

Title: Introduction to Symmetries in Quantum Field Theory

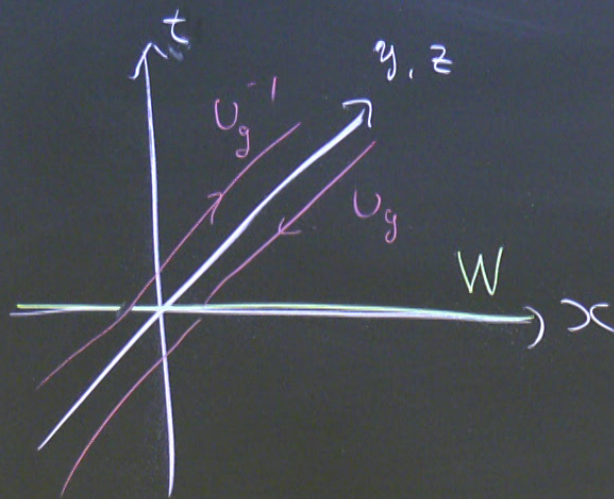
Speakers:

Collection: Global Categorical Symmetries

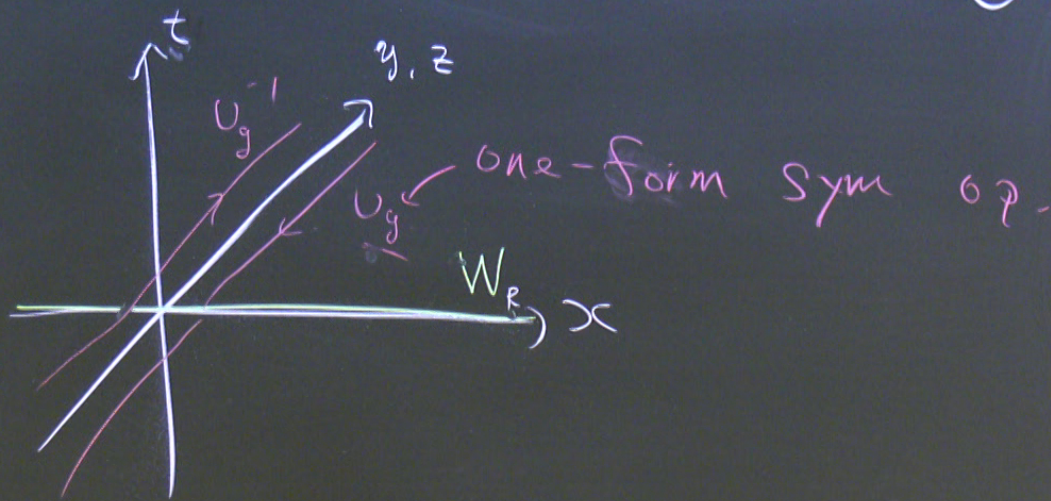
Date: June 16, 2022 - 11:00 AM

URL: <https://pirsa.org/22060035>

Confinement & One-form Sym



Confinement & One-form Sym



If deconfined
 $\rightarrow \langle \widetilde{W} \rangle \neq 0$

rm Sym

m Sym op.

If deconfined $W \times e$ ^{local} odd
 $\rightarrow \langle \tilde{W} \text{ (x-axis)} \rangle \neq 0$

$\rightarrow U_g |0\rangle \not\propto |0\rangle$

one-form

Spontaneous Sym Breaking

$$e^{c \oint \omega}$$

$$> \neq 0$$

$$\propto |0\rangle$$

$$\text{Maxwell theory} \xrightarrow{2+1d} \frac{\log L}{L} \text{ : perimeter}$$

$$\text{Electric one-form, Mag.}$$

$$Z(U(1)) = U(1), \underline{U(1)}$$

Sym Breaking $\rightarrow$ photon.

as NG-boson!

theory $\xrightarrow{\text{2Hd}} \frac{\log L}{L}$
 perimeter

form, Mag.
 $= \langle \psi \rangle, \langle \psi \rangle$

on.
 $\sqrt{G}$ -boson!

Confined No local counterterm
 to cancel.
 $W_\gamma \propto e^{\text{area}(\Sigma, \partial\Sigma = \gamma)}$

$\rightarrow \langle W(x\text{-axis}) \rangle$

2Hd
theory $\leftarrow \frac{\log L}{L}$
perimeter

form, Mag.

$U(1)$, $U(1)$

on

$U(1)$ -boson!

Confined No local counterterm
to cancel
 $W_\gamma \propto e^{-\text{area}(\Sigma, \partial\Sigma = \gamma)}$

$$\rightarrow \lim_{t, L} \langle W(\text{rectangle with width } L \text{ and height } t) \rangle$$

$$\sim \langle \underline{W(\text{xaxis})} \rangle^2 = 0$$

$\rightarrow$ One-form
preservation

Confinement & One-form Sym

Phase	Wilson line	One-form
de-confinement	perimeter	SSB
confinement	area	preserved

If deconf
→ $\langle \bar{W} \rangle$

→ $\langle \bar{W} \rangle$
one

m Sym

One-form

SSB

preserved

what we expect
for YM

Temp

deconfinement

SSB

sharp phase transition

preserved

confinement

Gauge th. with matter
 ψ

Electric one-form is explicitly
broken

$$Z(G) \rightarrow \{g \mid g\psi = \psi\}$$

Conf

W

$\rightarrow \lim_{t \rightarrow \infty}$

$\sim$

$\rightarrow$

th matter

ψ

m is explicitly
broken

$$|g\psi = \psi\rangle$$



Ex.

0-form $\mathbb{Z}_n$ sym

$\mathcal{L} \supset B$ & gapped

$\rightsquigarrow$ n vacua

What is the analog of this statement
for $\mathbb{Z}_n$ one-form sym?
($B\mathbb{Z}_n$)

What is the analog of this statement
for $\mathbb{Z}_n$ one-form sym?
(B $\mathbb{Z}_n$)

hint: Consider $U(1)$ with charge q
scalar field.

① What is E , one-form?

hint: Consider $U(1)$ with charge g
scalar field.

① What is $E1$ one-form?

② find a phase where
gapped
 $E1$ one-form

What is the IR theory

is statement

Non-invertible chiral sym.

4 d ^{massless} QED

U(1) gauge th. charge 1
w/ massless electron

$\mathbb{H}$
" (ψ_R, ψ_L)

chiral sym.

charge 1

electron

$\mathbb{H}$

(ψ_R, ψ_L)

classical chiral sym.

$$\psi_R \rightarrow e^{i\alpha} \psi_R, \quad \alpha \in U(1)$$

$$\psi_L \rightarrow \psi_L$$

ABJ anomaly.

$$dj = \frac{1}{8\pi^2} F^2$$

Noether current

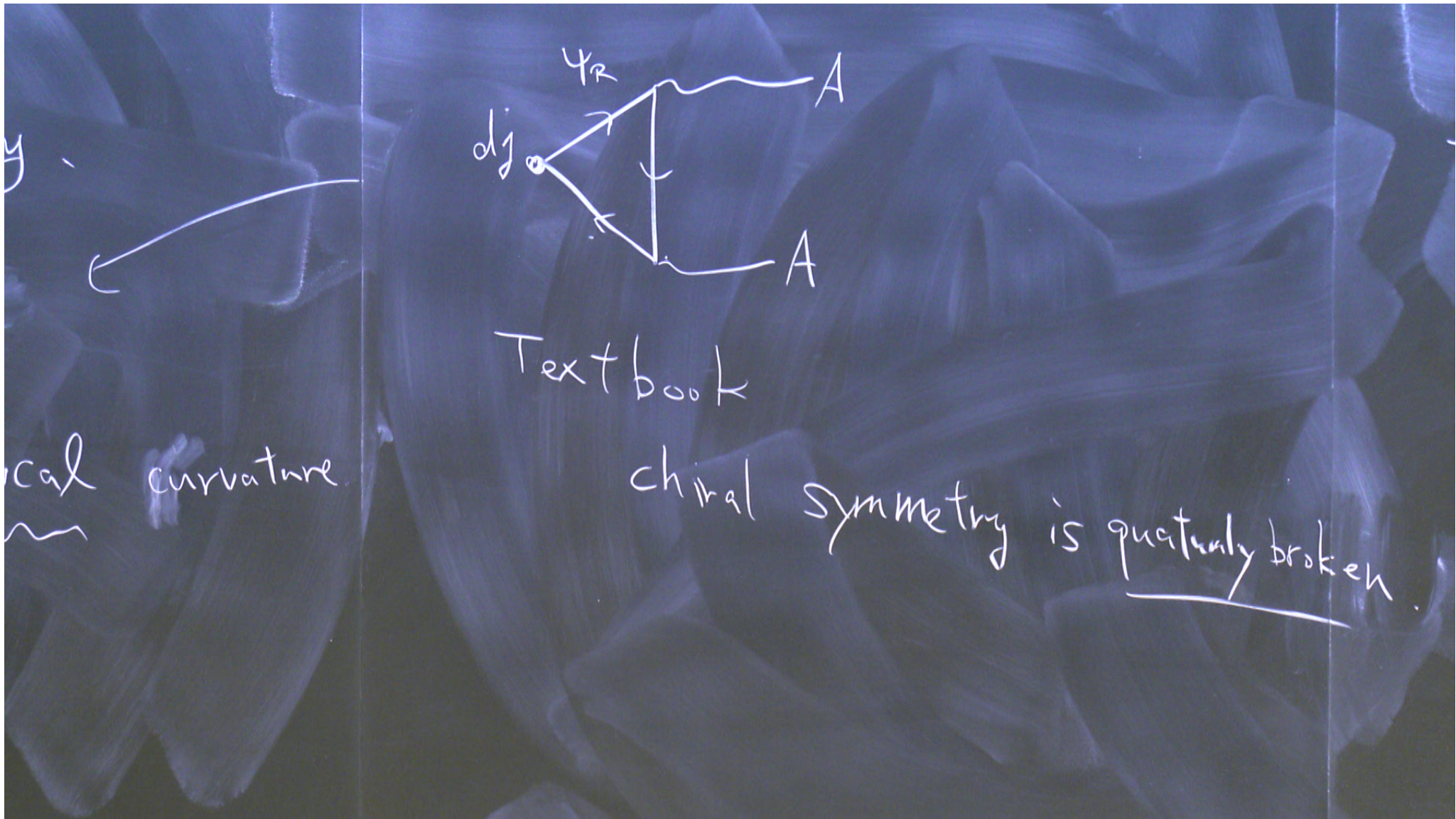
$$j_{\text{mine}} = \star J_{\text{chrs}}$$

dynamical

curvature

Wh
for

hint:



$$\text{Locally } F^2 = d \underbrace{C}_{\substack{\uparrow \\ \text{Chern-Simons}}} \underbrace{\int}_{\substack{\uparrow \\ \text{AdA}}} \text{Classical}$$

→ Locally modify

$$J \rightarrow J - \frac{1}{8\pi^2} C \int$$

quantumly broken

Locally $F^2 = d \underbrace{C}_{\uparrow} S$

Chern-Simons

→ Locally modify

$$j \rightarrow j - \frac{1}{8\pi^2} C S$$

quantumly broken

$$dj' = 0$$

$$AdA$$

$$= dC \int$$

Chern-Simons

diffy

$$- \frac{1}{8\pi^2} C \int$$

y'

on gen. manifold ✓

Globally, a general U(1) bundle.

has instanton number

$$\int_M F^2 \neq 0$$

Ex find a nonzero instanton configuration

$$\langle dj(x), \sigma(y), \sigma(z) \dots \rangle$$

$$= \sum_{\hat{I}} \langle dj(x), \sigma(y), \sigma(z) \dots \rangle_{\hat{I}}$$

$$= \underbrace{\langle \dots \rangle_{\hat{I}=0}}_{\hat{I} \neq 0} + \langle \dots \rangle_{\hat{I} \neq 0}$$



Textbook
chiral

$$\begin{aligned}
 & \langle dj(x), 0(y), 0(z) \rangle \\
 &= \sum_{\mathcal{I}} \langle dj(x), 0(y), 0(z) \rangle_{\mathcal{I}} \\
 &= \underbrace{\langle \dots \rangle_{\mathcal{I}=0}}_0 + \underbrace{\langle \dots \rangle_{\mathcal{I} \neq 0}}_{\text{non-zero}}
 \end{aligned}$$

No $U(1)$ -bundle w/ $I \neq 0$

→ Selection rule on $\mathbb{R}^4$

is satisfied

X. If

$\hookrightarrow$ gauge

: non-ab

$\exists$

instanton even on $\mathbb{R}^4$

Consider $C_\alpha = e^{i\alpha \int_\Sigma j}$, non-topological
 $\alpha \in \mathbb{R}$

We want

$$D_\alpha = e^{i\alpha \int_\Sigma j - i\alpha \int_\Sigma CS[A]}$$

photon.

not globally
gauge inv.

$i d \int_{\Sigma} j$ non-topological

$e \in \mathbb{R}$ photon.

$$-i \frac{\alpha}{\Theta} \int_{\Sigma} CS[A]$$

not globally
gauge inv.

BTW

If $\alpha \in 2\pi\mathbb{Z}$

correctly quantized.
CS

$$\widetilde{D}_{\alpha} = e^{i \frac{\alpha}{\Theta} \int_{\Sigma} CS[A]}$$

invertible defect

Integer Quantum Hall eff.

$$\alpha = \frac{P}{q} \cdot 2\pi$$

Fractional QHE

$$D_{\alpha}^{[\Sigma]} = C_{\alpha}^{[\Sigma]} \times A^{[P]}[\Sigma] : \text{topological}$$

TQFT desc. FQHE

on $\mathbb{R}^4$

No

→ Sel

× If $\exists$

← gauge group is abelian.

abelian

ABJ-anomalous
chiral symmetry

is a symmetry!!

We

$\mathcal{L} \supset \bar{\psi} \not{D} \psi$

q, p

$\mathcal{L} \supset [\bar{\psi} \psi]$

topological

QFT

desc.

$$\alpha = \frac{P}{q} \cdot 2\pi$$

Fractional QHE

$$D_q^{[\Sigma]}$$

$$C_\alpha^{[\Sigma]} \times$$

q, P

$$A^{[\Sigma]}$$

$$e^{i\int \gamma}$$

TQFT

desc.

FQHE

topological

ABJ- and
chiral

is