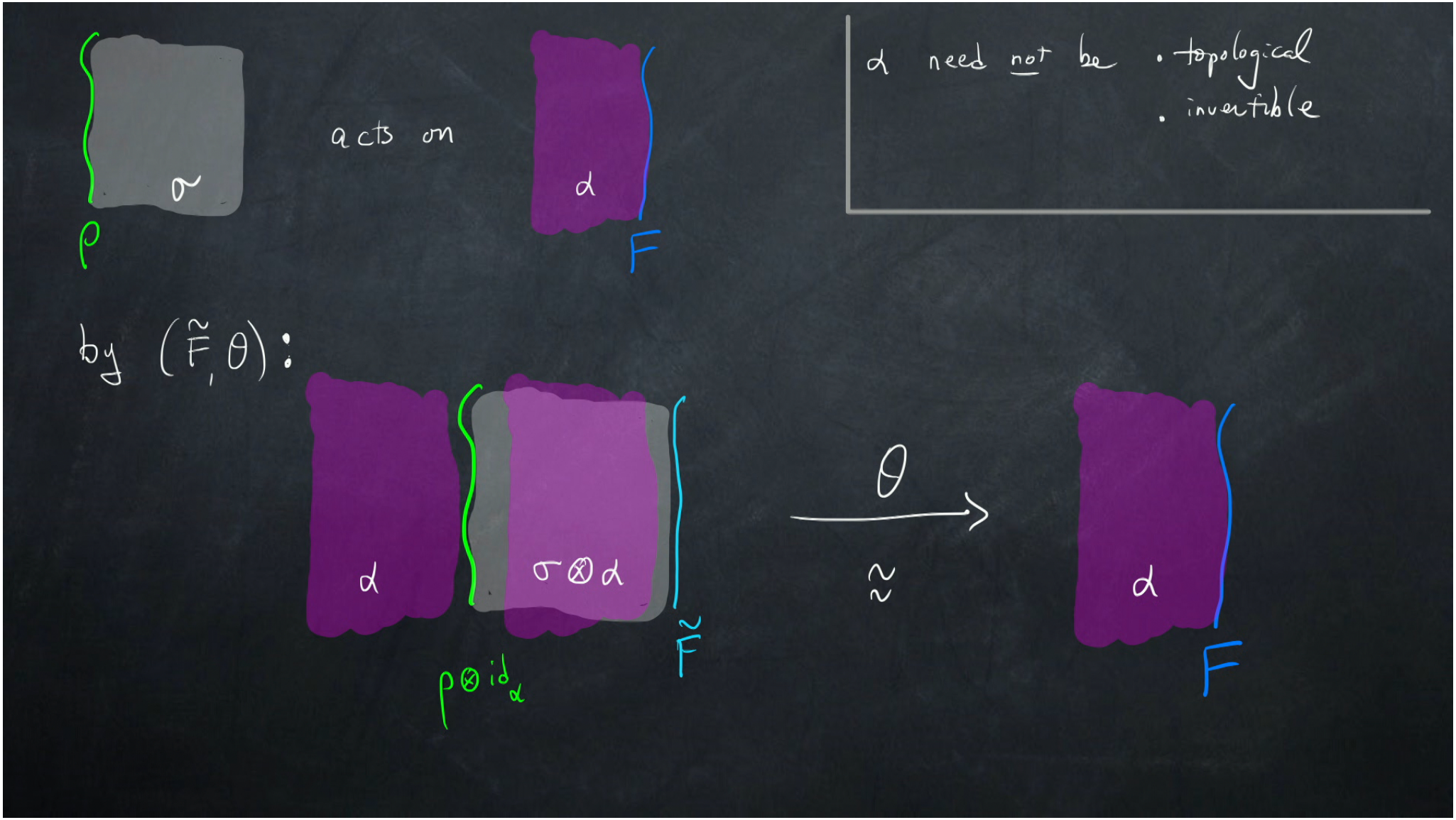
Title: Finite symmetry in QFT

Speakers: Dan Freed

Collection: Global Categorical Symmetries

Date: June 15, 2022 - 11:00 AM

URL: https://pirsa.org/22060034


ρ
σ
acts on
α
F
α need not be
• topological
• invertible
by $(\tilde{F}, \theta)$:
α
σ ⊗ α
$\tilde{F}$
ρ ⊗ id_α
θ
≃
α
F

Defn: A algebra. An augmentation of A is an algebra homomphism $\varepsilon_A : A \to \mathbb{C}$.

Ex: G finite group

$A = \mathbb{C}[G]$.

$\varepsilon_A : \mathbb{C}[G] \to \mathbb{C}$

$\sum \lambda_g g \mapsto \sum \lambda_g$

Quotients

A algebra. An augmentation of A is an algebra homomphism $\varepsilon_A : A \to \mathbb{C}$.

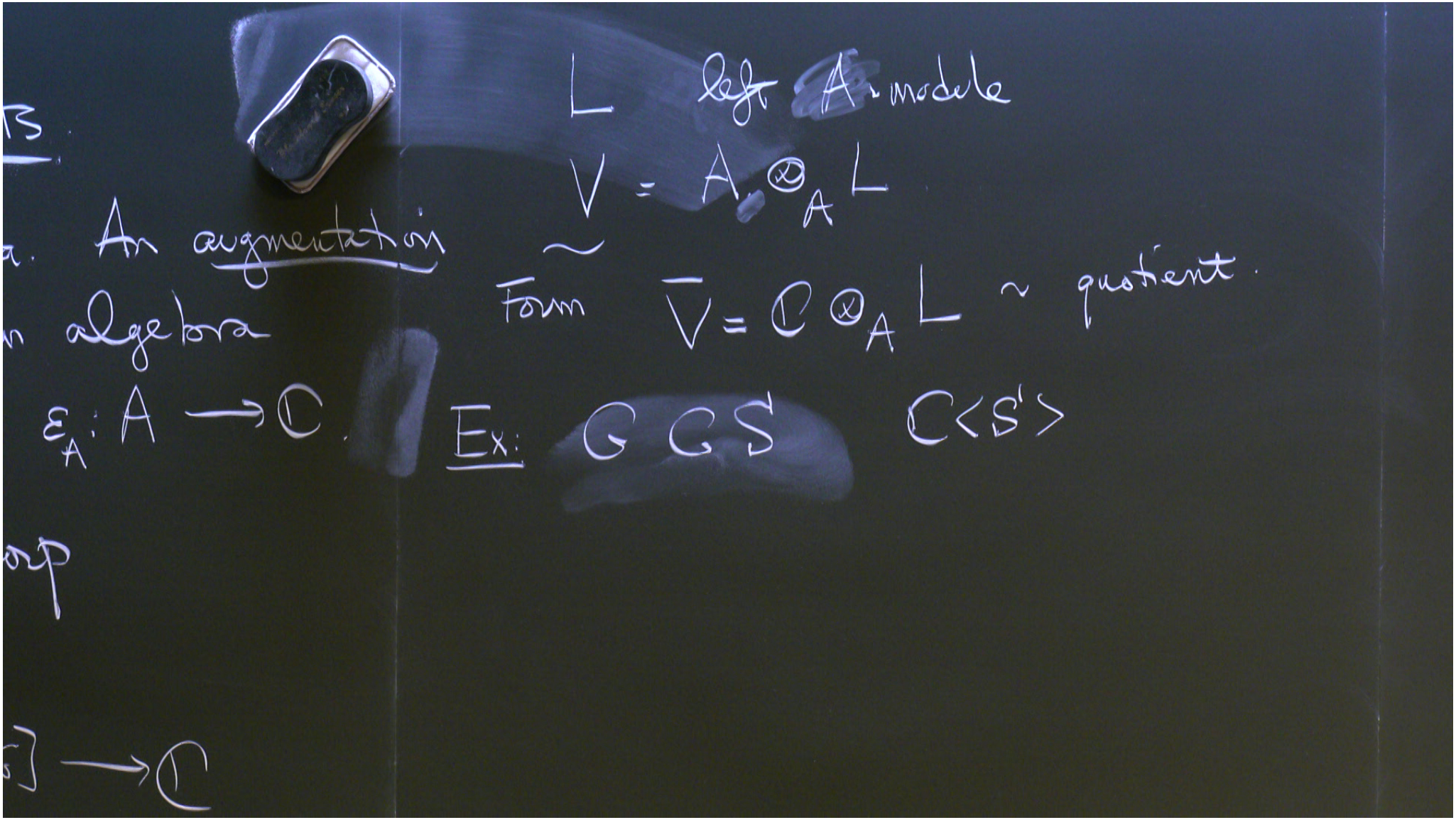
G finite group

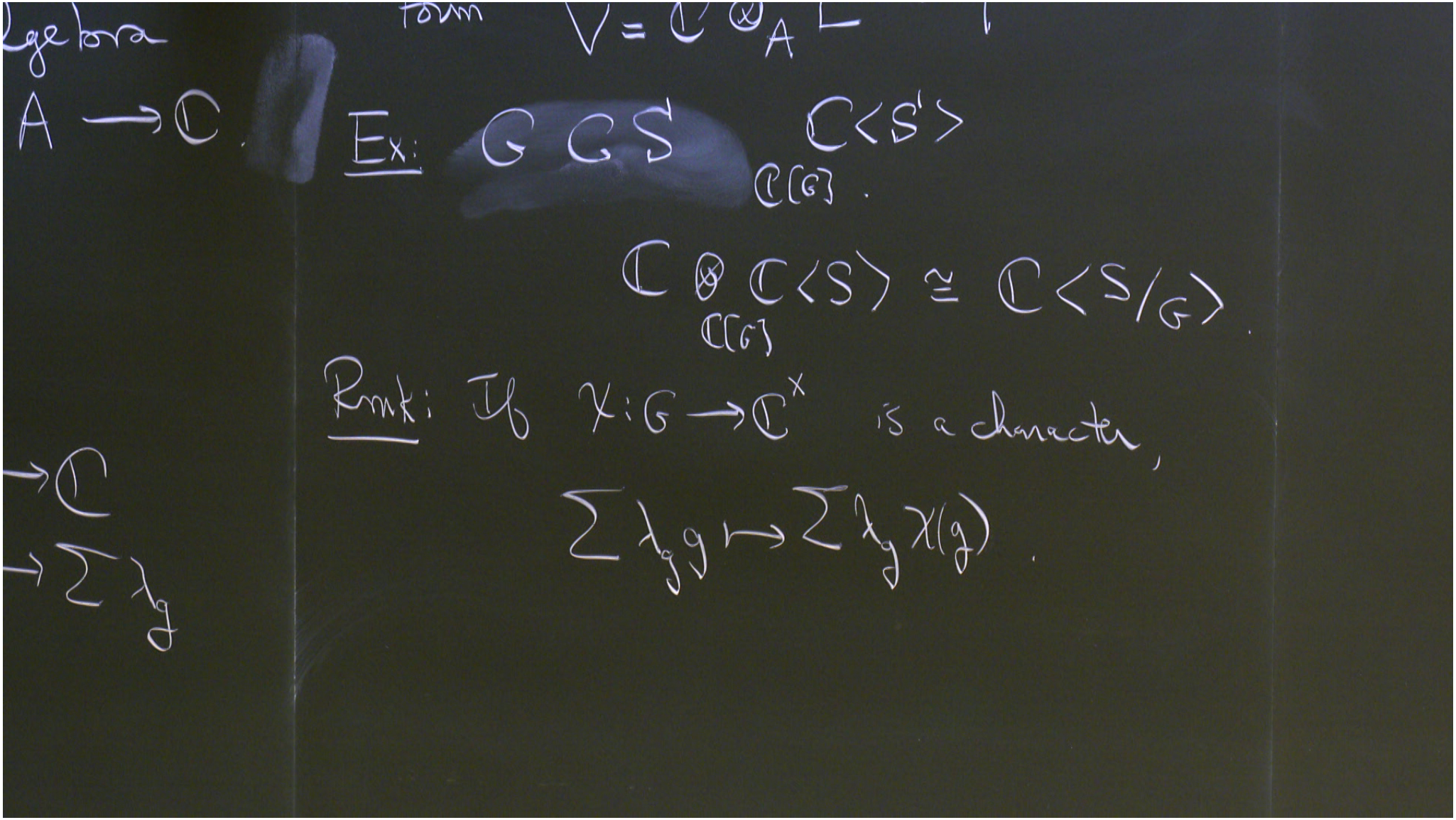
$A = \mathbb{C}[G]$

$\varepsilon_A : \mathbb{C}[G] \to \mathbb{C}$

L left A-module

$V = A \otimes_A L$


L left A-module
V = A ⊗_A L
An augmentation
algebra
form V̄ = ℂ ⊗_A L ~ quotient.
ε_A : A → ℂ
Ex: G GS ℂ⟨S⟩


form $V = \mathbb{C} \otimes_A$
$A \to \mathbb{C}$
Ex: G G S $\mathbb{C}\langle S \rangle$
$\mathbb{C}[G]$.
$\mathbb{C} \otimes_{\mathbb{C}[G]} \mathbb{C}\langle S \rangle \cong \mathbb{C}\langle S/G \rangle$.
Rmk: If $\chi : G \to \mathbb{C}^\times$ is a character,
$\sum \lambda_g g \mapsto \sum \lambda_g \chi(g)$.
$\to \mathbb{C}$
$\to \sum \lambda_g$

~ quotient.

$S'>$

$\cong \mathbb{C}\langle S/G \rangle$.

a character,

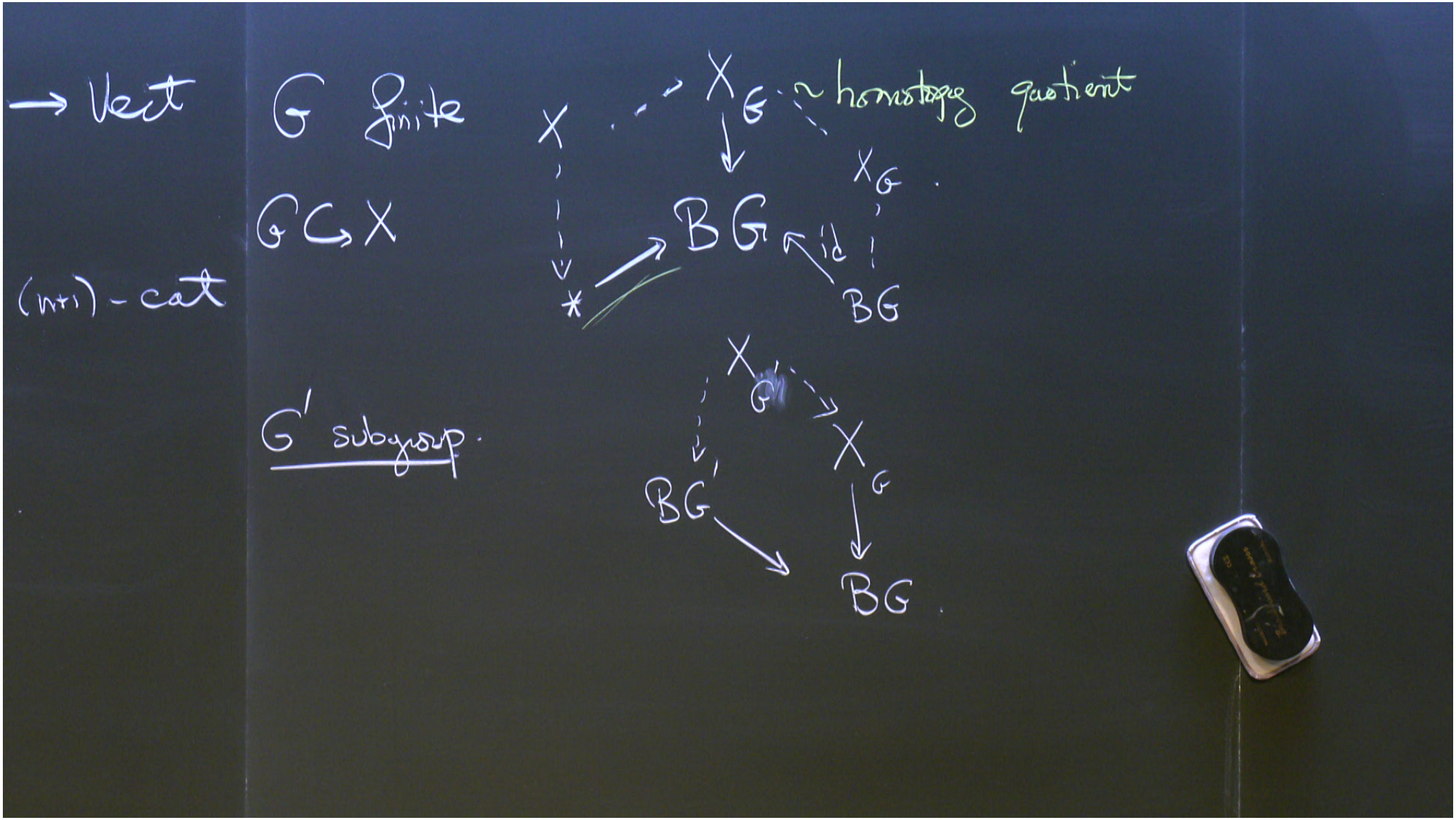
$\mathcal{A}$ fusion cat. Augmentation: $\varepsilon_{\mathcal{A}} : \mathcal{A} \to \mathrm{Vect}$

Defn: $\mathcal{C}'$ symm monoidal n-category

$\mathcal{C} = \mathrm{Alg}_{\mathcal{A}}(\mathcal{C}')$ Symm monoidal $(n+1)$-cat

$\mathcal{A}$

Augmentation: $\varepsilon_{\mathcal{A}} : \mathcal{A} \longrightarrow \mathbb{1}$.


→ Vect
G finite
G ↷ X
(n+1) - cat
G' subgroup.
X
X_G
≃ homotopy quotient
BG
id
BG
X_G
X_G'
BG
X_G
BG

nstage quotient

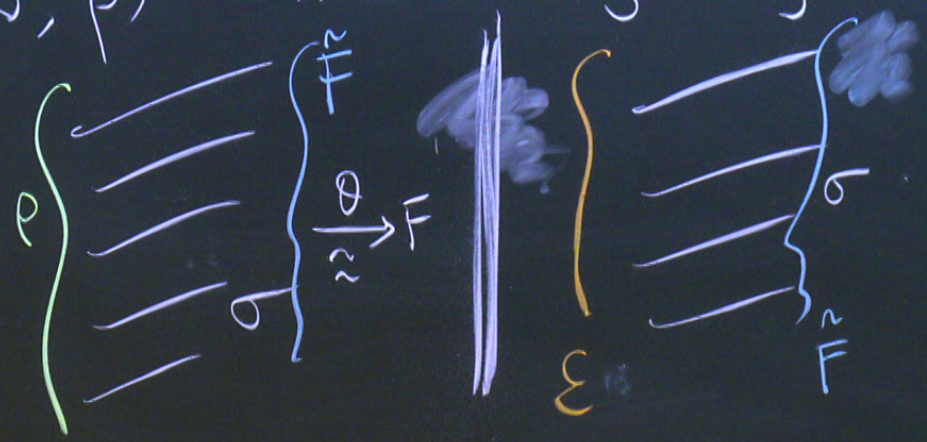
$\mathcal{G}$.

G

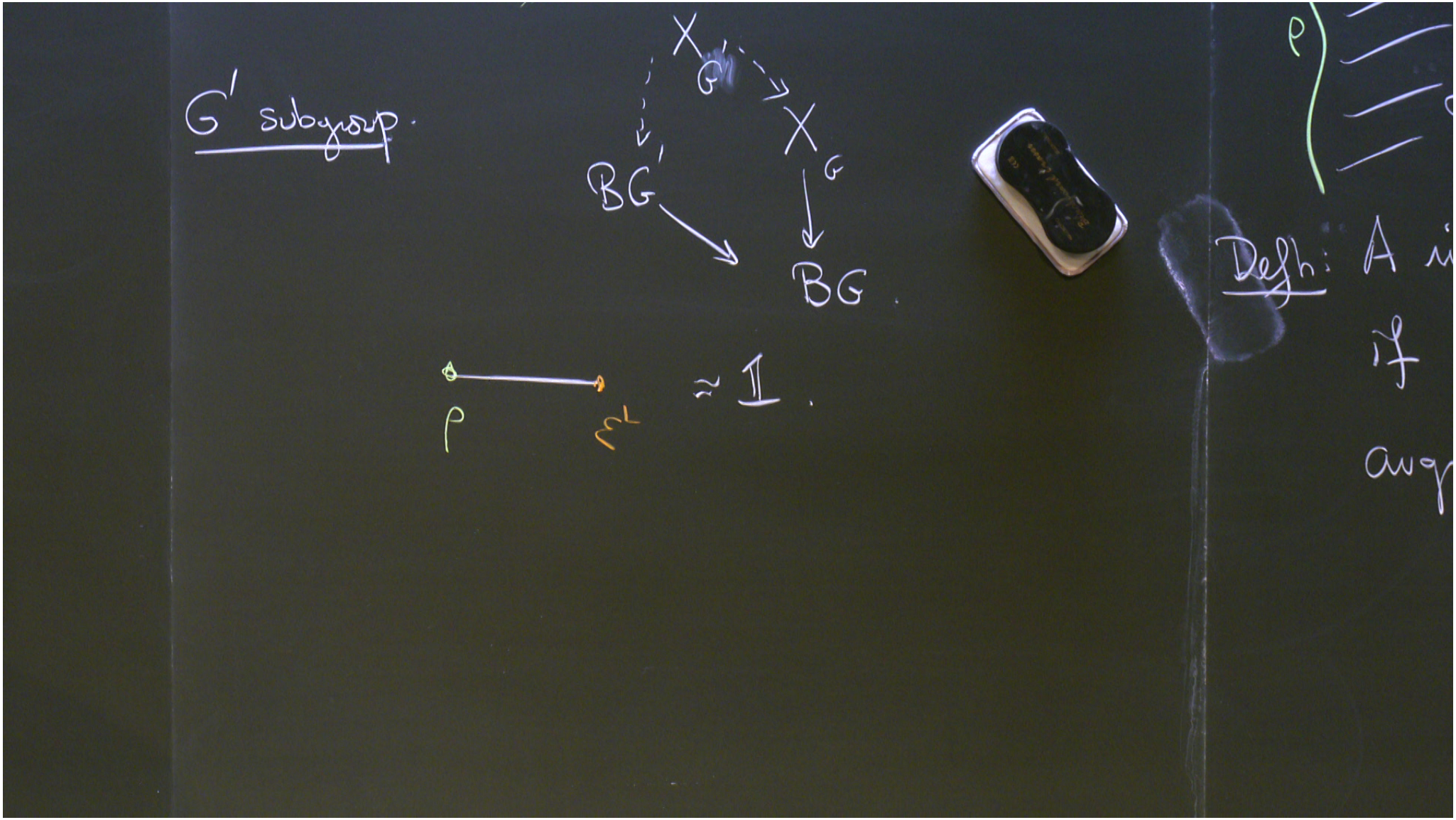
G .

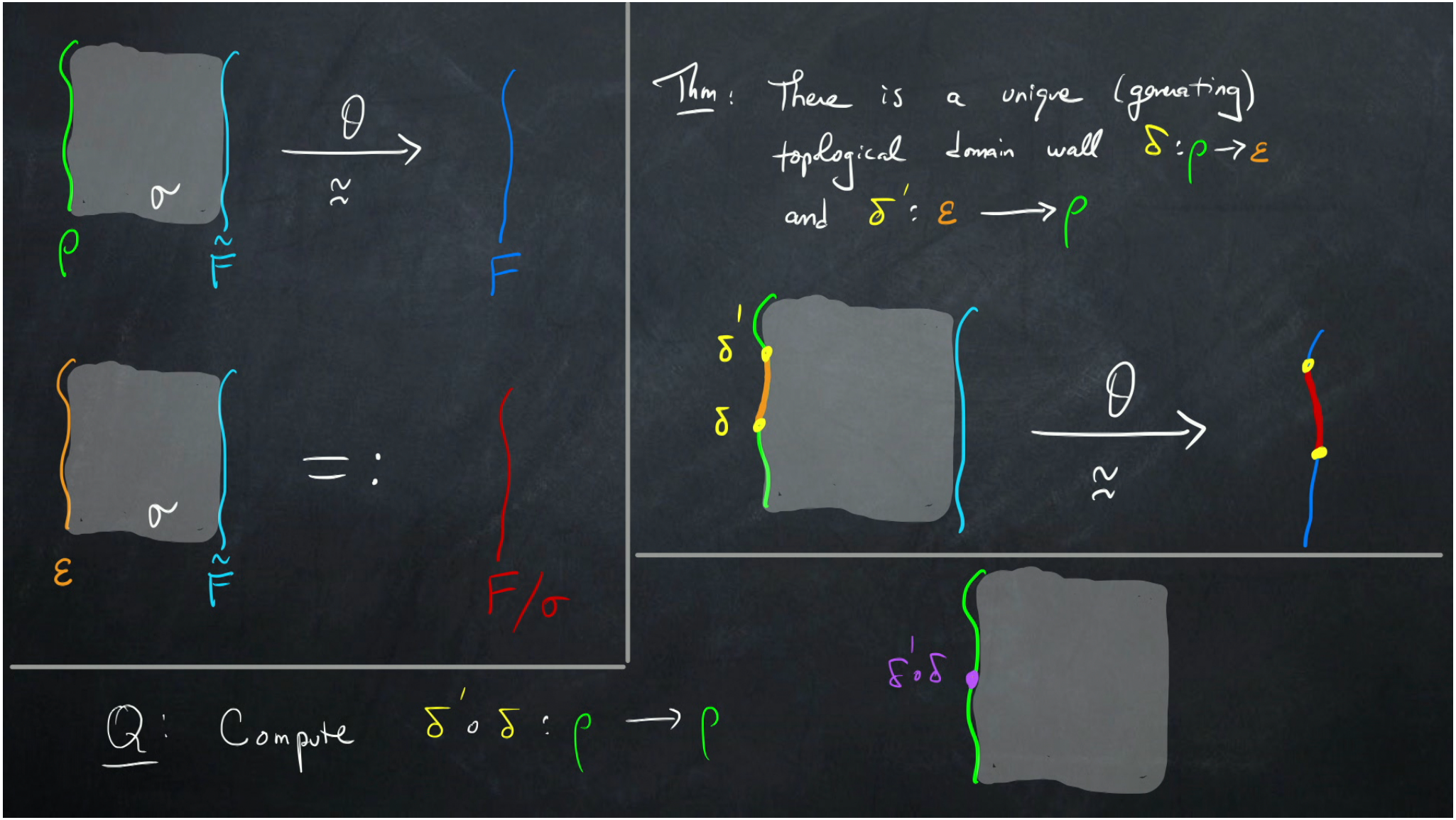
Field Theory.

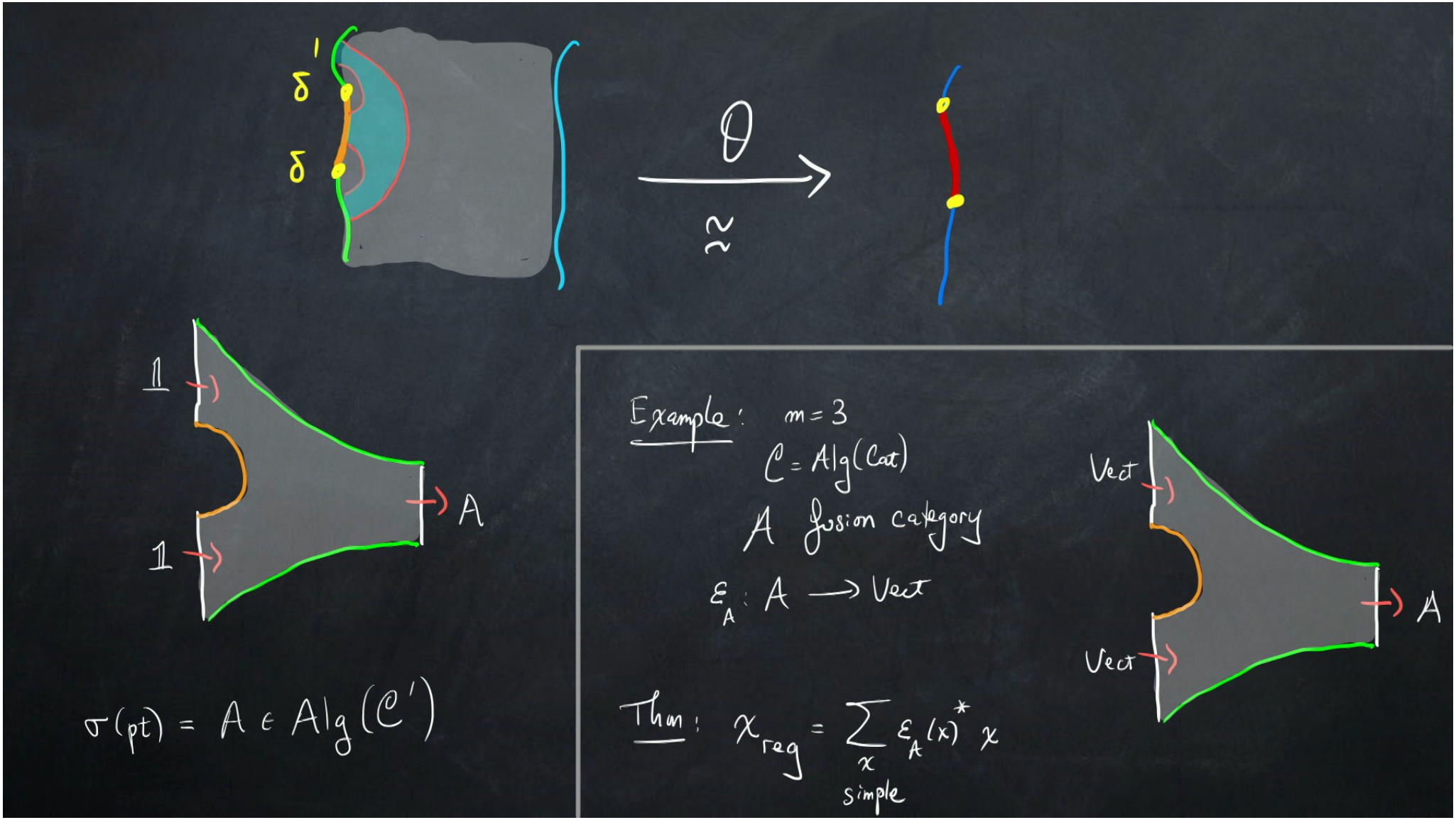
(σ, ρ) n-dim'l finite symmetry data



Defn: A right σ-module $\mathcal{E}$ is an augmentation if $\mathcal{E}(pt): \sigma(pt) \longrightarrow 1$ is an augmentation A of A.

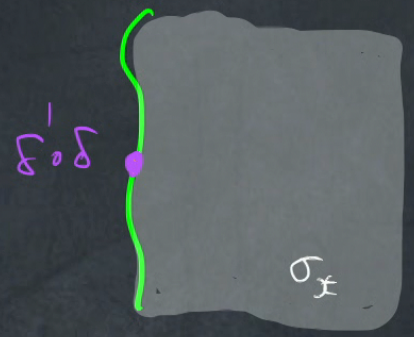

G' subgroup.
X
G'
X
G
BG
BG
ρ
≃ 𝟙.
Defn: A
if
aug

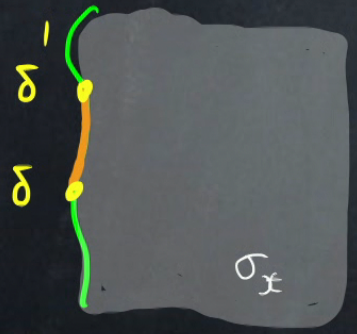

ρ
α
$\tilde{F}$
θ
≈
F
ε
α
$\tilde{F}$
=:
F/σ
Thm: There is a unique (generating) topological domain wall δ : ρ → ε and δ' : ε → ρ
δ'
δ
θ
≈
Q: Compute δ' ∘ δ : ρ → ρ
δ' ∘ δ


δ'
δ
θ
≃
𝟙
𝟙
A
σ(pt) = A ∈ Alg(C')
Example: m = 3
C = Alg(Cat)
A fusion category
$\mathcal{E}_A$: A ⟶ Vect
Thm: $\chi_{reg} = \sum_{x \text{ simple}} \mathcal{E}_A(x)^* \chi$
Vect
Vect
A

Finite Homotopy Theory Case

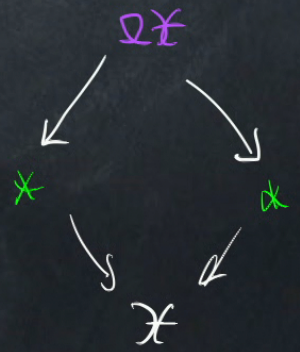
$\mathcal{X}$ π-finite





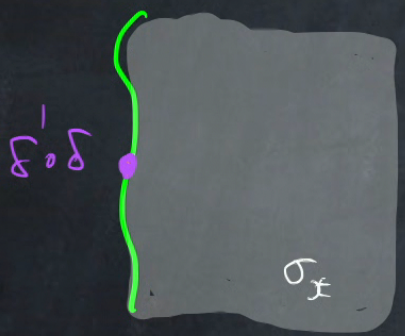


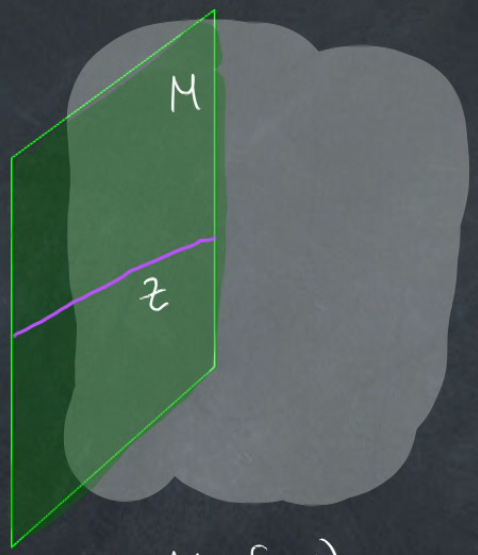
Compose



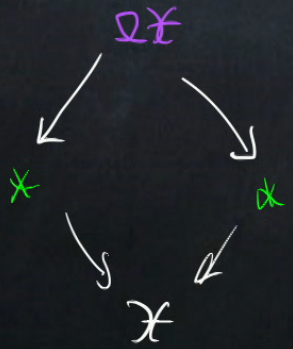
Example: $\mathcal{X} = B^{p+1}A$

$\Omega\mathcal{X} = B^{p}A$





$(\mathrm{Codim}_M Z = 1)$



Ex: $\mathcal{X} = BG$

For $g \in G$ have defect $\eta(g)$ where trivialization jumps by g.

Then

$$\delta' \circ \delta = \sum_{g \in G} \eta(g)$$

Quotient Defects

(σ, ρ), ε augmentation

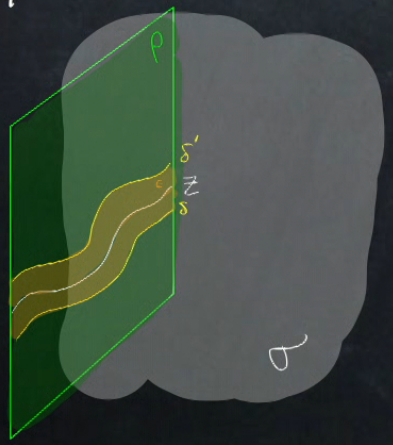
$Z \subset M$ submanifold

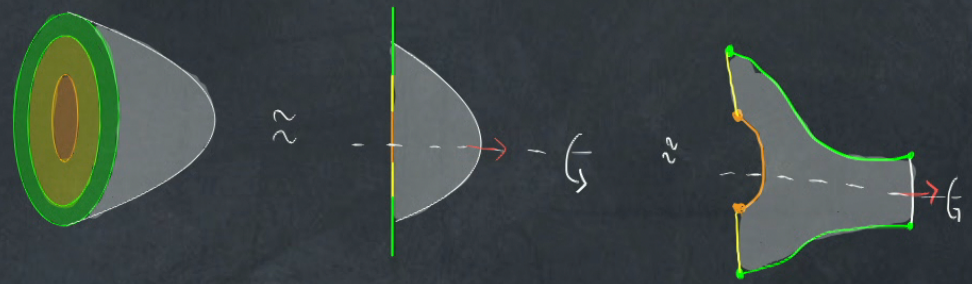
$\nu \subset M$ tubular neighborhood

$\downarrow$

Z

Def'n: The quotient defect $\varepsilon(Z)$ is:

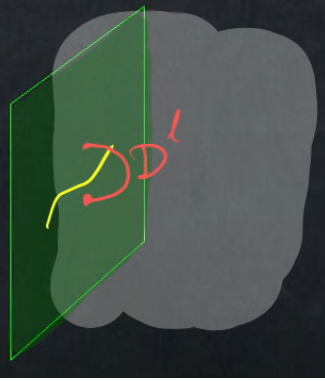




$\operatorname{Codim}_M Z = 2$

Finite homotopy Theory σ_x

Codim $= l$ ρ-defect

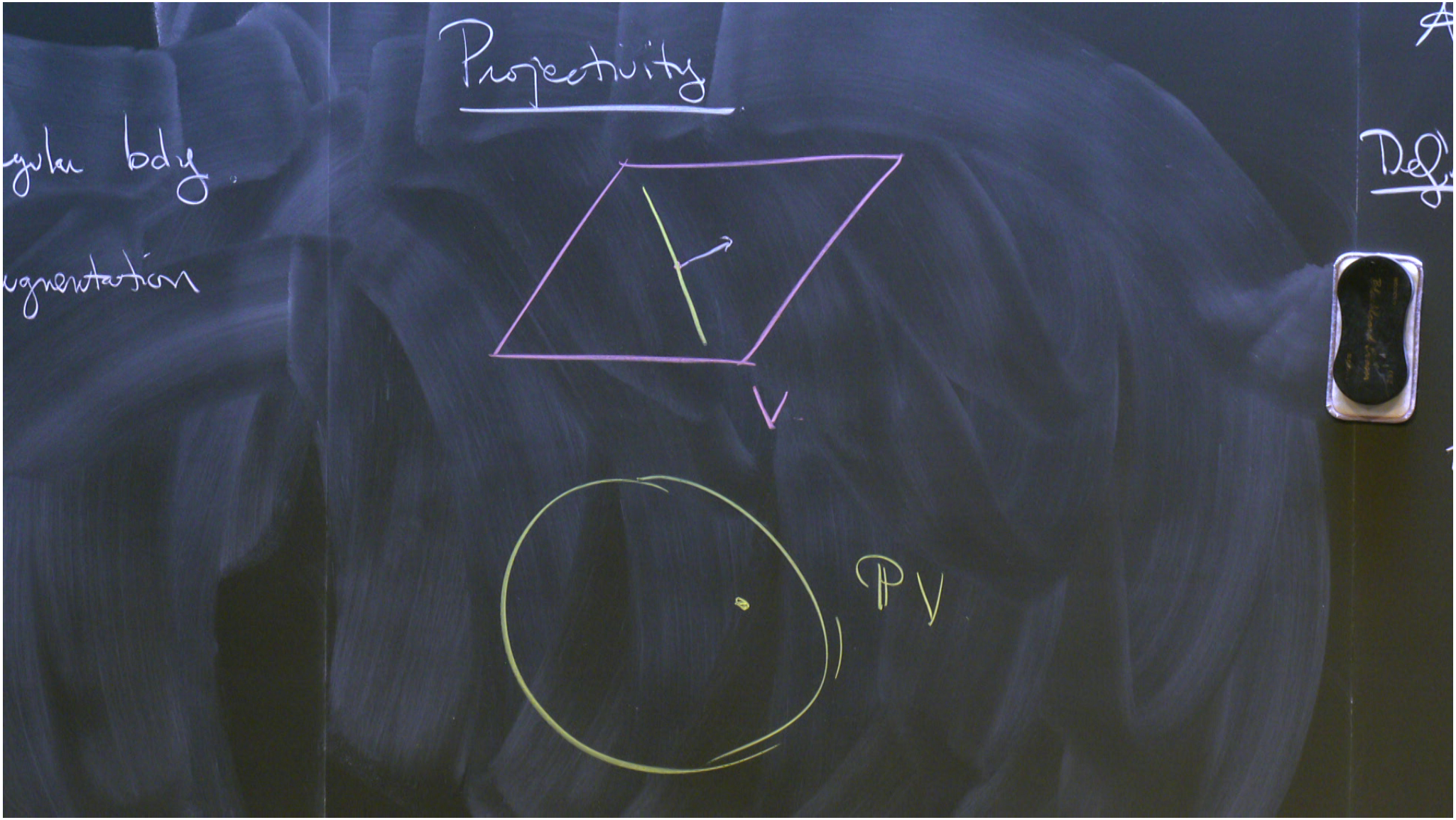


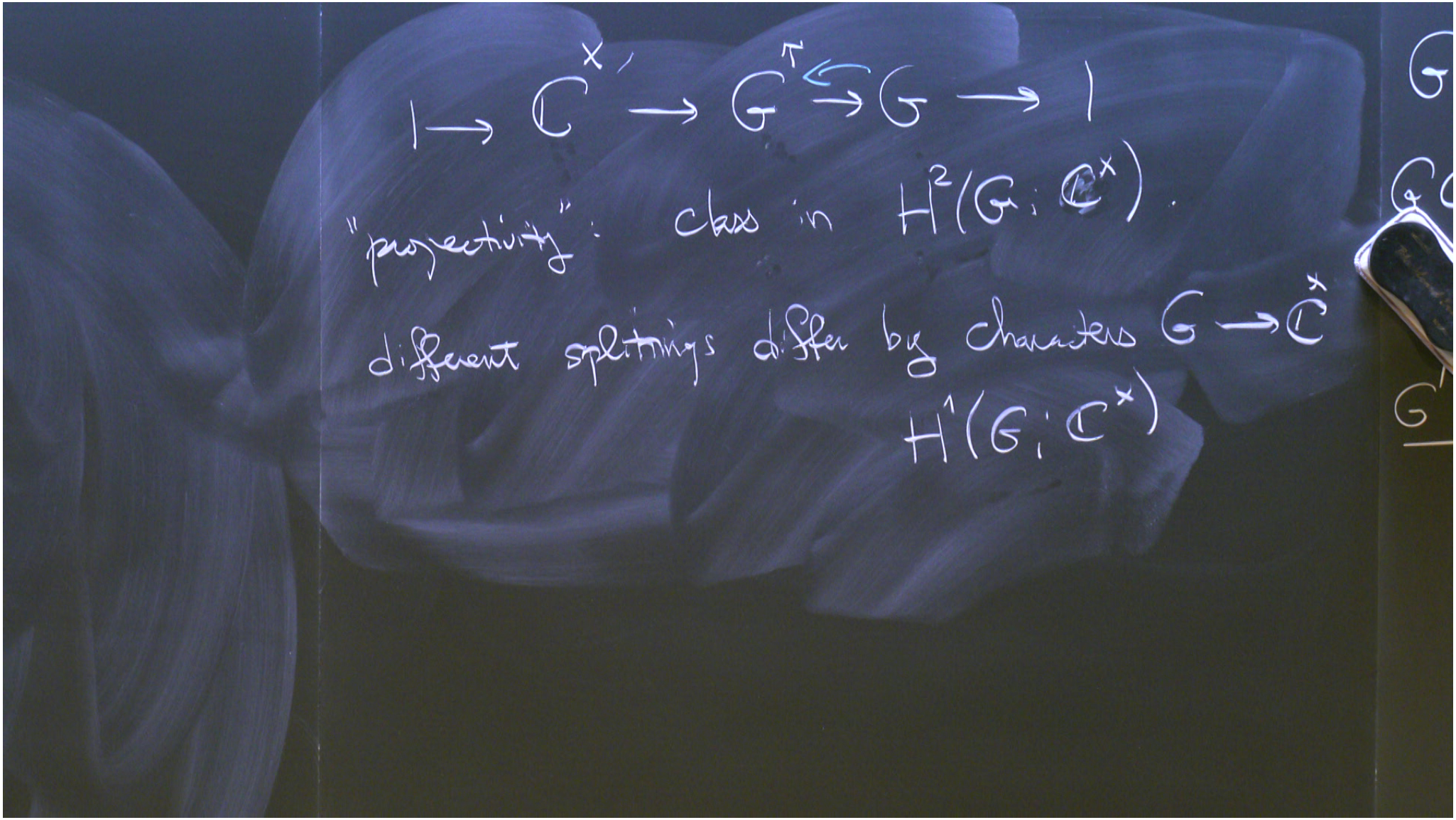
semiclassical defect

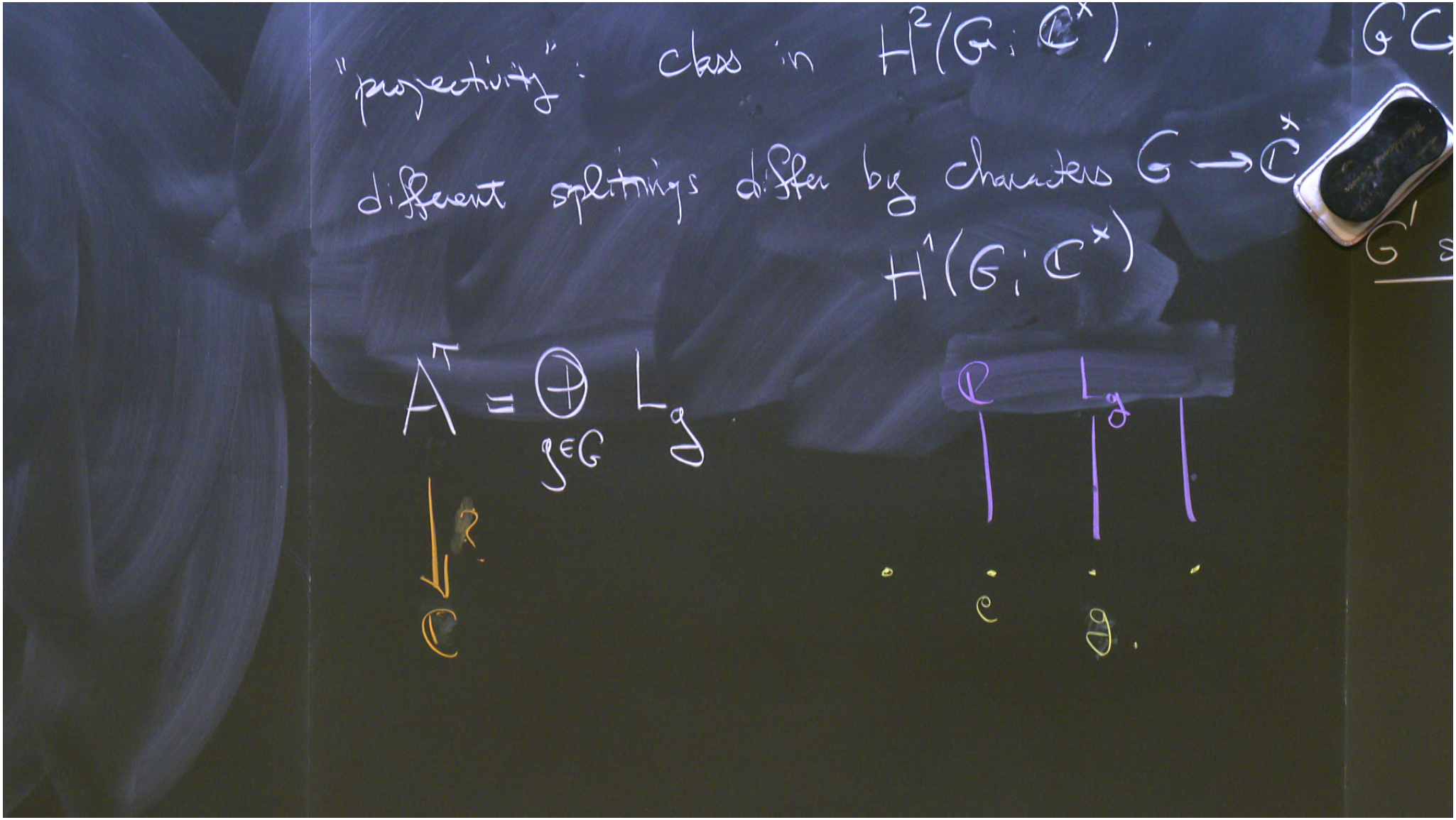
$$\begin{array}{c} Y \\ \downarrow \\ \Omega^l X \end{array}$$

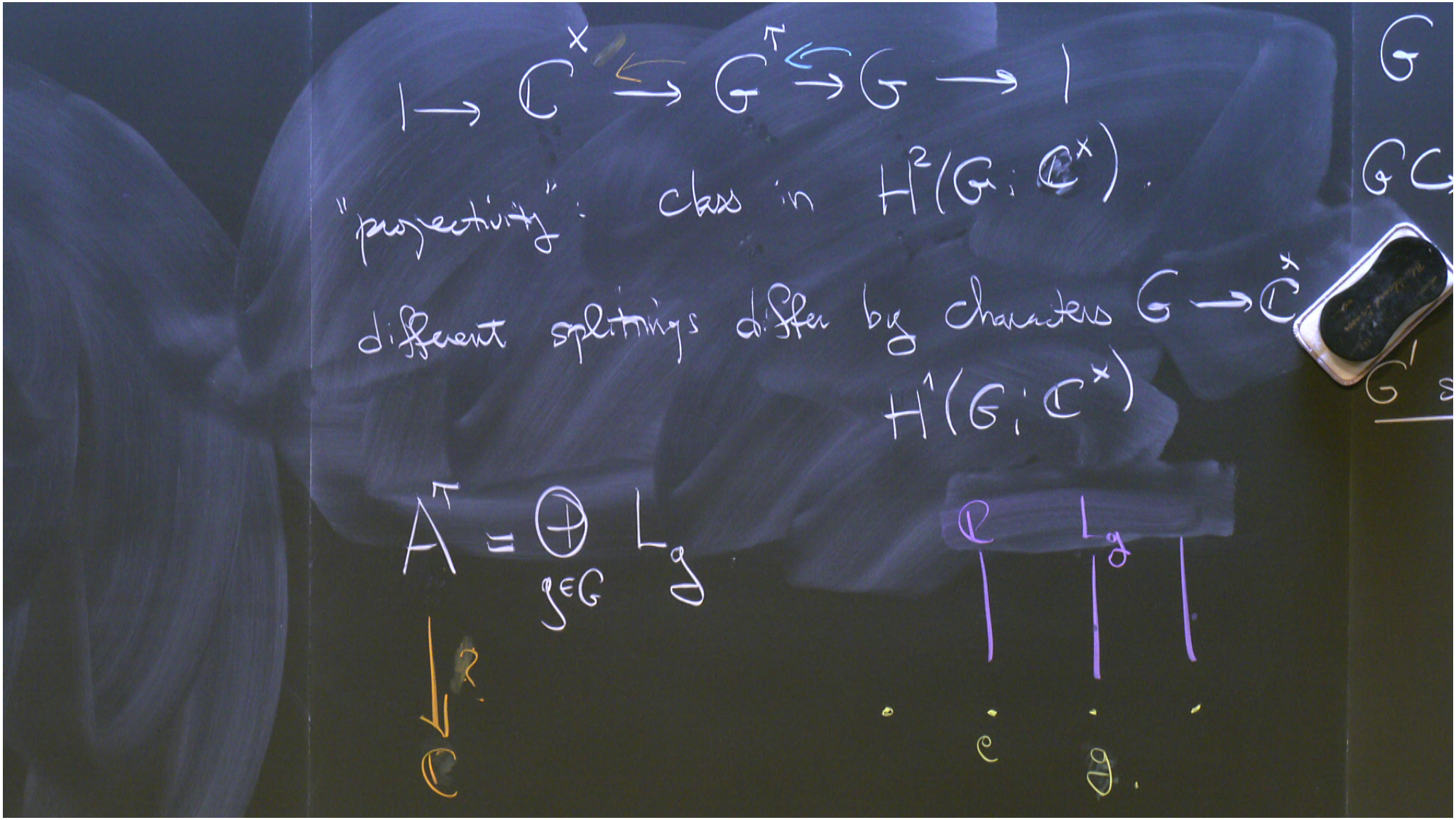
Problem: quotient def

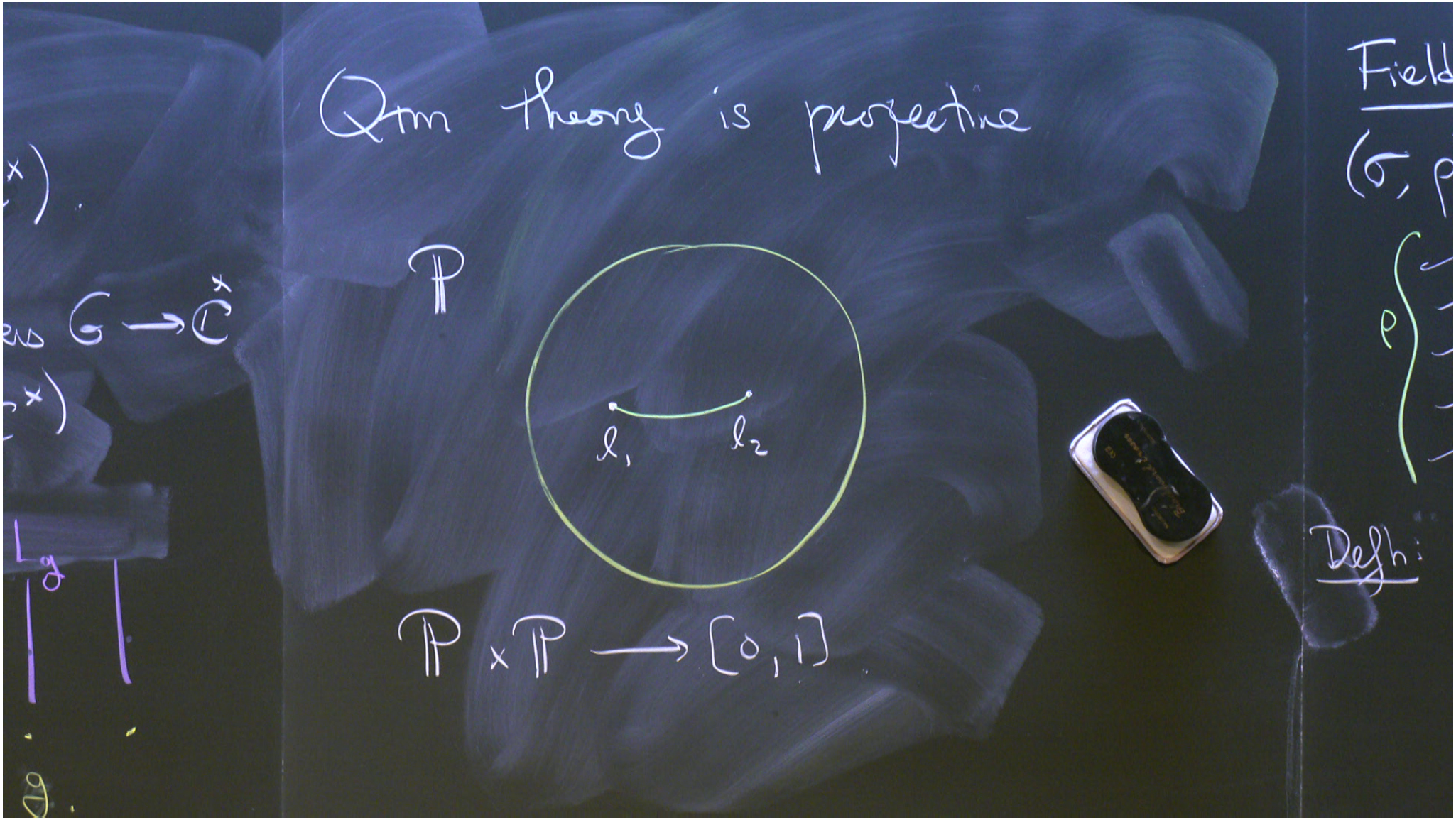
$$\begin{array}{c} \Omega^l X \\ \downarrow \mathrm{id} \\ \Omega^l X \end{array}$$

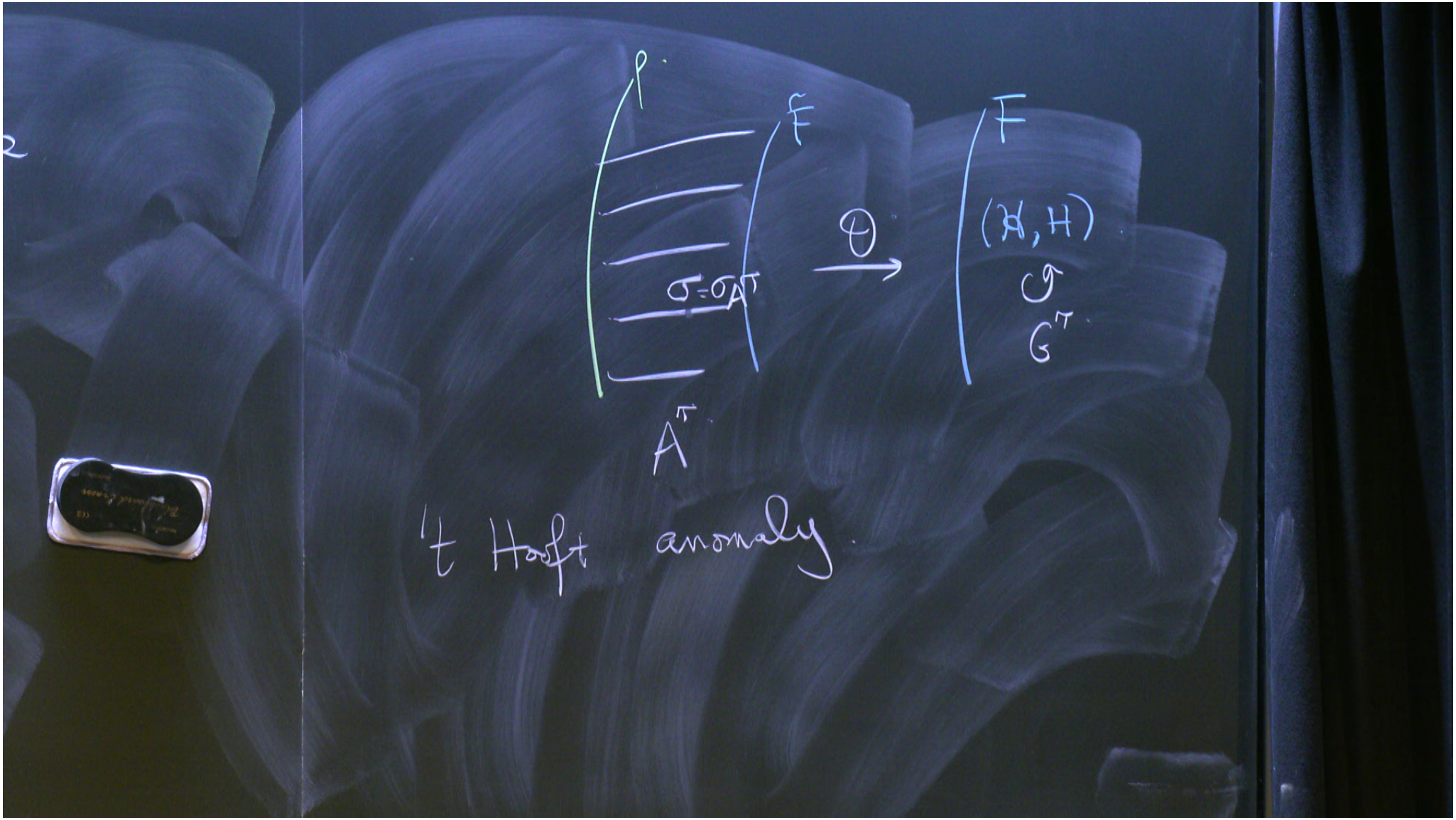

Projectivity
V
PV


$1 \to \mathbb{C}^\times \to G \xrightarrow{\pi} G \to 1$
"projectivity": class in $H^2(G; \mathbb{C}^\times)$.
different splittings differ by characters $G \to \mathbb{C}^\times$
$H^1(G; \mathbb{C}^\times)$


"projectivity": class in $H^2(G; \mathbb{C}^\times)$.
different splittings differ by characters $G \to \mathbb{C}^\times$
$H^1(G; \mathbb{C}^\times)$
$A = \bigoplus_{g \in G} L_g$
$\mathbb{C}$
L_g
e
g


$1 \to \mathbb{C}^\times \to G^\tau \to G \to 1$
"projectivity": class in $H^2(G;\mathbb{C}^\times)$.
different splittings differ by characters $G \to \mathbb{C}^\times$
$H^1(G;\mathbb{C}^\times)$
$A^\tau = \bigoplus_{g\in G} L_g$
L_g


Qm theory is projective
$\mathbb{P}$
ℓ_1
ℓ_2
$\mathbb{P} \times \mathbb{P} \longrightarrow [0,1]$
Field
Defn:

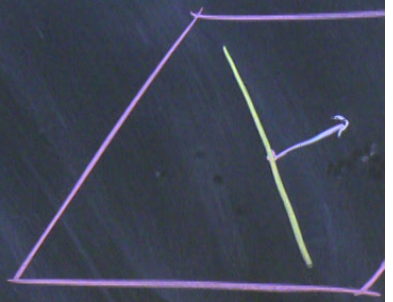

P
F
$\sigma = \sigma_{A^T}$
A^T
Θ
F
$(\mathcal{H}, H)$
$\mathcal{G}$
G^T
't Hooft anomaly

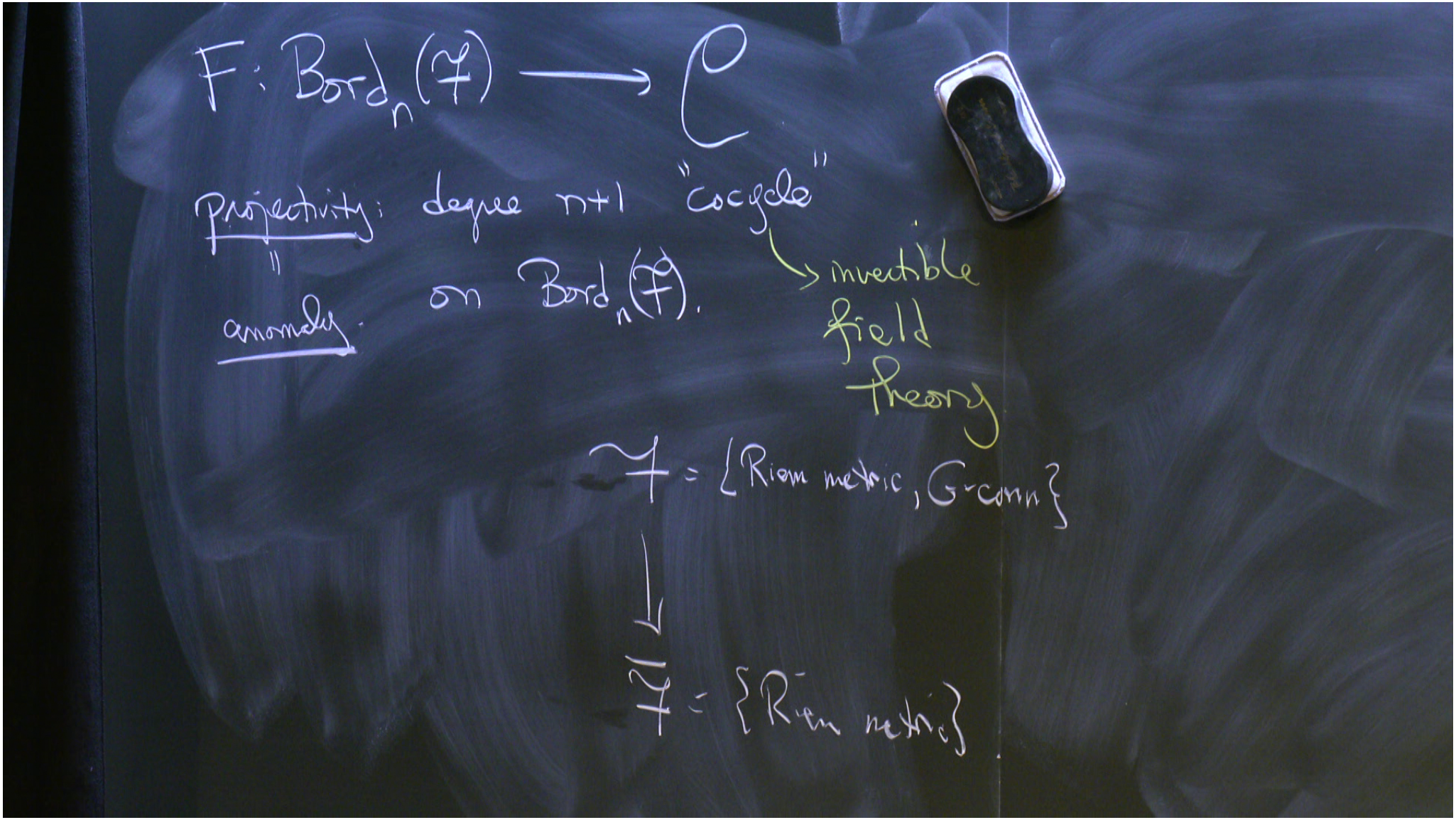
$\mathrm{Bord}_n(\mathcal{T}) \longrightarrow \mathbb{C}$

Projectivity

Projectivity: degree $n+1$ "cocycle" on $\mathrm{Bord}_n(\mathcal{T})$.

↳ invertible field theory




F : Bord_n(?) ⟶ C
Projectivity: degree n+1 "cocycle"
anomaly
on Bord_n(?).
invertible field theory
? = {Riem metric, G-conn}
? = {Riem metric}

cycle

↳ invertible field theory

{Riem metric, G-conn}

{Riem metric}

$\mathcal{X}$ π-finite space.

$m = n+1$

$(\mathcal{X}, \lambda)$ λ cocycle in degree m

(Y, μ) μ trivalizes $f_2^* \lambda_2 - f_1^* \lambda_1$.

f_1 f_2

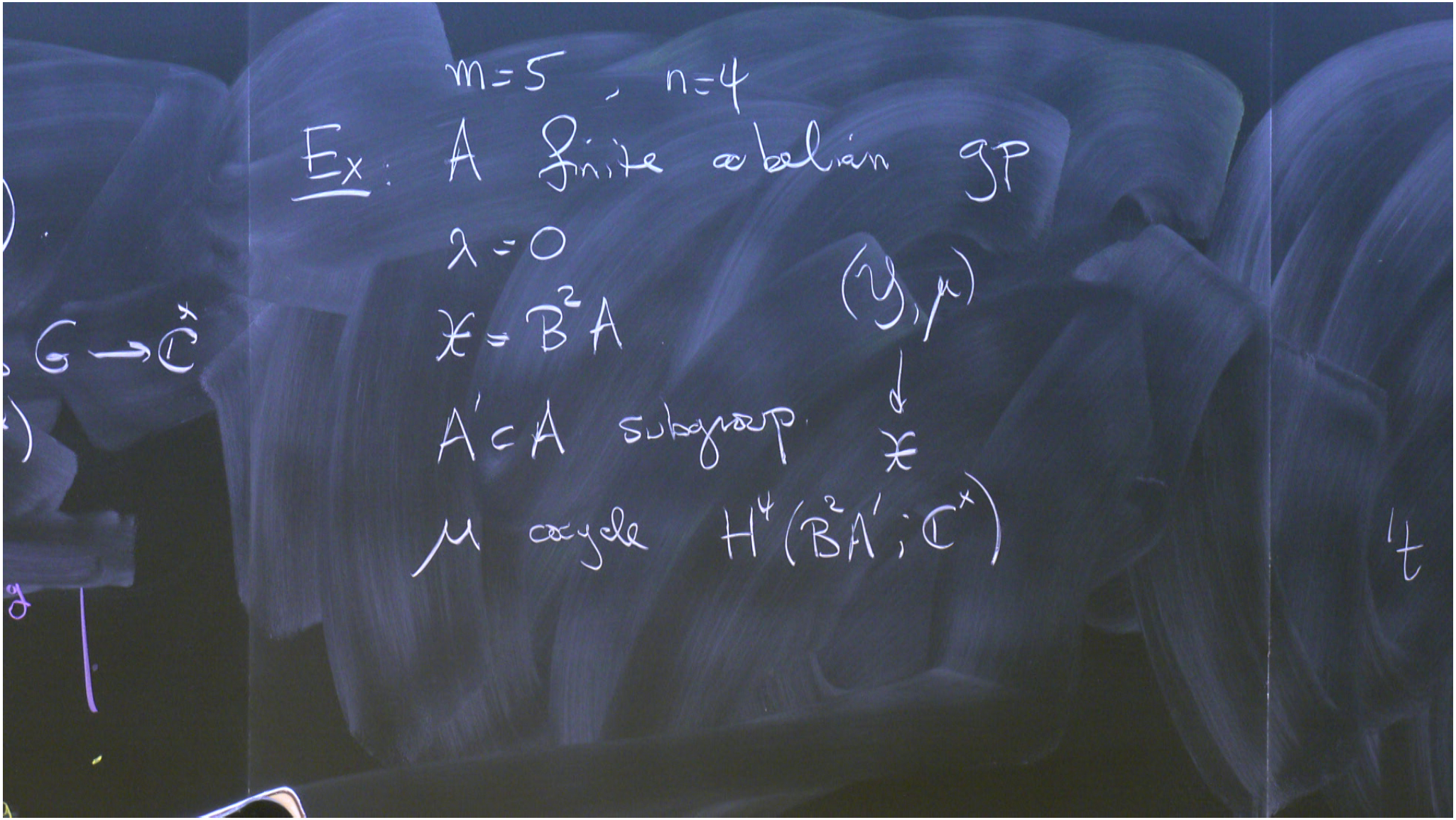
$(\mathcal{X}_1, \lambda_1)$ $(\mathcal{X}_2, \lambda_2)$

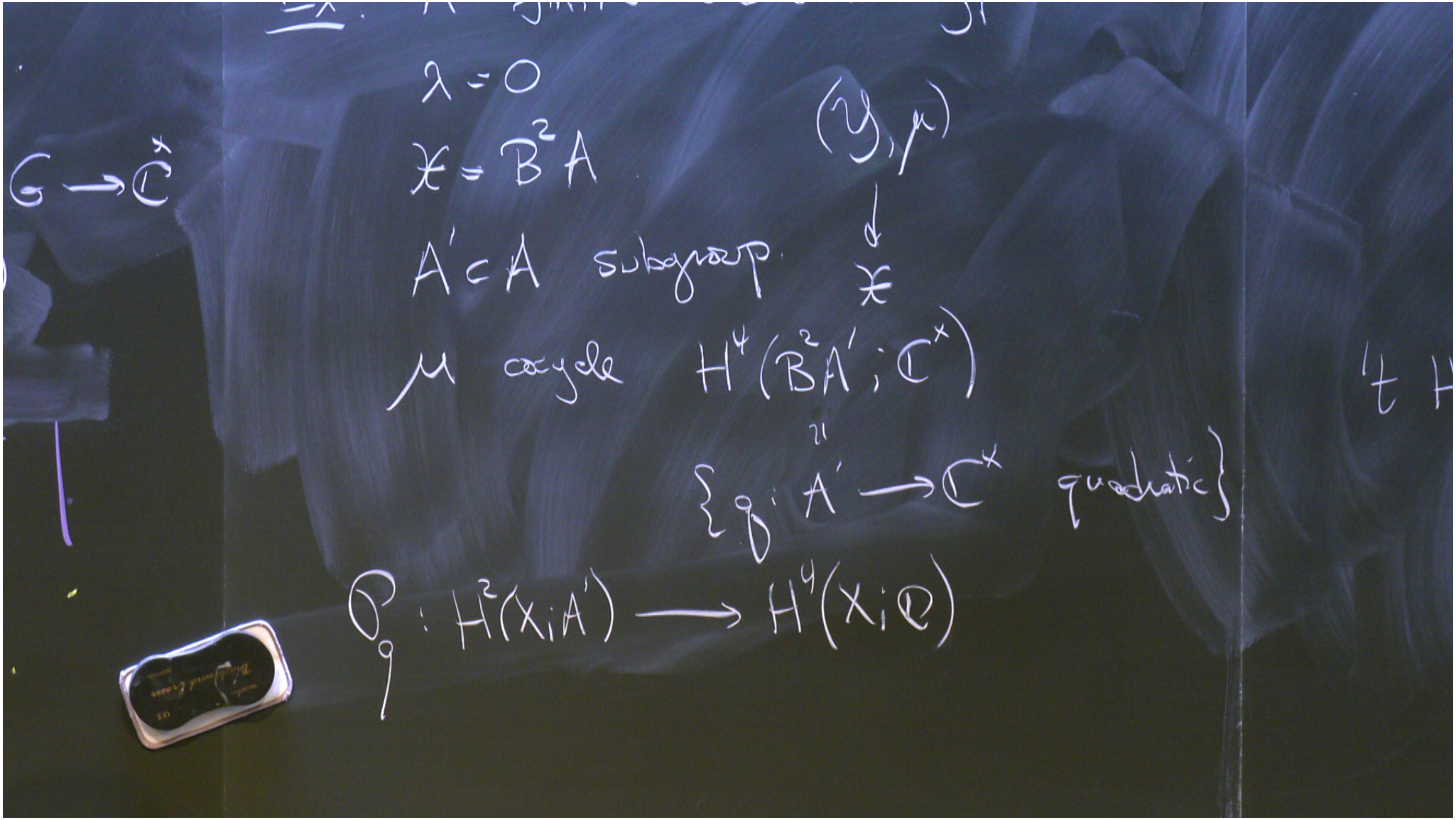
"proper

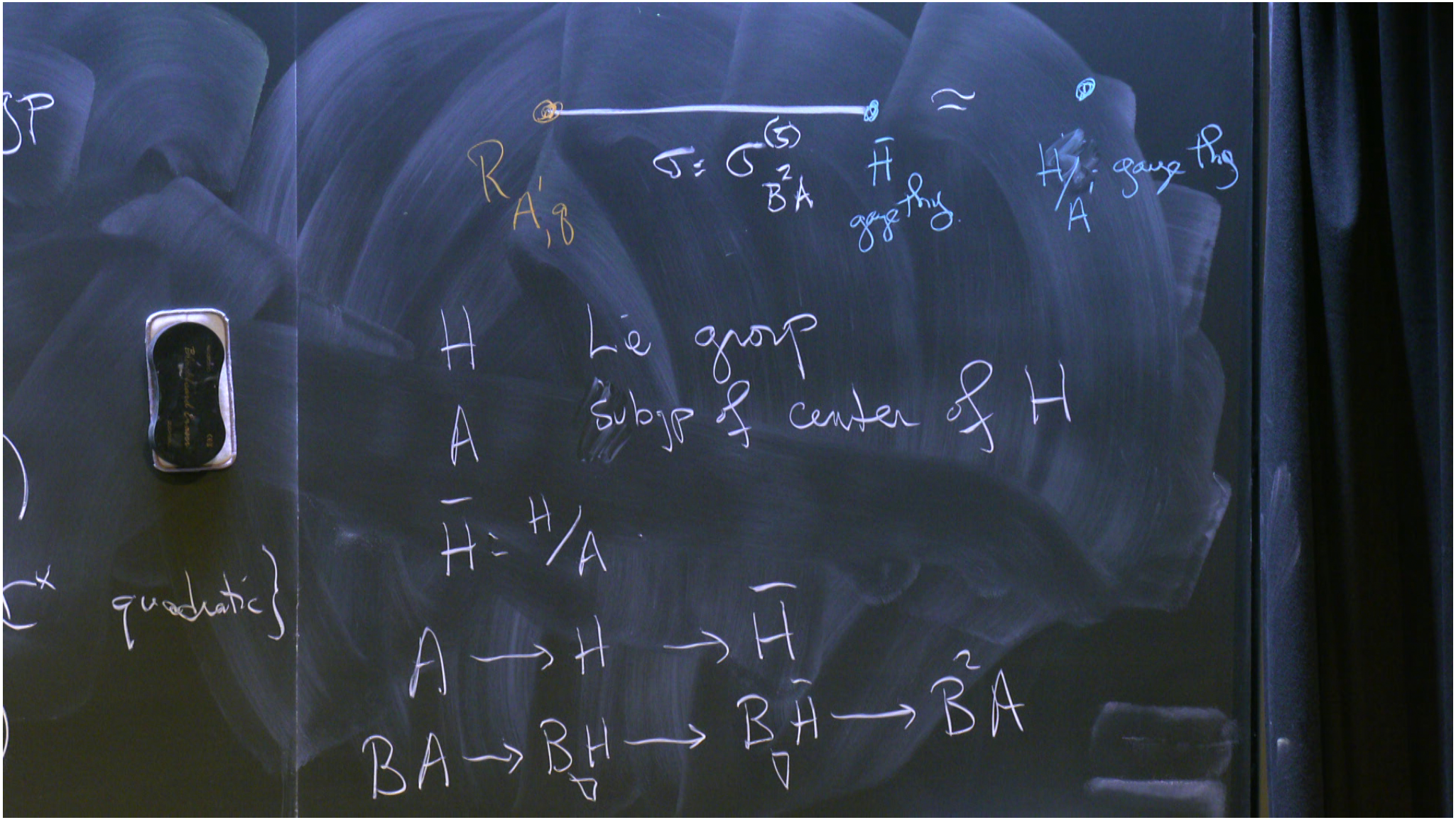
diffe

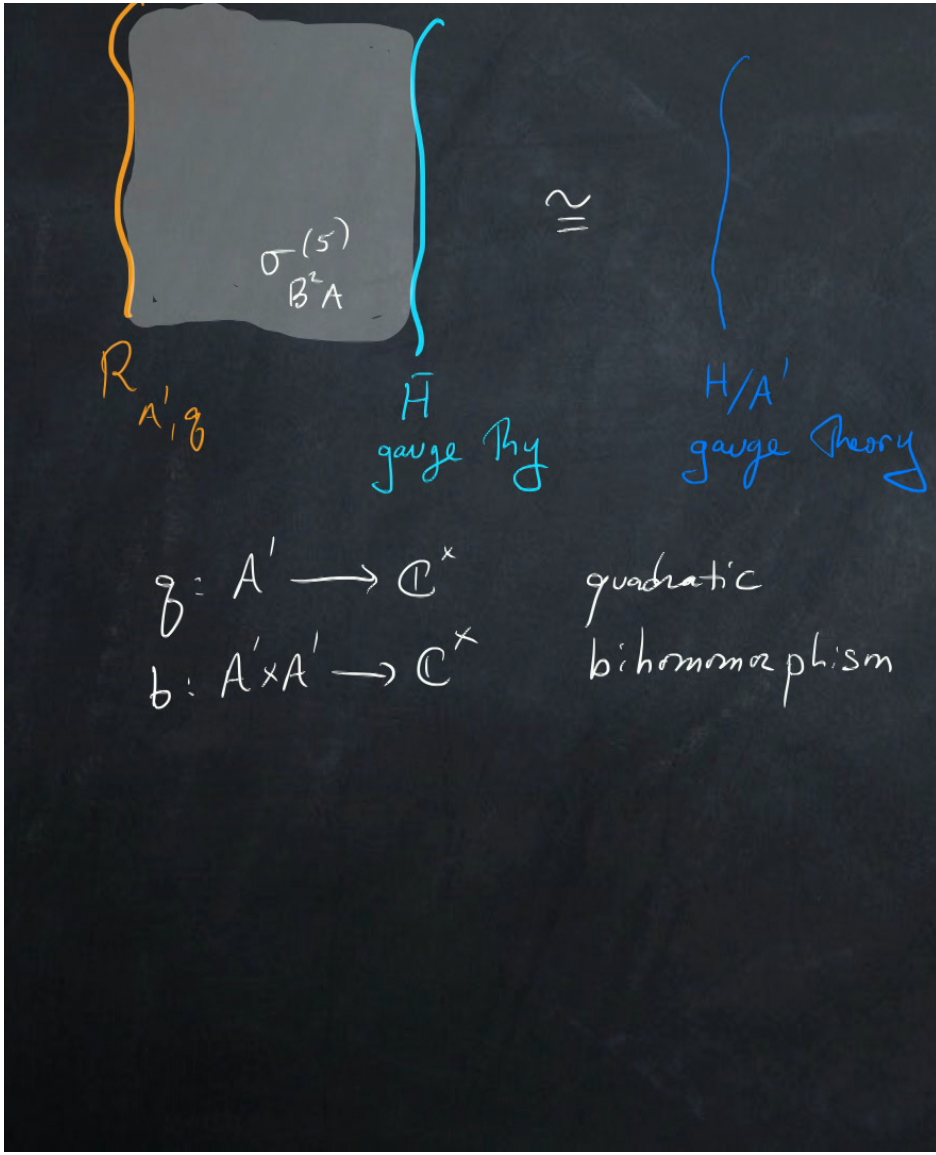
$A^{\top}$

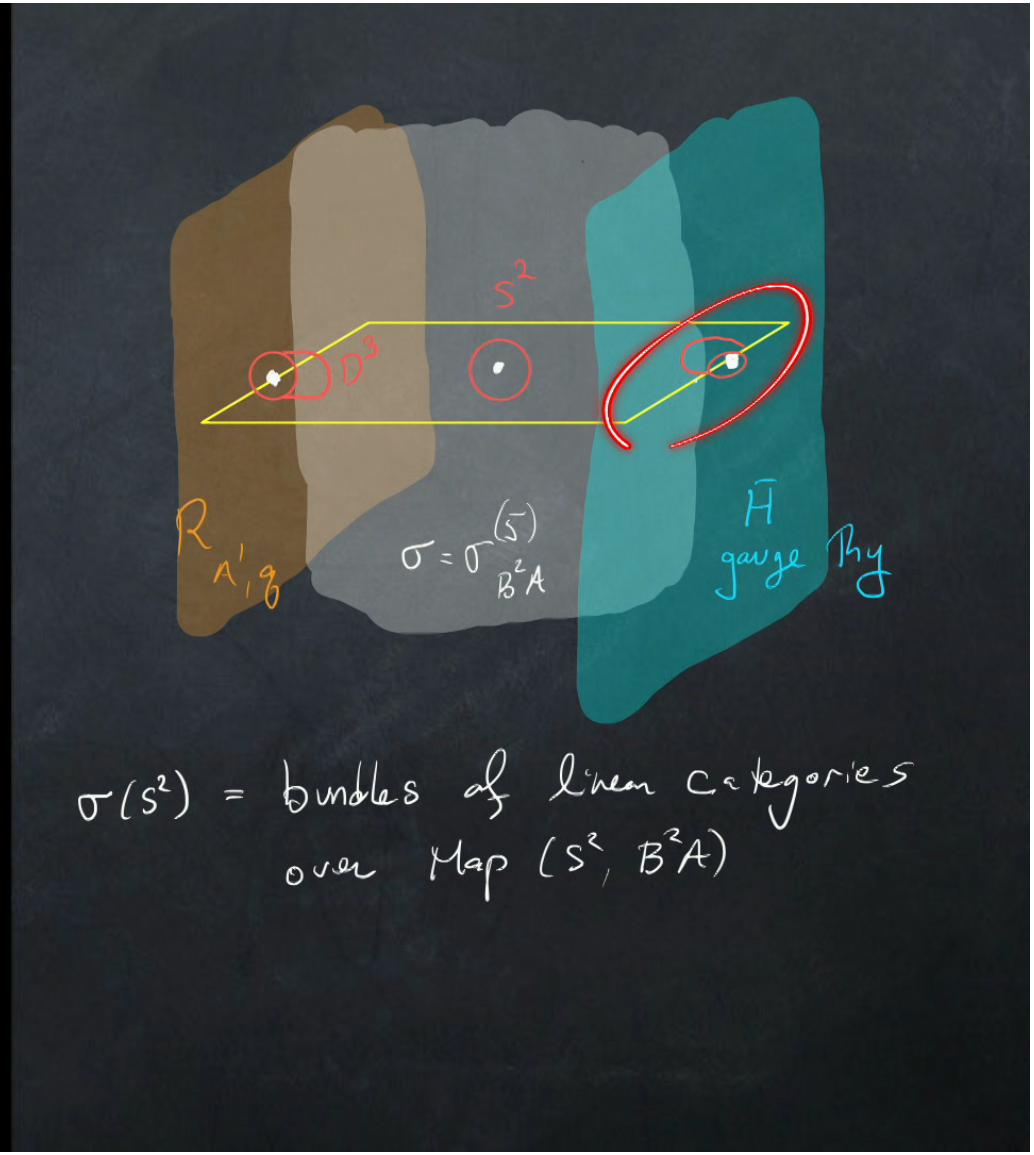
$\mathbb{C}$

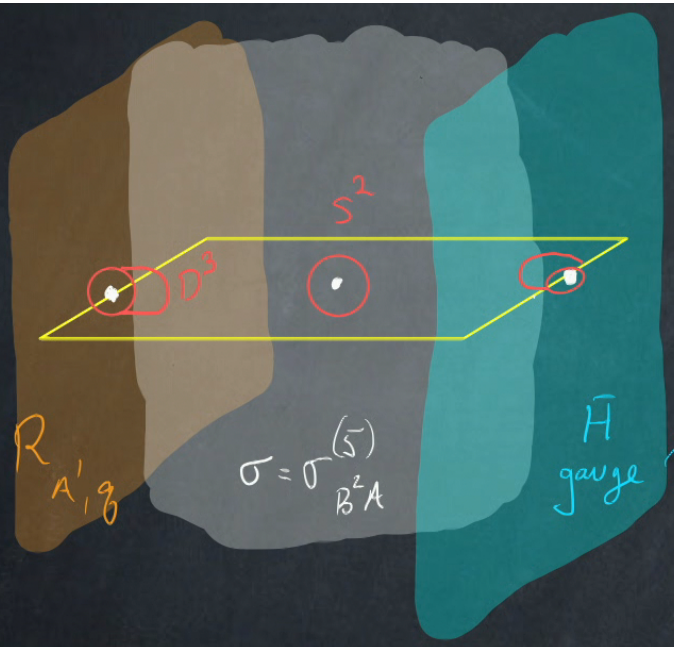

m=5 , n=4
Ex: A finite abelian gp
λ=0
χ=B²A
(γ,μ)
A'⊂A subgroup
μ cocycle H⁴(B²A';ℂˣ)
G→ℂˣ


$\lambda = 0$
$\mathcal{X} = B^2 A$
$A' \subset A$ subgroup
μ cocycle
(Y, μ)
$\downarrow$
$\mathcal{X}$
$H^4(B^2A'; \mathbb{C}^\times)$
$\{q : A' \to \mathbb{C}^\times$ quadratic$\}$
$\rho_q : H^2(X; A') \longrightarrow H^4(X; \mathbb{Q})$


$R_{A',\beta}$
$\sigma = \sigma^{(2)}_{BA}$
$\bar{H}$ gauge thy
$\simeq$
H/A ; gauge thy
H Lie group
A subgp of center of H
$\bar{H} = H/A$
$A \to H \to \bar{H}$
$BA \to BH \to B\bar{H} \to B^2A$


$\sigma^{(S^1)}_{B^2A}$
$\simeq$
$R_{A',q}$
$\bar{H}$ gauge Thy
H/A' gauge Theory
$q : A' \longrightarrow \mathbb{C}^\times$ quadratic
$b : A' \times A' \longrightarrow \mathbb{C}^\times$ bihomomorphism


S^2
D^3
$R_{A',q}$
$\sigma = \sigma^{(S^1)}_{B^2A}$
$\bar{H}$ gauge Thy
$\sigma(S^2)$ = bundles of linear categories over Map (S^2, B^2A)



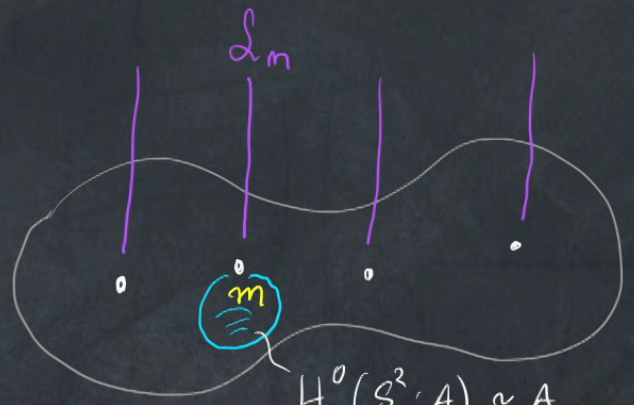
$\sigma(S^2)$ = bundles of linear categories over Map (S^2, B^2A)

Map (S^2, B^2A) :

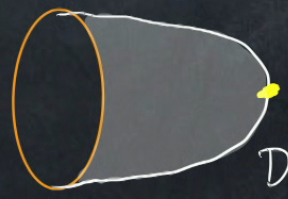
$\pi_0 = H^2(S^2; A) \simeq A \ni m$

$\pi_1 = H^1(S^2; A) = 0$

$\pi_2 = H^0(S^2; A) \simeq A$



$\mathcal{L}_m \simeq \bigoplus \mathcal{L}_{m,e} \cdot e$



- $m \in A'$
- $e|_{A'} = e'(m)^{-1}$

$H^2(S^2; A') \times H^0(S^2; A') \longrightarrow \mathbb{C}^\times$

$\simeq A'$ $\simeq A'$

$e' : A' \longrightarrow (A')^\vee$