Title: Lattice systems and topological field theories

Speakers: Michael Hopkins

Collection: Global Categorical Symmetries

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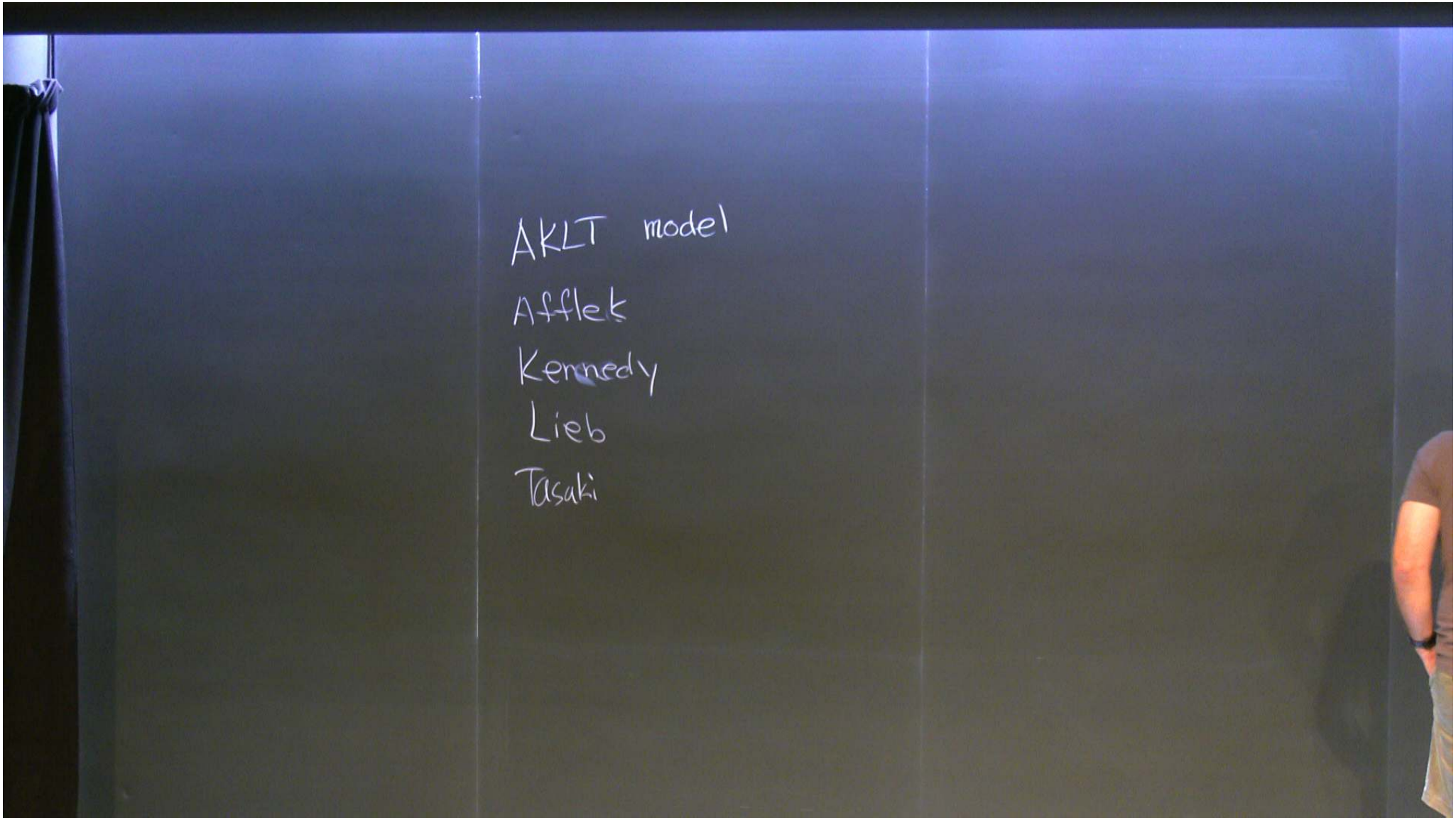

AKLT model
Affleck
Kennedy
Lieb
Tasaki

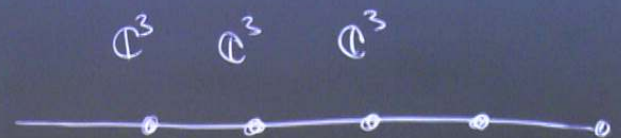
AKLT model

Affleck

Kennedy

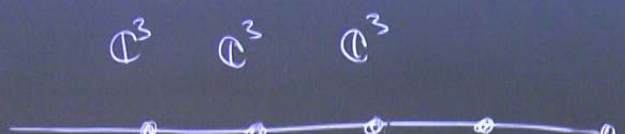
Lieb

Tasaki



$H = \sum_s H_s$

H_s projection operator



$H = \sum_s H_s$

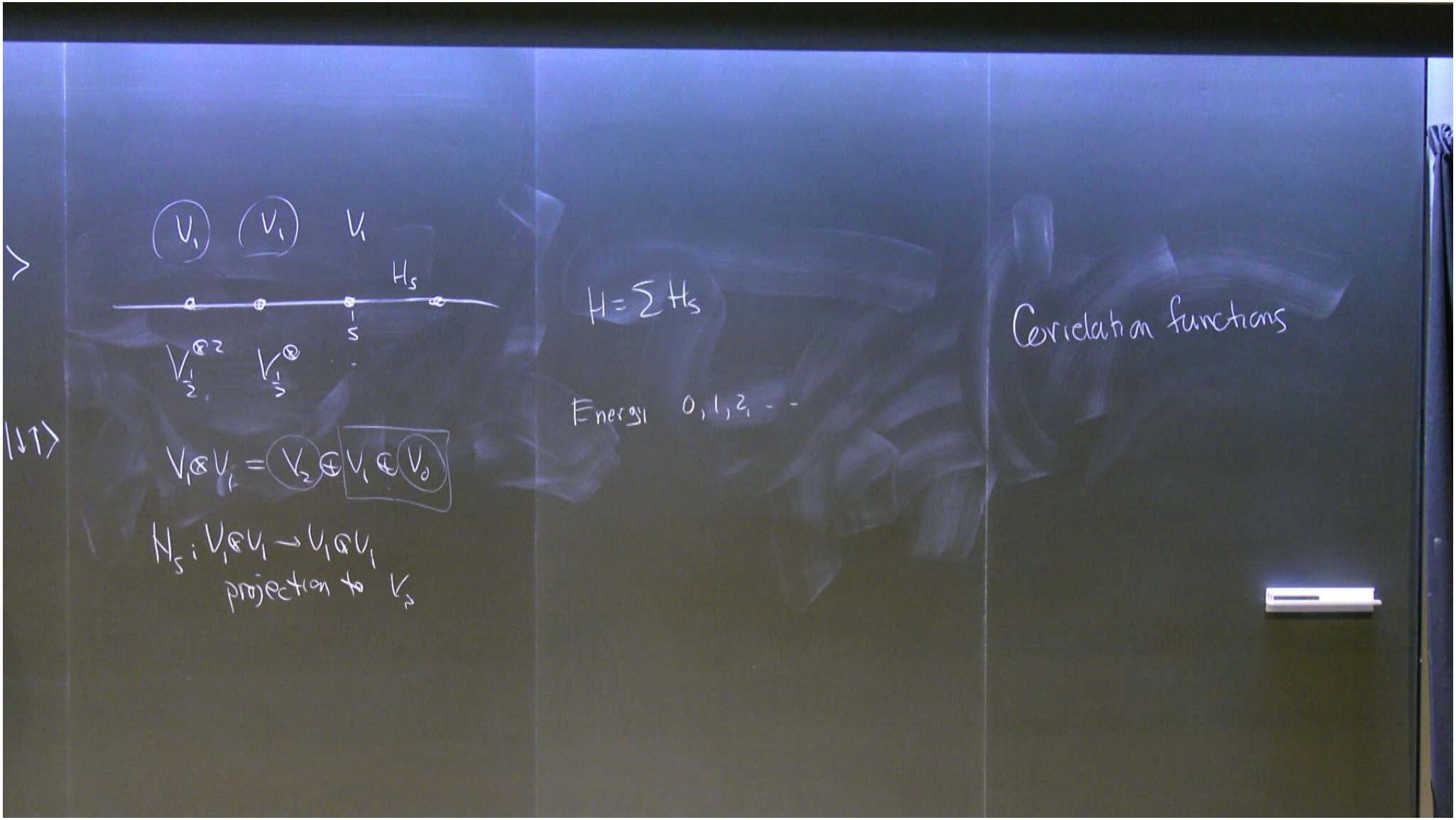
H_s projection operator

Valence Bond Solid

$SU(2)$ $\mathbb{C}^2$ Spin $\frac{1}{2}$

$Sym^n(\mathbb{C}^2)$ Spin $(\frac{n}{2})$

V_n = Spin-n rep


V_1 V_1 V_1
H_s
s
$V_1^{\otimes 2}$ $V_1^{\otimes}$
$|\psi\rangle$
$V_1 \otimes V_1 = V_2 \oplus V_1 \oplus V_0$
$N_s : V_1 \otimes V_1 \to V_1 \otimes V_1$
projection to V_2
$H = \sum H_s$
Energy 0, 1, 2, - -
Correlation functions

$\mathcal{H}$

A operator dynamical variables

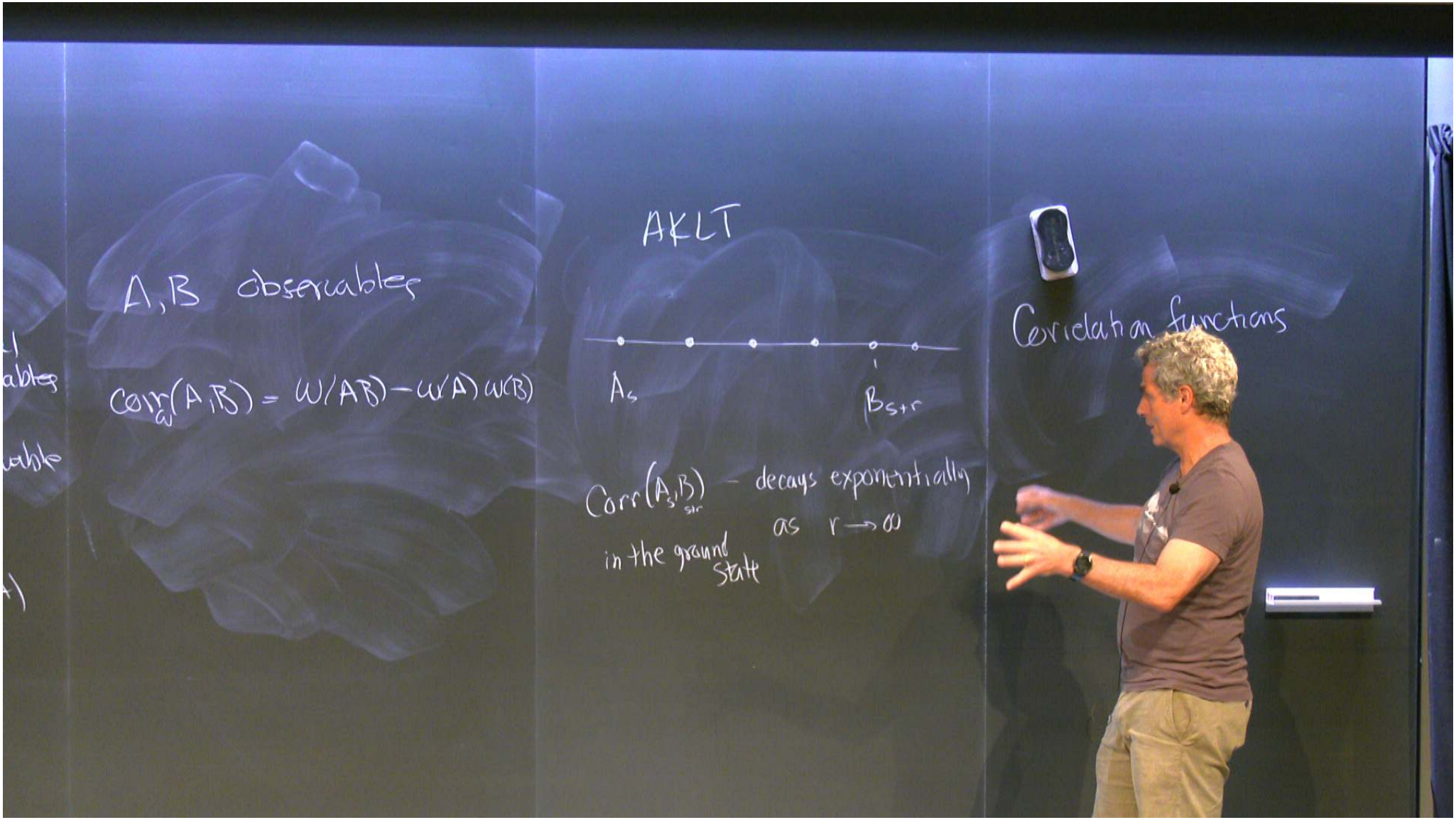
self-adjoint A — observable

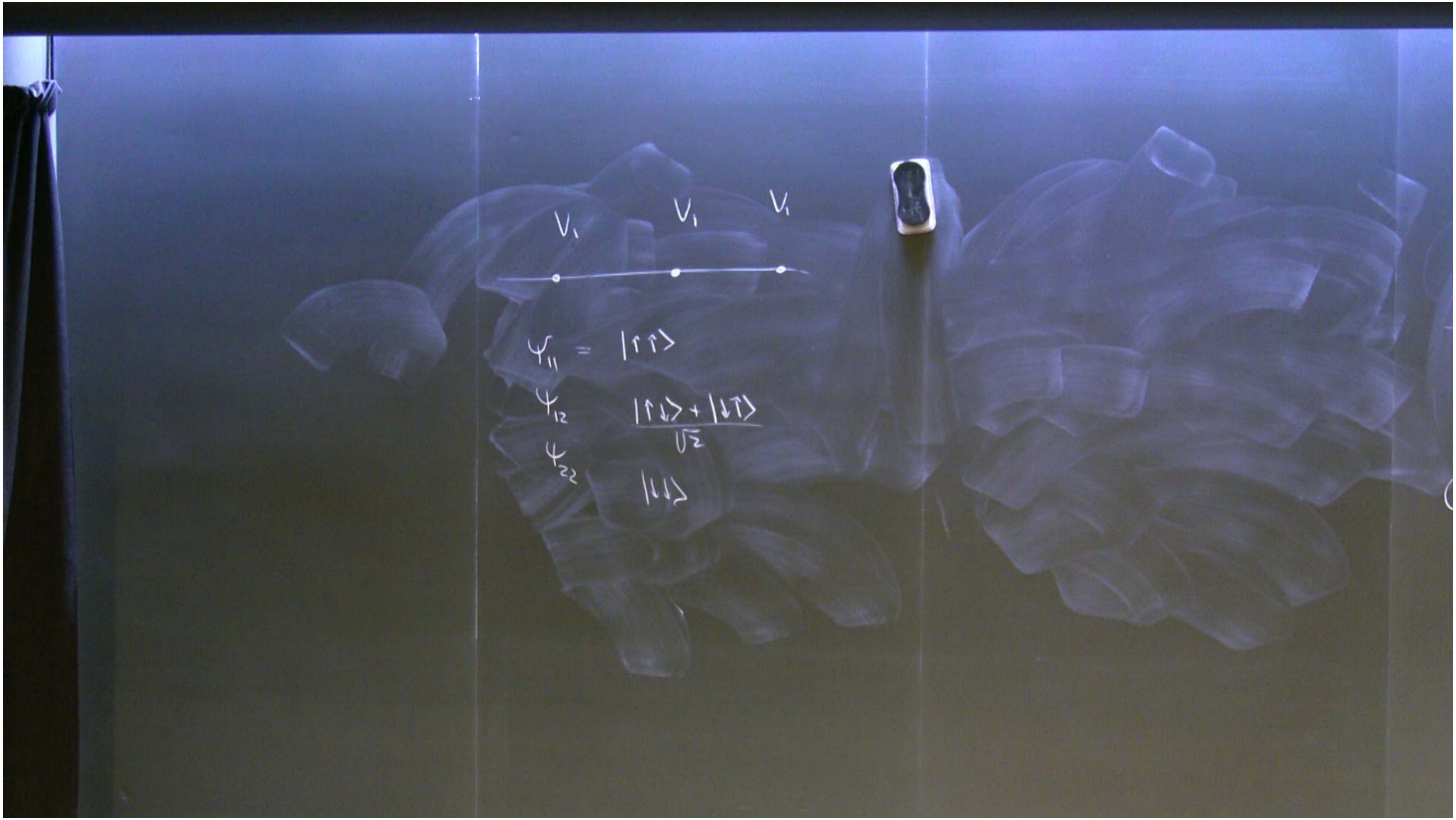
$|\psi\rangle \in \mathcal{H}$

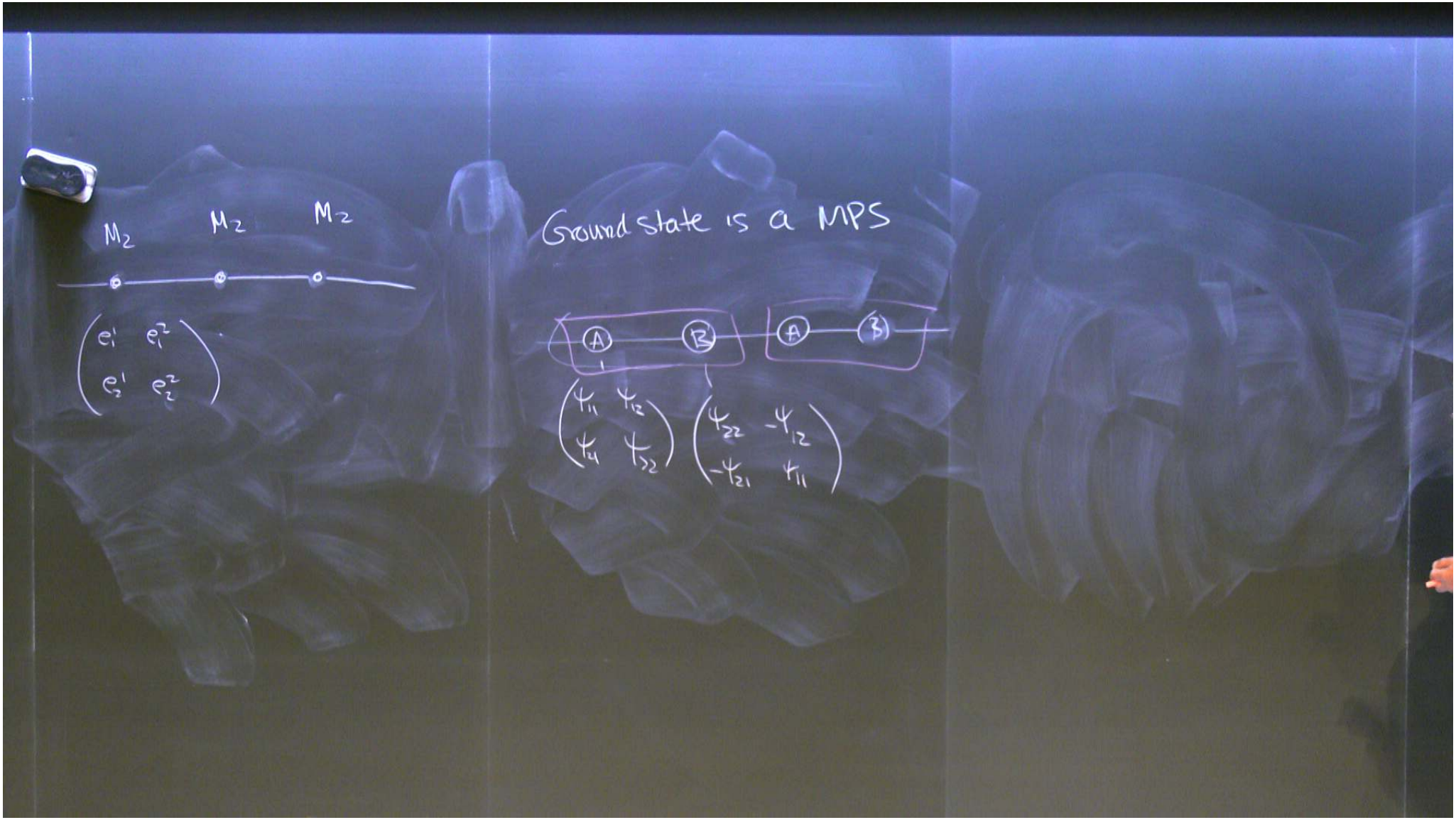
$\frac{\langle\psi|A|\psi\rangle}{\langle\psi|\psi\rangle} = w(A) = E(A)$

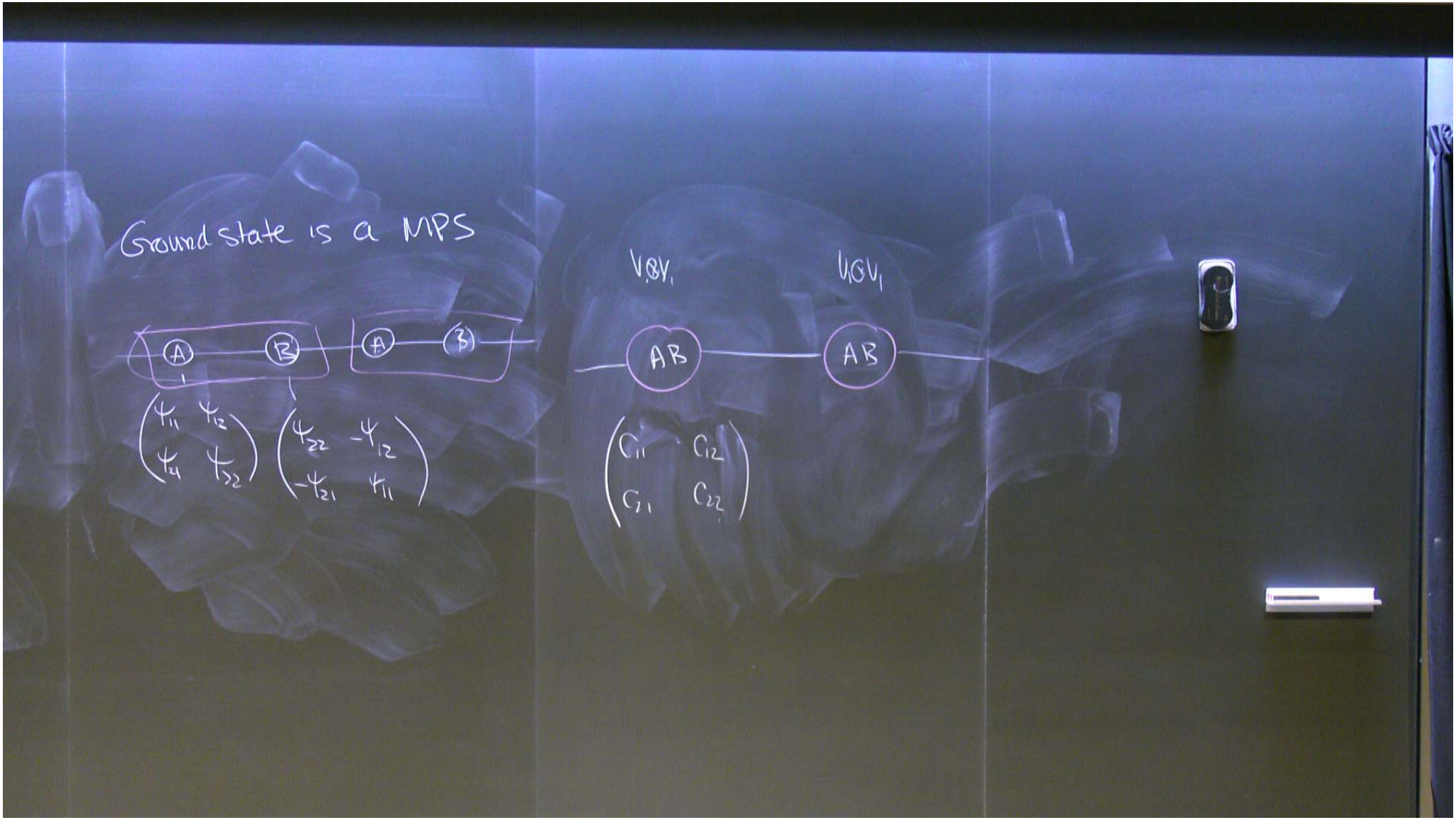
A, B observables

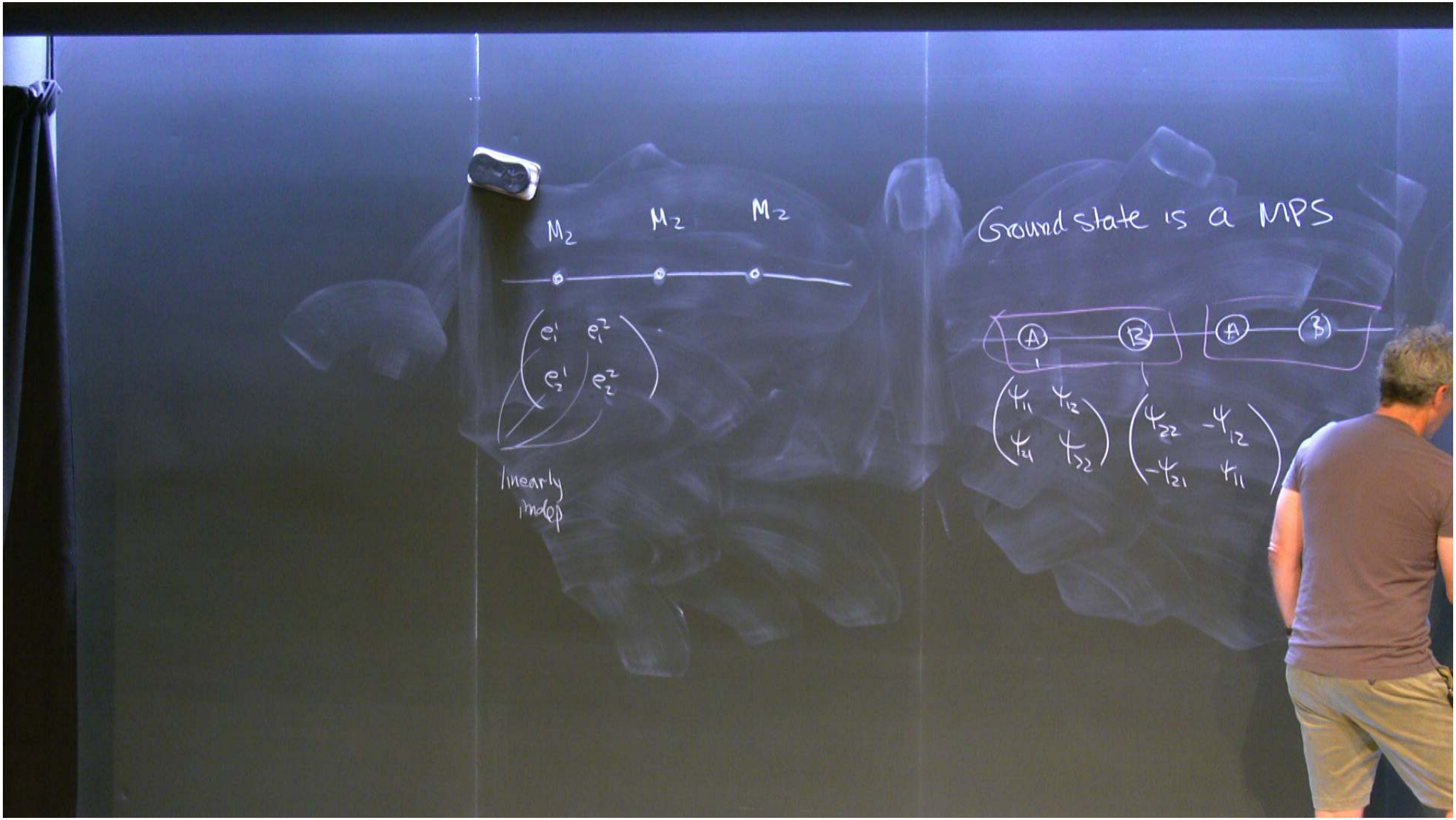
$Corr(A,B) = w(AB) - w(A)\,w(B)$

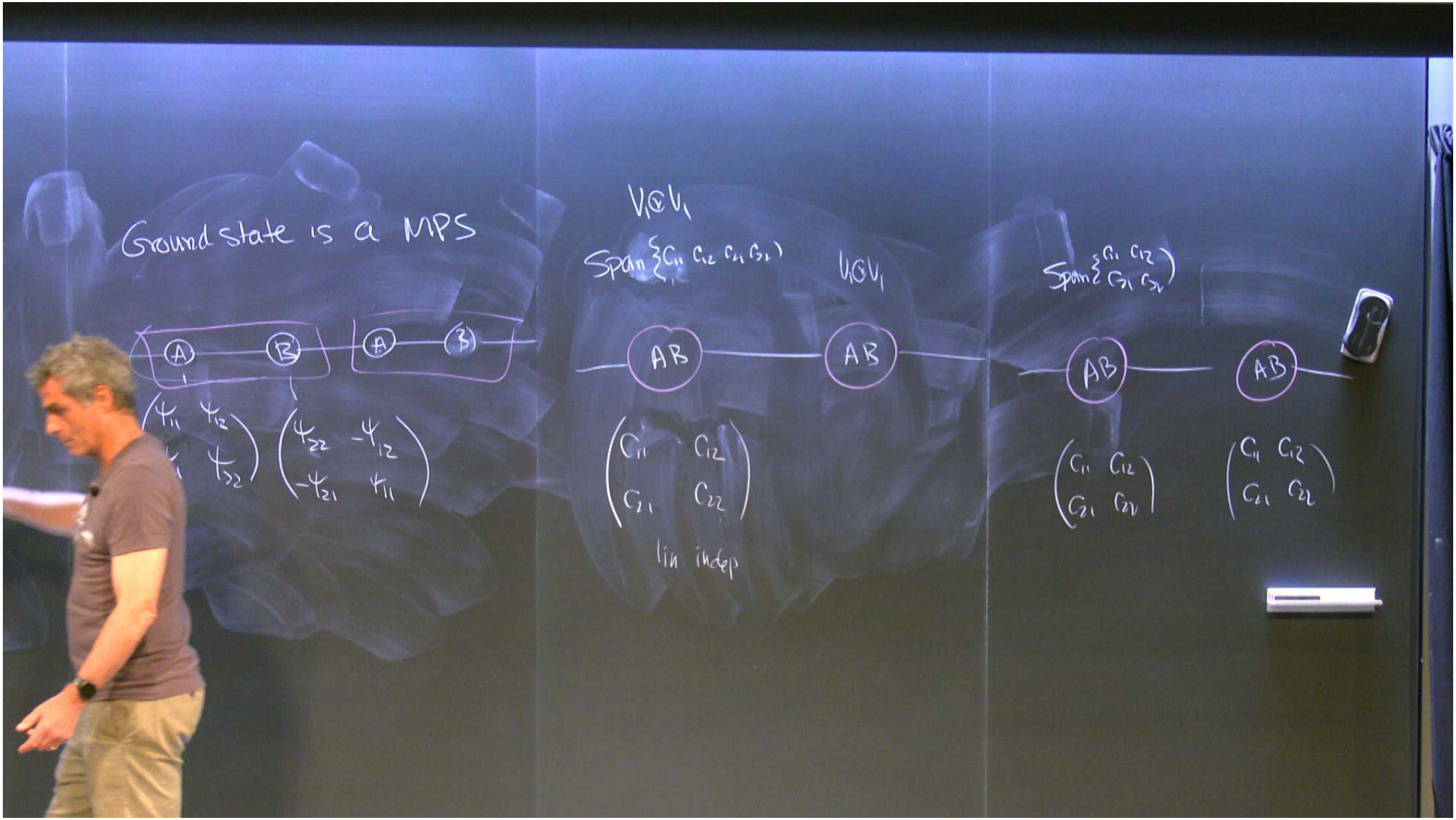

A, B observables
$\mathrm{Corr}_\psi(A,B) = \omega(AB) - \omega(A)\omega(B)$
AKLT
A_s
B_{s+r}
$\mathrm{Corr}(A_s, B_{s+r})$ – decays exponentially as $r \to \infty$
in the ground state
Correlation functions

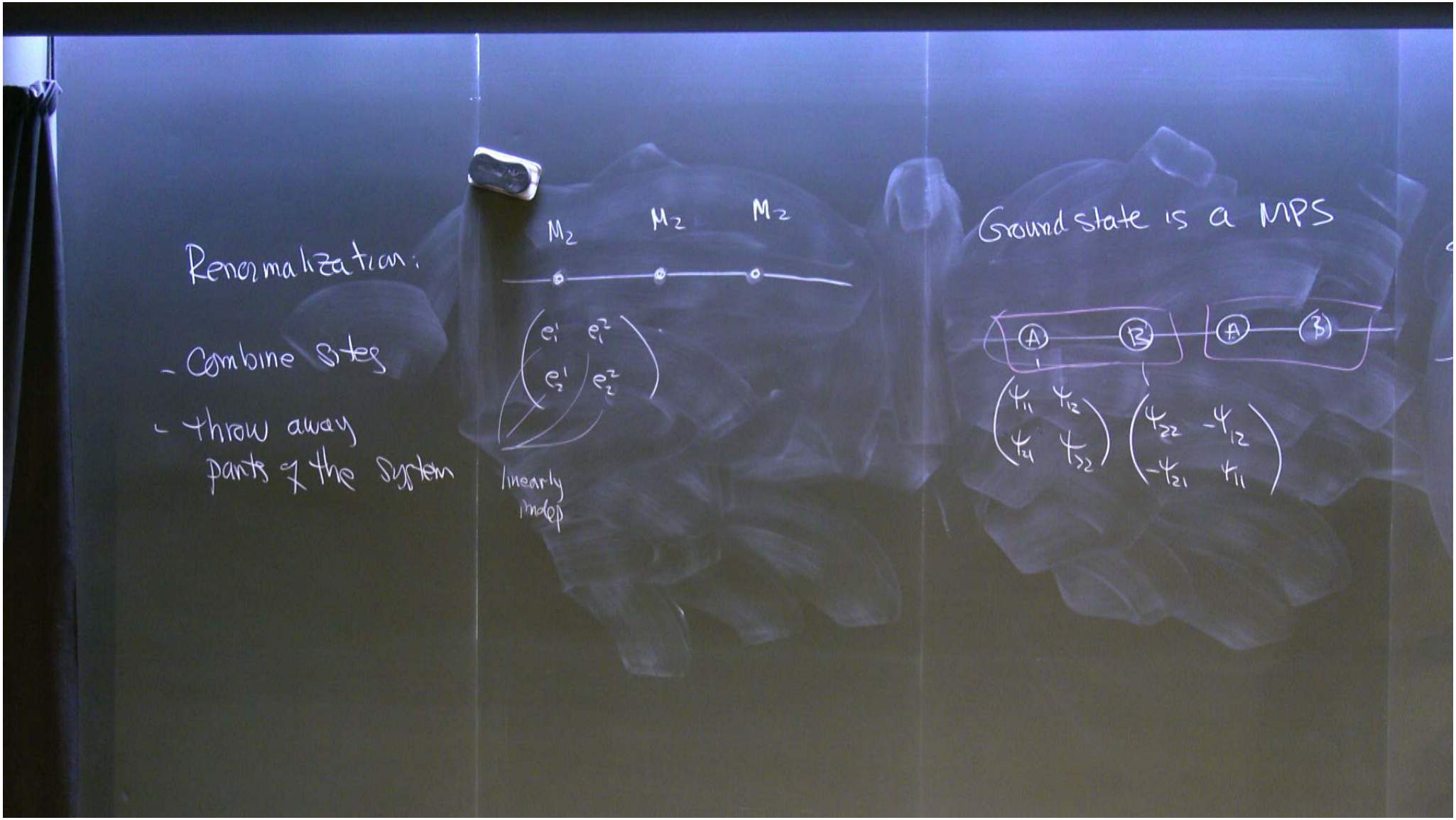

V_1 V_1 V_1
$\psi_{11} = |\uparrow\uparrow\rangle$
$\psi_{12} \quad \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$
$\psi_{22} \quad |\downarrow\downarrow\rangle$

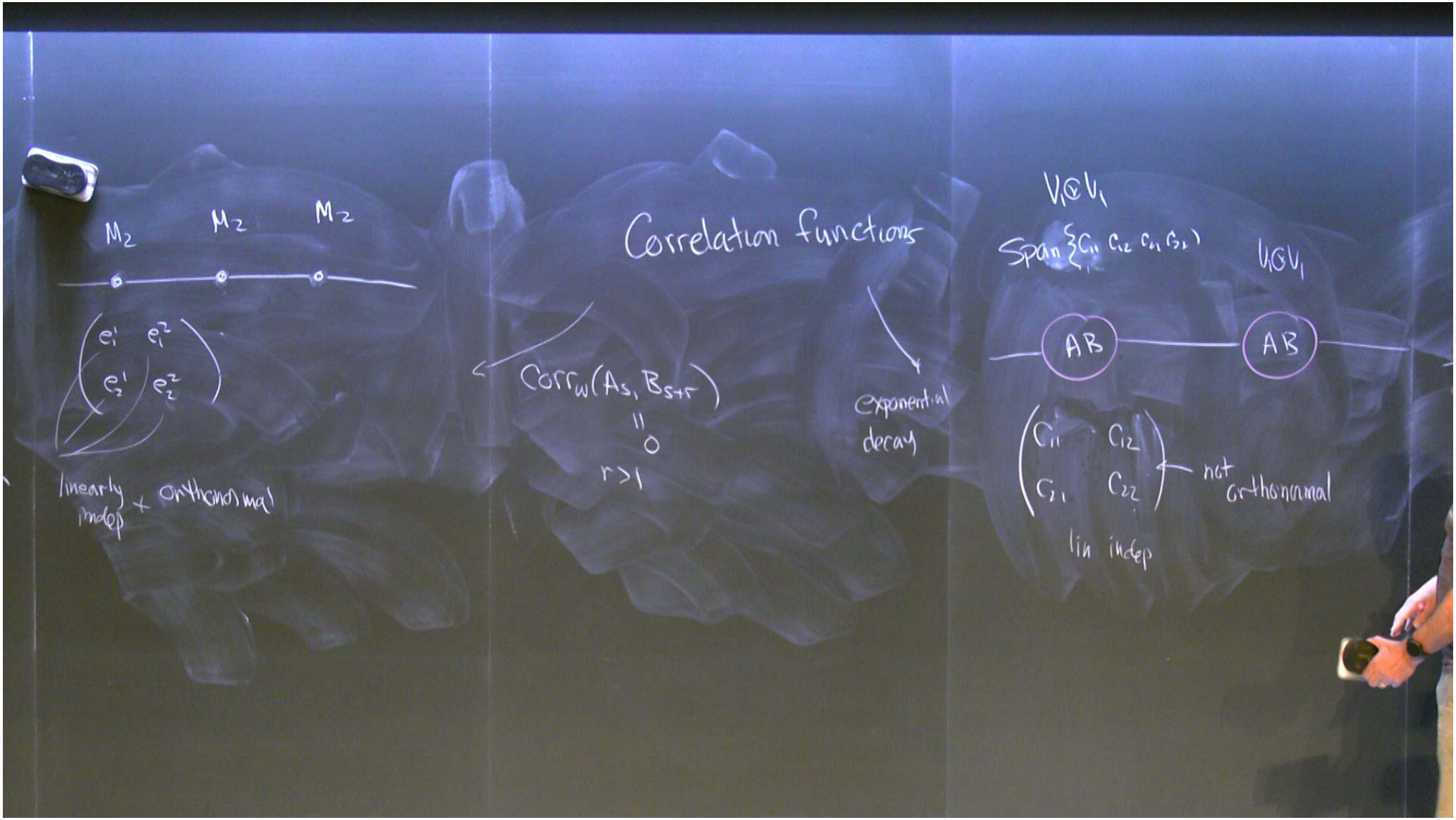

M2
M2
M2
Ground state is a MPS
A
B
A
B

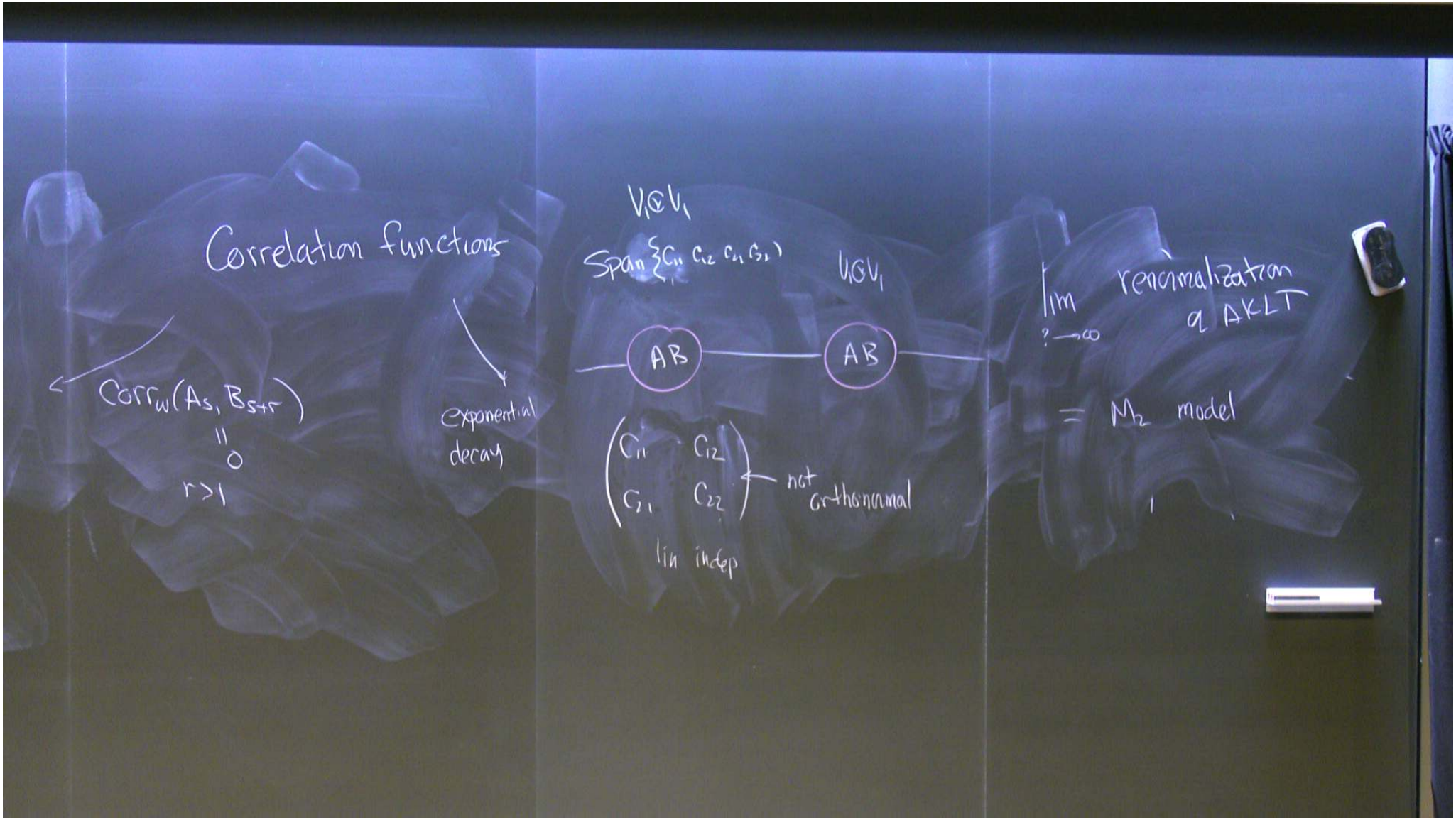

Ground state is a MPS
A
B
A
B
AB
AB

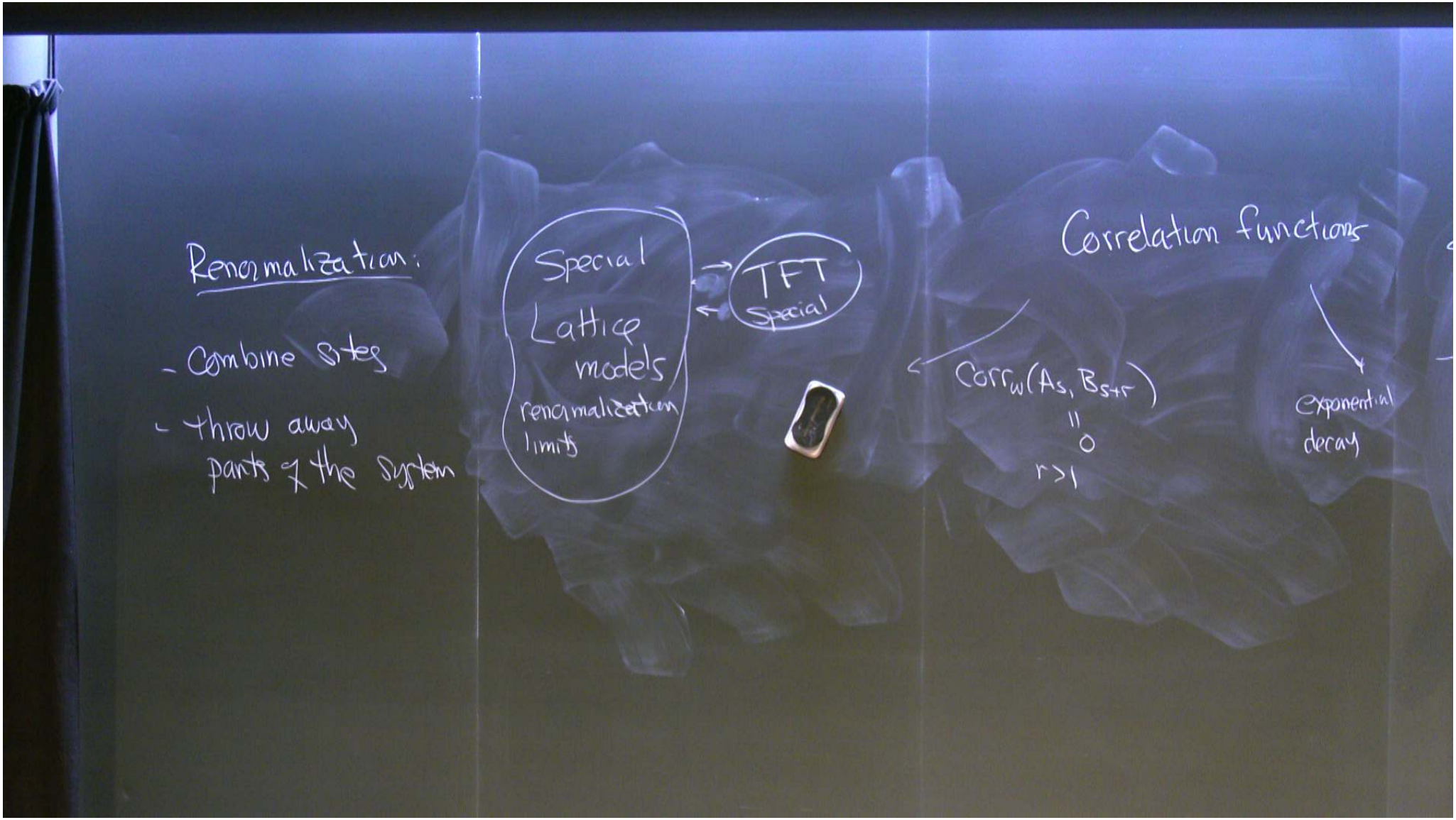

M_2 M_2 M_2
linearly indep
Ground state is a MPS
A B A B

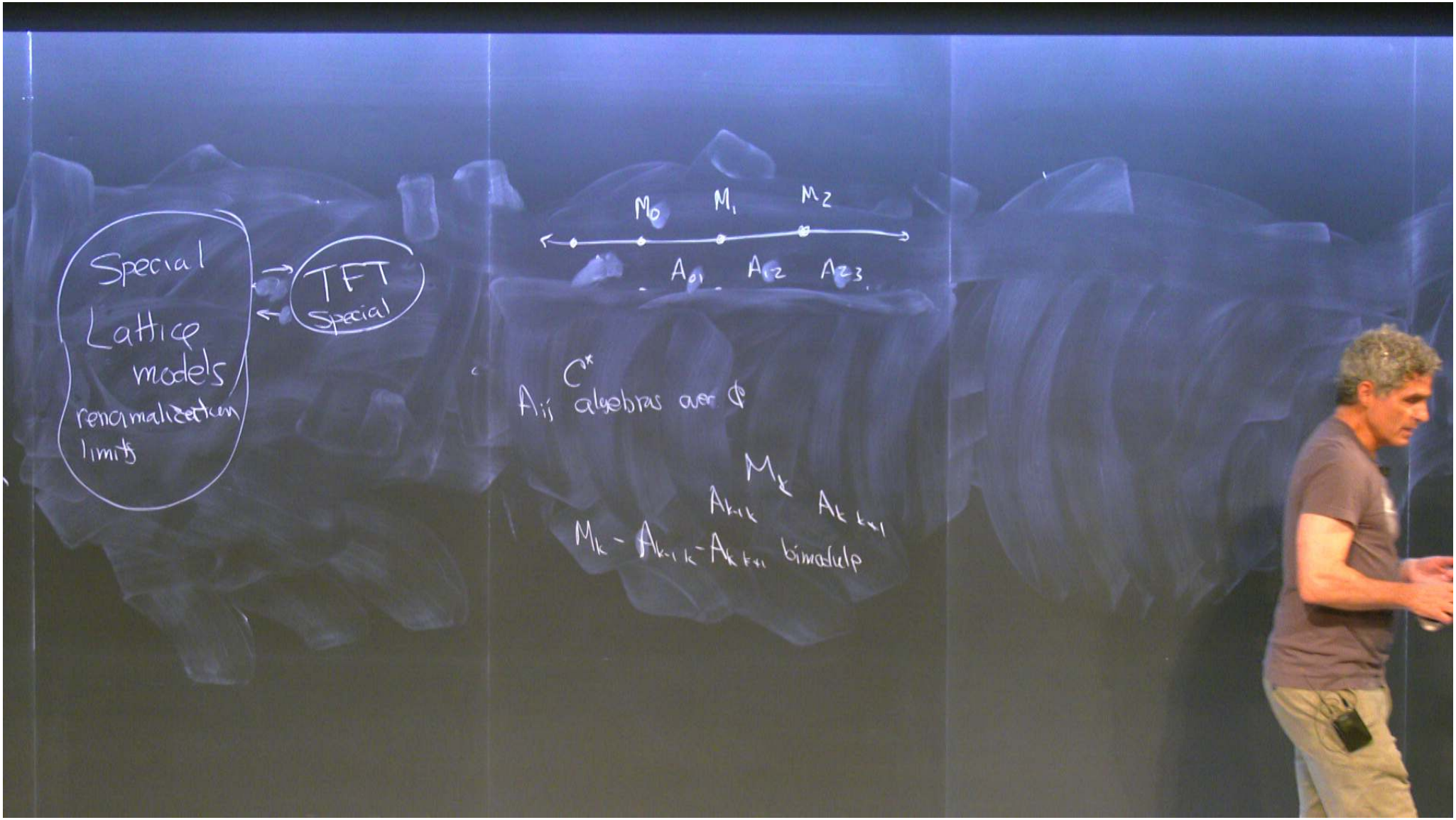

Ground state is a MPS
A
B
A
B
AB
AB
AB
AB
lin indep

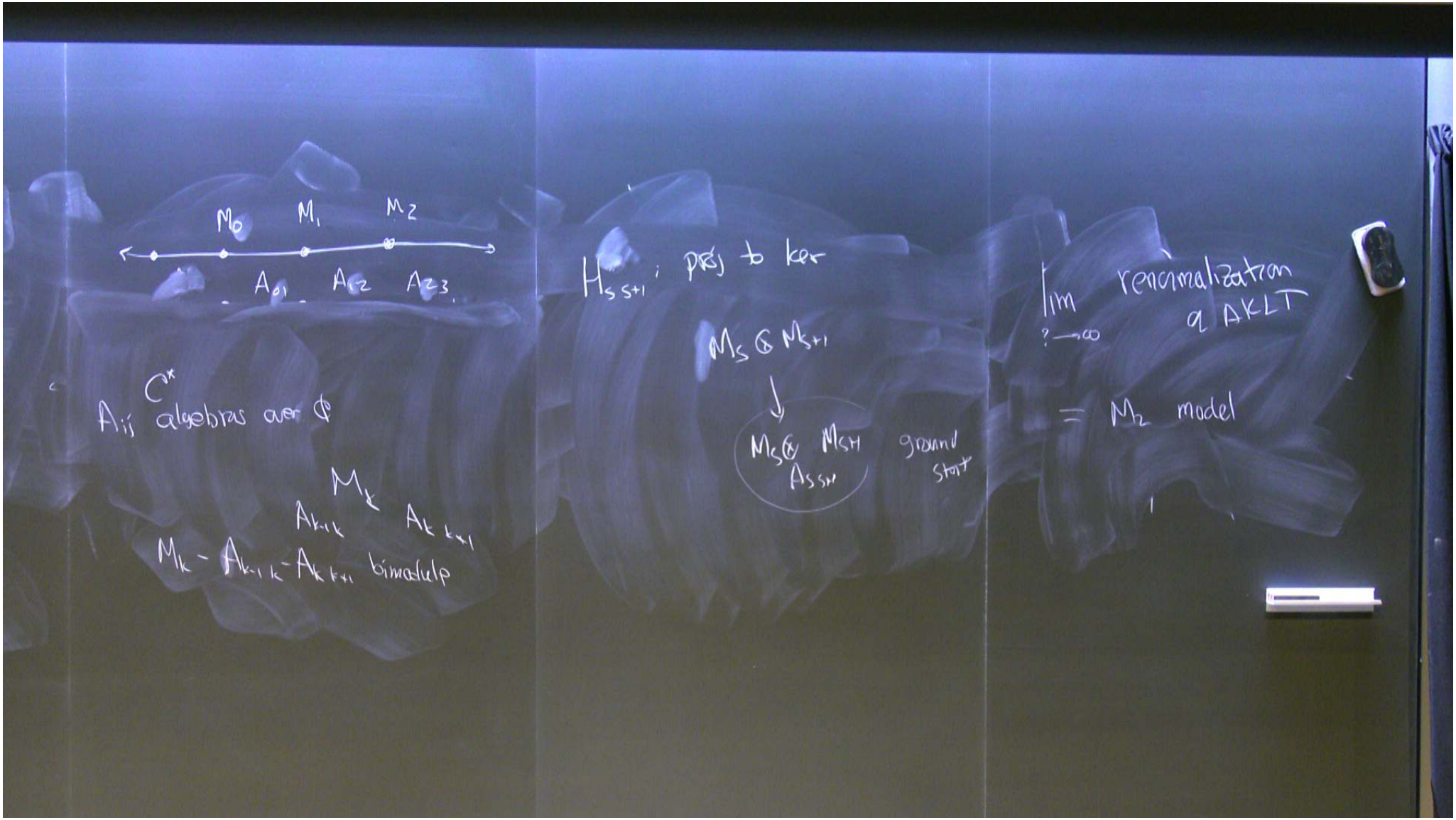

Renormalization:
- Combine sites
- throw away parts of the system
M_2 M_2 M_2
linearly indep
Ground state is a MPS
A B A B


M2 M2 M2
linearly indep + orthonormal
Correlation functions
Corr_w(A_s, B_{s+r})
= 0
r > 1
exponential decay
V ⊗ V
Span {C11 C12 C21 C22}
V ⊗ V
AB
AB
C11 C12
C21 C22
not orthonormal
lin indep


Correlation functions
$Corr_w(A_s, B_{s+r})$
= 0
r>1
exponential decay
$V_1 \otimes V_1$
Span $\{c_{11}\ c_{12}\ c_{21}\ c_{22}\}$
$V_1 \otimes V_1$
AB
AB
$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$
not orthonormal
lin indep
lim $? \to \infty$
renormalization of AKLT
= M_2 model

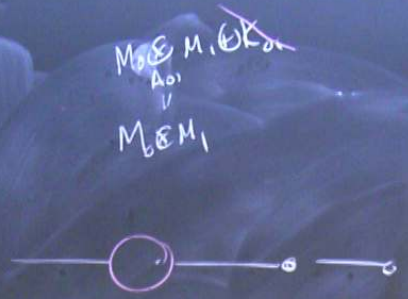

Renormalization:
- Combine sites
- throw away parts of the system
Special
Lattice models
renormalization limits
TFT
Special
Correlation functions
$Corr_w(A_s, B_{s+r})$
=
0
r>1
exponential decay

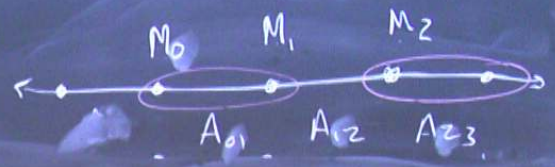

Special
Lattice
models
renormalization
limits
TFT
Special
M_0 M_1 M_2
A_{01} A_{12} A_{23}
C^*
A_{ij} algebras over $\mathbb{C}$
M_k
$A_{k-1\,k}$ $A_{k\,k+1}$
$M_k - A_{k-1\,k} - A_{k\,k+1}$ bimodule


M_0 M_1 M_2
A_{01} A_{12} A_{23}
C^*
A_{ij} algebras over $\mathbb{C}$
M_k
$A_{k-1\,k}$ $A_{k\,k+1}$
$M_k - A_{k-1\,k} - A_{k\,k+1}$ bimodule
$H_{s\,s+1}$: proj to ker
$M_s \otimes M_{s+1}$
$M_s \otimes M_{s+1}$ ground state
$A_{s\,s+1}$
$\lim_{? \to \infty}$ renormalization of AKLT
$= M_2$ model

Renormalization:

- Combine sites
- throw away parts of the system





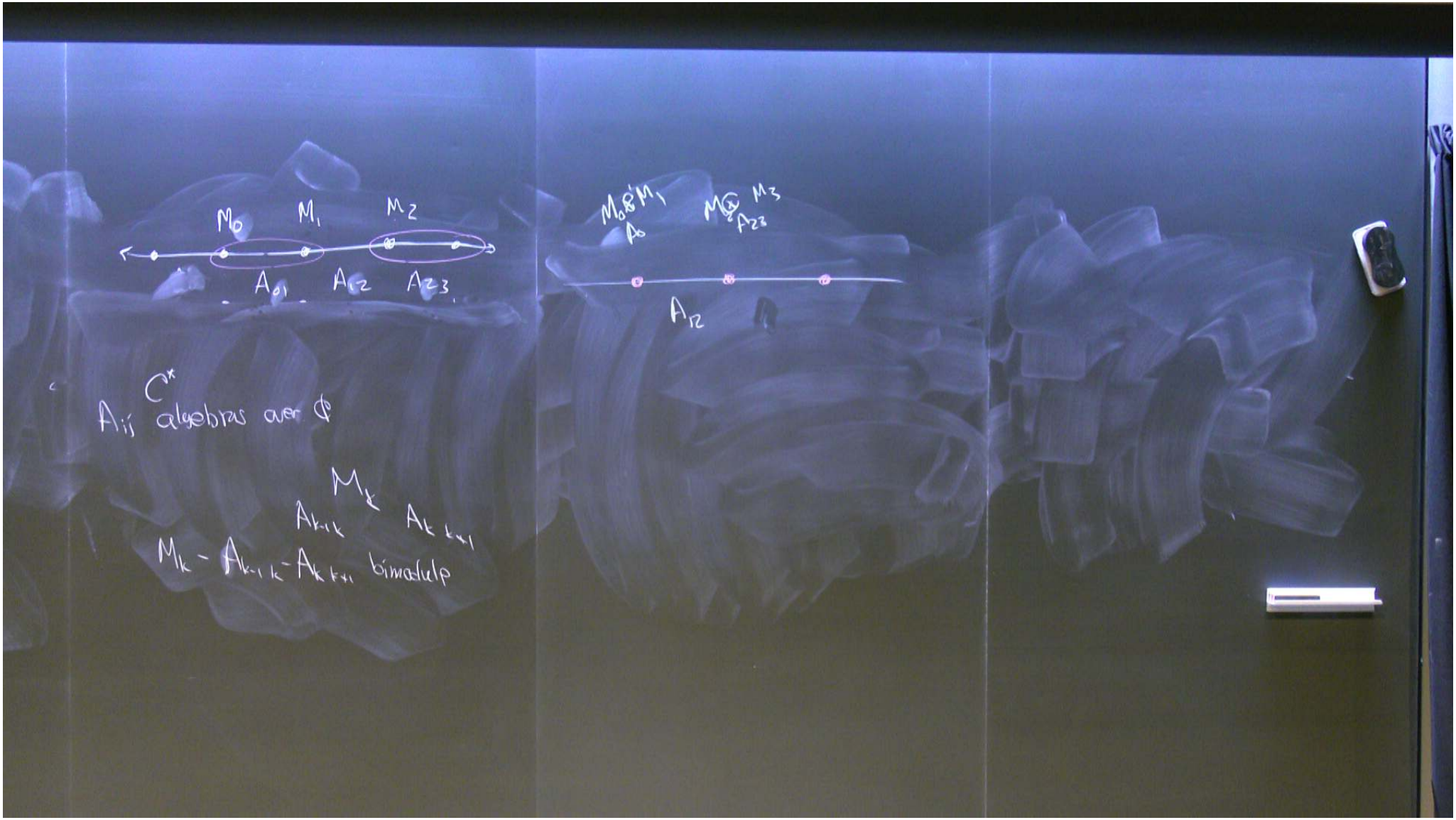
C^*

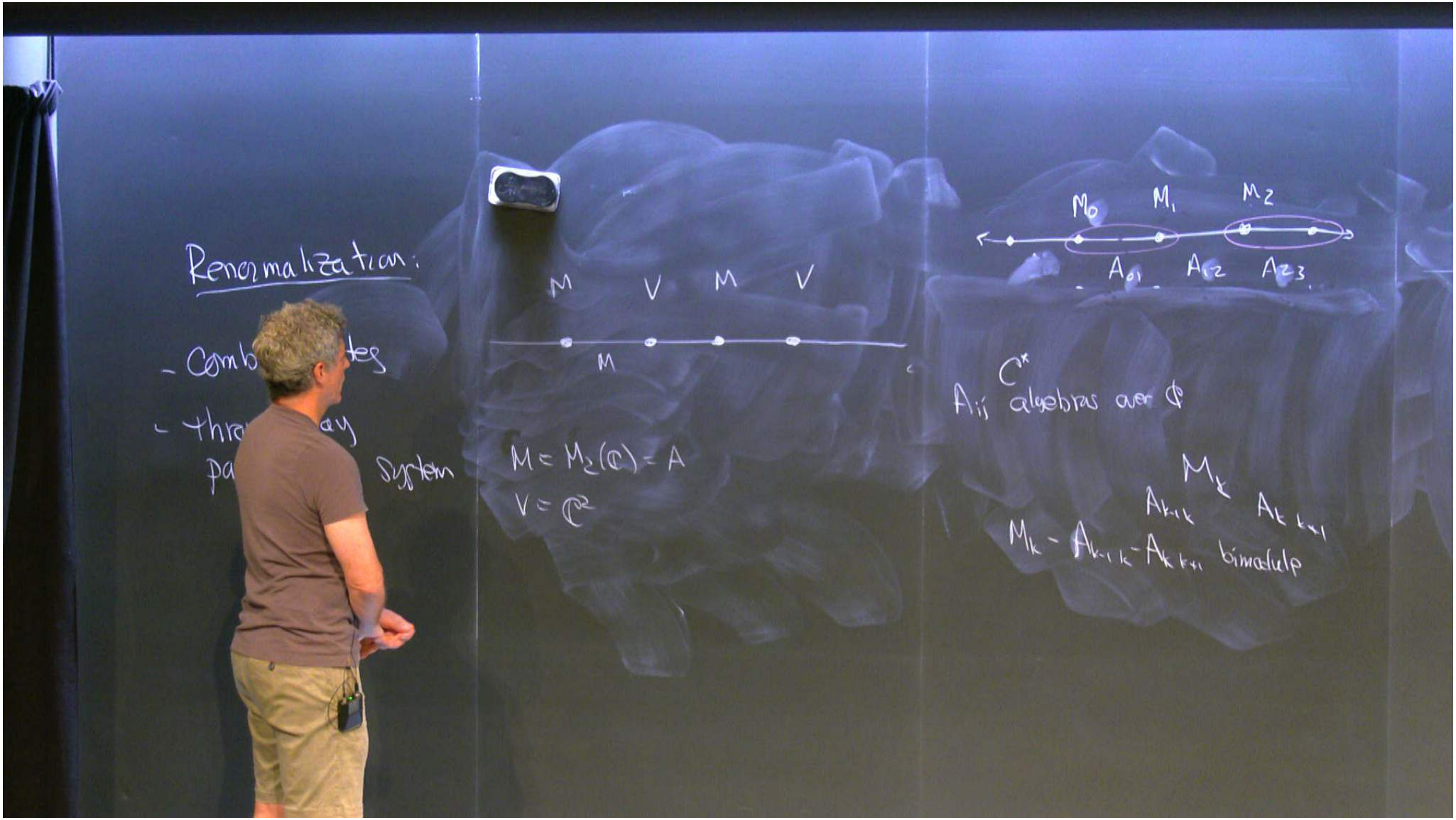
A_{ij} algebras over $\mathbb{C}$

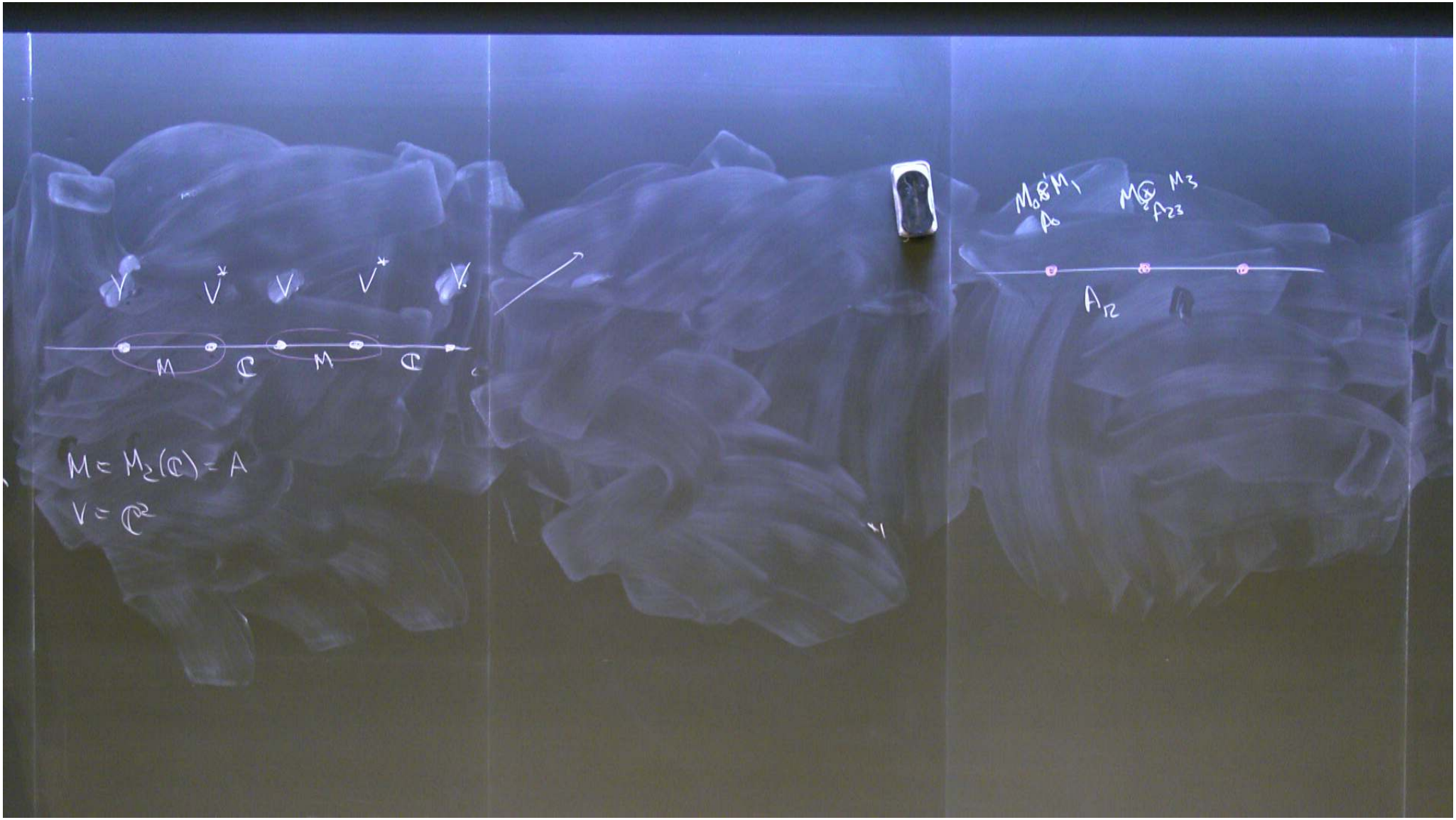
M_k

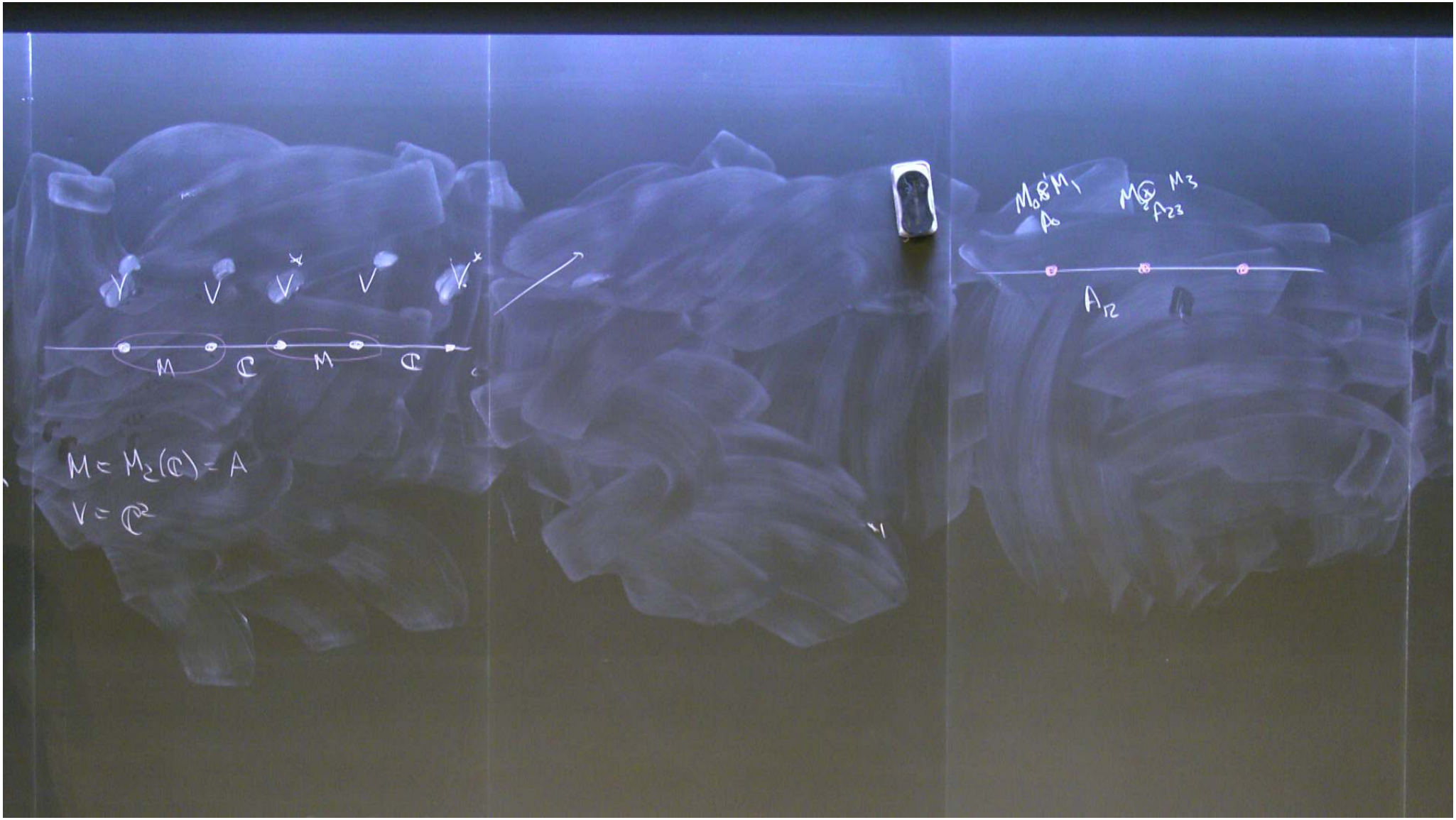
$A_{k-1\,k}$ $A_{k\,k+1}$

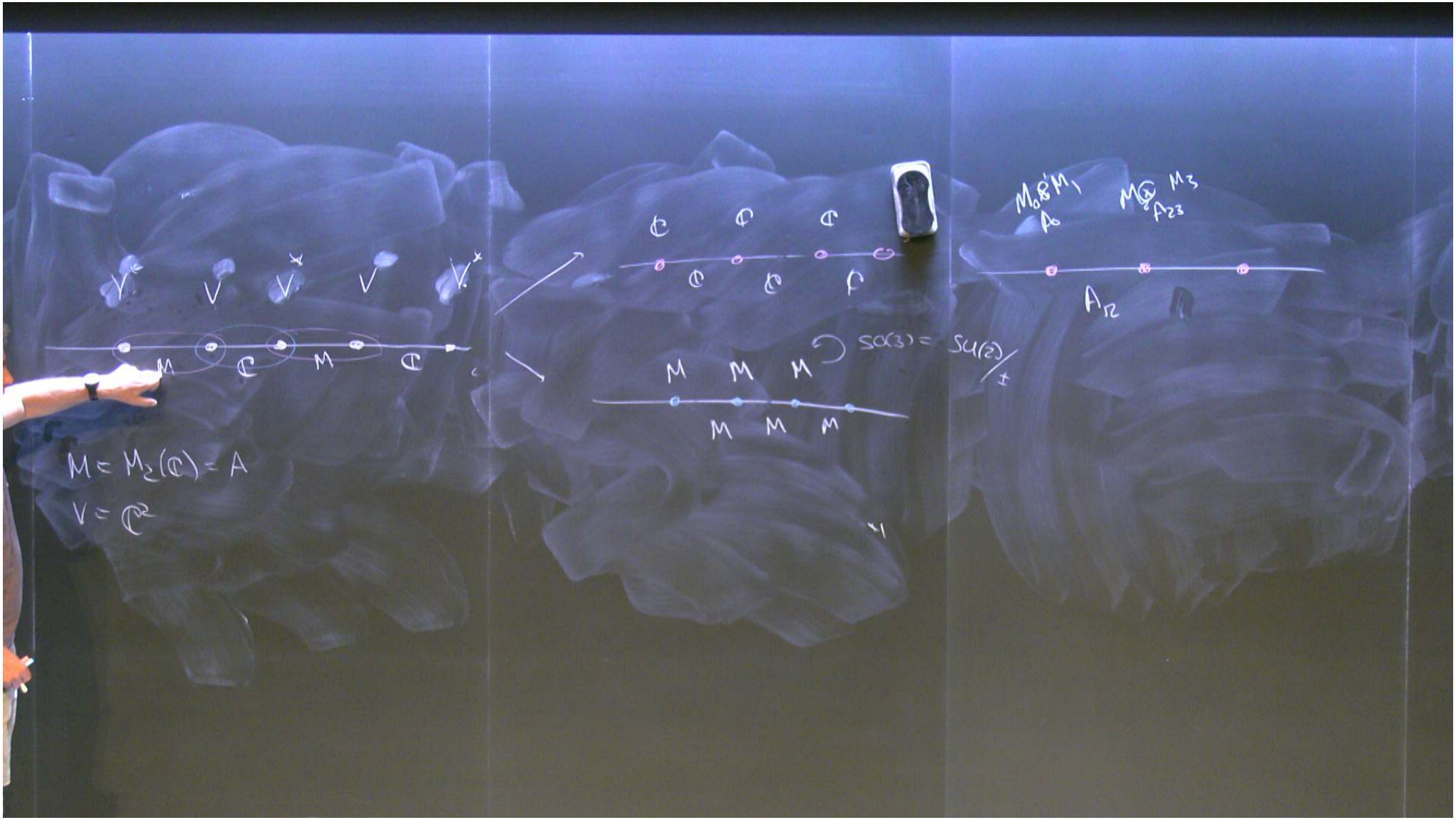
$M_k - A_{k-1\,k} - A_{k\,k+1}$ bimodule


M_0 M_1 M_2
A_{01} A_{12} A_{23}
$M_0 \& M_1$
A_0
$M_2 \& M_3$
A_{23}
A_{12}
C^*
A_{ij} algebras over $\mathbb{C}$
M_k
$A_{k-1\,k}$ $A_{k\,k+1}$
$M_k - A_{k-1\,k} - A_{k\,k+1}$ bimodule


Renormalization:
- Comb teg
- thro ay
pa System
M V M V
M
M = M₂(ℂ) = A
V = ℂ²
M0 M1 M2
A01 A12 A23
C*
Aij algebras over ℂ
Mk
Ak-1k Akk+1
Mk - Ak-1k - Akk+1 bimodule


M
ℂ
M
ℂ
M = M₂(ℂ) = A
V = ℂ²
M₀ & M₁
A₀
M₂ & M₃
A₂₃
A₁₂


V V V V V
M ℂ M ℂ
M = M₂(ℂ) = A
V = ℂ²
M₀ & M₁
A₀
M₂ & M₃
A₂₃
A₁₂


M = M_2(ℂ) = A
V = ℂ^2
SO(3) = SU(2)/±1
A_12
A_23