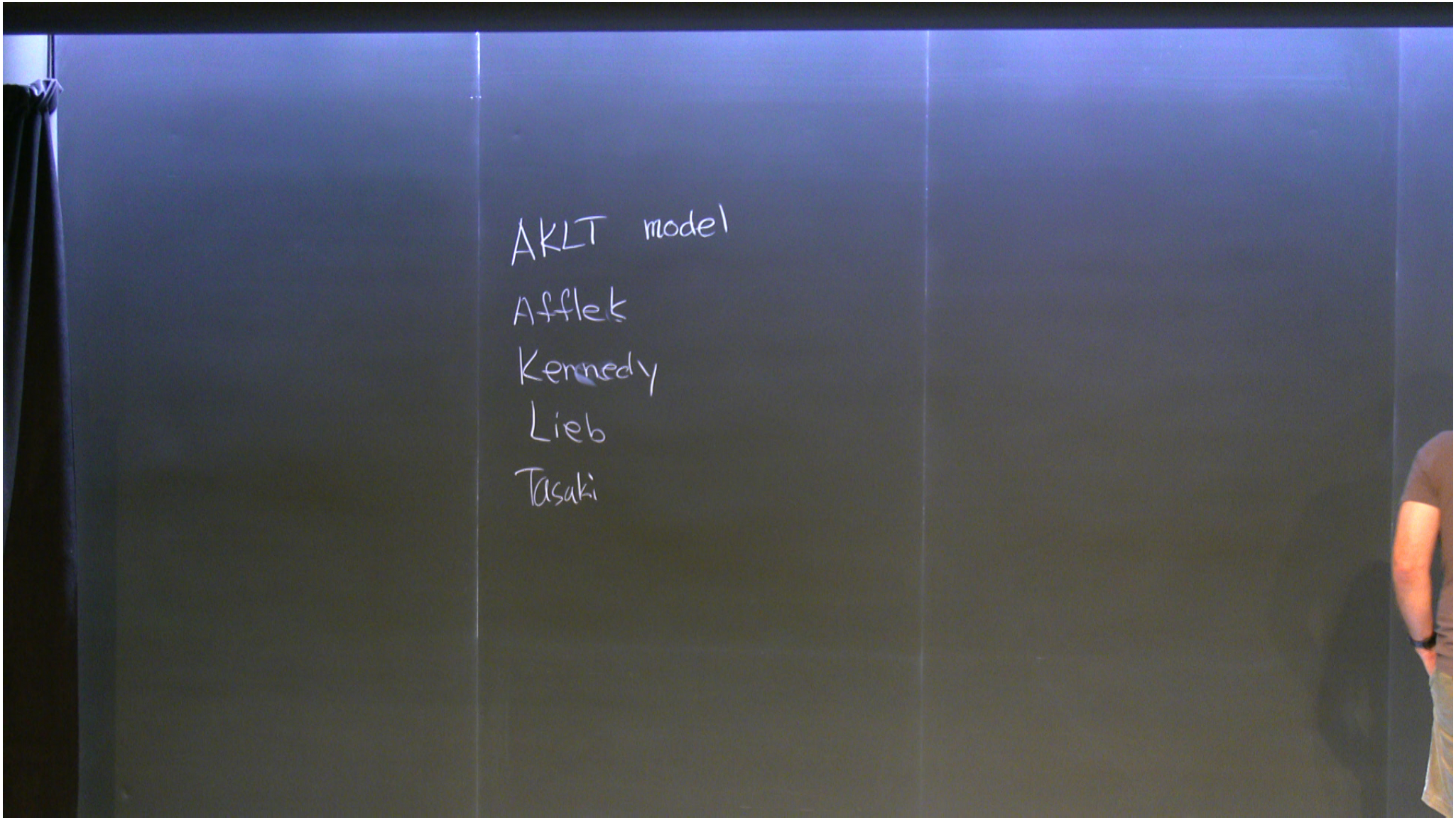
Title: Lattice systems and topological field theories

Speakers: Michael Hopkins

Collection: Global Categorical Symmetries

Date: June 15, 2022 - 9:00 AM

URL: https://pirsa.org/22060028


AKLT model
Affleck
Kennedy
Lieb
Tasaki

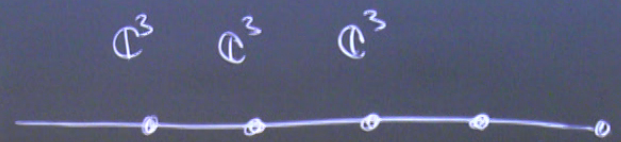
AKLT model

Affleck

Kennedy

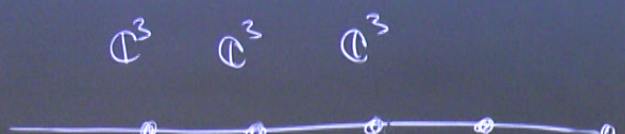
Lieb

Tasaki



$H = \sum_s H_s$

H_s projection operator



$H = \sum_s H_s$

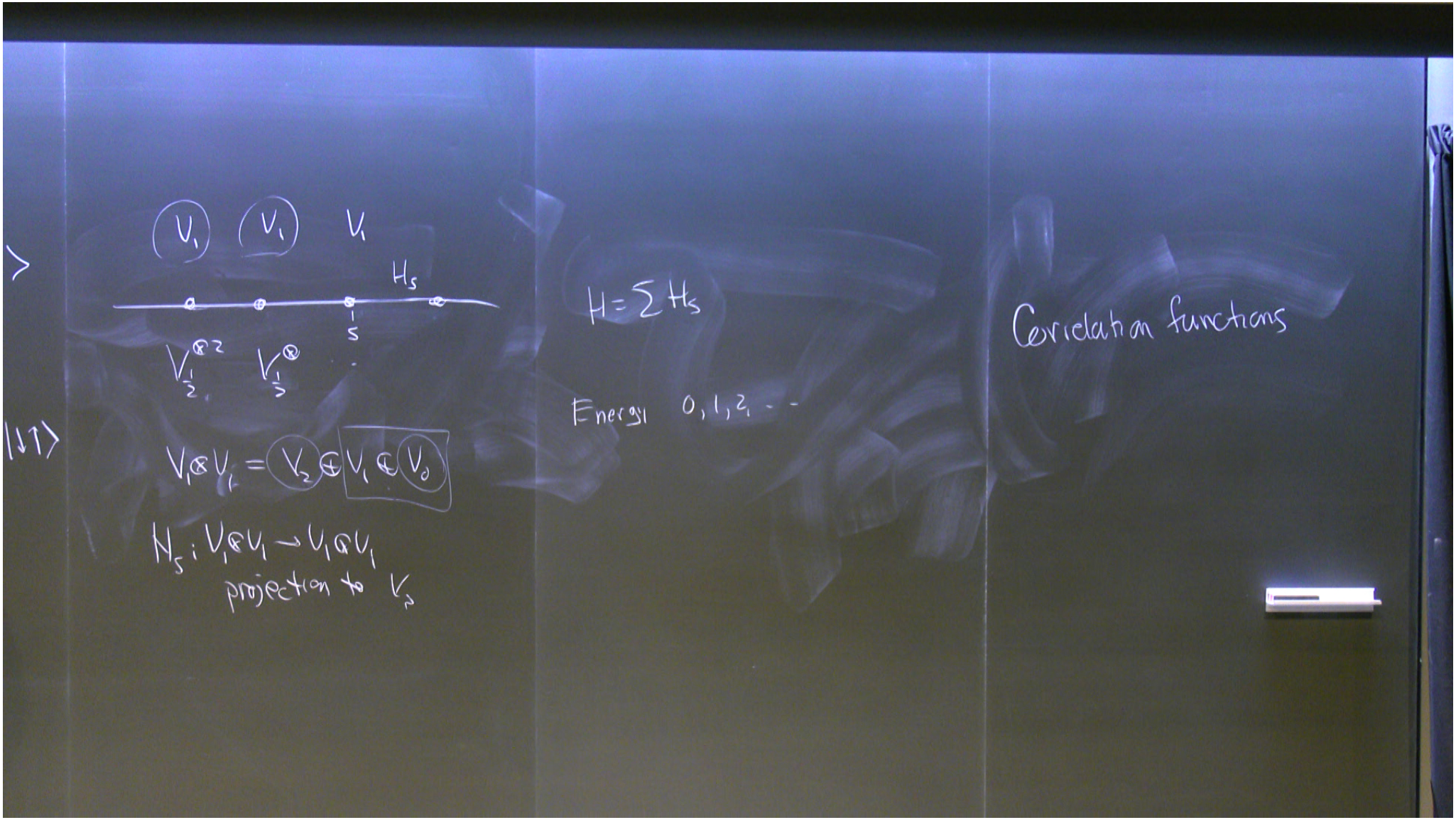
H_s projection operator

Valence Bond Solid

$SU(2)$ $\mathbb{C}^2$ Spin $\frac{1}{2}$

$Sym^n(\mathbb{C}^2)$ Spin $(\frac{n}{2})$

V_n = Spin-n rep


V_1 V_1 V_1
H_s
s
$V_{\frac{1}{2}}^{\otimes 2}$ $V_{\frac{1}{2}}^{\otimes}$
$|\psi\rangle$
$V_1 \otimes V_1 = V_2 \oplus V_1 \oplus V_0$
$N_s : V_1 \otimes V_1 \to V_1 \otimes V_1$
projection to V_2
$H = \sum H_s$
Energy $0, 1, 2, \cdots$
Correlation functions

$\mathcal{H}$

A operator dynamical variables

self-adjoint A — observable

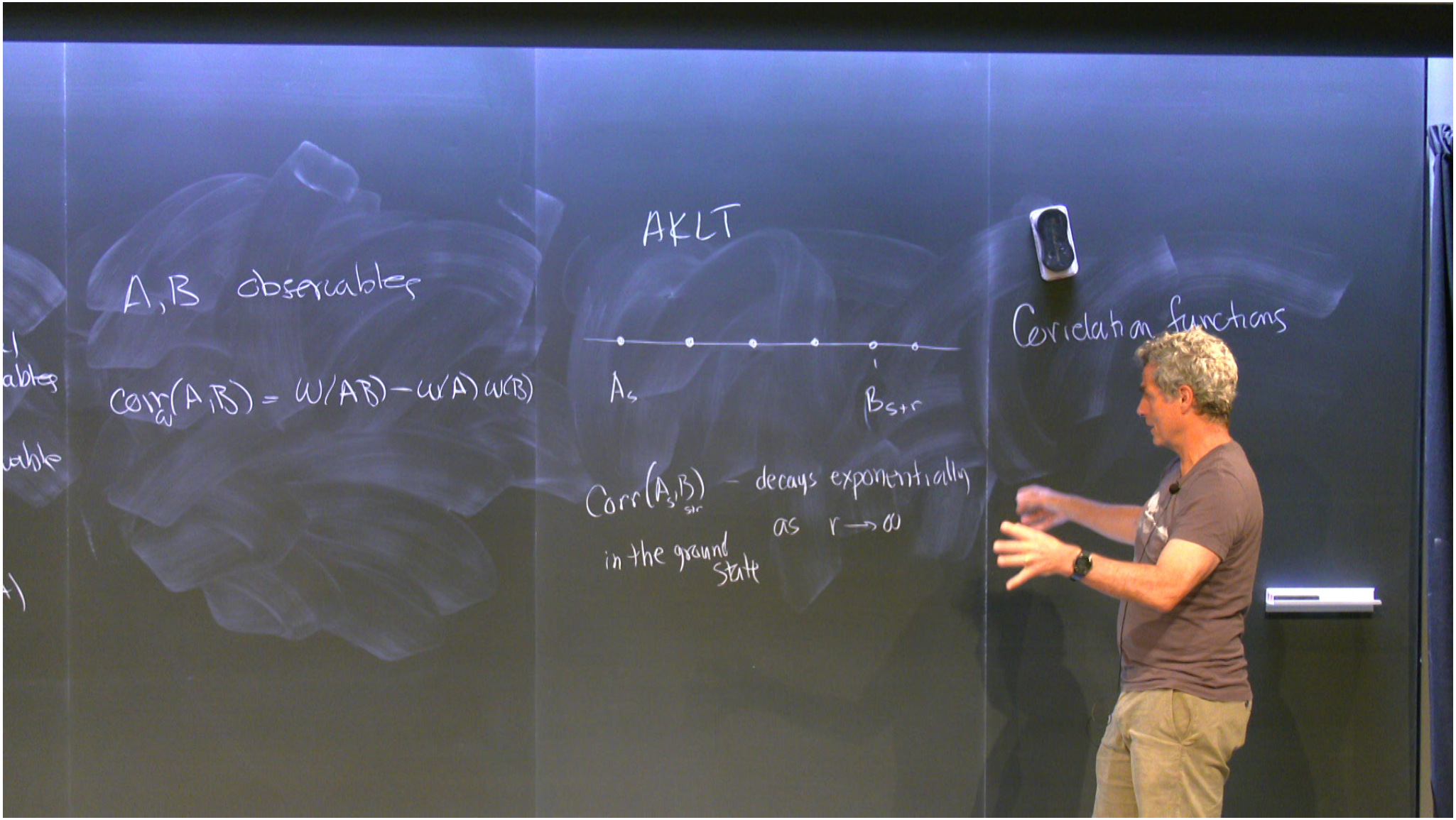
$|\psi\rangle \in \mathcal{H}$

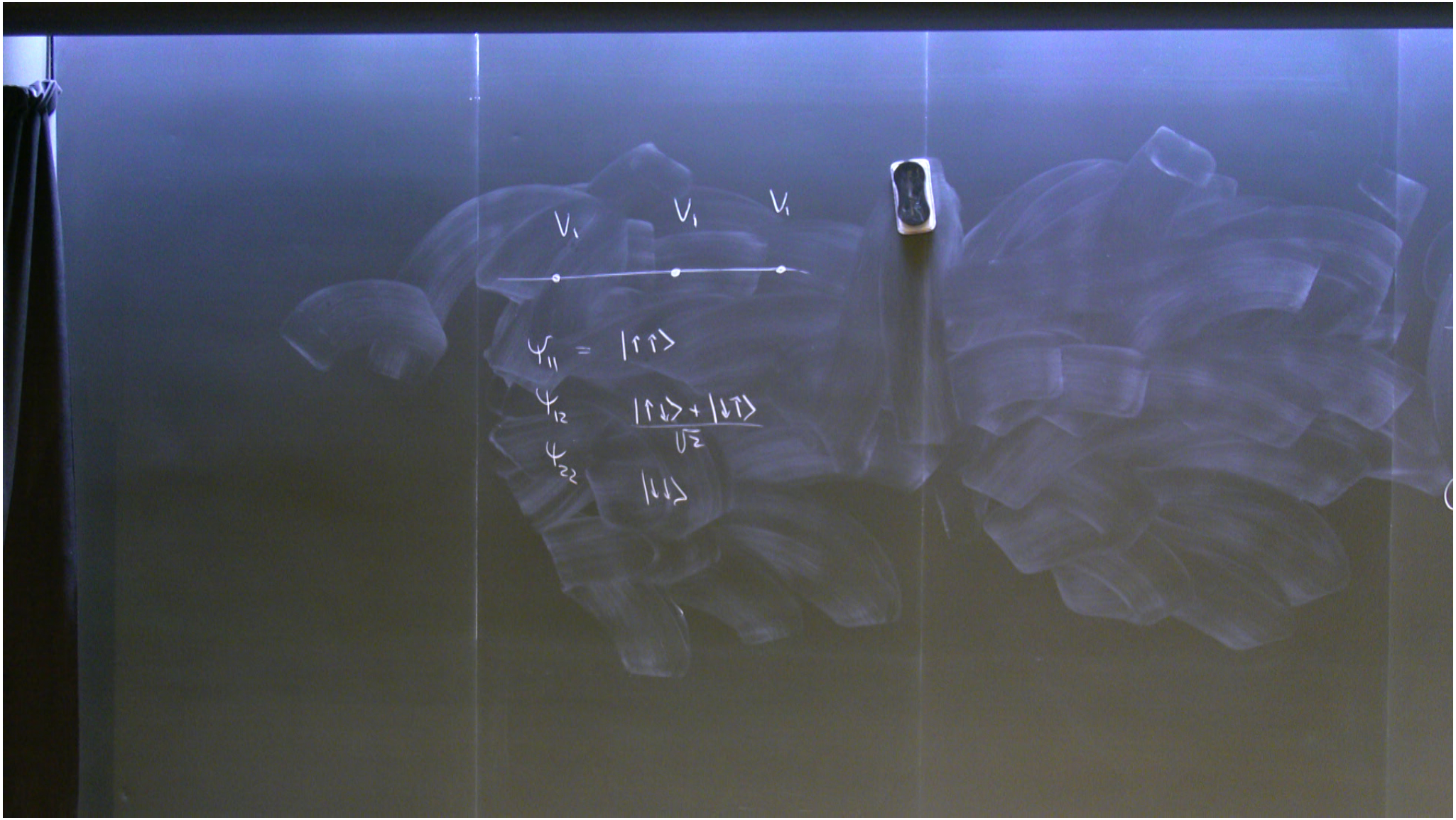
$\frac{\langle\psi|A|\psi\rangle}{\langle\psi|\psi\rangle} = \omega(A) = E(A)$

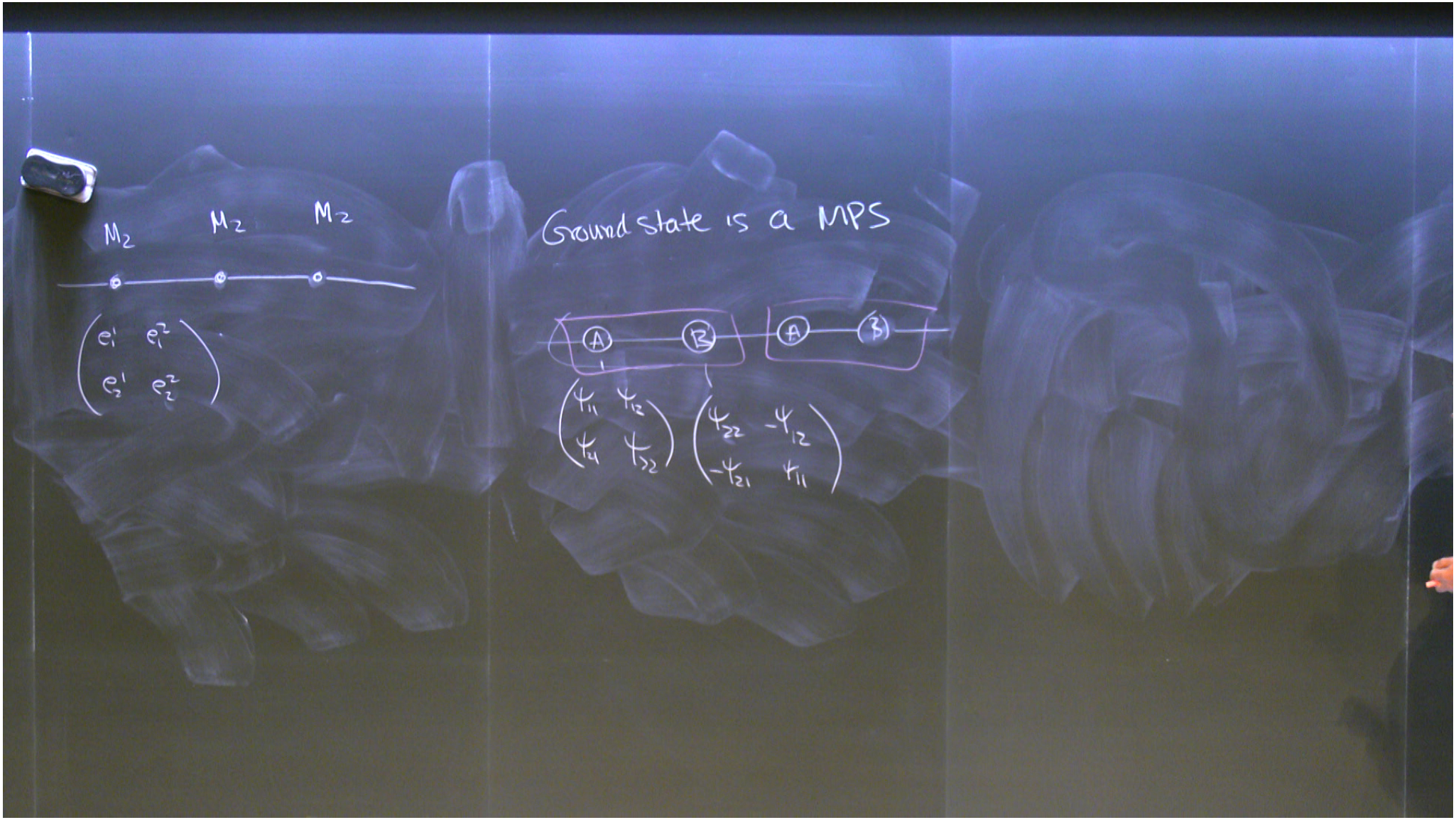
A, B observables

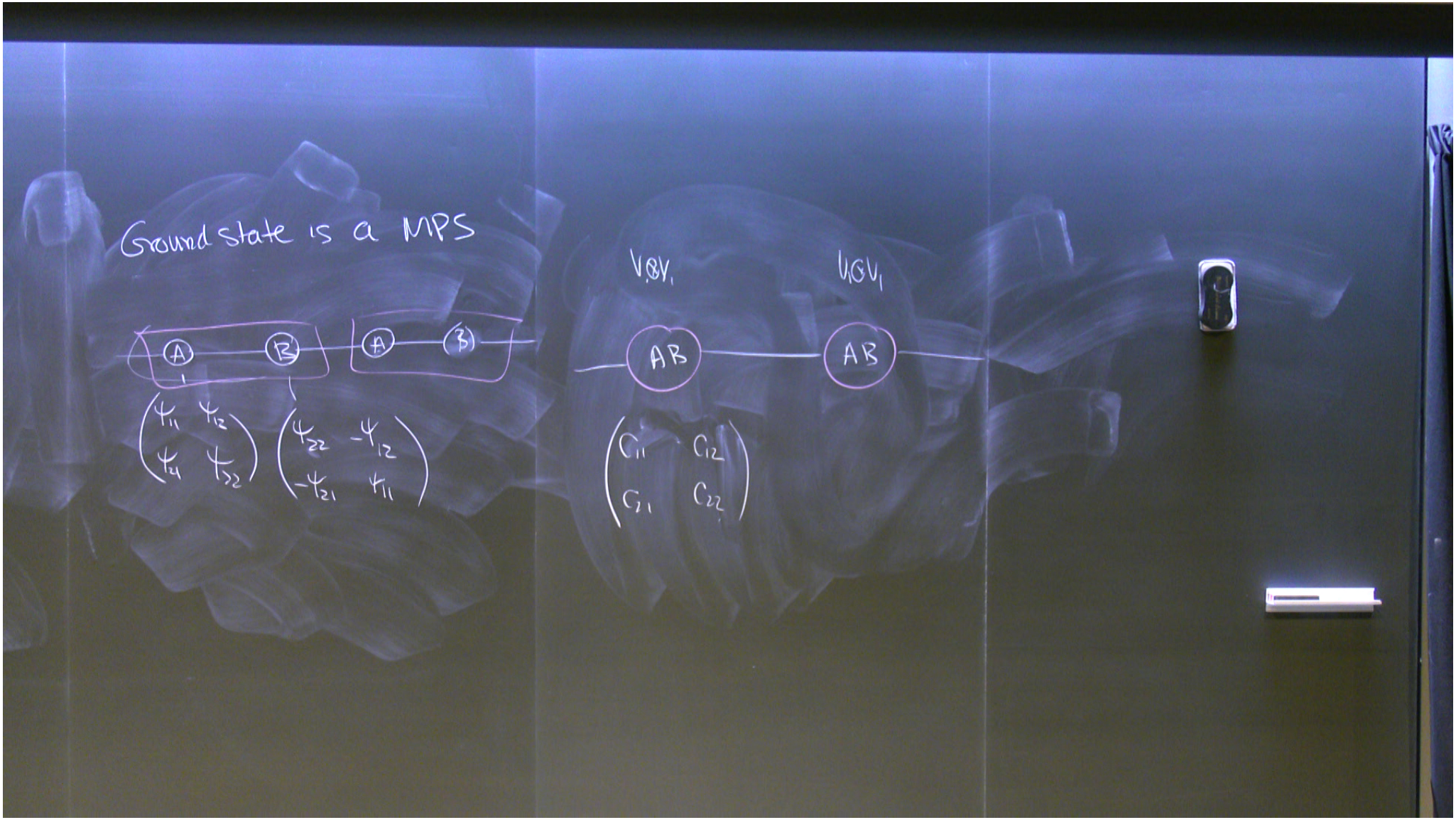
$Corr(A,B) = \omega(AB) - \omega(A)\omega(B)$

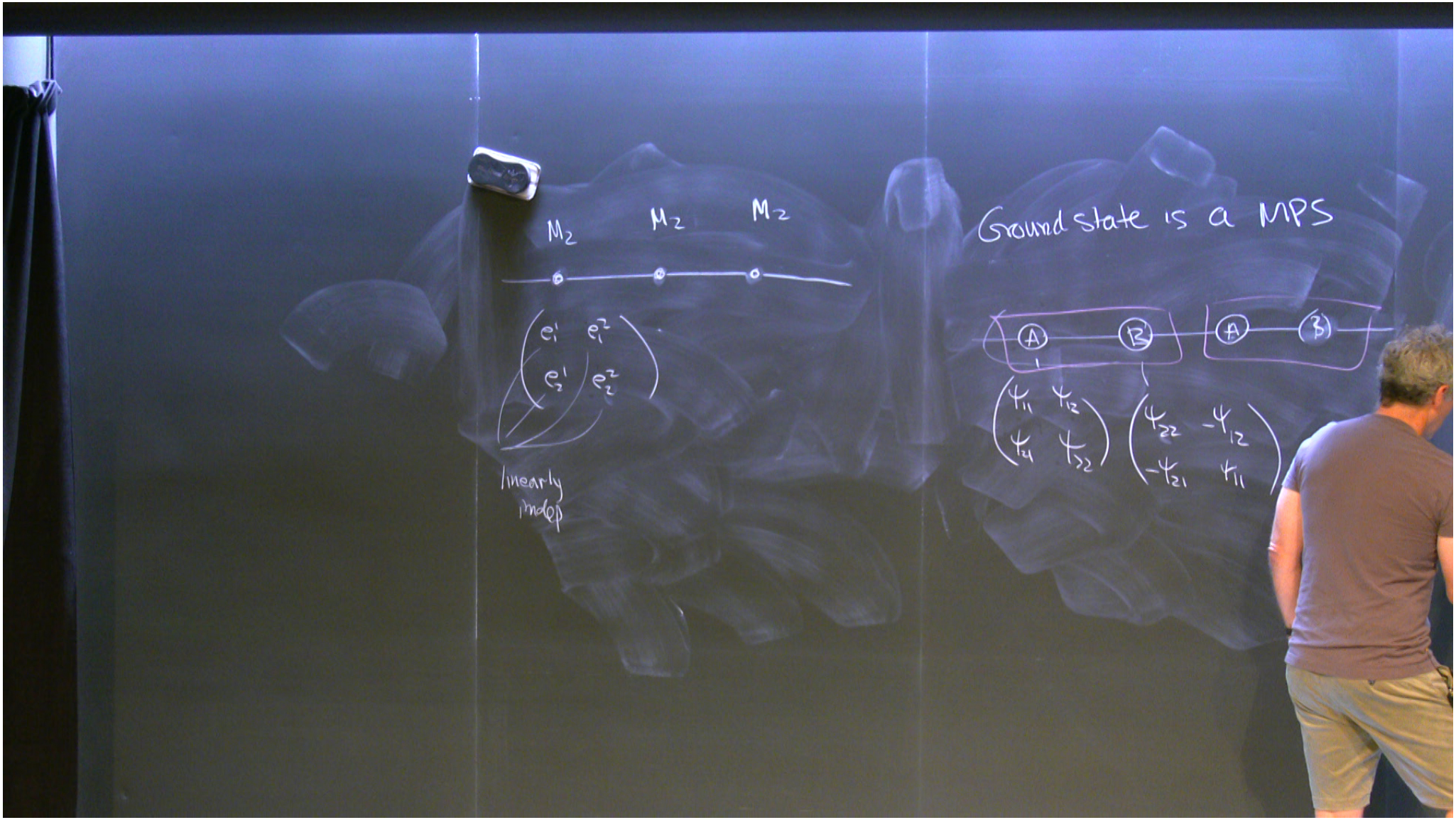


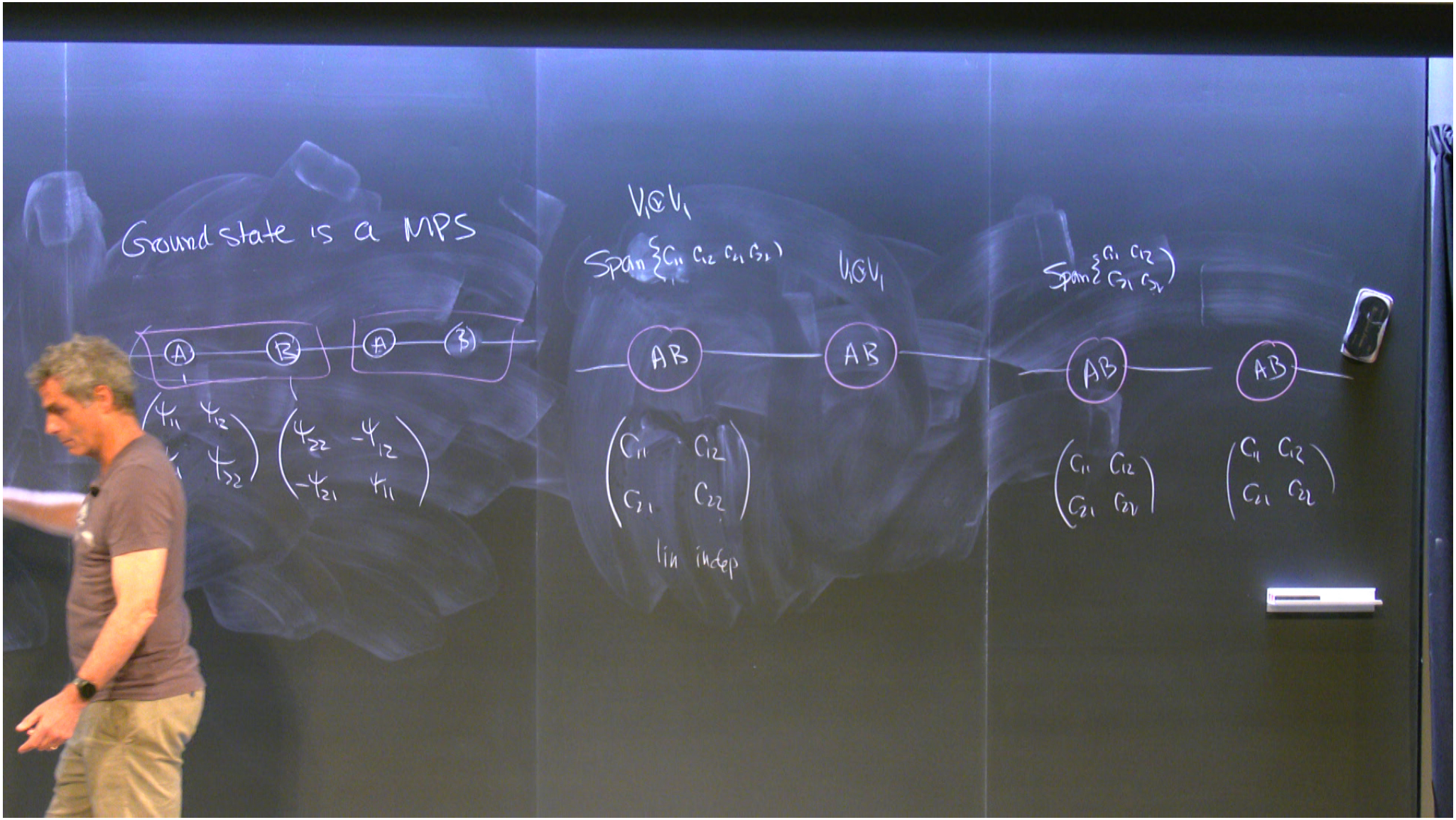

A, B observables
$Corr_\omega(A,B) = \omega(AB) - \omega(A)\omega(B)$
AKLT
A_s
B_{s+r}
$Corr(A_s, B_{s+r})$ – decays exponentially as $r \to \infty$
in the ground state
Correlation functions

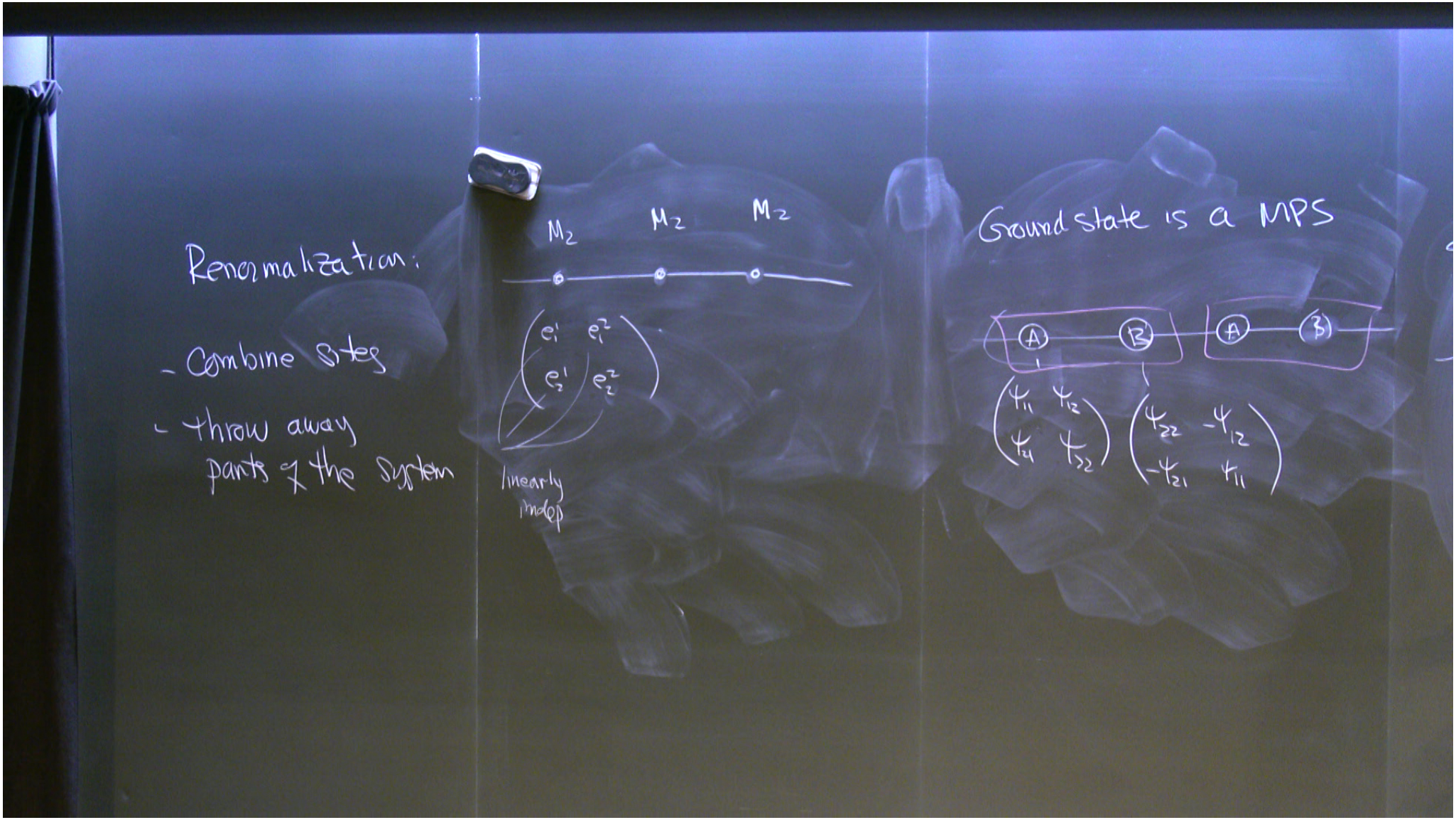

V_1 V_1 V_1
$\psi_{11} = |\uparrow\uparrow\rangle$
$\psi_{12} \quad \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$
$\psi_{22} \quad |\downarrow\downarrow\rangle$

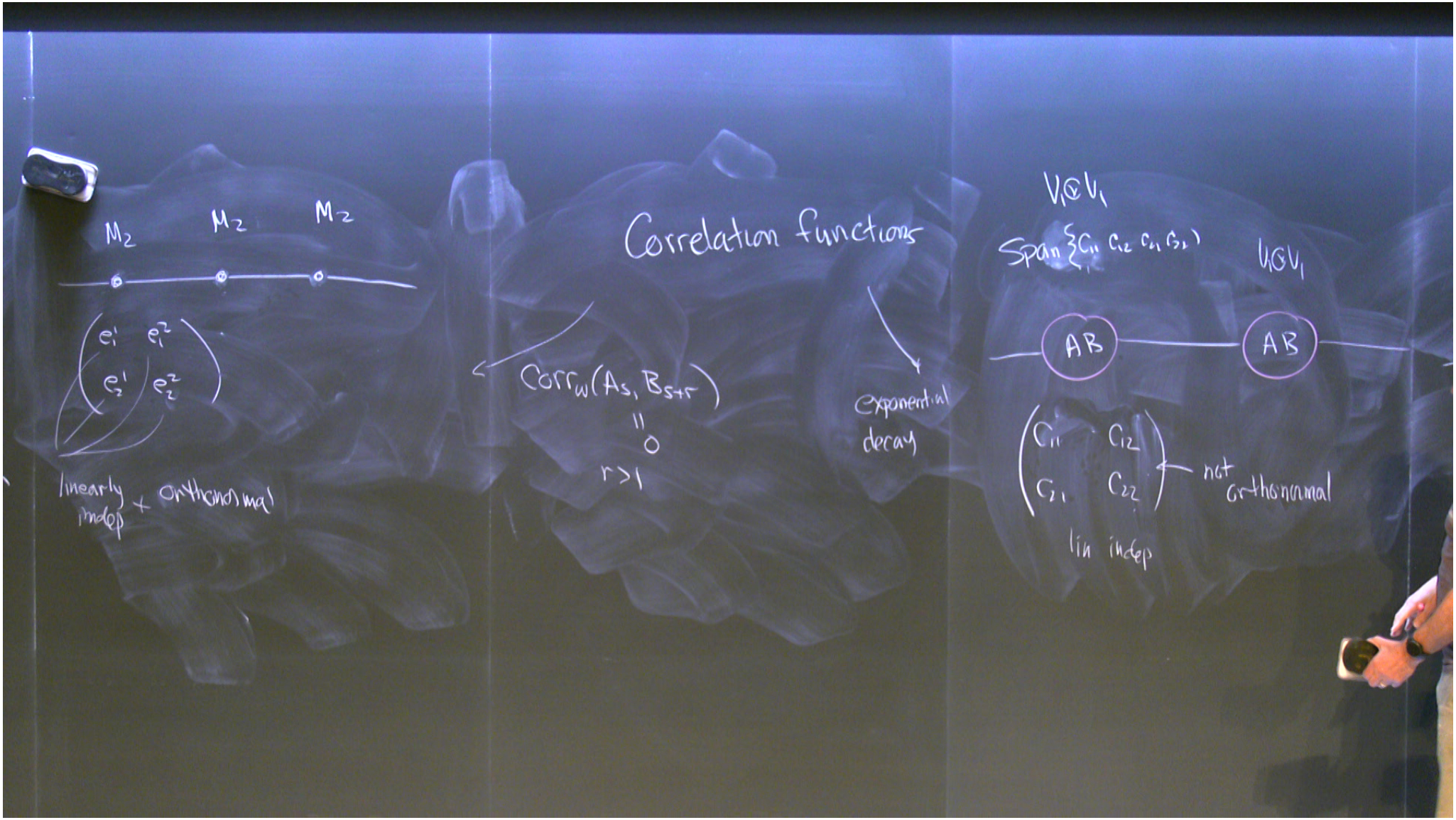

M2 M2 M2
Ground state is a MPS
A B A B

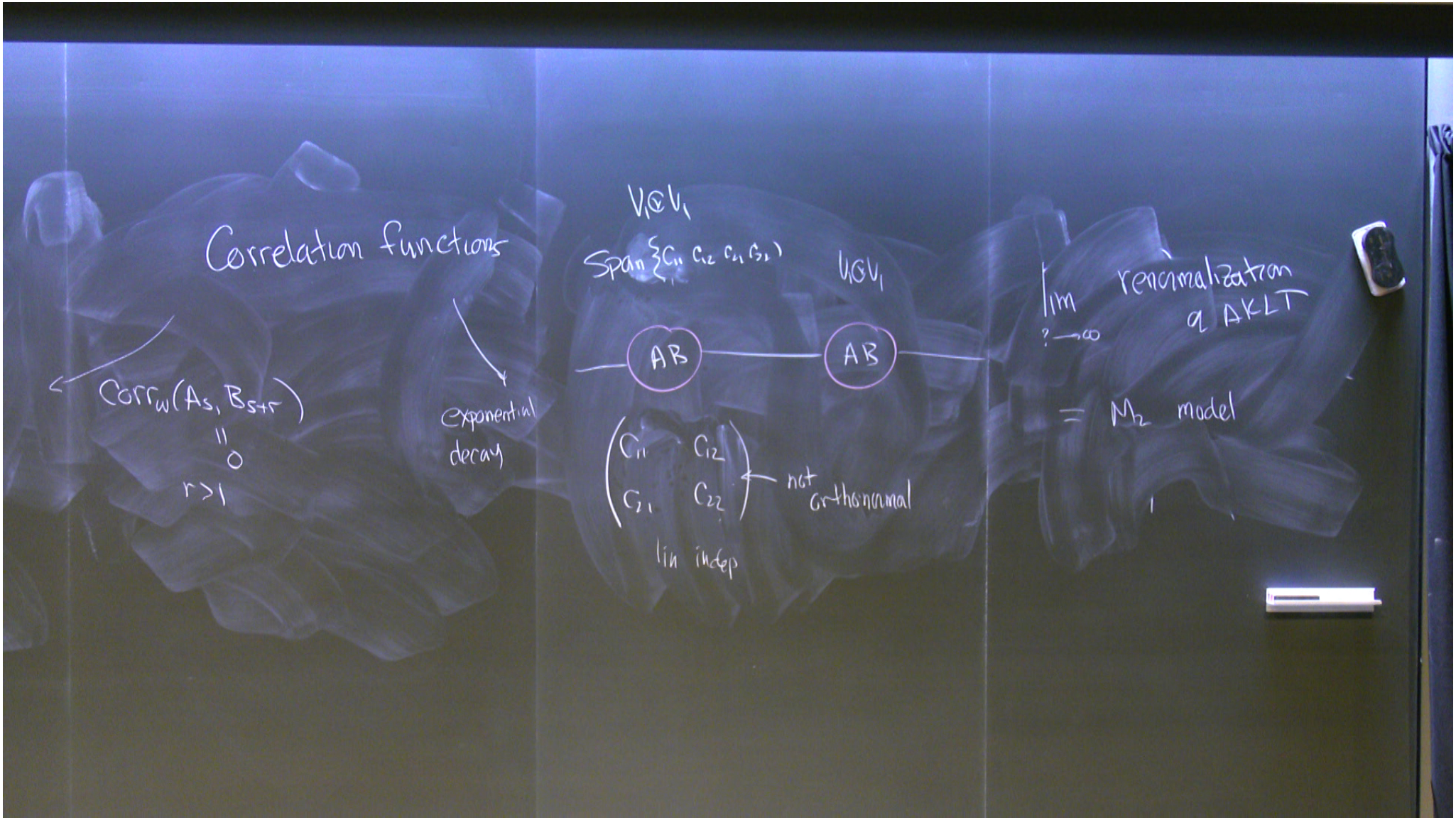

Ground state is a MPS
A
B
A
B
AB
AB

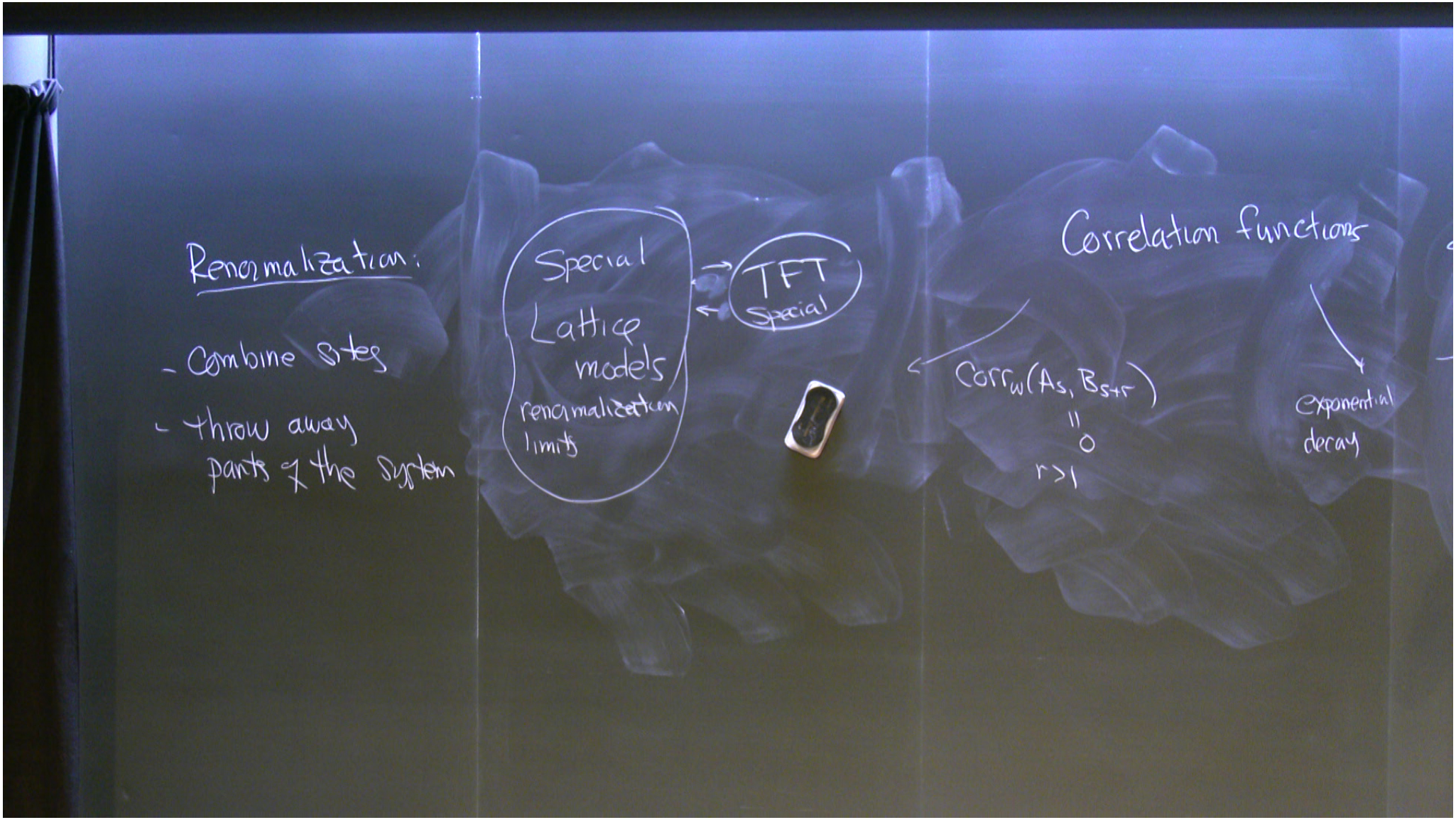

M_2 M_2 M_2
linearly indep
Ground state is a MPS
A B A B

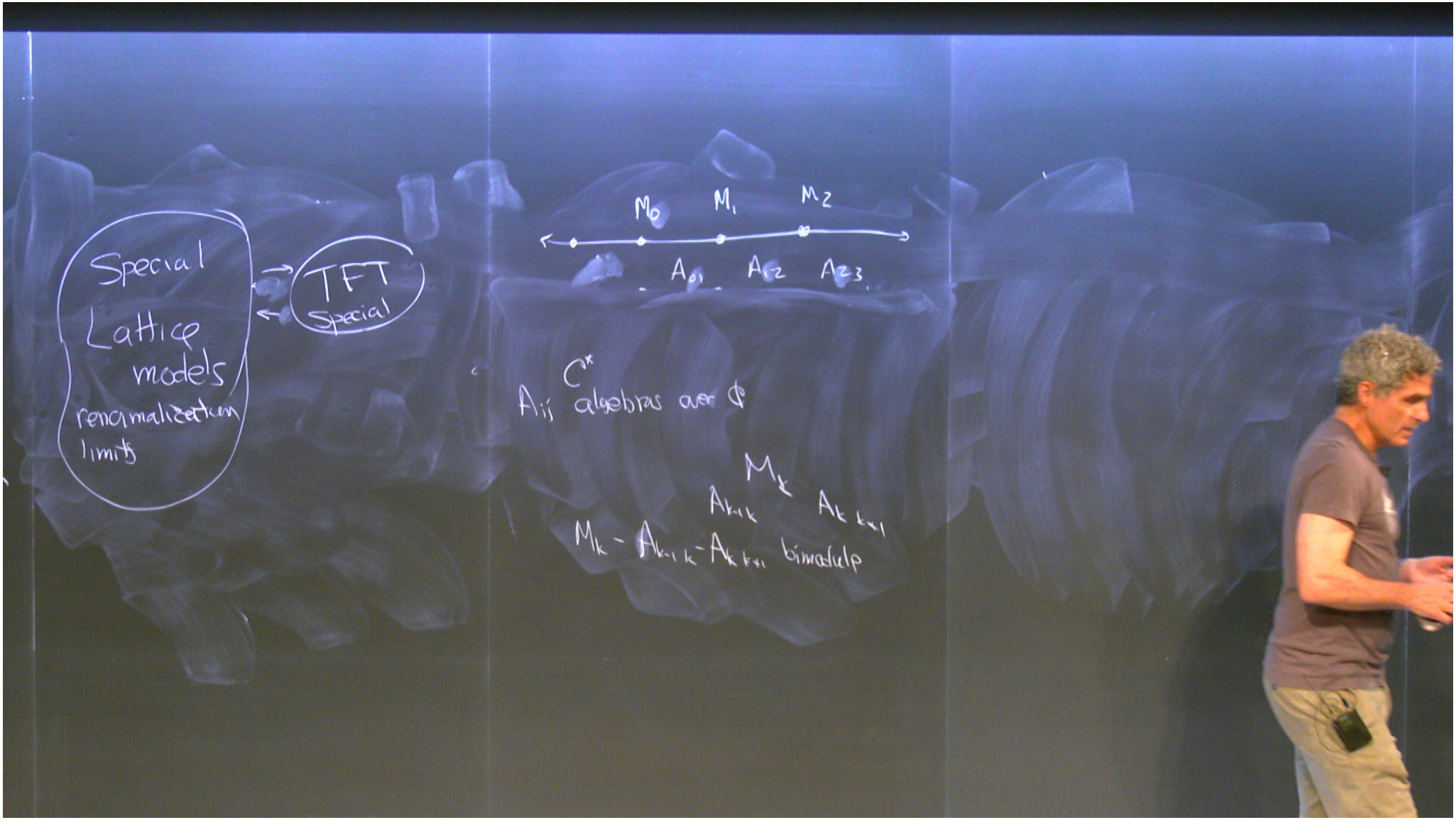

Ground state is a MPS
A
B
A
B
AB
AB
lin indep
AB
AB

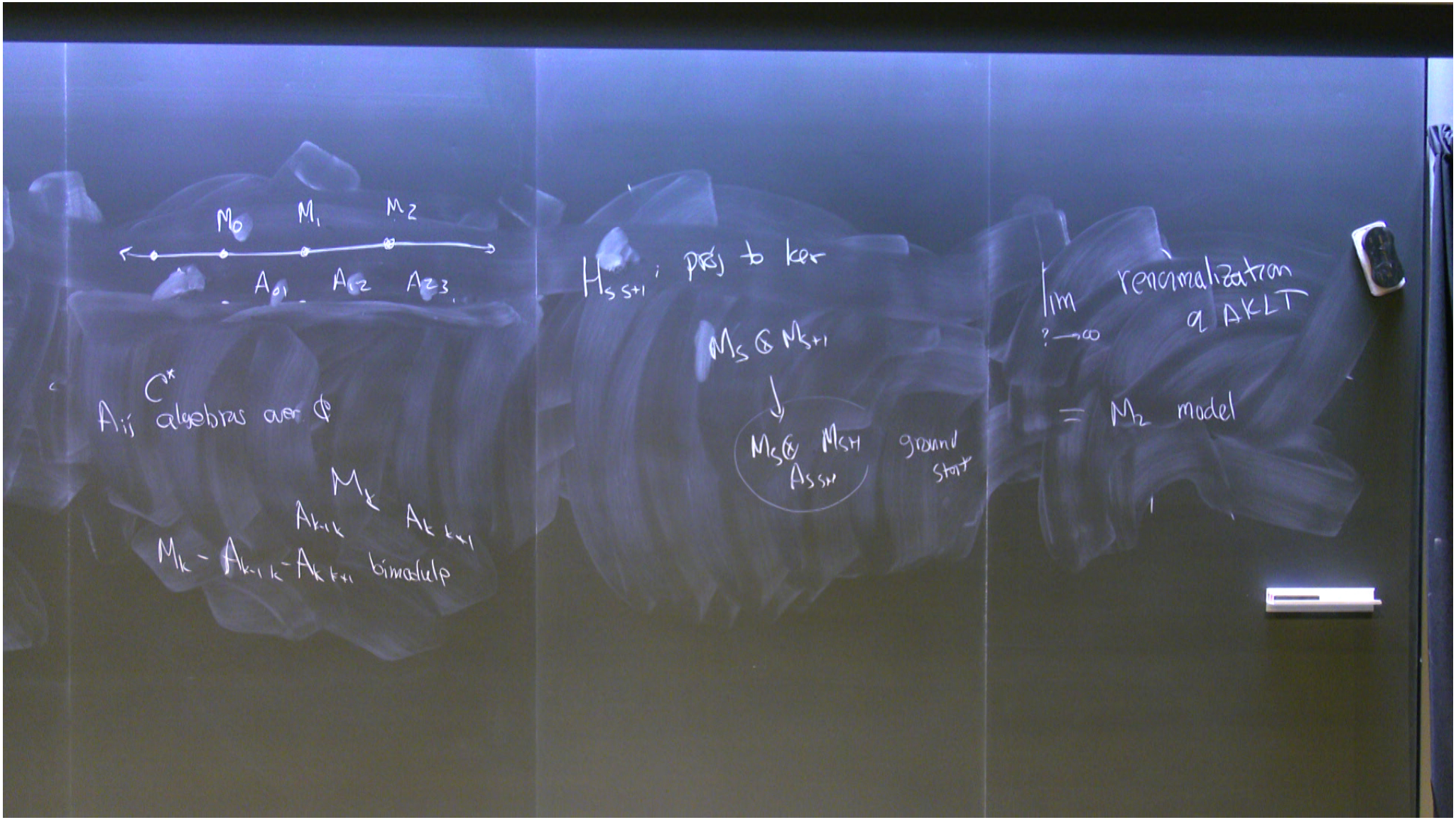

Renormalization:
- Combine sites
- throw away parts of the system
M_2 M_2 M_2
linearly map
Ground state is a MPS
A B A B


M2 M2 M2
linearly indep + orthonormal
Correlation functions
$Corr_w(A_s, B_{s+r})$
= 0
r>1
exponential decay
$V_1 \otimes V_1$
Span $\{C_{11}\ C_{12}\ C_{21}\ C_{22}\}$
$V_0 \otimes V_1$
AB
AB
not orthonormal
lin indep


Correlation functions
$Corr_w(A_s, B_{s+r})$
= 0
r>1
exponential decay
$V_1 \otimes V_1$
Span $\{c_{11}\ c_{12}\ c_{21}\ c_{22}\}$
$V_1 \otimes V_1$
AB
AB
$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$
not orthonormal
lin indep
lim renormalization of AKLT
$? \to \infty$
$= M_2$ model

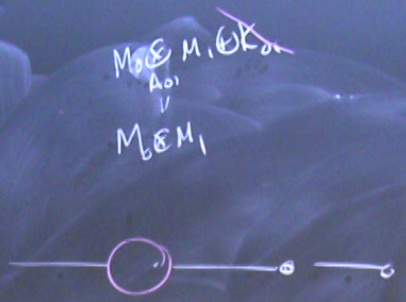

Renormalization:
- Combine sites
- throw away parts of the system
Special
Lattice models
renormalization limits
TFT
Special
Correlation functions
$Corr_w(A_s, B_{s+r})$
=
0
r>1
exponential decay

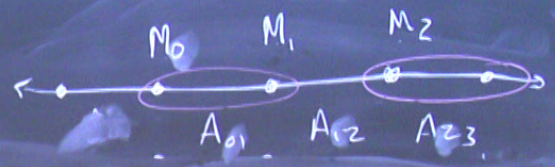

Special
Lattice
models
renormalization
limits
TFT
Special
M_0 M_1 M_2
A_{01} A_{12} A_{23}
C^*
A_{ij} algebras over $\mathbb{C}$
M_k
$A_{k-1\,k}$ $A_{k\,k+1}$
$M_k - A_{k-1\,k} - A_{k\,k+1}$ bimodule


M0 M1 M2
A01 A12 A23
C*
Aij algebras over Φ
Mk
Ak-1k Akk+1
Mk - Ak-1k - Akk+1 bimodule
Hss+1 ; proj to ker
Ms ⊗ Ms+1
Ms ⊗ Ms+1
Ass+1
ground state
lim
? → ∞
renormalization
of AKLT
= M2 model

Renormalization:

- Combine sites
- throw away parts of the system





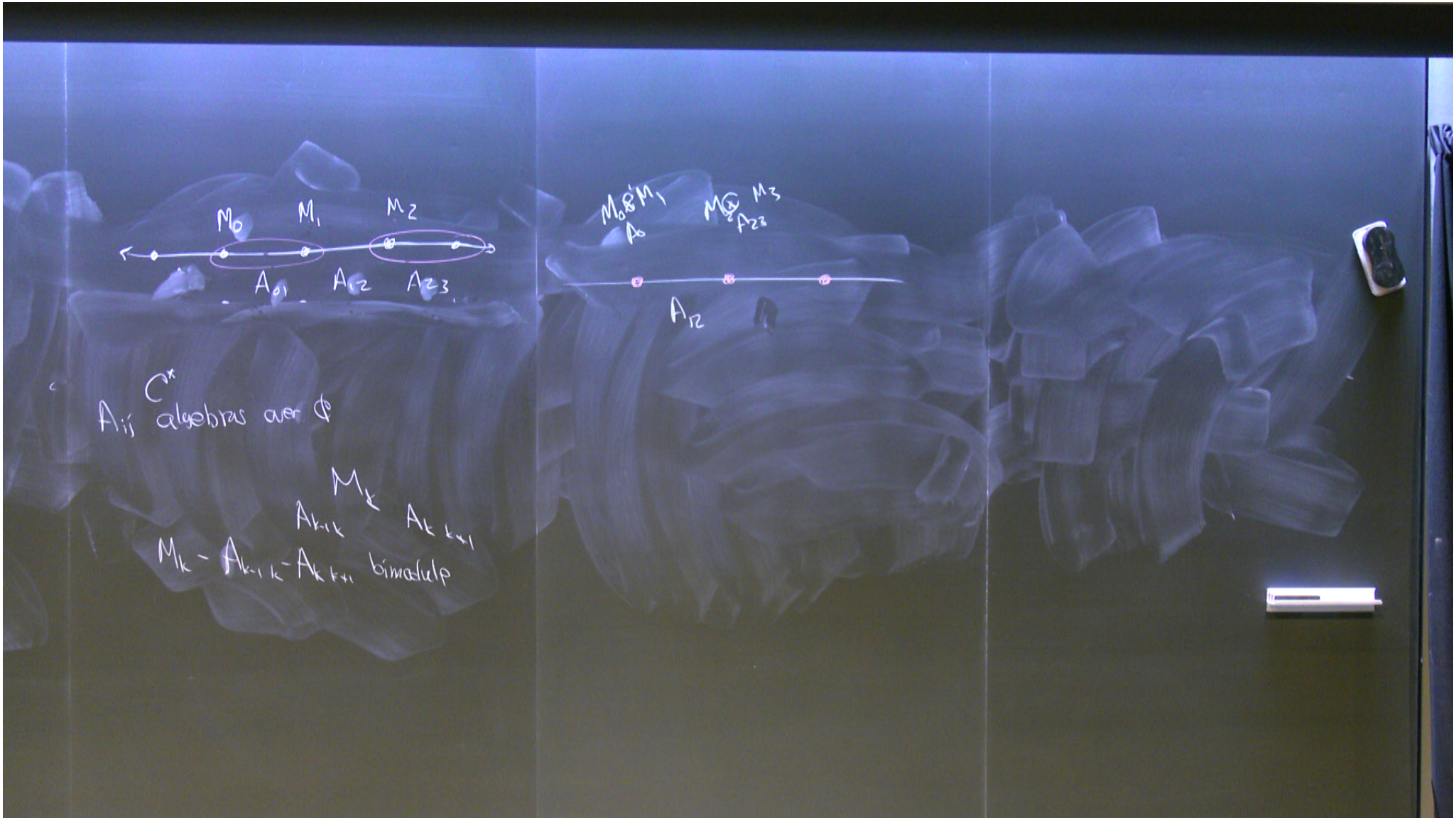
C^*

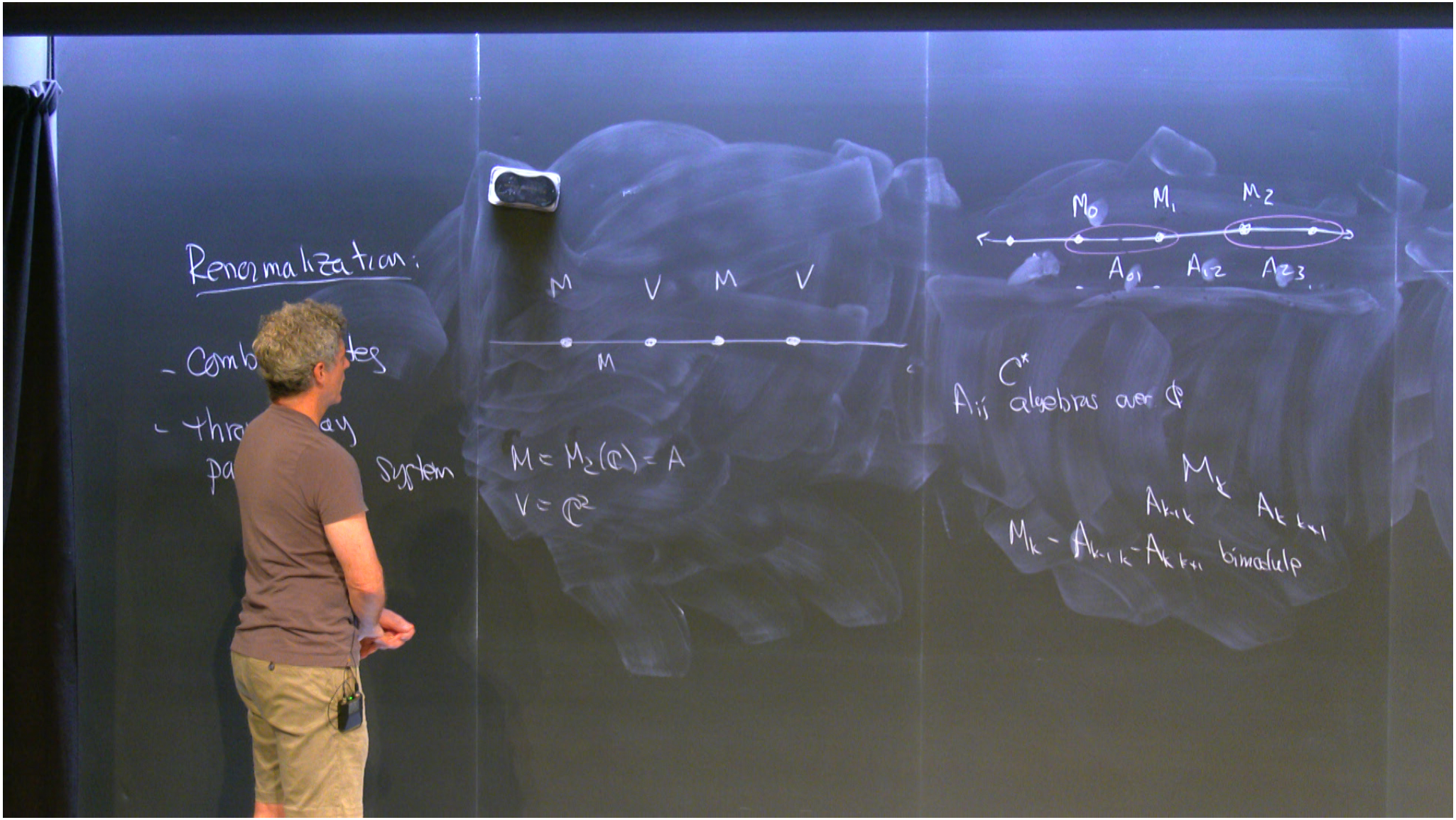
A_{ij} algebras over $\mathbb{C}$

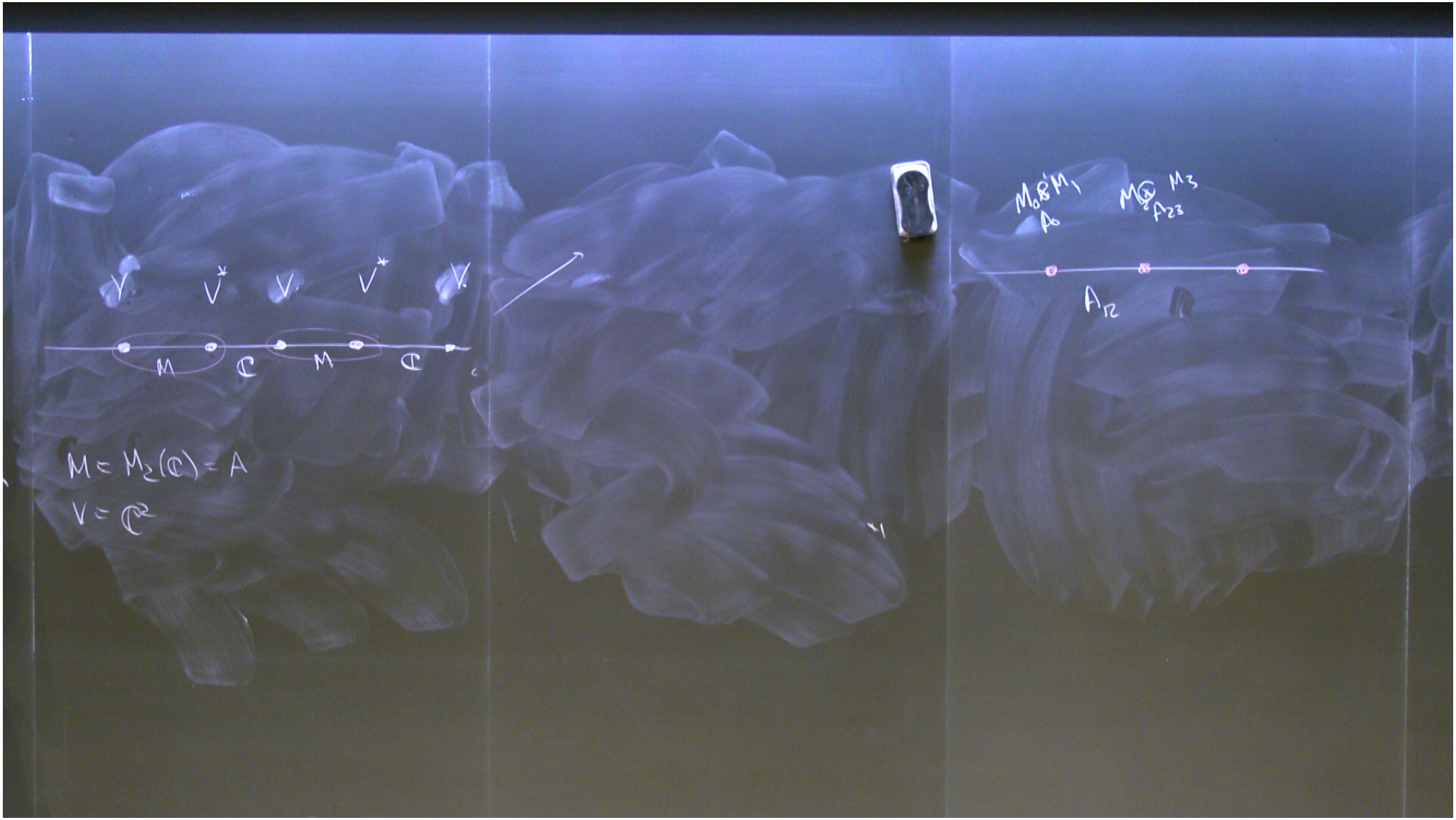
M_k

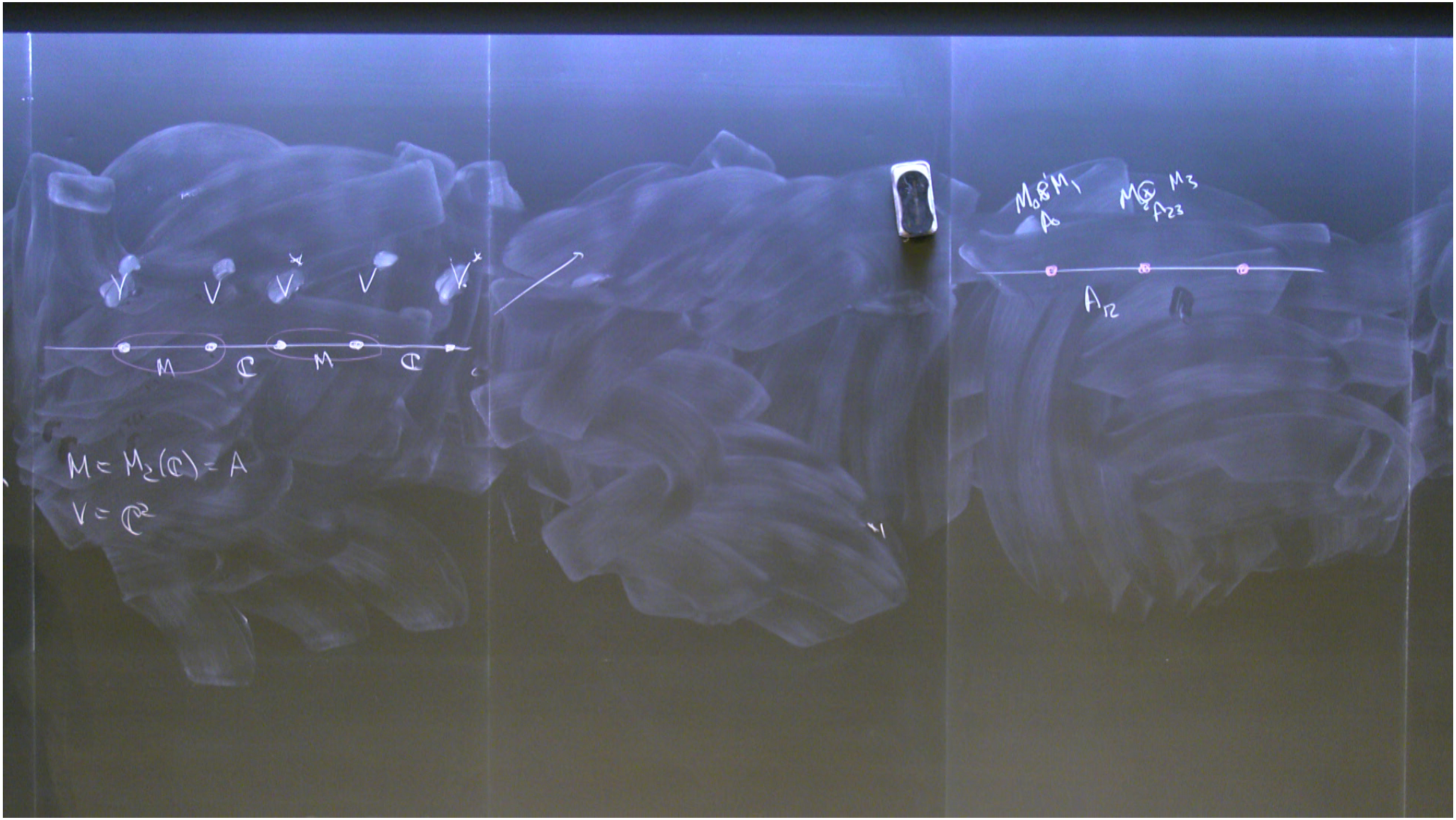
$A_{k-1\,k}$ $A_{k\,k+1}$

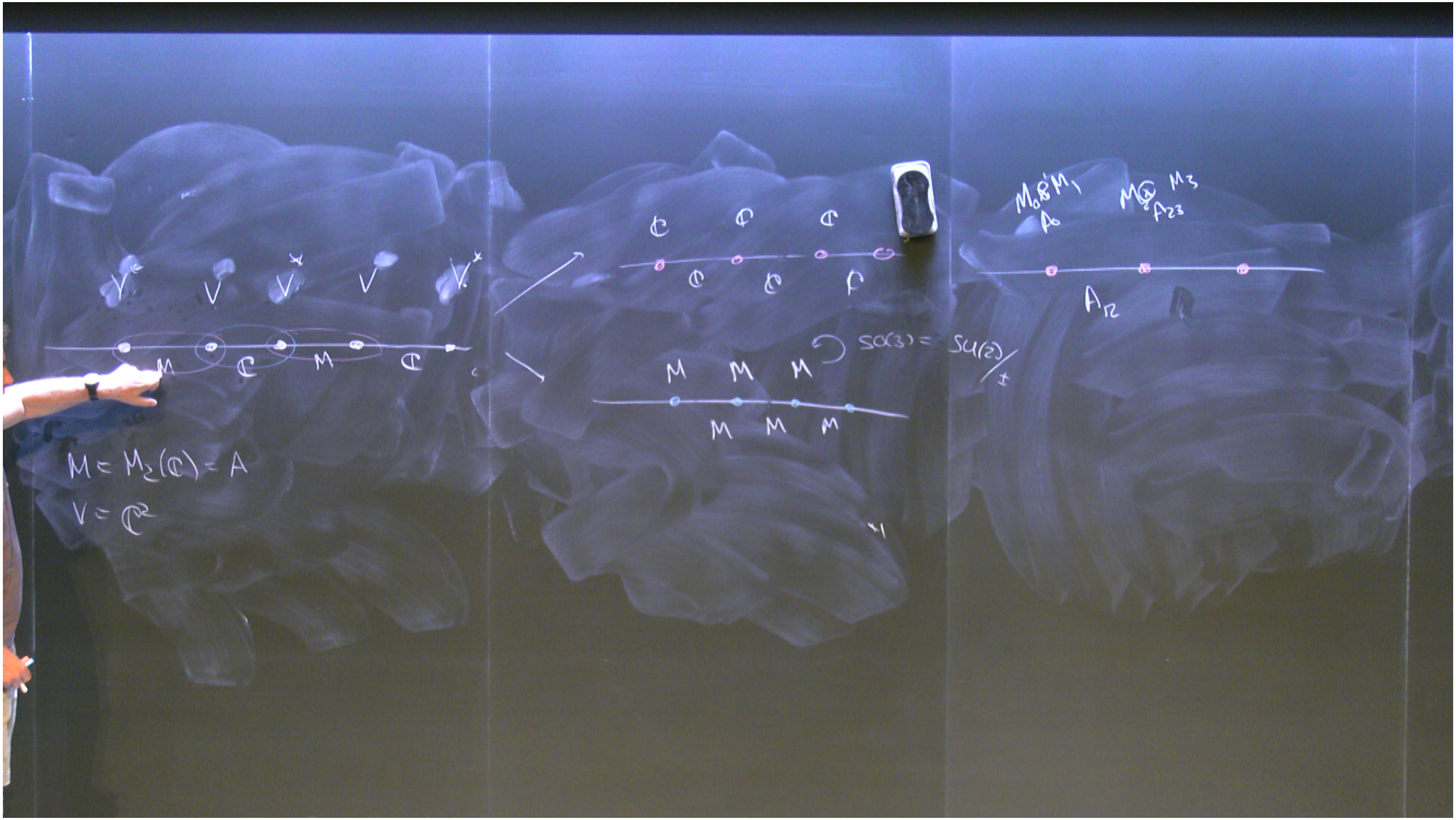
M_k - $A_{k-1\,k}$ - $A_{k\,k+1}$ bimodule


M_0 M_1 M_2
A_{01} A_{12} A_{23}
$M_0 \& M_1$
A_0
M_2 M_3
A_{23}
A_{12}
C^*
A_{ij} algebras over $\mathbb{C}$
M_k
$A_{k-1\,k}$ $A_{k\,k+1}$
$M_k - A_{k-1\,k} - A_{k\,k+1}$ bimodule


Renormalization:
- Comb
teg
- thre
ay
pa
System
M V M V
M
M = M₂(ℂ) = A
V = ℂ²
M₀ M₁ M₂
A₀₁ A₁₂ A₂₃
C*
Aᵢⱼ algebras over ℂ
Mₖ
Aₖ₋₁ₖ Aₖ ₖ₊₁
Mₖ - Aₖ₋₁ₖ - Aₖ ₖ₊₁ bimodule


M
ℂ
M
ℂ
M = M₂(ℂ) = A
V = ℂ²
A₀
A₂₃
A₁₂


V V V V V
M ℂ M ℂ
M = M₂(ℂ) = A
V = ℂ²
M₀ & M₁
A₀
M₂ & M₃
A₂₃
A₁₂


M = M₂(ℂ) = A
V = ℂ²
SO(3) = SU(2)/±1