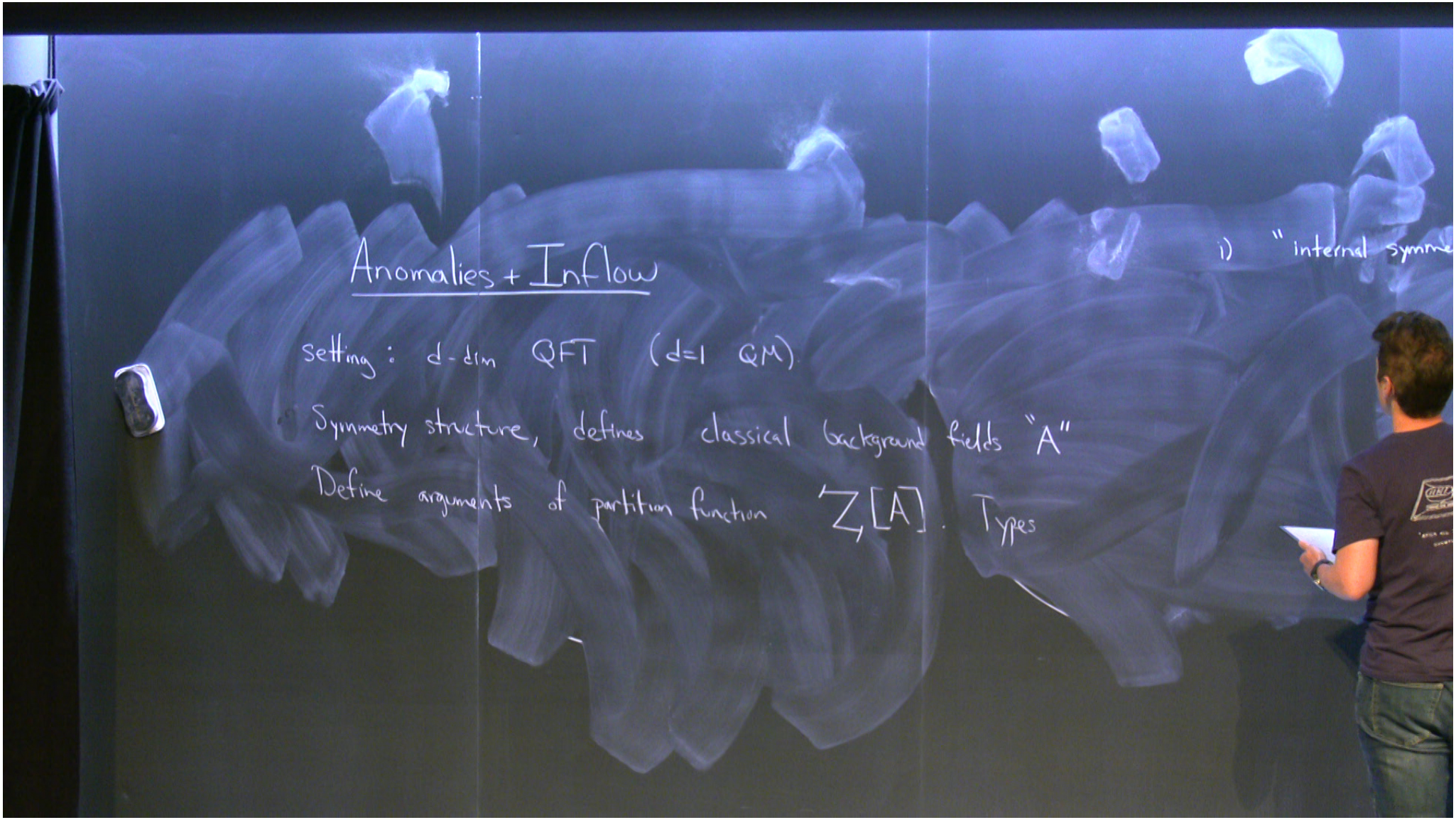
Title: Introduction to Anomalies in Quantum Field Theory

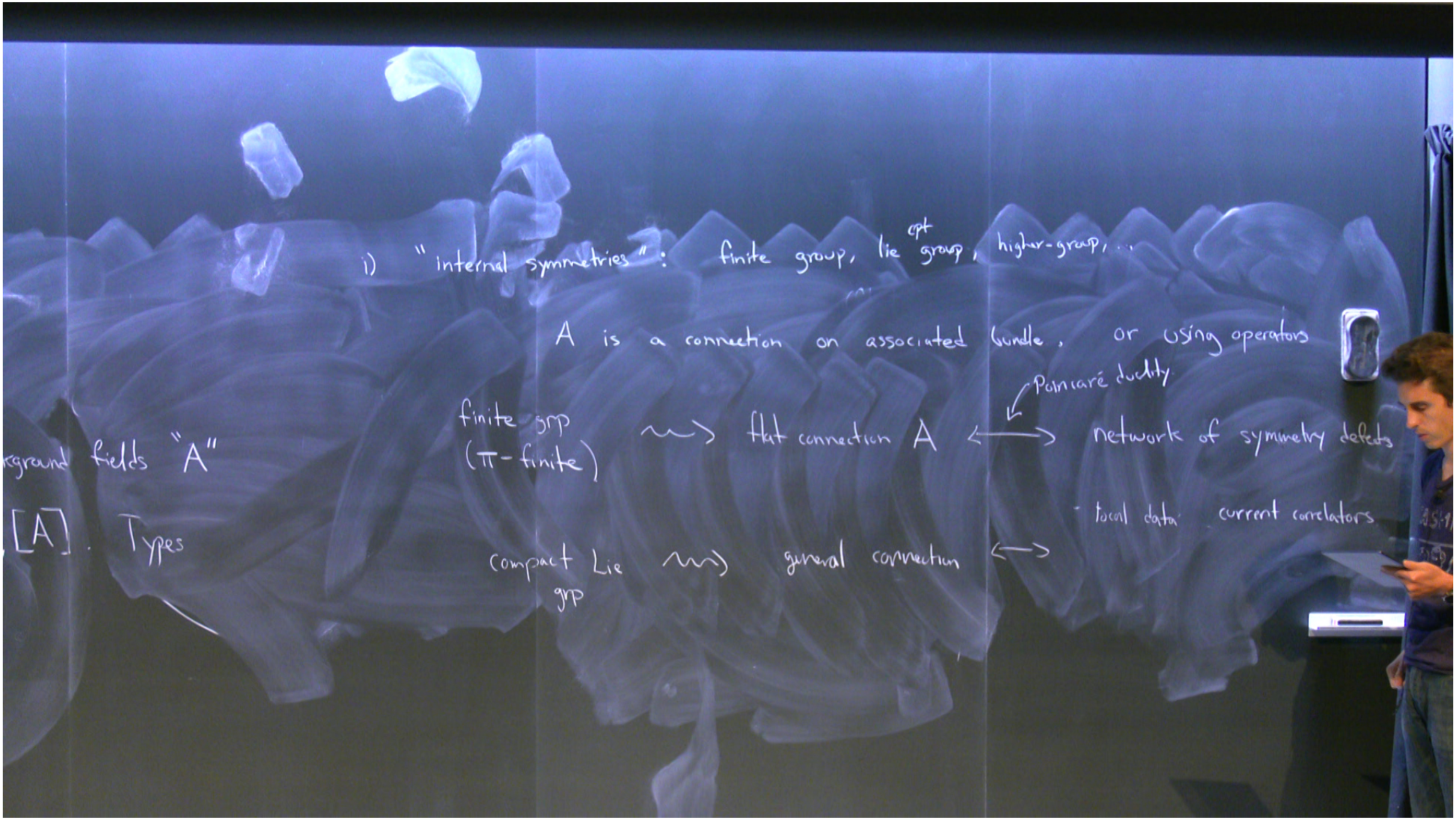
Speakers:

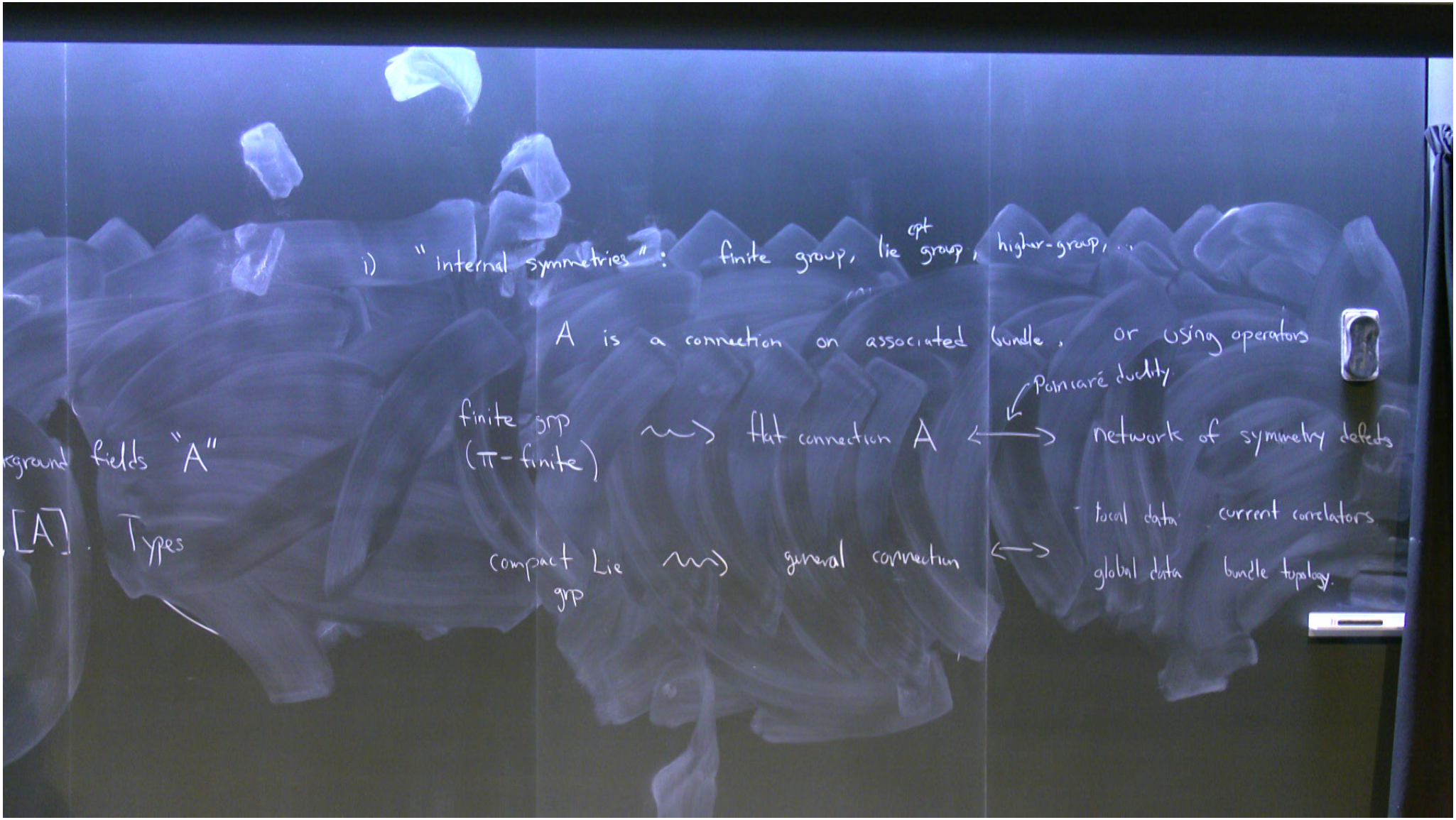
Collection: Global Categorical Symmetries

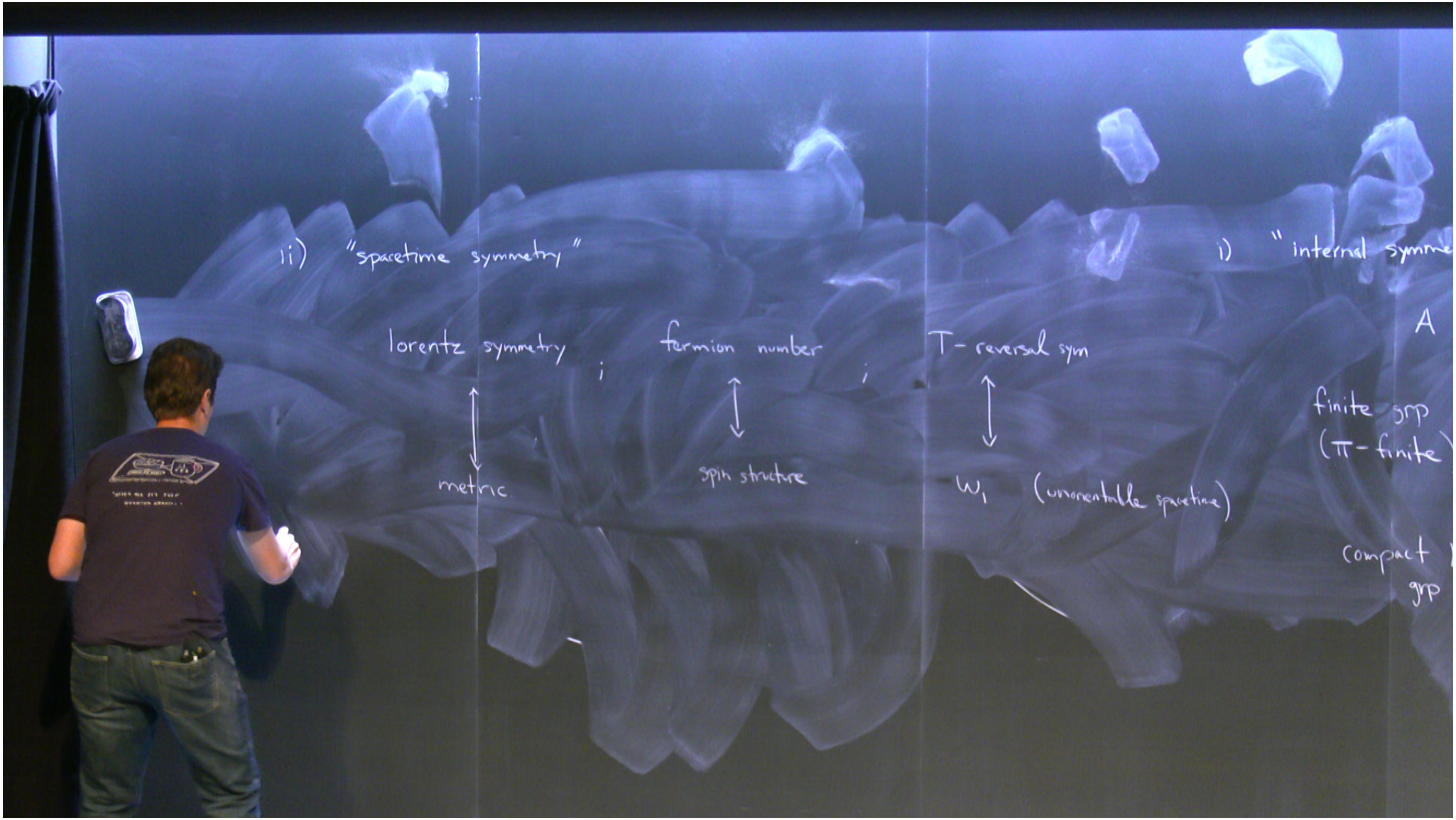
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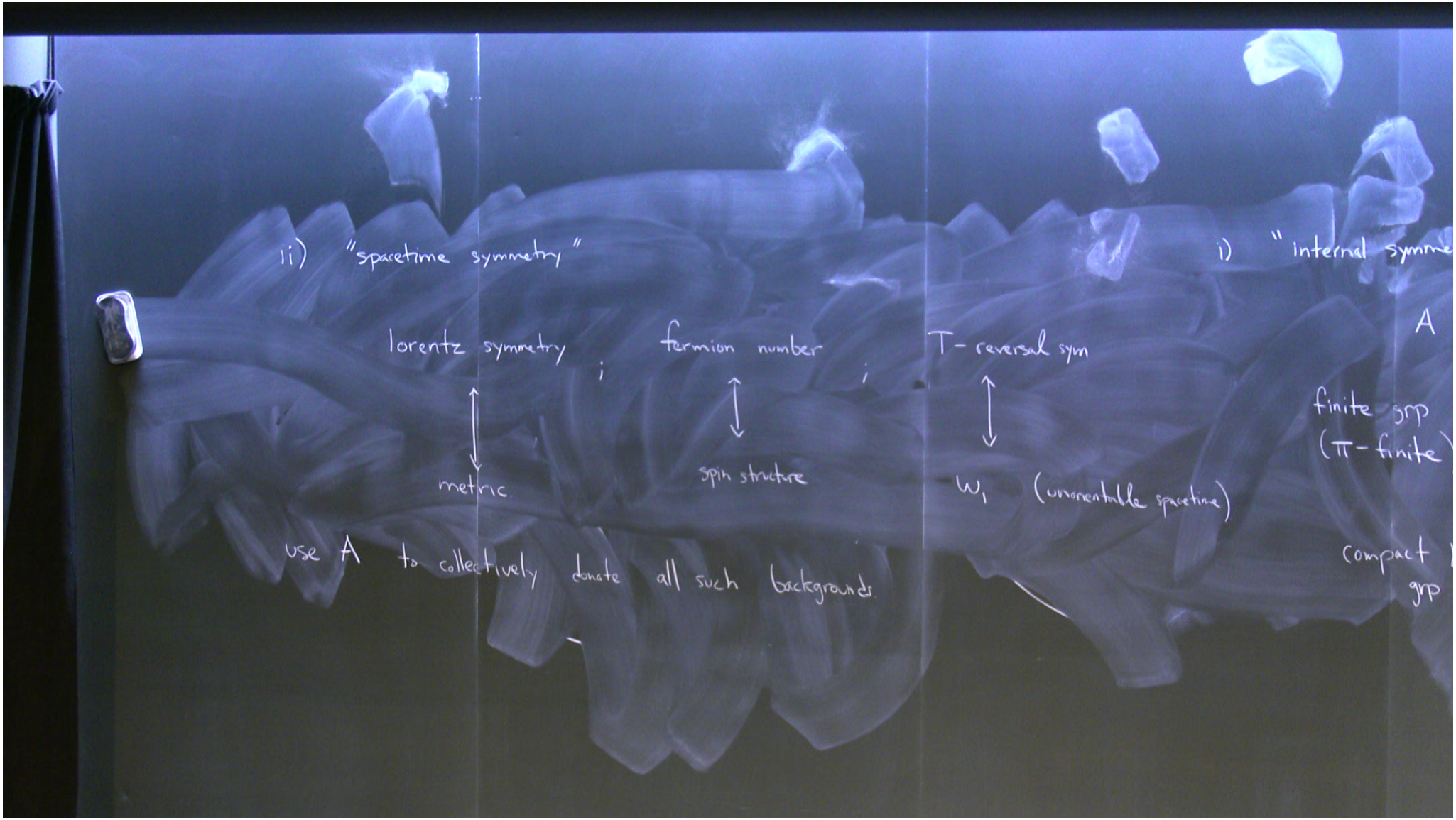
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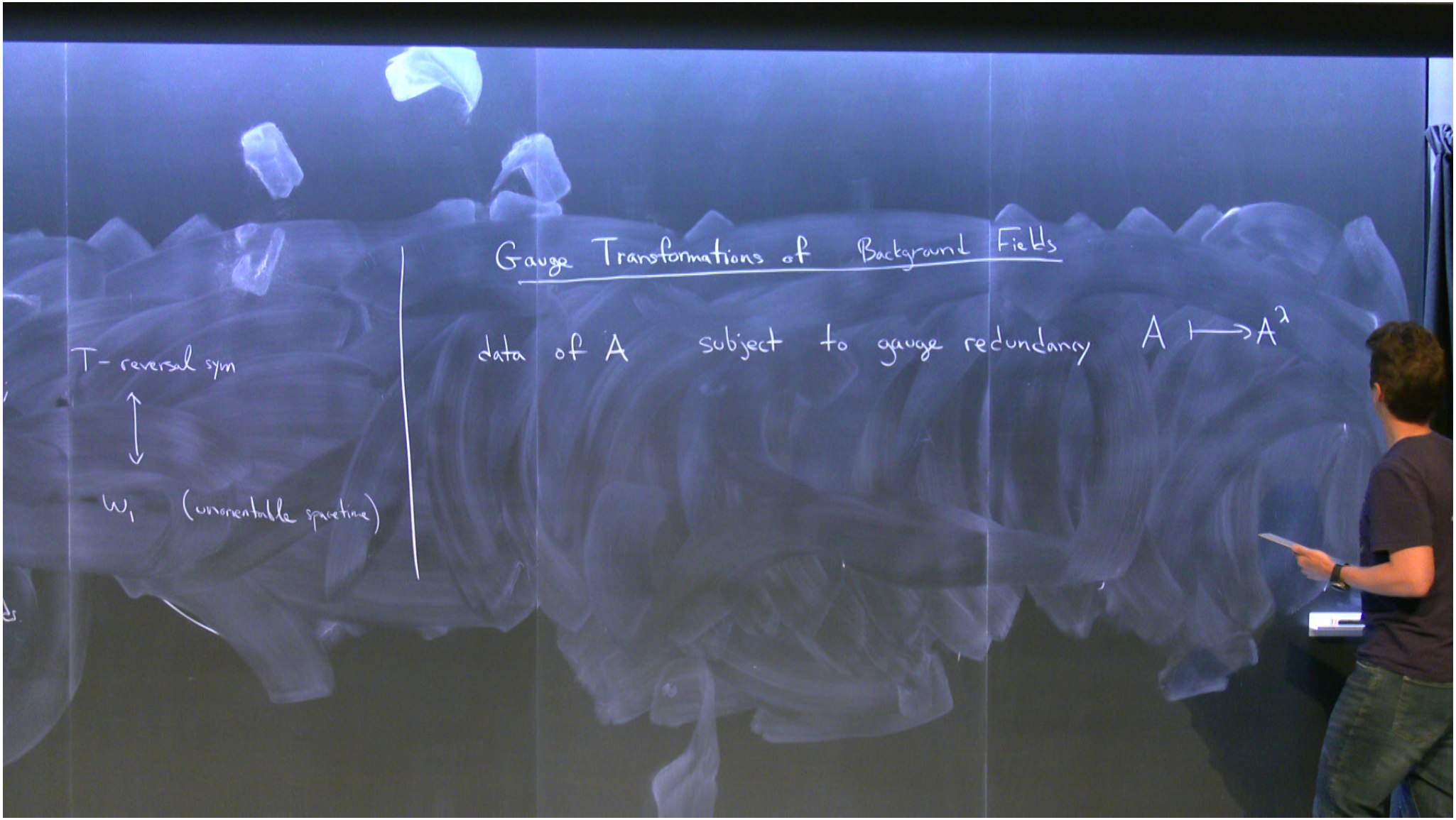

Anomalies + Inflow
setting: d-dim QFT (d=1 QM)
Symmetry structure, defines classical background fields "A"
Define arguments of partition function Z[A]. Types
i) "internal symme

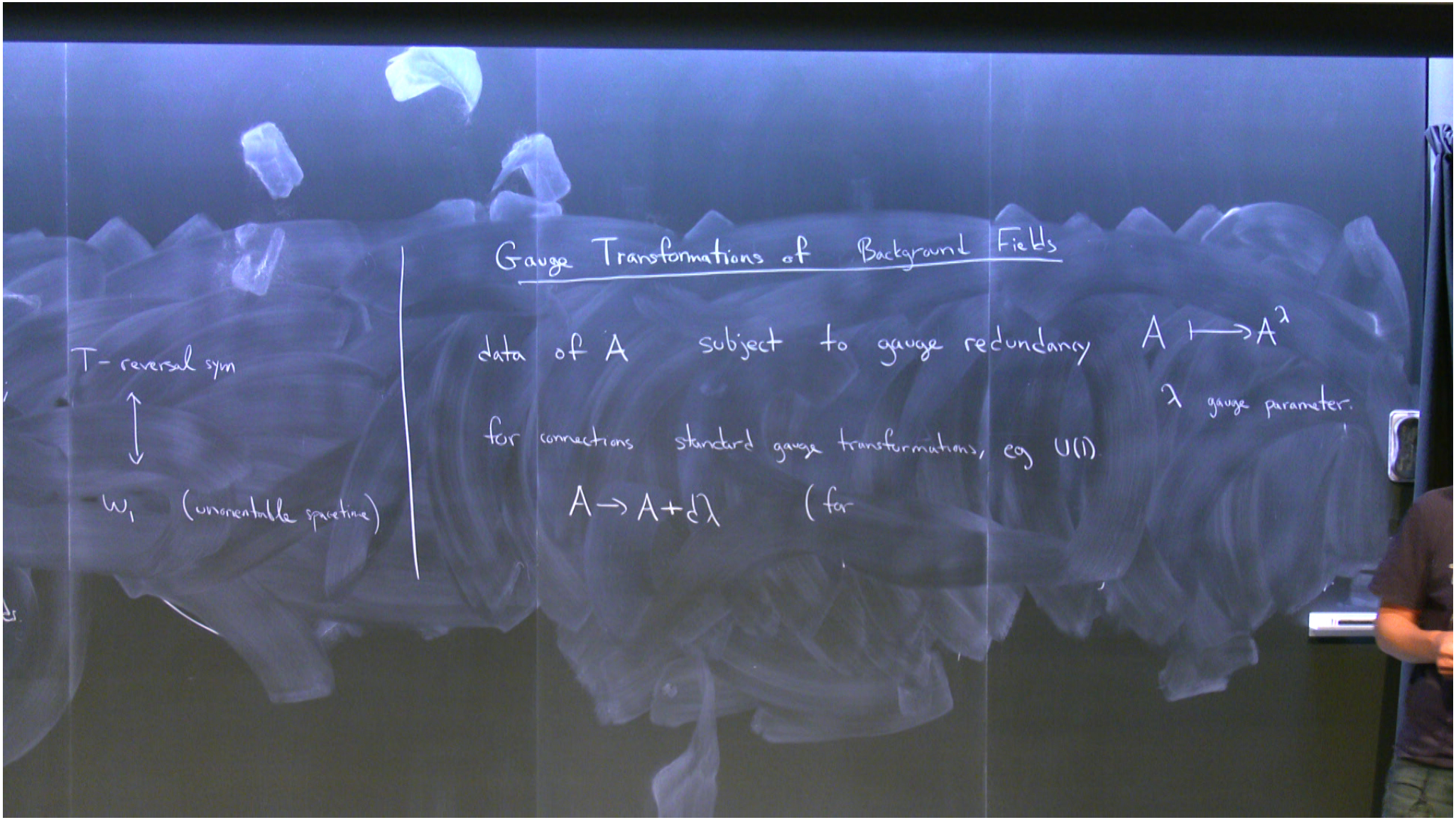

1) "internal symmetries": finite group, lie (cpt) group, higher-group, ...
A is a connection on associated bundle, or using operators
Poincaré duality
finite grp (π-finite) ⇝ flat connection A ⟷ network of symmetry defects
[A]. Types
compact Lie grp ⇝ general connection ⟷
current correlators

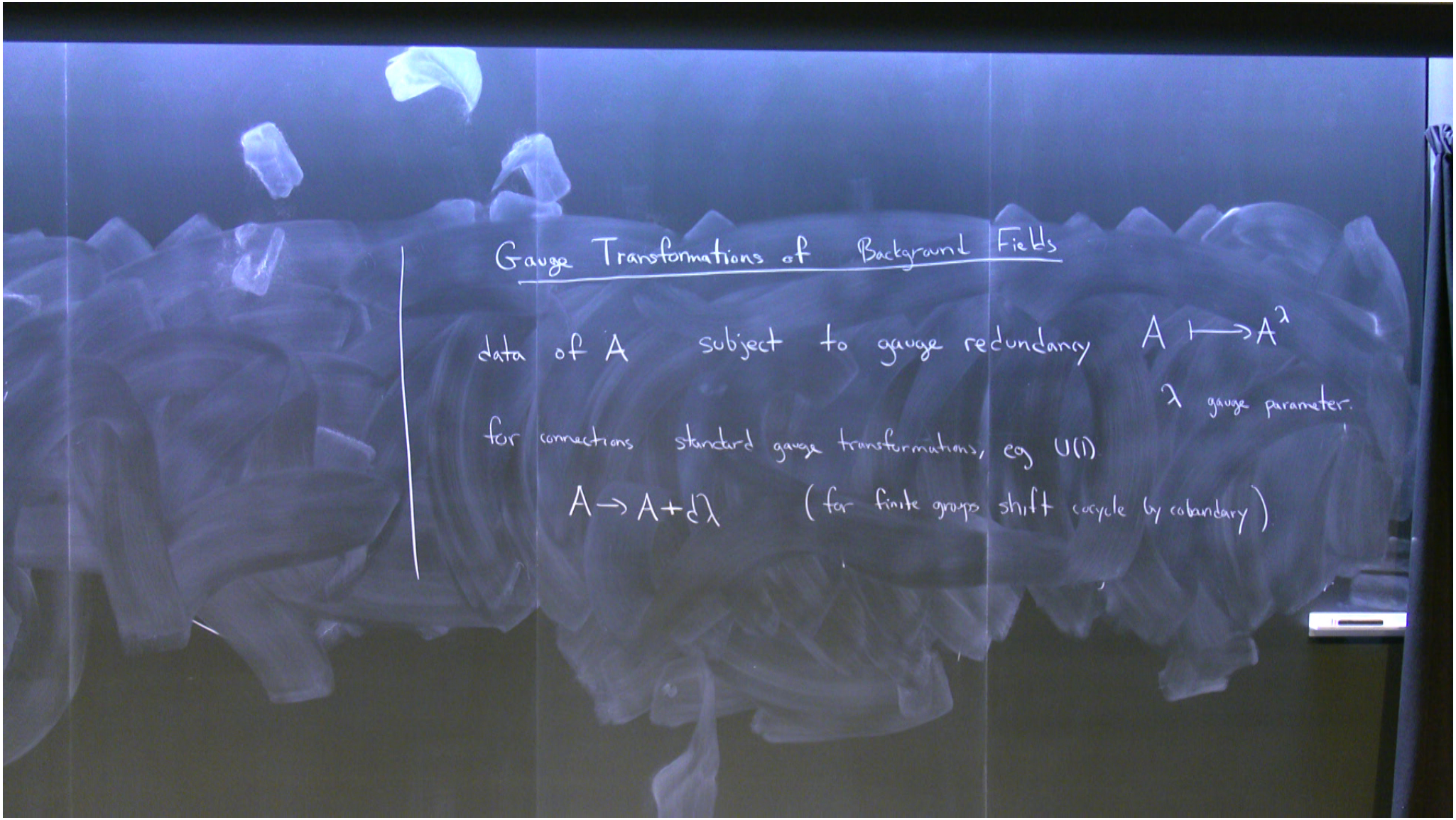

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[A]. Types
compact Lie grp ⇝ general connection ⟷
local data current correlators
global data bundle topology.

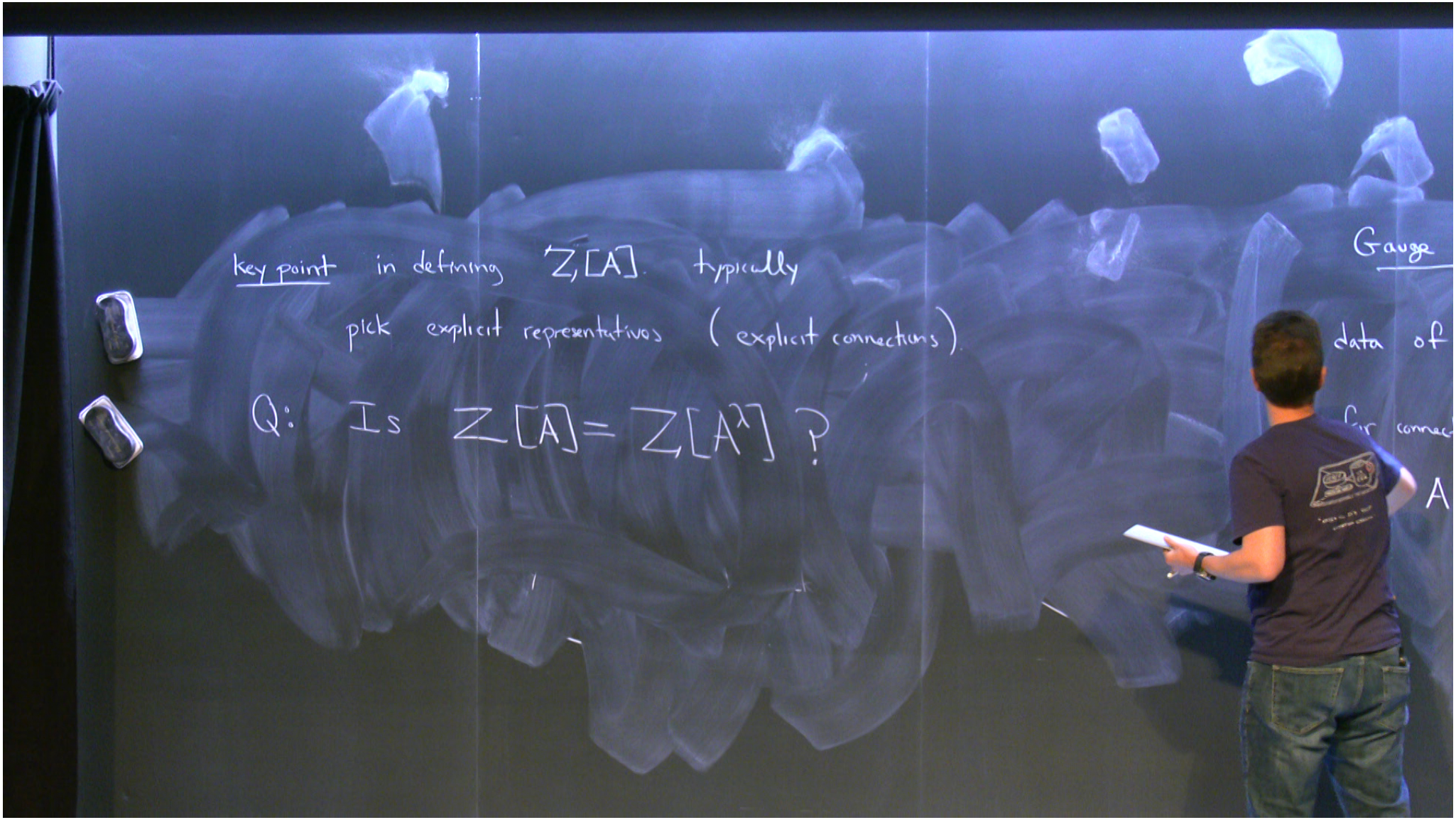

ii) "spacetime symmetry"
lorentz symmetry
metric
fermion number
spin structure
T-reversal sym
w_1 (unorientable spacetime)
i) "internal symme
A
finite grp
(π-finite
compact
grp


ii) "spacetime symmetry"
i) "internal symme
lorentz symmetry
fermion number
T-reversal sym
A
finite grp
(π-finite
metric
spin structure
w_1 (unorientable spacetime)
use A to collectively denote all such backgrounds
compact
grp


Gauge Transformations of Background Fields
data of A subject to gauge redundancy $A \longmapsto A^{\lambda}$
T-reversal sym
w_1 (unorientable spacetime)


T - reversal sym
w_1 (unorientable spacetime)
Gauge Transformations of Background Fields
data of A subject to gauge redundancy $A \longmapsto A^\lambda$
λ gauge parameter.
for connections standard gauge transformations, eg U(1)
$A \to A + d\lambda$ (for

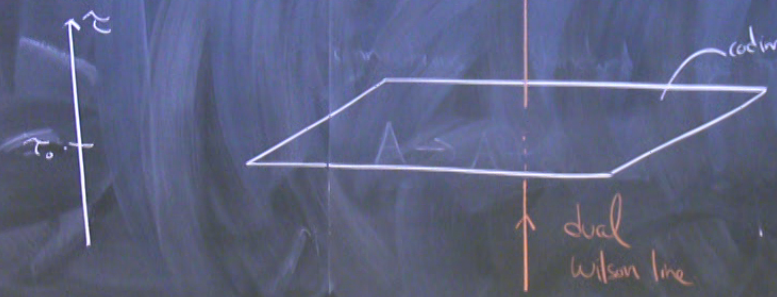

Gauge Transformations of Background Fields
data of A subject to gauge redundancy $A \longmapsto A^{\lambda}$
λ gauge parameter.
for connections standard gauge transformations, eg U(1)
$A \to A + d\lambda$ (for finite groups shift cocycle by coboundary)


key point in defining $\bar{Z}[A]$ typically
pick explicit representatives (explicit connections)
Q: Is $Z[A] = Z[A^\lambda]$?
Gauge
data of

gauge invariance of $Z[A]$ closely related to topological invariance of sym defects

nections)

eg symmetry group $U(1) \ni \exp(i\varphi)$ → codim 1 defect



associated background field

$$A = \varphi\, \delta(\tau - \tau_0)\, d\tau$$

$$\exp\left(i \oint A\right) = \exp(i\varphi)$$

defect at specific location $\tau = \tau_0$ ⟶ explicit choice of connection

gauge invariance of Z

move defect from τ_0 to τ_1 requires gauge trans.

$$\lambda = \varphi\left(\Theta(\tau-\tau_1) - \Theta(\tau-\tau_0)\right)$$

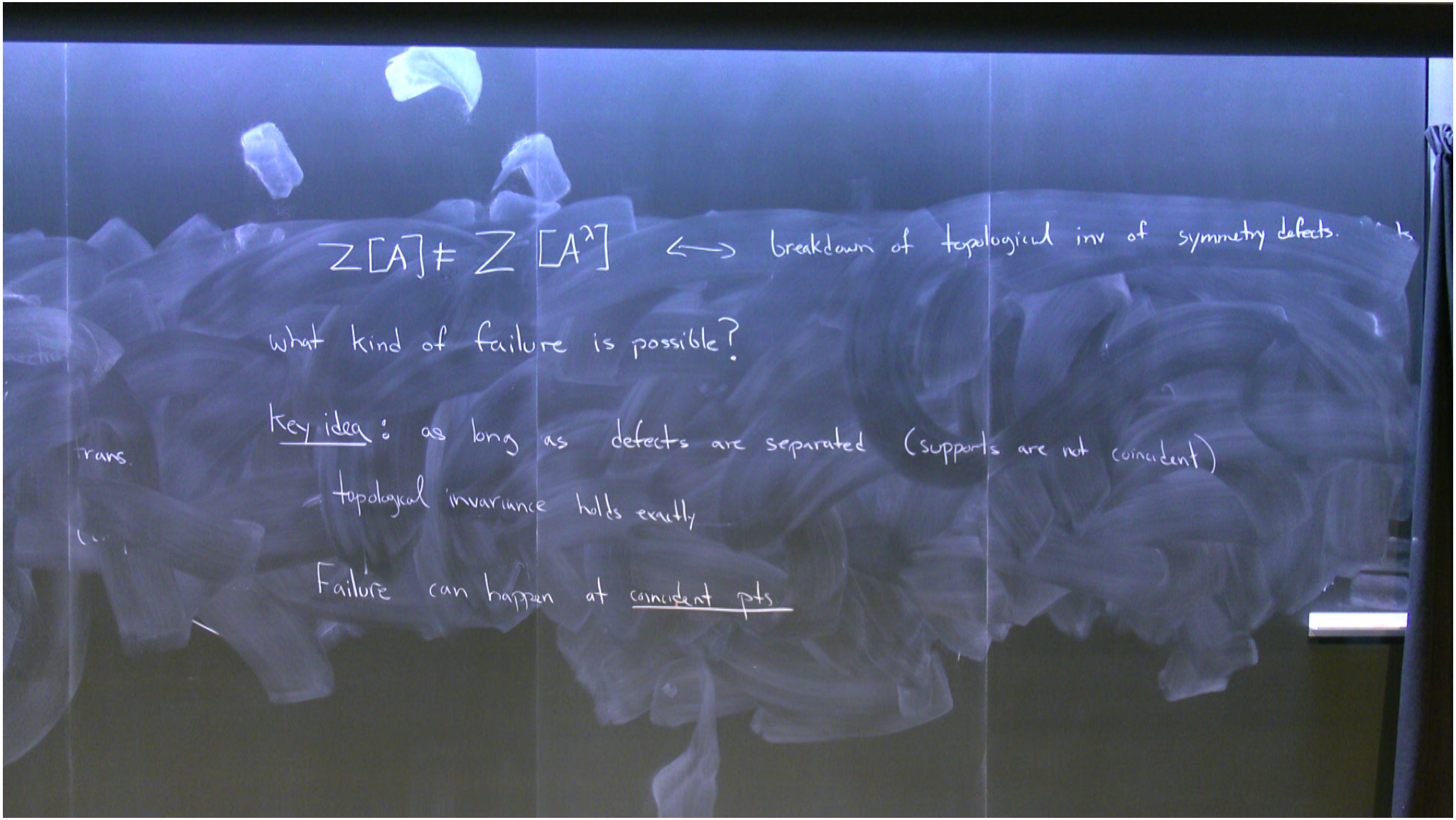
(compact support)

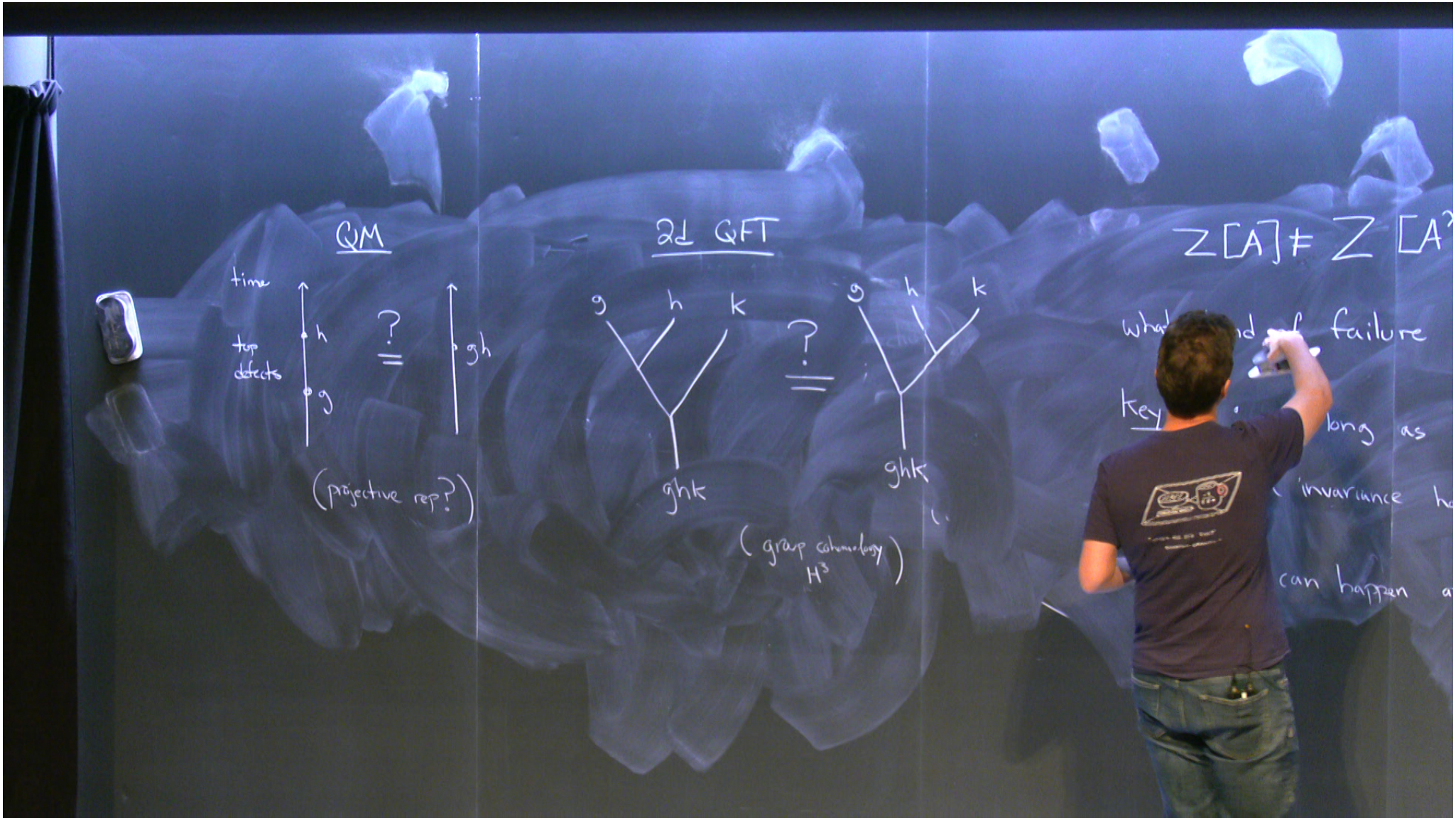
$\partial\Theta = \delta$

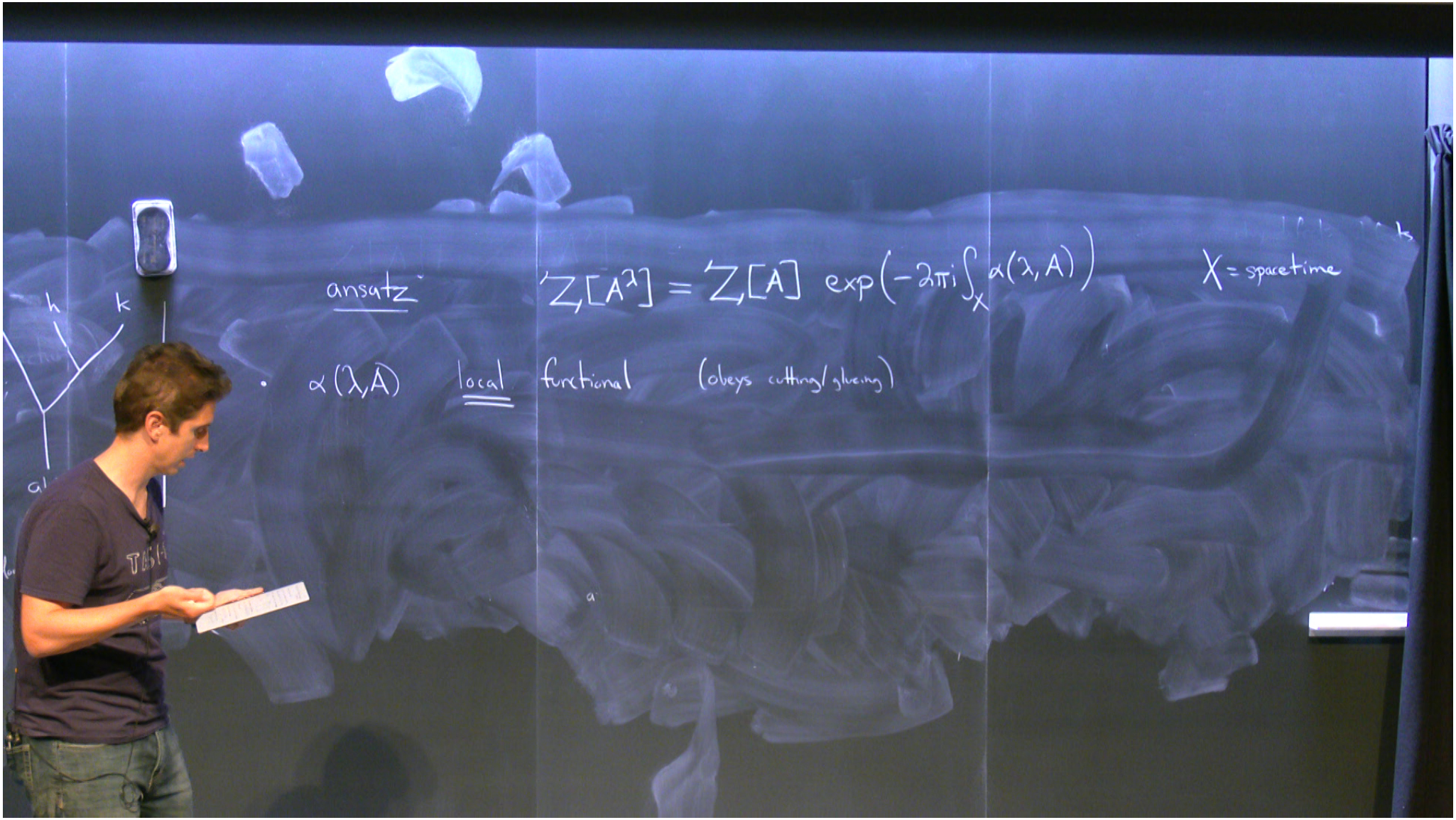
eg symmetry grou

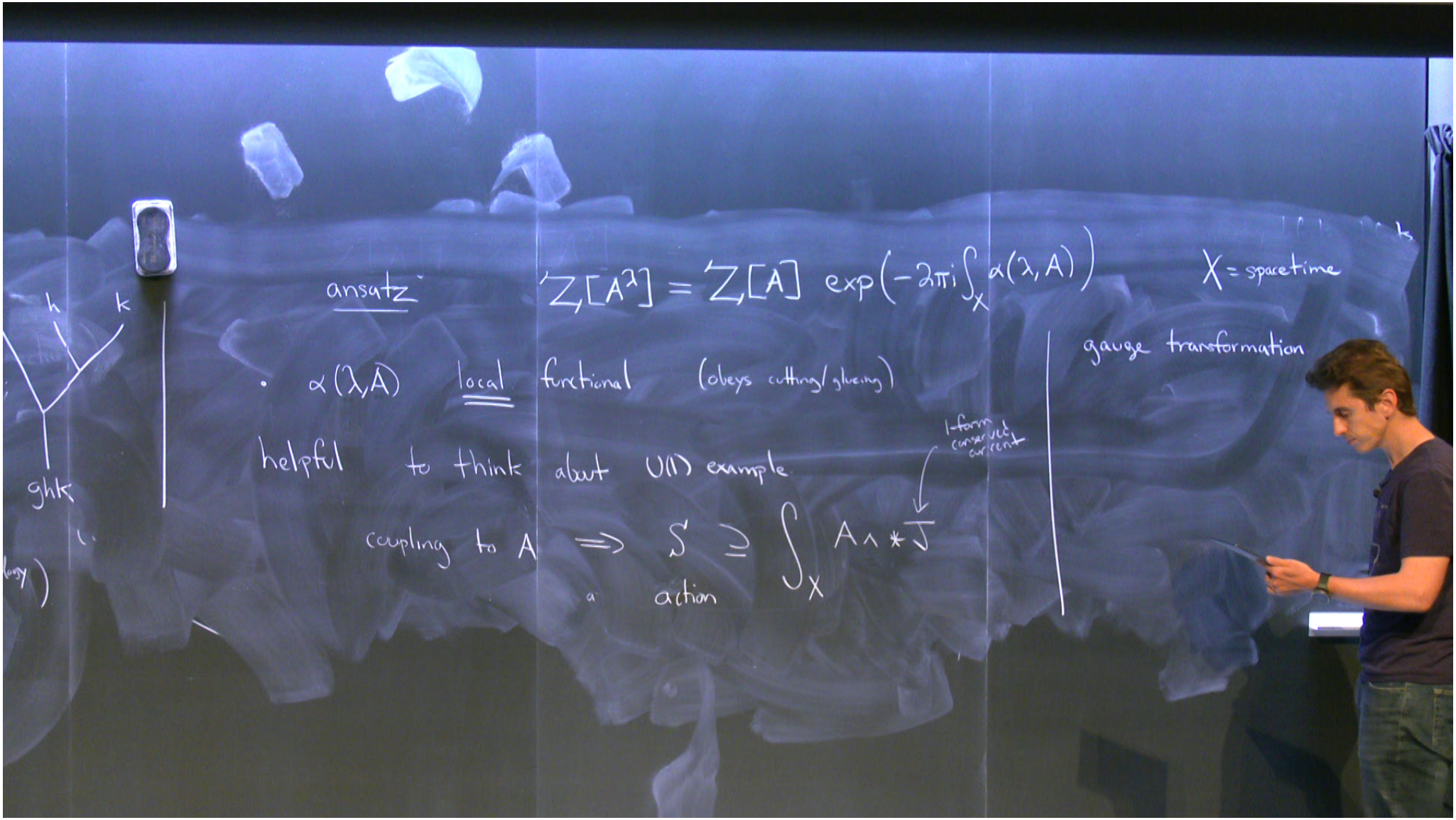


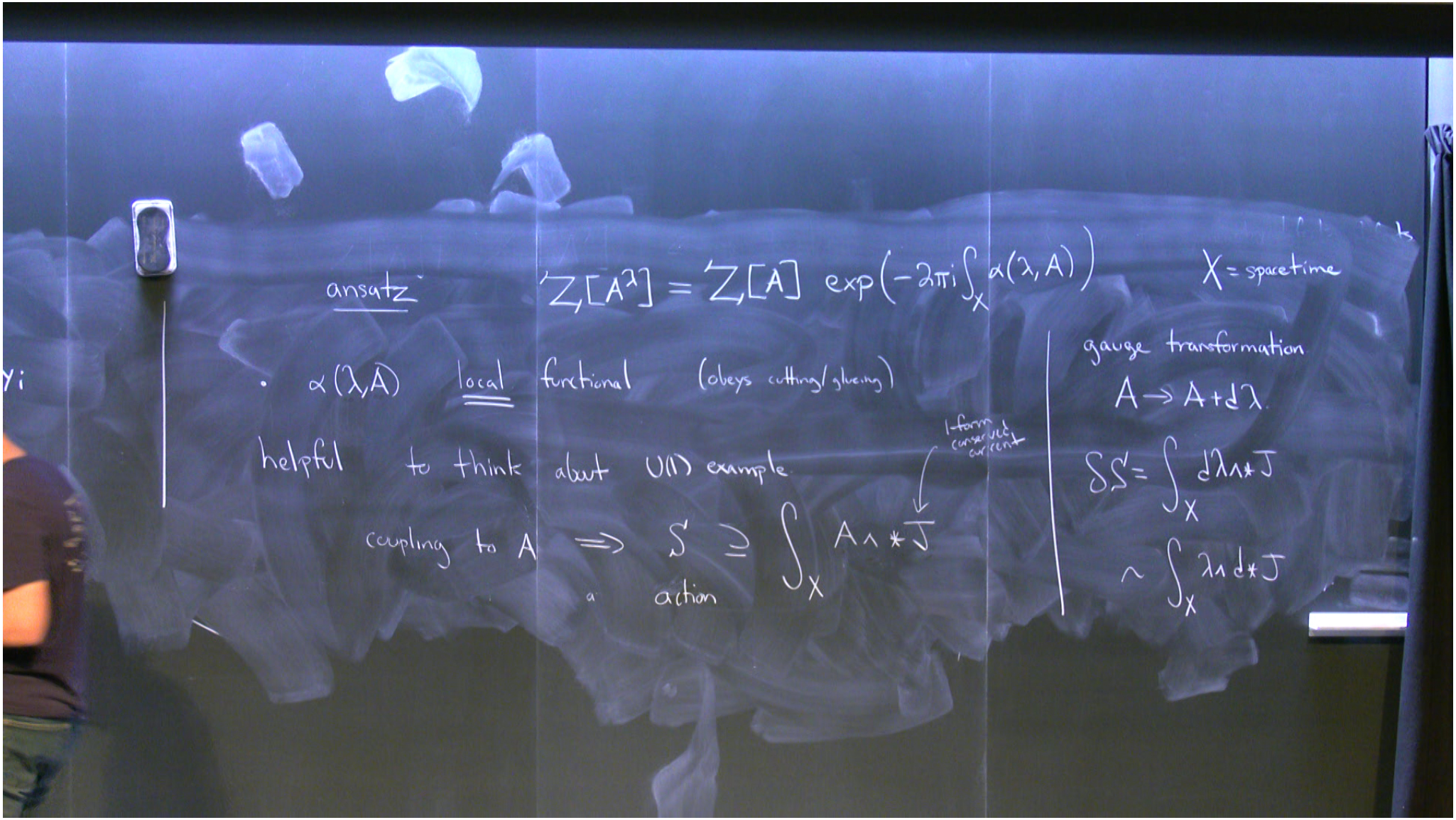


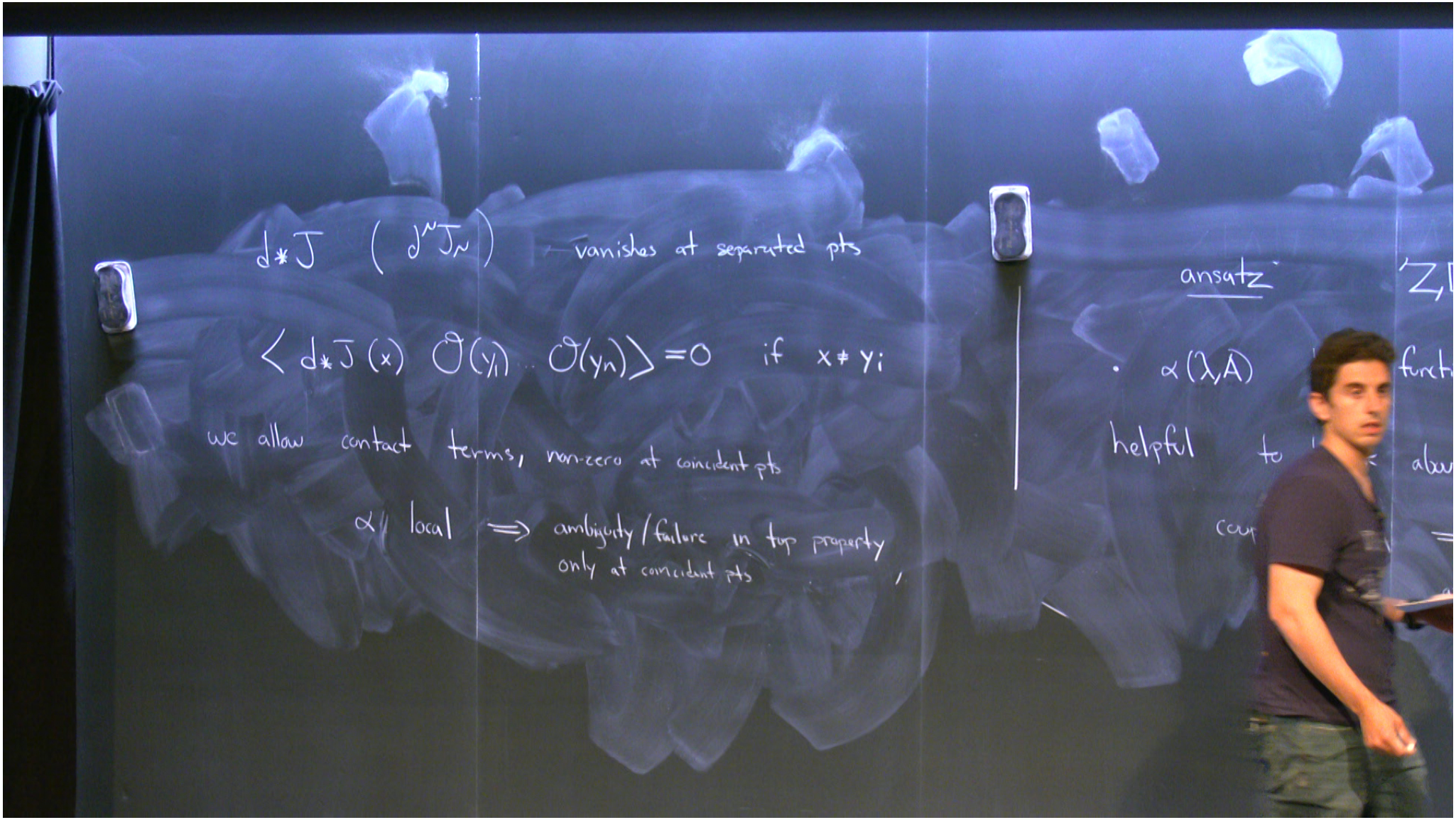

$Z[A] \neq Z[A^\lambda]$ ⟷ breakdown of topological inv of symmetry defects.
What kind of failure is possible?
Key idea: as long as defects are separated (supports are not coincident)
topological invariance holds exactly
Failure can happen at coincident pts

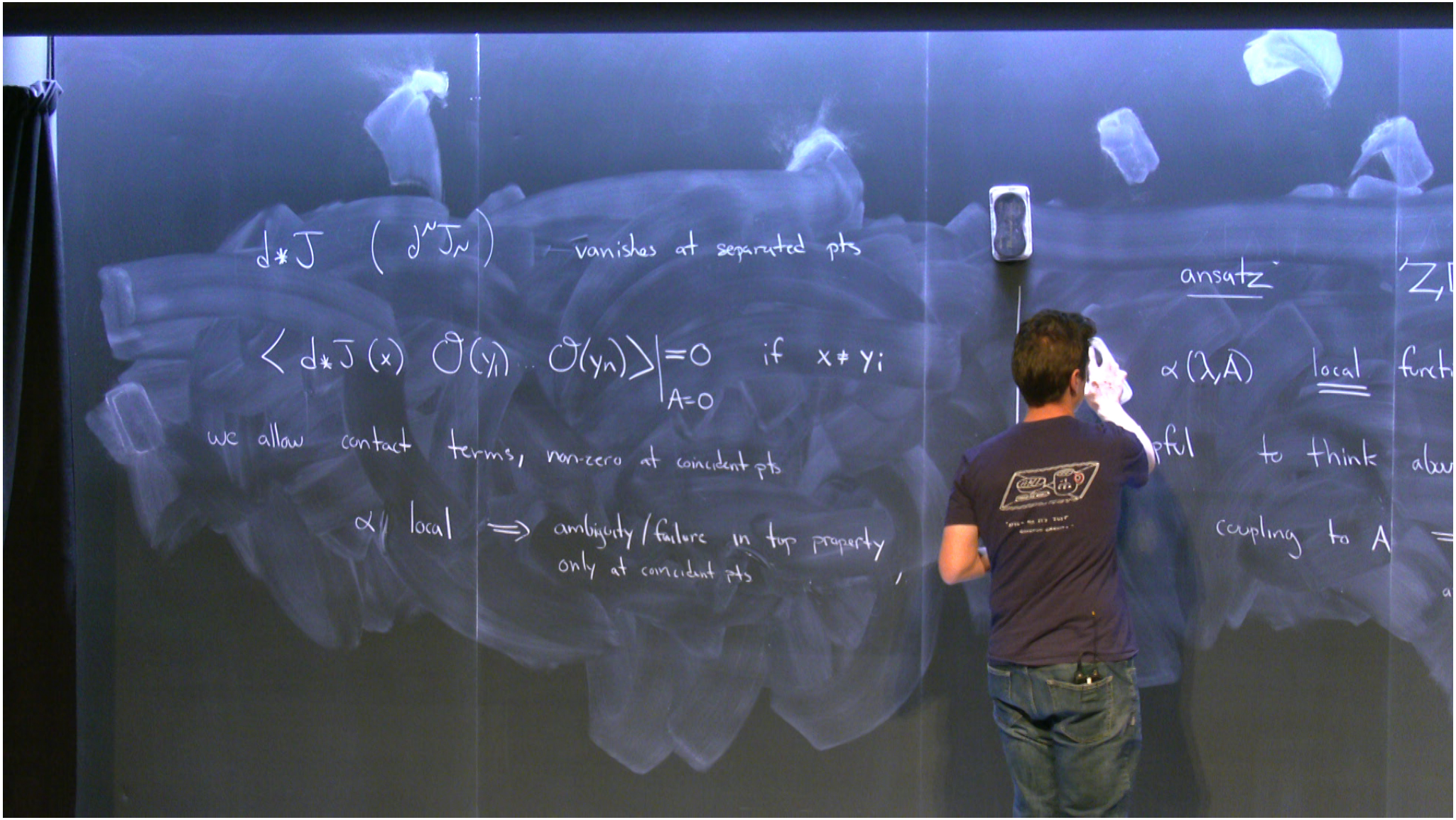

QM
time
top defects
h
g
?
gh
(projective rep?)
2d QFT
g
h
k
ghk
?
g
h
k
ghk
(group cohomology H^3)
Z[A] ≠ Z [A'
failure
key
long as
invariance
can happen

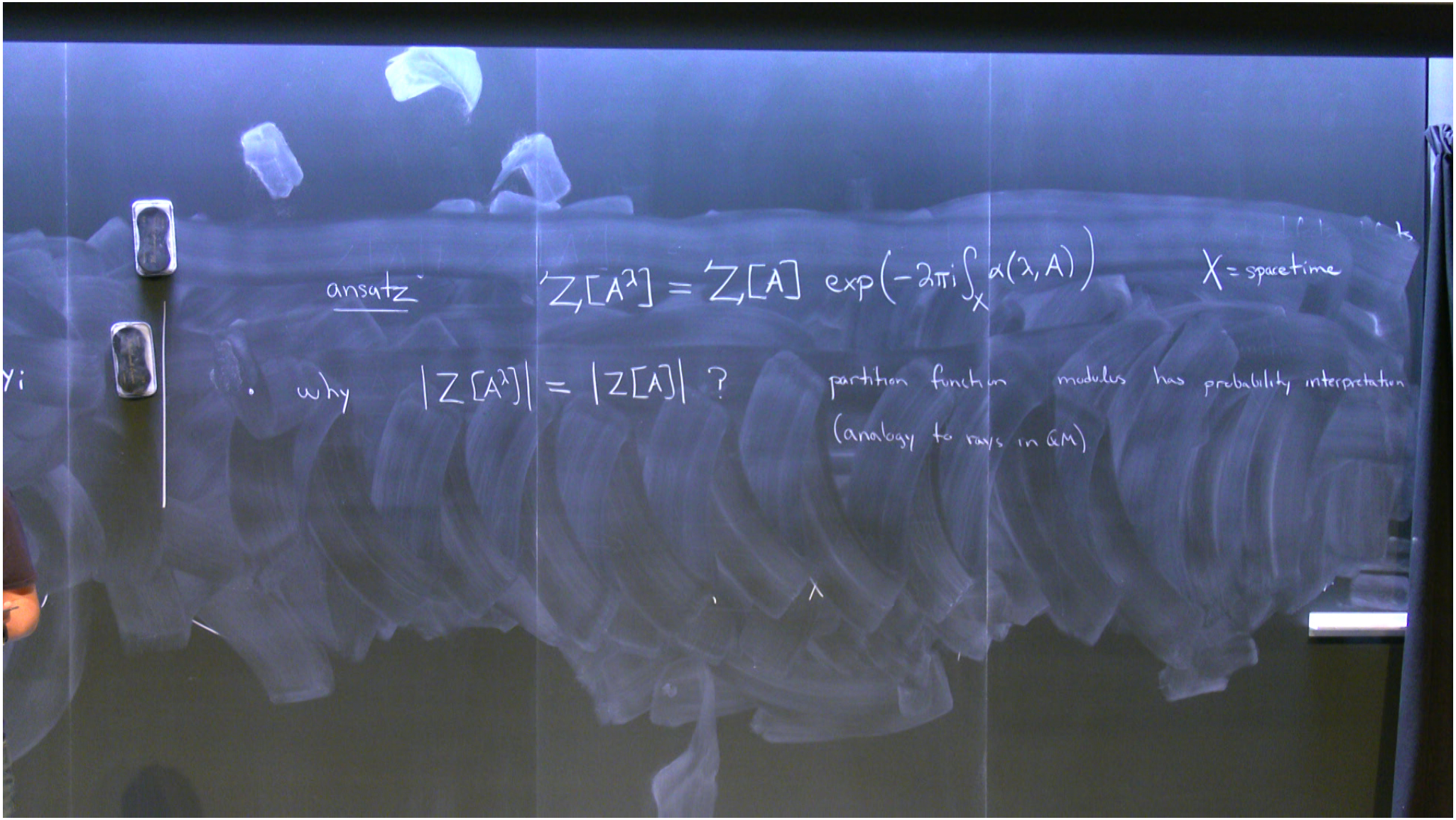

ansatz
$Z[A^\lambda] = Z[A] \exp\left(-2\pi i \int_X \alpha(\lambda, A)\right)$
X = spacetime
$\alpha(\lambda, A)$ local functional (obeys cutting/gluing)
h
k

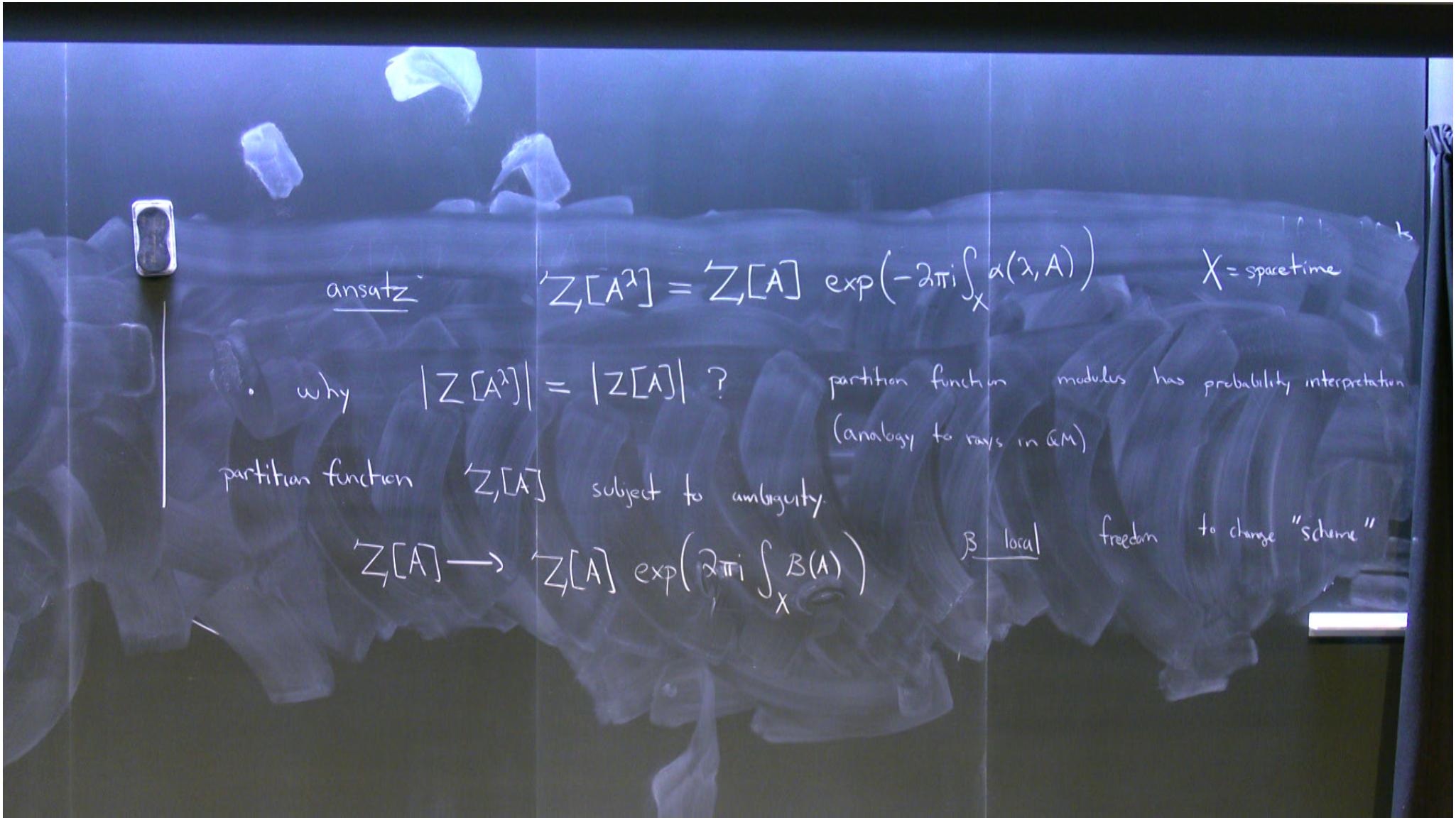

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ansatz
$Z[A^\lambda] = Z[A] \exp\left(-2\pi i \int_X \alpha(\lambda, A)\right)$
X = spacetime
gauge transformation
$\alpha(\lambda, A)$ local functional (obeys cutting/gluing)
helpful to think about U(1) example
1-form conserved current
coupling to A $\Rightarrow$ $S \supseteq \int_X A \wedge *J$
a action

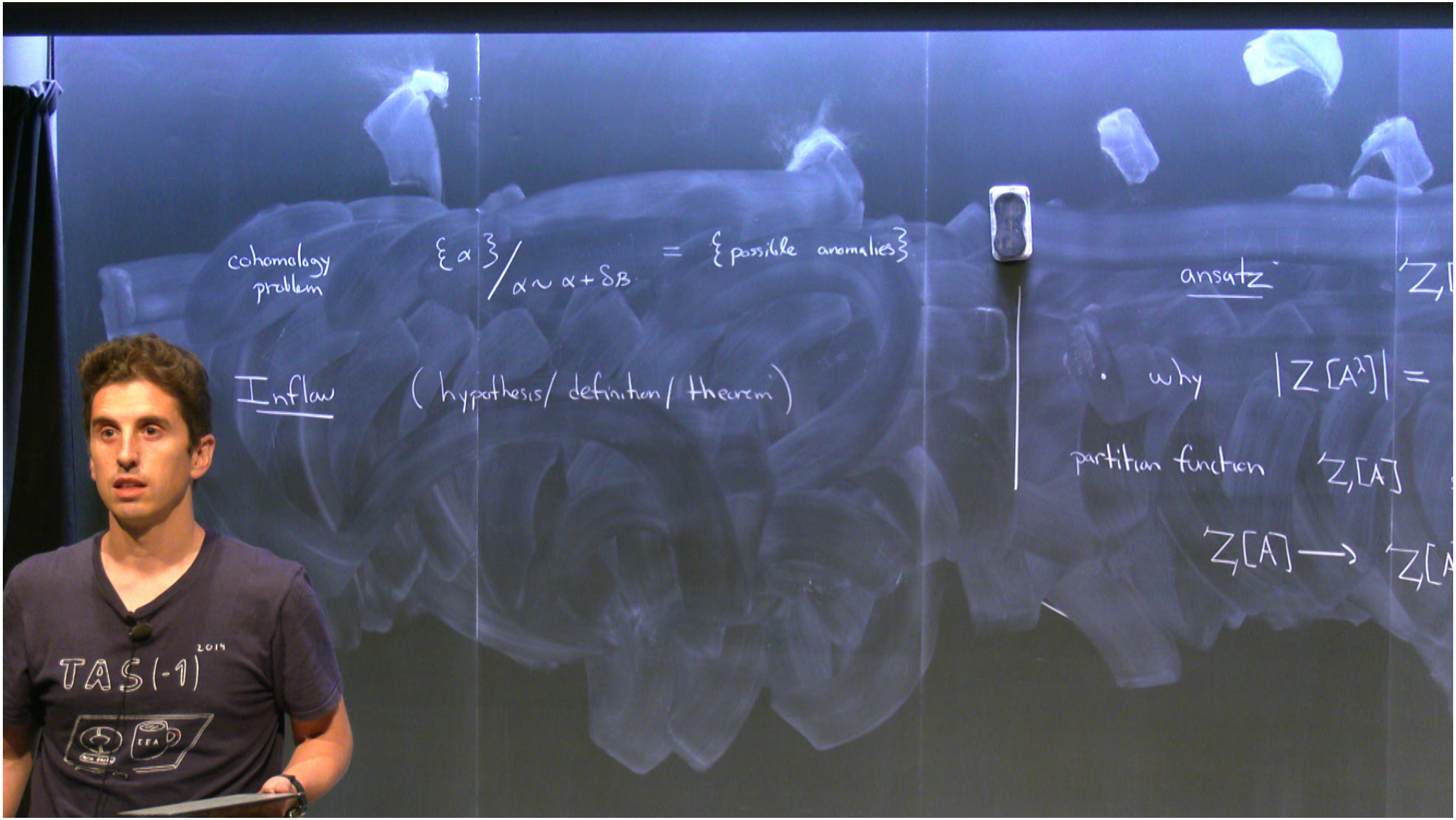

ansatz
$Z[A^\lambda] = Z[A] \exp\left(-2\pi i \int_X \alpha(\lambda, A)\right)$
X = spacetime
• $\alpha(\lambda, A)$ local functional (obeys cutting/gluing)
helpful to think about U(1) example
coupling to A $\Rightarrow$ $S' \supseteq \int_X A \wedge *J$
1-form conserved current
a action
gauge transformation
$A \to A + d\lambda$
$\delta S = \int_X d\lambda \wedge *J$
$\sim \int_X \lambda \wedge d*J$

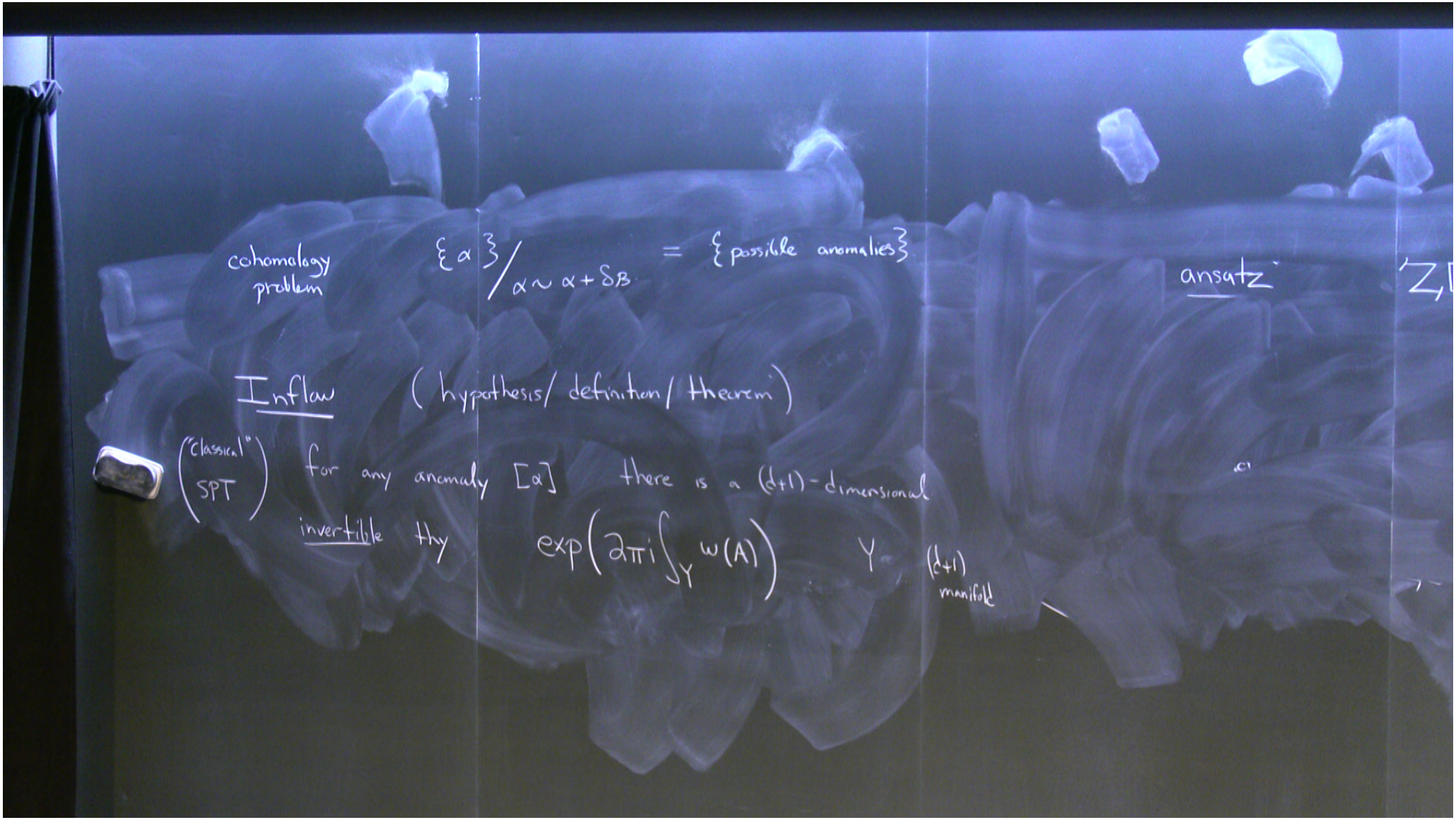

$d*J$ $(J^\mu J_\mu)$ — vanishes at separated pts
$\langle d*J(x)\ \mathcal{O}(y_1) \ldots \mathcal{O}(y_n)\rangle = 0$ if $x \neq y_i$
we allow contact terms, non-zero at coincident pts
α // local $\Rightarrow$ ambiguity / failure in top property only at coincident pts
ansatz
$\alpha(\lambda, A)$
helpful

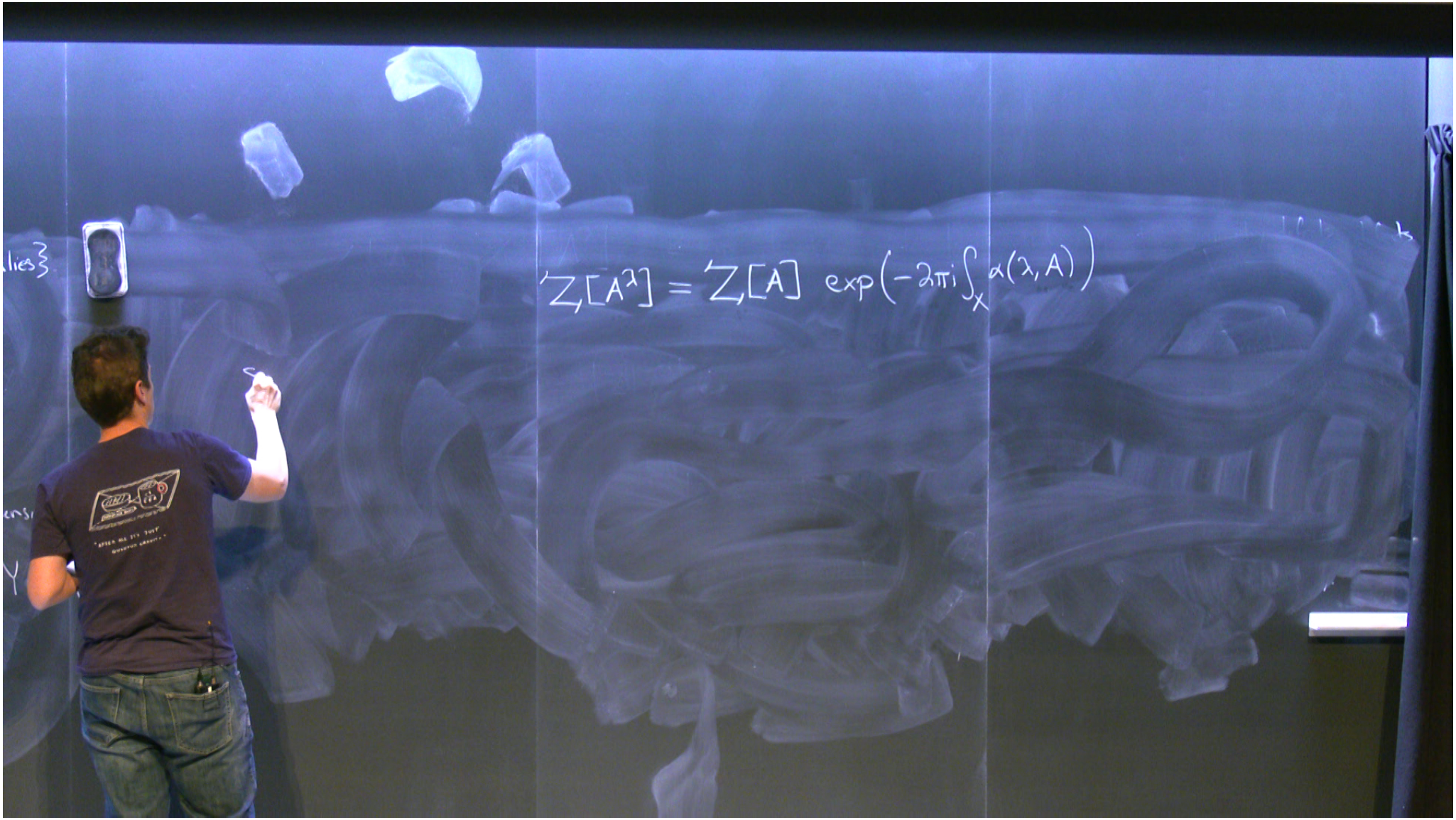

$d*J$ $(\partial^\mu J_\mu)$ — vanishes at separated pts
$\langle d*J(x)\ \mathcal{O}(y_1) \cdots \mathcal{O}(y_n)\rangle\big|_{A=0} = 0$ if $x \neq y_i$
we allow contact terms, non-zero at coincident pts
α local $\Rightarrow$ ambiguity / failure in top property only at coincident pts
ansatz
$\alpha(\lambda, A)$ local
coupling to A

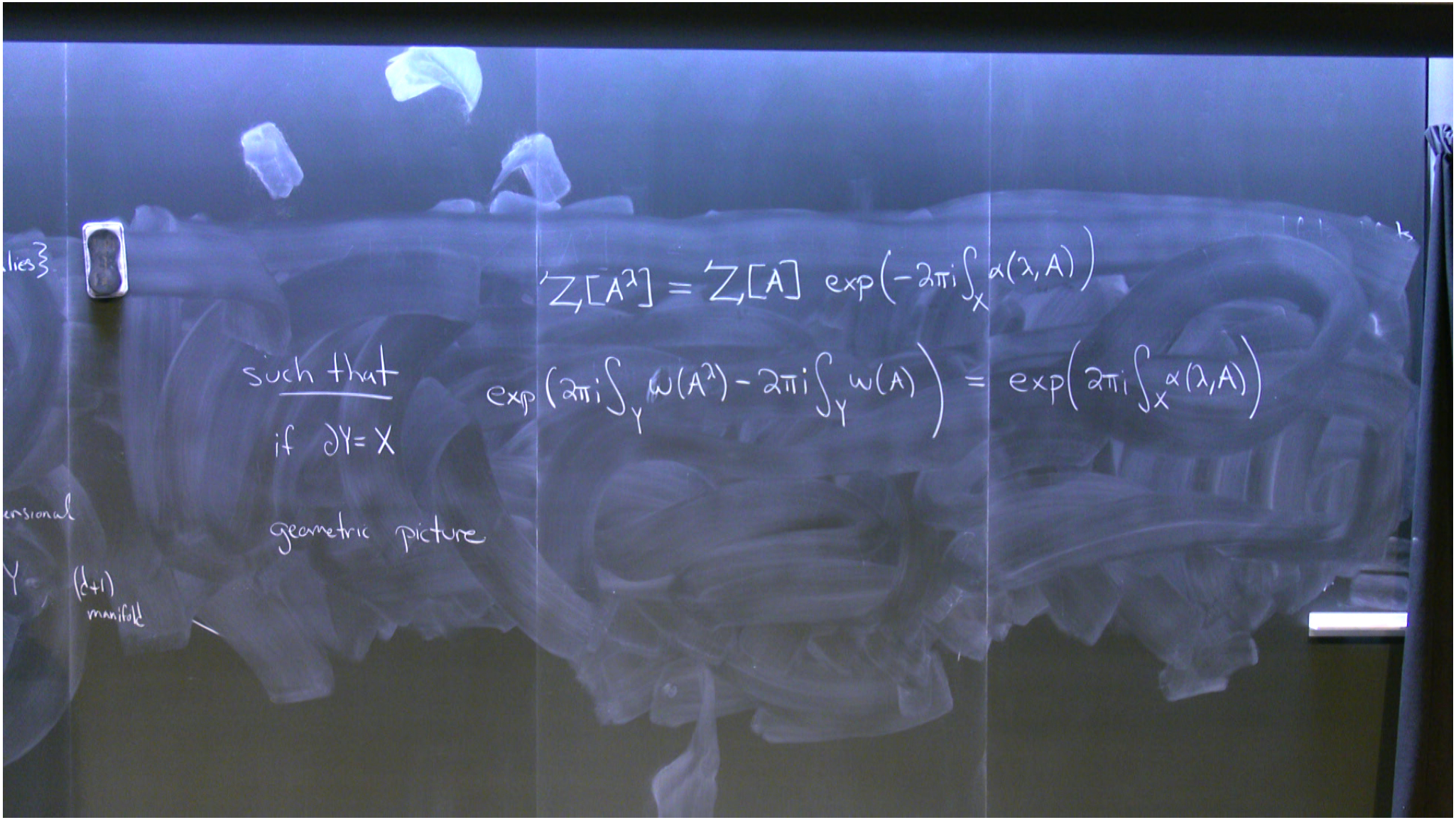

ansatz
$Z[A^\lambda] = Z[A] \exp\left(-2\pi i \int_X a(\lambda, A)\right)$
X = spacetime
• why $|Z[A^\lambda]| = |Z[A]|$?
partition function modulus has probability interpretation
(analogy to rays in QM)


ansatz
$Z[A^\lambda] = Z[A] \exp\left(-2\pi i \int_X a(\lambda, A)\right)$
X = spacetime
• why $|Z[A^\lambda]| = |Z[A]|$?
partition function modulus has probability interpretation
(analogy to rays in QM)
partition function $Z[A]$ subject to ambiguity
$Z[A] \longrightarrow Z[A] \exp\left(2\pi i \int_X B(A)\right)$
B local freedom to change "scheme"


cohomology problem
$\{\alpha\}/_{\alpha \sim \alpha + \delta\beta} = \{\text{possible anomalies}\}$
Inflow (hypothesis/definition/theorem)
ansatz
• why $|Z[A^\lambda]| =$
partition function $Z[A]$
$Z[A] \longrightarrow$
TAS(-1) 2019


cohomology problem
$\{\alpha\}/_{\alpha \sim \alpha + \delta\beta} = \{\text{possible anomalies}\}$
ansatz
Inflow (hypothesis/definition/theorem)
("classical" SPT) for any anomaly $[\alpha]$ there is a $(d+1)$-dimensional
invertible thy $\exp\left(2\pi i \int_Y \omega(A)\right)$ Y $(d+1)$ manifold


$Z[A^\lambda] = Z[A] \exp\left(-2\pi i \int_X a(\lambda, A)\right)$


$Z[A^\lambda] = Z[A] \exp\left(-2\pi i \int_X \alpha(\lambda, A)\right)$
such that
if $\partial Y = X$
$\exp\left(2\pi i \int_Y \omega(A^\lambda) - 2\pi i \int_Y \omega(A)\right) = \exp\left(2\pi i \int_X \alpha(\lambda, A)\right)$
geometric picture
(d+1) manifold

$$Z[A^\lambda] = Z[A]\ \exp\left(-2\pi i \int_X \alpha(\lambda, A)\right)$$

such that

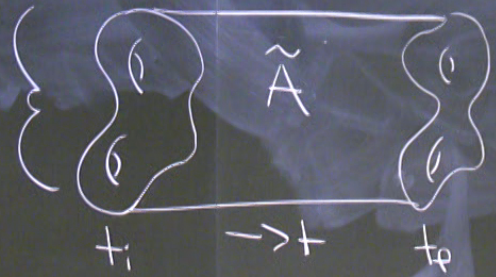
if $\partial Y = X$

$$\exp\left(2\pi i \int_Y \omega(A^\lambda) - 2\pi i \int_Y \omega(A)\right) = \exp\left(2\pi i \int_X \alpha(\lambda, A)\right)$$

geometric picture

(d+1) manifold $X \times_\lambda S^1$ mapping torus for λ

(d+1) manifold

$\tilde{A}\big|_{t_i} = A$ $\times$  $\tilde{A}\big|_{t_f} = A^\lambda$ gluing t_i to t_f

anomalous phase $= \exp\left(2\pi i \int_X \alpha(\lambda, A)\right) = \exp\left(2\pi i \int_{X \times_2 S^1} w(\tilde{A})\right)$

such that

allows us to define a modified but fully gauge invariant partition fnctn

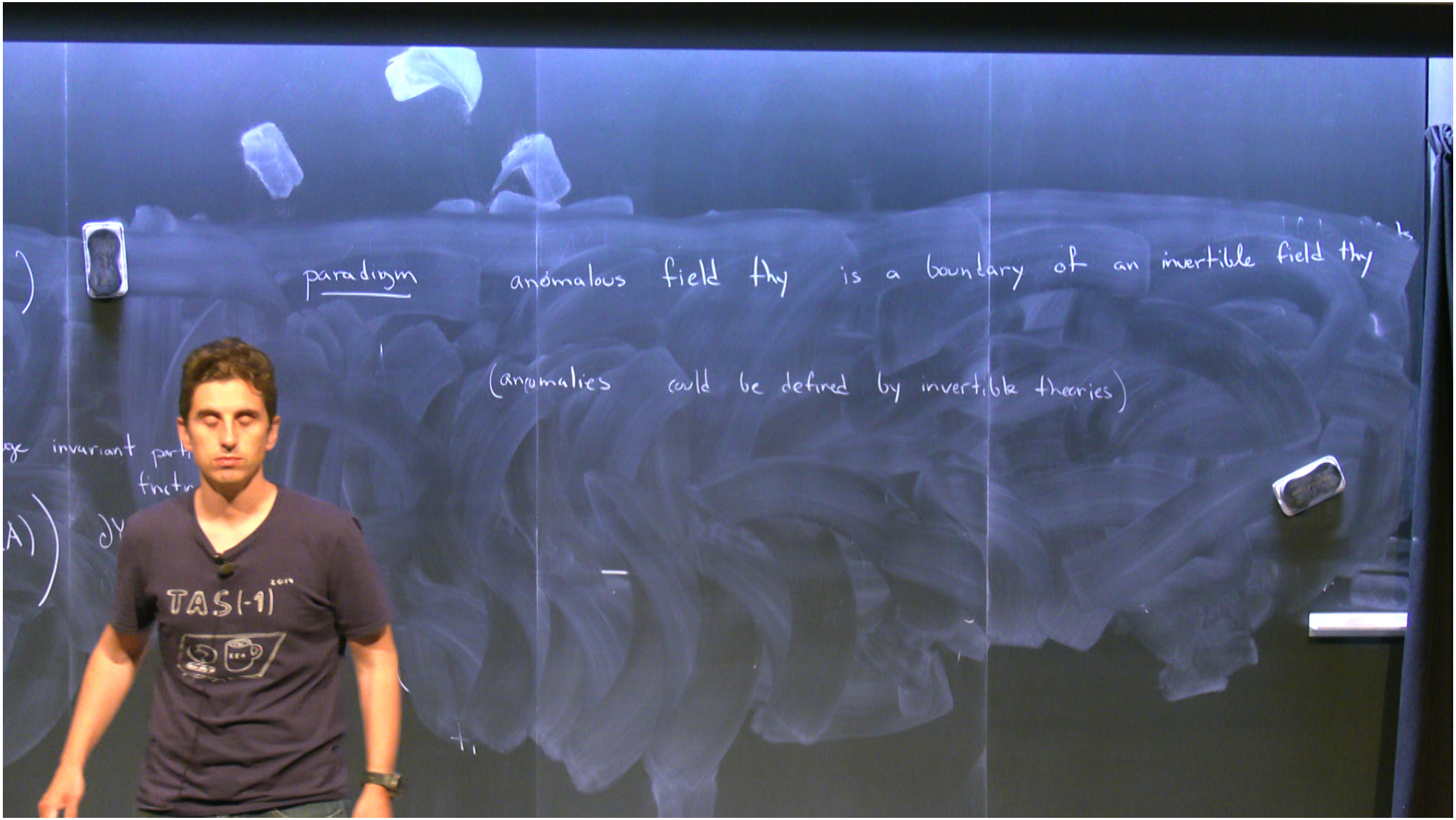
$$\widetilde{Z}[A] = Z[A] \exp\left(2\pi i \int_Y w(A)\right) \quad \partial Y = X$$

$$\widetilde{Z}[A'] = \widetilde{Z}[A]$$

geometric pict

$\tilde{A}|_{t_i} = A \times \{$

t_i


paradigm
anomalous field thy is a boundary of an invertible field thy
(anomalies could be defined by invertible theories)
invariant
TAS(-1)