Title: Introduction to Anomalies in Quantum Field Theory

Speakers:

Collection: Global Categorical Symmetries

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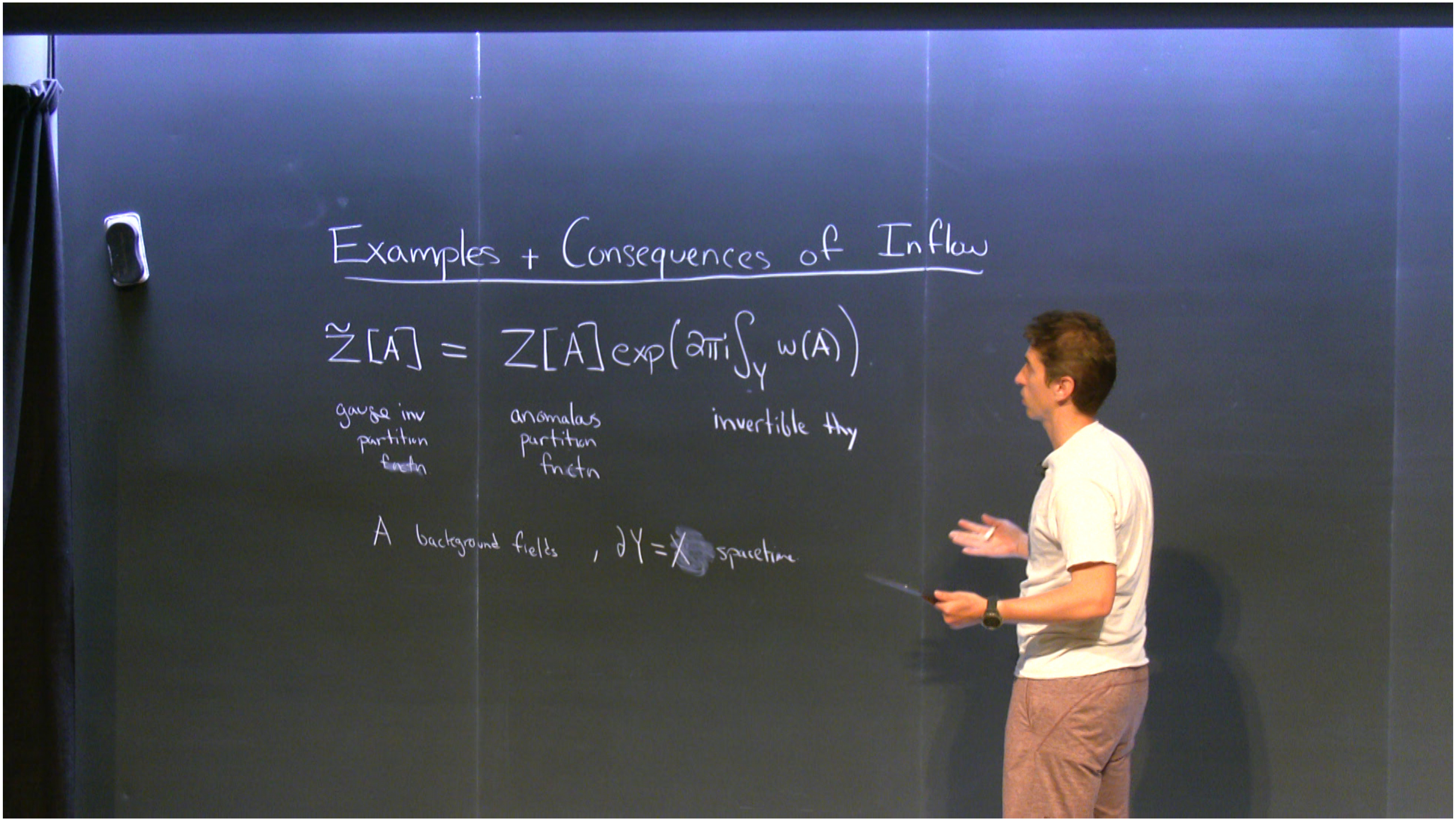
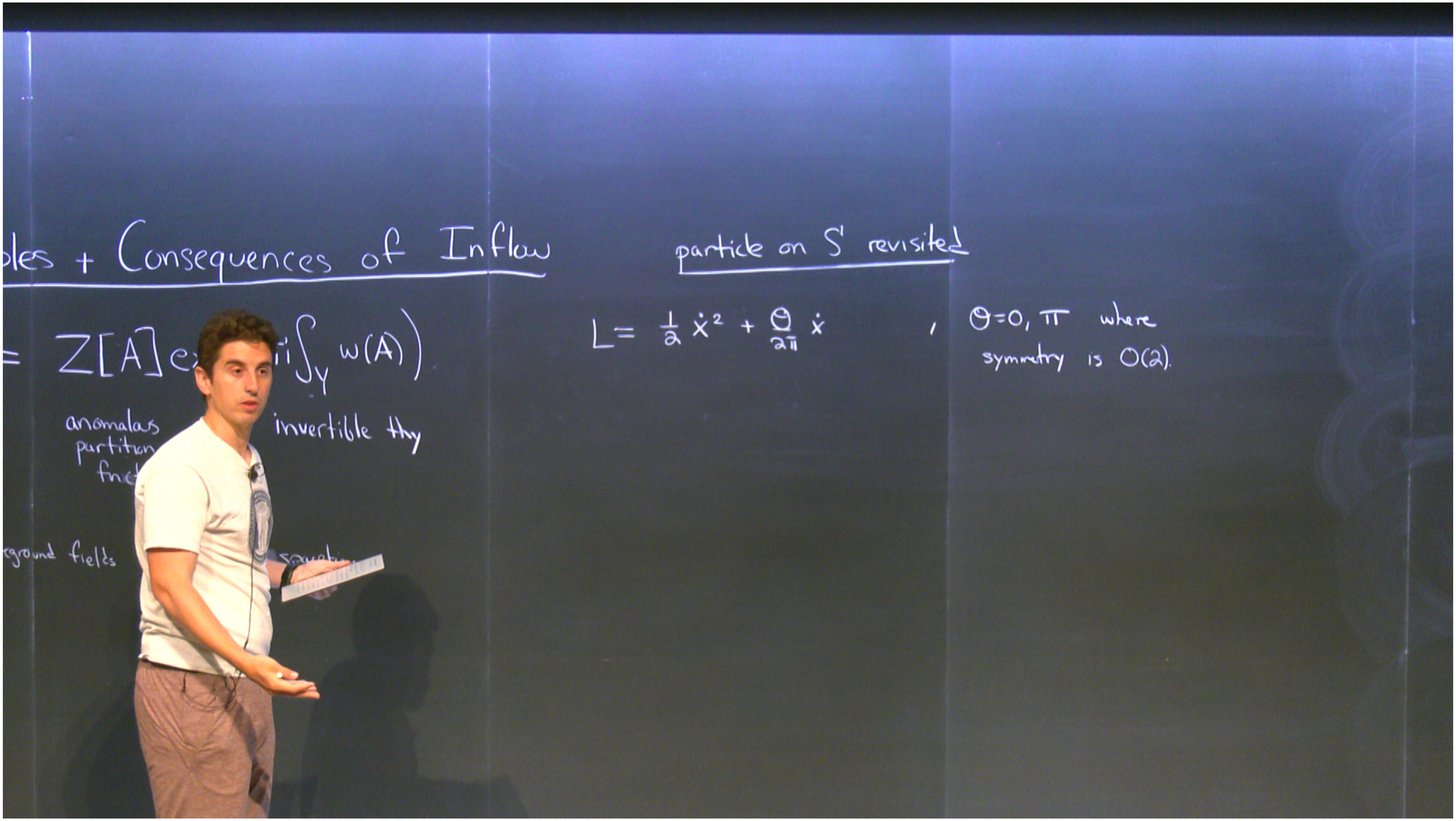
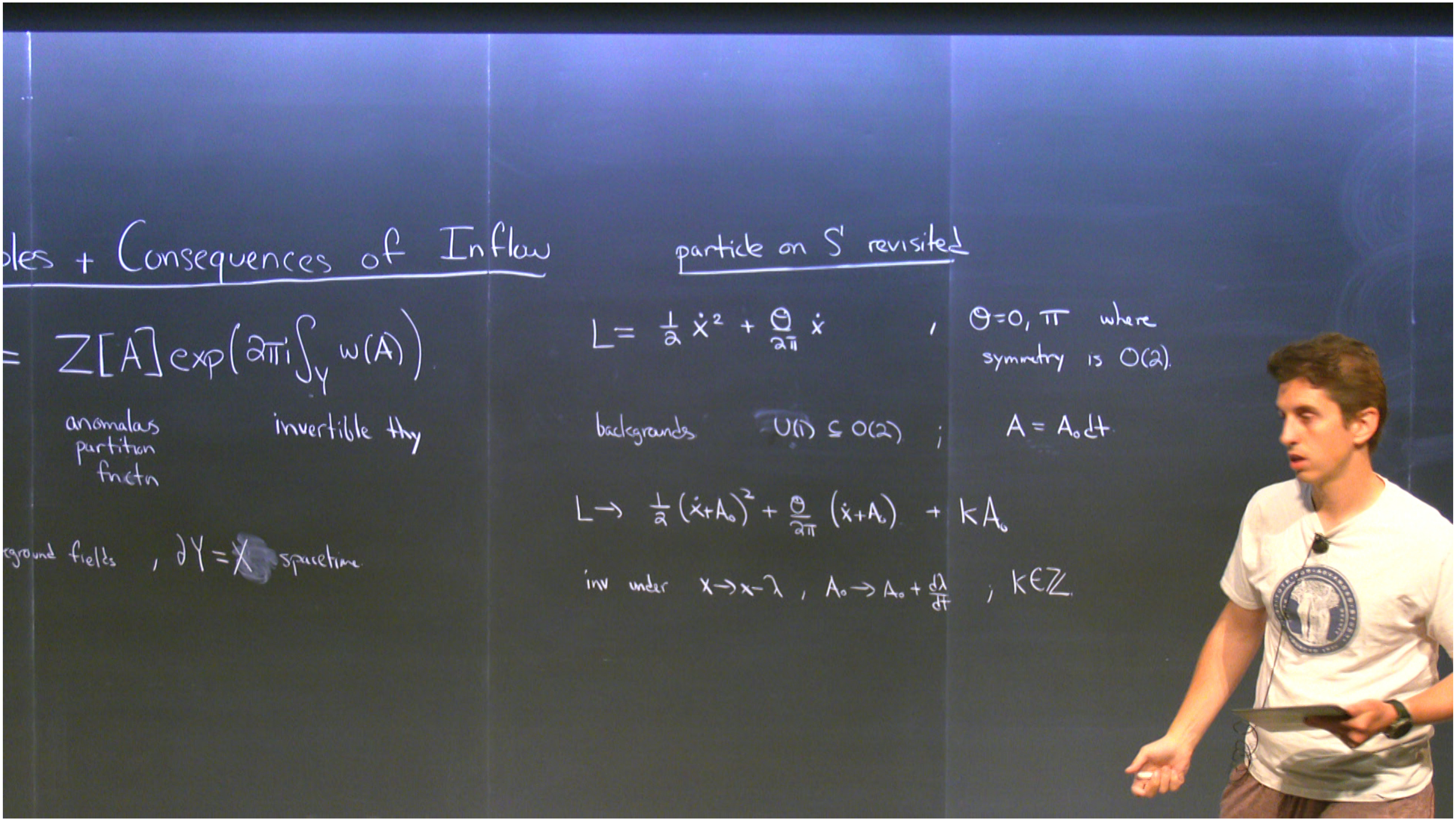
Examples + Consequences of Inflow


Examples + Consequences of Inflow
$\tilde{Z}[A] = Z[A] \exp\left(2\pi i \int_Y \omega(A)\right)$
gauge inv partition fnctn
anomalous partition fnctn
invertible thy
A background fields , $\partial Y = X$ spacetime


les + Consequences of Inflow
Z[A]
w(A)
anomalous partition
invertible thy
ground fields
particle on S' revisited
L = 1/2 ẋ² + θ/2π ẋ , θ = 0, π where
symmetry is O(2).


…les + Consequences of Inflow
$= Z[A] \exp\left(2\pi i \int_Y \omega(A)\right)$
anomalous partition fnctn
invertible thy
…ground fields , $\partial Y = X$ spacetime
particle on S^1 revisited
$L = \frac{1}{2}\dot{x}^2 + \frac{\theta}{2\pi}\dot{x}$, $\theta = 0, \pi$ where symmetry is $O(2)$.
backgrounds $U(1) \subseteq O(2)$; $A = A_0 dt$
$L \to \frac{1}{2}(\dot{x}+A_0)^2 + \frac{\theta}{2\pi}(\dot{x}+A_0) + kA_0$
inv under $x \to x - \lambda$, $A_0 \to A_0 + \frac{d\lambda}{dt}$; $k \in \mathbb{Z}$

Inflow

particle on S^1 revisited

$$L = \frac{1}{2}\dot{x}^2 + \frac{\theta}{2\pi}\dot{x} \quad , \quad \theta = 0, \pi \text{ where symmetry is } O(2).$$

backgrounds $U(1) \subseteq O(2)$; $A = A_0 dt$

$$L \to \frac{1}{2}(\dot{x}+A_0)^2 + \frac{\theta}{2\pi}(\dot{x}+A_0) + kA_0$$

inv under $x \to x - \lambda$, $A_0 \to A_0 + \frac{d\lambda}{dt}$; $k \in \mathbb{Z}$.

is e^{-S} invariant under C? ($x \to -x$, $A_0 \to -A_0$).

i) $\theta = 0$, $L = \frac{1}{2}(\dot{x}+A_0)^2 + kA_0$ inv if $k=0$

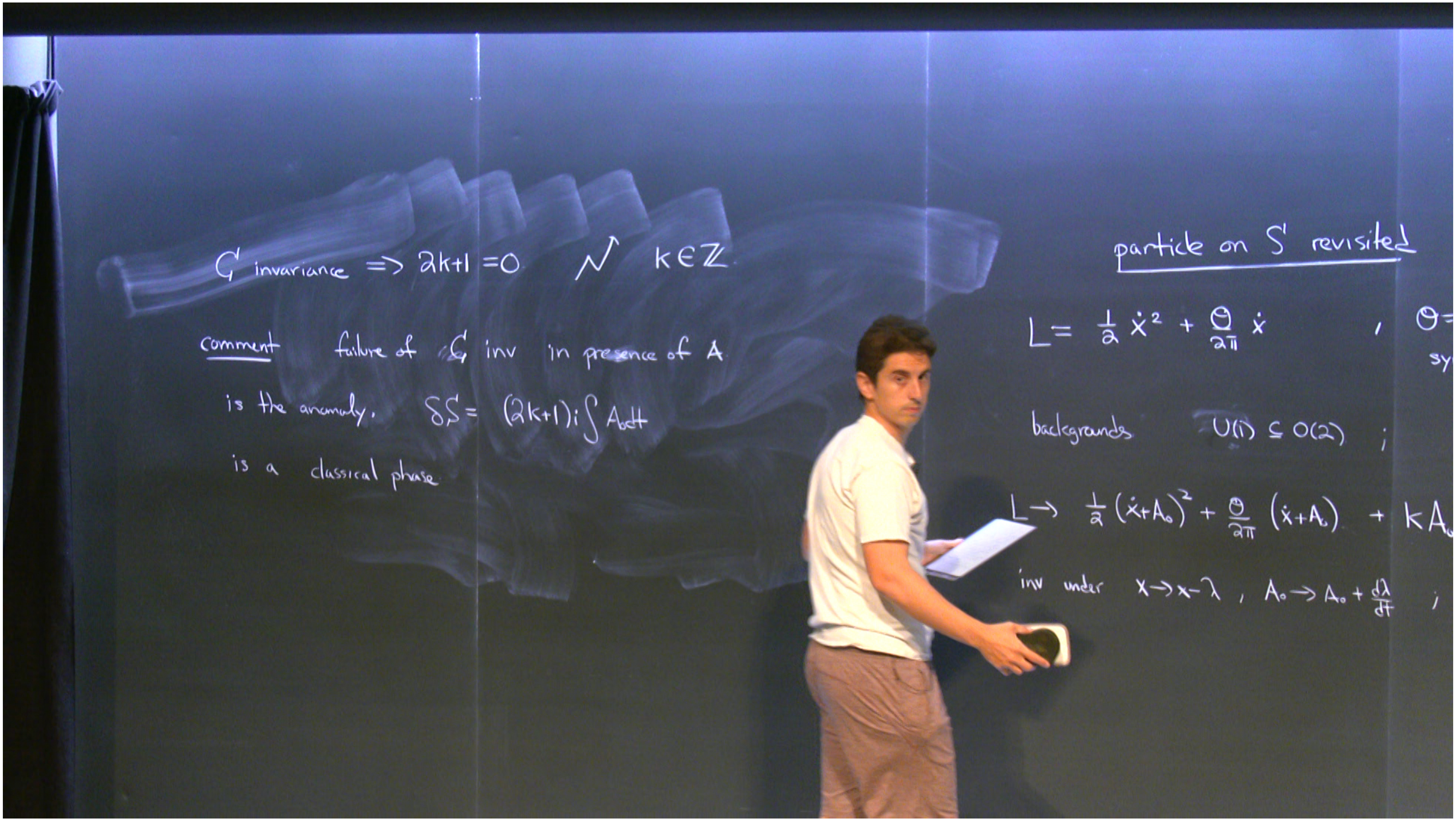
ii) $\theta = \pi$; $L = \frac{1}{2}(\dot{x}+A_0)^2 + \frac{\pi}{2\pi}(\dot{x}+A_0) + kA_0$

Exercise show under C:

$$e^{-S} \longrightarrow e^{-S} \exp\left(i(2k+1)\oint A_0 dt\right)$$

S = Euclidean action

time $= S^1$ compact


C invariance => 2k+1 = 0 ↯ k ∈ ℤ
comment failure of C inv in presence of A
is the anomaly. δS = (2k+1) i ∫ A₀dt
is a classical phase
particle on S¹ revisited
L = ½ ẋ² + θ/2π ẋ , θ=
sy
backgrounds U(1) ⊆ O(2) ;
L → ½ (ẋ+A₀)² + θ/2π (ẋ+A₀) + kA
inv under x → x−λ , A₀ → A₀ + dλ/dt ;

$+1 = 0 \quad \nearrow \quad k \in \mathbb{Z}$

C inv in presence of A

$S = (2k+1)\, i \int A_0 dt$

<u>inflow view</u> extend $S^1 = \partial Y$

A into Y

$$\tilde{Z}[A] = Z_1[A] \cdot \underbrace{\exp\left(\frac{i}{2}\int_Y F\right)}_{\text{invertible if } \partial Y = \phi\,,\ (-1)^{c_1}} \qquad F = dA \,,$$

$\theta = 0, \pi$ where symmetry is $O(2)$.

$A = A_0 dt$

<u>verify that $\tilde{Z}[A]$ is C inv.</u>

is e^{-S} invari

i) $\theta = 0$, $L = \frac{1}{2}($

ii) $\theta = \pi$; $L = \frac{1}{2}($

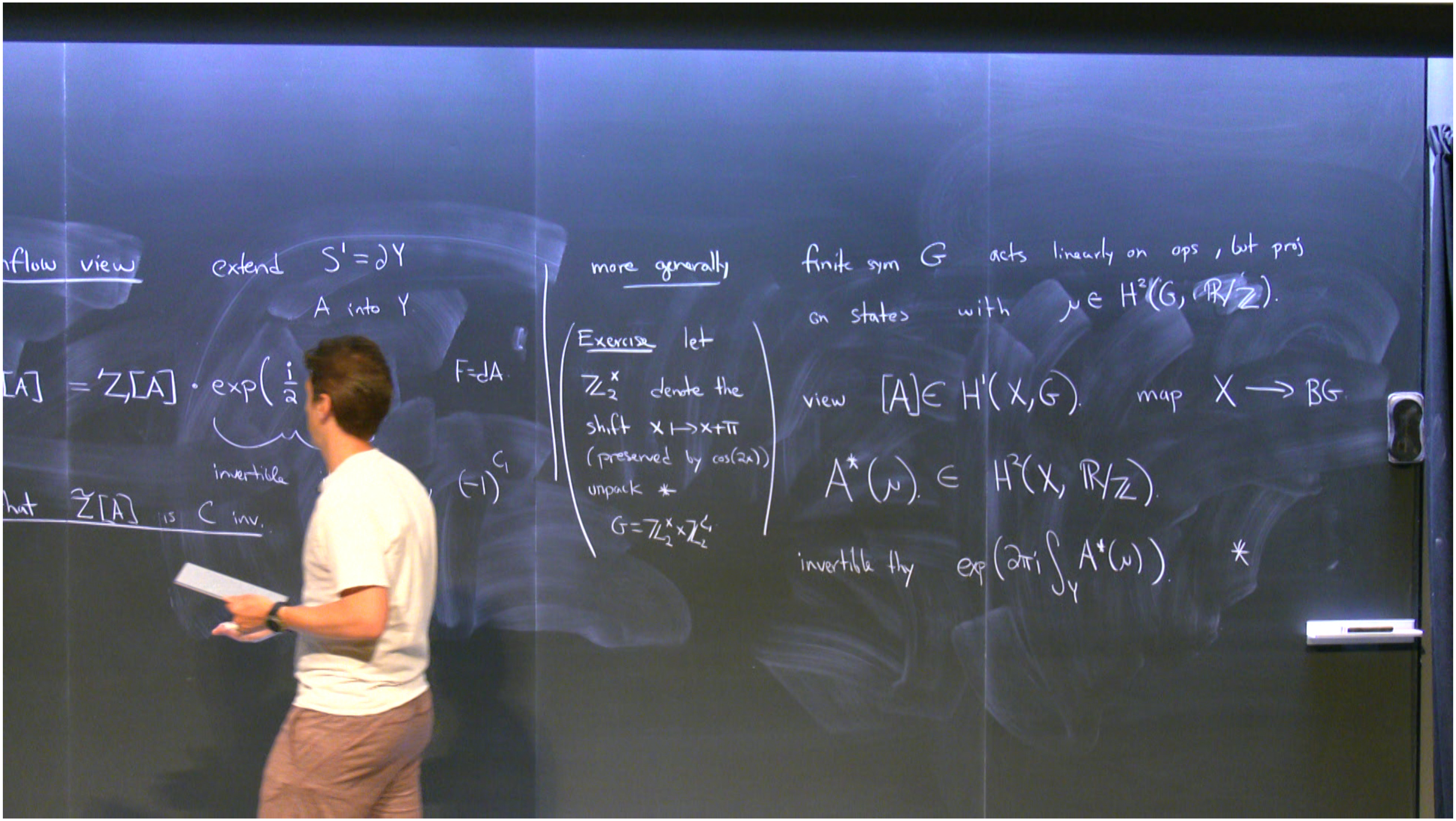
<u>Exercise</u> show under

$e^{-S} \longrightarrow e^{-S'}$

S = Euclidean action

tim

comp


flow view
extend S' = ∂Y
A into Y
[A] = Z,[A] · exp(i/2
F=dA
invertible
(-1)^C_1
hat Z[A] is C inv.
more generally
Exercise let
Z_2^x denote the
shift x ↦ x+π
(preserved by cos(2x))
unpack *
G = Z_2^x × Z_2^C_1
finite sym G acts linearly on ops, but proj
on states with μ ∈ H^2(G, R/Z).
view [A] ∈ H^1(X, G). map X → BG.
A*(μ) ∈ H^2(X, R/Z).
invertible thy exp(2πi ∫_Y A*(μ)) *

T-invariant fermions

real ferm $\{\psi^i, \psi^j\} = 2\delta^{ij}$

$i = 1, \dots, N$. T inv $\Rightarrow$ degenerate ground state unless $N = 0 \bmod 8$

inflow view

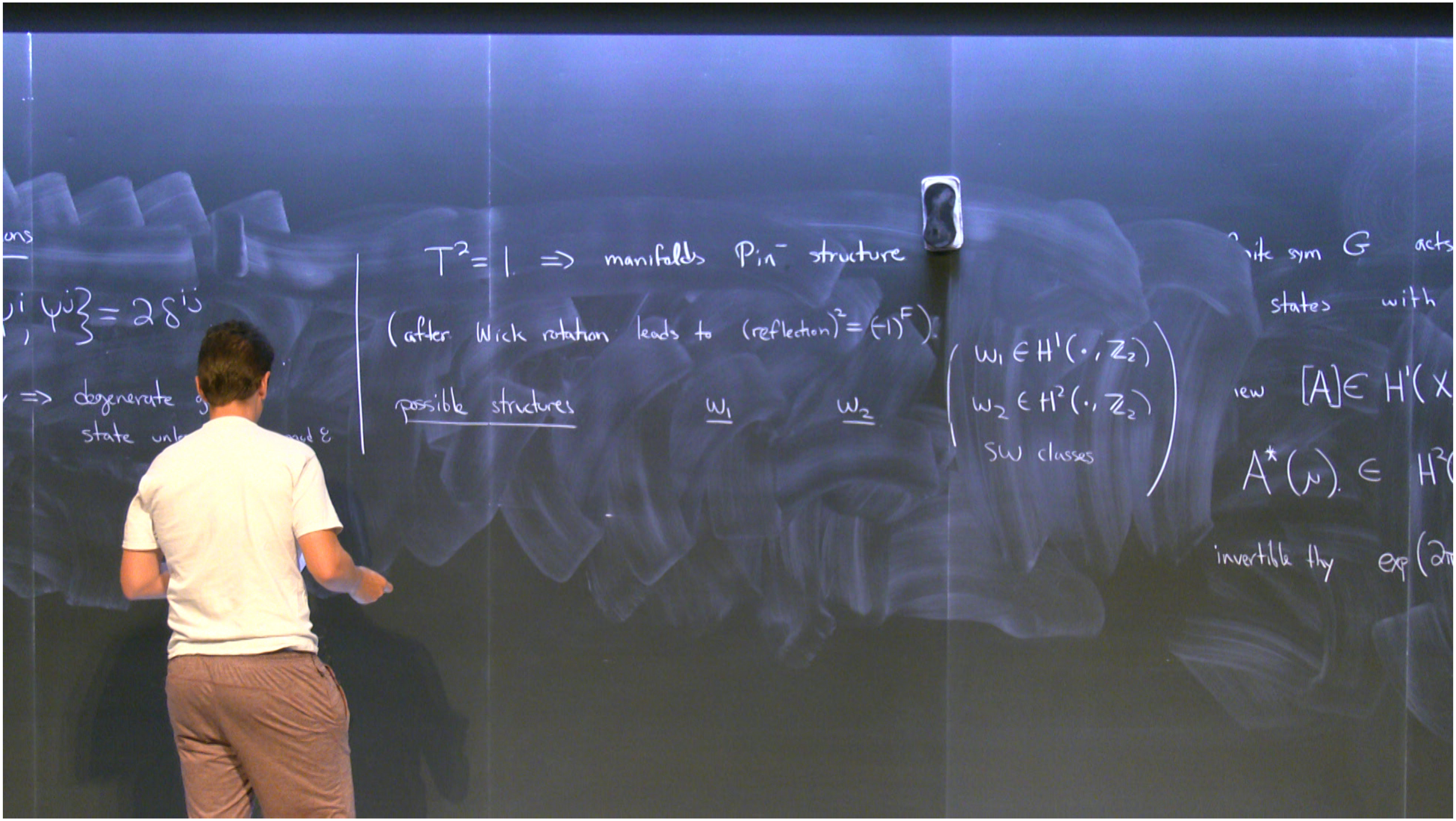
extend $S^1 = \partial Y$

A into Y

$$\tilde{Z}[A] = Z_1[A] \cdot \underbrace{\exp\left(\frac{i}{2}\int_Y F\right)}_{\text{invertible}} \qquad F = dA$$

if $\partial Y = \phi$, $(-1)^{c_1}$

verify that $\tilde{Z}[A]$ is C inv.


T² = 1 ⇒ manifolds Pin⁻ structure
(after Wick rotation leads to (reflection)² = (−1)^F)
possible structures
w₁
w₂
w₁ ∈ H¹(·, Z₂)
w₂ ∈ H²(·, Z₂)
SW classes
states with
invertible thy

...ons

$\{\psi^i, \psi^j\} = 2\delta^{ij}$

$\Rightarrow$ degenerate ground state unless $N = 0 \mod 8$

$T^2 = 1 \Rightarrow$ manifolds Pin^- structure

(after Wick rotation leads to $(\text{reflection})^2 = (-1)^F$)

possible structures	w_1	w_2
SO	0	any
O	any	any
Spin	0	0
Pin^+	any	0
Pin^-	any	$w_1 \cup w_1$

$\left(\begin{array}{l} w_1 \in H^1(\cdot, \mathbb{Z}_2) \\ w_2 \in H^2(\cdot, \mathbb{Z}_2) \\ \text{SW classes} \end{array} \right)$

$T^2 = 1 \Rightarrow$ manifolds Pin^- structure

...er. Wick rotation leads to $(\text{reflection})^2 = (-1)^F$.

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Closed 2-manifold Y, $w_2 = \underset{\text{mod } 2}{\text{euler}} = w_1 \cup w_1$ $**$

$\Rightarrow$ any such Y has a Pin^- structure

Ex Prove $**$. Which 2-manifolds admit Pin^+ struc?

abstract description of invertible thy (gen $\mathbb{RP}^2$)

2-dim Pin^- bordism grp is $\Omega_2^{Pin^-} \cong \mathbb{Z}_8$

invertible thy $Z[Y] \in U(1)$ compatible with bordism

$$Z[Y] \in \mathrm{Hom}\left(\Omega_2^{Pin^-}, U(1)\right) \cong \mathbb{Z}_8$$

(generator: ABK inv of associated quadratic form)

$T^2 = 1 \Rightarrow$ manifolds Pin^- structure

(after Wick rotation leads to $(\text{reflection})^2 = (-1)^F$)

possible structures	w_1	w_2
SO	0	any
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(w_1

w_2

S

...vertible thy (gen $\mathbb{RP}^2$)

...sm grp is $\Omega_2^{Pin^-} \cong \mathbb{Z}_8$

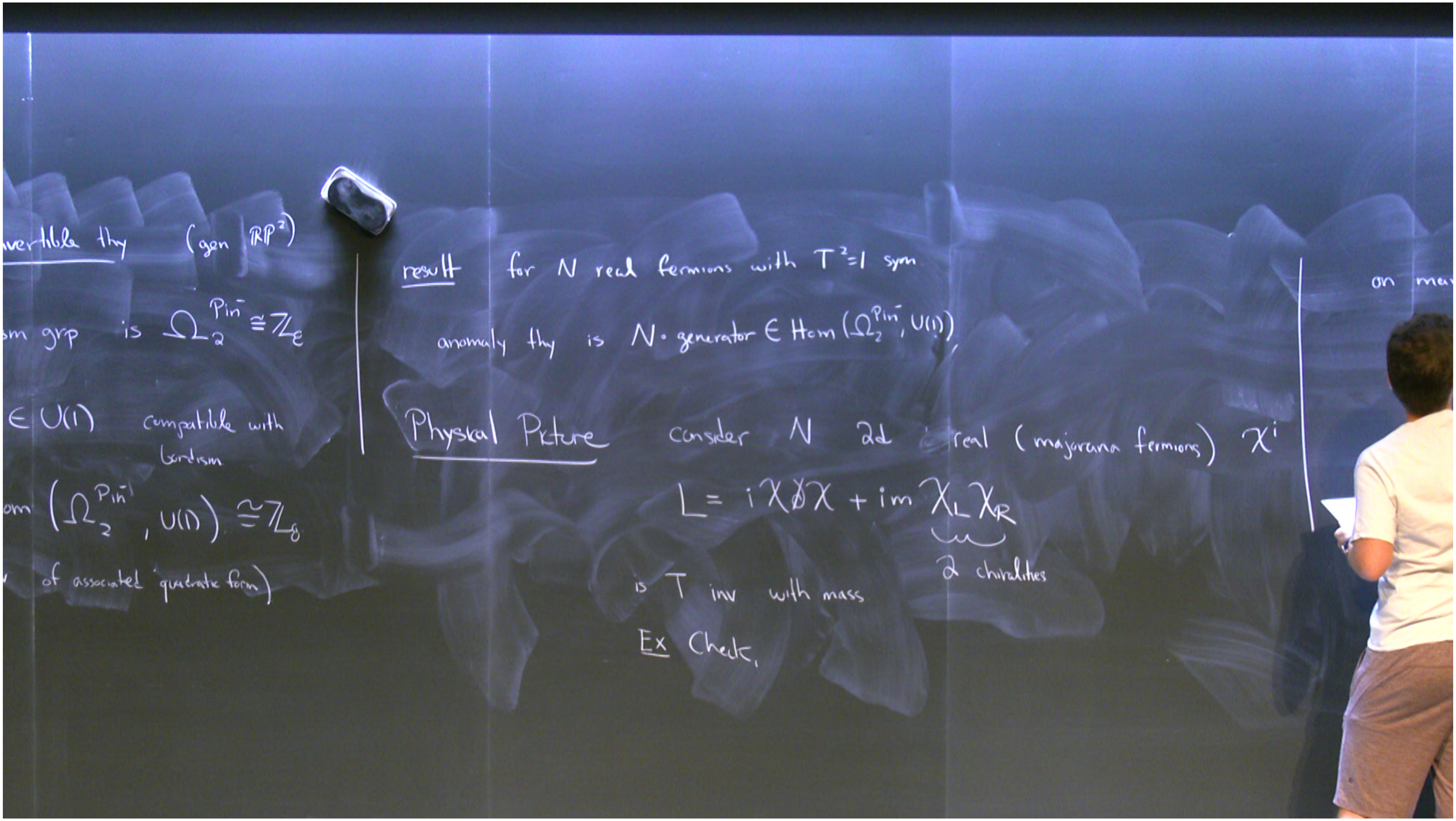
$\in U(1)$ compatible with bordism

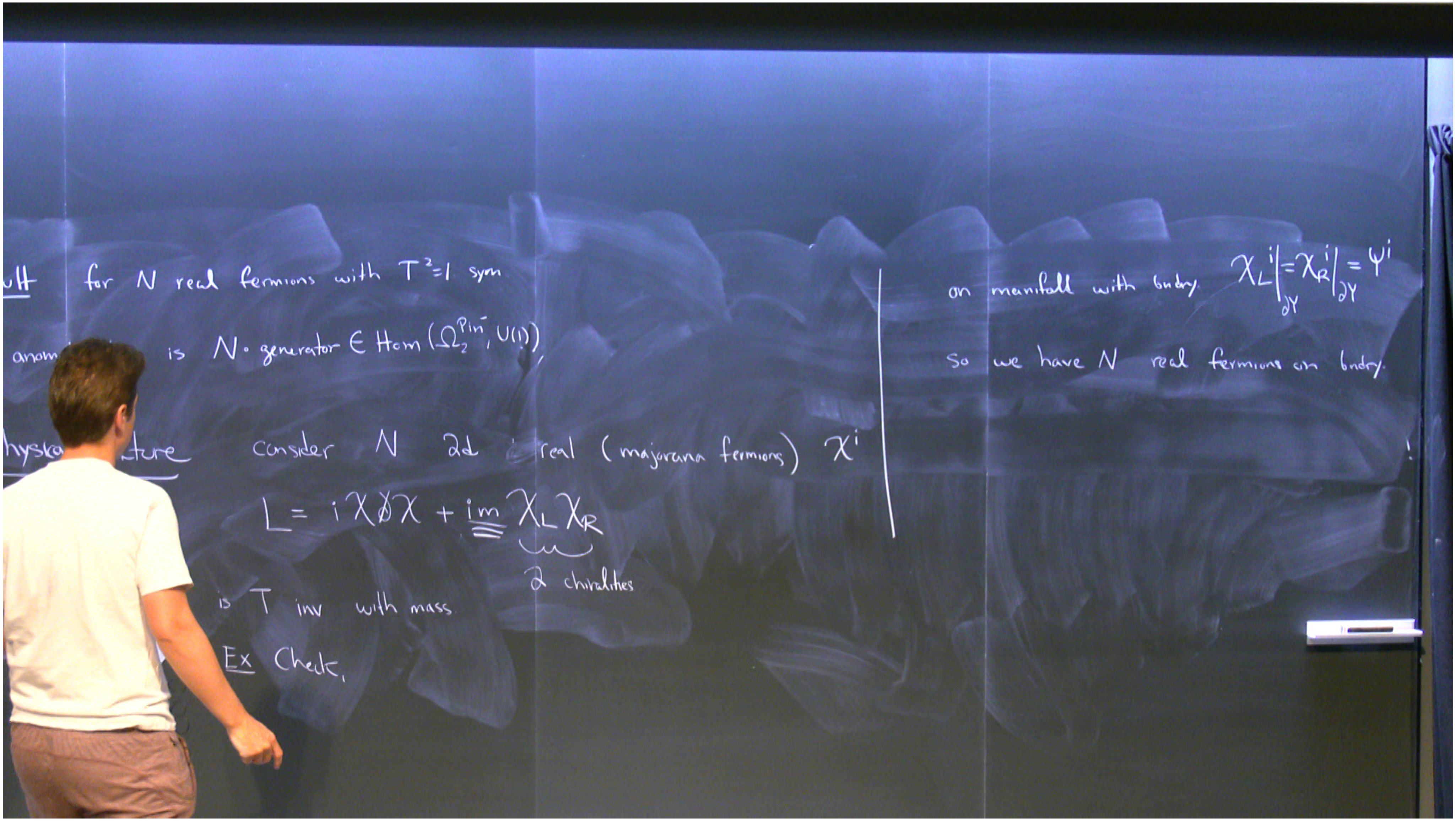
...om $\left(\Omega_2^{Pin^-}, U(1)\right) \cong \mathbb{Z}_8$

... of associated 'quadratic form')

<u>result</u> for N real fermions with $T^2=1$ sym

anomaly thy is $N \cdot$ generator $\in \mathrm{Hom}(\Omega_2^{Pin^-}, U(1))$


vertible thy (gen $\mathbb{RP}^2$)
grp is $\Omega_2^{Pin^-} \cong \mathbb{Z}_8$
$\in U(1)$ compatible with bordism
$(\Omega_2^{Pin^-}, U(1)) \cong \mathbb{Z}_8$
of associated quadratic form)
result for N real fermions with $T^2=1$ sym
anomaly thy is $N \cdot$ generator $\in \mathrm{Hom}(\Omega_2^{Pin^-}, U(1))$
Physical Picture consider N 2d real (majorana fermions) χ^i
$L = i\chi \not{\partial} \chi + im\, \underbrace{\chi_L \chi_R}_{}$
2 chiralities
is T inv with mass
Ex Check,
on


for N real fermions with T²=1 sym
is N · generator ∈ Hom(Ω₂^Pin⁻, U(1))
consider N 2d real (majorana fermions) χⁱ
L = iχ∂̸χ + im χ_L χ_R
2 chiralities
is T inv with mass
Ex Check.
on manifold with bndry χ_Lⁱ|∂Y = χ_Rⁱ|∂Y = ψⁱ
so we have N real fermions on bndry.

ult for N real fermions with $T^2=1$ sym

anomaly thy is $N \cdot$ generator $\in \mathrm{Hom}(\Omega_2^{Pin^-}, U(1))$

hysical Picture consider N 2d real (majorana fermions) χ^i

$$L = i\chi \partial\!\!\!/ \chi + \underline{im}\, \underbrace{\chi_L \chi_R}_{2 \text{ chiralities}}$$

is T inv with mass

Ex Check.

on manifold with bndry $\chi_L^i\big|_{\partial Y} = \chi_R^i\big|_{\partial Y} = \psi^i$

so we have N real fermions on bndry.

for large m (equivalently low energies)

χ very massive

$\Rightarrow \lim_{|m| \to \infty} Z_\chi(Y)$ invertible

(suitable eta invariant).

direct construction of invertible thy

with bndry N ψ's

<u>result</u> for N real fermions with $T^2=1$ sym

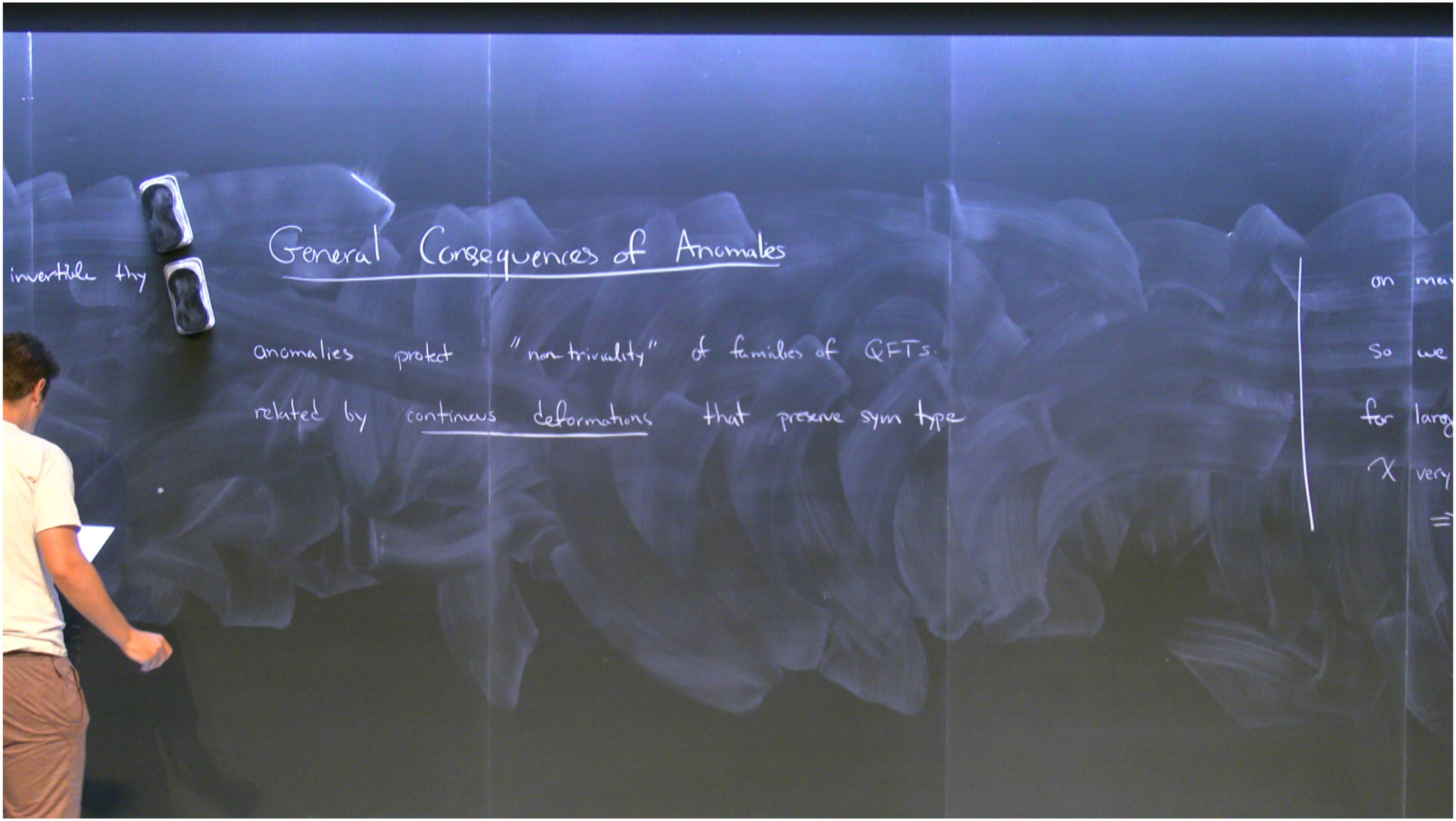
anomaly thy is $N \cdot$ generator $\in \mathrm{Hom}(\Omega_2^{Pin^-}, U(1))$

<u>Physical Picture</u> consider N 2d real

$$L = i\chi \not{\partial} \chi + \underline{im}\, \underbrace{\chi_L \chi_R}_{\text{2 chir}}$$

is T inv with mass

<u>Ex</u> Check.


invertible thy
General Consequences of Anomalies
anomalies protect "non-triviality" of families of QFTs.
related by continuous deformations that preserve sym type
on
so we
for
very

invertible thy

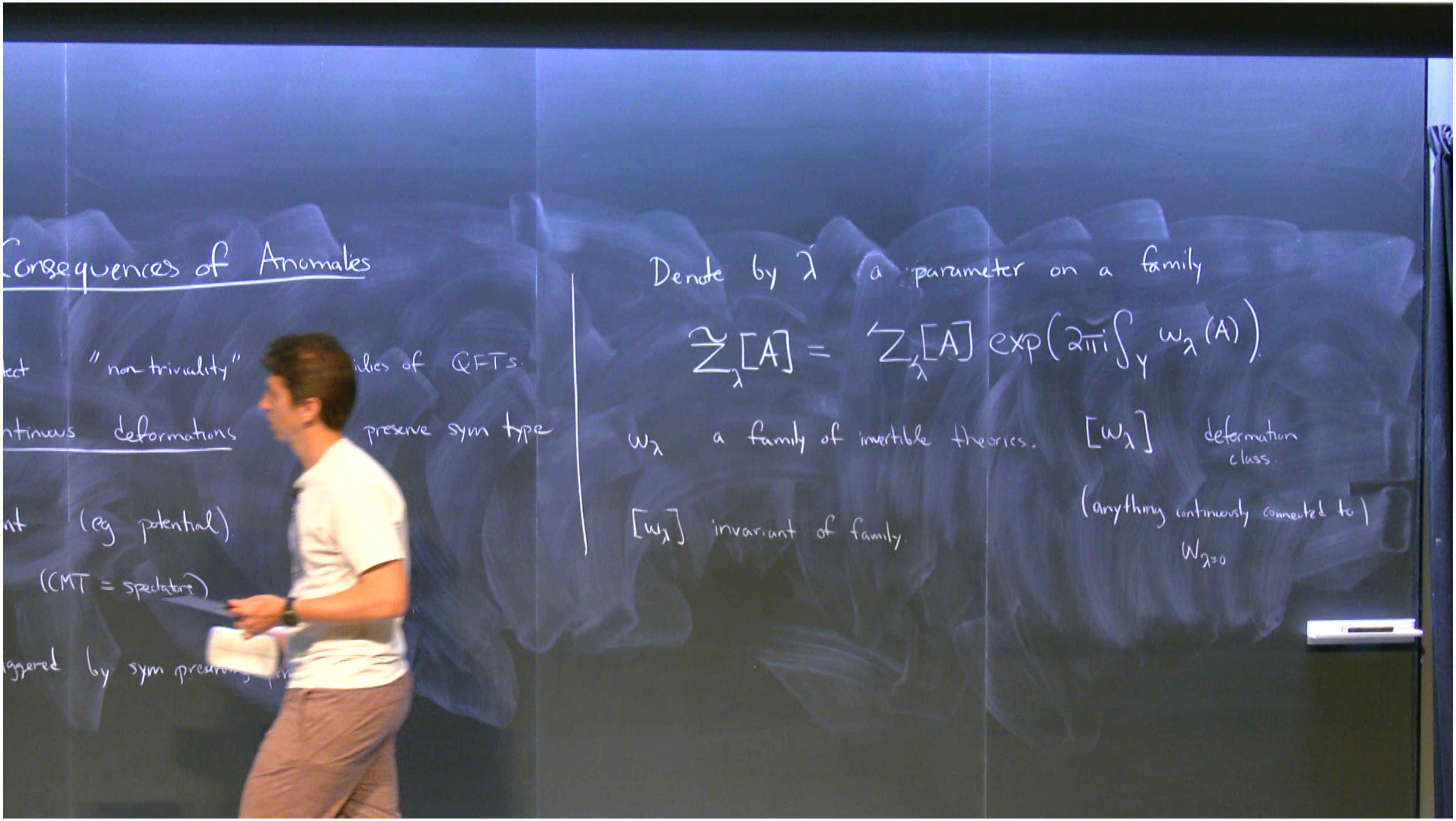
on ma

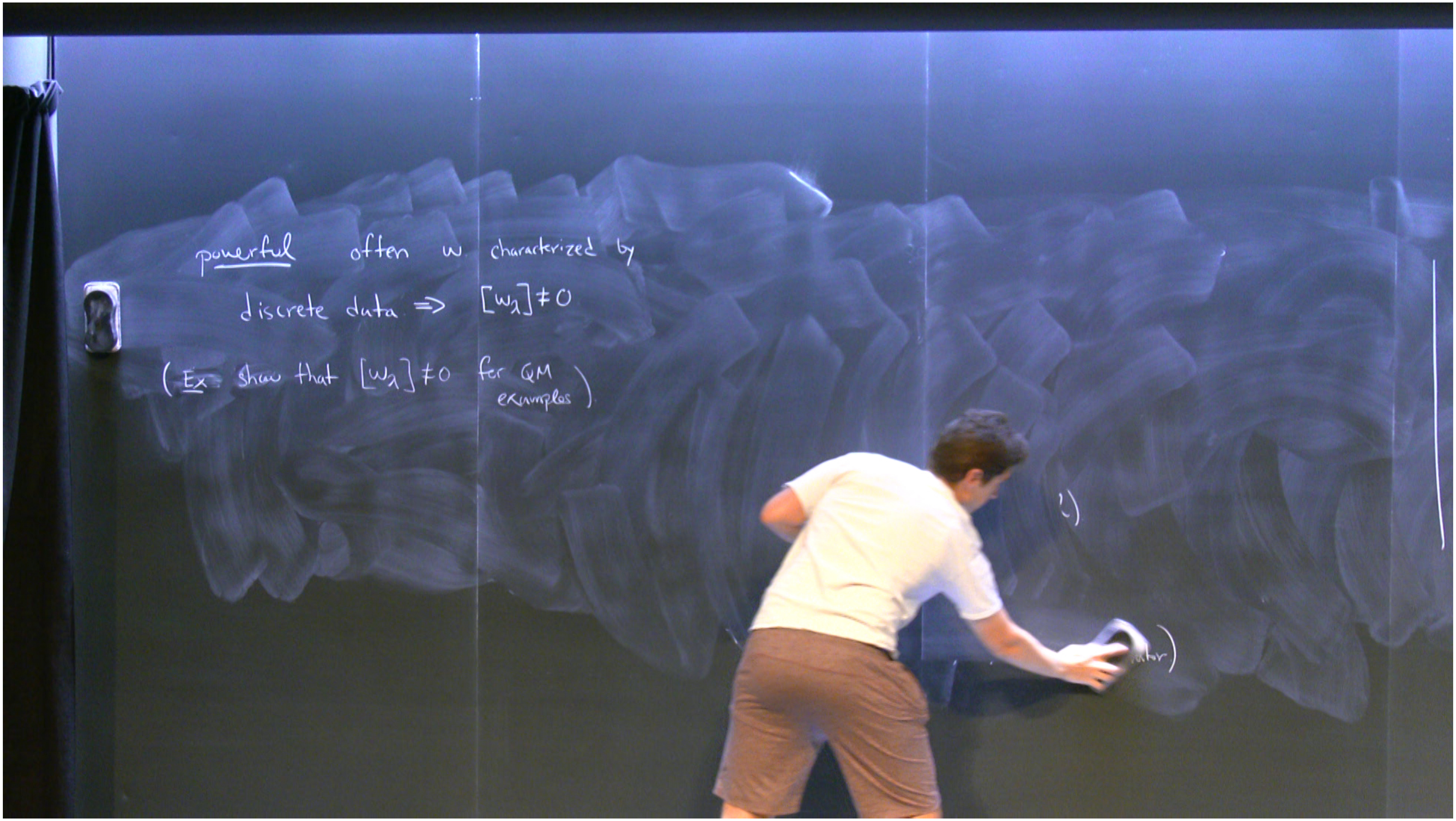
General Consequences of Anomalies

anomalies protect "non-triviality" of families of QFTs.

related by continuous deformations that preserve sym type

- dialing coupling constant (eg potential)
- add massive fields (CMT = spectator)
- RG flows triggered by sym preserving operator.


Consequences of Anomalies
"non-trivality"
ilies of QFTs.
deformations
preserve sym type
(eg potential)
(CMT = spectator)
by sym preserving
Denote by λ a parameter on a family
Z̃_λ[A] = Z_λ[A] exp(2πi ∫_Y ω_λ(A))
ω_λ a family of invertible theories.
[ω_λ] deformation class.
(anything continuously connected to)
ω_λ=0
[ω_λ] invariant of family


powerful often ω characterized by
discrete data ⇒ [ω_λ] ≠ 0
(Ex: show that [ω_λ] ≠ 0 for QM examples)

<u>powerful</u> often ω characterized by

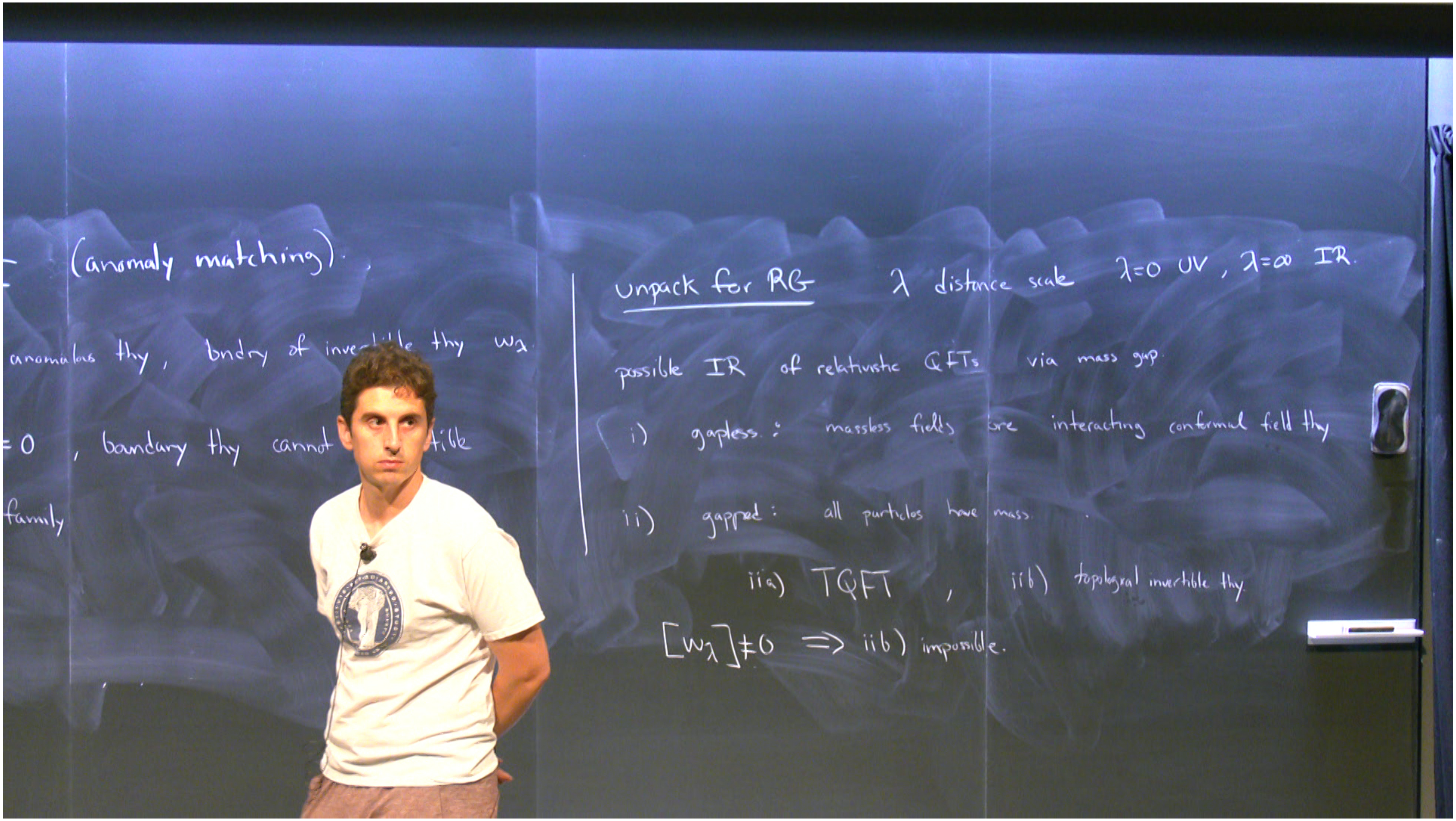
discrete data $\Rightarrow$ $[\omega_\lambda] \neq 0$

(<u>Ex</u>: show that $[\omega_\lambda] \neq 0$ for QM examples)

<u>Key point</u> (anomaly matching).

$Z_\lambda[A]$, anomalous thy, bndry of invertible thy ω_λ

If $[\omega_\lambda] \neq 0$, boundary thy cannot be invertible anywhere in family


(anomaly matching).
Unpack for RG
λ distance scale λ=0 UV, λ=∞ IR.
possible IR of relativistic QFTs via mass gap
i) gapless: massless fields are interacting conformal field thy
ii) gapped: all particles have mass.
iia) TQFT, iib) topological invertible thy
[ωλ]≠0 ⇒ iib) impossible.
anomalous thy, bndry of invertible thy ωλ.
boundary thy cannot
family