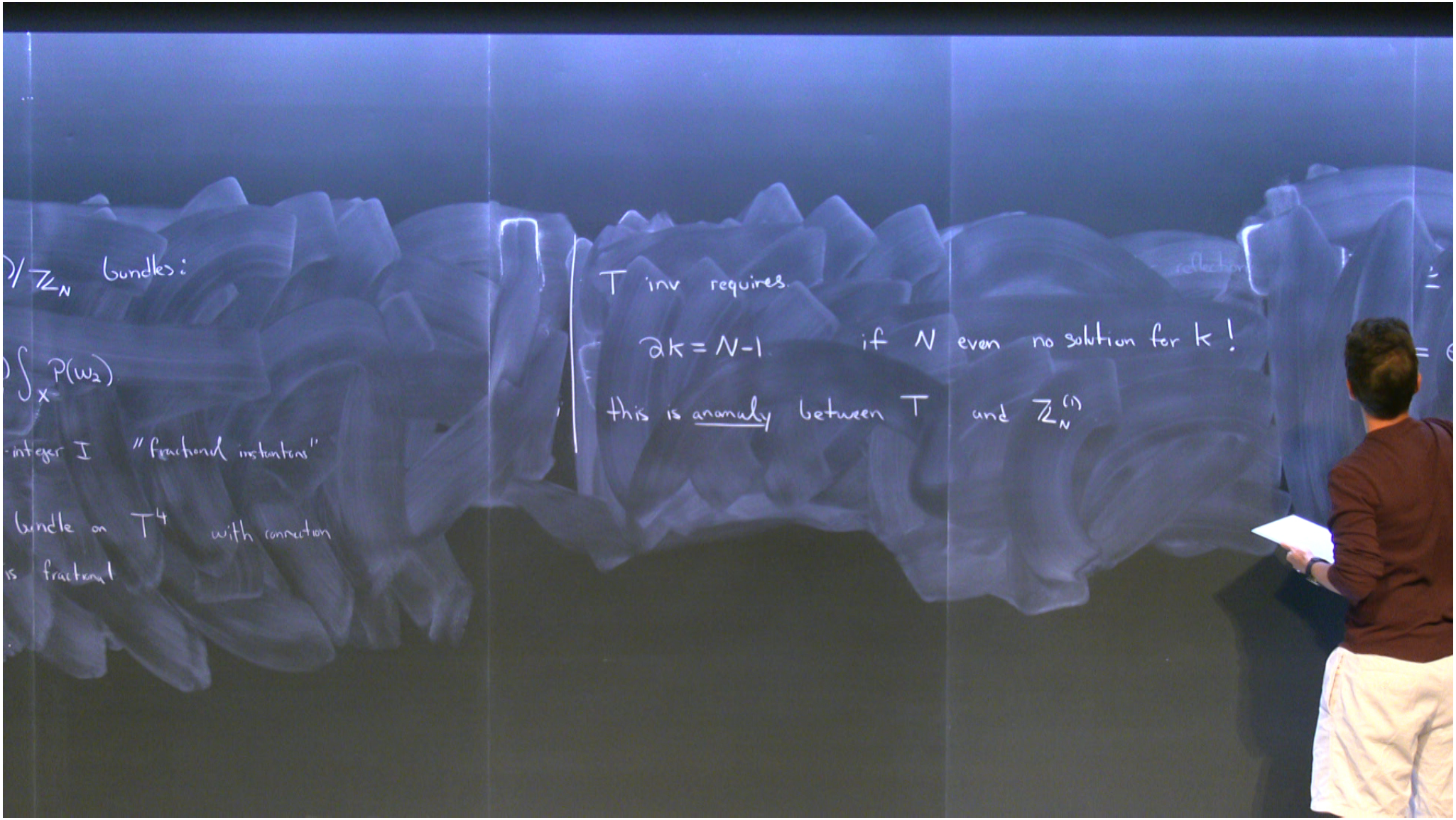

$\mathbb{Z} \ni c_2 = \frac{1}{8\pi^2}\int_X \mathrm{Tr}(f \wedge f) - \mathrm{Tr}(f)\wedge\mathrm{Tr}(f)$
$SU(N)$ and $\frac{SU(N)}{\mathbb{Z}_N}$
$\Rightarrow$ for $SU(N)/\mathbb{Z}_N$ bundle
$I = c_2 - \frac{(N-1)}{2N}\int \widetilde{w}_2 \cup \widetilde{w}_2$
integral lifts
what is this?
Pontryagin Square
$P: H^2(X, \mathbb{Z}_N) \to H^4(X, \mathbb{Z}_{\gcd(2,N)N})$
• mod N $P(w) = w \cup w$
• if lifts exist P is cup of lifts
• if N even, X spin P divisible by two


formula for $SU(N)/\mathbb{Z}_N$ bundles:
$I = \text{integer} - \frac{(N-1)}{2N}\int_X P(w_2)$
configurations with non-integer I "fractional instantons"
$\mathbb{Z} \ni c_2 = \frac{1}{8\pi^2}\int_X \mathrm{Tr}(f\wedge f) - \mathrm{Tr}(f)\wedge\mathrm{Tr}(f)$
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formula for $SU(N)/\mathbb{Z}_N$ bundles:
$I = \text{integer} - \frac{(N-1)}{2N}\int_X P(w_2)$
configurations with non-integer I "fractional instantons"
HW construct an $SO(3)$ bundle on T^4 with connection such that I is fractional
$\mathbb{Z} \ni c_2 = \frac{1}{8\pi^2}\int_X \mathrm{Tr}(f\wedge f) - \mathrm{Tr}(f)\wedge\mathrm{Tr}(f)$
$\Rightarrow$ for $SU(N)/\mathbb{Z}_N$ bundle
$I = c_2 - \frac{(N-1)}{2N}\int \widetilde{w}_2 \cup \widetilde{w}_2$
inte


$/\mathbb{Z}_N$ bundles:
$\int_X P(w_2)$
integer I "fractional instantons"
bundle on T^4 with connection
is fractional
Anomaly $\Theta=\pi$
couple to a background $\mathbb{Z}_N^{(1)}$ field. Check T-invariance
$Im(-S) = i\pi I + \frac{2\pi i k}{2N}\int_X P(A)$
new counterterm
$k \in \mathbb{Z}$
$k \sim k + \gcd(2,N)\,N$.


Anomaly $\Theta=\pi$
couple to a b nd $\mathbb{Z}_N^{(1)}$ field. Check T-invariance
$Im(-S) = i\pi$ + $\frac{2\pi i k}{2N}\int_X P(A)$
new counterterm
$k \in \mathbb{Z}$
$k \sim k + \gcd(2,N)\,N$.
Evaluate
$T(e^S) = e^S \cdot \exp(2\pi i(I + \frac{2k}{2N}\int_X P(A)))$
Ex derive this formula
use fractionalization formula for I


$/\mathbb{Z}_N$ bundles:
$\int_X P(w_2)$
integer I "fractional instantons"
bundle on T^4 with connection
is fractional
T inv requires
$2k = N-1$ if N even no solution for k!
this is anomaly between T and $\mathbb{Z}_N^{(1)}$

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$2k = N-1$ if N even no solution for k!

this is anomaly between T and $\mathbb{Z}_N^{(1)}$

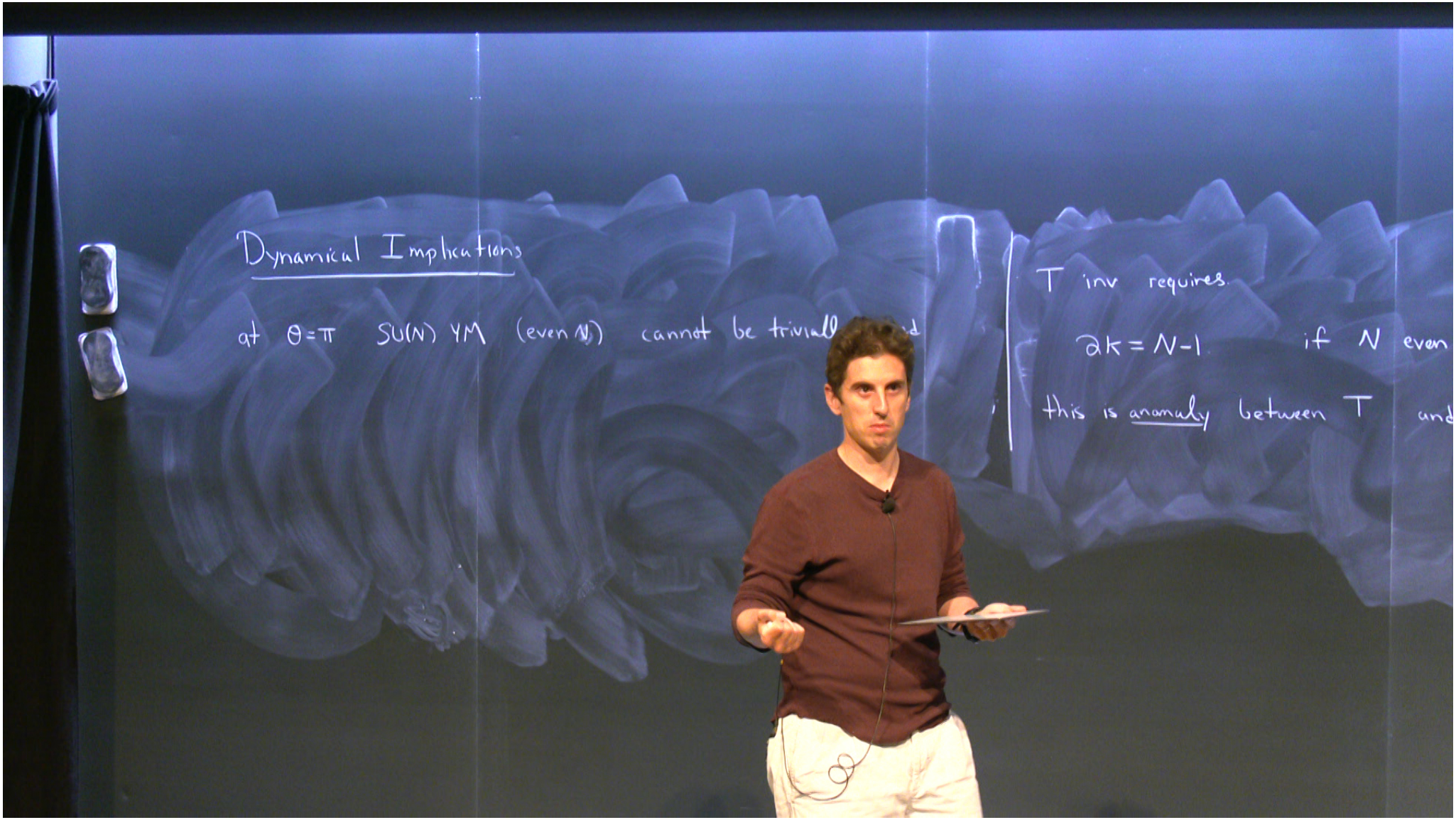
note: failure of T inv is only by background fields.

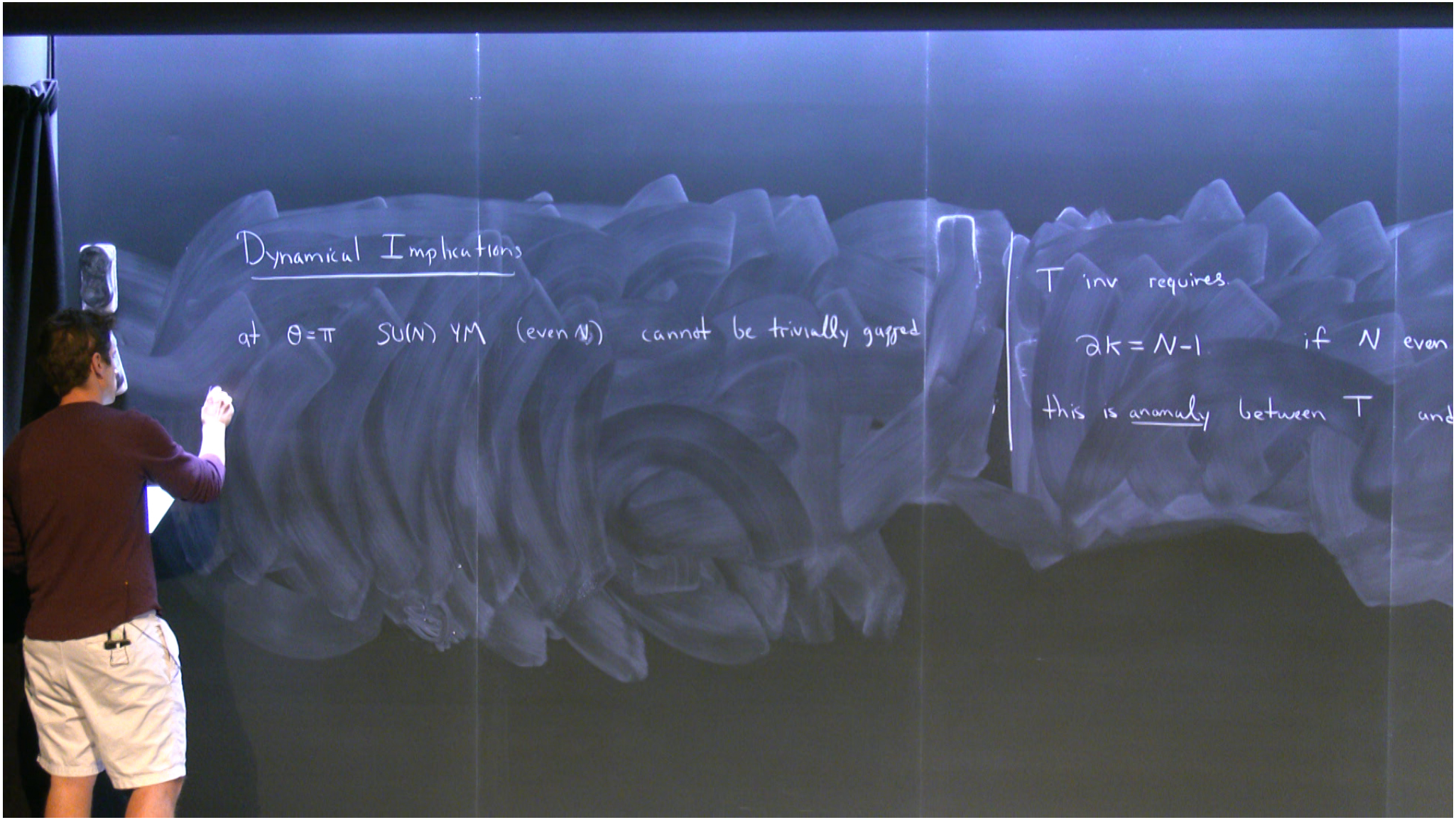
associated 5d invertible theory defined on manifolds without orientation (T sym) and $\mathbb{Z}_N^{(1)}$

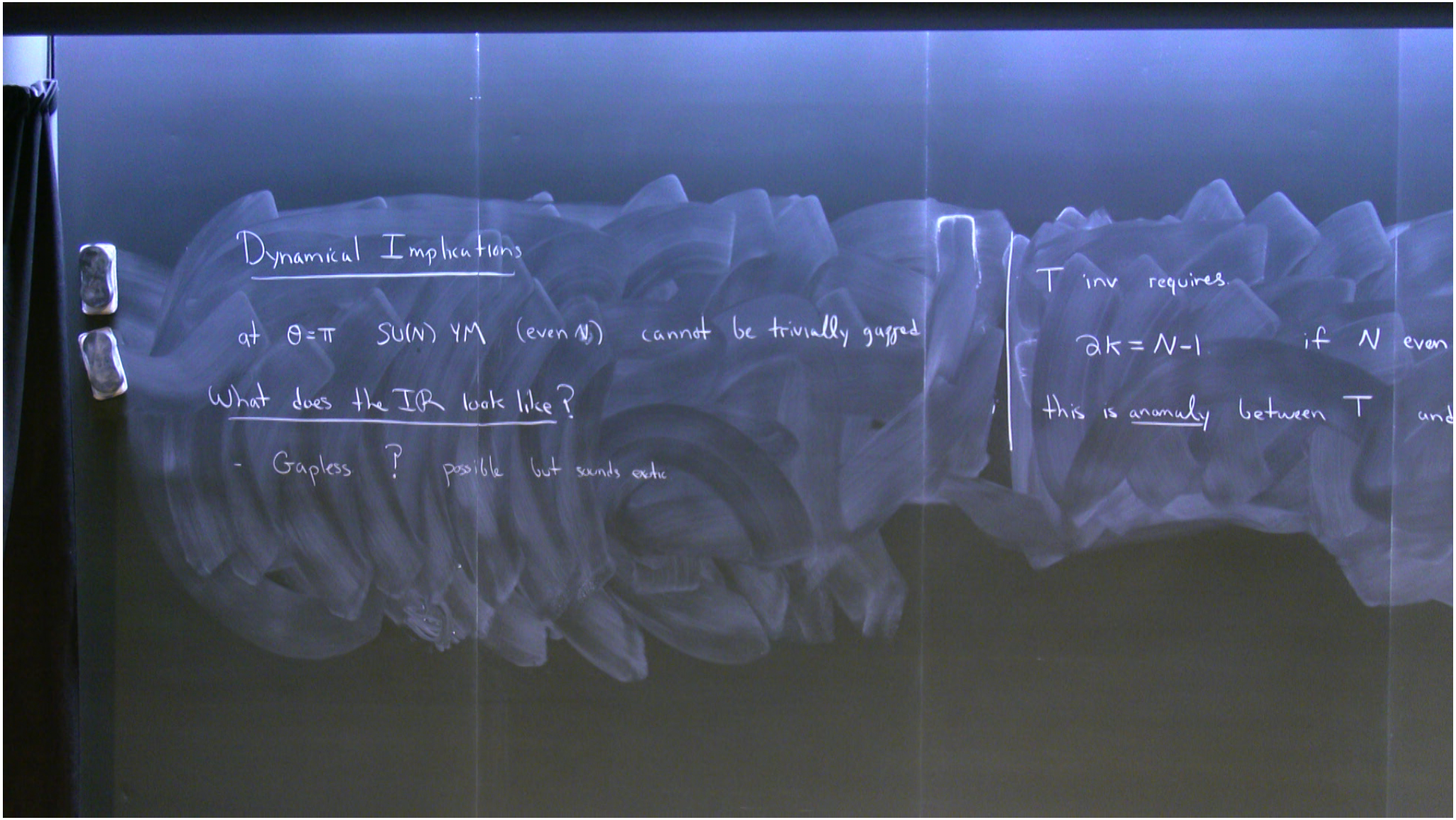
($\mathbb{Z}_2^{(1)}$ subgroup)

$$\exp\left(i\pi \int_Y \widehat{w}_1(TY) \cup P(A)\right)$$

integral uplift of w_1 with twisted integral coeffs.


Dynamical Implications
at θ=π SU(N) YM (even N) cannot be trivial
T inv requires
2k = N-1 if N even
this is anomaly between T


Dynamical Implications
at θ=π SU(N) YM (even N) cannot be trivially gapped
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Dynamical Implications
at θ=π SU(N) YM (even N) cannot be trivially gapped
What does the IR look like?
- Gapless ? possible but sounds exotic
T inv requires
2k = N-1 if N even
this is anomaly between T and

Implications

SU(N) YM (even N) cannot be trivially gapped

IR look like?

? possible but sounds exotic

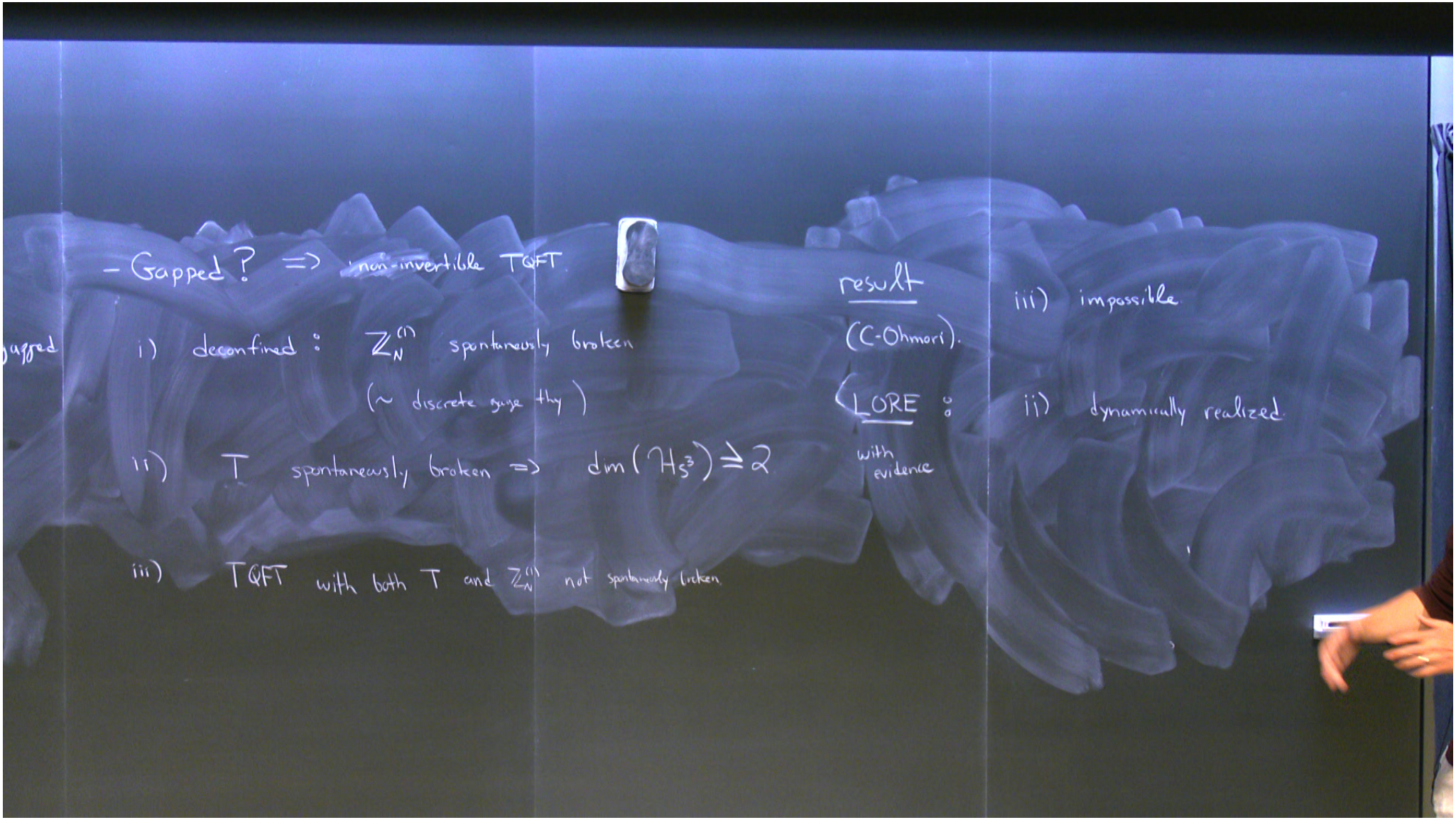
- Gapped ? $\Rightarrow$ non-invertible TQFT

i) deconfined : $\mathbb{Z}_N^{(1)}$ spontaneously broken

($\sim$ discrete gauge thy)

ii) T spontaneously broken $\Rightarrow$ $\dim(\mathcal{H}_{S^3}) \geq 2$

iii) TQFT with both T and $\mathbb{Z}_N^{(1)}$ not spontaneously broken


- Gapped ? => non-invertible TQFT
result
iii) impossible
(C-Ohmori).
i) deconfined : $\mathbb{Z}_N^{(1)}$ spontaneously broken
(~ discrete gauge thy)
(LORE :
ii) dynamically realized
with evidence
ii) T spontaneously broken => $\dim(\mathcal{H}_{S^3}) \geq 2$
iii) TQFT with both T and $\mathbb{Z}_N^{(1)}$ not spontaneously broken