

Title: Lattice systems and topological field theories

Speakers: Michael Hopkins

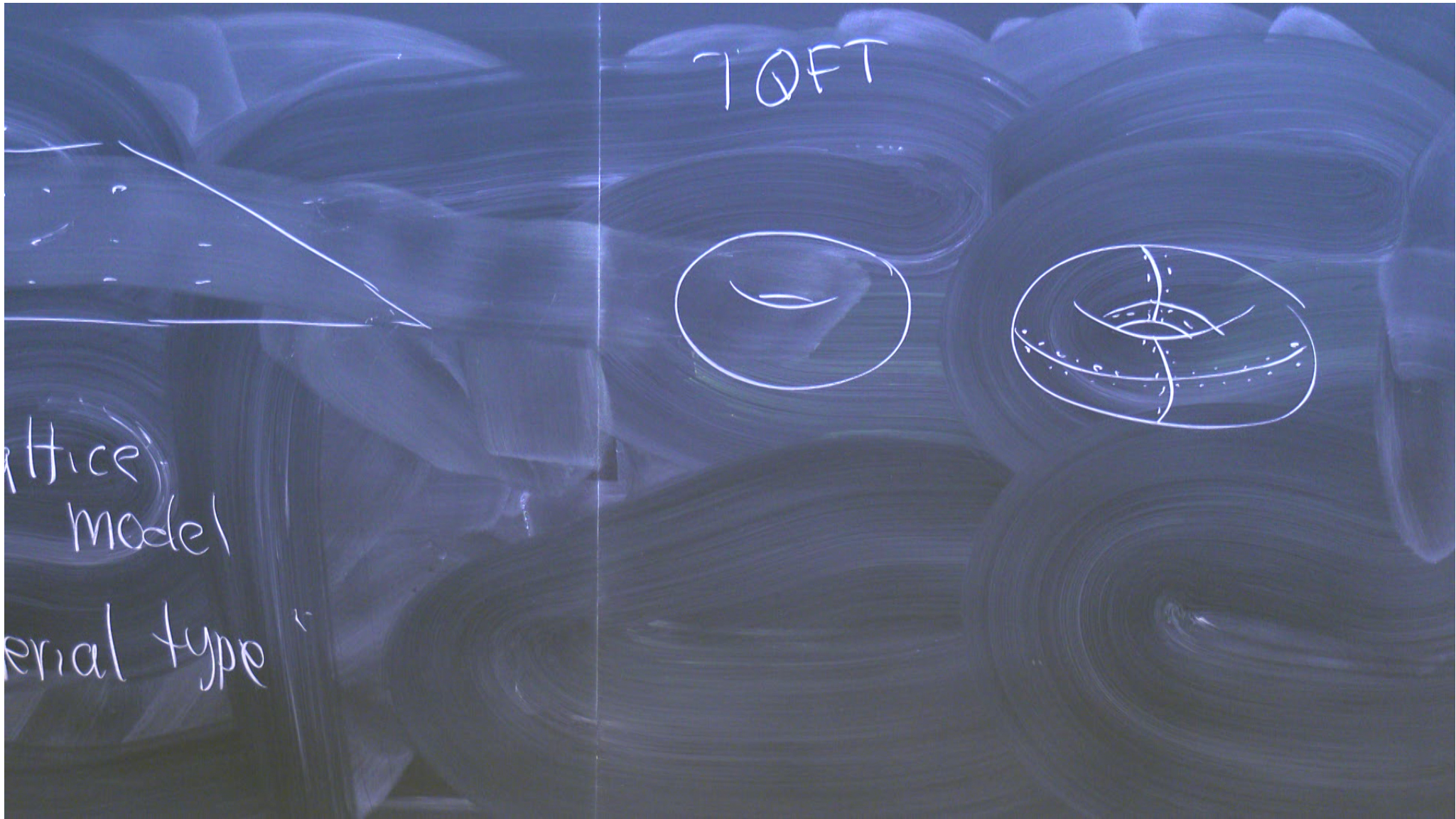
Collection: Global Categorical Symmetries

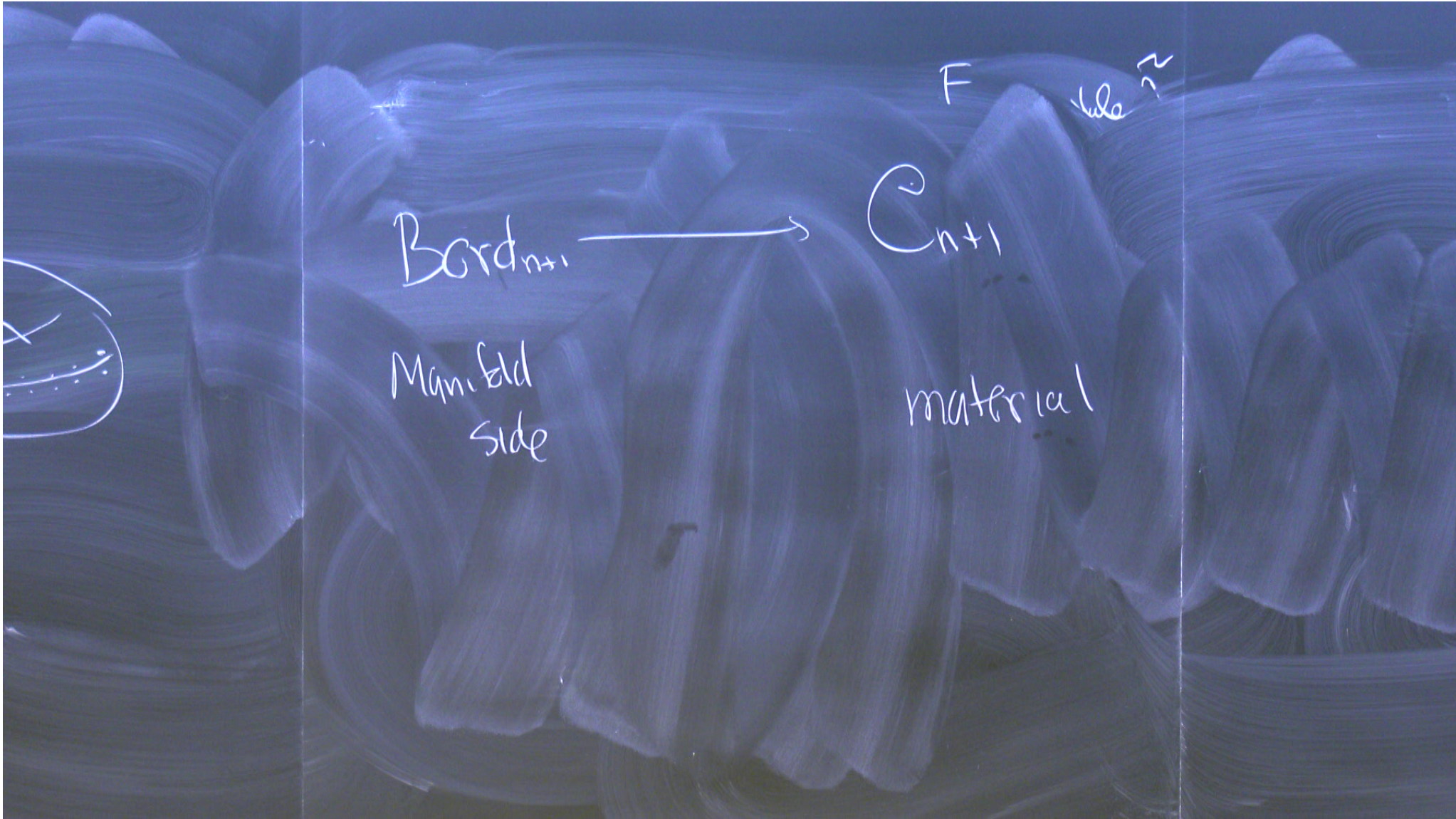
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Lattice
model

TQFT





Bord_n



$$\text{Bord}_n^X = \text{Bord}_n \otimes X$$

$$M = n\text{-manifold}$$

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$$\text{Bord}_{n+1} \xrightarrow{F} C_{n+1}$$

$$\text{Bord}_{n+1} \otimes M \rightarrow C_{n+1} \otimes M$$

$$M \xrightarrow{u} M$$

$\text{Bord}_n \otimes \mathbb{Z}$

$$\Sigma^n F \otimes M$$

$$\Sigma^n \text{Bord}_{n+1} \otimes M \rightarrow \Sigma^n C_{n+1} \otimes M$$

$$\downarrow \psi$$

$$F[M]$$

$$\nearrow$$

$$F[M]$$

$$\Sigma^n F \otimes M$$

$$\text{ord}_{n+1} \otimes M \rightarrow \Sigma^n C_{n+1} \otimes M$$

ψ
 M

$F[M]$

$$C_1 \otimes M$$

modelled by
lattice systems
on M

$$M \rightarrow pt$$

$$C_1 M \rightarrow C_1(pt) = C_1 = Vect$$

$$\bigcup_{i \in \mathbb{N}} \mathbb{R}^n = \varprojlim \bigcup_{i \in \mathbb{N}} \mathbb{R}^n / \mathbb{R}^n \cdot A$$

$A \subset \mathbb{C}^n$

lattice
moeb's
on $\mathbb{R}^n$

$$= \bigcup_{i \in \mathbb{N}} (\mathbb{R}^n, *)$$

$$= \bigcup_{i \in \mathbb{N}} \mathbb{R}^n$$

$$\mathbb{C}_1 \otimes \mathbb{R}^n = \varprojlim_{A \subset \mathbb{C}^1} \mathbb{C}_1 \otimes \mathbb{R}^n / \mathbb{R}^n \cdot A$$

lattice
moeb's
on $\mathbb{R}^n$

$$= \mathbb{C}_1(\mathbb{S}^n, *)$$

$$= \mathbb{C}_{n+1}$$

$$\mathbb{P}^n / \mathbb{P}^{n-1} \cong \mathbb{A}^1$$

$$A \subset \mathbb{C}P^1$$

$$(\mathbb{S}^n, x)$$

141

objects of $\mathcal{C}_{n+1}$



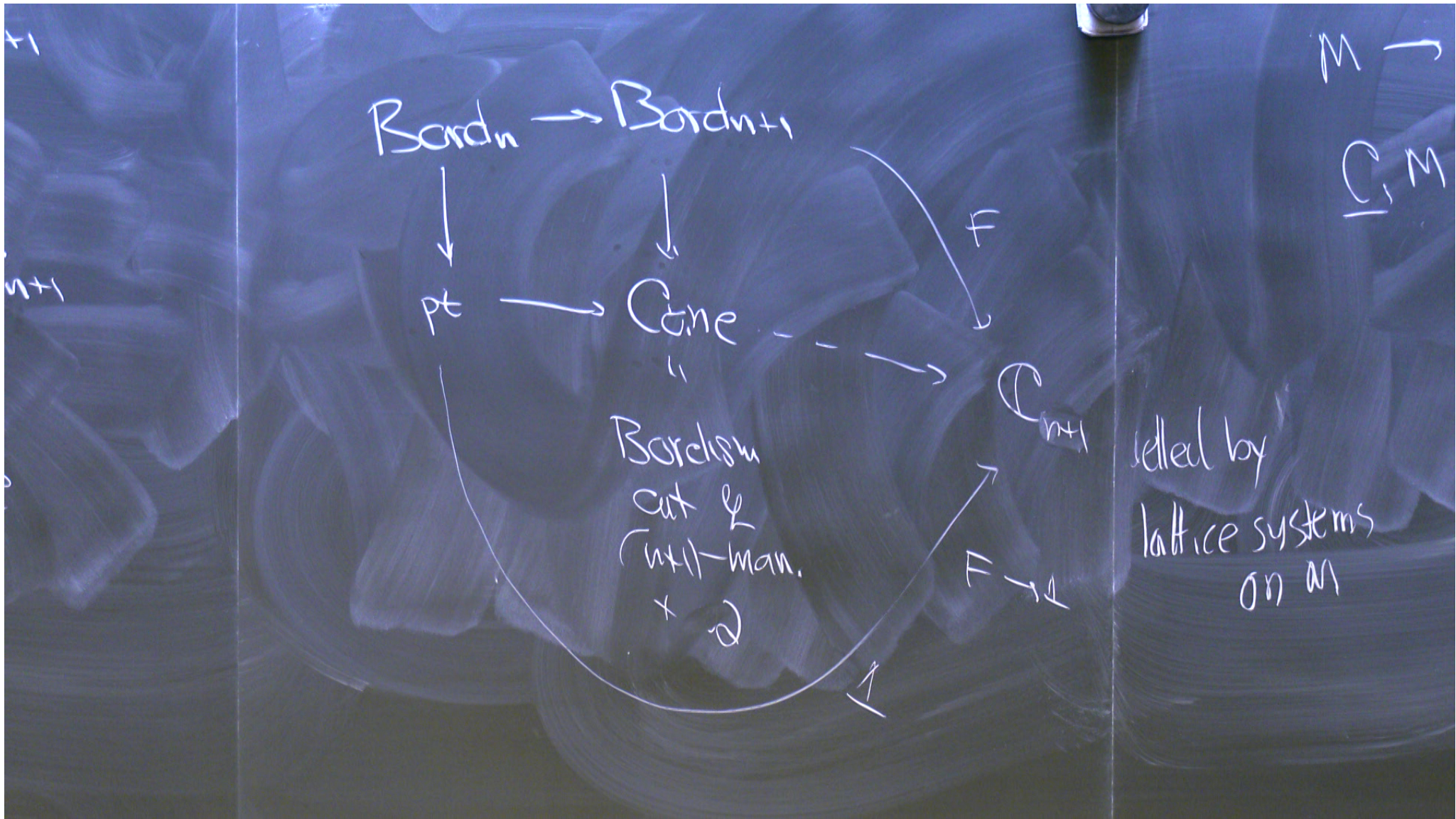
Sym
mon
func

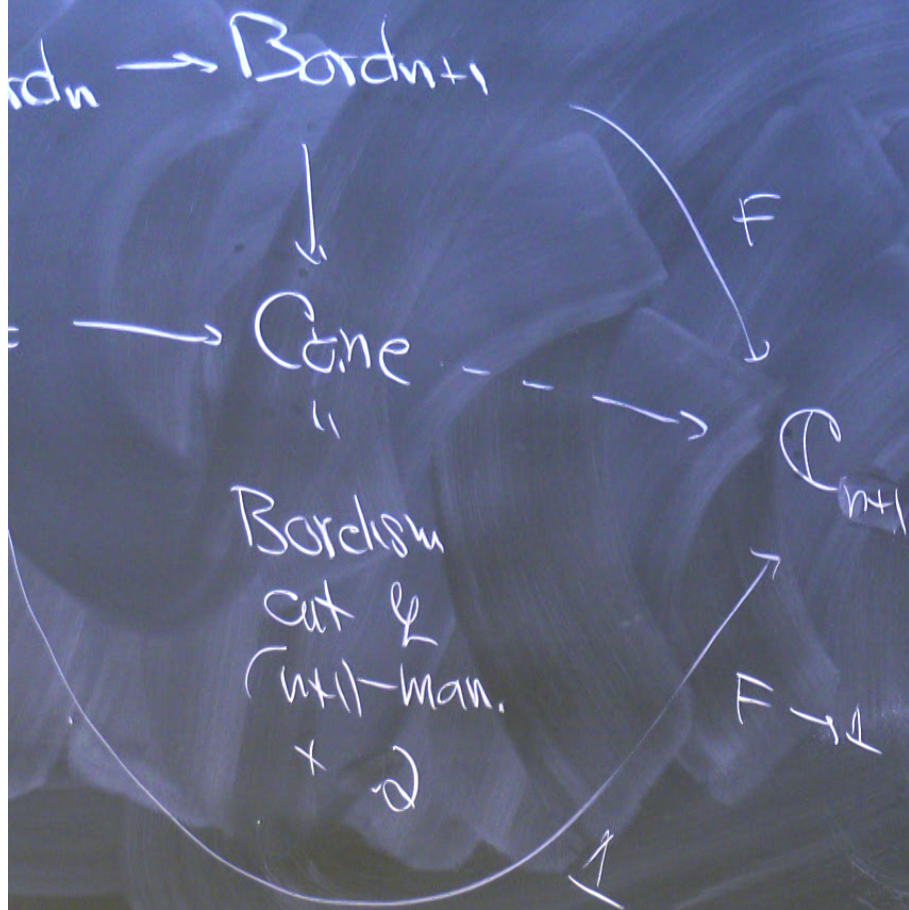
$$\text{Bord}_{n+1} \rightarrow \mathcal{C}_{n+1}$$

What about the morphisms?

$$\mathbb{S}^n, \text{Bord}$$

$$\mathbb{S}^n$$





$$1 \xrightarrow{D} F$$

$$F \xrightarrow{D} 1$$

left boundary
condition

right
boundary
condition

$$\sigma \mid F \rightarrow F$$

$$F \rightarrow G$$

$$\mathbb{I} \rightarrow G \otimes F^\vee$$

$\Rightarrow F \rightarrow G$ Domain walls

$$\begin{array}{ccc} C_1 & \xrightarrow{\quad} & C_0\text{-mod} \\ & \searrow & \text{"} \\ X & \rightarrow & \text{Vect} \\ & & \text{Hom}_C(1, X) \end{array}$$

$$F \rightarrow G$$

$$\mathbb{I} \rightarrow G \otimes F^\vee$$

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$$1 \otimes M$$

$$C_n$$

M

$$C_{n+1}$$

$$C_n - \text{mod}$$

$$\text{Hom}_{C_{n+1}}(1, -)$$

$$F$$

cont of boundary
conditions
for F

$$1 \xrightarrow{D} F$$

$$F \xrightarrow{D} 1$$

$\rightarrow C_n - \text{mod}$

, -)

cont of boundary
conditions
for F

Def the image
of $C_n - \text{mod}$ in C_{n+1}
is the co-localization
of C_{n+1} at 1

$F \in C_{n+1} - \text{mod}$ F is "local"

$\sigma \mid F \rightarrow F$

F local

$$\text{Hom}_{C_{n+1}}(F, G)$$

$$= C_n\text{-mod}(\text{Hom}(1, F), \text{Hom}(1, G))$$

$$D \in \text{Hom}(1, F)$$

$$1 \begin{array}{c} \xrightarrow{D^* \text{ left}} \\ \xleftarrow{D} \end{array} F$$

alg in

$$DD^* : 1 \rightarrow 1 \in C_n$$

$$(DD^*)DD^* \rightarrow DD^*$$

Def D is a Dirichlet
boundary condition

If

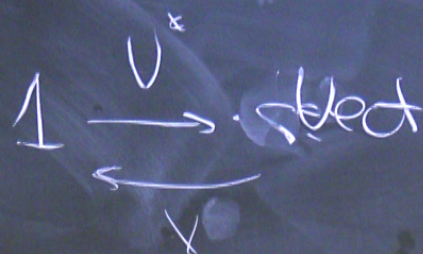
$$C_{n+1}(F, G)$$

is

DP^* -module in $C_{n+1}(I, \mathbb{R})$

Def the im
of C_n -m
is the co
of C_{n+1} at
 $F \in C_{n+1}$ -mod

Ex $C_2 \Rightarrow \text{Vect} = \mathbb{K}$



$$A = \text{End}(U)$$

$D \in$

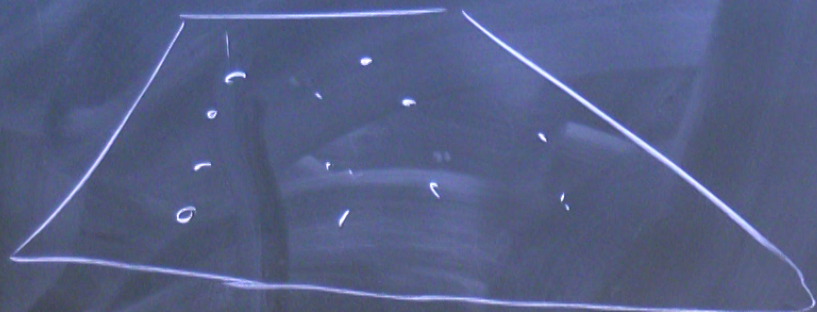
1

alg in

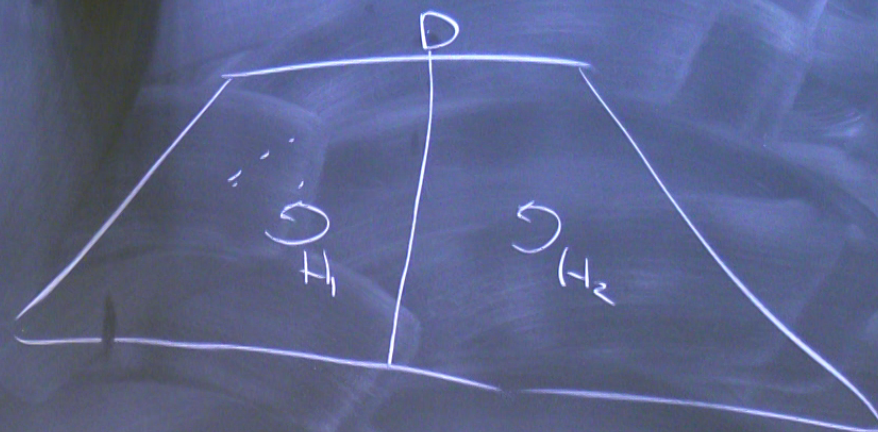
DD^*

(DD^*)

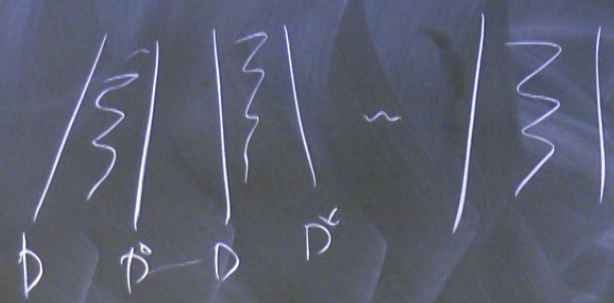
Homology
Picture



ob ∂_{n-1}



D $D = \text{Dirichlet boundary}$



D' - Dirichlet boundary
cone

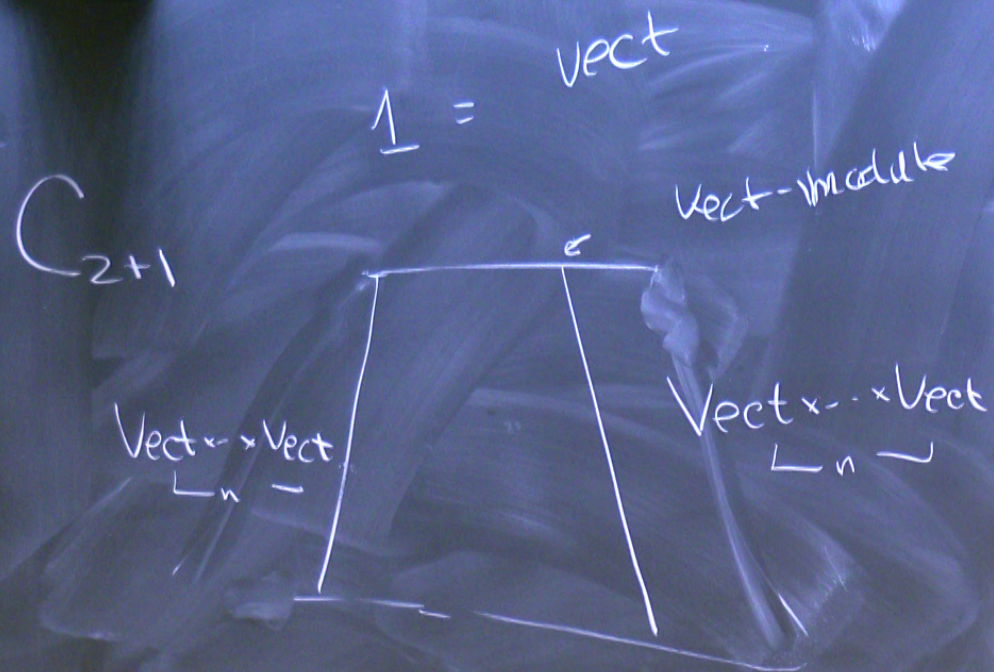


C_n

C_{n-1}



$\in C_1$



an algebra object in $\mathcal{C}$
 $\mathcal{C}$ -linear cat

$$n \begin{bmatrix} V_1 & \dots & V_n \\ \vdots & & \vdots \\ V_1 & \dots & V_n \end{bmatrix}$$

$$n \begin{bmatrix} v_{11} & \dots & v_{1n} \\ \vdots & & \vdots \\ v_{n1} & \dots & v_{nn} \end{bmatrix}$$

$$V = \mathbb{R}^n$$

W

$$\begin{bmatrix} w & \dots & w \\ \vdots & & \vdots \\ w & \dots & w \end{bmatrix}$$

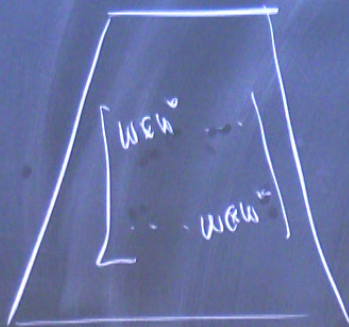
$$\begin{bmatrix} w \otimes w^\vee & \dots & w \otimes w^\vee \\ \vdots & & \vdots \\ w \otimes w^\vee & \dots & w \otimes w^\vee \end{bmatrix}$$

$$\begin{bmatrix} w^\vee & \dots & w^\vee \\ \vdots & & \vdots \\ w^\vee & \dots & w^\vee \end{bmatrix}$$

$$W \otimes W^\vee \otimes V \otimes V^\vee$$

$\downarrow$

$$\begin{bmatrix} W & \dots & W \\ \vdots & & \vdots \\ W & \dots & W \end{bmatrix}$$



$$\begin{bmatrix} W^* & \dots & W^* \\ \vdots & & \vdots \\ W & \dots & W \end{bmatrix}$$

$\mathcal{D}$

$$W \otimes W^* \otimes V \otimes V^*$$

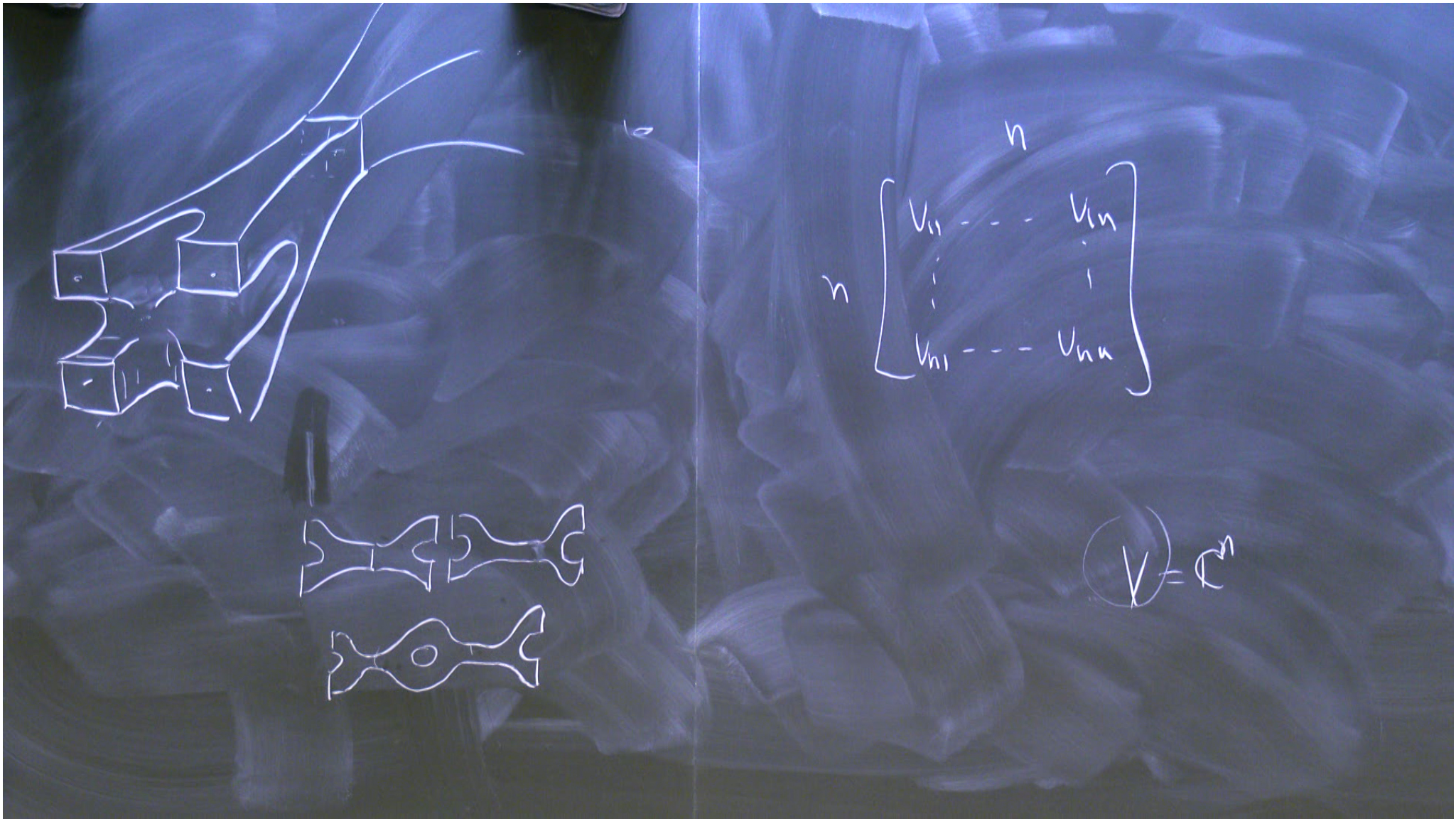


C_n

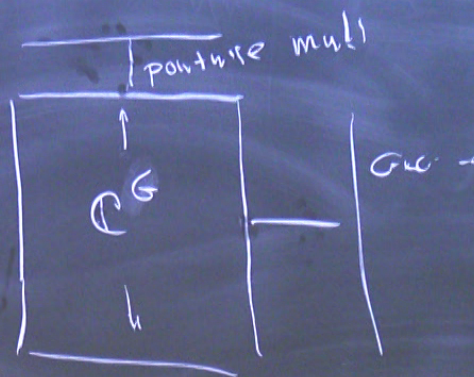


$\in C_1$

$\rightarrow F$



Ex Toric code



← Vector →

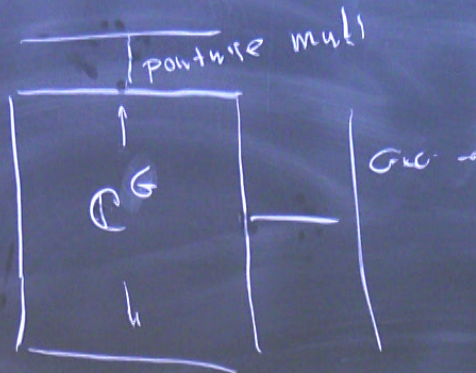
$$\begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

$$\begin{bmatrix} w_1 w_2 \\ \vdots \\ w_{n-1} w_n \end{bmatrix}$$

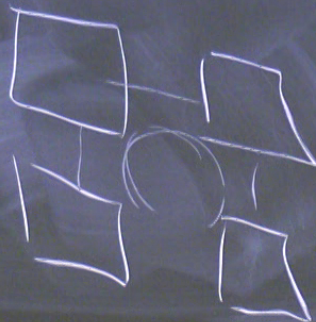
$$\begin{bmatrix} w_1^x \\ \vdots \\ w_n^x \end{bmatrix}$$

Ex Toric code

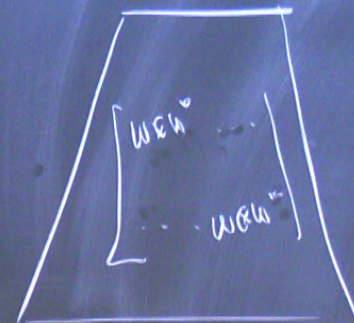
$$G = \mathbb{Z}_2$$



← Vector →



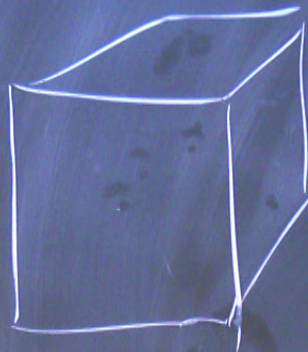
$$\begin{bmatrix} w & \dots & w \\ \vdots & & \vdots \\ w & \dots & w \end{bmatrix}$$



$$\begin{bmatrix} w^* & \dots & w^* \\ \vdots & & \vdots \\ w^* & \dots & w^* \end{bmatrix}$$

$$G = \mathbb{Z}_2$$

$$G \times \mathbb{R}^n$$



$$C_{n+1} \otimes \mathbb{R}^n$$



$$\hookrightarrow C_n$$