Title: Introduction to Symmetries in Quantum Field Theory

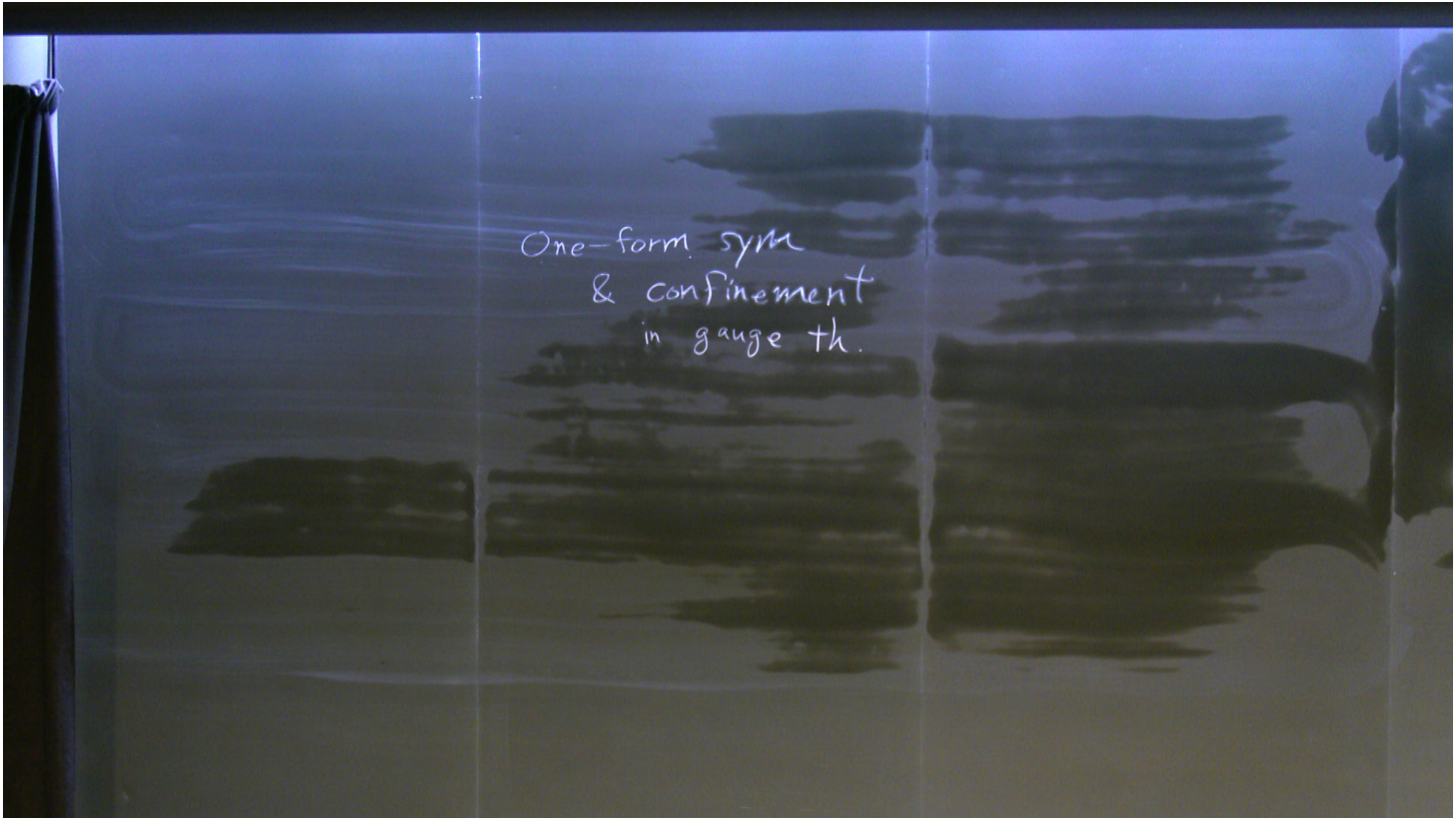
Speakers:

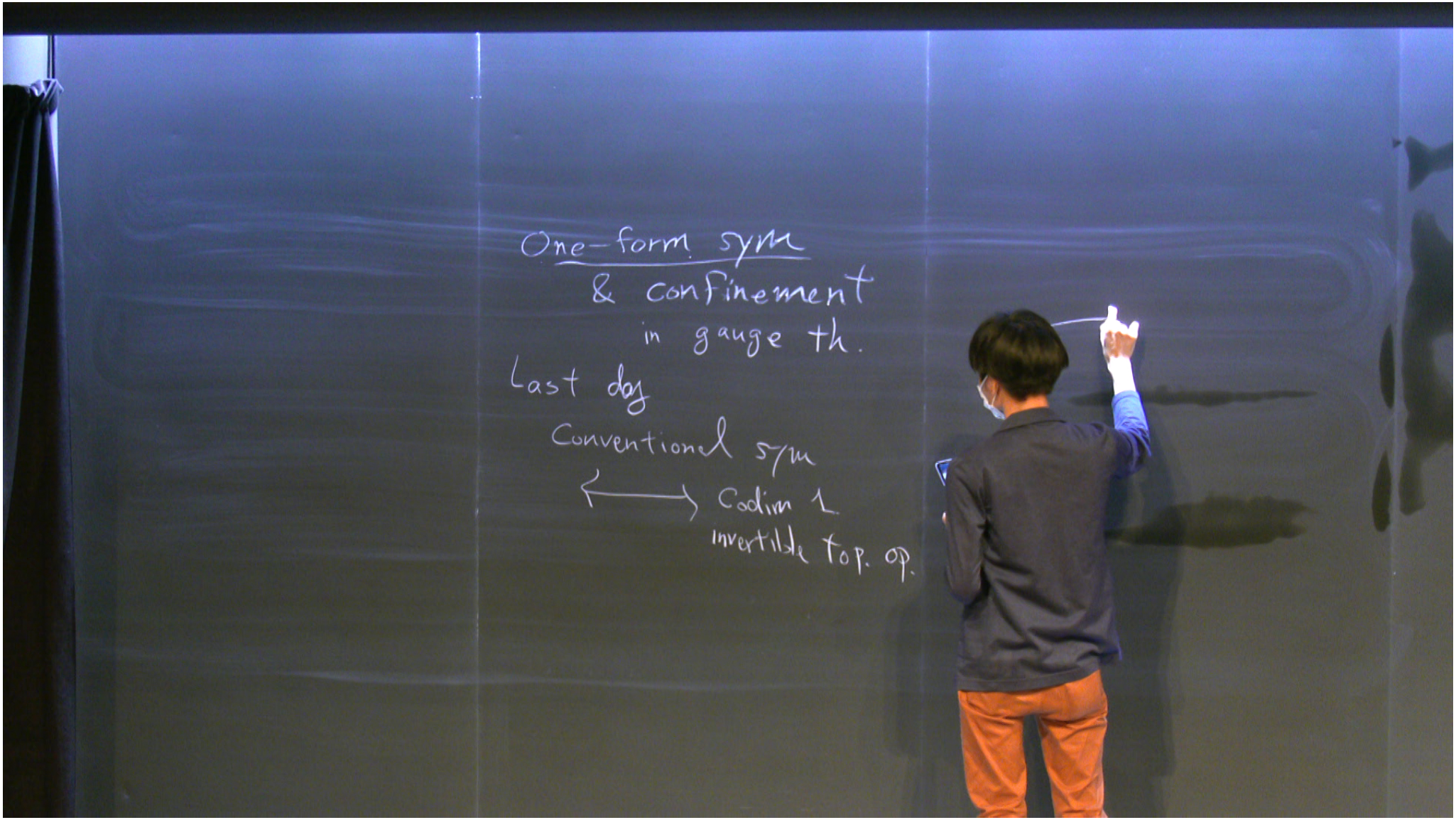
Collection: Global Categorical Symmetries

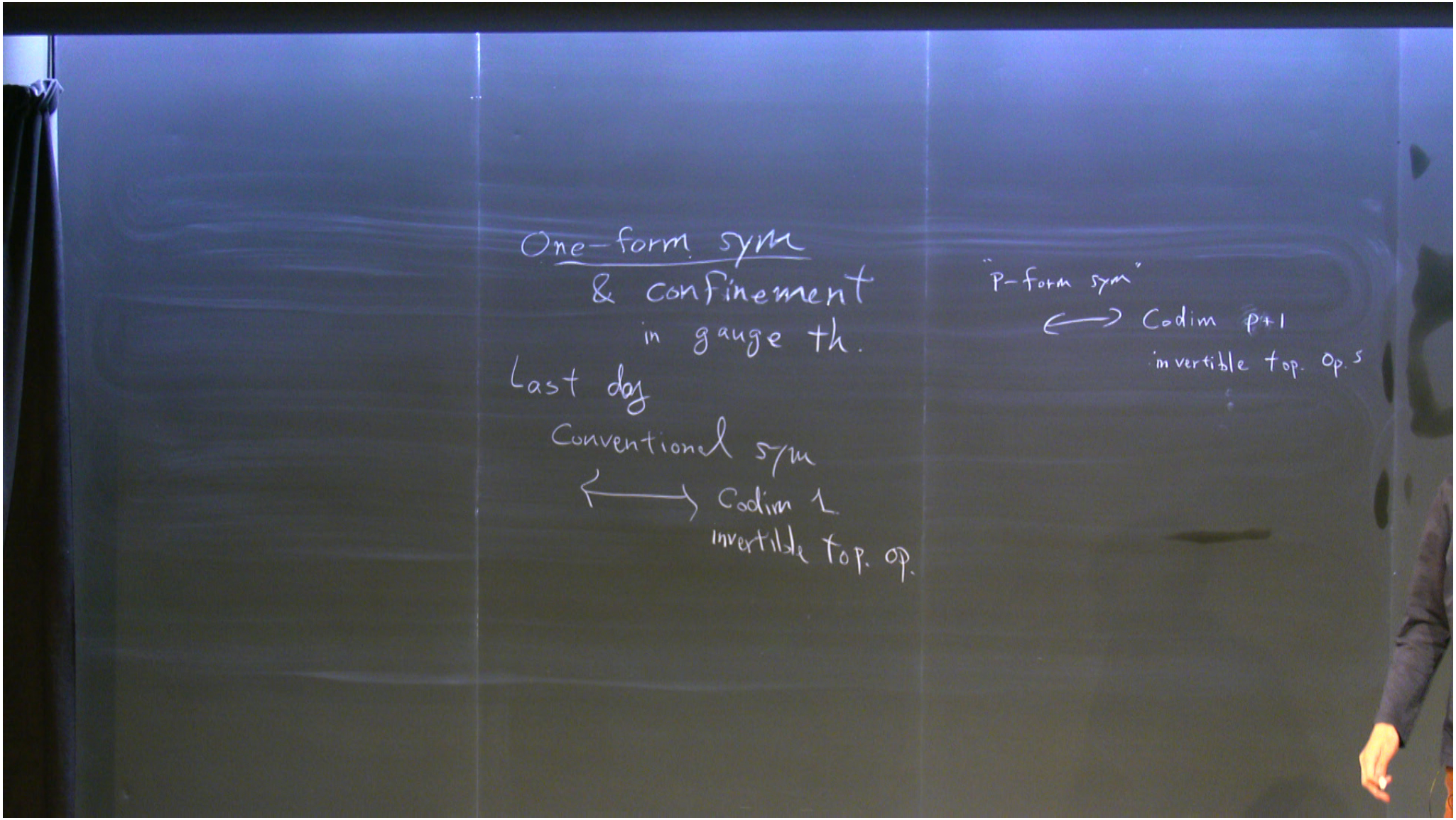
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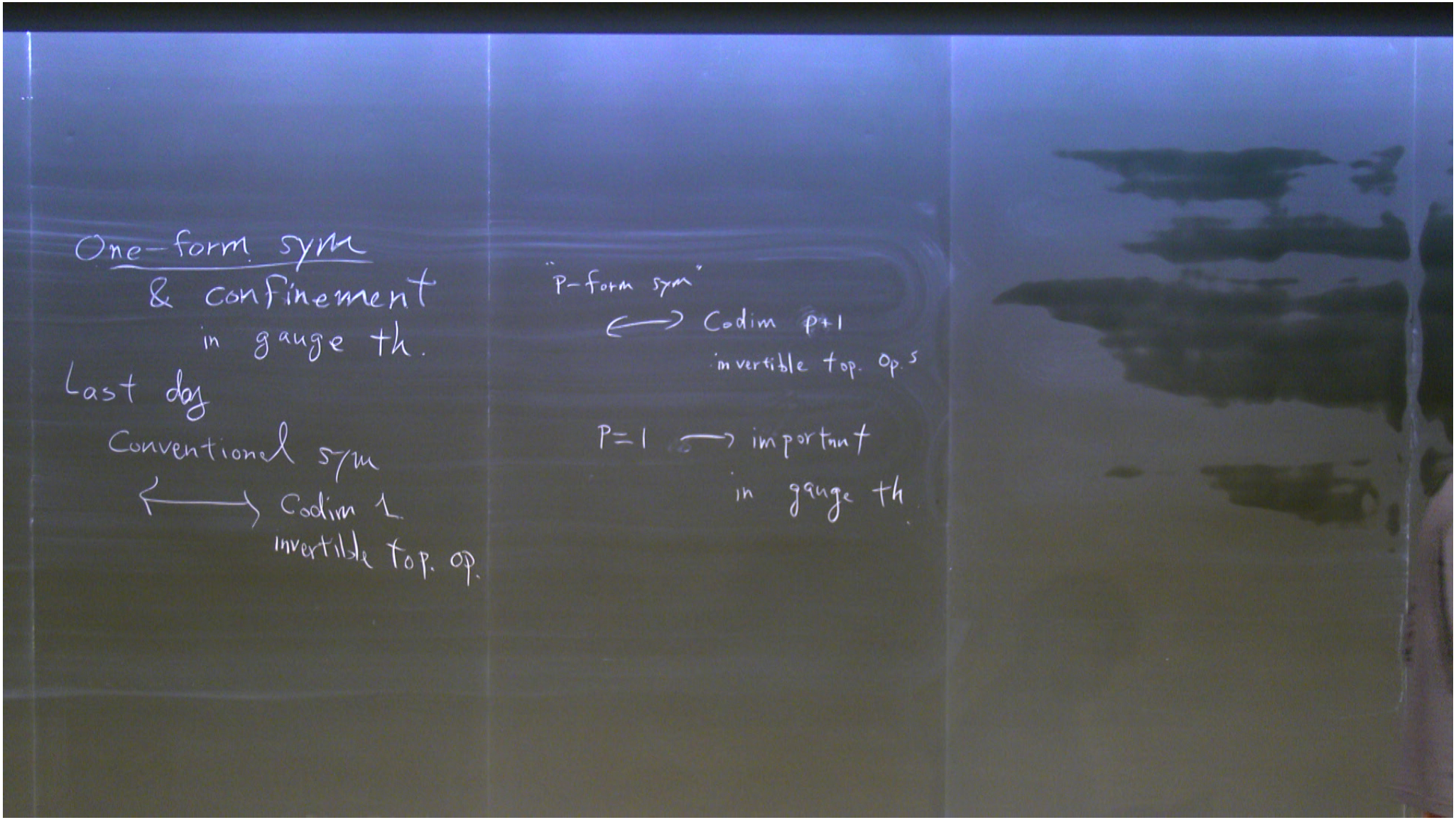
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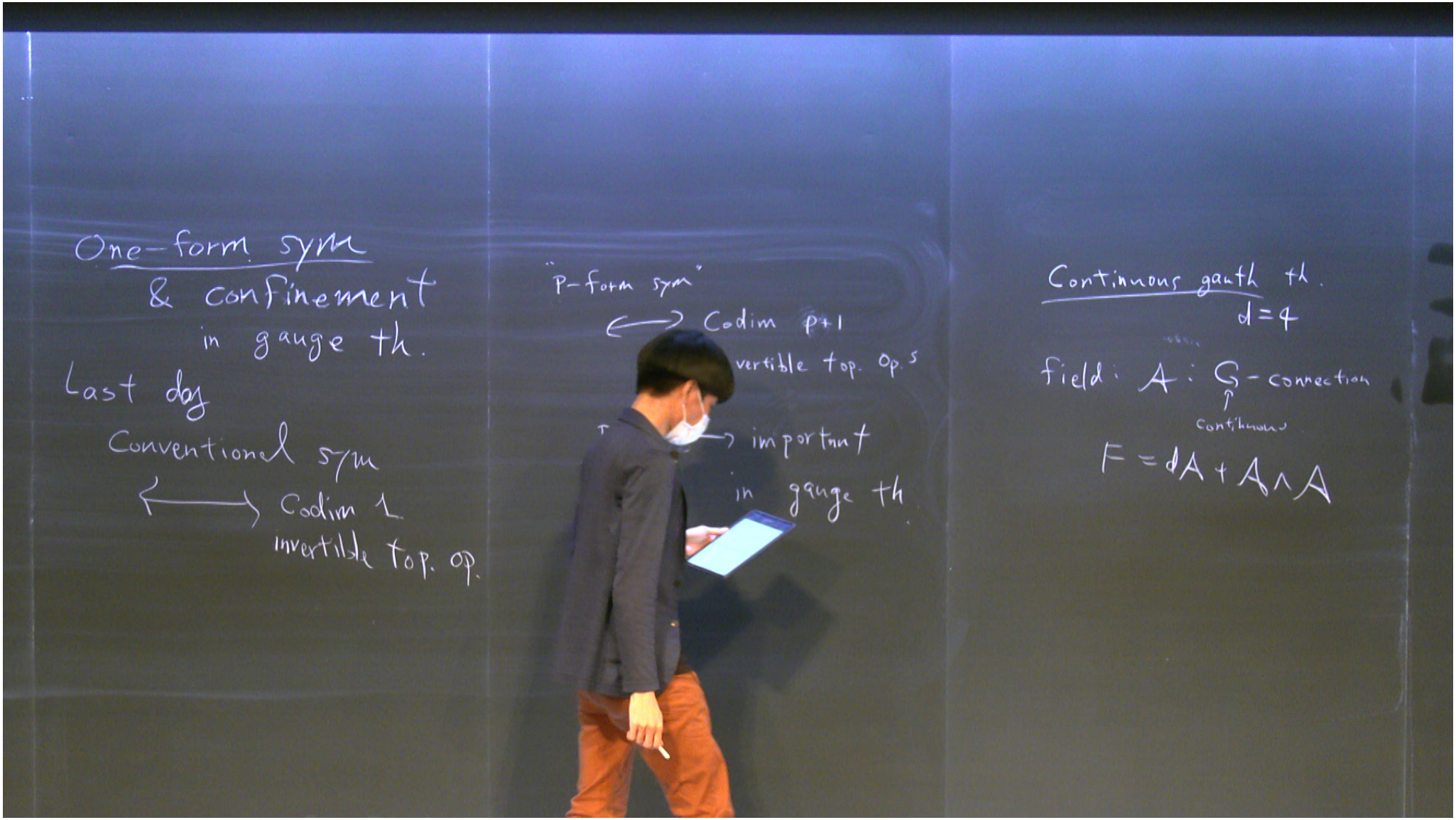

PERIMETER
INSTITUTE

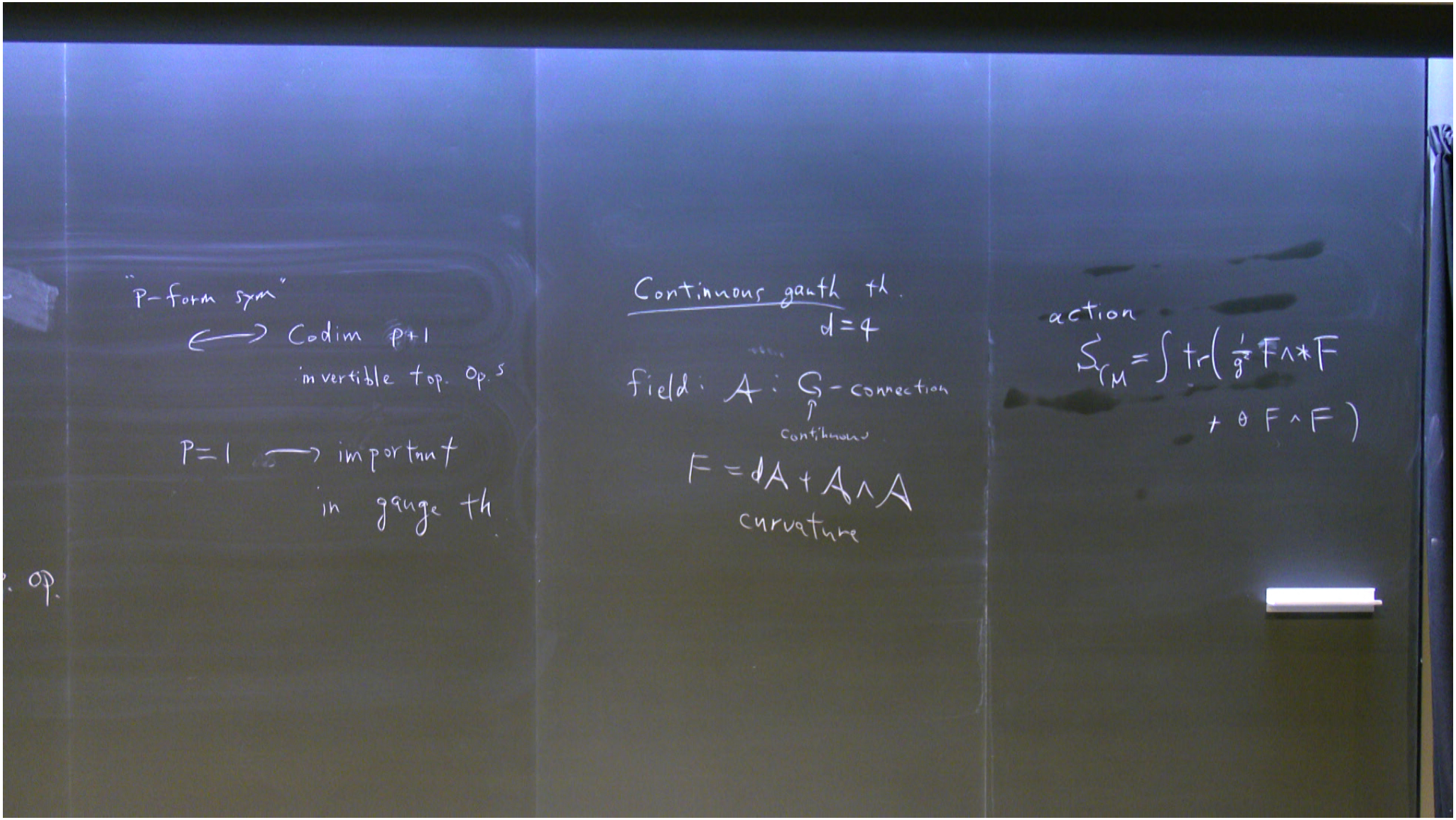

One-form sym
& confinement
in gauge th.

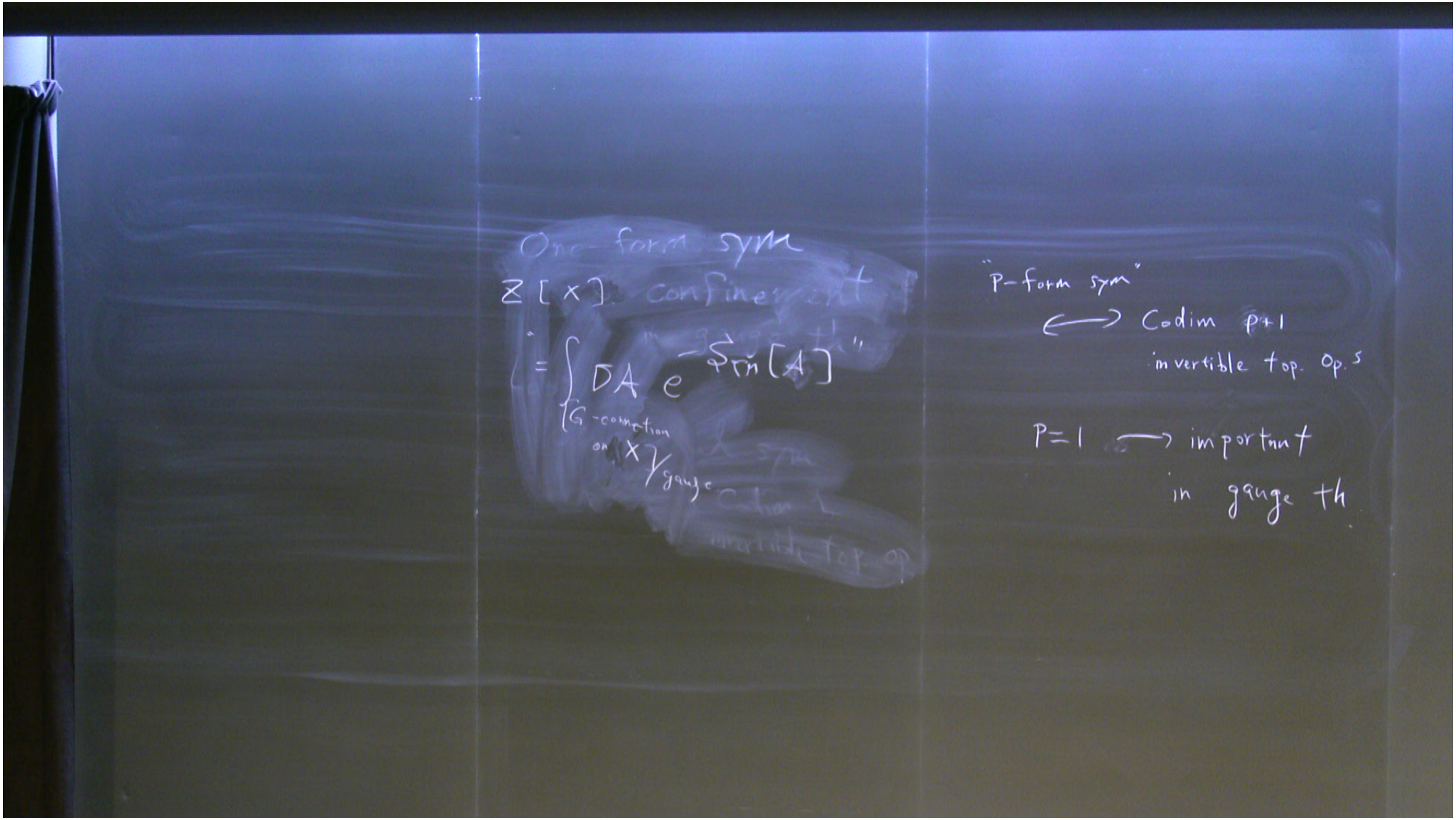

One-form sym
& confinement
in gauge th.
Last day
Conventional s/m
Codim 1
invertible top. op.

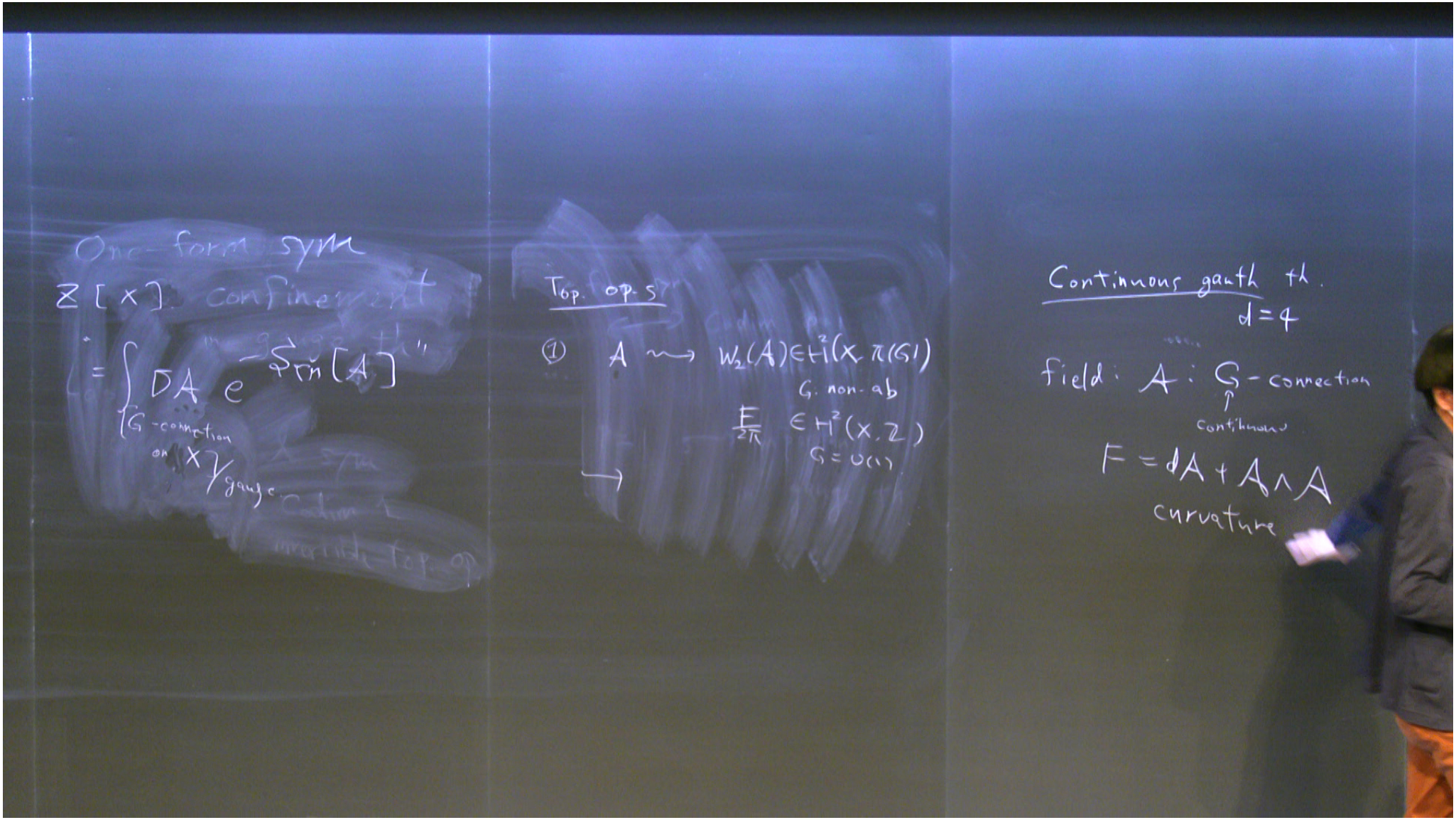

One-form sym
& confinement
in gauge th.
Last day
Conventional s/m
⟷ Codim 1
invertible top. op.
"p-form sym"
⟷ Codim p+1
invertible top. ops


One-form sym
& confinement
in gauge th.
Last day
Conventional sym
⟷ Codim 1
invertible top. op.
"p-form sym"
⟷ Codim p+1
invertible top. op.s
p=1 ⟶ important
in gauge th.


One-form sym
& confinement
in gauge th.
Last day
Conventional sym
⟷ Codim 1
invertible top. op.
"p-form sym"
⟷ Codim p+1
vertible top. op.s
→ important
in gauge th.
Continuous gauge th.
d=4
field: A : G-connection
continuous
F = dA + A∧A


"p-form sym"
⟷ Codim p+1
invertible top. op.s
p=1 ⟶ important
in gauge th
. op.
Continuous gauth th.
d=4
field: A : G-connection
↑
Continuous
F = dA + A∧A
curvature
action
S_M = ∫ tr(1/g² F∧*F
+ θ F∧F)


One-form sym
Z[X] confinement
$= \int_{G\text{-connection on } X/\text{gauge}} DA\, e^{-S_{YM}[A]}$
"p-form sym"
⟷ Codim p+1
invertible top. op.s
p=1 ⟶ important
in gauge th


One-form sym
Z[X] confinement
$= \int_{G\text{-connection on } X} DA\, e^{-S_{YM}[A]}$
gauge
Top. op.s
① $A \rightsquigarrow w_2(A) \in H^2(X, \pi(G))$
G: non-ab
$\frac{F}{2\pi} \in H^2(X, \mathbb{Z})$
$G = U(1)$
Continuous gauth th.
$d = 4$
field: A: G-connection
continuous
$F = dA + A \wedge A$
curvature

One-form sym

$Z[X]$ confinement

$= \int_{G\text{-connection on } X} DA\, e^{-S_{YM}[A]}$ / gauge

Top. op.s

① $A \rightsquigarrow W_2(A) \in H^2(X, \pi_1(G))$

G: non-ab

$\frac{F}{2\pi} \in H^2(X, \mathbb{Z})$

$G = U(1)$

dim 2 top. op $\quad \phi\left(\int_D W_2(A)\right)$

$\phi \in \pi_1(G)^\vee$

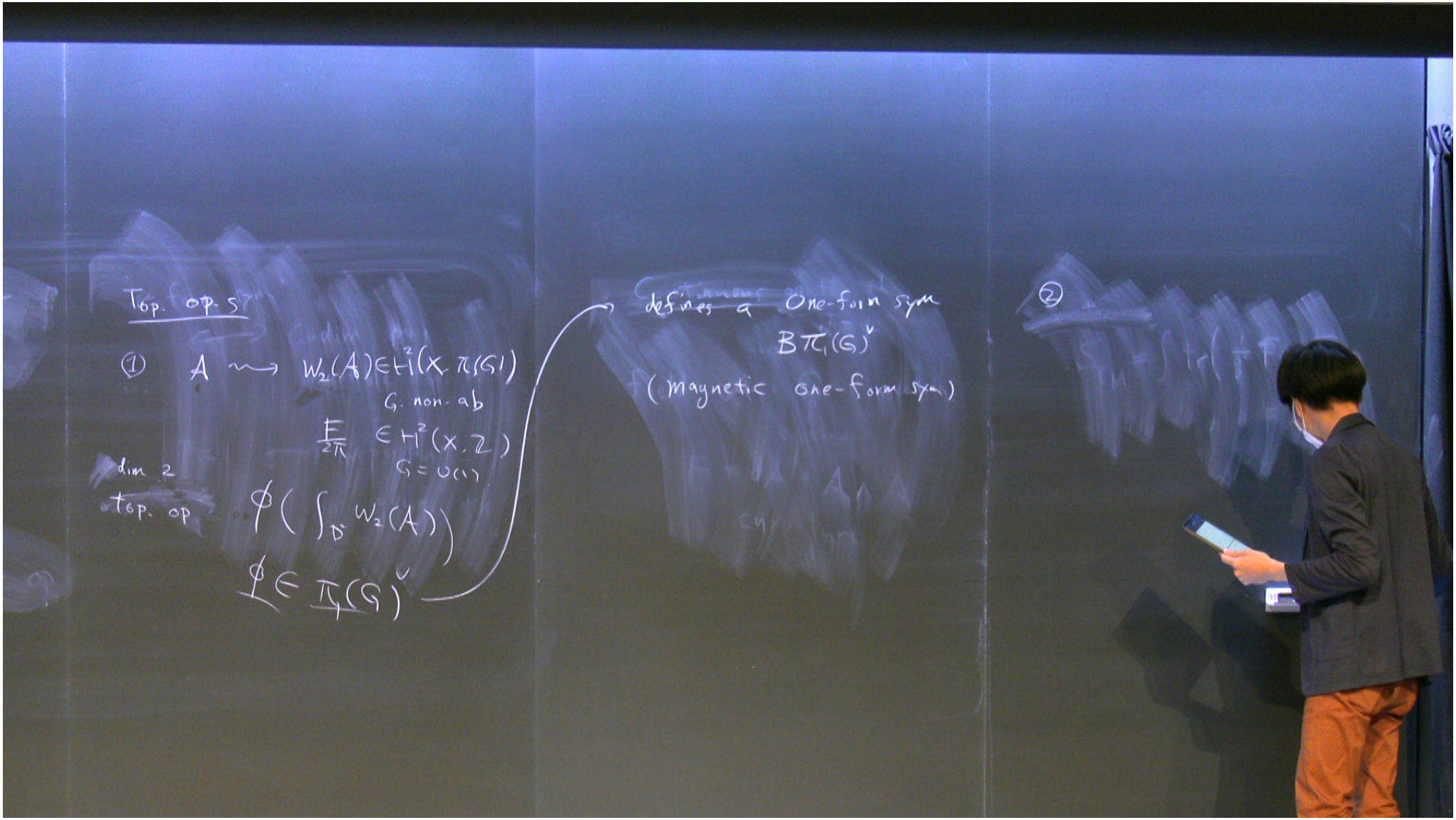
Continuous gauth th.

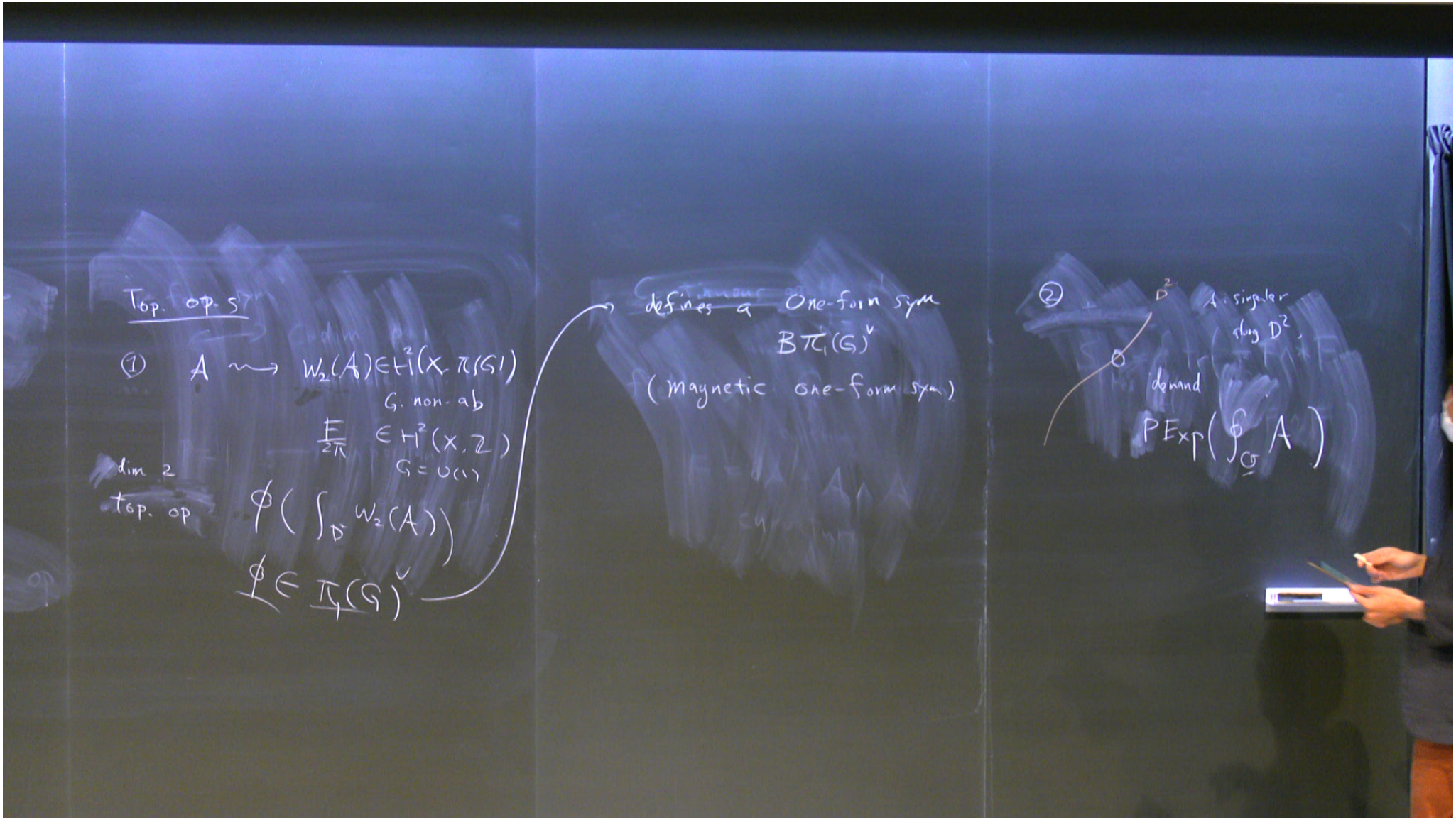
$d = 4$

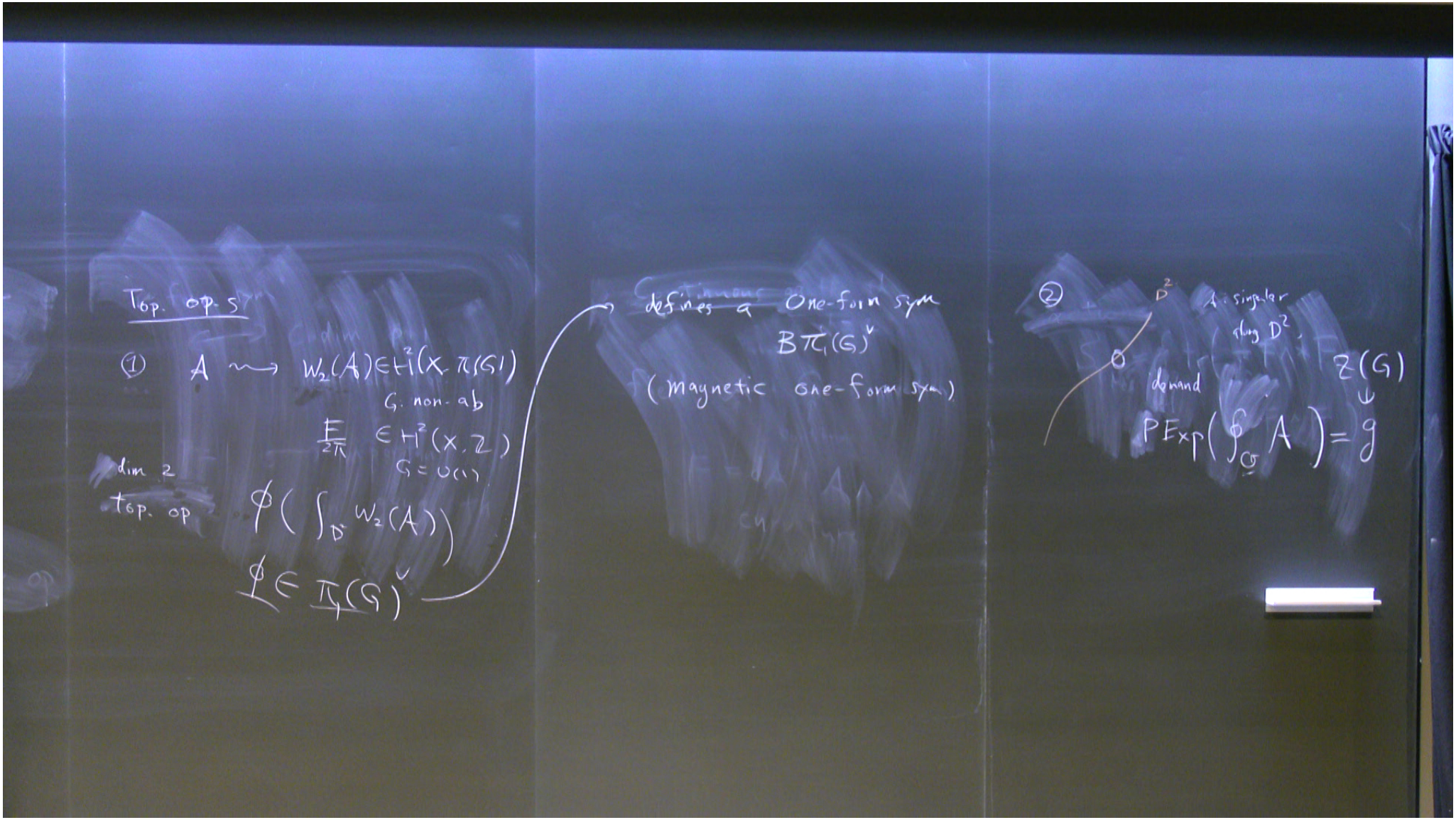
field: A: G-connection (continuous)

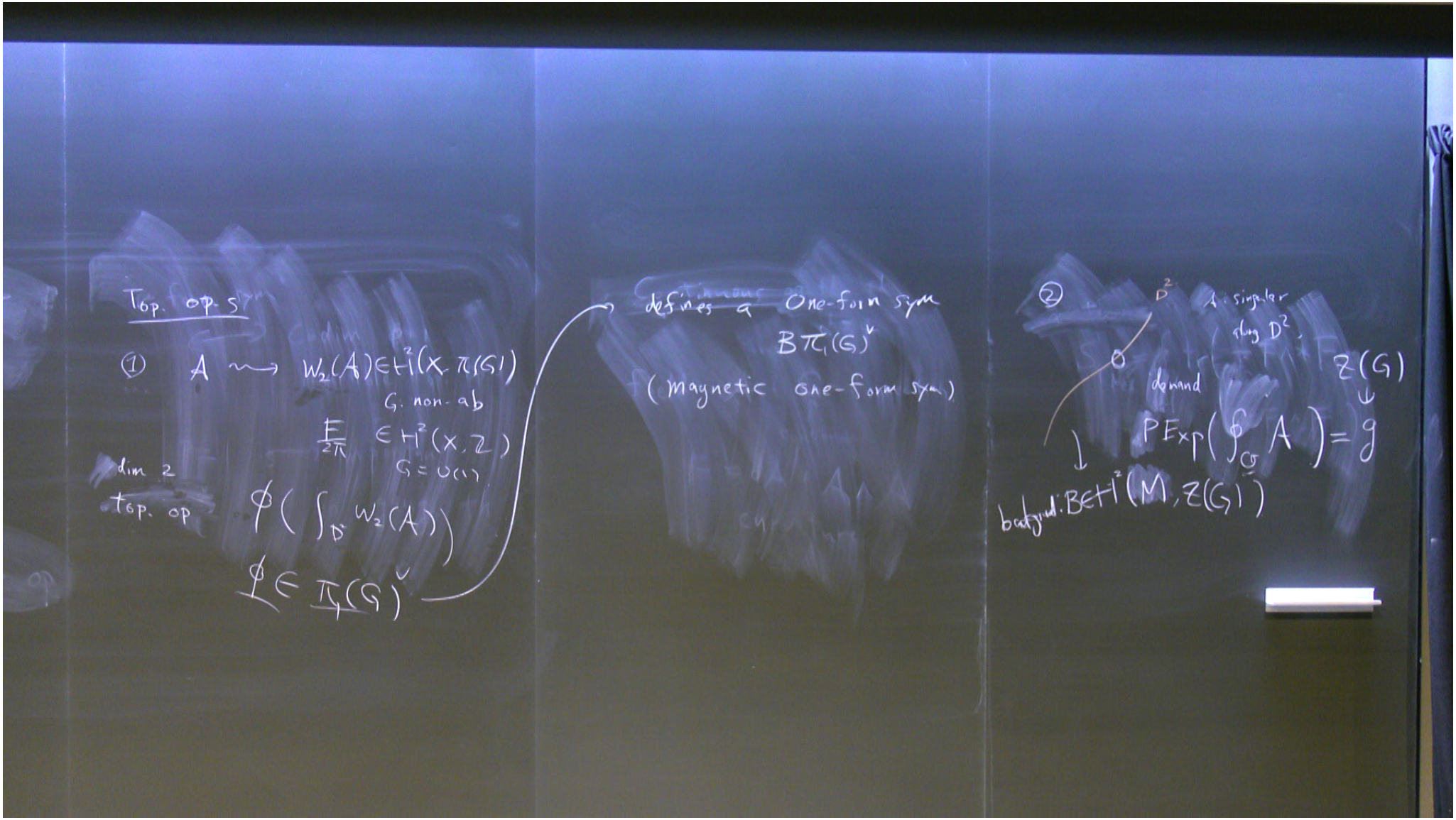
$F = dA + A \wedge A$

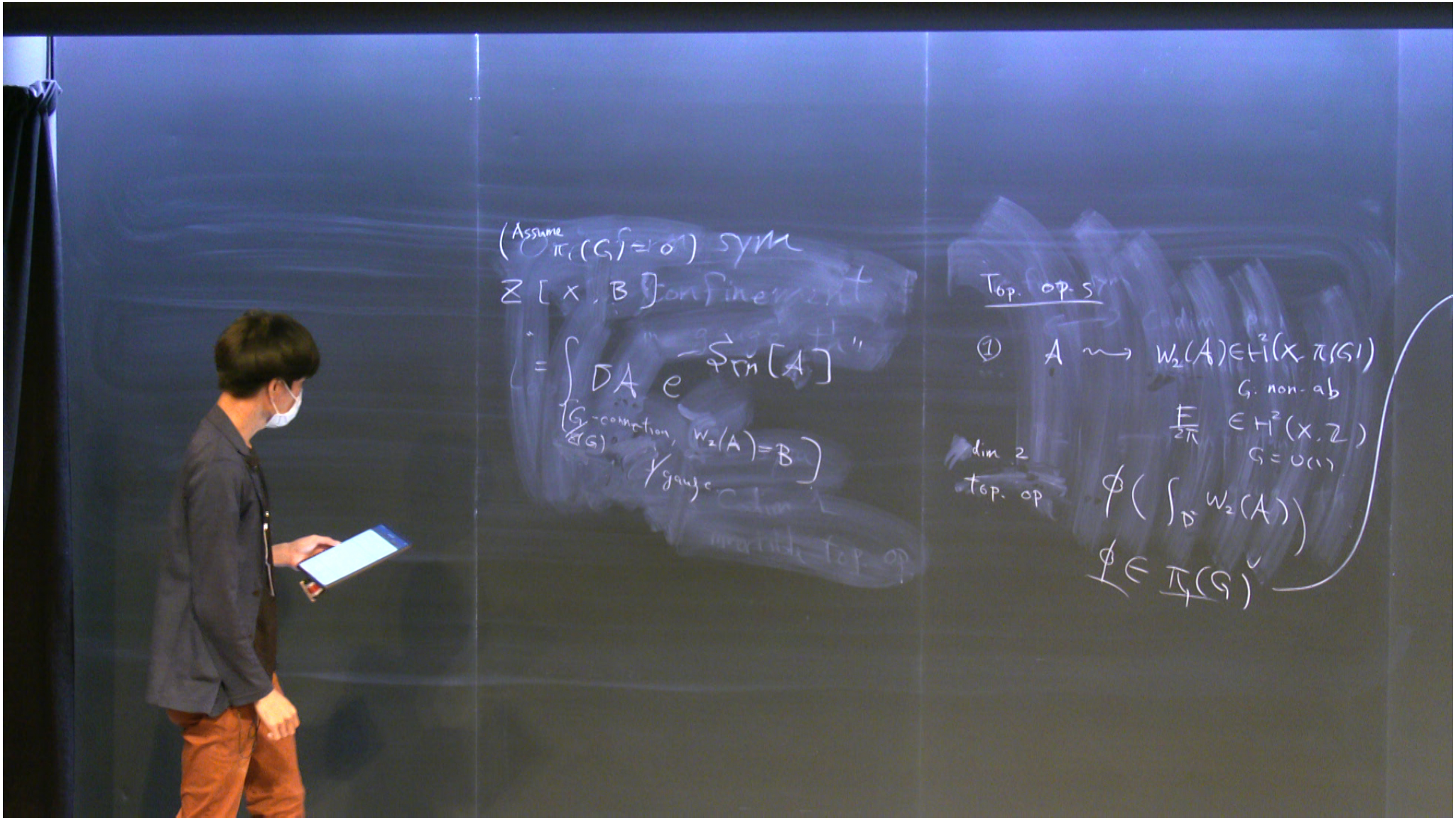
curvature

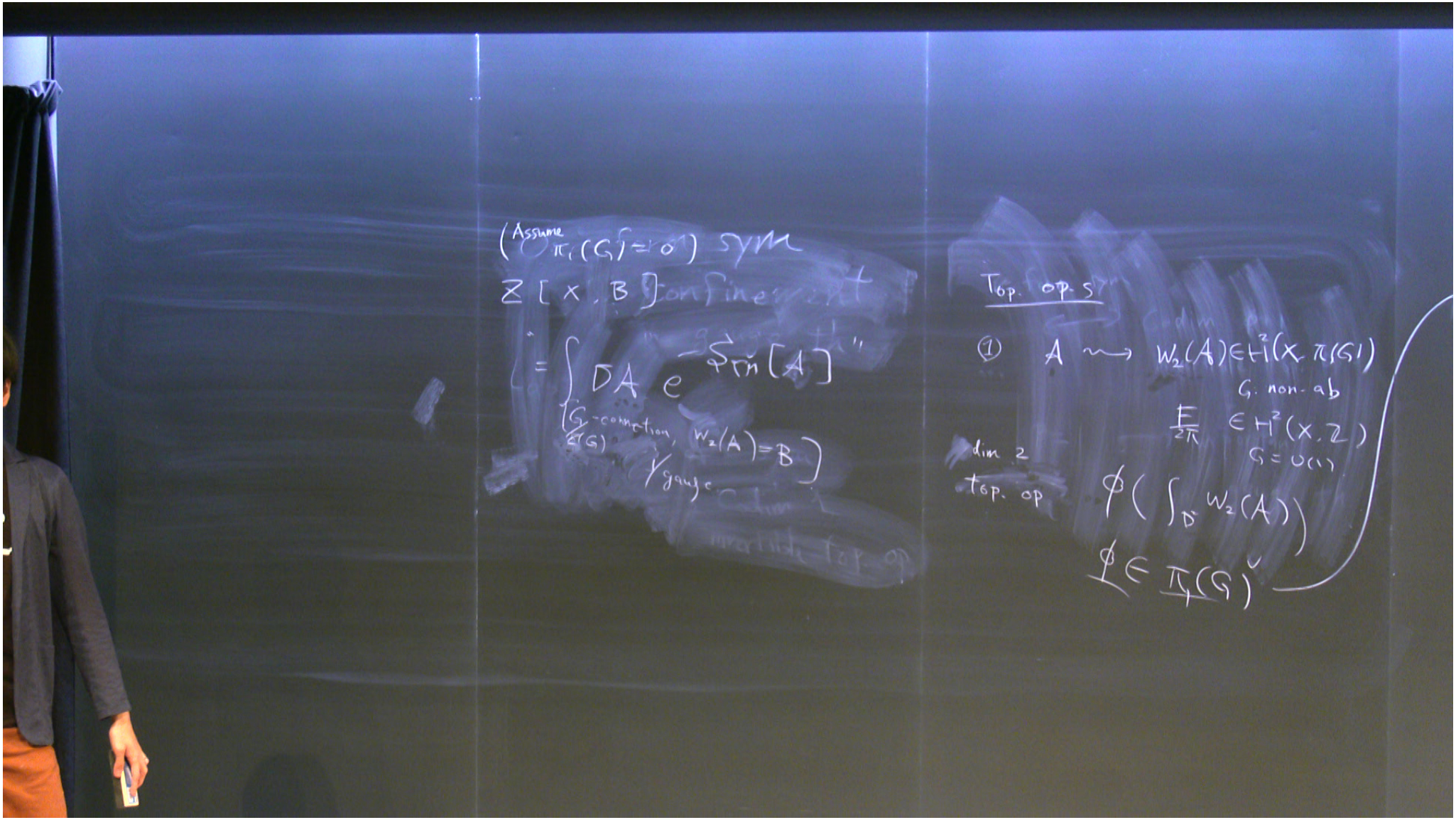

Top. op-s
① A ⇝ $W_2(A) \in H^2(X, \pi_1(G))$
G. non-ab
$\frac{F}{2\pi} \in H^2(X, \mathbb{Z})$
$G = U(1)$
dim 2
top. op $\phi\left(\int_D W_2(A)\right)$
$\phi \in \pi_1(G)^\vee$
defines a One-form sym
$B\pi_1(G)^\vee$
(magnetic one-form sym)
②

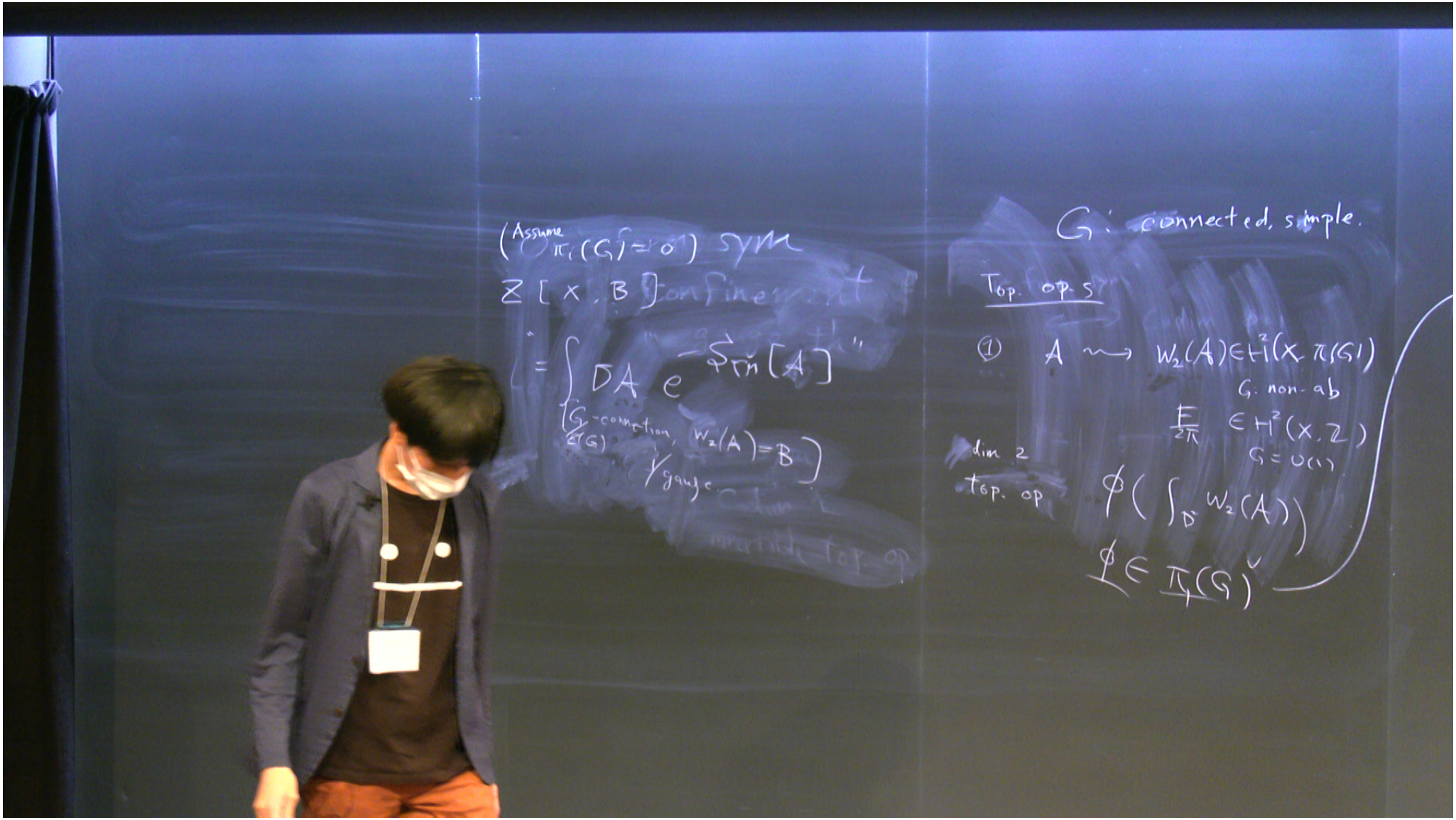

Top. op. 5
① A ⇝ $W_2(A) \in H^2(X, \pi_1(G))$
G non-ab
$\frac{F}{2\pi} \in H^2(X, \mathbb{Z})$
$G = U(1)$
dim 2
top. op $\phi\left(\int_D W_2(A)\right)$
$\phi \in \pi_1(G)^\vee$
defines a One-form sym
$B\pi_1(G)^\vee$
(magnetic one-form sym)
②
D^2
A singular
along D^2
demand
$Pexp\left(\oint_\sigma A\right)$

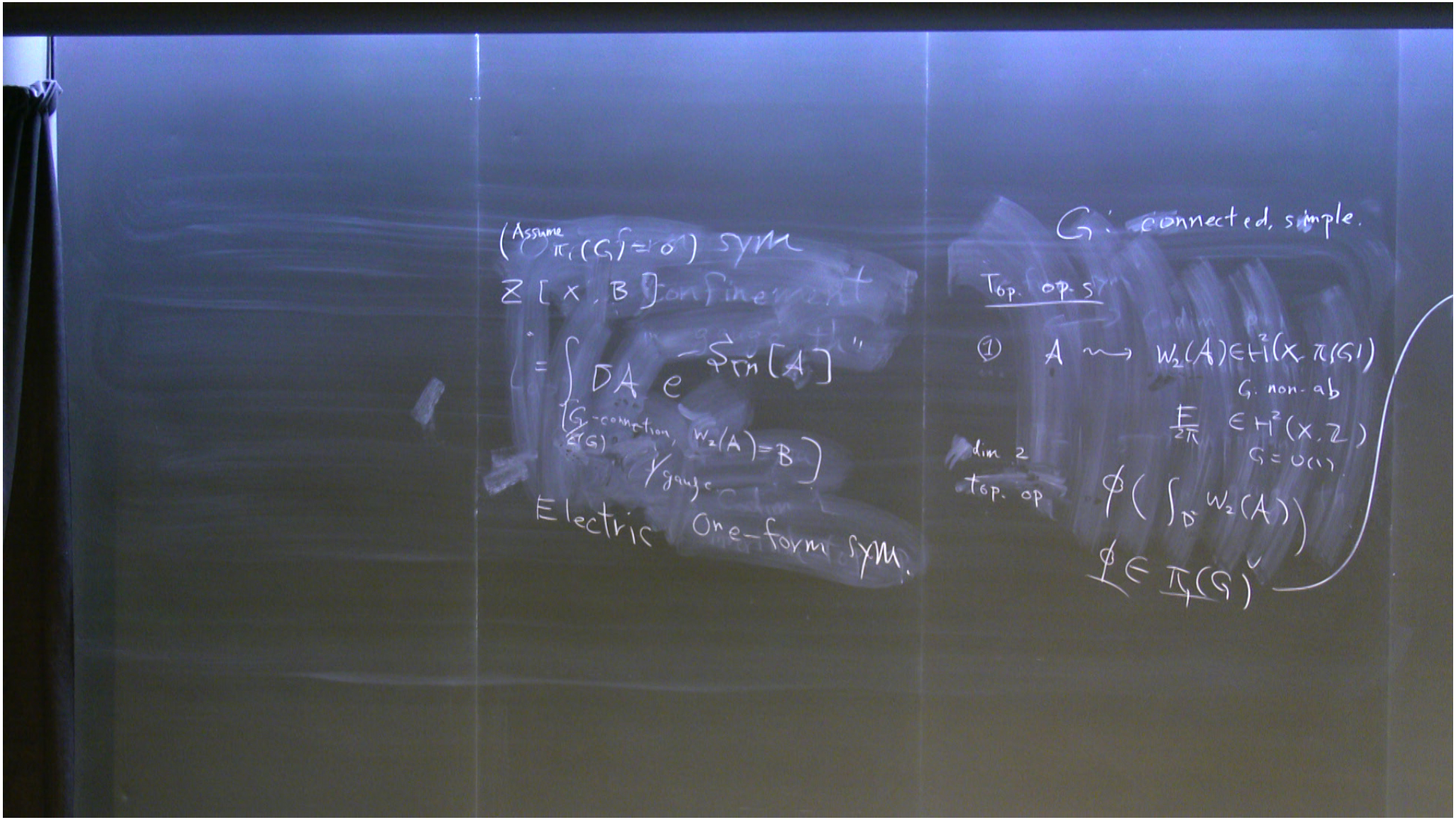

Top. op-s
① $A \rightsquigarrow w_2(A) \in H^2(X, \pi_1(G))$
G non-ab
$\frac{F}{2\pi} \in H^2(X, \mathbb{Z})$
$G = U(1)$
dim 2 top. op $\phi\left(\int_{D^2} w_2(A)\right)$
$\phi \in \pi_1(G)^\vee$
defines a one-form sym
$B\pi_1(G)^\vee$
(magnetic one-form sym)
②
D^2
A singular along D^2
demand
$Z(G)$
$\mathrm{Pexp}\left(\oint_C A\right) = g$

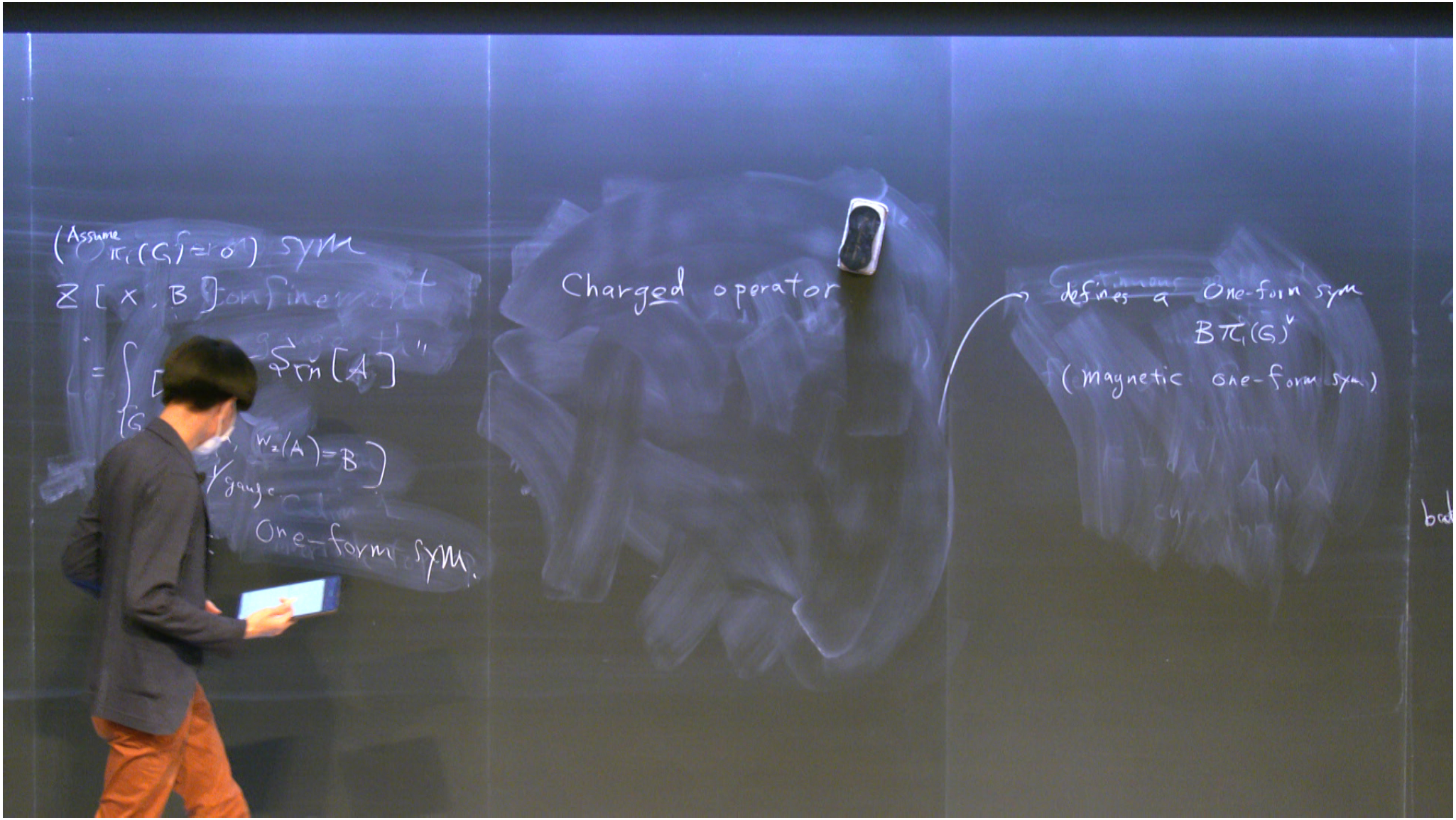

Top. op-s
① A ⇝ $w_2(A) \in H^2(X, \pi_1(G))$
G non-ab
$\frac{F}{2\pi} \in H^2(X, \mathbb{Z})$
$G = U(1)$
dim 2 top. op $\phi\left(\int_{D^2} w_2(A)\right)$
$\phi \in \pi_1(G)^\vee$
defines a one-form sym
$B\pi_1(G)^\vee$
(magnetic one-form sym)
②
D^2
A singular
plug D^2,
0
demand
$Z(G)$
$P\mathrm{Exp}\left(\oint_G A\right) = g$
bdry pred. $Bet^2(M, Z(G))$

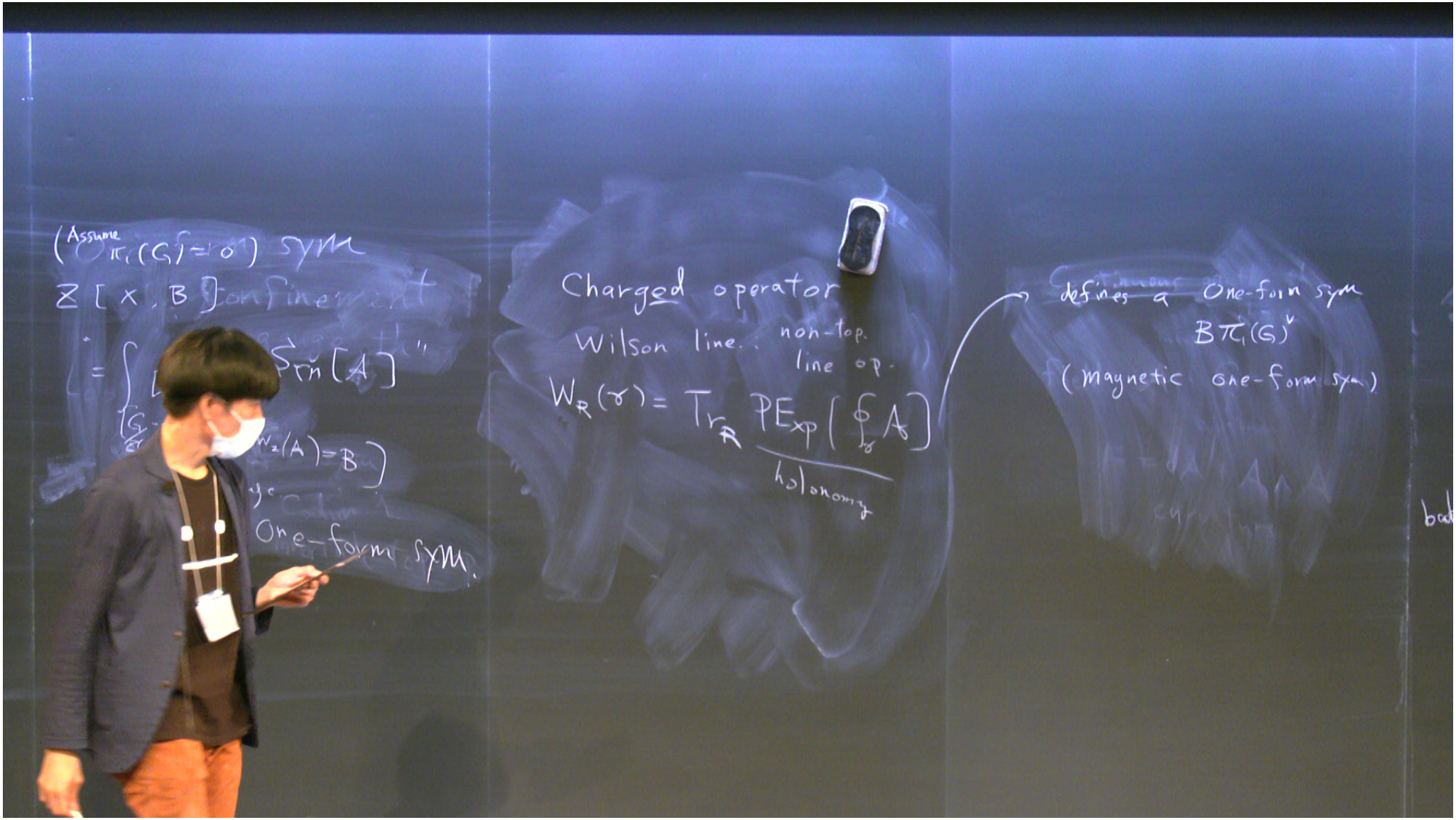

(Assume $\pi_1(G)=0$) sym
$Z[X,B]$ confinement
$= \int DA\, e^{-S_{YM}[A]}$
G-connection, $w_2(A)=B$
/gauge
Top. op. s
① $A \rightsquigarrow w_2(A) \in H^2(X, \pi_1(G))$
G non-ab
$\frac{F}{2\pi} \in H^2(X,\mathbb{Z})$
$G=U(1)$
dim 2
top. op
$\phi\left(\int_D w_2(A)\right)$
$\phi \in \pi_1(G)^\vee$

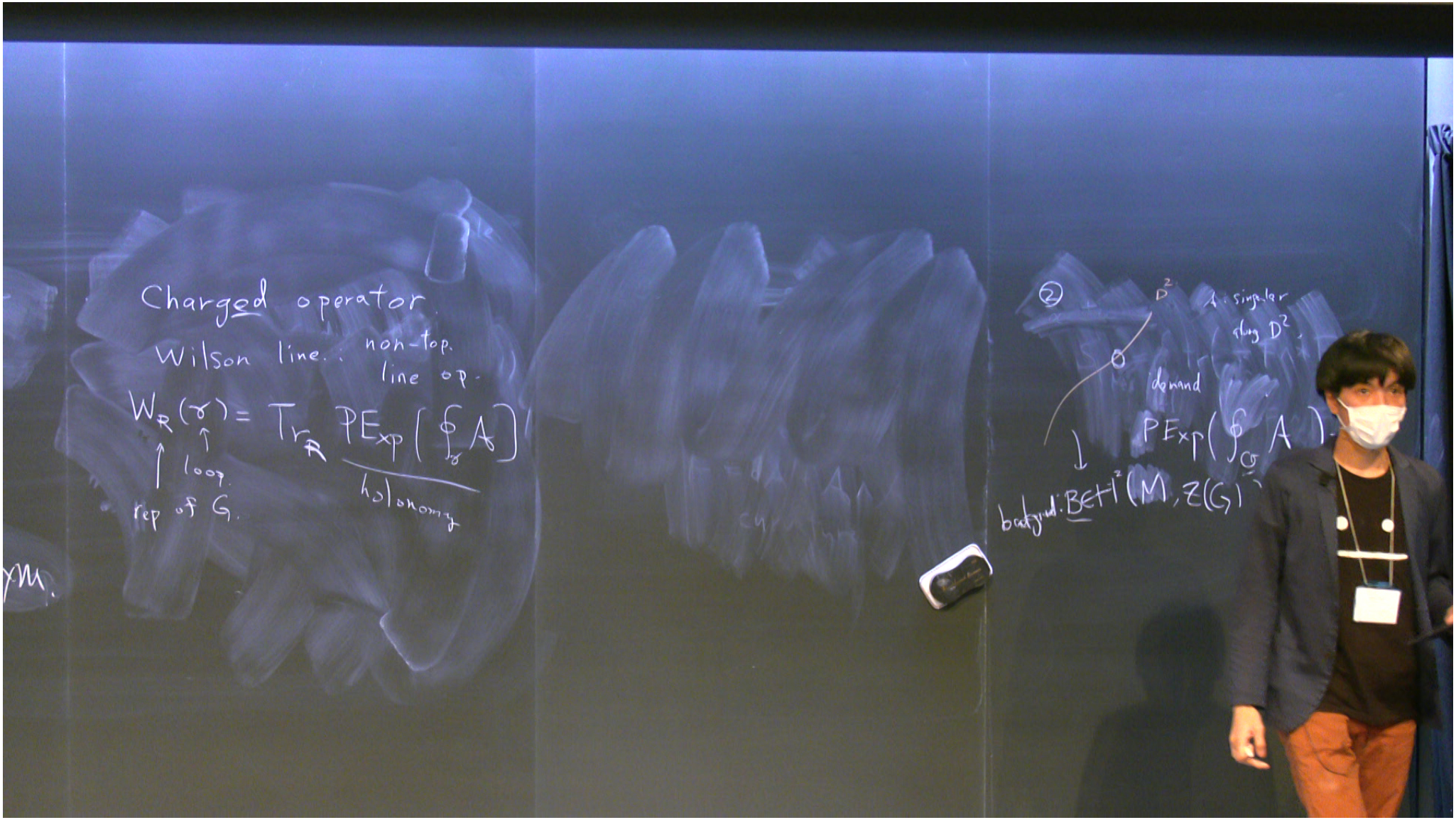

(Assume $\pi_1(G)=0$) sym
$Z[X, B]$ confinement
$= \int_{G\text{-connection}, W_2(A)=B} DA\, e^{-S_{YM}[A]}$
/gauge
Top. op.s
① $A \rightsquigarrow W_2(A) \in H^2(X, \pi_1(G))$
G: non-ab
$\frac{F}{2\pi} \in H^2(X, \mathbb{Z})$
$G = U(1)$
dim 2
top. op
$\phi\left(\int_D W_2(A)\right)$
$\phi \in \pi_1(G)^\vee$

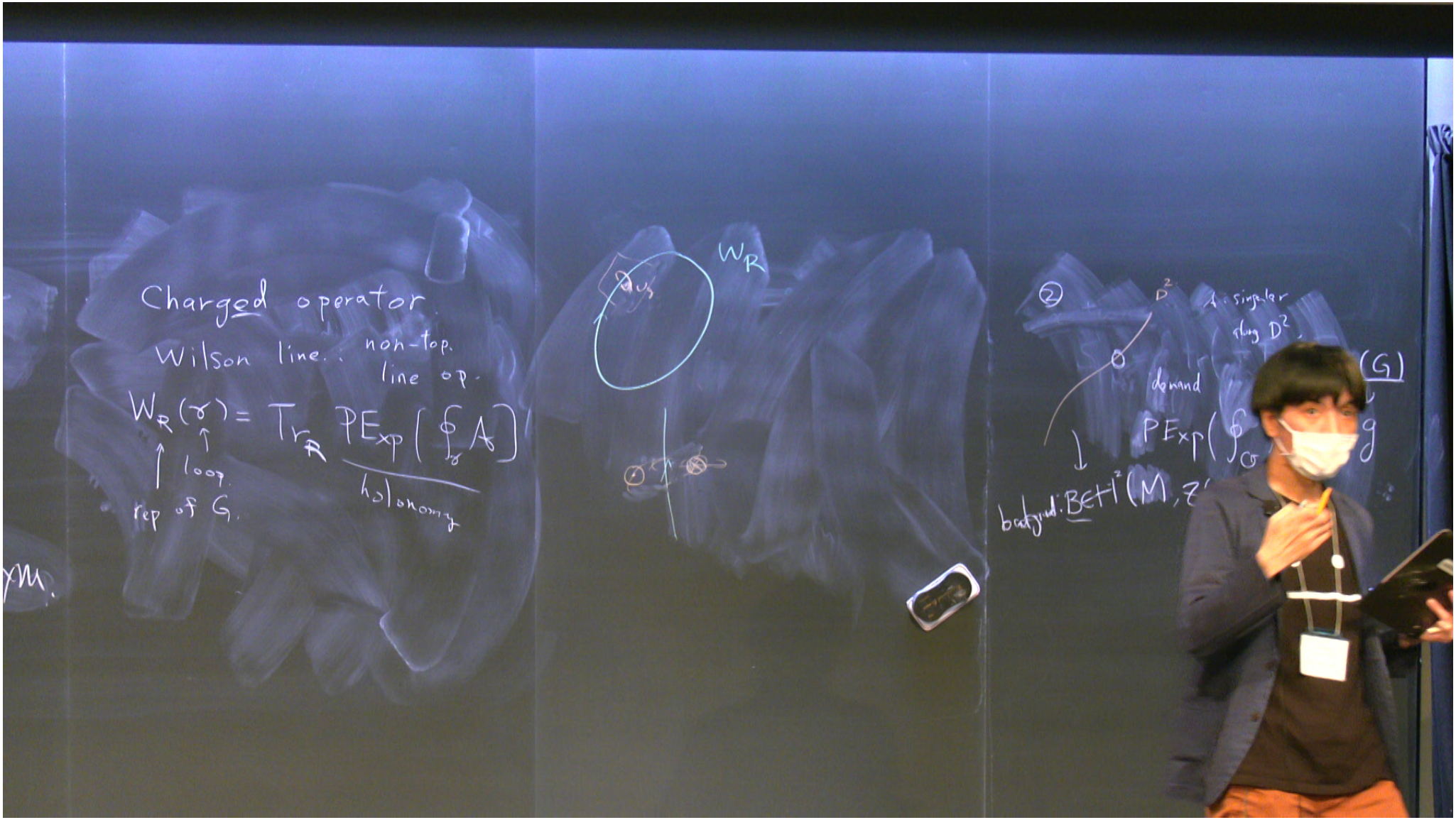

(Assume $\pi_1(G)=0$) sym
$Z[X, B]$ confinement
$= \int_{G\text{-connection}, W_2(A)=B} DA\, e^{-S_{YM}[A]}$
gauge
G: connected, simple.
Top. op.s
① $A \rightsquigarrow W_2(A) \in H^2(X, \pi_1(G))$
G: non-ab
$\frac{F}{2\pi} \in H^2(X, \mathbb{Z})$
$G = U(1)$
dim 2 top. op
$\phi\left(\int_D W_2(A)\right)$
$\phi \in \pi_1(G)^\vee$

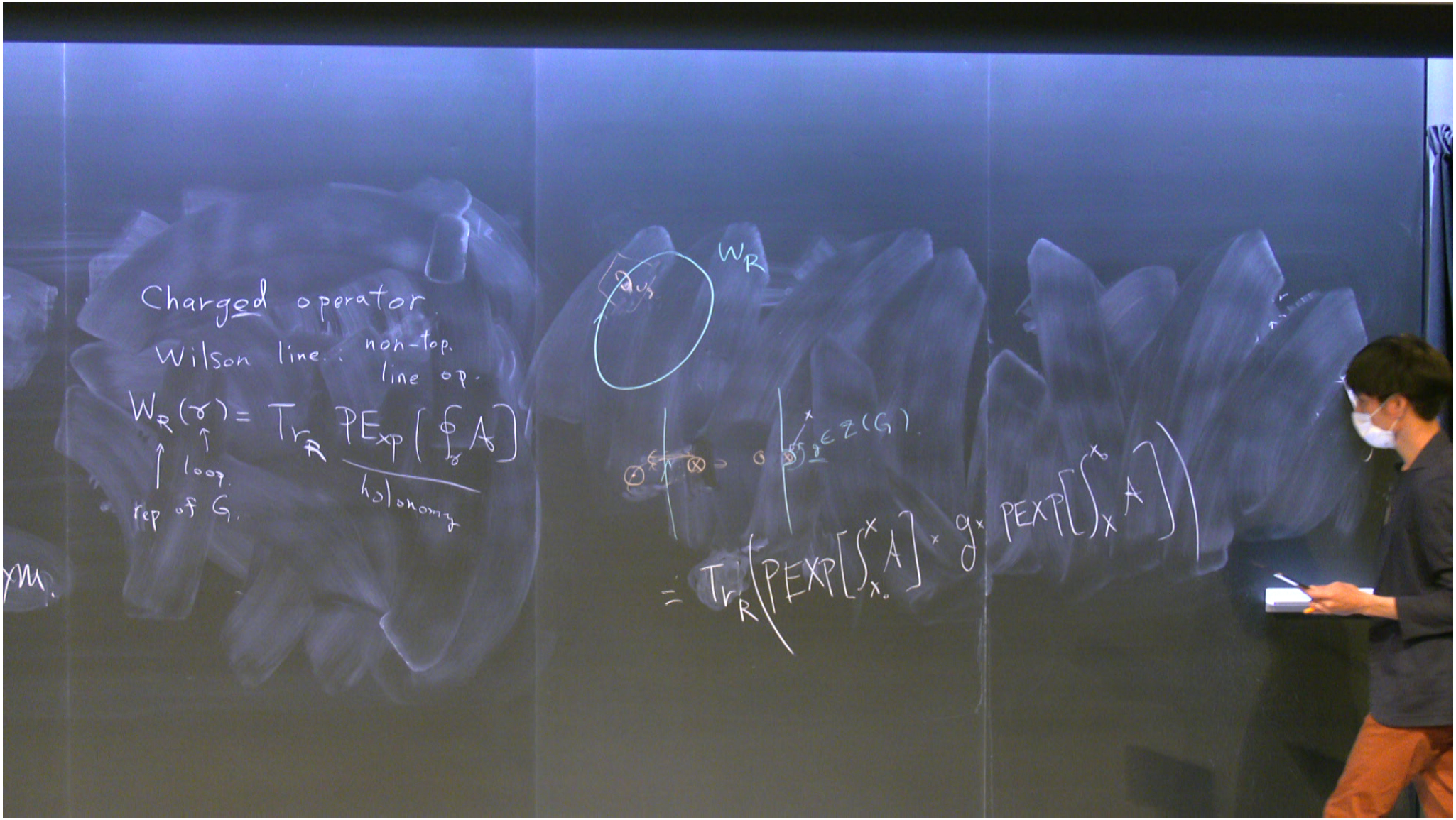

(Assume $\pi_1(G)=0$) sym
$Z[X,B]$ confinement
$= \int_{G\text{-connection}, W_2(A)=B} DA\, e^{-S_{YM}[A]}$ / gauge
Electric one-form sym.
G: connected, simple.
Top. op.s
① $A \rightsquigarrow W_2(A) \in H^2(X, \pi_1(G))$
G: non-ab
$\frac{F}{2\pi} \in H^2(X,\mathbb{Z})$
$G = U(1)$
dim 2 top. op
$\phi\left(\int_D W_2(A)\right)$
$\phi \in \pi_1(G)$

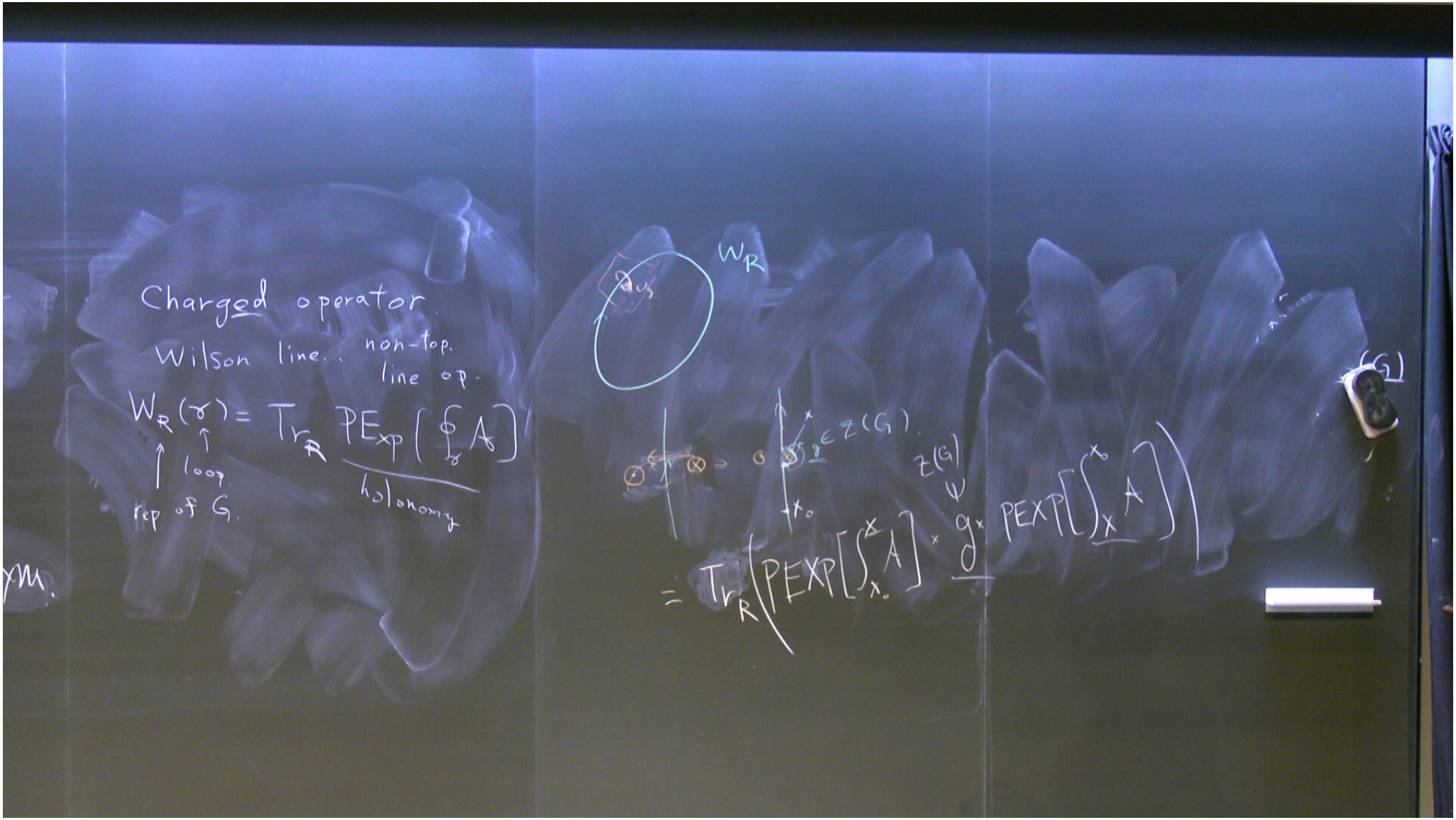

(Assume $\pi_1(G)=0$) sym
$Z[X, B]$ on fine
$= \int_G [$ $S_{YM}[A]$
$w_2(A)=B$)
gauge
One-form sym.
Charged operator
defines a One-form sym
$B\pi_1(G)^\vee$
(Magnetic one-form sym)

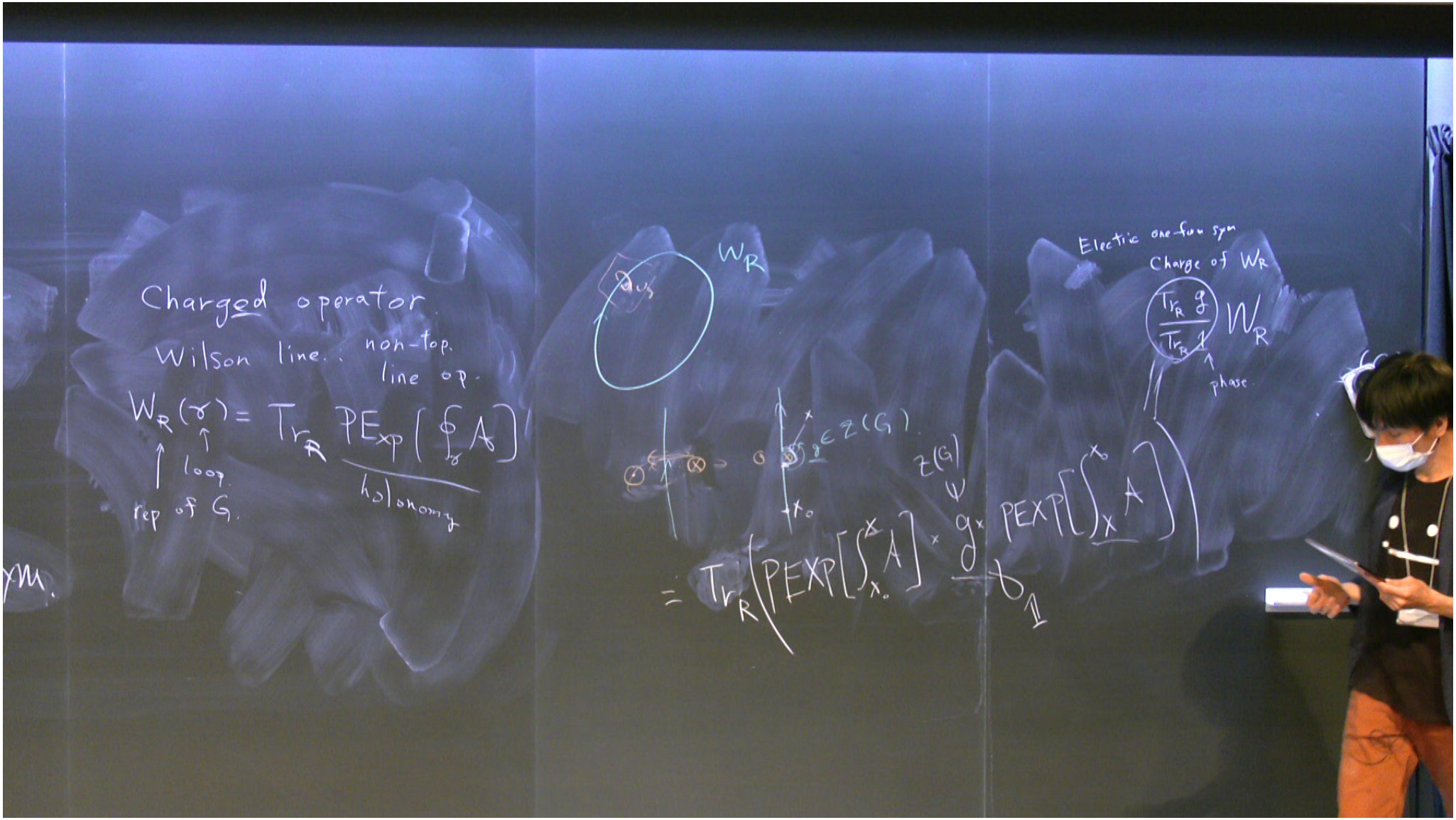

(Assume $\pi_1(G)=0$) sym
Charged operator
Wilson line. non-top. line op.
$W_R(\gamma) = \mathrm{Tr}_R\, \mathrm{PExp}\left(\oint_\gamma A\right)$
holonomy
defines a One-form sym
$B\pi_1(G)^\vee$
(Magnetic one-form sym)
One-form sym.

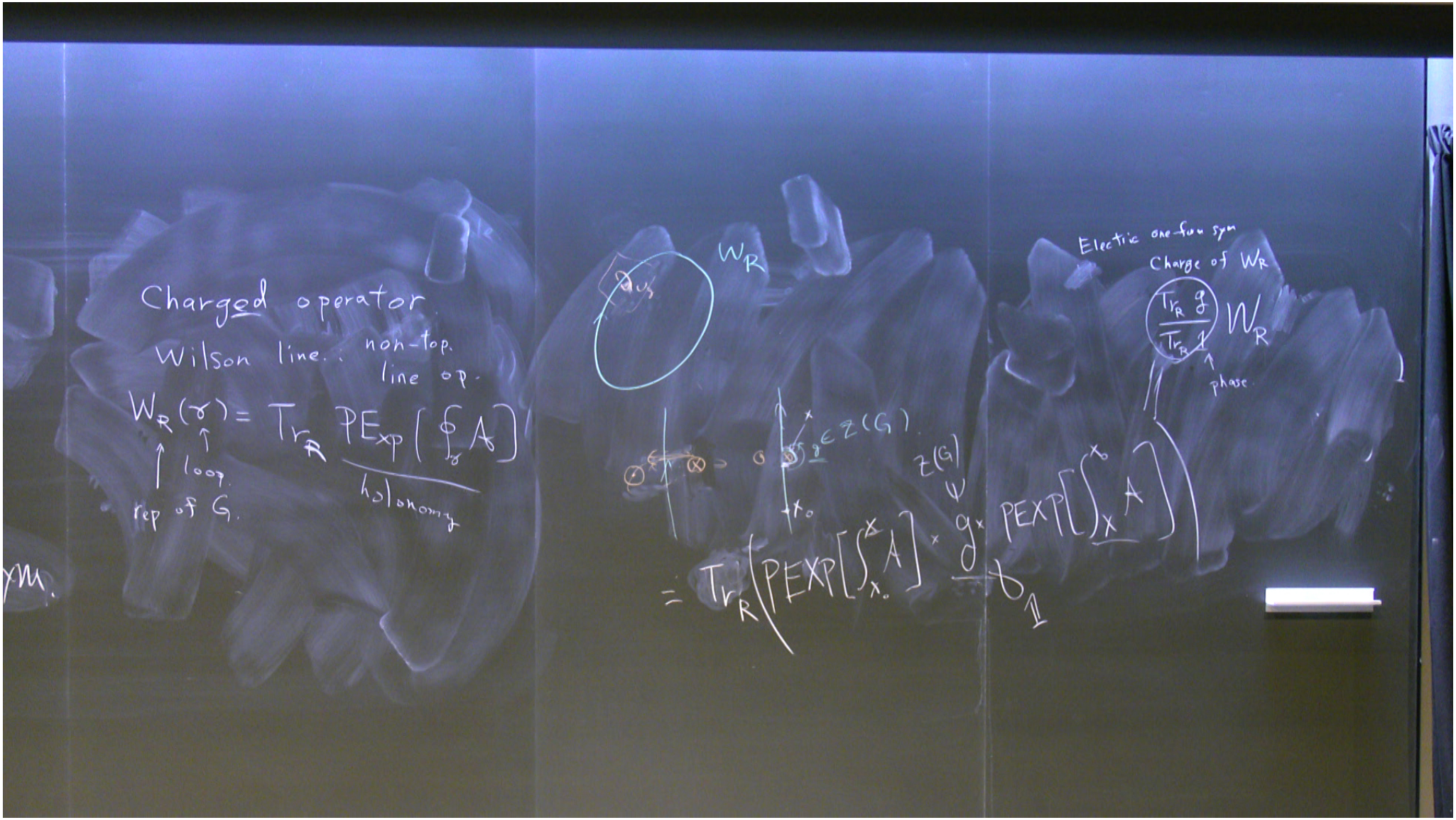

Charged operator
Wilson line: non-top. line op.
$W_R(\gamma) = \mathrm{Tr}_R\, \mathcal{P}\mathrm{Exp}\left(\oint_\gamma A\right)$
loop
rep of G.
holonomy
②
D^2
A: singular along D^2
demand
$\mathcal{P}\mathrm{Exp}\left(\oint_{S^1} A\right)$
background: $\mathrm{Bett}^2(M, Z(G))$


Charged operator
Wilson line: non-top. line op.
$W_R(\gamma) = \mathrm{Tr}_R\, \mathrm{PExp}\left(\oint_\gamma A\right)$
loop
rep of G.
holonomy
W_R
②
D^2
: singular
plug D^2
demand
(G)
$\mathrm{PExp}\left(\oint_C$
boundary: Bett^1(M,
g

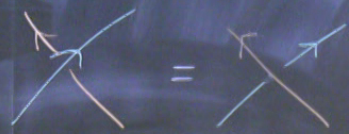

Charged operator
Wilson line: non-top. line op.
$W_R(\gamma) = \mathrm{Tr}_R\, \mathrm{PExp}\left(\oint_\gamma A\right)$
loop
rep of G
holonomy
W_R
$g \in Z(G)$
$= \mathrm{Tr}_R\left(\mathrm{PEXP}\left[\int_{x_0}^{x} A\right] \cdot g \cdot \mathrm{PEXP}\left[\int_{x}^{x_0} A\right]\right)$


Charged operator
Wilson line: non-top. line op.
$W_R(\gamma) = \mathrm{Tr}_R\, \mathrm{PExp}\left(\oint_\gamma A\right)$
loop
rep of G
holonomy
W_R
$z \in Z(G)$
x_0
$Z(G)$
g^x
$= \mathrm{Tr}_R\left(\mathrm{PEXP}\left[\int_{x_0}^{x} A\right] \cdot g^x \cdot \mathrm{PEXP}\left[\int_x^{x_0} A\right]\right)$


Charged operator
Wilson line: non-top. line op.
$W_R(\gamma) = \mathrm{Tr}_R\, \mathrm{PExp}\left(\oint_\gamma A\right)$
loop
rep of G
holonomy
W_R
$g \in Z(G)$
x_0
$Z(G)$
$= \mathrm{Tr}_R\left(\mathrm{PEXP}\left[\int_{x_0}^{x} A\right] \cdot g \cdot \mathrm{PEXP}\left[\int_{x}^{x_0} A\right]\right)$
$= g \cdot \mathbb{1}$
Electric one-form sym
Charge of W_R
$\frac{\mathrm{Tr}_R\, g}{\mathrm{Tr}_R\, \mathbb{1}}$
W_R
phase


Charged operator
Wilson line.. non-top. line op.
$W_R(\gamma) = \mathrm{Tr}_R\, \mathrm{PExp}\left(\oint_\gamma A\right)$
loop
rep of G.
holonomy
W_R
$g \in Z(G)$
x_0
x
$Z(G)$
g^x
$= \mathrm{Tr}_R\left(\mathrm{PEXP}\left[\int_{x_0}^{x} A\right] \cdot g^x \cdot \mathrm{PEXP}\left[\int_x^{x_0} A\right]\right)$
$\to$
$\mathbb{1}$
Electric one-form sym
Charge of W_R
$\frac{\mathrm{Tr}_R\, g}{\mathrm{Tr}_R\, \mathbb{1}}$
W_R
phase

Alternatively, locally



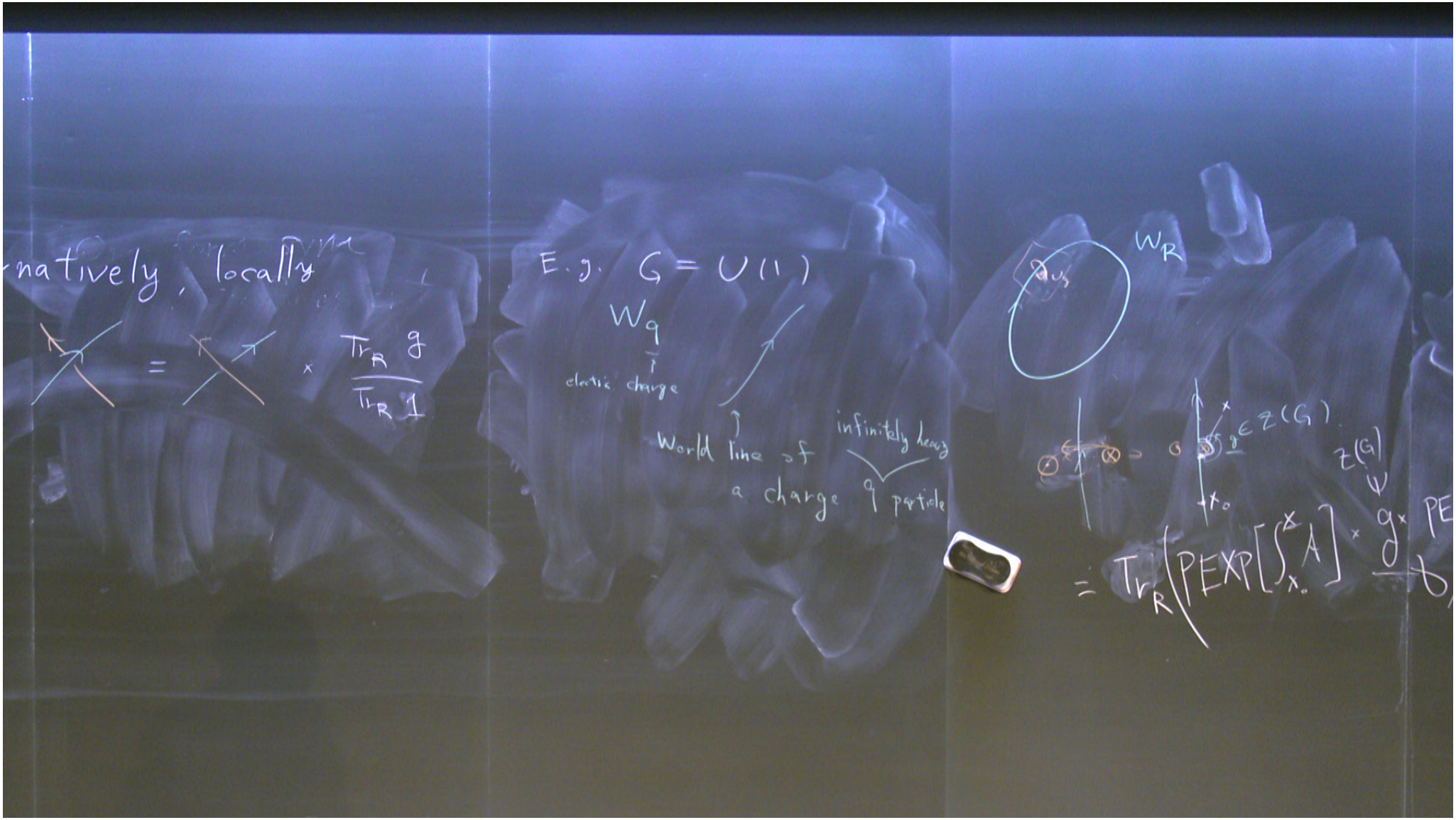
$\times \frac{\mathrm{Tr}_R\, g}{\mathrm{Tr}_R\, 1}$

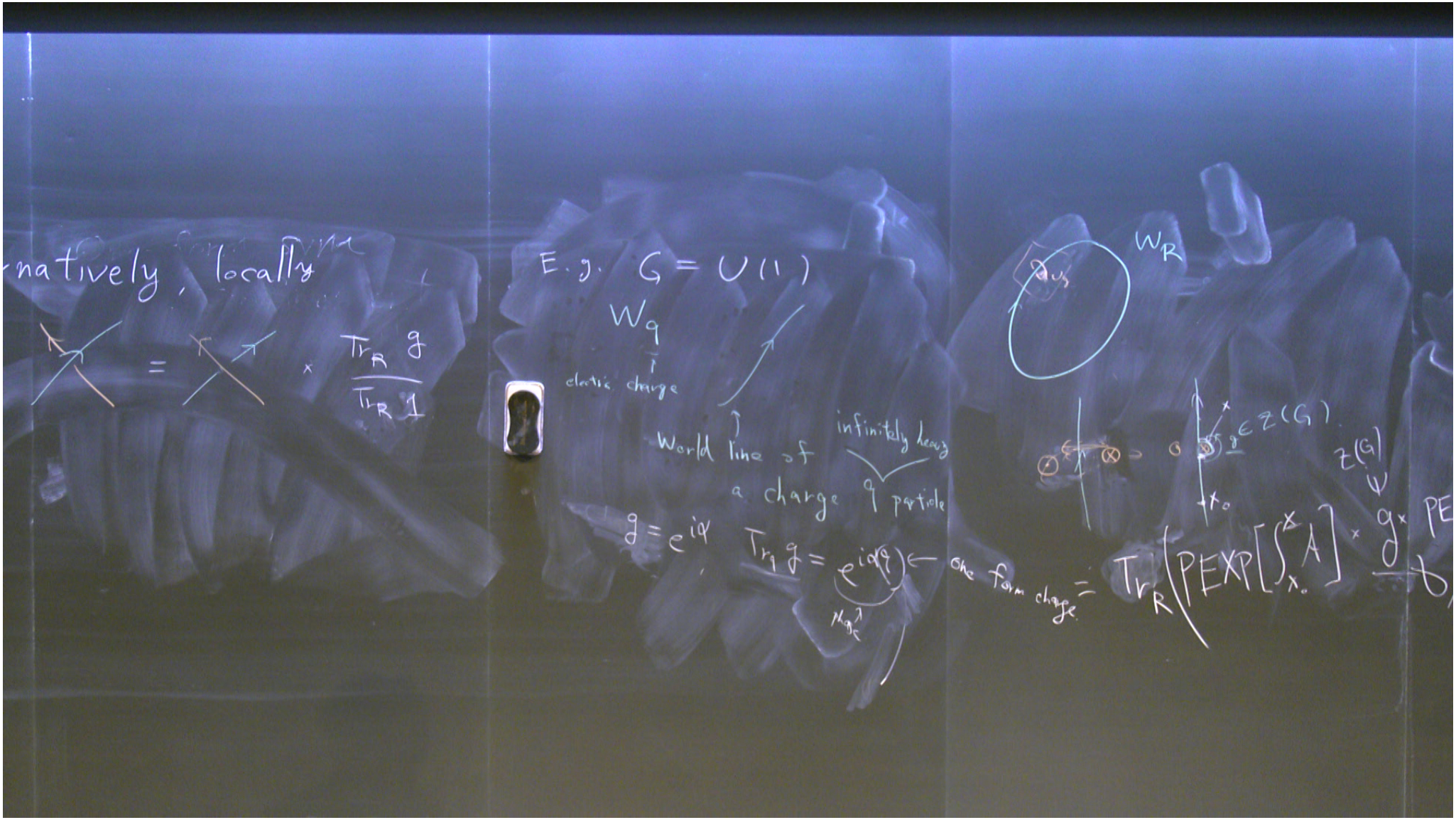
Charged operator.

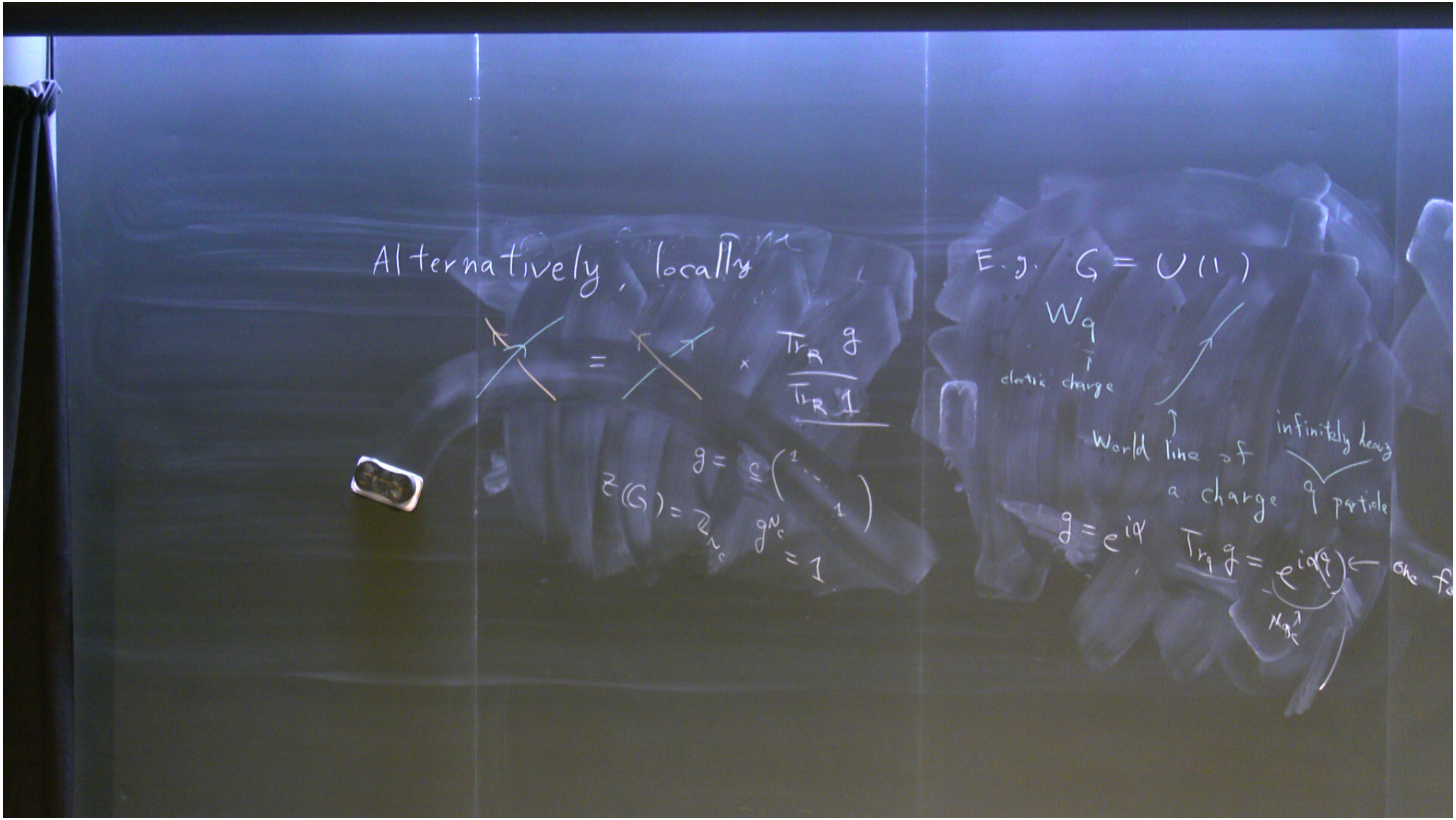
Wilson line.: non-top. line op.

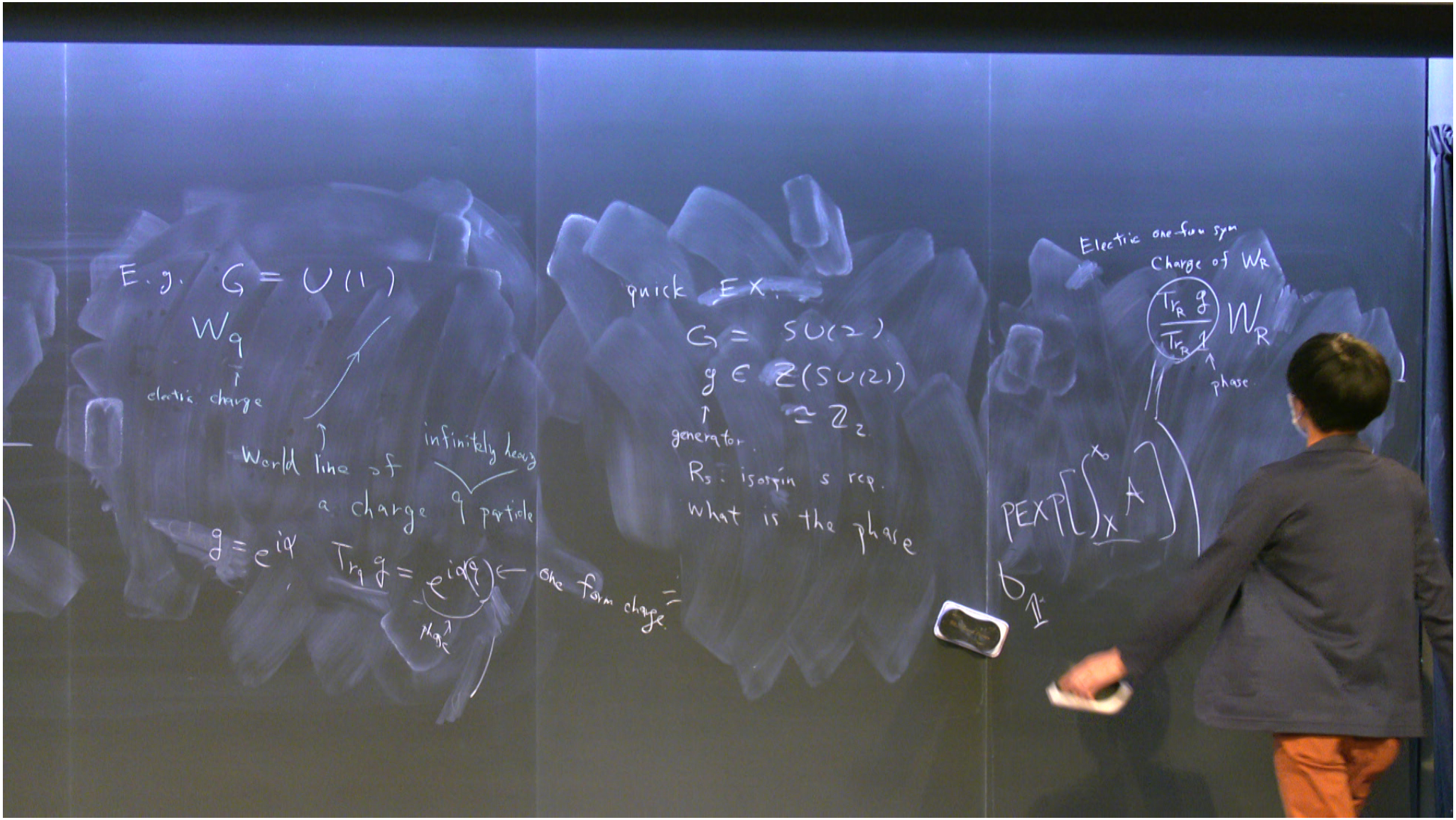
$W_R(\gamma) = \mathrm{Tr}_R\, \mathcal{P}\mathrm{Exp}\left(\oint_\gamma A\right)$

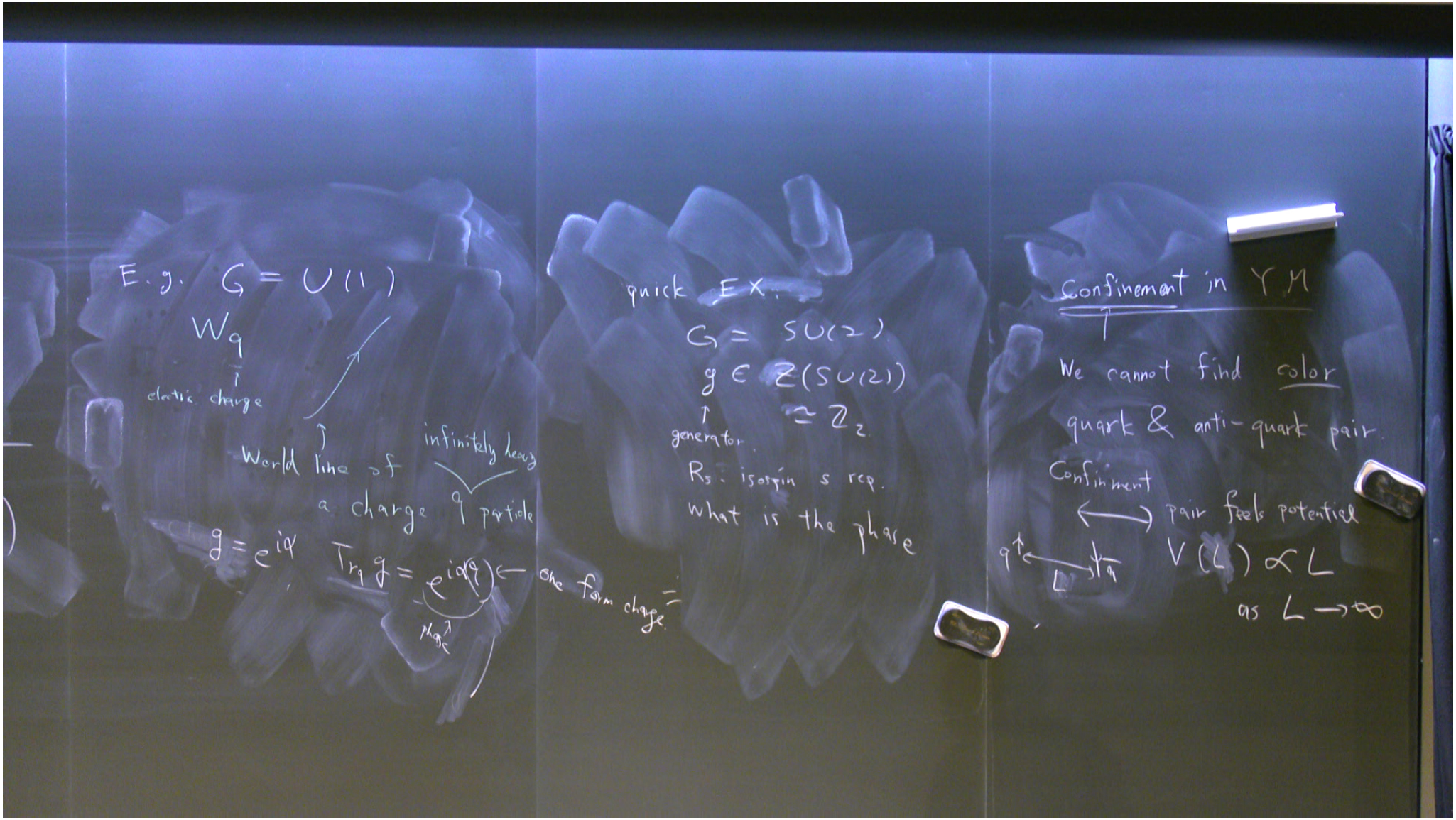
R: rep of G. γ: loop. holonomy


natively, locally
$\mathrm{Tr}_R\, g$
$\mathrm{Tr}_R\, 1$
E.g. $G = U(1)$
W_q
electric charge
World line of a charge q particle
infinitely heavy
W_R
$g \in Z(G)$
$Z(G)$
$= \mathrm{Tr}_R \left(P \exp \left[\int_{x_0}^{x} A \right] \times g \times P \right.$


natively, locally
E.g. G = U(1)
W_q
elastic charge
World line of a charge q particle
infinitely heavy
$g = e^{iq}$
$Tr_q g = e^{iq(g)}$
one form charge
W_R
∂_{U_3}
$g \in Z(G)$
$Z(G)$
$Tr_R\left(PEXP\left[\int_{x_0}^{x} A\right] \times g \times PE\right.$


Alternatively, locally
$= \times \frac{\mathrm{Tr}_R\, g}{\mathrm{Tr}_R\, 1}$
$g = \in \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix}$
$z(G) = \mathbb{Z}_{N_c}$
$g^{N_c} = 1$
E.g. $G = U(1)$
W_q
electric charge
World line of a charge q particle
infinitely heavy
$g = e^{iq}$
$\mathrm{Tr}_q\, g = e^{iq(q)}$
N_c


E.g. G = U(1)
W_q
elastic charge
World line of a charge q particle
infinitely heavy
$g = e^{iq}$
$T_{rq} g = e^{iqq}$
charge
quick Ex.
G = SU(2)
$g \in Z(SU(2)) \simeq Z_2$
generator
R_s: isospin s rep.
What is the phase
one form charge =
Electric one-form sym
Charge of W_R
$\frac{T_{r_R} g}{T_{r_R}}$
W_R
phase
$P\exp\left[\int_X A\right]$
$b \in$


E.g. G = U(1)
W_q
elastic charge
World line of a charge q particle
infinitely heavy
$g = e^{iq}$
$Tr_q g = e^{iqq}$
charge
one form charge
quick EX.
G = SU(2)
$g \in Z(SU(2)) \simeq \mathbb{Z}_2$
generator
R_s: isospin s rep.
What is the phase
Confinement in YM
We cannot find color quark & anti-quark pair.
Confinement
pair feels potential
$V(L) \propto L$
as $L \to \infty$

E.g. $G = U(1)$

W_q

↑ elastic charge

World line of a charge q particle (infinitely heavy)

$g = e^{iq}$ $\quad Tr_q\, g = e^{iqq}$ ← one form charge

↑ charge

quick EX.

$G = SU(2)$

$g \in Z(SU(2)) \simeq \mathbb{Z}_2$

↑ generator

R_s : isospin s rep.

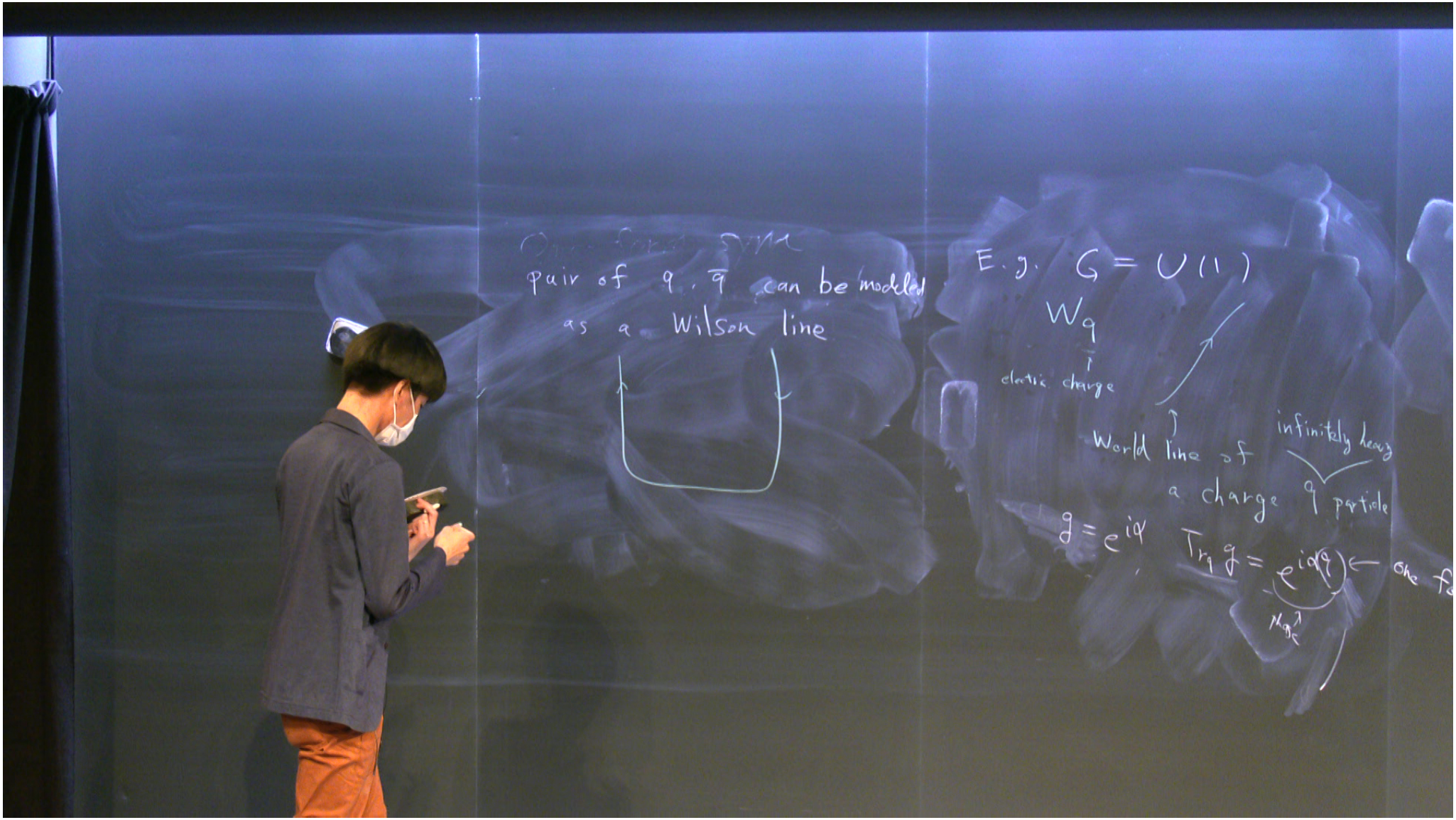
What is the phase

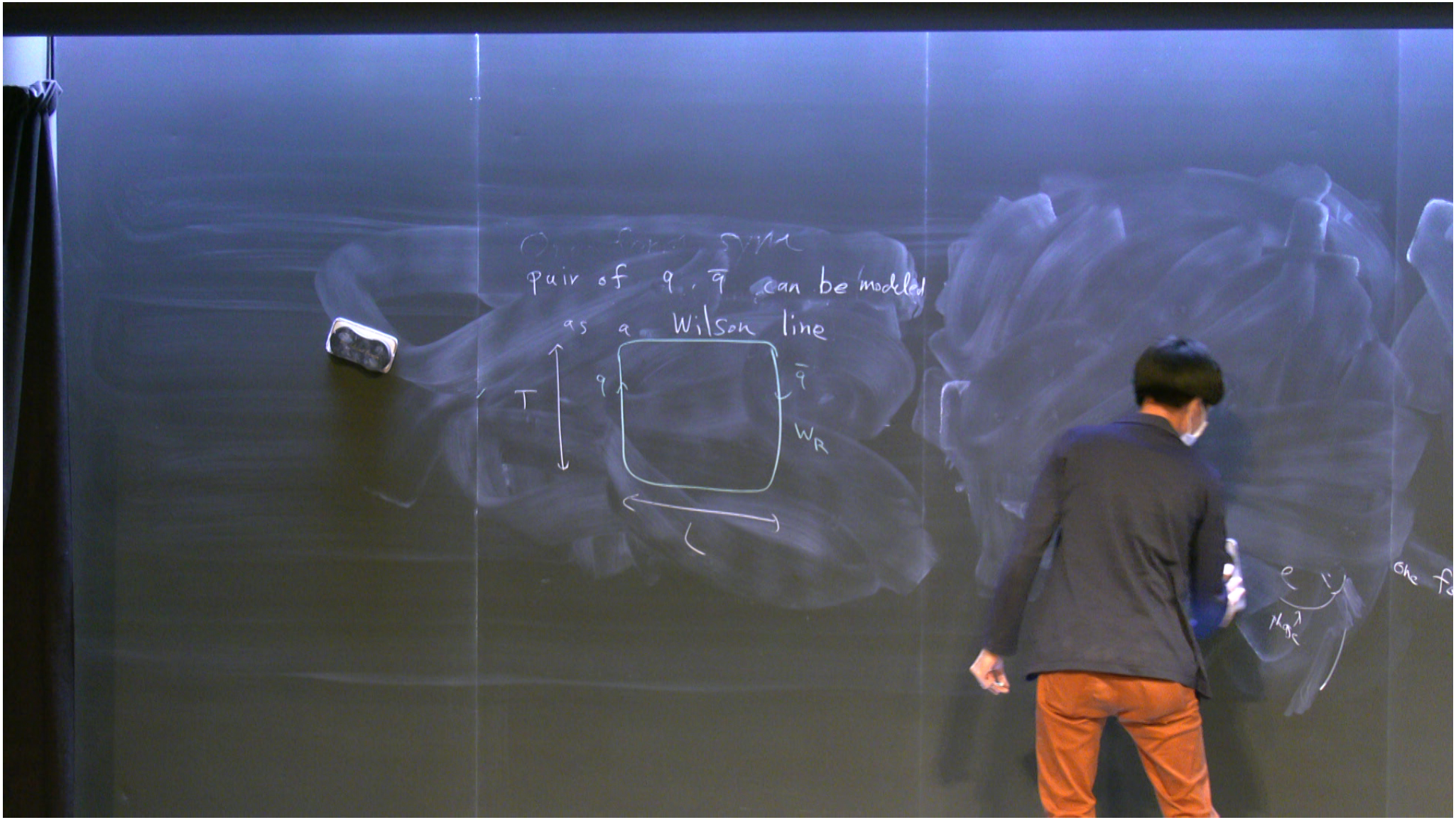
Confinement in YM

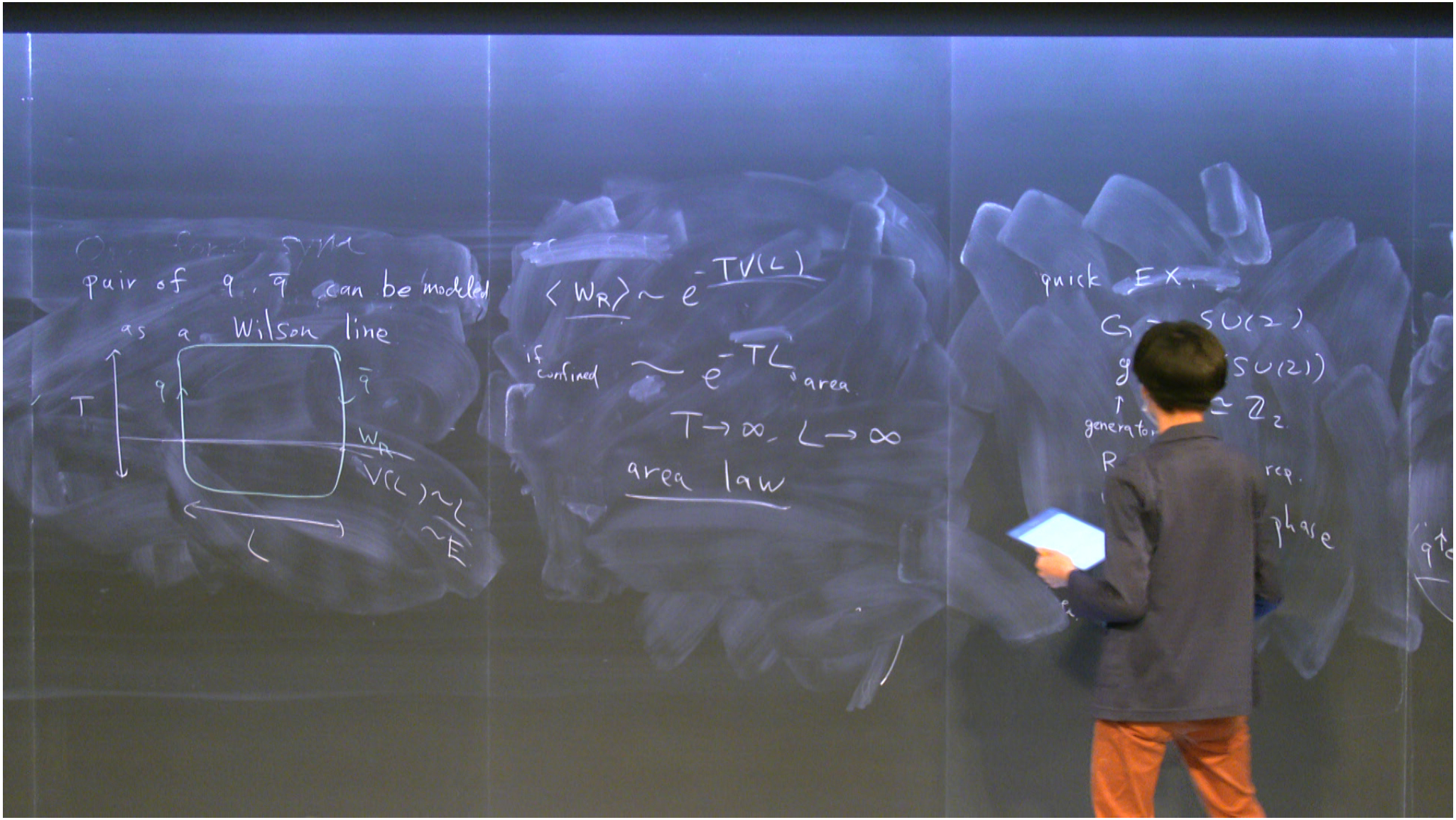
We cannot find color quark & anti-quark pair.

Confinement $\longleftrightarrow$ pair feels potential

$V(L) \propto L$ as $L \to \infty$


pair of $q, \bar{q}$ can be modeled
as a Wilson line
E.g. $G = U(1)$
W_q
electric charge
World line of
a charge q particle
infinitely heavy
$g = e^{i\alpha}$
$\mathrm{Tr}_q\, g = e^{iq\alpha}$


pair of $q, \bar{q}$ can be modeled
as a Wilson line
T
q
$\bar{q}$
W_R


pair of $q, \bar{q}$ can be modeled
as a Wilson line
T
q
$\bar{q}$
W_R
$V(L) \sim L$
$\sim E$
$\langle W_R \rangle \sim e^{-TV(L)}$
if confined $\sim e^{-TL_{\text{area}}}$
$T \to \infty, L \to \infty$
area law
quick Ex.
$G = SU(2)$
$g = SU(2)$
$\simeq \mathbb{Z}_2$
generator
R
rep.
phase

modeled

$\langle W_R \rangle \sim e^{-TV(L)}$

if confined $\sim e^{-TL}$, area

$T \to \infty, L \to \infty$

area law

If de-confined / screened

$V(L) \to$ constant

$L \to \infty$

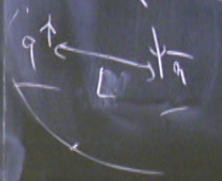
$\langle W_R \rangle \sim e^{-\text{Perimeter}}$

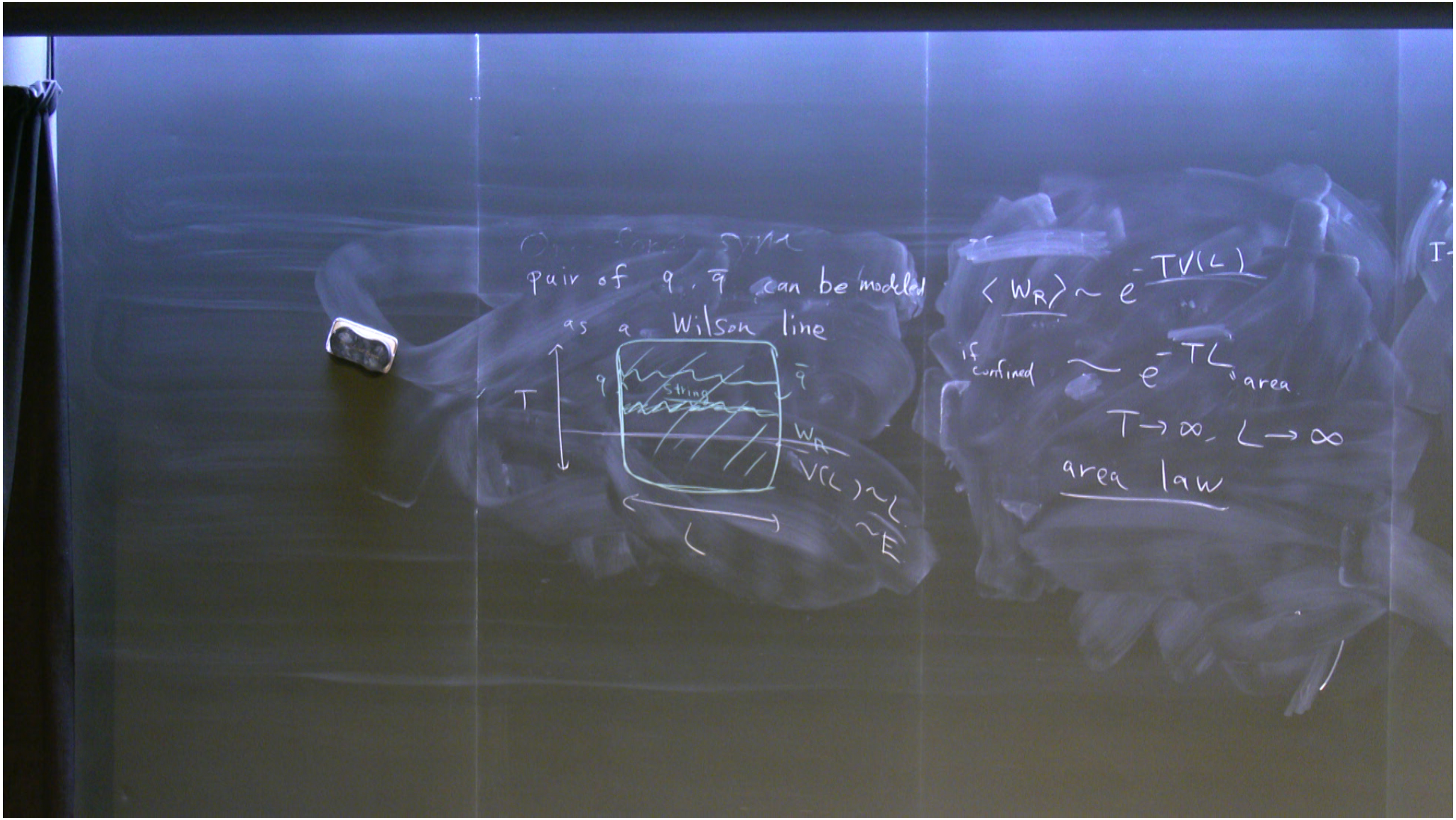
Perimeter law

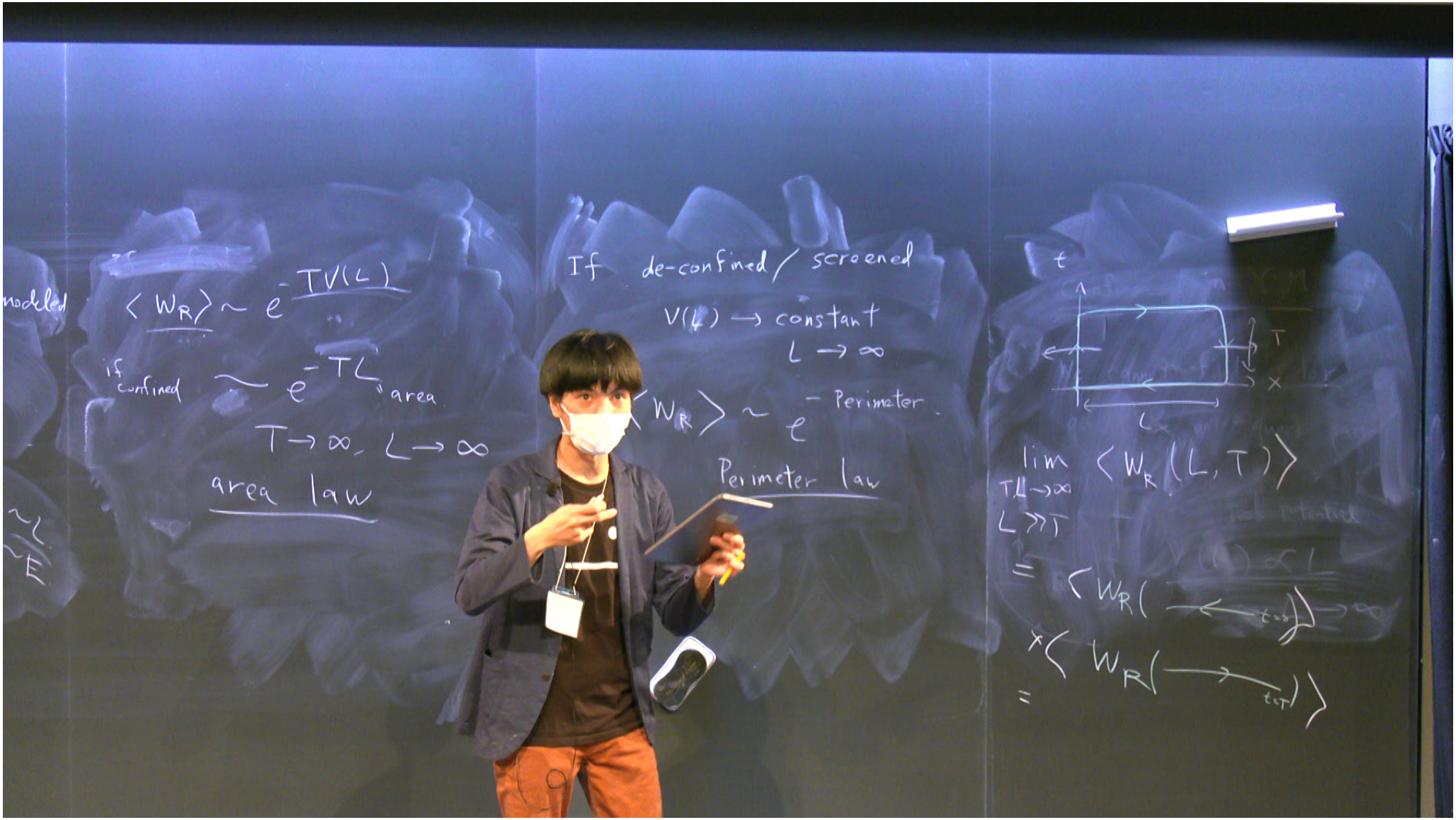
Confinement in YM

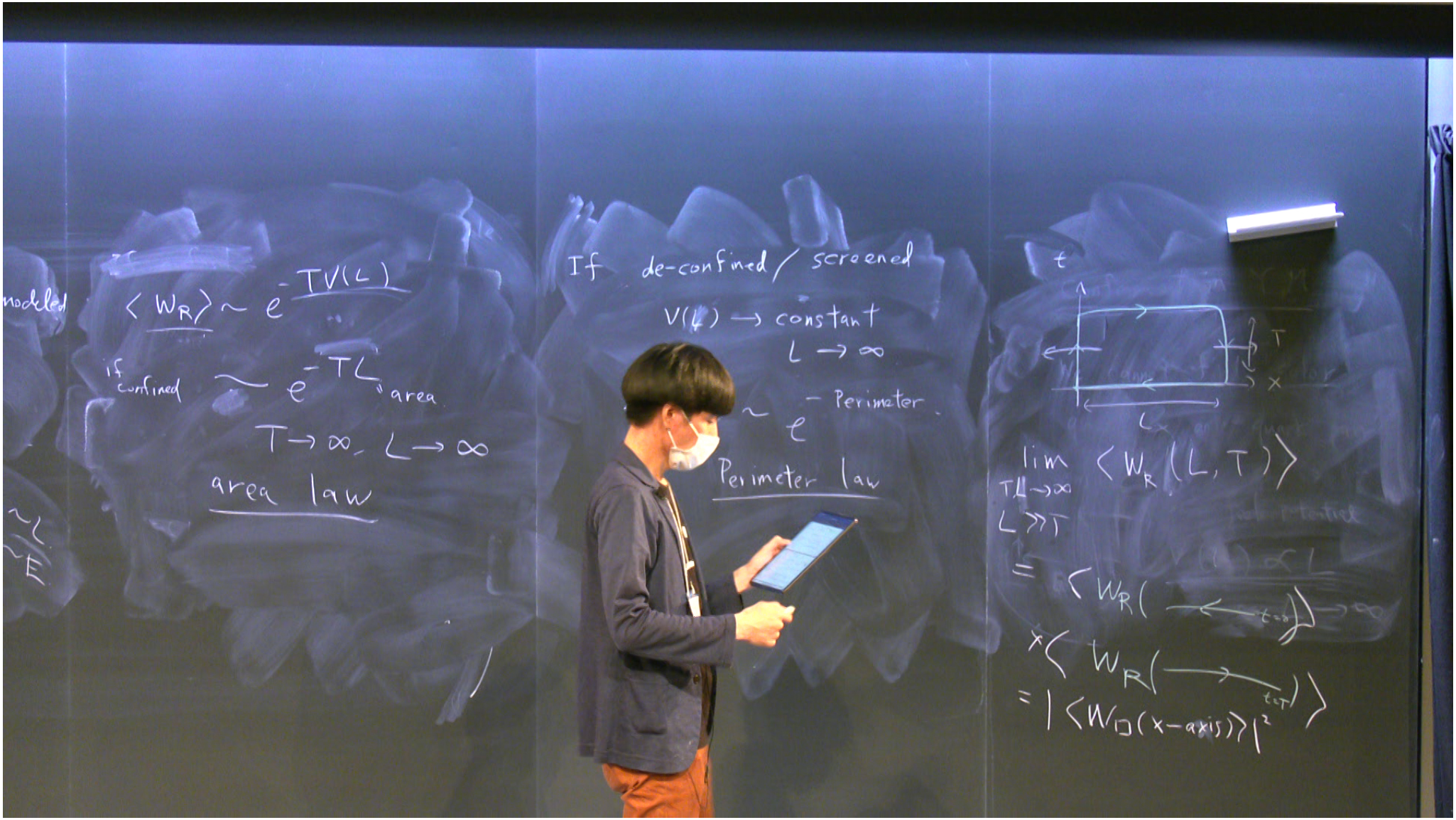
We cannot find color quark & anti-quark pair

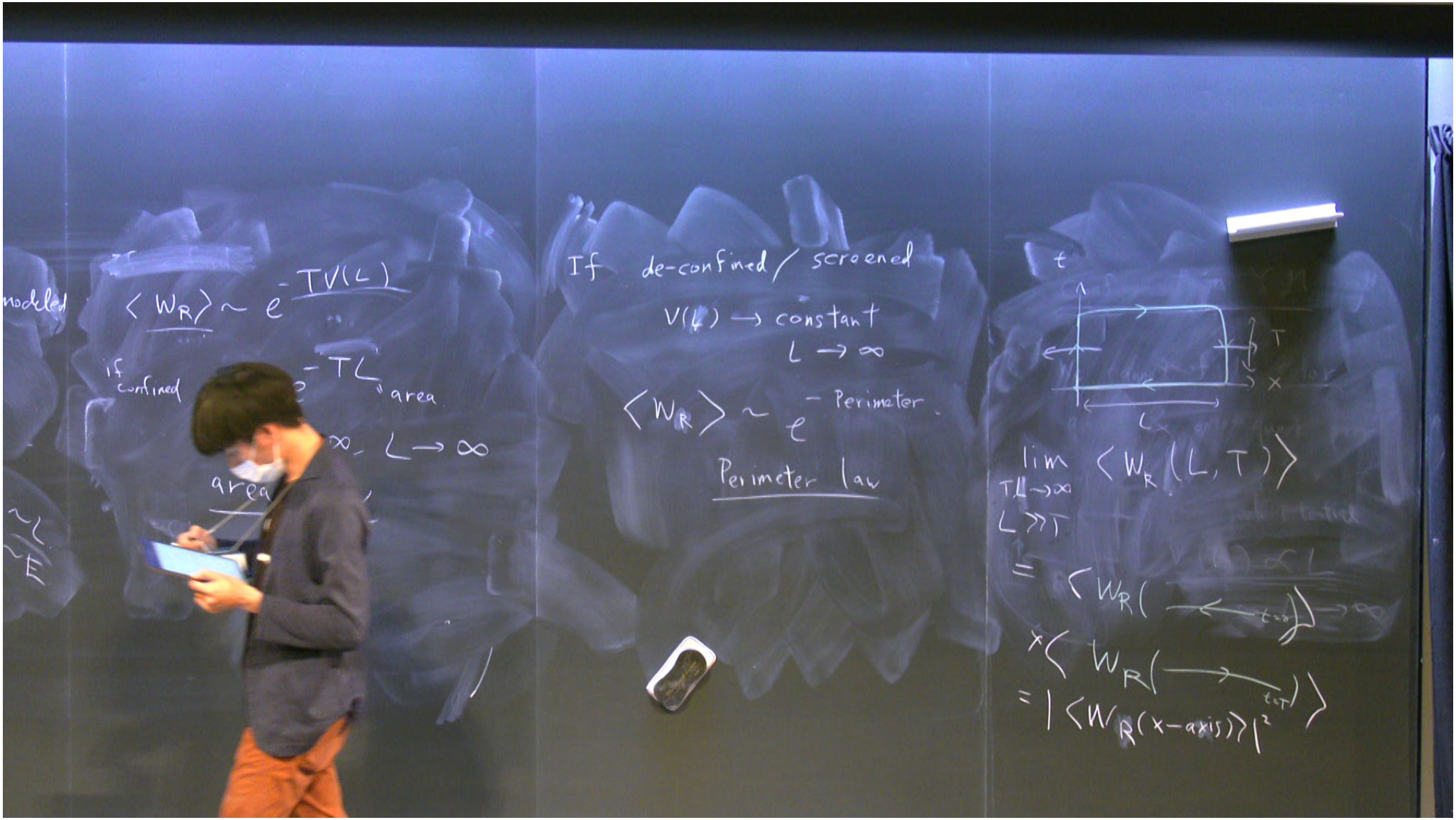
Confinement $\Longleftrightarrow$ pair feels potential $V(L) \propto L$ as $L \to \infty$

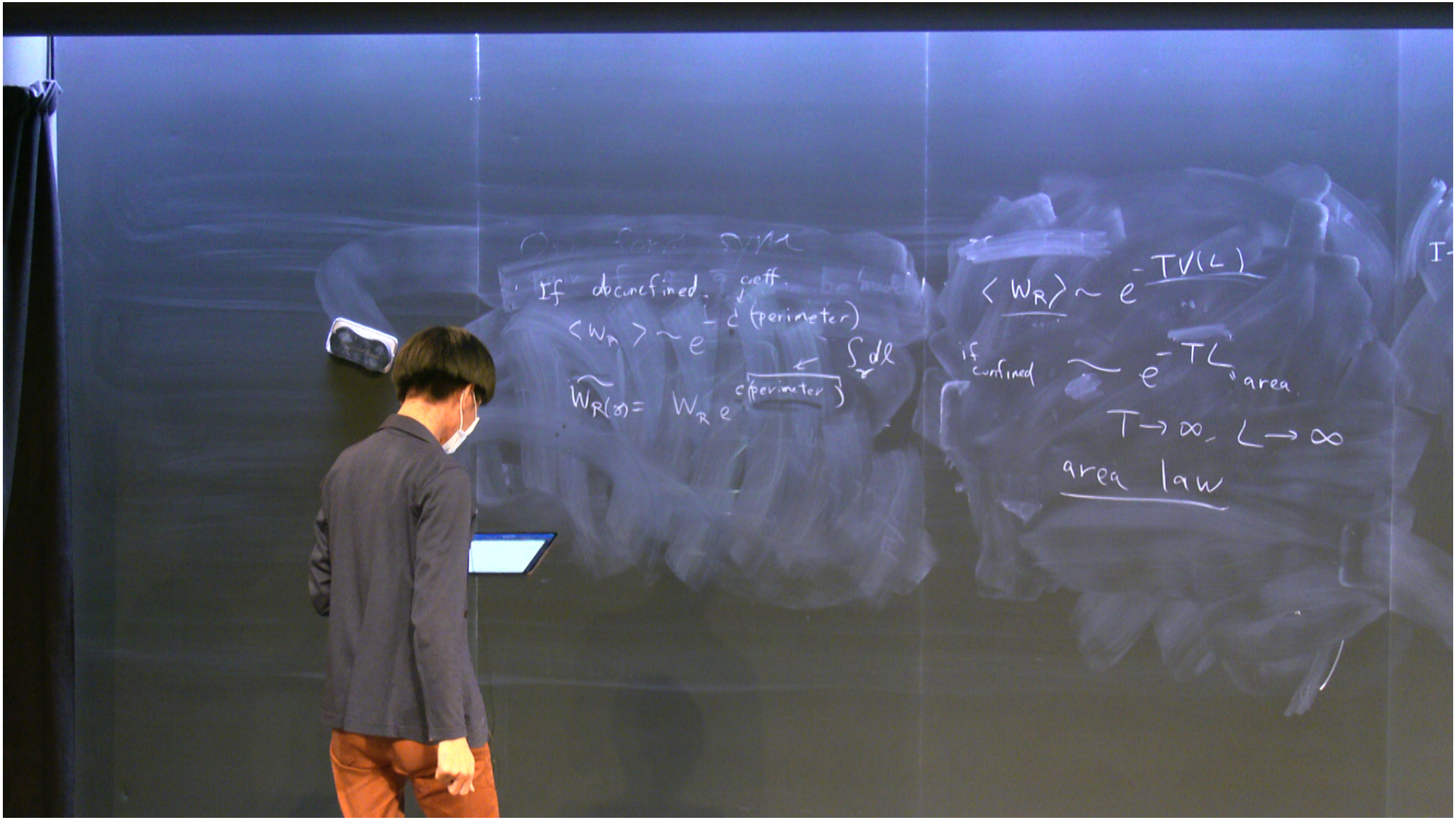


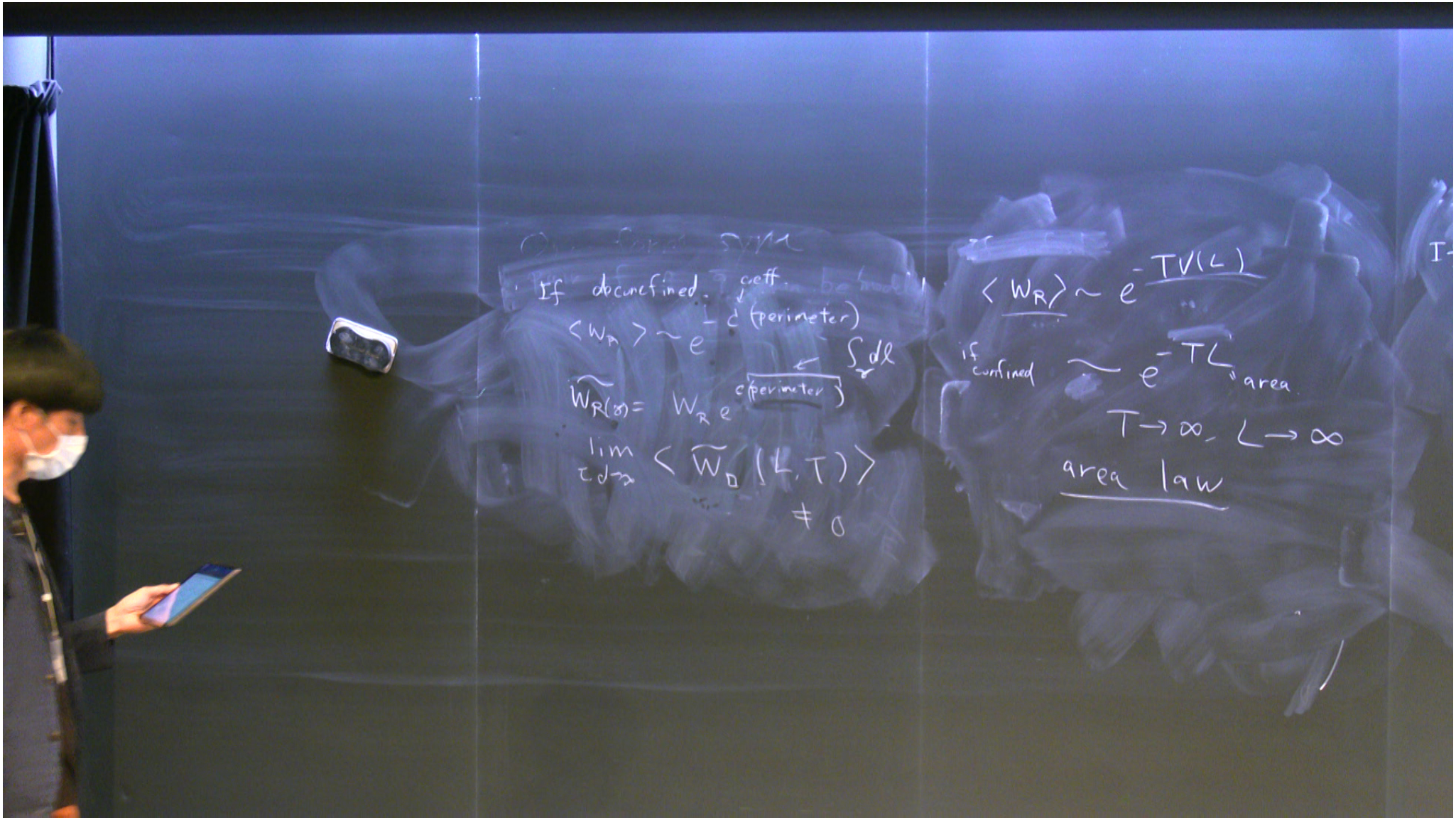

pair of $q, \bar{q}$ can be modeled as a Wilson line
q
$\bar{q}$
string
T
W_R
$V(L) \sim L$
$\sim E$
L
$\langle W_R \rangle \sim e^{-TV(L)}$
if confined $\sim e^{-TL}$, area
$T \to \infty, L \to \infty$
area law

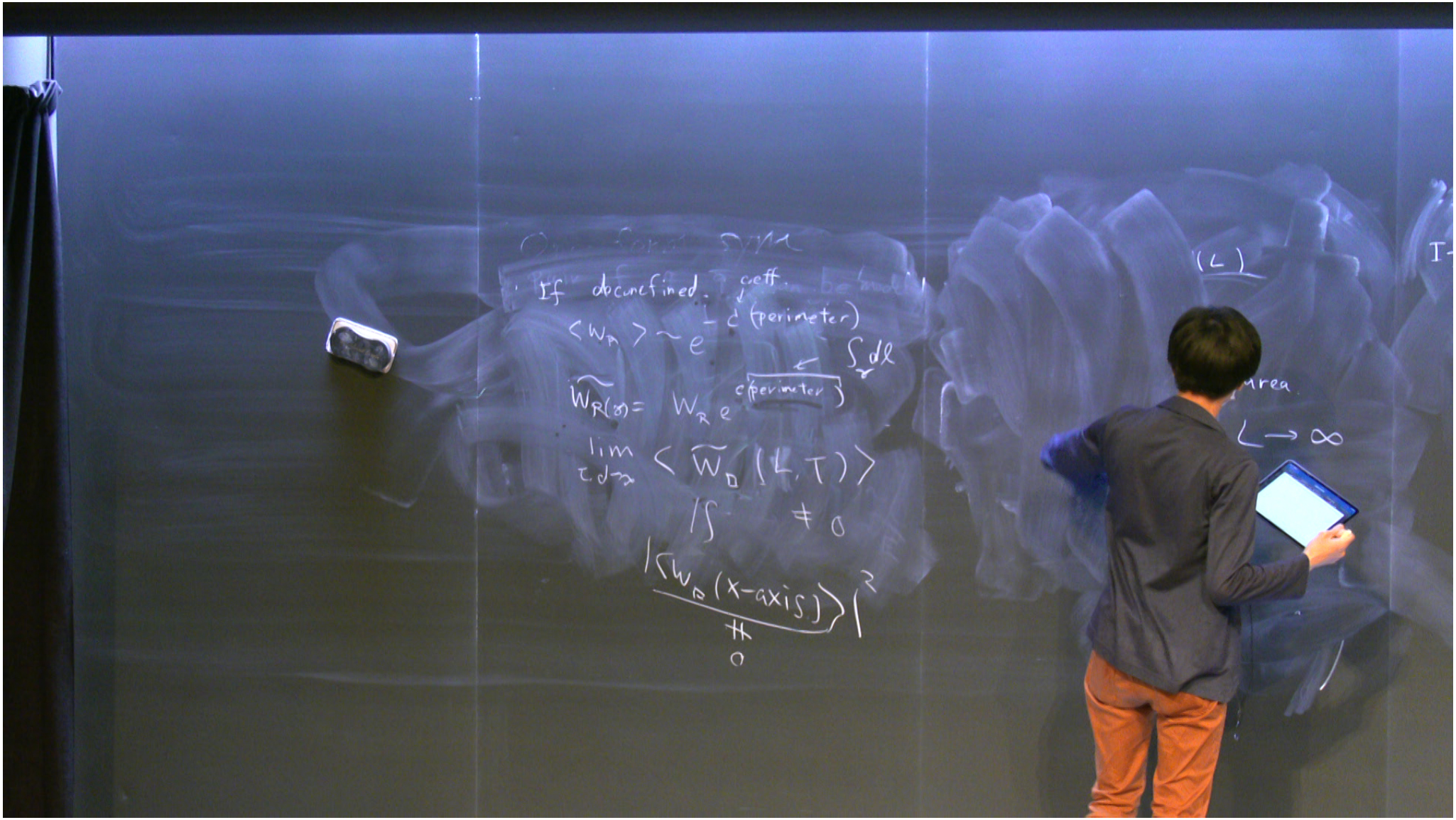

modeled
$\langle W_R \rangle \sim e^{-TV(L)}$
if confined $\sim e^{-TL}$, area
$T \to \infty$, $L \to \infty$
area law
If de-confined / screened
$V(L) \to$ constant
$L \to \infty$
$\langle W_R \rangle \sim e^{-\text{Perimeter}}$
Perimeter law
t
T
x
L
$\lim_{T,L \to \infty,\ L \gg T} \langle W_R(L,T) \rangle$
$= \langle W_R($ $t=\infty$ $) \rangle \times \langle W_R($ $t=T$ $) \rangle$
=


modeled
$\langle W_R \rangle \sim e^{-TV(L)}$
if confined $\sim e^{-TL}$, area
$T \to \infty, L \to \infty$
area law
If de-confined / screened
$V(L) \to$ constant
$L \to \infty$
$\sim e^{-\text{Perimeter}}$
Perimeter law
t
T
x
L
$\lim_{TL \to \infty, L \gg T} \langle W_R(L, T) \rangle$
$= \langle W_R($ $t=\infty$ $) \rangle^{*} \langle W_R($ $t=T$ $) \rangle$
$= |\langle W_{\square}(x\text{-axis}) \rangle|^2$


$\langle W_R \rangle \sim e^{-TV(L)}$
If confined
$-TL$, area
$L \to \infty$
area
If de-confined / screened
$V(L) \to$ constant
$L \to \infty$
$\langle W_R \rangle \sim e^{-\text{Perimeter}}$
Perimeter law
t
T
x
L
$\lim_{T,L \to \infty} \langle W_R(L,T) \rangle$
$L \gg T$
$= \langle W_R($
$t=\infty$
$\times \langle W_R($
$t=T$
$= |\langle W_R(x\text{-axis}) \rangle|^2$


If deconfined.
coeff.
(perimeter)
area
area law


If deconfined.
coeff.
$\langle W_R \rangle \sim e^{-c(\text{perimeter})}$
$\int dl$
$\widetilde{W_{R(\sigma)}} = W_R e^{c(\text{perimeter})}$
$\lim_{\tau \to \infty} \langle \widetilde{W_\square}(L,T) \rangle \neq 0$
$\langle W_R \rangle \sim e^{-TV(L)}$
if confined $\sim e^{-TL}$, area
$T \to \infty,\ L \to \infty$
area law


If deconfined.
coeff.
(perimeter)
$\langle W_R \rangle \sim e^{-c(\text{perimeter})}$
$\int dl$
$\widetilde{W_{R(\sigma)}} = W_R e^{c(\text{perimeter})}$
$\lim_{\tau \to \infty} \langle \widetilde{W}_\square(L,T) \rangle$
$\neq 0$
$|\langle W_\square(x\text{-axis}) \rangle|^2$
$\neq 0$
(L)
area
$L \to \infty$

If deconfined. coeff.

$\langle W_R \rangle \sim e^{-c(\text{perimeter})}$

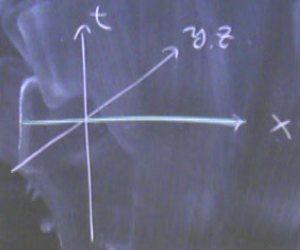
$\widetilde{W_{R(\sigma)}} = W_R e^{c(\text{perimeter})}$ ← $\int_\sigma dl$

$\lim_{T \to \infty} \langle \widetilde{W}_\square (L, T) \rangle$

$\neq 0$

$|\langle W_\square (x\text{-axis}) \rangle|^2$

$\neq 0$



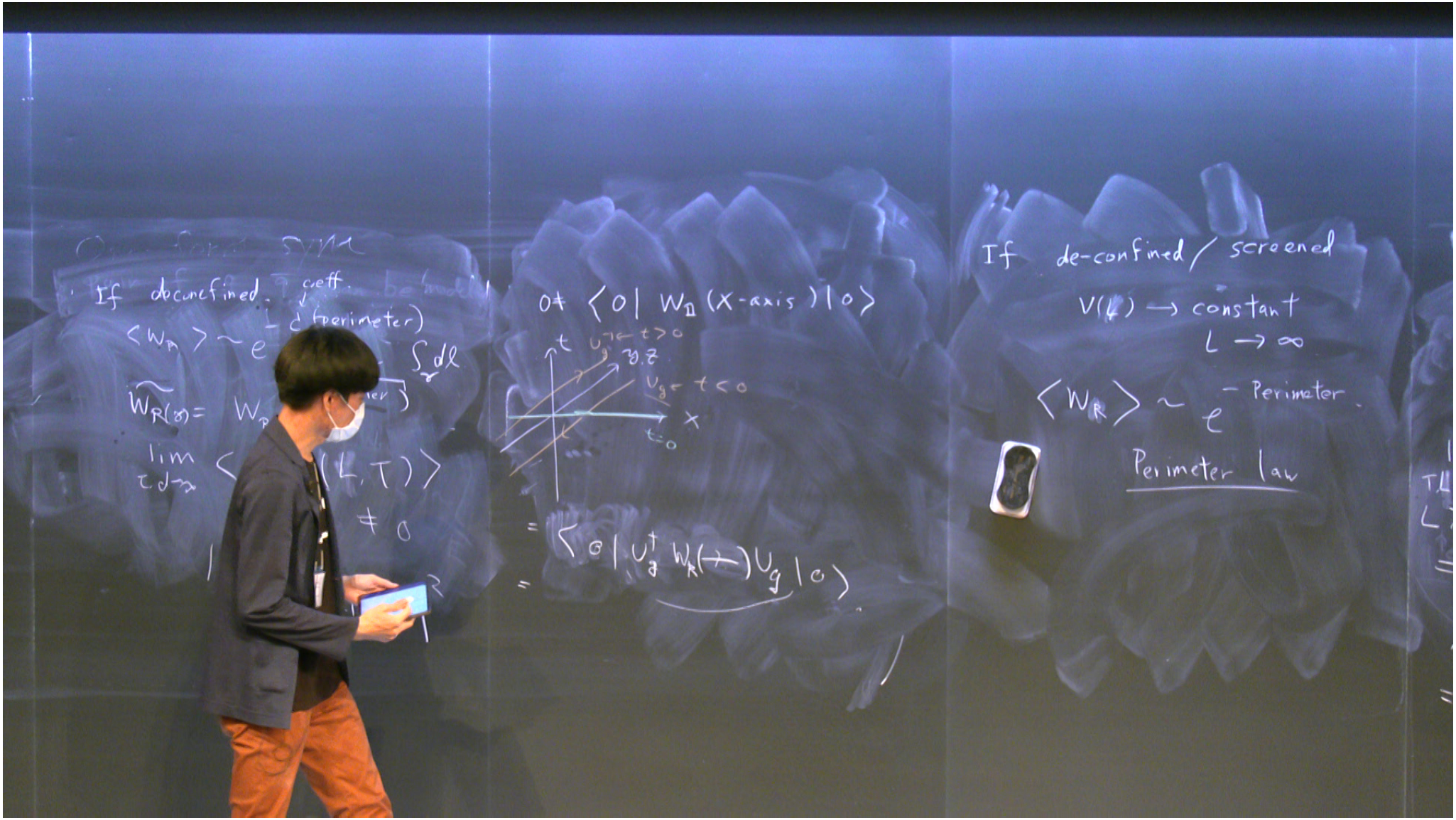
If de-confined / screened

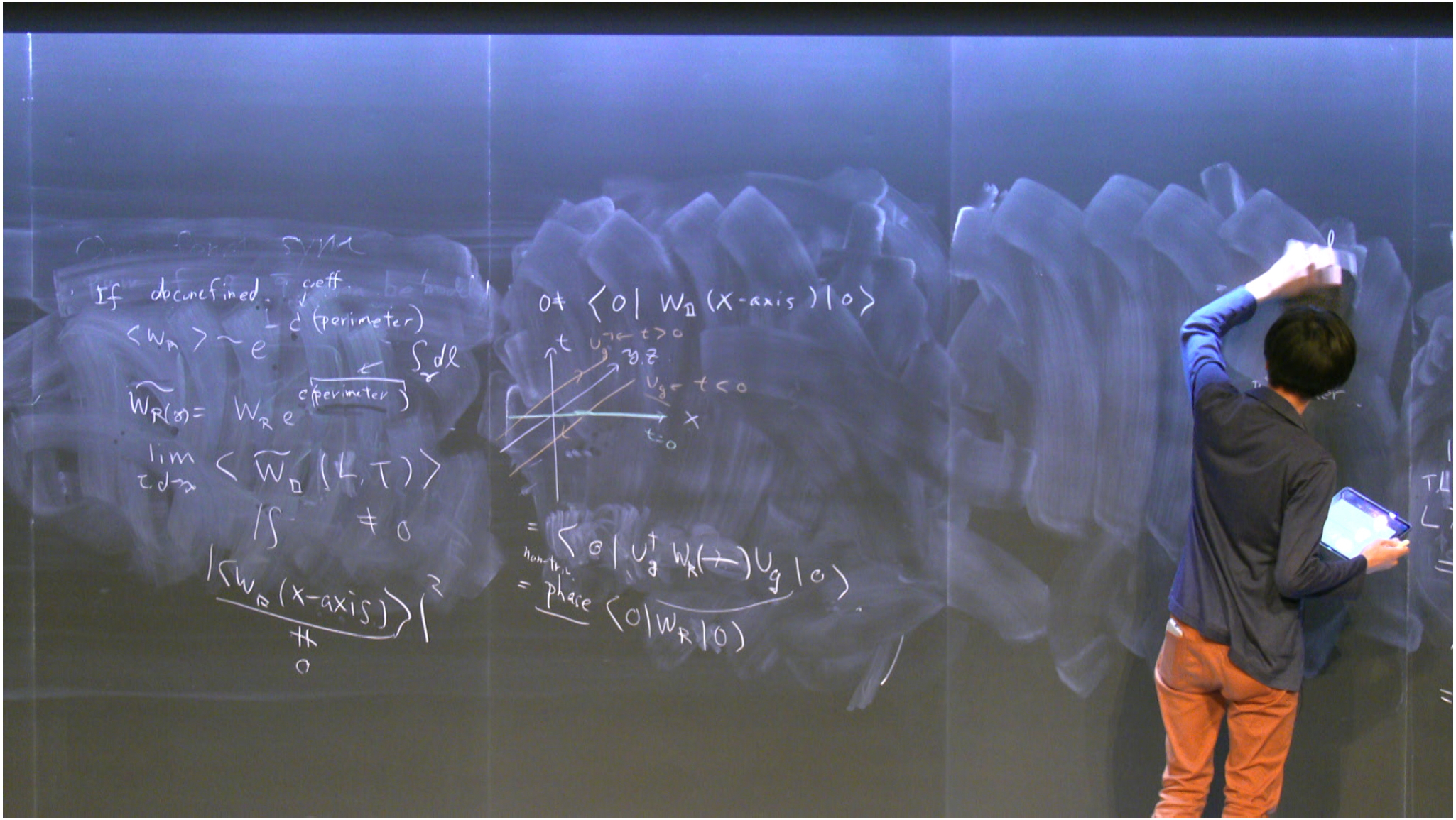
$V(L) \to$ constant

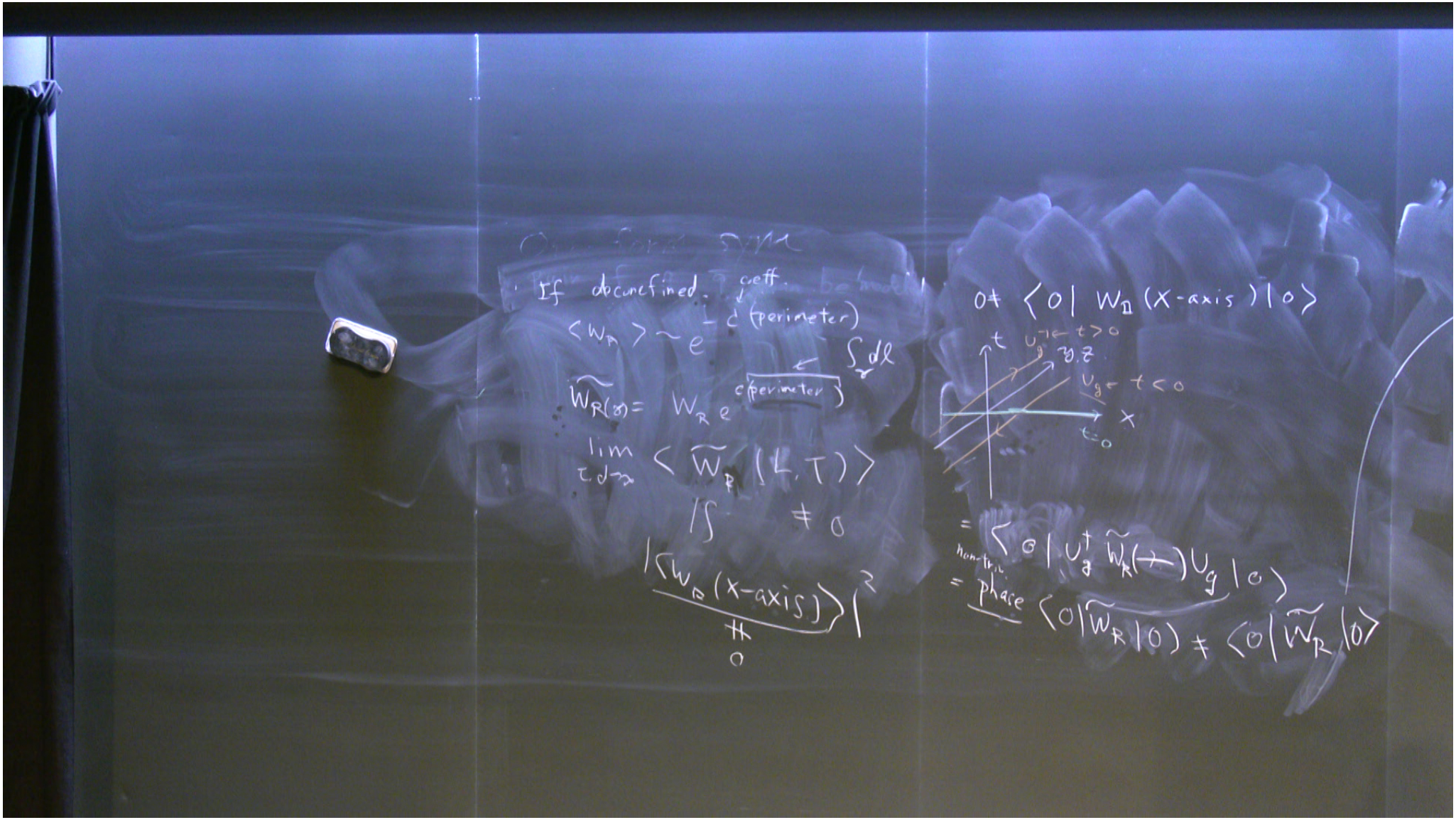
$L \to \infty$

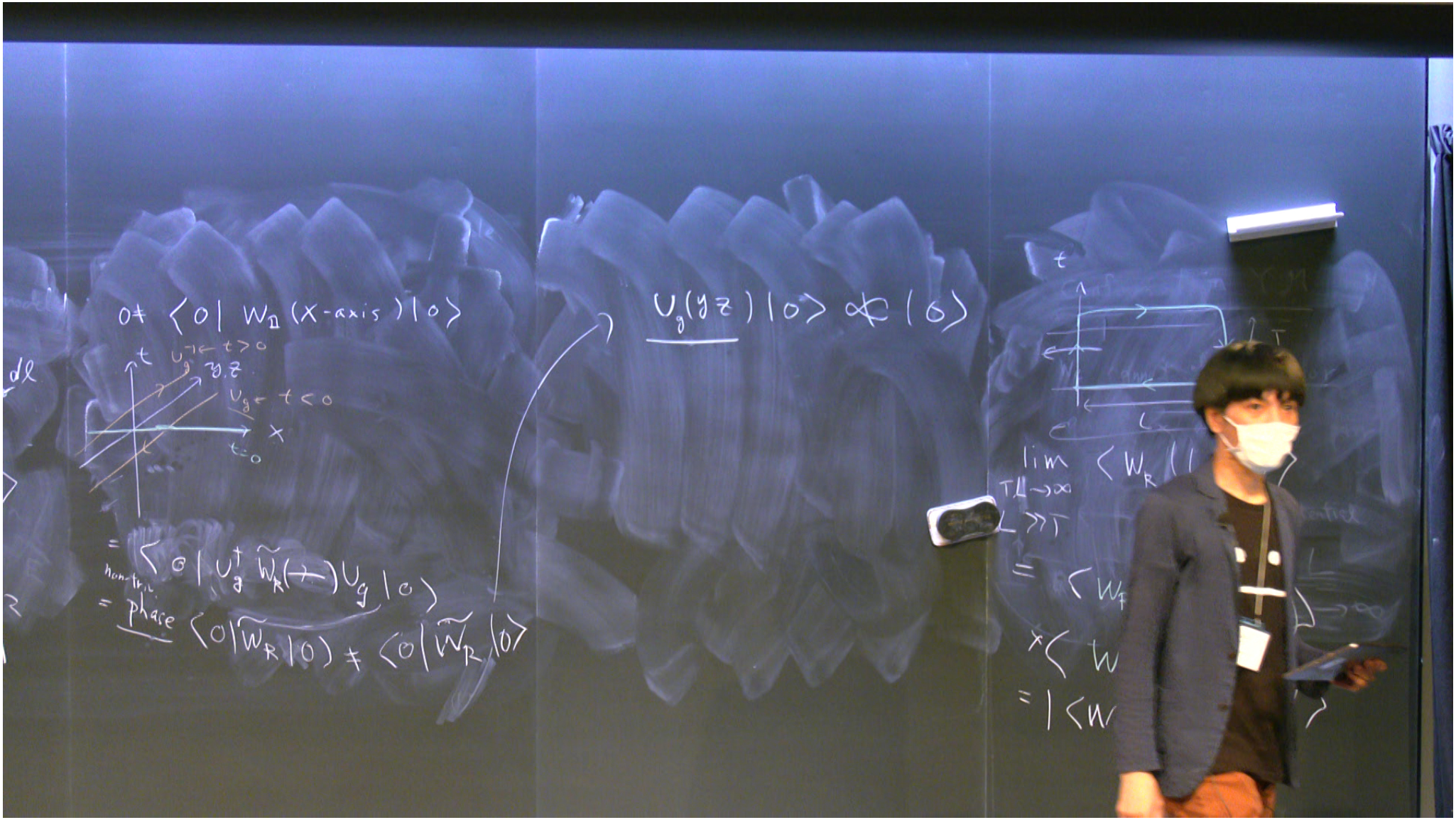
$\langle W_R \rangle \sim e^{-\text{Perimeter}}$

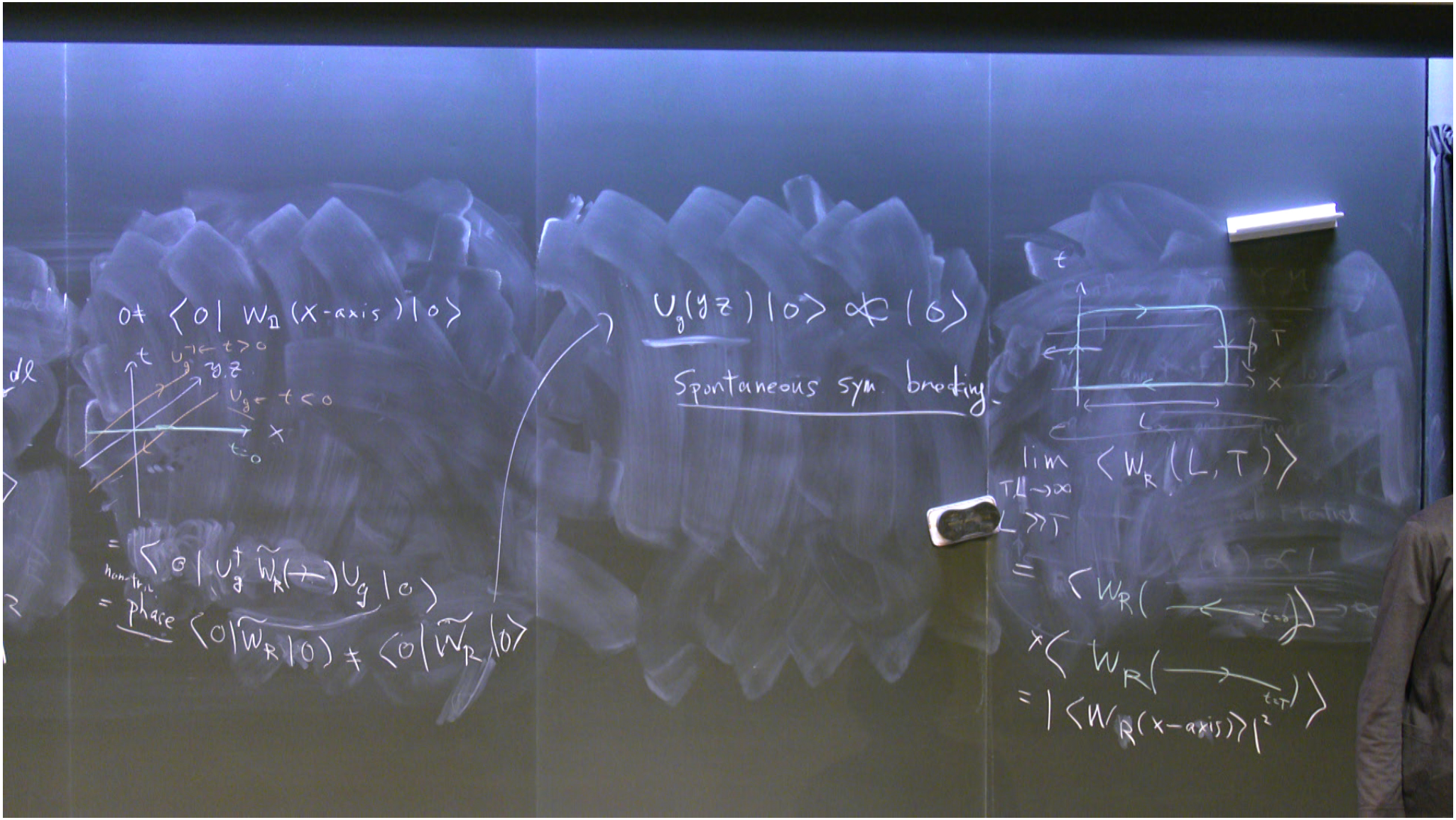
Perimeter law


If deconfined
coeff.
d (perimeter)
$\langle W_R \rangle \sim e$
$\int dl$
$\widetilde{W_R(\sigma)} = W_R$
$\lim_{T \to \infty} \langle (L, T) \rangle$
$\neq 0$
Of $\langle 0 | W_{\square}$ (x-axis) $| 0 \rangle$
t
U_g t>0
y,z
U_g t<0
x
t=0
$= \langle 0 | U_g^\dagger W_R(-) U_g | 0 \rangle$
If de-confined / screened
$V(L) \to$ constant
$L \to \infty$
$\langle W_R \rangle \sim e^{-\text{Perimeter}}$
Perimeter law


If deconfined.
coeff.
(perimeter)
(perimeter)
x-axis
t
x
y,z
t=0


If deconfined.
coeff.
(perimeter)
perimeter
$\neq 0$
$\neq 0$
$t \to -t > 0$
$t < 0$
$t = 0$
x
t
y, z
$= \langle 0 | U_g^\dagger \widetilde{W}_R(-) U_g | 0 \rangle$
phase
$\langle 0 | \widetilde{W}_R | 0 \rangle \neq \langle 0 | \widetilde{W}_R | 0 \rangle$


$\neq \langle 0 | W_R(x\text{-axis}) | 0 \rangle$
$U_g(yz) | 0 \rangle \propto | 0 \rangle$
$= \langle 0 | U_g^\dagger \tilde{W}_R(t) U_g | 0 \rangle$
non-trivial
= phase $\langle 0 | \tilde{W}_R | 0 \rangle \neq \langle 0 | \tilde{W}_R | 0 \rangle$
$\lim_{T \to \infty}$
$L \gg T$


$0 \neq \langle 0 | W_R(x\text{-axis}) | 0 \rangle$
$U_g(yz)|0\rangle \propto |0\rangle$
Spontaneous sym. breaking
$= \langle 0 | U_g^\dagger \tilde{W}_R(-) U_g | 0 \rangle$
non-trivial
= phase
$\langle 0 | \tilde{W}_R | 0 \rangle \neq \langle 0 | \tilde{W}_R | 0 \rangle$
$\lim_{T,L\to\infty} \langle W_R(L,T) \rangle$
$L \gg T$
$= \langle W_R($
$\times \langle W_R($
$= |\langle W_R(x\text{-axis}) \rangle|^2$