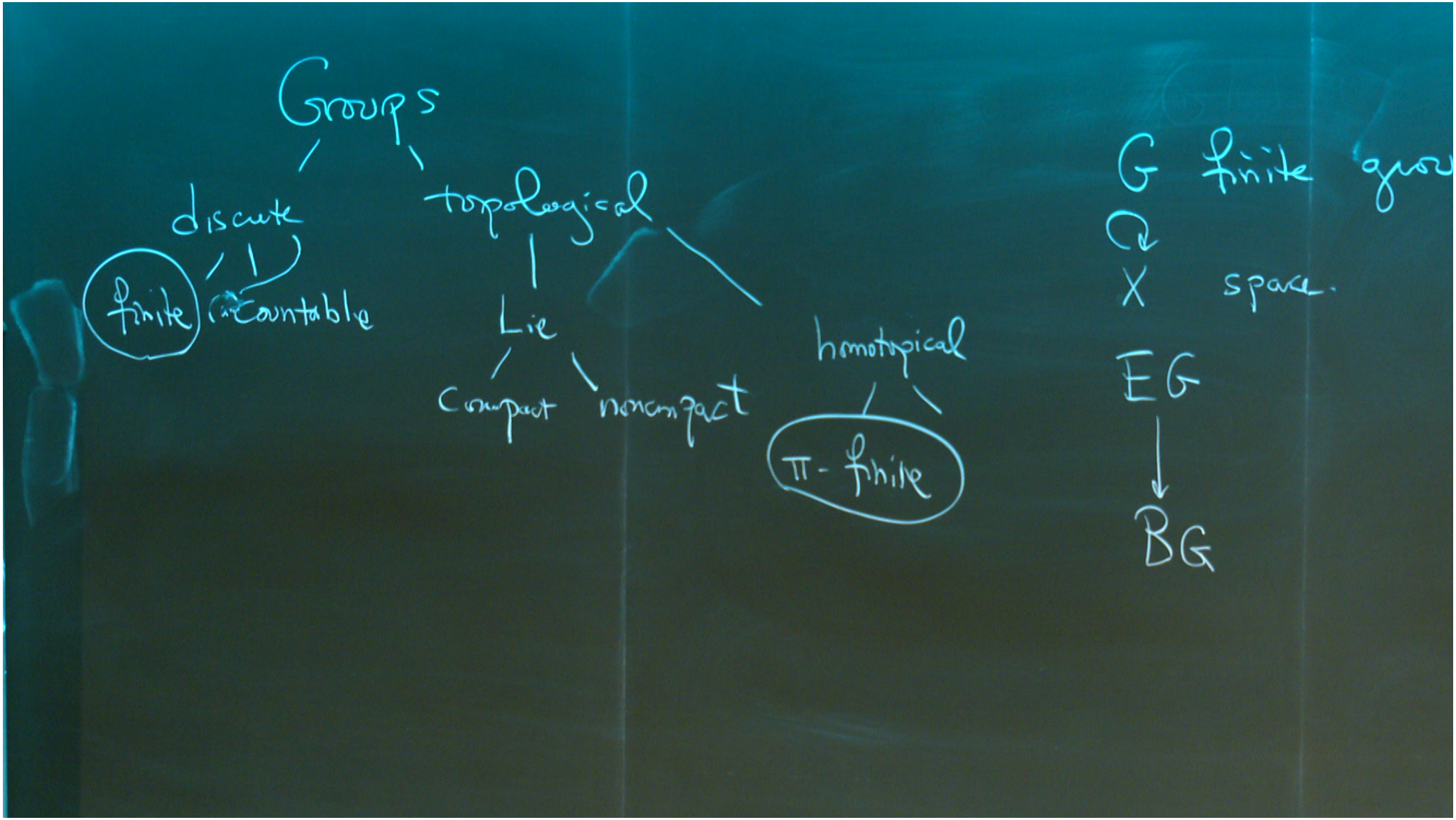
Title: Finite symmetry in QFT

Speakers: Dan Freed

Collection: Global Categorical Symmetries

Date: June 13, 2022 - 11:30 AM

URL: https://pirsa.org/22060026


Groups
discrete
topological
finite
countable
Lie
homotopical
Compact
noncompact
π - finite
G finite grou
G ↷ X space.
EG
BG

Symmetry

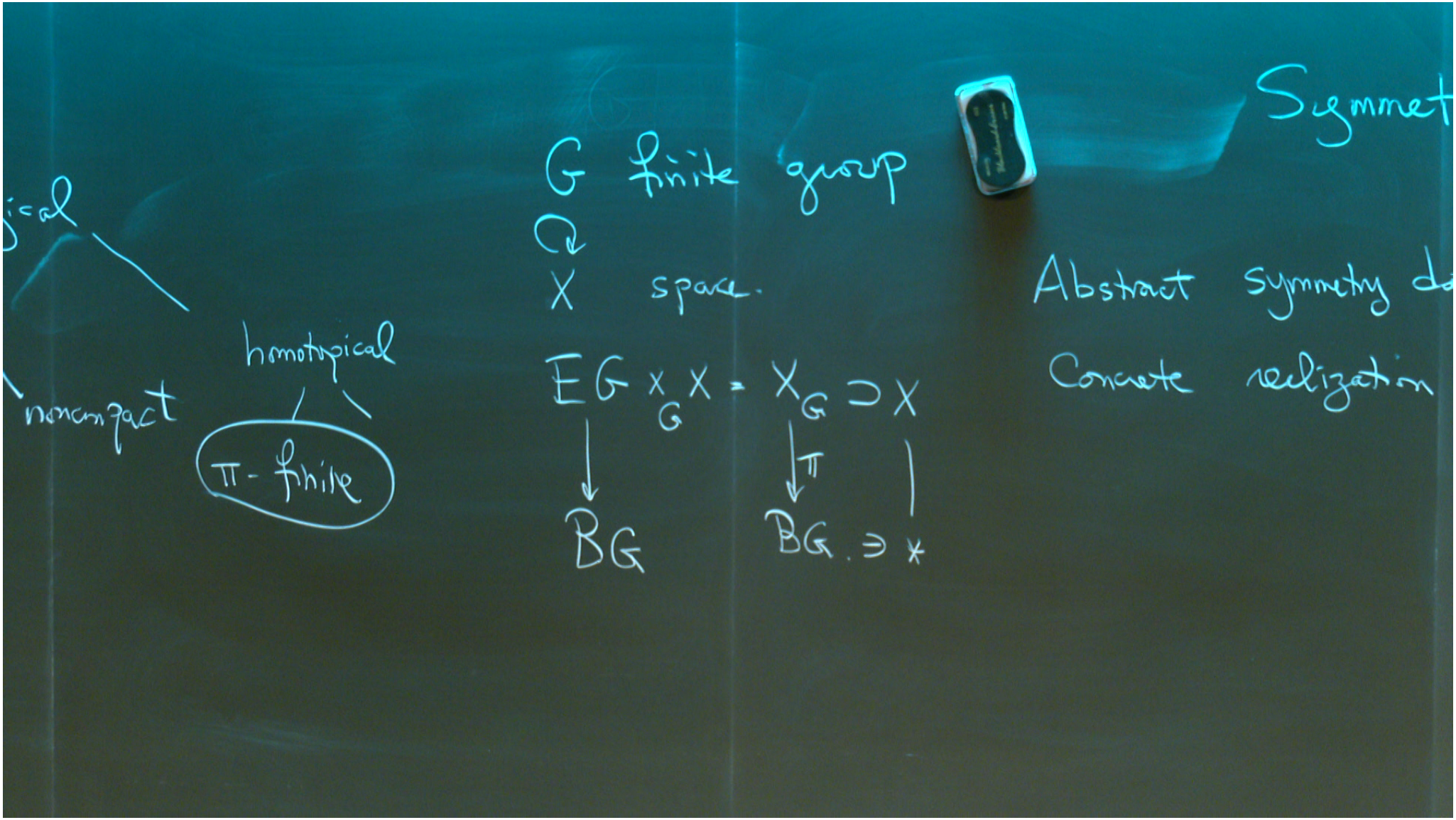
e group

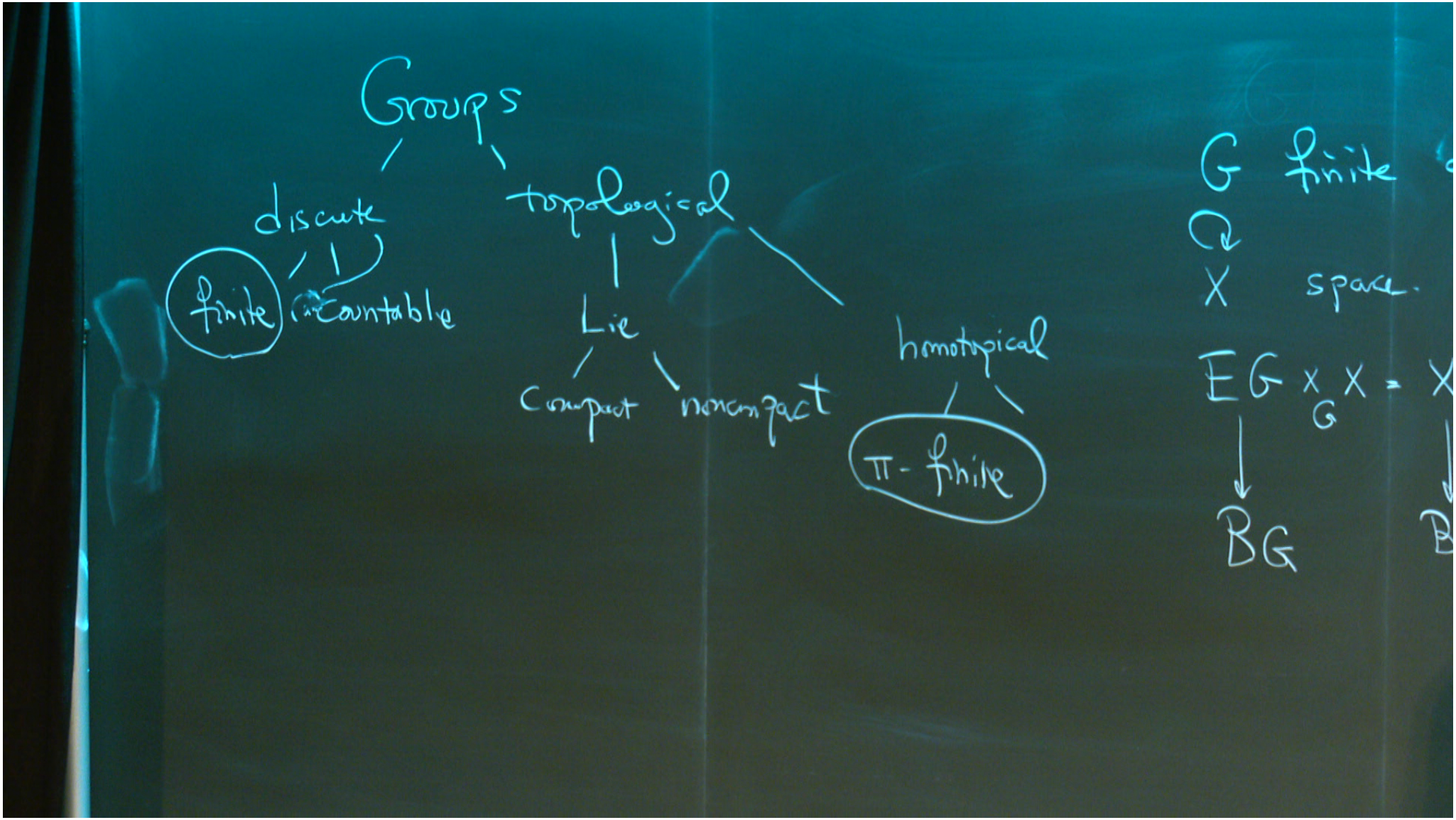
Abstract symmetry data: $(BG, *)$

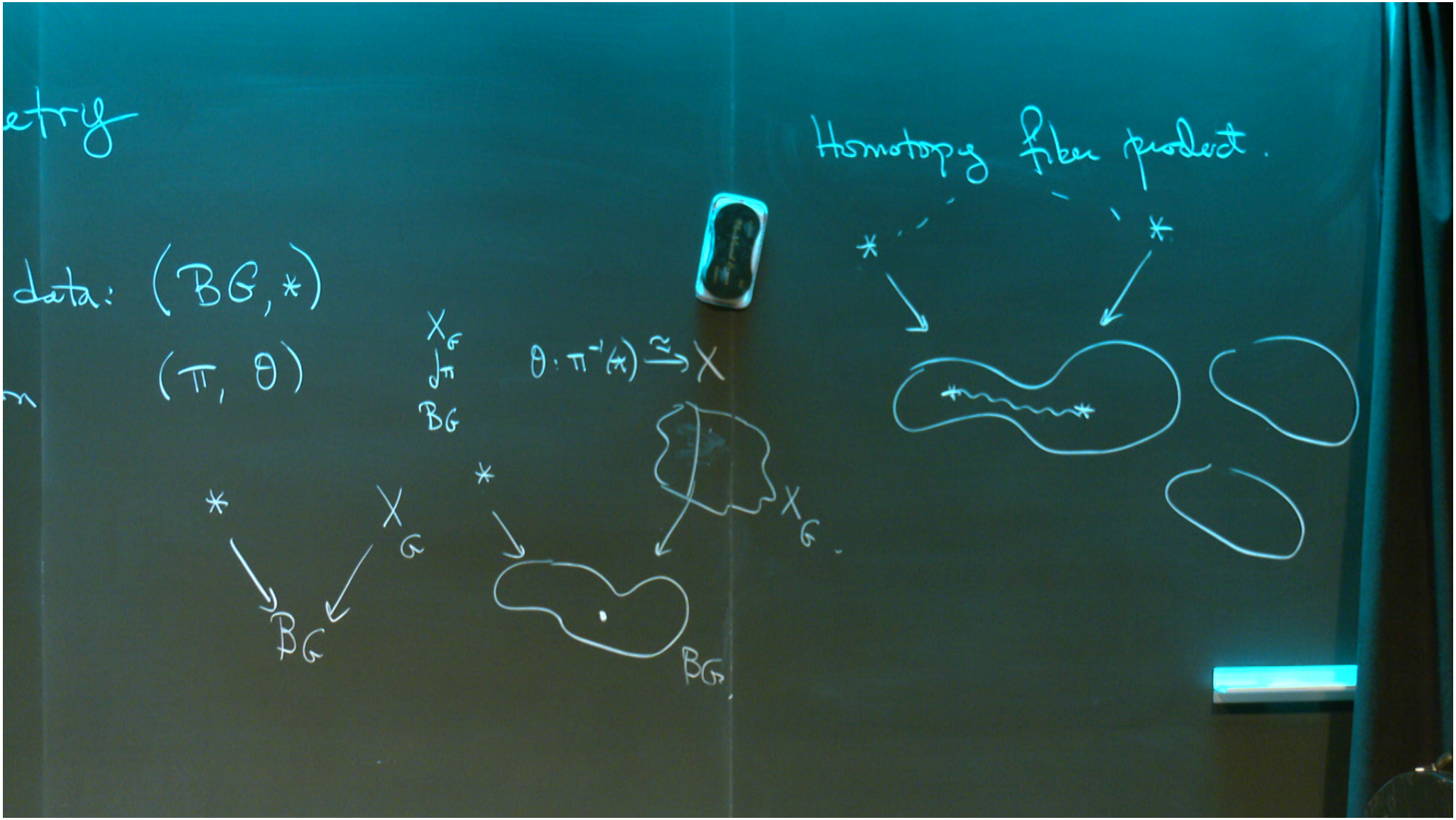
Concrete realization (π, θ)

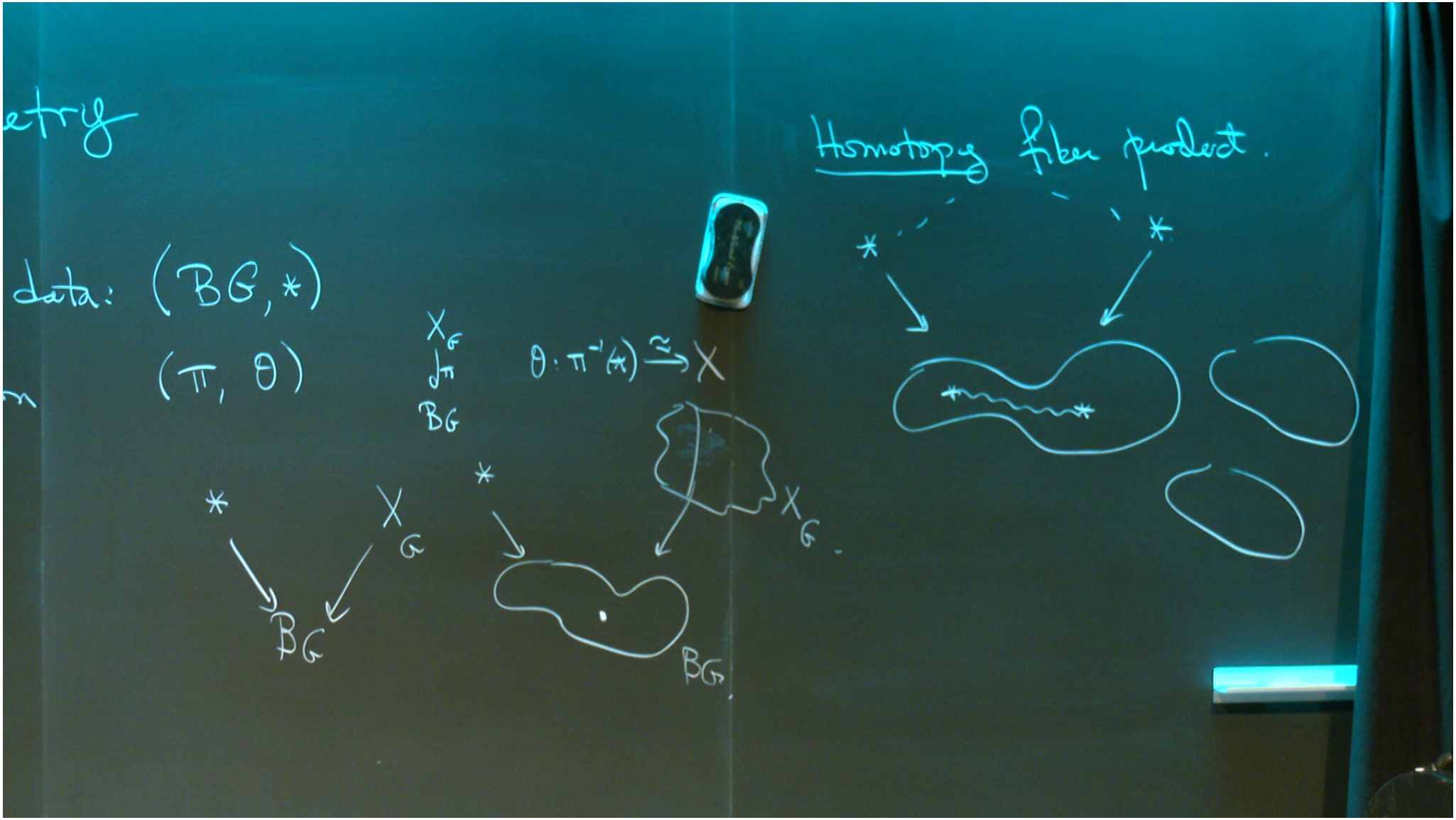
$$\begin{array}{ccc} X_G & \supset & X \\ \downarrow{\scriptstyle \pi} & & | \\ BG & \ni & * \end{array}$$

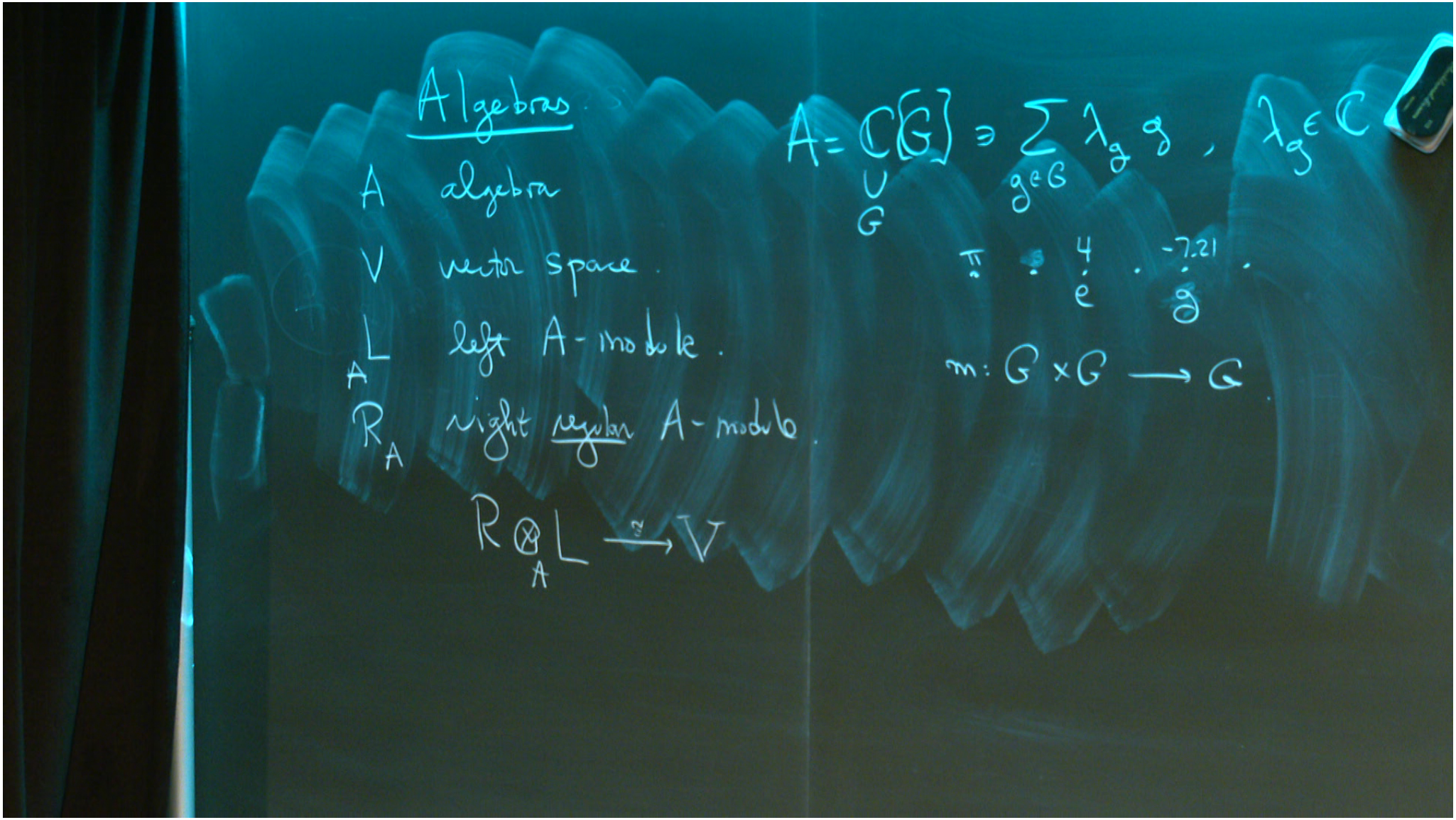
$$\begin{array}{c} X_G \\ \downarrow{\scriptstyle \pi} \\ BG \end{array} \qquad \theta : \pi^{-1}(*) \xrightarrow{\simeq} X$$

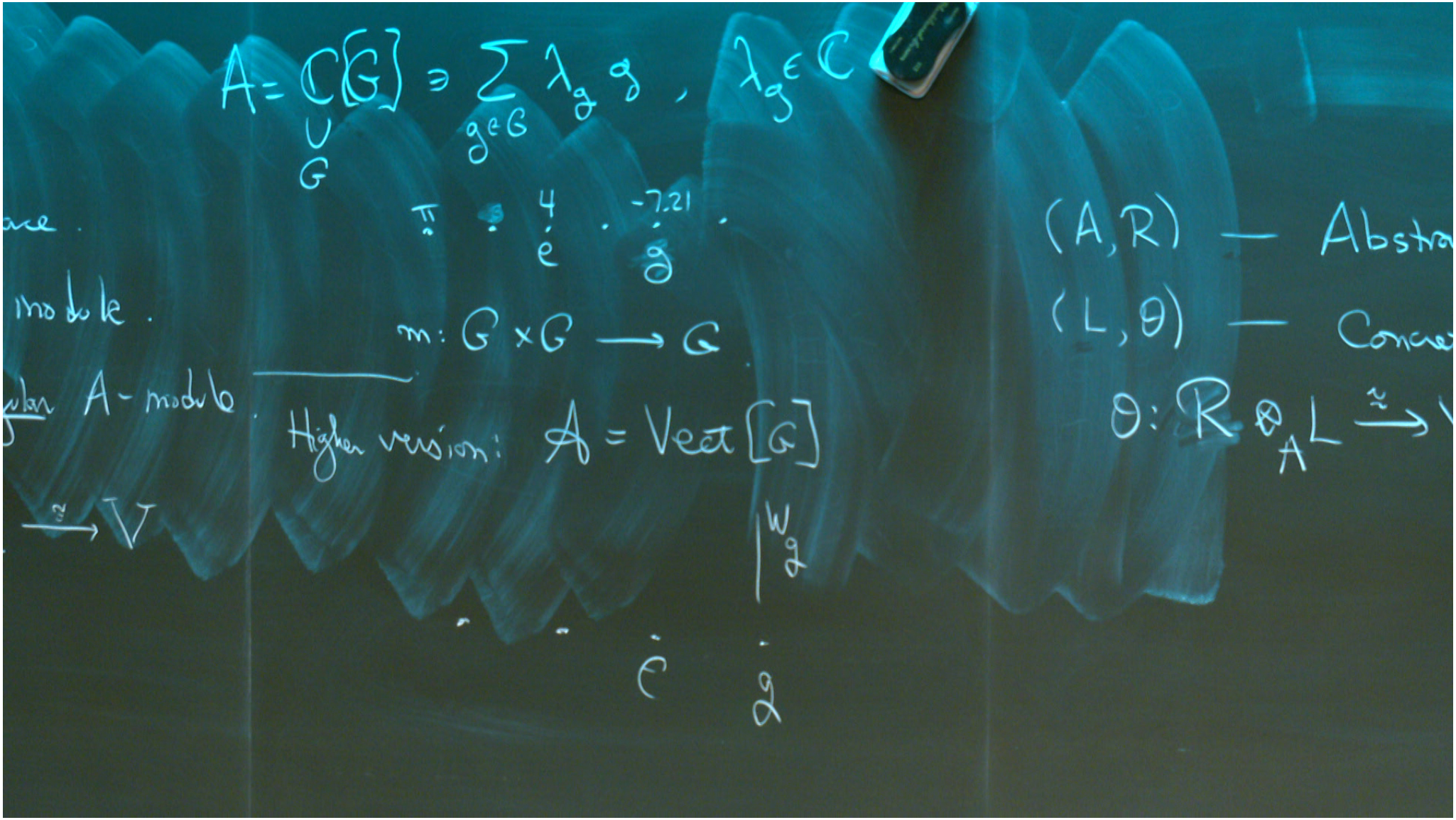

ical
homotopical
nonsmpact
π - finite
G finite group
X space.
$EG \times_G X = X_G \supset X$
BG
$BG \ni *$
Sgmmet
Abstract symmetry
Concrete realization


Groups
discrete
topological
finite
countable
Lie
homotopical
compact
noncompact
π - finite
G finite
G ↷ X space.
EG ×_G X = X
BG


etry
Homotopy fiber product.
data: $(BG, *)$
(π, θ)
X_G
$\downarrow$
BG
$\theta : \pi^{-1}(*) \xrightarrow{\simeq} X$
$*$
X_G
BG
$*$
X_G
BG
$*$
$*$


etry
data: $(BG, *)$
(π, θ)
X_G
$\downarrow$
BG
$\theta : \pi^{-1}(*) \xrightarrow{\simeq} X$
Homotopy fiber product.
*
*
X_G
*
X_G
BG
BG


Algebras
A algebra
V vector space
$_AL$ left A-module
R_A right regular A-module
$R \otimes_A L \xrightarrow{\sim} V$
$A = \mathbb{C}[G] = \sum_{g \in G} \lambda_g g, \quad \lambda_g \in \mathbb{C}$
$\pi : 4 \cdot -7.21$
$e \quad g$
$m : G \times G \longrightarrow G$


$A = \mathbb{C}[G] = \bigoplus_{g \in G} \lambda_g g, \quad \lambda_g \in \mathbb{C}$
$\pi : 4 \cdot -7.21$
$e \quad g$
$m : G \times G \longrightarrow G$
Higher version: $\mathcal{A} = \mathrm{Vect}[G]$
(A, R) — Abstra
(L, θ) — Conce
$\theta : R \otimes_A L \xrightarrow{\simeq}$
$\xrightarrow{\simeq} V$
W_g
$e \quad g$

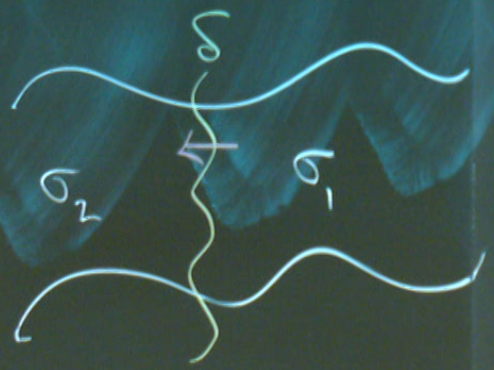
Theory

Parameters : • n dimension (spacetime)

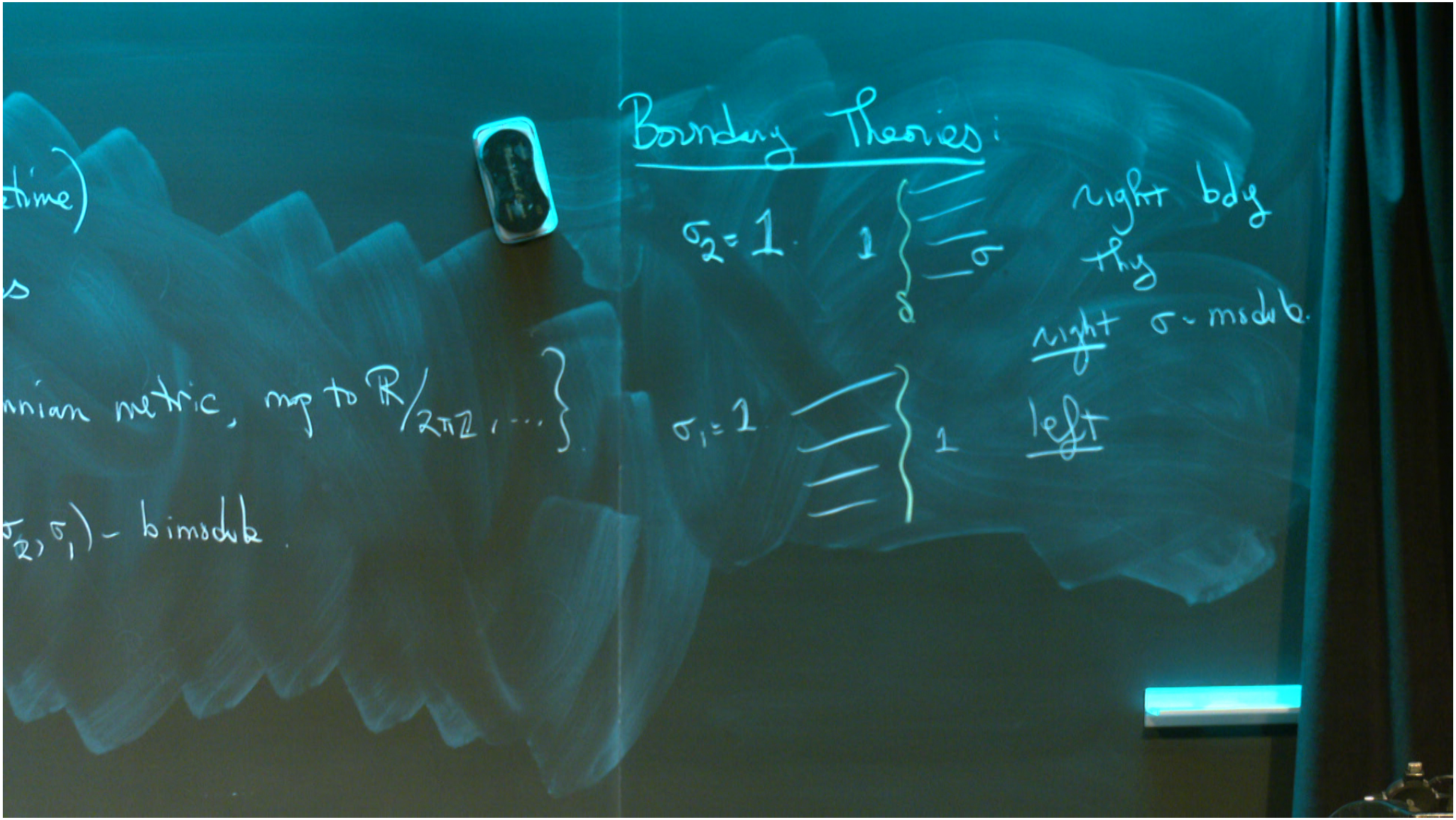
• $\mathcal{Y}$ background fields

$\mathcal{Y} = \{$orientation, Riemannian metric, map to $\mathbb{R}/2\pi\mathbb{Z}$, $\dots\}$

Wall :



(σ_2, σ_1) - bimodule


Boundary Theories:
$\sigma_2 = 1$
1
δ
σ
right bdy thy
right σ-msdule.
$\sigma_1 = 1$
1
left
nnian metric, mp to $\mathbb{R}/2\pi\mathbb{Z}$, ...}
σ_2, σ_1) - bimsdule.

Defn 1: Fix $n \in \mathbb{Z}^{\geq 1}$. An n-dim'l field-theoretic symmetry data is a pair (σ, ρ)

σ: $(n+1)$-dim'l topological field thy

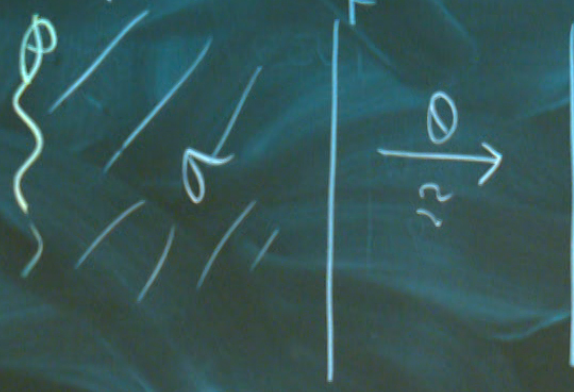
ρ: right boundary theory for σ, topological

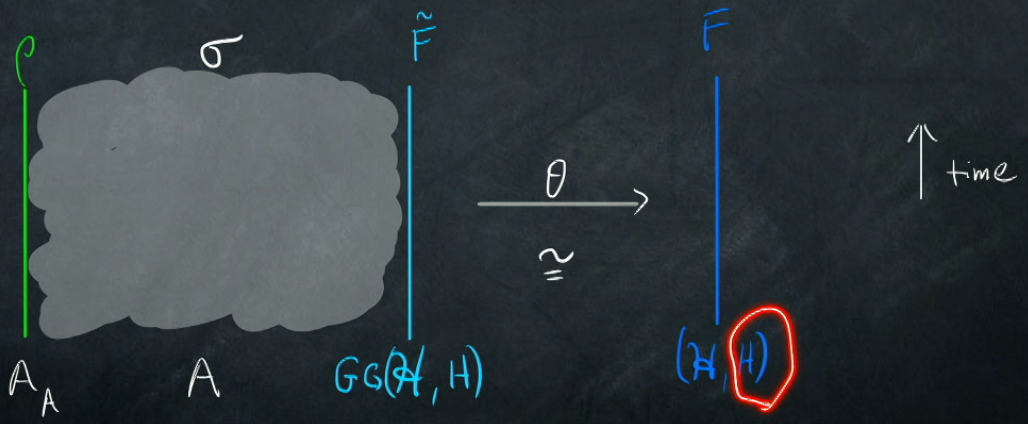
Ex: ρ regular bdy thy (Dirichlet)

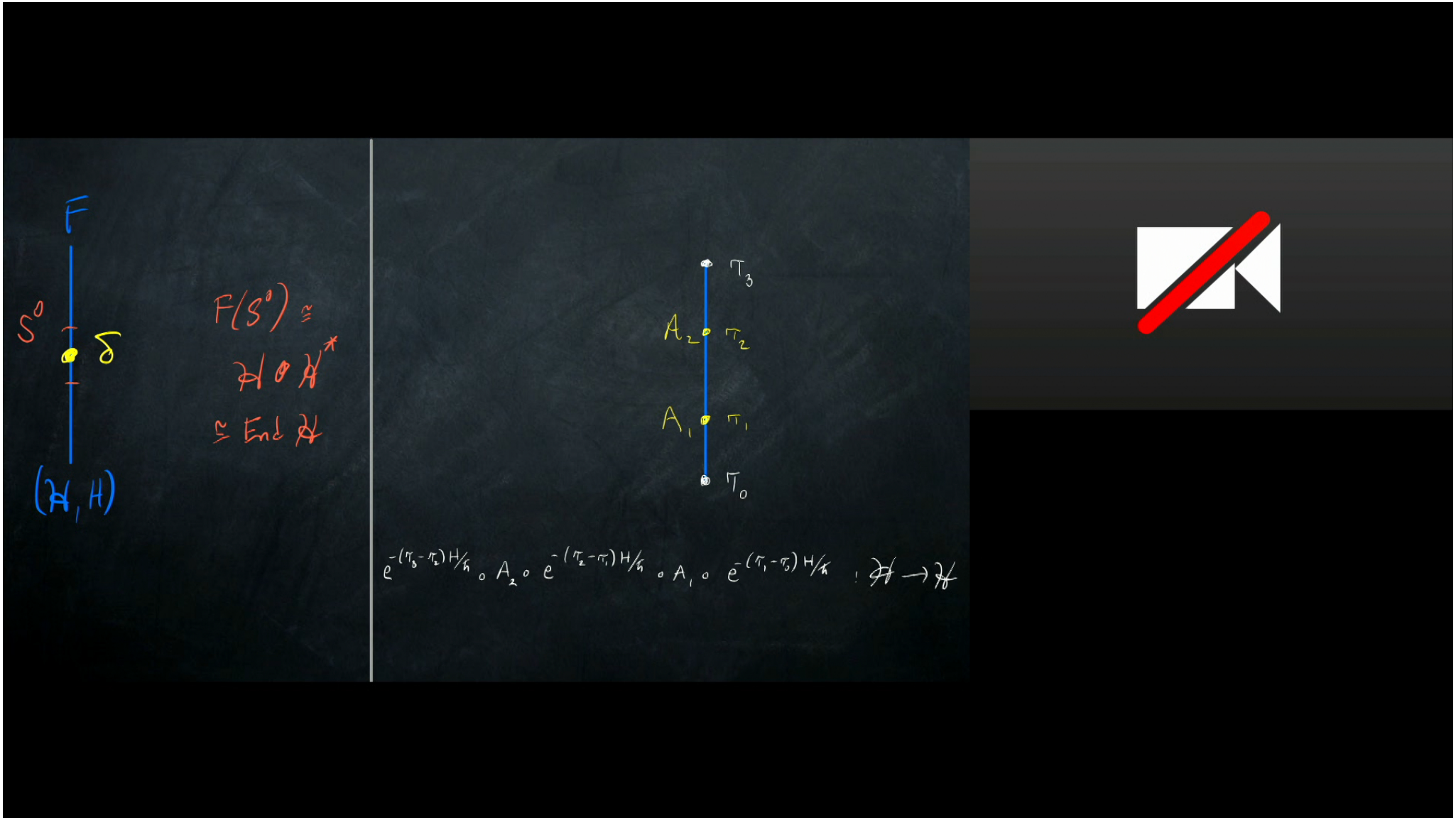
Defn 2: Fix (σ, ρ). A (σ,ρ)-module structure on an n-dim'l field thy F is $(\tilde{F}, \Theta)$.

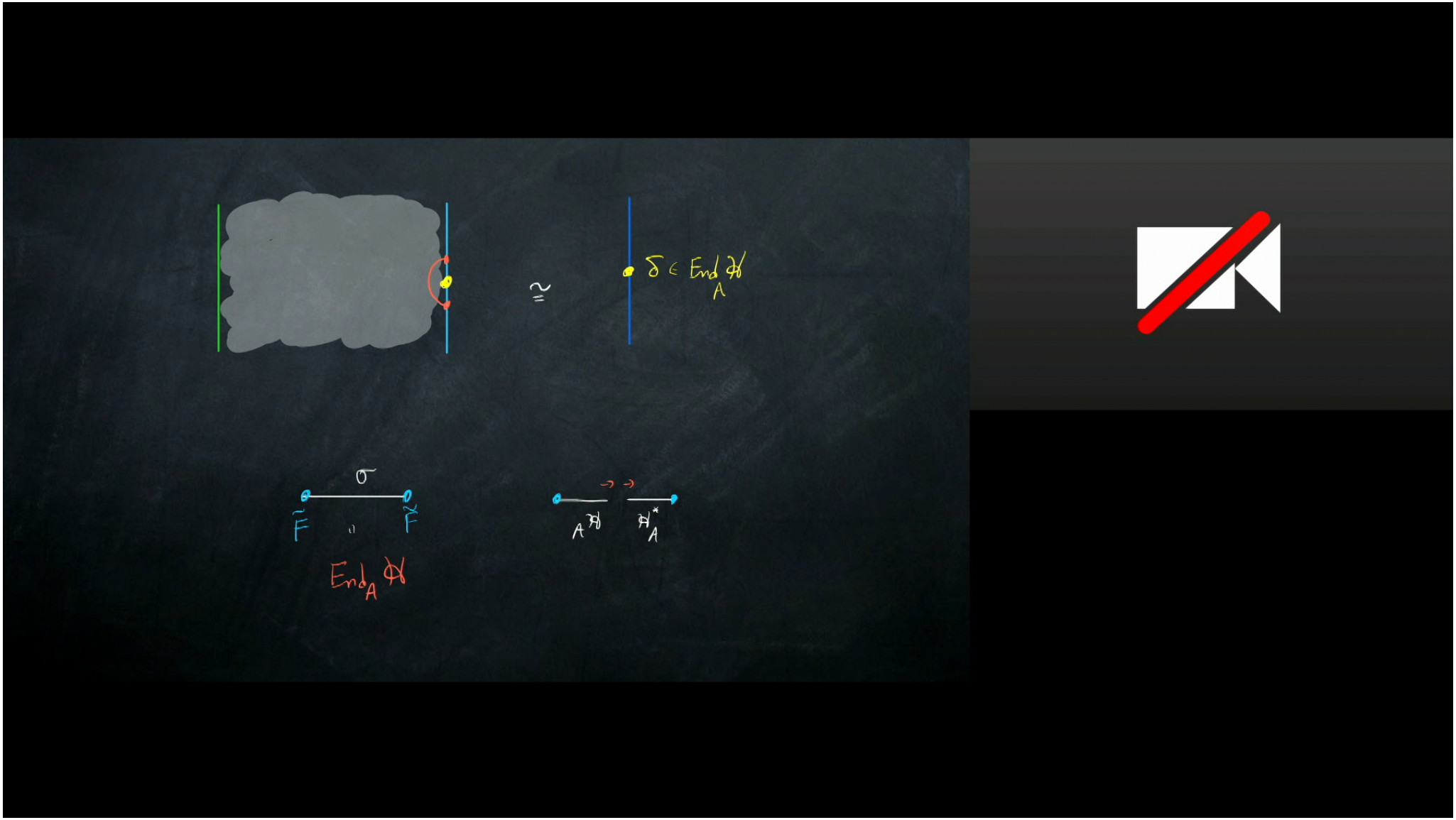
$\tilde{F}$: left bdy thy for σ

$\Theta: \rho \otimes_\sigma \tilde{F} \xrightarrow{\simeq} F$




σ
F̃
F
θ
≅
time
A
A
GG(H, H)
(H, H)
A = ℂ[G]


F
S^0
δ
$(\mathcal{H}, H)$
$F(S^0) \cong$
$\mathcal{H} \otimes \mathcal{H}^*$
$\cong End\ \mathcal{H}$
τ_3
A_2
τ_2
A_1
τ_1
τ_0
$e^{-(\tau_3-\tau_2)H/\hbar} \circ A_2 \circ e^{-(\tau_2-\tau_1)H/\hbar} \circ A_1 \circ e^{-(\tau_1-\tau_0)H/\hbar} : \mathcal{H} \to \mathcal{H}$

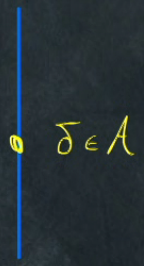

≃
$\delta \in \mathrm{End}_A \mathcal{H}$
σ
$\tilde{F}$
$\tilde{F}$
$\mathrm{End}_A \mathcal{H}$
$A\mathcal{H}$
$\mathcal{H}_A^*$



$\simeq A$ (as a vector space)



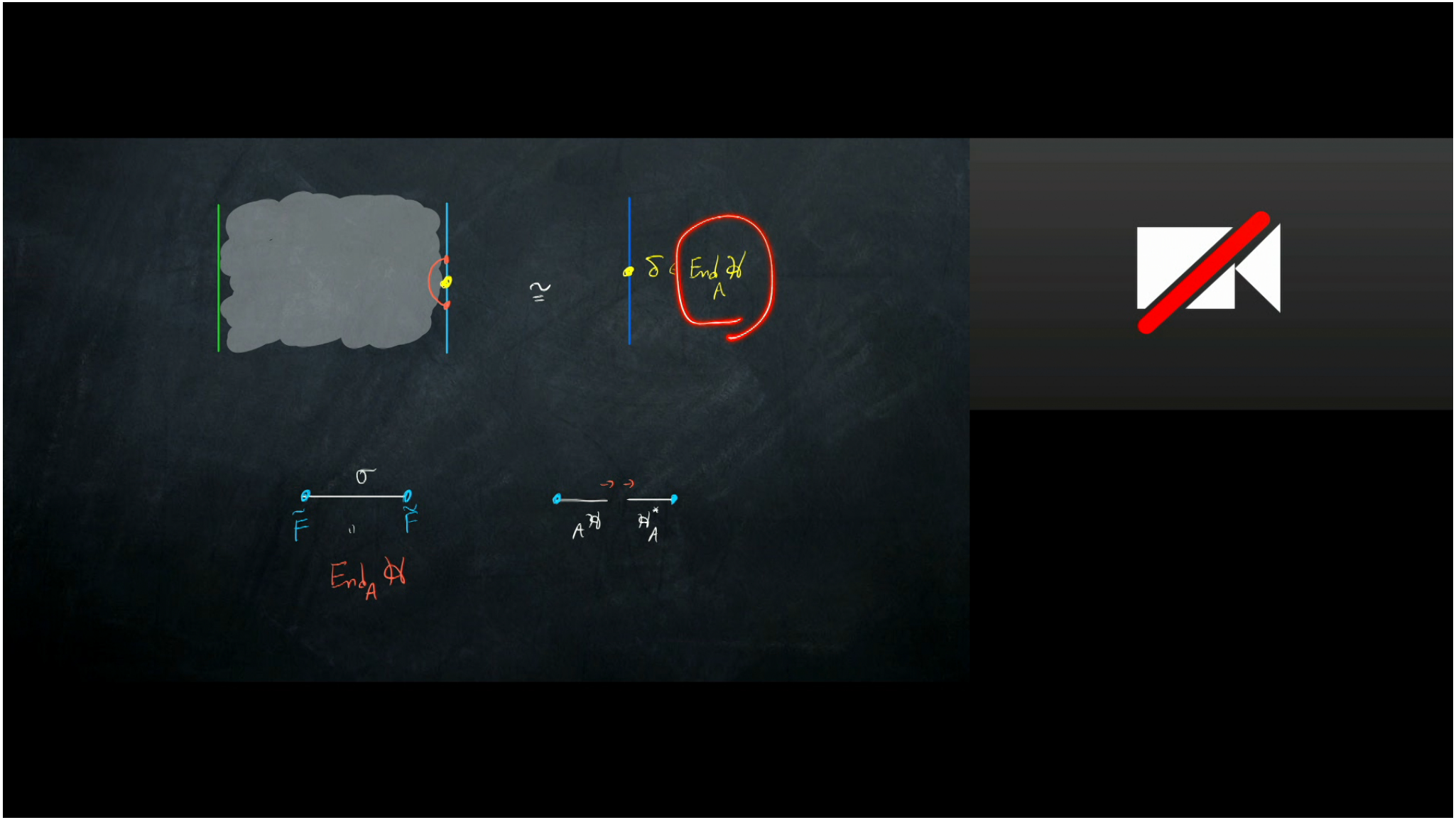
$\simeq$



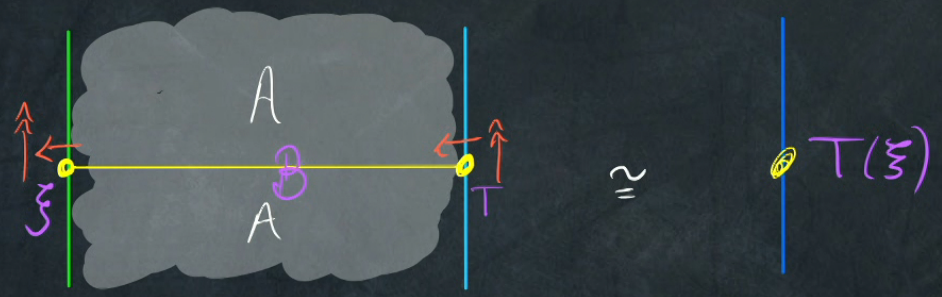


$\simeq A$ (as a vector space)


≃
$\delta \in \mathrm{End}_A \mathcal{H}$
σ
$\tilde{F}$
$\tilde{F}$
$\mathrm{End}_A \mathcal{H}$
$A\mathcal{H}$
$\mathcal{H}_A^*$


≃
$\delta \in \mathrm{End}_A \mathcal{H}$
σ
$\tilde{F}$
$\tilde{F}$
$\mathrm{End}_A \mathcal{H}$
$A\mathcal{H}$
$\mathcal{H}_A^*$


$\cong$
$\delta \in \mathbb{C}[G]^G$
$\sigma(S') \cong \mathbb{C}[G]^G$

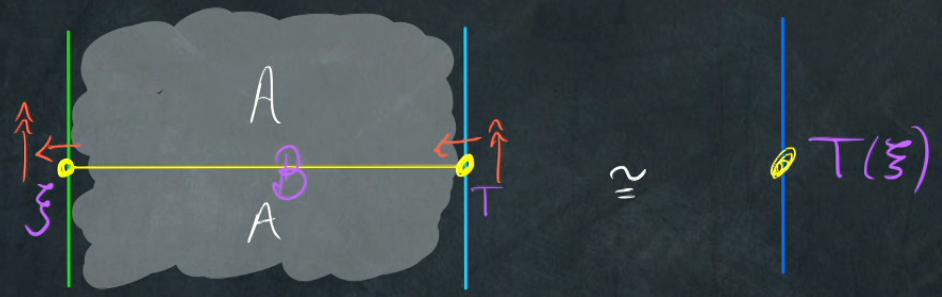


$\simeq$

B (A,A) - bimodule

$\xi \in B$

$T : B \longrightarrow End(\mathcal{H})$ (A,A) - bimodule map

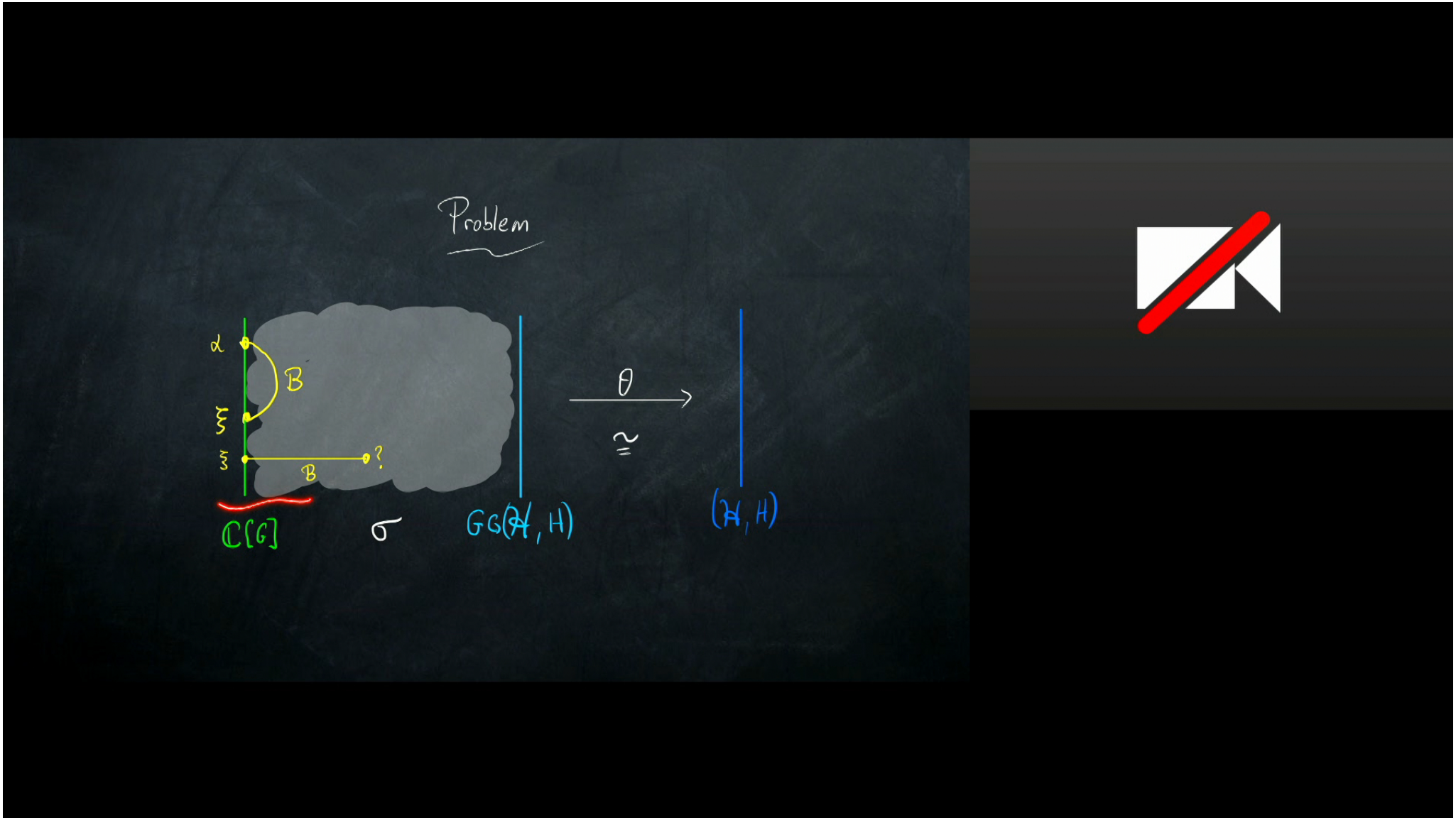


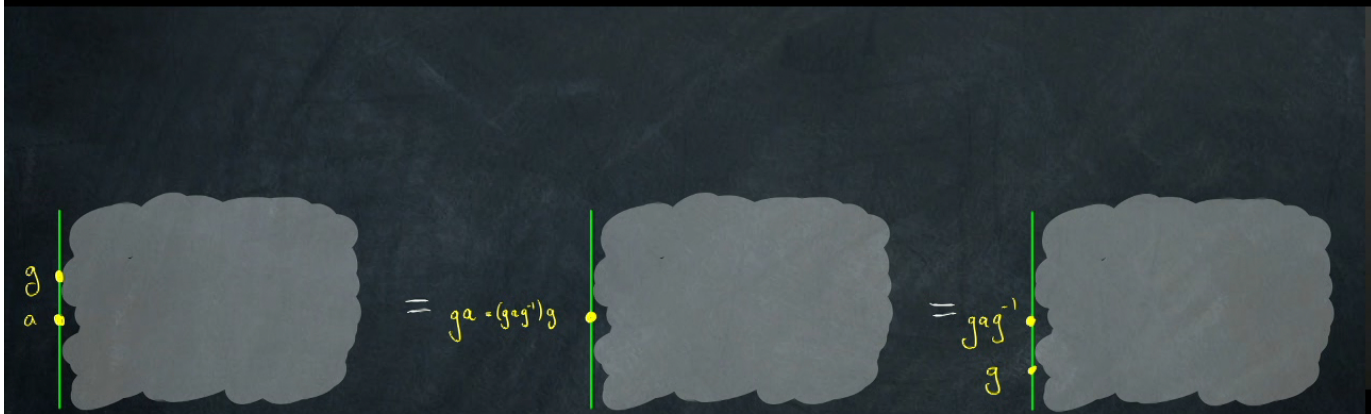
$\simeq$

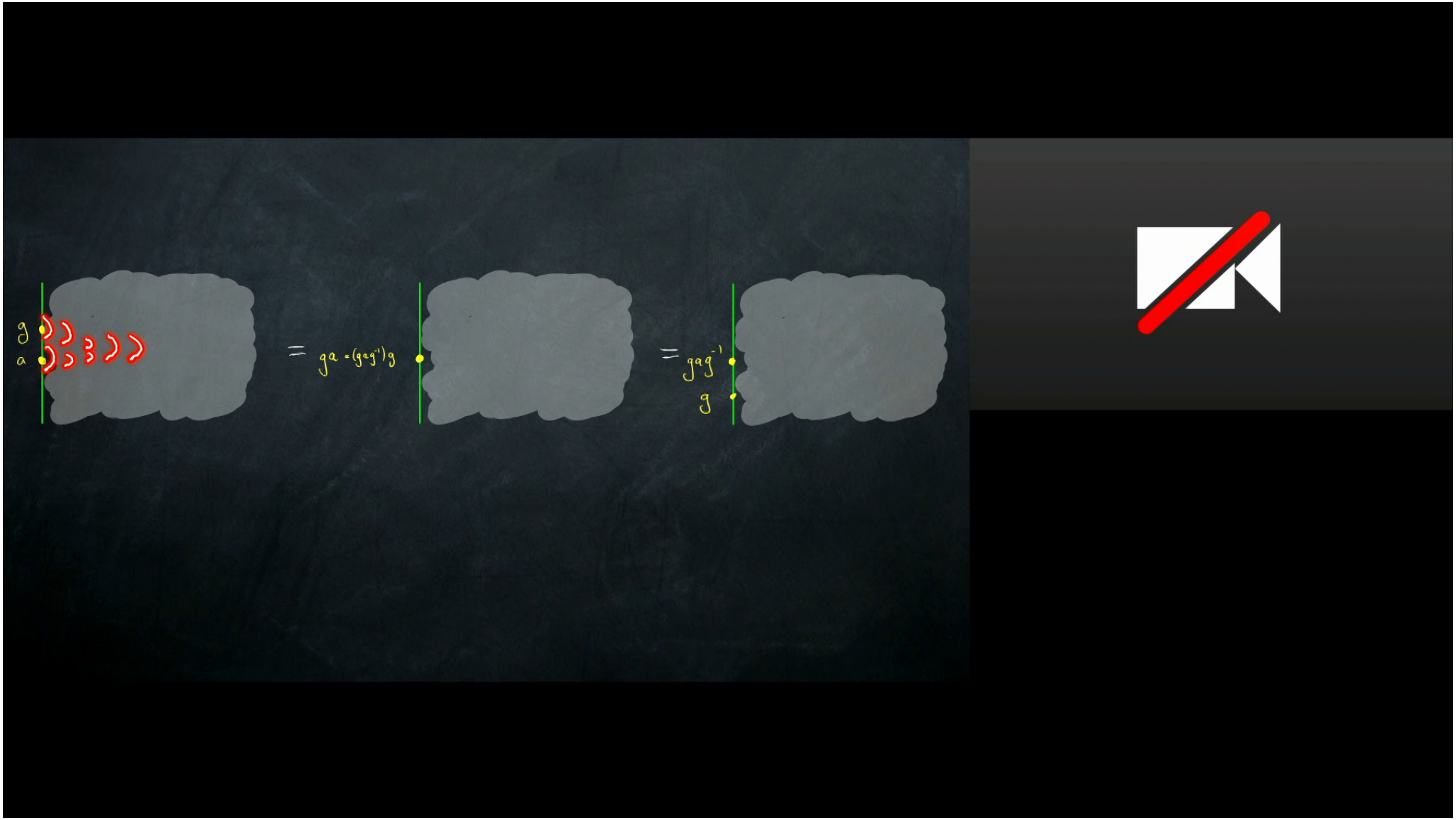
B (A,A) - bimodule

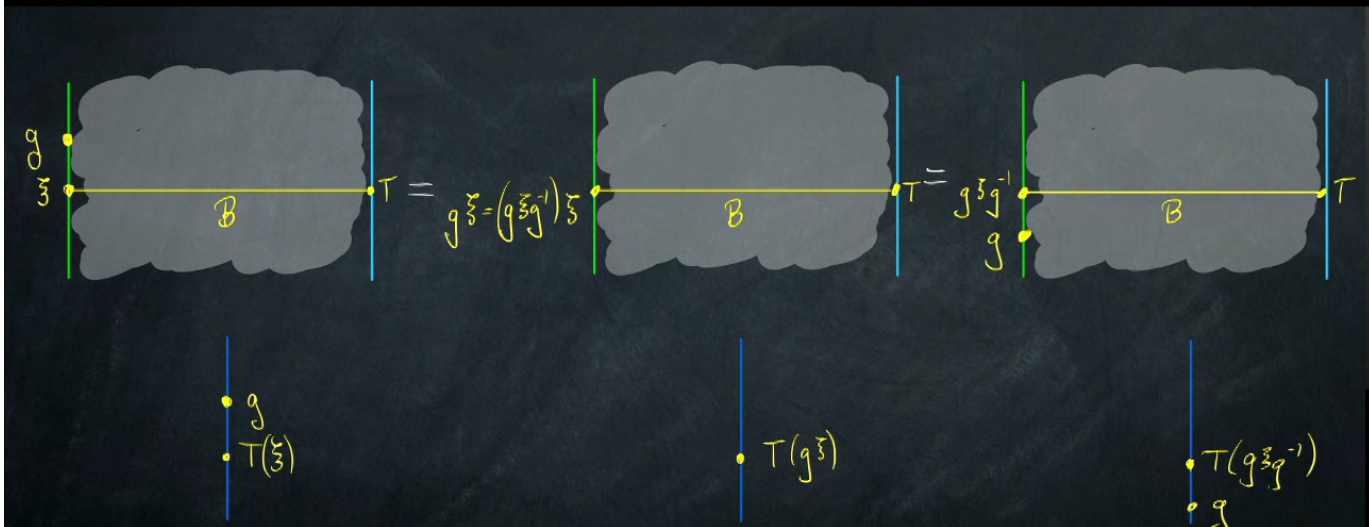
$\xi \in B$

$T : B \longrightarrow End(\mathcal{H})$ (A,A) - bimodule map


Problem
α
B
ξ
ξ
B
?
θ
≃
ℂ[G]
σ
GG(H, H)
(H, H)


g
a
= ga = (gag⁻¹)g
= gag⁻¹
g

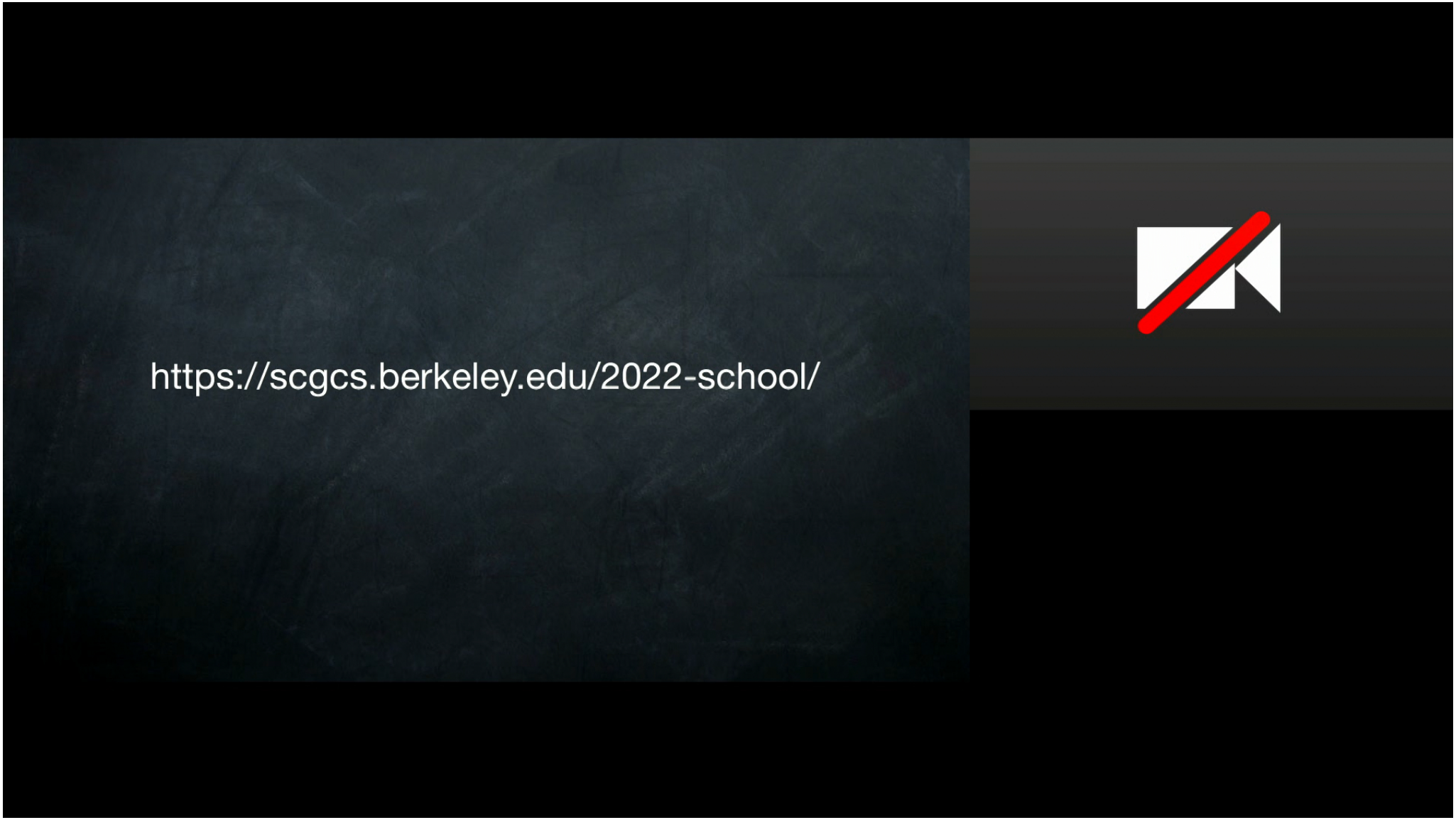

g
a
= ga = (gag⁻¹)g
= gag⁻¹
g



B (A,A)-bimodule

$\xi \in B$

$T : B \longrightarrow End(\mathcal{H})$ (A,A)-bimodule map


https://scgcs.berkeley.edu/2022-school/