Title: Introduction to Symmetries in Quantum Field Theory

Speakers:

Collection: Global Categorical Symmetries

Date: June 13, 2022 - 9:30 AM

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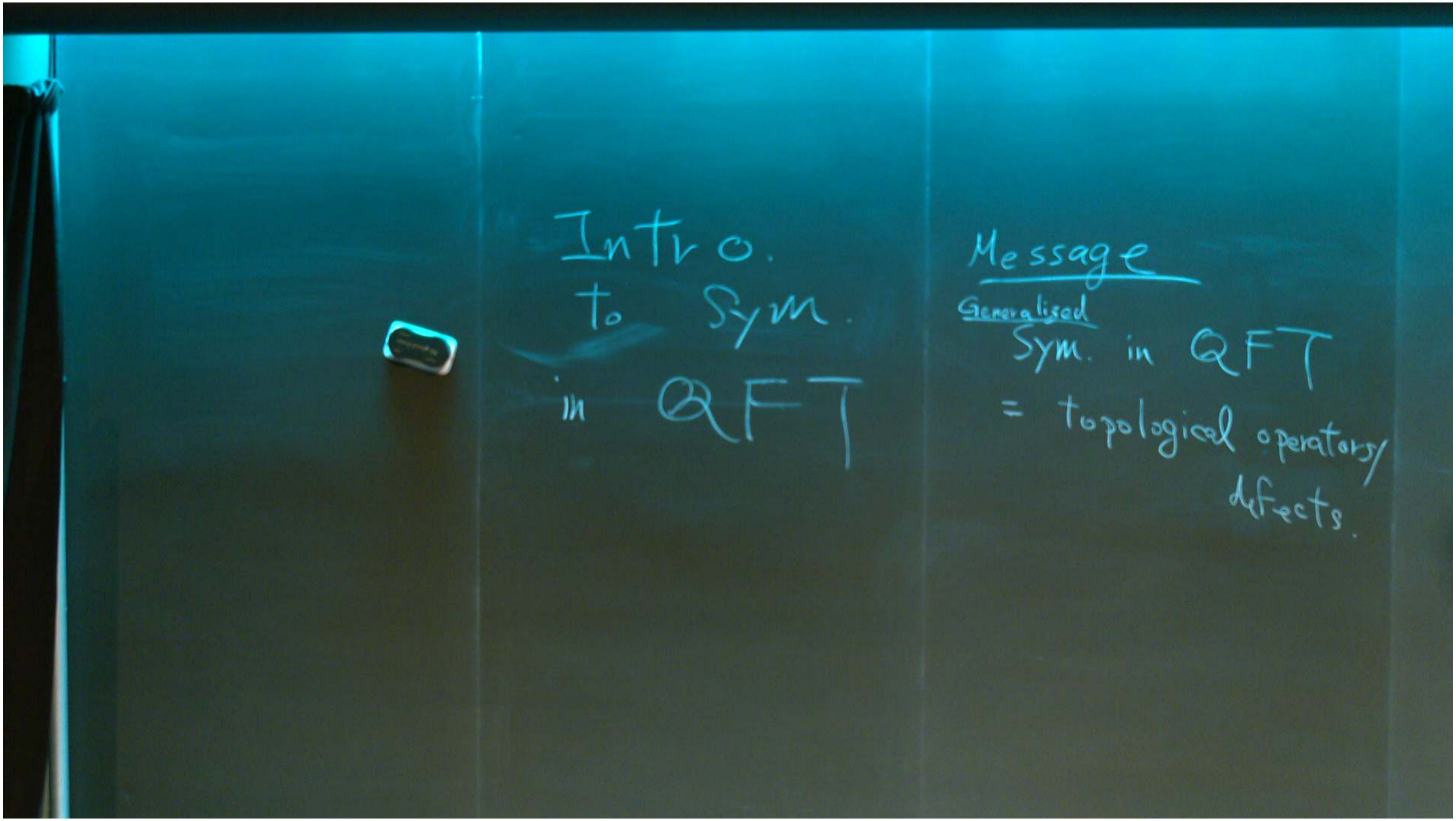
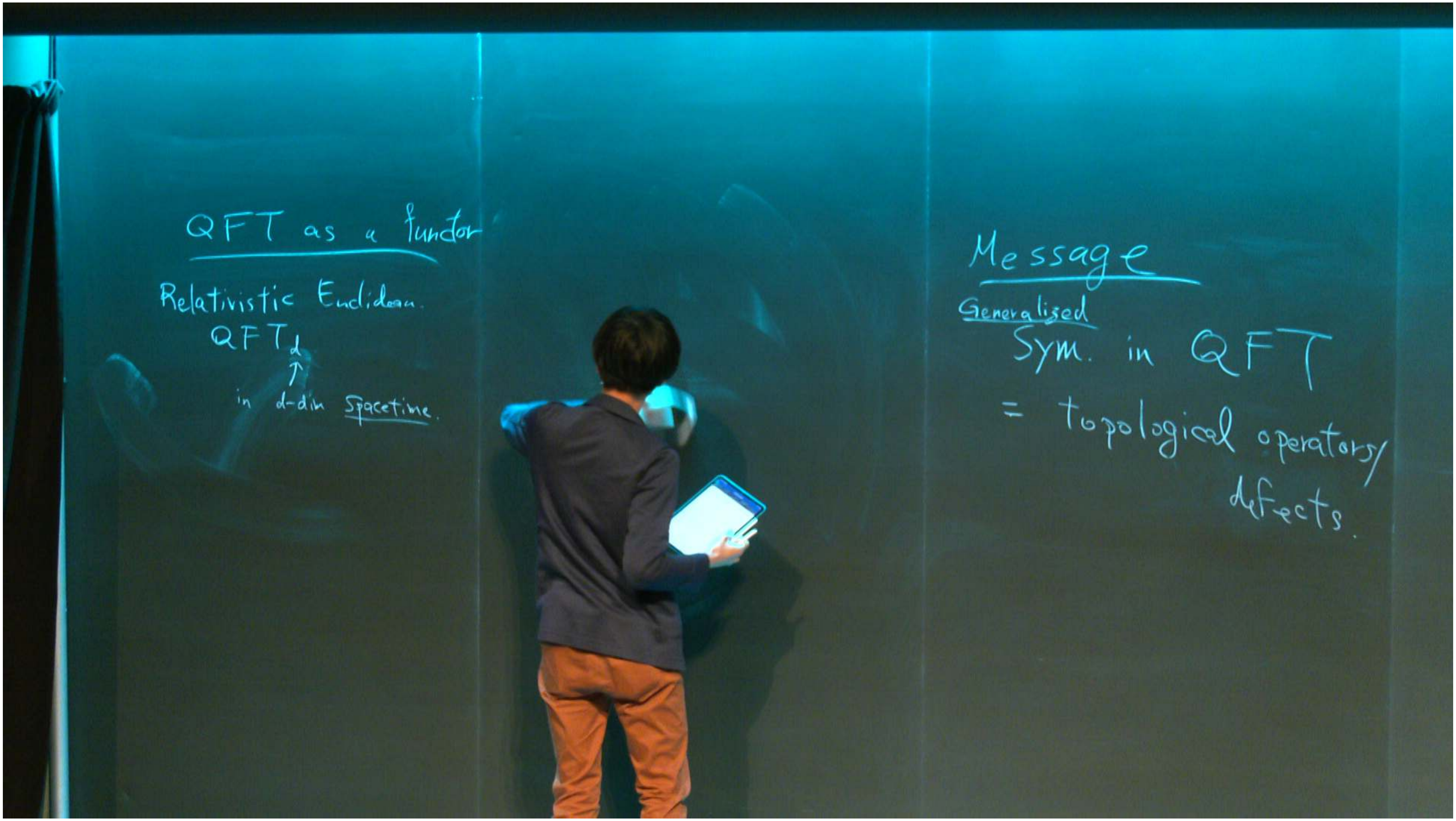

Intro.
to Sym.
in QFT
Message
Generalised
Sym. in QFT
= topological operators/
defects.


QFT as a functor
Relativistic Euclidean
QFT$_d$
in d-dim Spacetime.
Message
Generalised
Sym. in QFT
= topological operators/
defects.

for

"define" it as a functor

$Z : \underline{\mathrm{Bord}^{\langle d,d-1\rangle}_{\mathcal{S}}} \longrightarrow \underset{\substack{\uparrow \\ \text{cat. of vec. sp.s}}}{\mathrm{Vect}}$

$\mathcal{S}$: properties / structures of the manifolds.

e.g. diff, Riem, Spin-str, G-connections, ..., ...

$\mathrm{Obj}(\mathrm{Bord}^{\langle d,d-1\rangle}_{\mathcal{S}})$

$= \left\{ (d-1)\text{-dim closed } \mathcal{S}\text{-mfds} \right\}$

$\mathrm{Hom}_{\mathrm{Bord}^{\langle d,d-1\rangle}_{\mathcal{S}}}(M, N)$

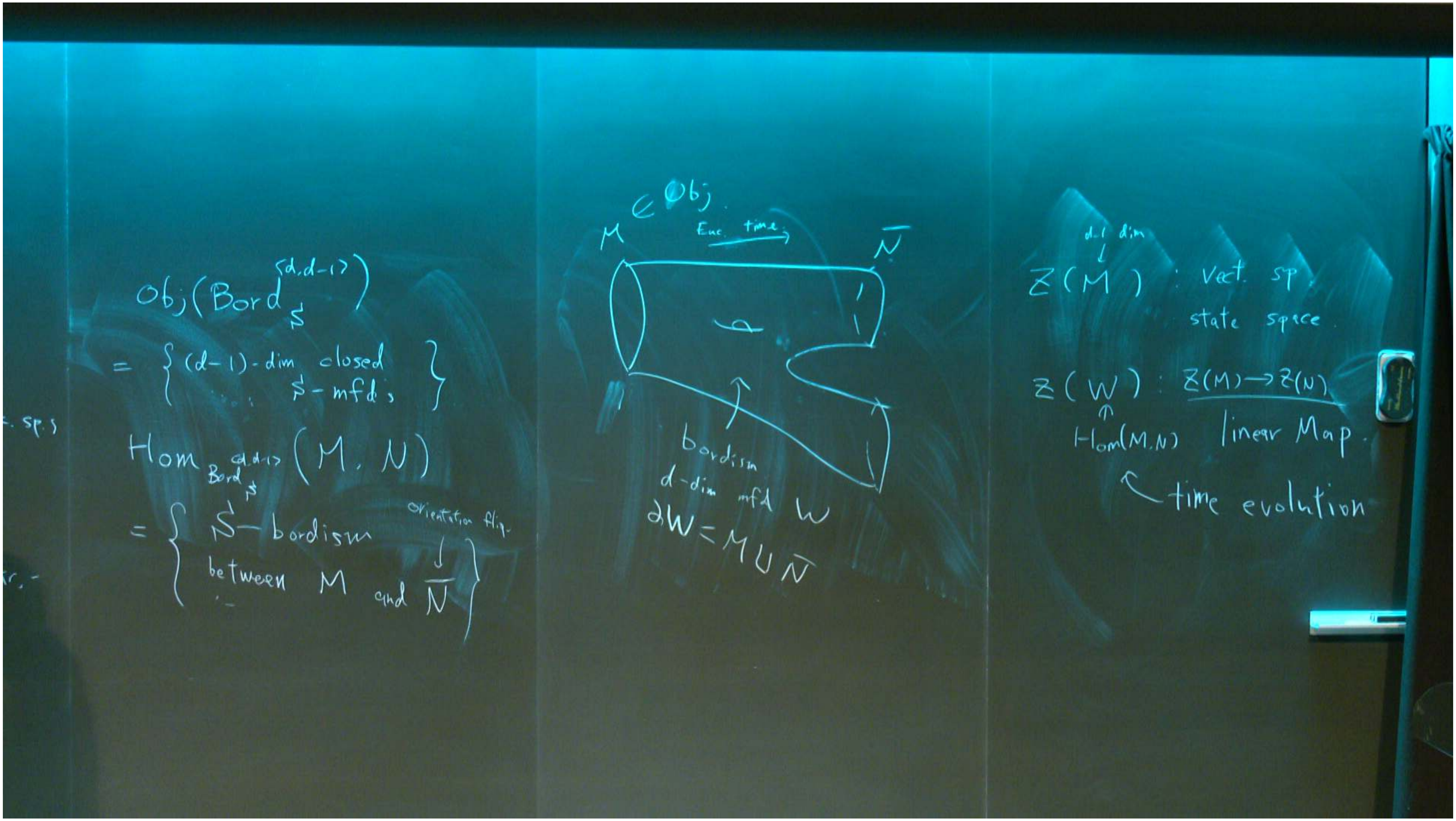
$= \left\{ \begin{array}{l} \mathcal{S}\text{-bordism} \\ \text{between } M \text{ and } \overset{\substack{\text{orientation flip} \\ \downarrow}}{\overline{N}} \end{array} \right\}$

Plan of talk

Day 1:

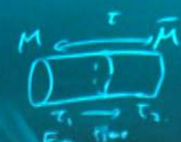
QFT as a fu

& extended oper


$\mathrm{Obj}(\mathrm{Bord}_{S}^{\langle d,d-1\rangle})$
$= \{ (d-1)$-dim closed S-mfd's $\}$
$\mathrm{Hom}_{\mathrm{Bord}_{S}^{\langle d,d-1\rangle}}(M, N)$
$= \{ S$-bordism between M and $\overline{N} \}$
orientation flip
$M \in \mathrm{Obj}$
Euc. time
$\overline{N}$
bordism
d-dim mfd W
$\partial W = M \sqcup \overline{N}$
d-1 dim
$Z(M)$: Vect. sp.
state space
$Z(W) : Z(M) \to Z(N)$
Hom(M,N)
linear Map.
time evolution

Z has to be monoidal

$$Z[M_1 \sqcup M_2] = Z(M_1) \otimes Z(M_2)$$



$$Z(M \times [0, \tau]) = U_M(\tau)$$

time ev. on M

$$U_M(\tau_2) \cdot U_M(\tau_1) = U_M(\tau_1 + \tau_2)$$

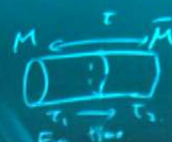
$$\longrightarrow U_M = e^{-\tau \underline{H_M}}$$

Hamiltonian

Z has to be monoidal

$Z[M_1 \sqcup M_2]$

$= Z(M_1) \otimes Z(M_2)$



$Z(\underset{d-1}{M} \times [0,\tau])$

$= U_M(\tau) : Z(M) \to Z(M)$

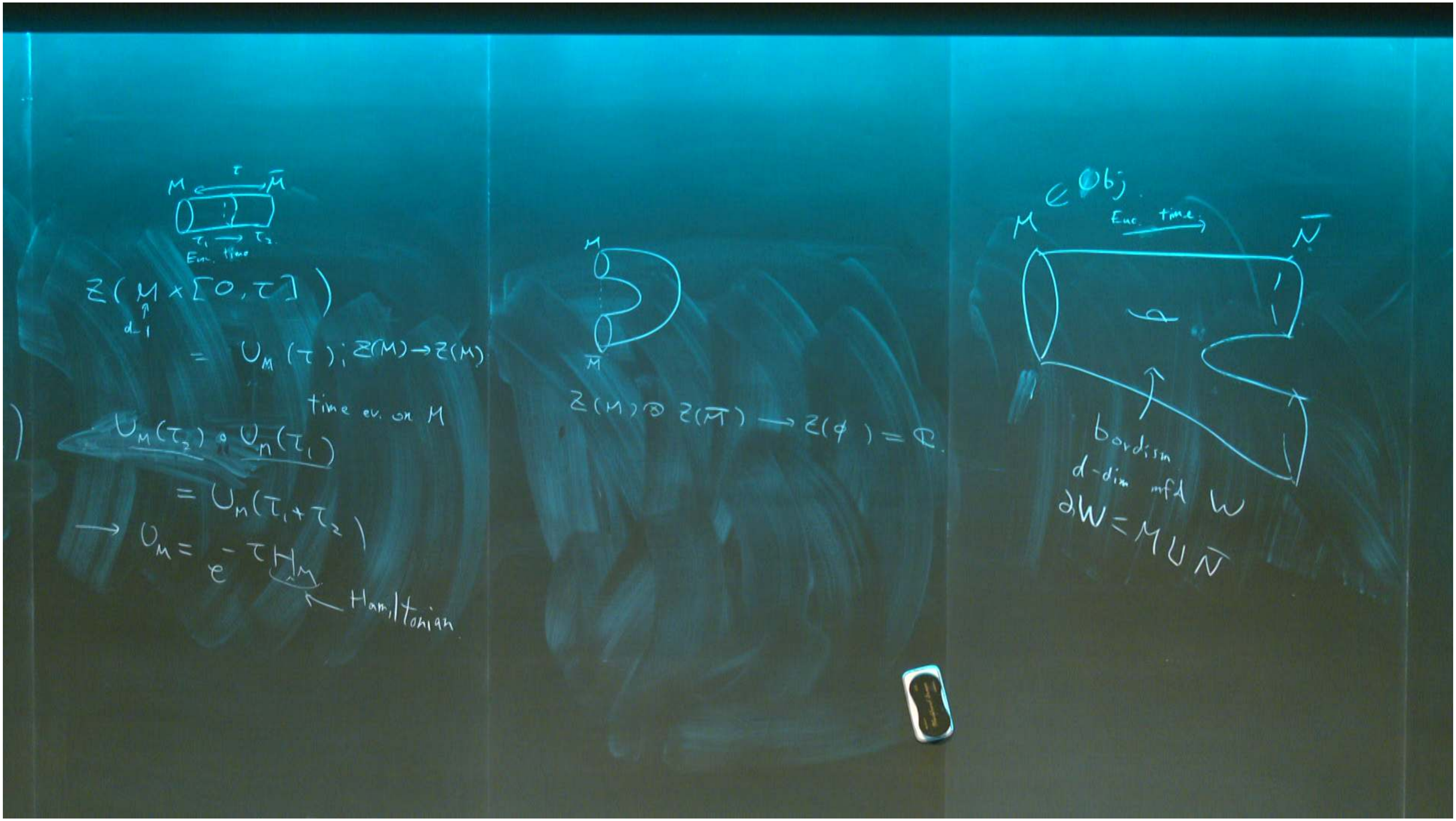
time ev. on M

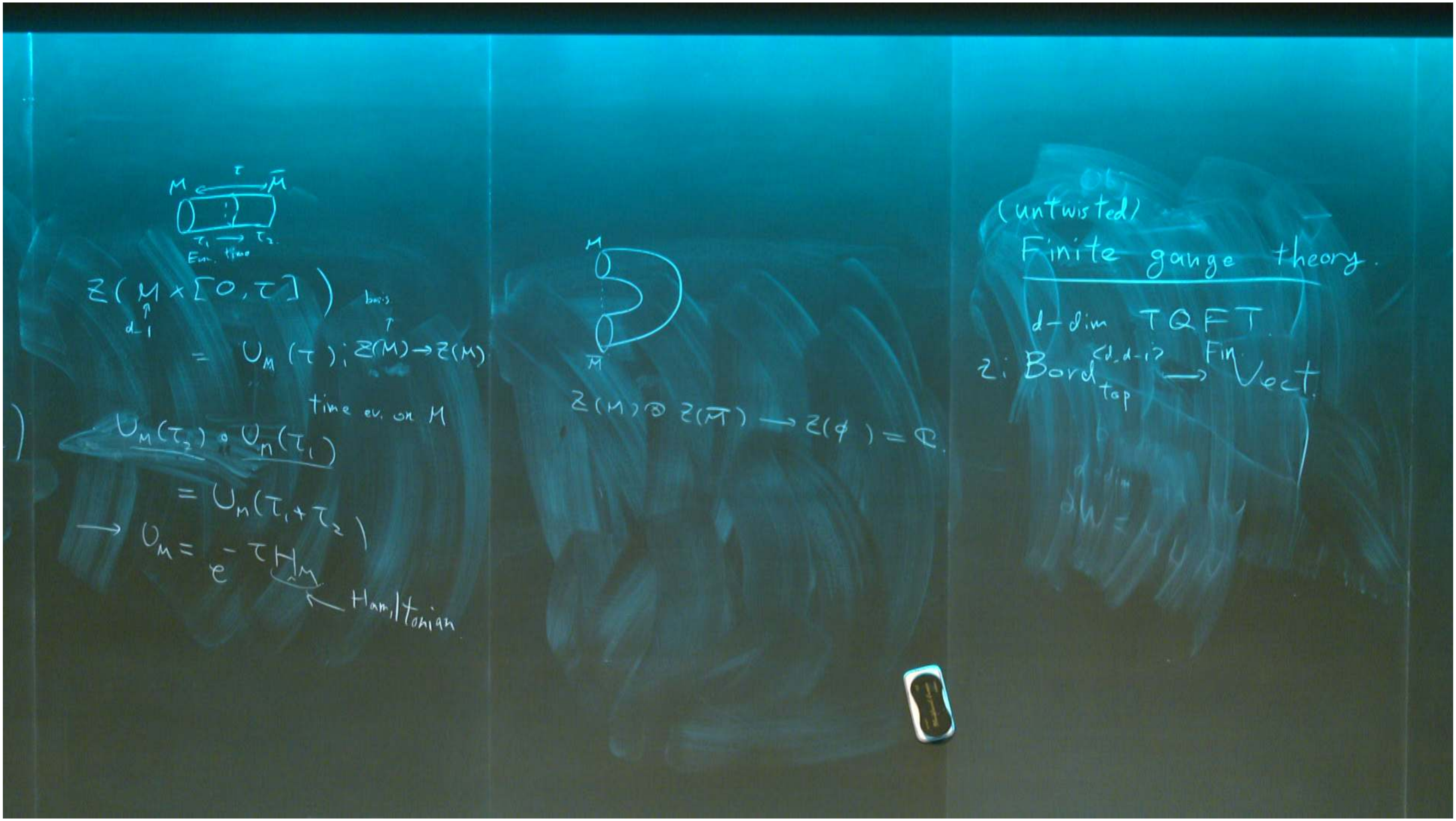
$U_M(\tau_2) \circ U_M(\tau_1)$

$= U_M(\tau_1 + \tau_2)$

$\longrightarrow U_M = e^{-\tau H_M}$

Hamiltonian


τ
M M̄
τ₁ → τ₂
Euc. time
Z(M × [0, τ])
d−1
= U_M (τ) : Z(M) → Z(M)
time ev. on M
U_M(τ₂) ∘ U_M(τ₁)
= U_M(τ₁ + τ₂)
→ U_M = e^{−τH_M}
Hamiltonian
M
M̄
Z(M) ⊗ Z(M̄) → Z(∅) = ℂ
M ∈ Obj.
Euc. time
N̄
bordism
d-dim mfd W
∂W = M ⊔ N̄


M τ M̄
τ₁ → τ₂
Euc. time
Z(M × [0, τ])
d−1
basis
= U_M (τ) ; Z(M) → Z(M)
time ev. on M
U_M(τ₂) ∘ U_M(τ₁)
= U_M(τ₁ + τ₂)
→ U_M = e^{−τ H_M}
Hamiltonian
M
M̄
Z(M) ⊗ Z(M̄) → Z(∅) = ℂ.
(untwisted)
Finite gauge theory.
d-dim TQFT.
Z : Bord_top^{⟨d, d−1⟩} → FinVect.

$Z(M)$

M

ian

$Z(M) \otimes Z(\overline{M}) \longrightarrow Z(\emptyset) = \mathbb{C}$.

(untwisted)

Finite gauge theory.

d-dim TQFT

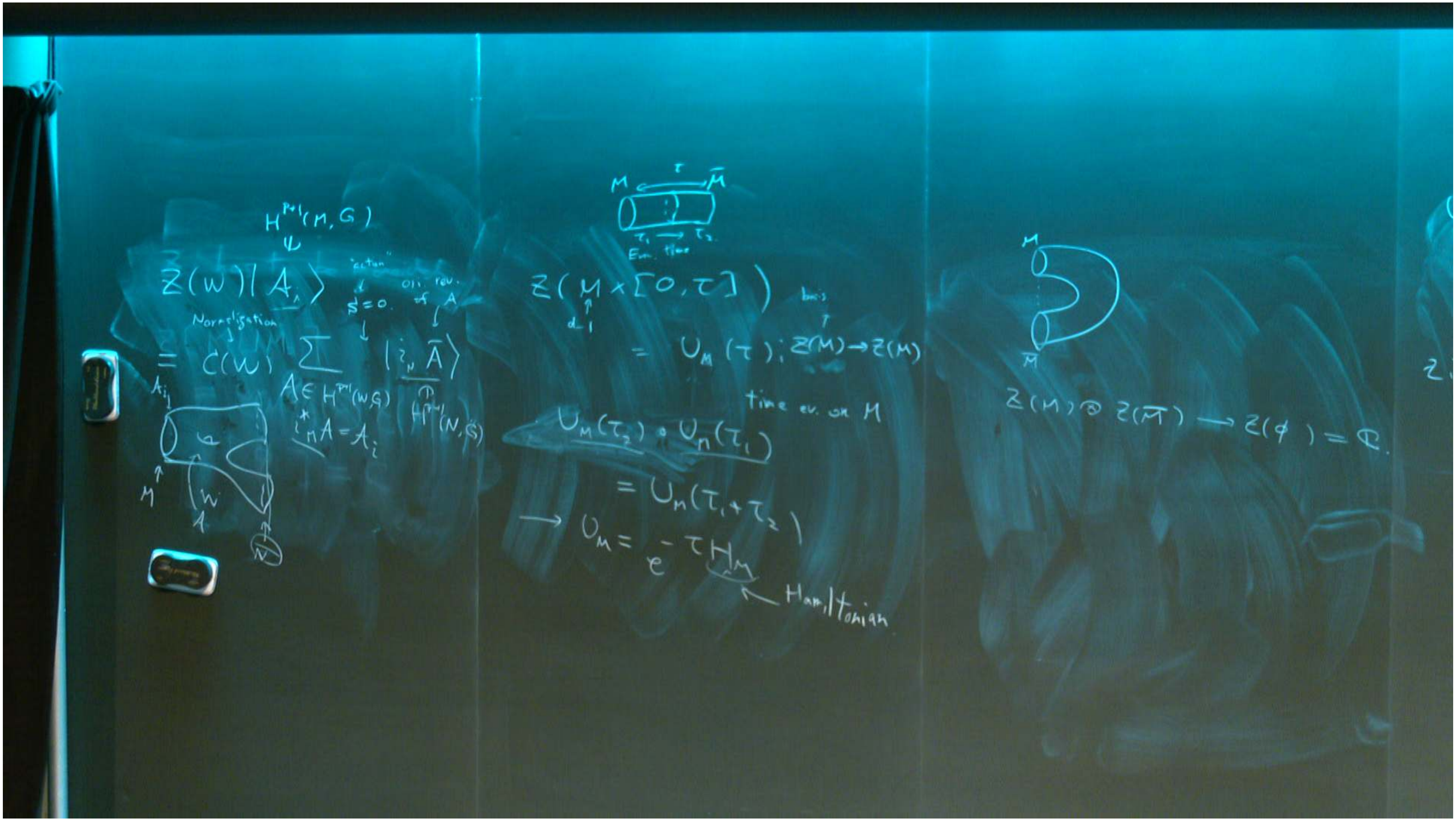
$Z: \mathrm{Bord}^{\langle d, d-1\rangle}_{top} \longrightarrow \mathrm{Fin}\,\mathrm{Vect}$

fix $p \in \mathbb{Z}$ $\quad$ G: finite group

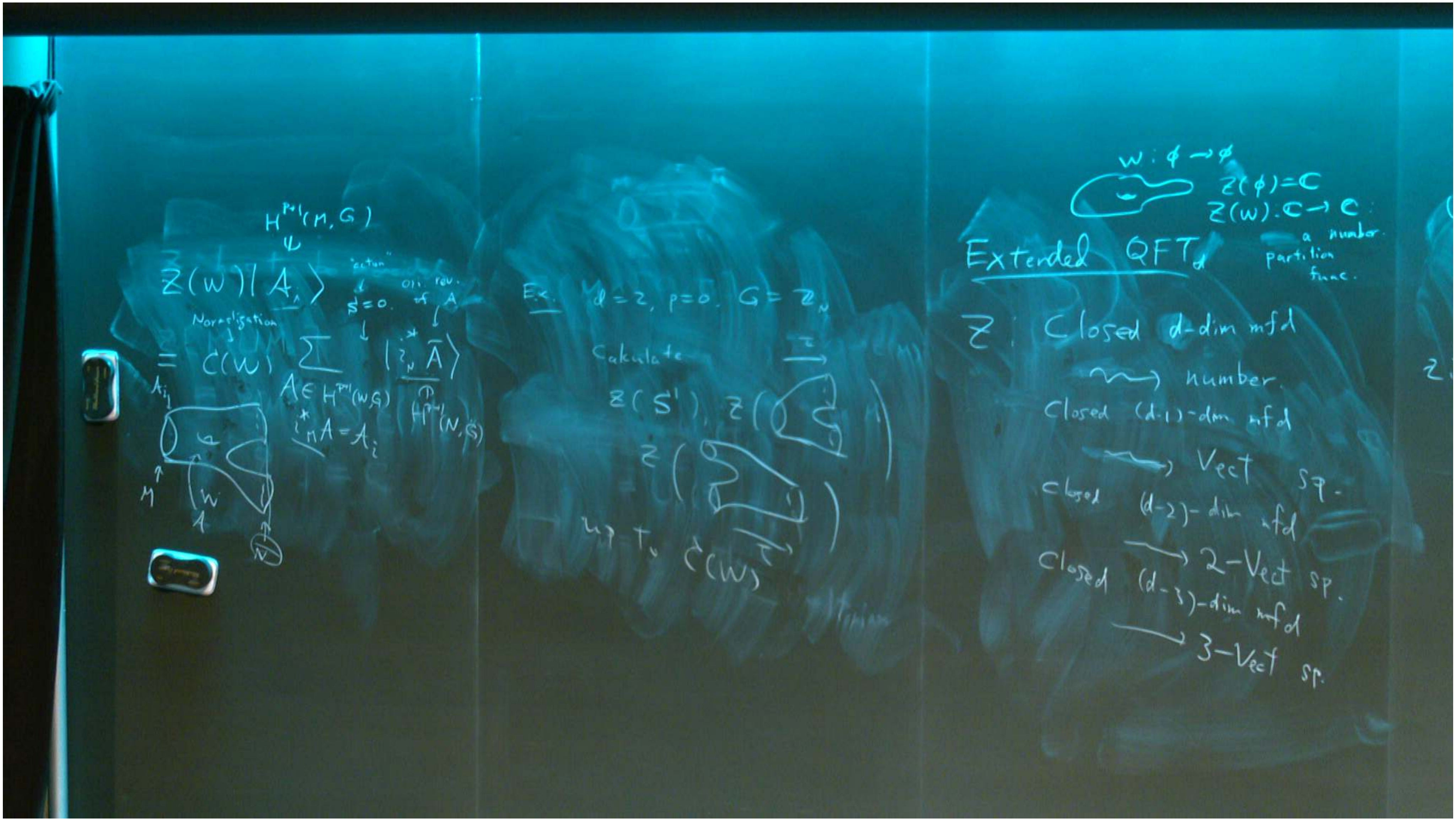
$p \leq d-1$ $\quad$ abelian if $p > 0$

$Z(\underset{\substack{\cap \\ \mathrm{Ob}_j(\mathrm{Bord})}}{M}) = \langle \overbrace{H^{p+1}(M, G)}^{\text{finite set}} \rangle_{\mathbb{C}}$

$\psi = c_1|A_1\rangle + c_2|A_2\rangle + \cdots$, $\quad A_1 \in H^{p+1}(M, G)$



Normalization
action
Euc. time
Hamiltonian
time ev. on M



W: ϕ → ϕ
Z(ϕ) = C
Z(W): C → C
a number
partition func.
Extended QFT_d
Z: Closed d-dim mfd
→ number.
Closed (d-1)-dim mfd
→ Vect sp.
Closed (d-2)-dim mfd
→ 2-Vect sp.
Closed (d-3)-dim mfd
→ 3-Vect sp.
Ex. d = 2, p = 0. G = Z_N
Calculate
Z(S^1), Z(
Z(
Normalization
"action"

$W : \phi \to \phi$

$Z(\phi) = \mathbb{C}$

$Z(W) : \mathbb{C} \to \mathbb{C}$: a number. partition func.

Extended QFT_d

Z : Closed d-dim mfd

$\rightsquigarrow$ number.

Closed $(d-1)$-dim mfd

$\rightsquigarrow$ Vect sp. — Category

Closed $(d-2)$-dim mfd

$\longrightarrow$ 2-Vect sp.

Closed $(d-3)$-dim mfd

$\longrightarrow$ 3-Vect sp. — higher-Category

Finite set

$$Z(M) = \langle H^{p+1}(M;G) \rangle_{\mathbb{C}}$$

$M \in Ob_j(\mathrm{Bord})$

$\psi = c_1|A_1\rangle + c_2|A_2\rangle + \cdots$

$A_i \in H^{p+1}(M;G)$

$W : \emptyset \to \emptyset$

$Z(\emptyset) = \mathbb{C}$

$Z(W) : \mathbb{C} \to \mathbb{C}$: a number
partition func.

Extended QFT_d

Z : Closed d-dim mfd $\rightsquigarrow$ number

Closed $(d-1)$-dim mfd $\rightsquigarrow$ Vect sp.

Closed $(d-2)$-dim mfd $\longrightarrow$ 2-Vect sp. $\leftarrow$ Category

Closed $(d-3)$-dim mfd $\longrightarrow$ 3-Vect sp. $\leftarrow$ higher-Category

In particular $\left(\text{Non top.} \; \lim_{r \to 0} Z(S_r^{q-1}) \right)$

$\mathrm{Ob}(Z(S^{q-1})) \ni a$

sphere

codim q defect / extended op.

Supposedly equivalent formulation given by "decorated Bordisms".