Title: â€™t Hooft anomalies of QFTs realized in string theory

Speakers: Federico Bonetti

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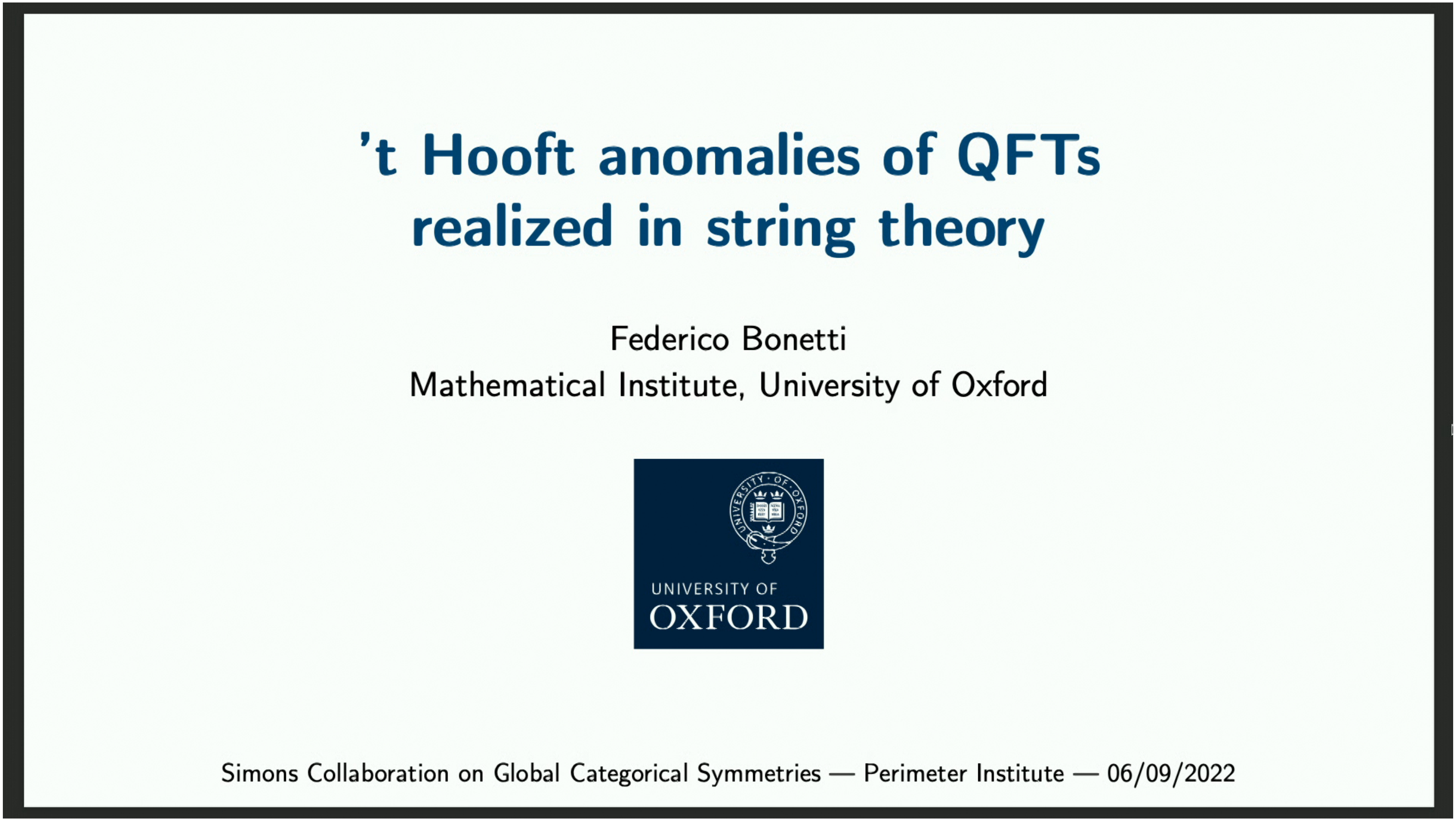
Abstract: String theory constructions allow one to realize vast classes of non-trivial quantum field theories (QFTs), including many strongly coupled models that elude a conventional Lagrangian description. 't Hooft anomalies for global symmetries are robust observables that are particularly well-suited to explore QFTs realized in string theory. In this talk, I will discuss systematic methods to compute anomalies of theories engineered with branes, using as input the geometry and flux configuration transverse to the non-compact directions of the branes worldvolume. Examples from M-theory and Type IIB string theory illustrate the versatility of this approach, which can capture both ordinary and generalized symmetries, continuous or discrete.



’t Hooft anomalies of QFTs realized in string theory

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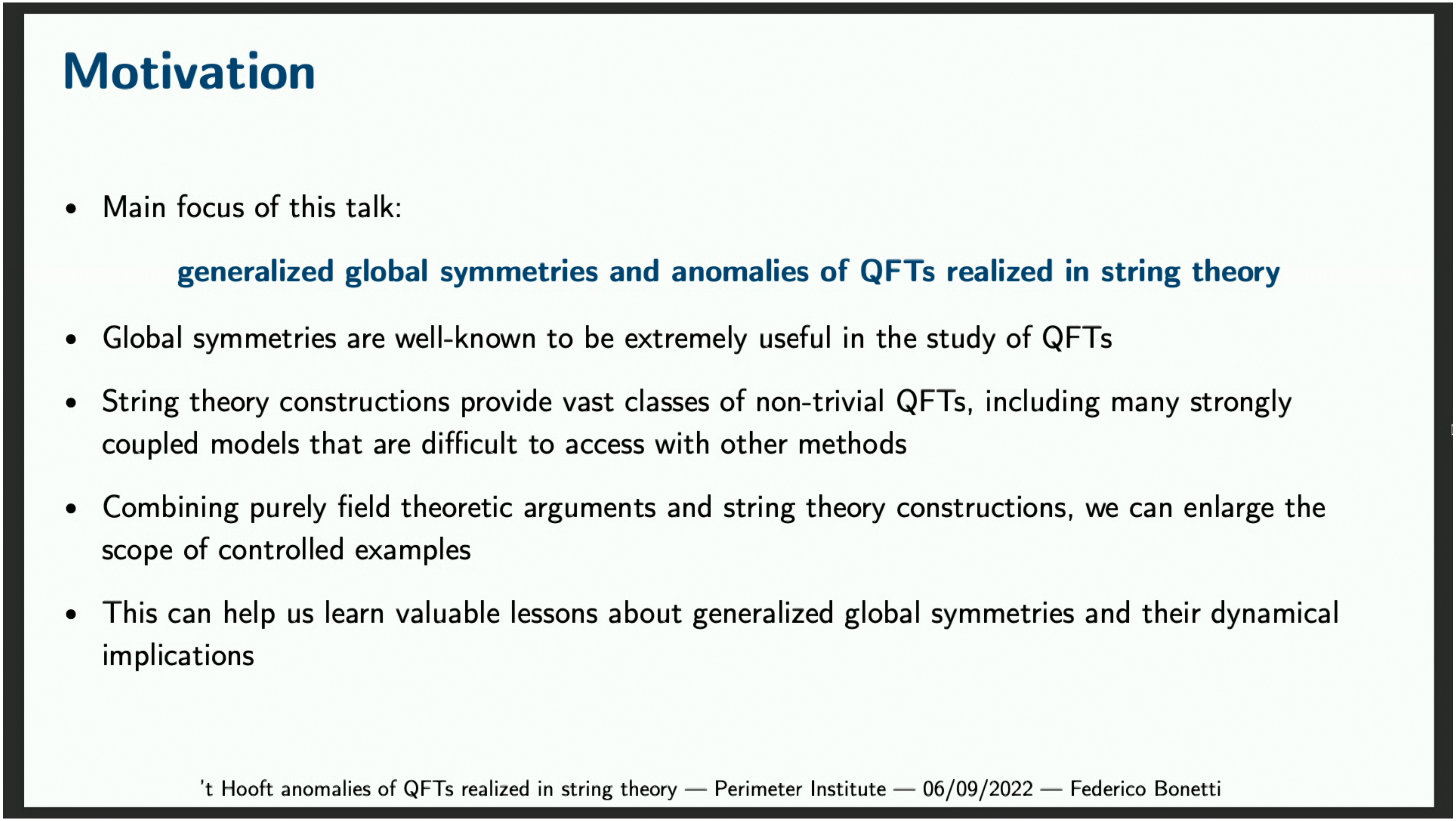
Simons Collaboration on Global Categorical Symmetries — Perimeter Institute — 06/09/2022

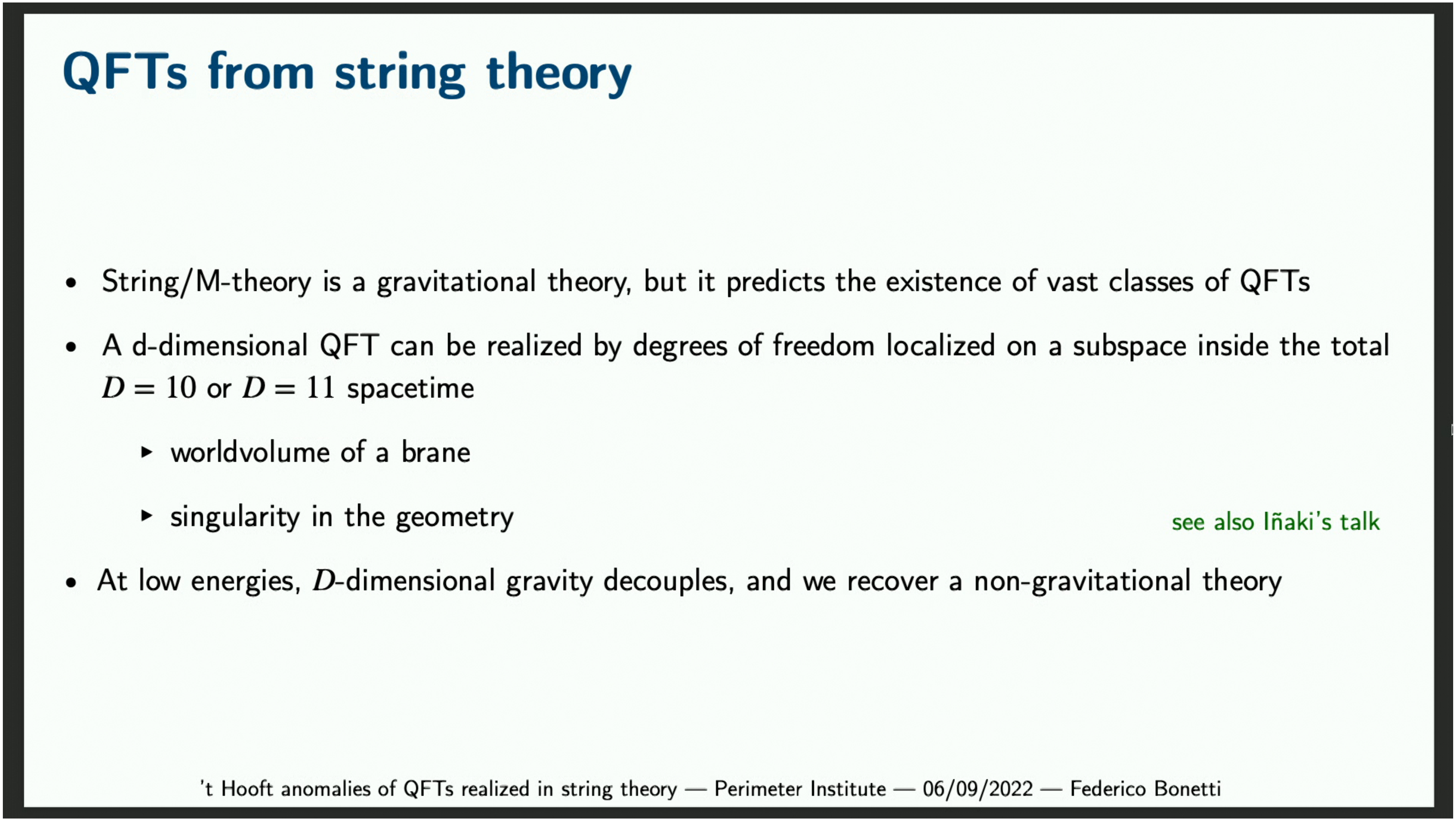


Motivation

- Main focus of this talk:

generalized global symmetries and anomalies of QFTs realized in string theory

- Global symmetries are well-known to be extremely useful in the study of QFTs
- String theory constructions provide vast classes of non-trivial QFTs, including many strongly coupled models that are difficult to access with other methods
- Combining purely field theoretic arguments and string theory constructions, we can enlarge the scope of controlled examples
- This can help us learn valuable lessons about generalized global symmetries and their dynamical implications



QFTs from string theory

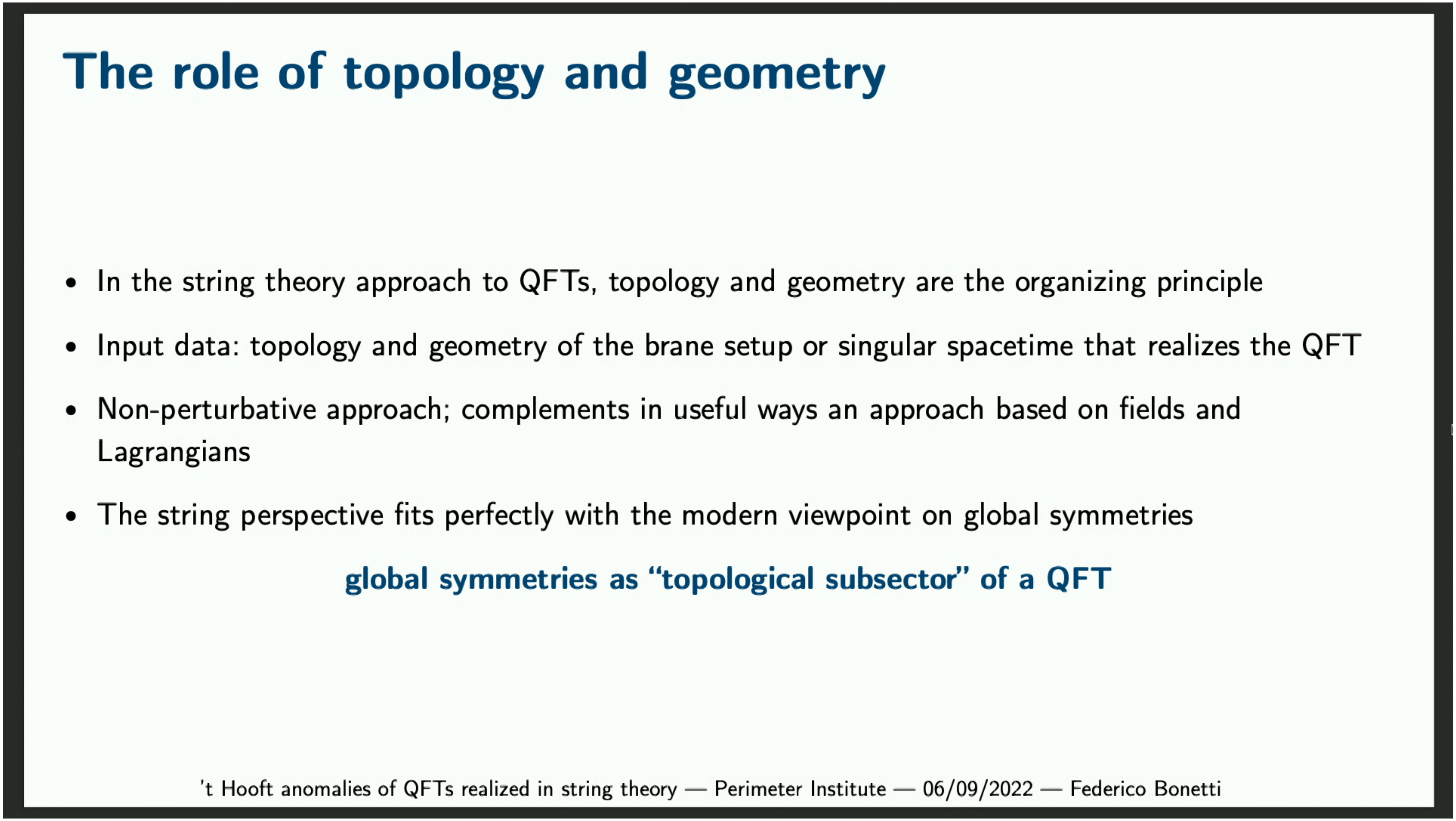
- String/M-theory is a gravitational theory, but it predicts the existence of vast classes of QFTs
- A d-dimensional QFT can be realized by degrees of freedom localized on a subspace inside the total $D = 10$ or $D = 11$ spacetime
    - worldvolume of a brane
    - singularity in the geometry

see also Iñaki's talk

- At low energies, D-dimensional gravity decouples, and we recover a non-gravitational theory

QFTs from string theory

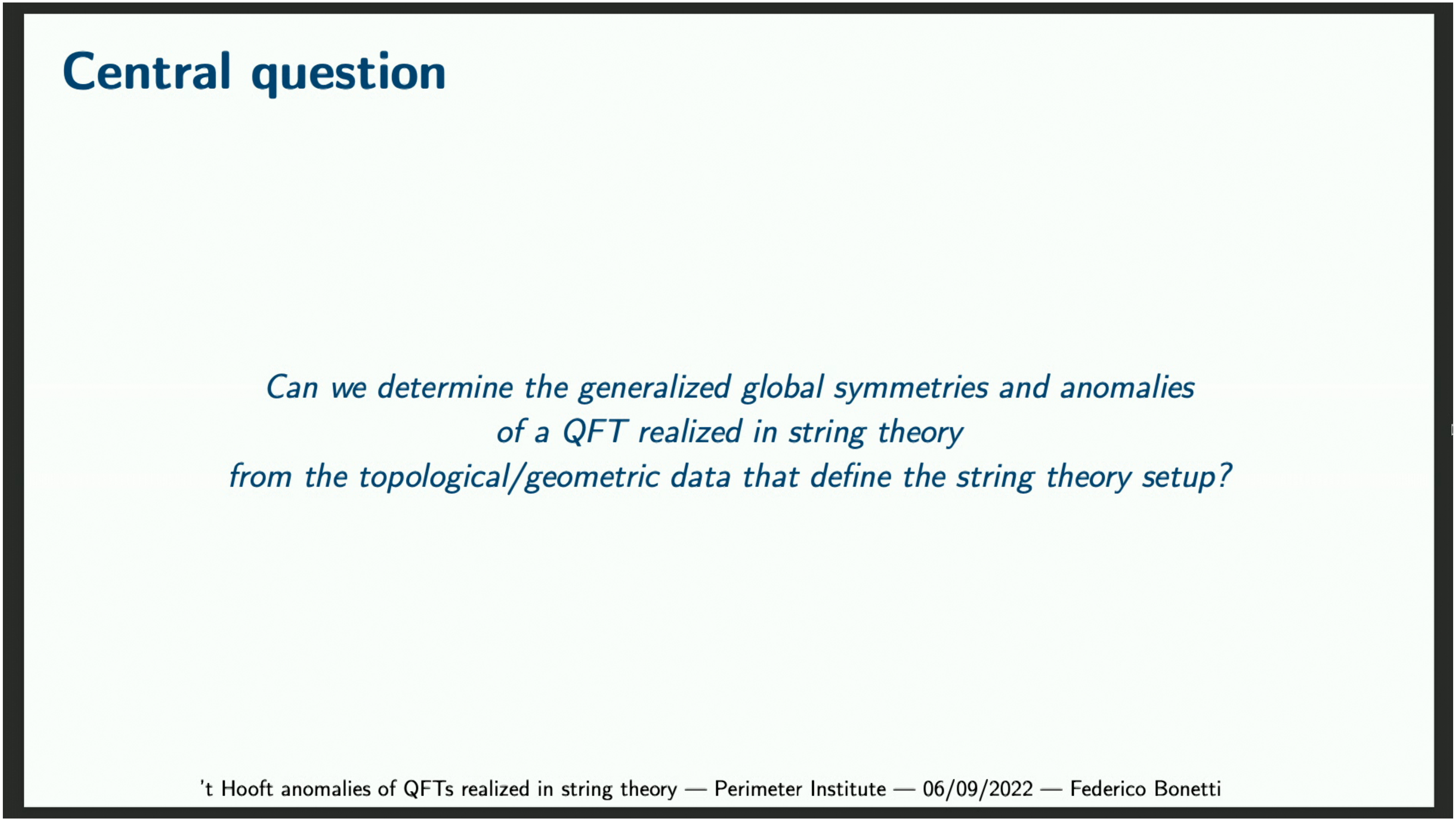
- String theory constructions yield vast classes of non-trivial QFTs, including intrinsically strongly coupled theories that do not admit a conventional Lagrangian description
- String/M-theory predicts the existence of interacting SCFTs in 5d and 6d, which would be hard to establish using purely field-theoretic methods
- These higher-dimensional fixed points can be reduced to lower dimensions
- Example: 6d SCFTs reduced to 4d SCFTs on a Riemann surface (class S program and its generalizations)



The role of topology and geometry

- In the string theory approach to QFTs, topology and geometry are the organizing principle
- Input data: topology and geometry of the brane setup or singular spacetime that realizes the QFT
- Non-perturbative approach; complements in useful ways an approach based on fields and Lagrangians
- The string perspective fits perfectly with the modern viewpoint on global symmetries

global symmetries as "topological subsector" of a QFT



Central question

Can we determine the generalized global symmetries and anomalies of a QFT realized in string theory from the topological/geometric data that define the string theory setup?

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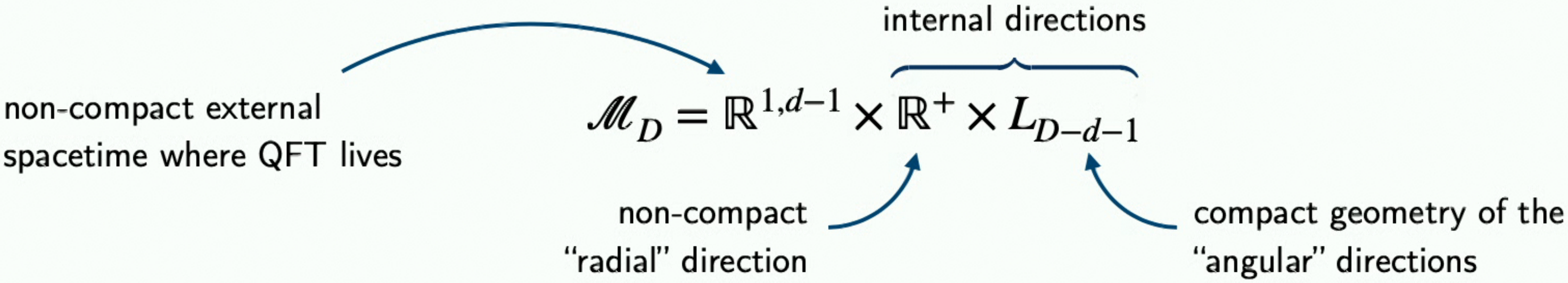
Strategy

- The d-dimensional QFT we want to study is realized by localized degrees of freedom inside a larger D-dimensional ambient space ($D = 10$ or $D = 11$)
- Schematically



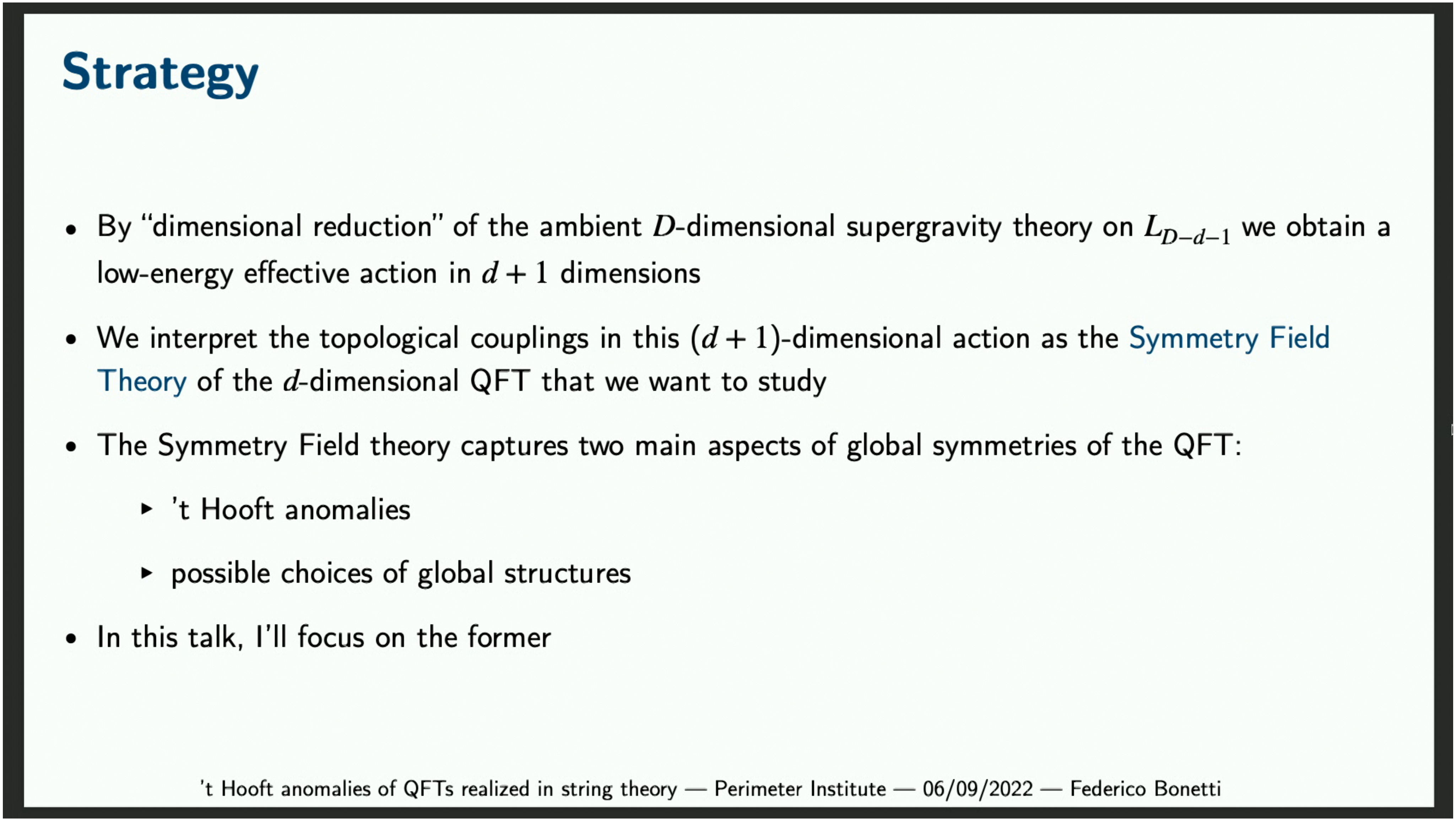
$$\mathscr{M}_D = \mathbb{R}^{1,d-1} \times \overbrace{\mathbb{R}^+ \times L_{D-d-1}}^{\text{internal directions}}$$

non-compact external spacetime where QFT lives

non-compact "radial" direction

compact geometry of the "angular" directions

- At low-energy the D-dimensional ambient space is described by a supergravity theory
- We can access the global symmetries and anomalies of the d-dimensional QFT by analyzing the couplings of the D-dimensional supergravity theory on the compact space L_{D-d-1}

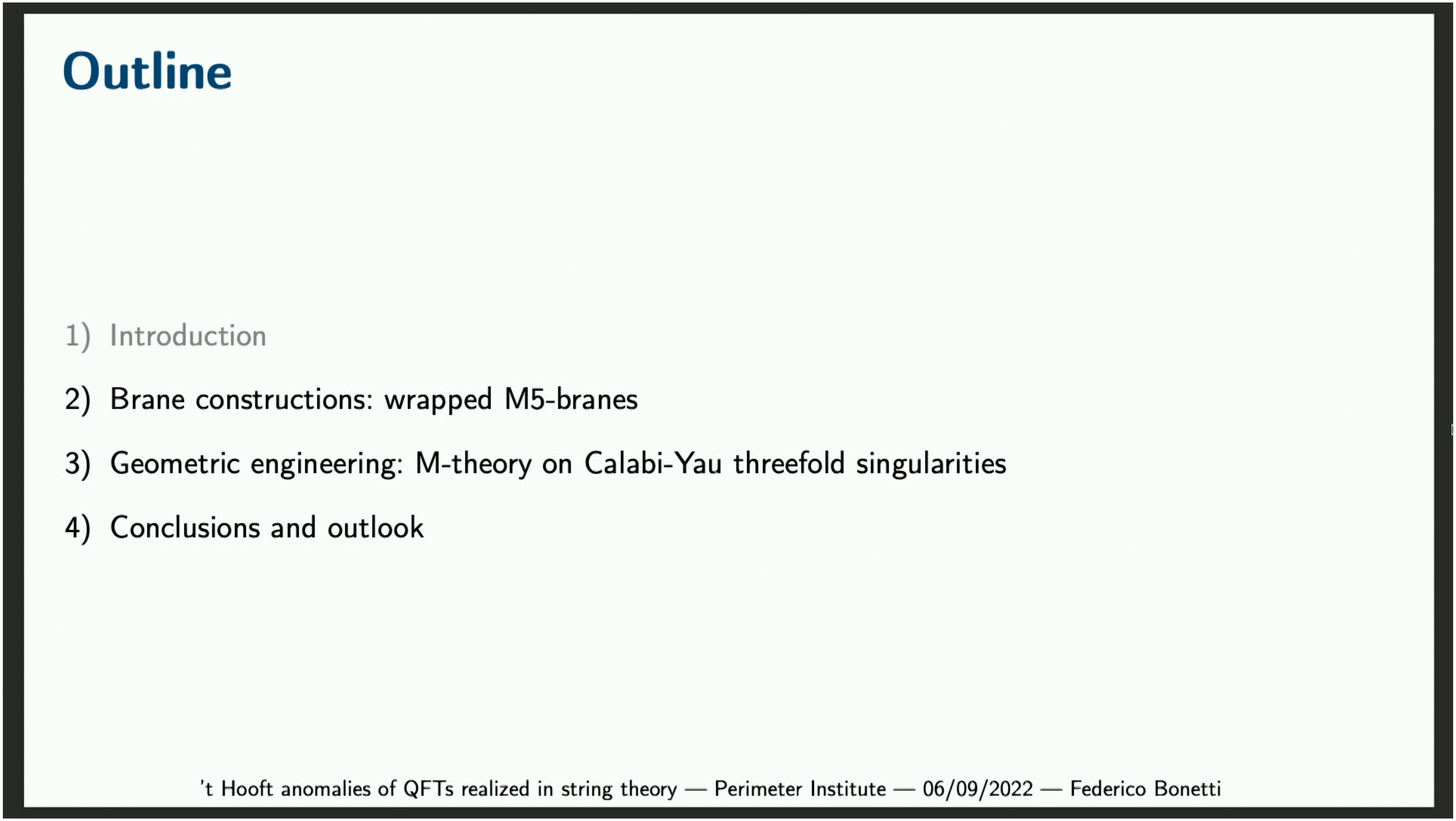


Strategy

- By "dimensional reduction" of the ambient D-dimensional supergravity theory on L_{D-d-1} we obtain a low-energy effective action in $d+1$ dimensions
- We interpret the topological couplings in this $(d+1)$-dimensional action as the Symmetry Field Theory of the d-dimensional QFT that we want to study
- The Symmetry Field theory captures two main aspects of global symmetries of the QFT:
  - 't Hooft anomalies
  - possible choices of global structures
- In this talk, I'll focus on the former

Strategy

- This approach can be applied both to brane constructions and to geometric engineering
- Ongoing program to extend this paradigm
  - include as many string theory constructions as possible (in M-theory, Type II string theory, F-theory, etc.)
  - identify/develop finer and finer tools to analyze the D-dimensional supergravity theory on L_{D-d-1} in order to capture subtler and subtler aspects of generalized global symmetries
- Today's talk: illustrate the versatility of this circle of ideas using two case studies
  - wrapped M5-branes (to study 4d SCFTs)
  - M-theory on Calabi-Yau threefold canonical singularity (to study 5d SCFTs)



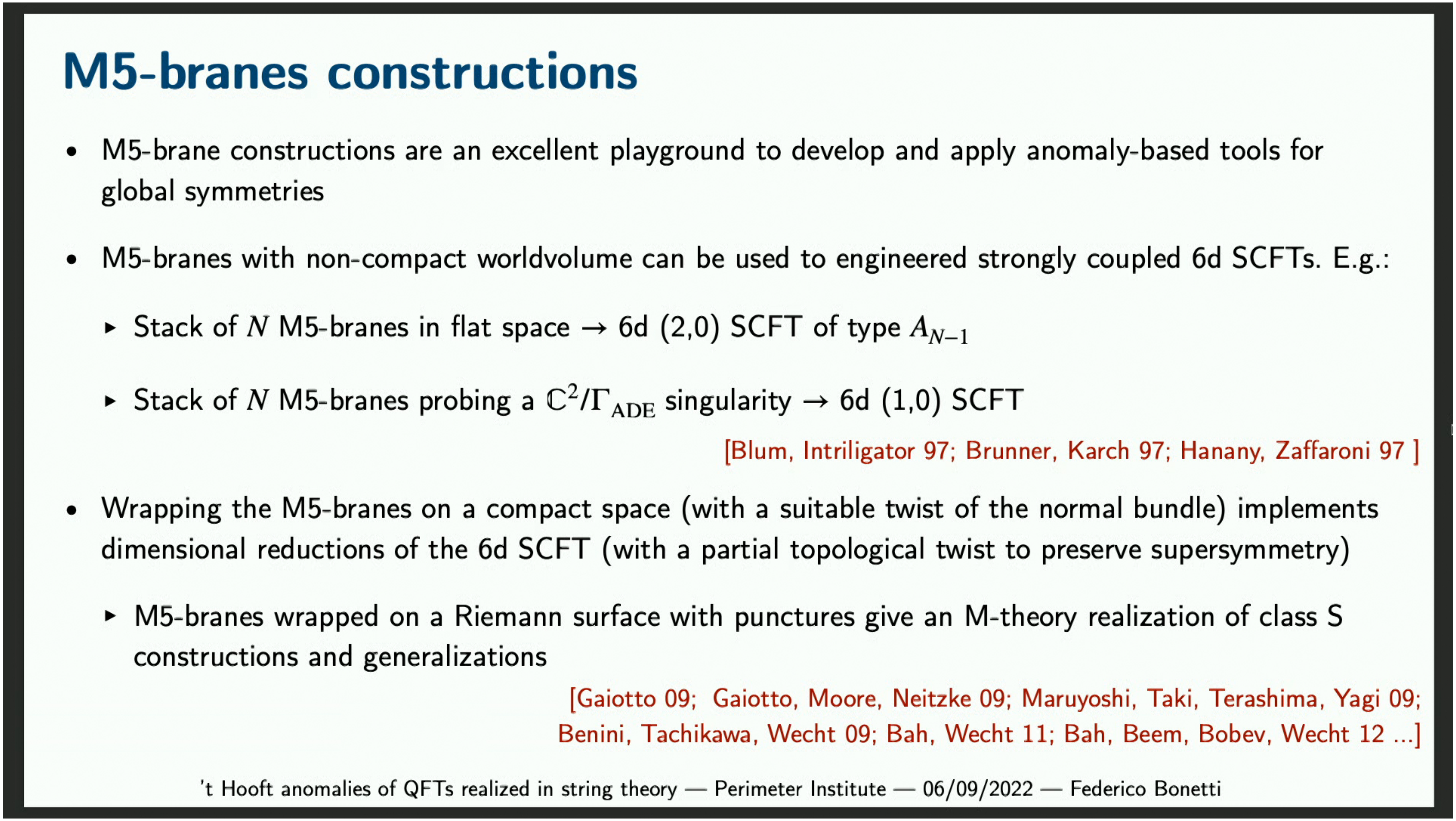
Outline

1) Introduction
2) Brane constructions: wrapped M5-branes
3) Geometric engineering: M-theory on Calabi-Yau threefold singularities
4) Conclusions and outlook

Brane constructions: wrapped M5-branes

Brane constructions and anomaly inflow

- For QFTs realized via brane constructions, **anomaly inflow** methods provide access to generalized global symmetries and the symmetry field theory
- The *global* symmetries of the worldvolume QFT correspond to *gauge* symmetries in the ambient space supergravity
- In the presence of the brane, the effective action of the ambient space supergravity is no longer gauge invariant
- This effect is precisely cancelled by the 't Hooft anomalies of the global symmetries on the brane worldvolume



M5-branes constructions

- M5-brane constructions are an excellent playground to develop and apply anomaly-based tools for global symmetries
- M5-branes with non-compact worldvolume can be used to engineered strongly coupled 6d SCFTs. E.g.:
  - Stack of N M5-branes in flat space → 6d (2,0) SCFT of type A_{N-1}
  - Stack of N M5-branes probing a $\mathbb{C}^2/\Gamma_{\text{ADE}}$ singularity → 6d (1,0) SCFT

[Blum, Intriligator 97; Brunner, Karch 97; Hanany, Zaffaroni 97]

- Wrapping the M5-branes on a compact space (with a suitable twist of the normal bundle) implements dimensional reductions of the 6d SCFT (with a partial topological twist to preserve supersymmetry)
  - M5-branes wrapped on a Riemann surface with punctures give an M-theory realization of class S constructions and generalizations

[Gaiotto 09; Gaiotto, Moore, Neitzke 09; Maruyoshi, Taki, Terashima, Yagi 09; Benini, Tachikawa, Wecht 09; Bah, Wecht 11; Bah, Beem, Bobev, Wecht 12 ...]

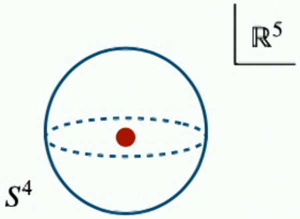
M5-branes as sources in supergravity

- M5-branes are magnetically charged under the 3-form gauge field C_3 of 11d supergravity

0	1	2	3	4	5	6	7	8	9	10
X	X	X	X	X	X					

$\mathbb{R}^{1,5}$ worldvolume

transverse $\mathbb{R}^5$



$$\int_{S^4} G_4 = N$$

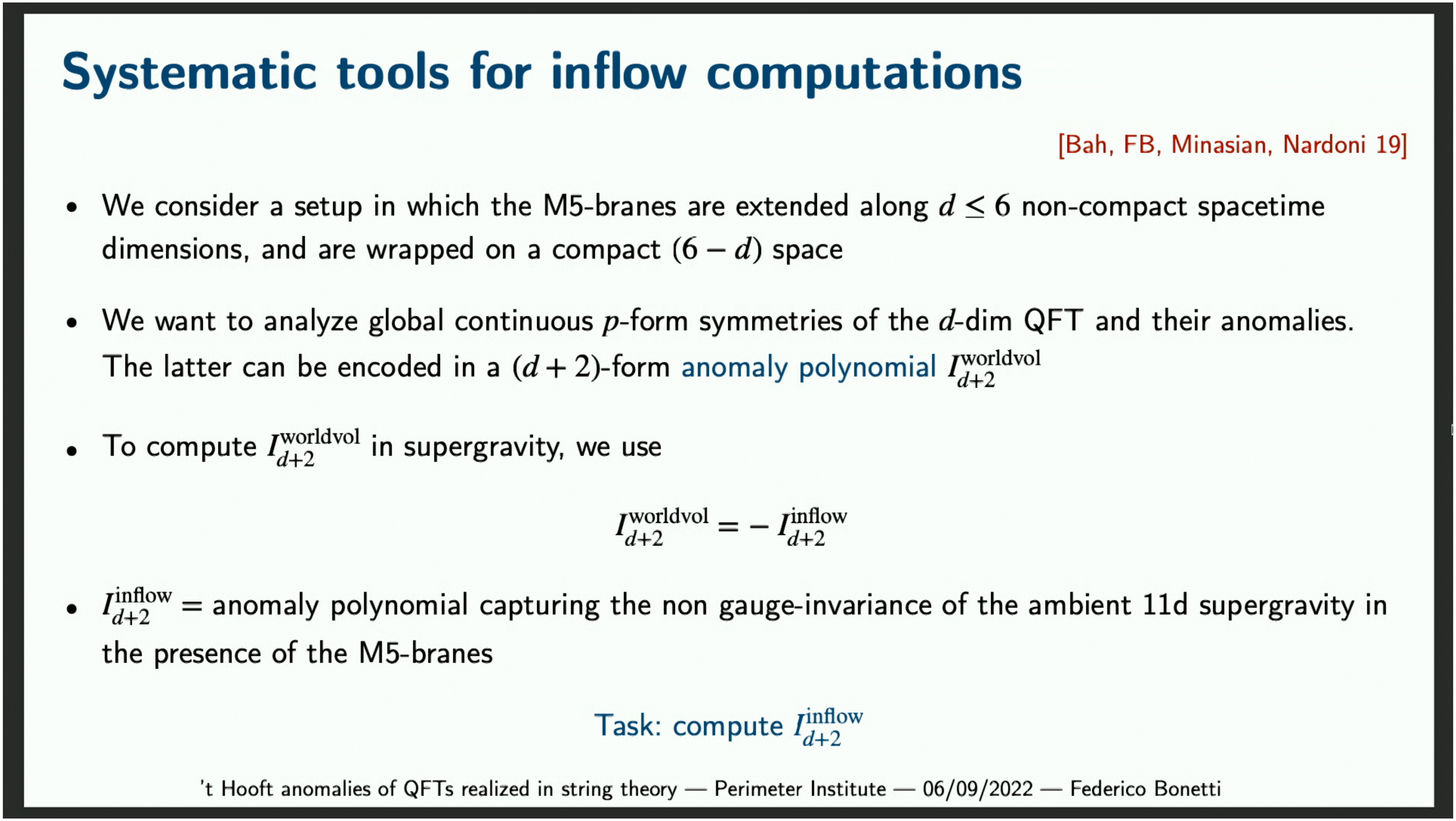
$$G_4 = dC_3$$

- The topological couplings in the M-theory low-energy effective action are written in terms of C_3

$$S_{\mathrm{M}}^{\mathrm{top}} = \int_{M_{11}} \left[-\frac{1}{6} C_3 \wedge G_4 \wedge G_4 - C_3 \wedge X_8 \right] \ , \qquad G_4 = dC_3 \ , \qquad X_8 = \frac{1}{192} \left[p_1(TM_{11})^2 - 4p_2(TM_{11}) \right]$$

- In the presence of M5-brane sources, these couplings require a careful treatment. They acquire anomalous gauge variations that cancel against the 't Hooft anomalies of the QFT on the worldvolume of the M5-branes

[Freed, Harvey, Minasian, Moore 98; Harvey, Minasian, Moore 98]



Systematic tools for inflow computations

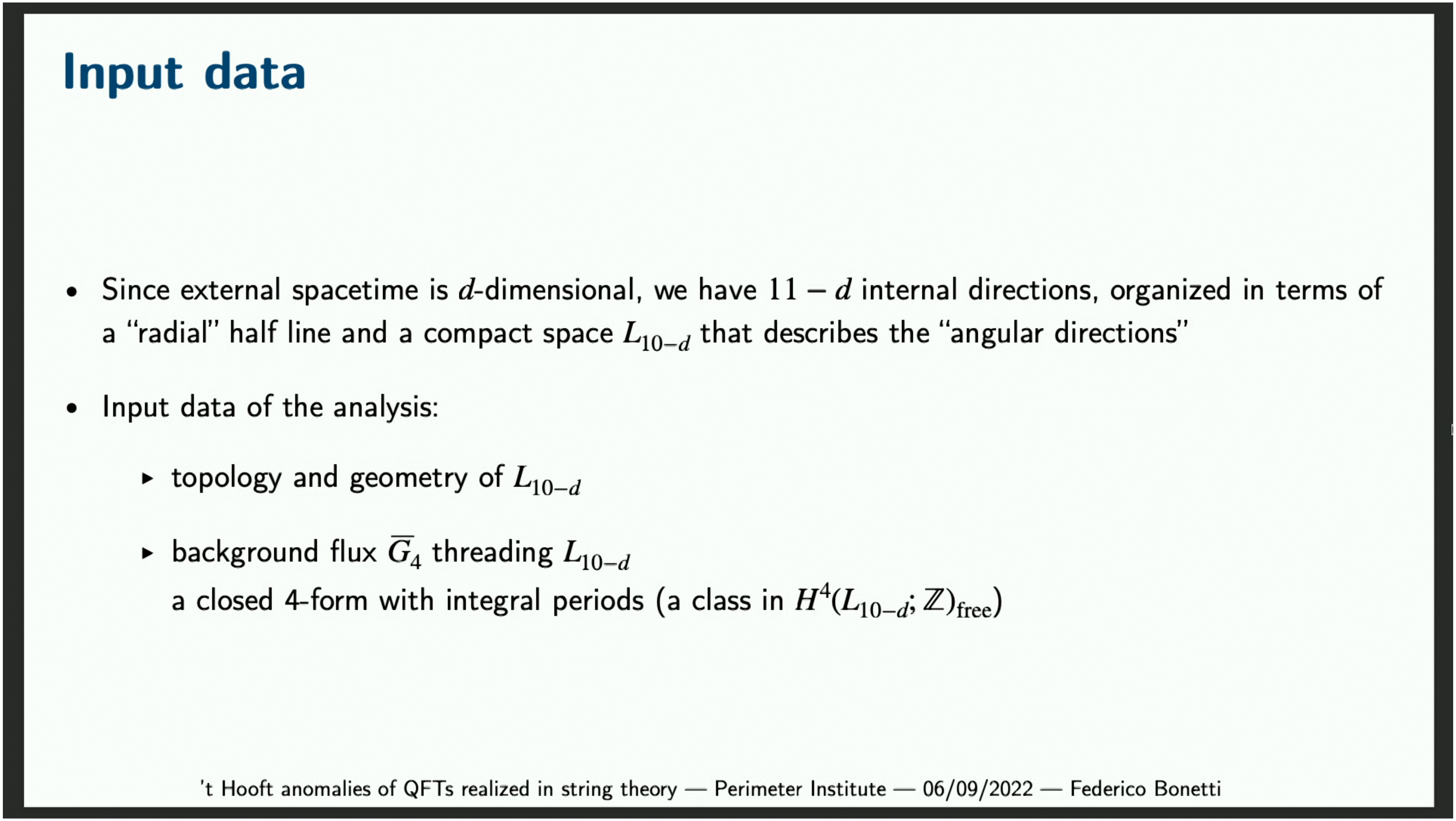
[Bah, FB, Minasian, Nardoni 19]

- We consider a setup in which the M5-branes are extended along $d \leq 6$ non-compact spacetime dimensions, and are wrapped on a compact $(6-d)$ space
- We want to analyze global continuous p-form symmetries of the d-dim QFT and their anomalies. The latter can be encoded in a $(d+2)$-form anomaly polynomial $I_{d+2}^{\text{worldvol}}$
- To compute $I_{d+2}^{\text{worldvol}}$ in supergravity, we use

$$I_{d+2}^{\text{worldvol}} = -\, I_{d+2}^{\text{inflow}}$$

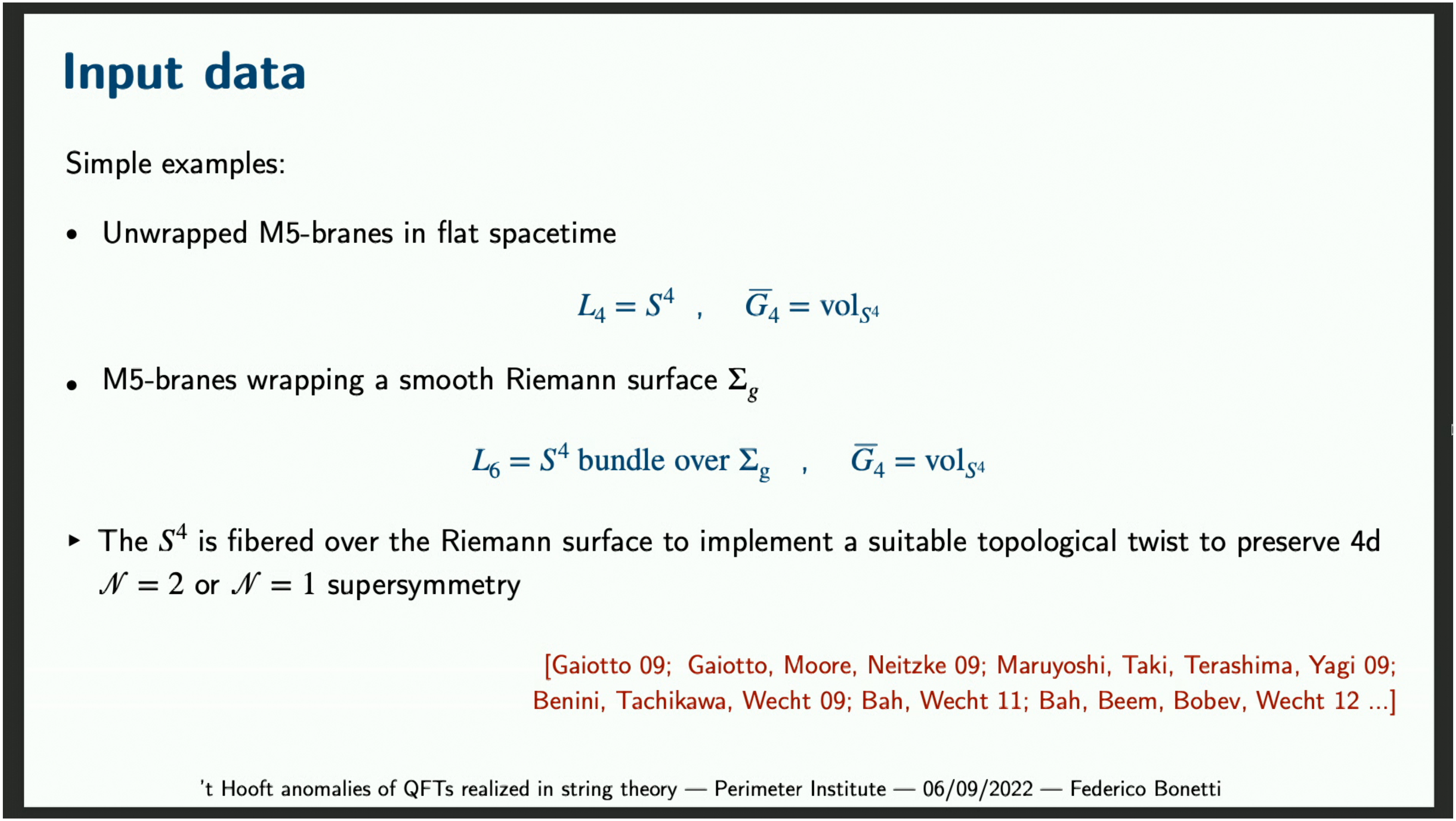
- I_{d+2}^{inflow} = anomaly polynomial capturing the non gauge-invariance of the ambient 11d supergravity in the presence of the M5-branes

Task: compute I_{d+2}^{inflow}



Input data

- Since external spacetime is d-dimensional, we have $11 - d$ internal directions, organized in terms of a "radial" half line and a compact space L_{10-d} that describes the "angular directions"
- Input data of the analysis:
  - topology and geometry of L_{10-d}
  - background flux $\overline{G}_4$ threading L_{10-d}
    a closed 4-form with integral periods (a class in $H^4(L_{10-d}; \mathbb{Z})_{\text{free}}$)



Input data

Simple examples:

- Unwrapped M5-branes in flat spacetime

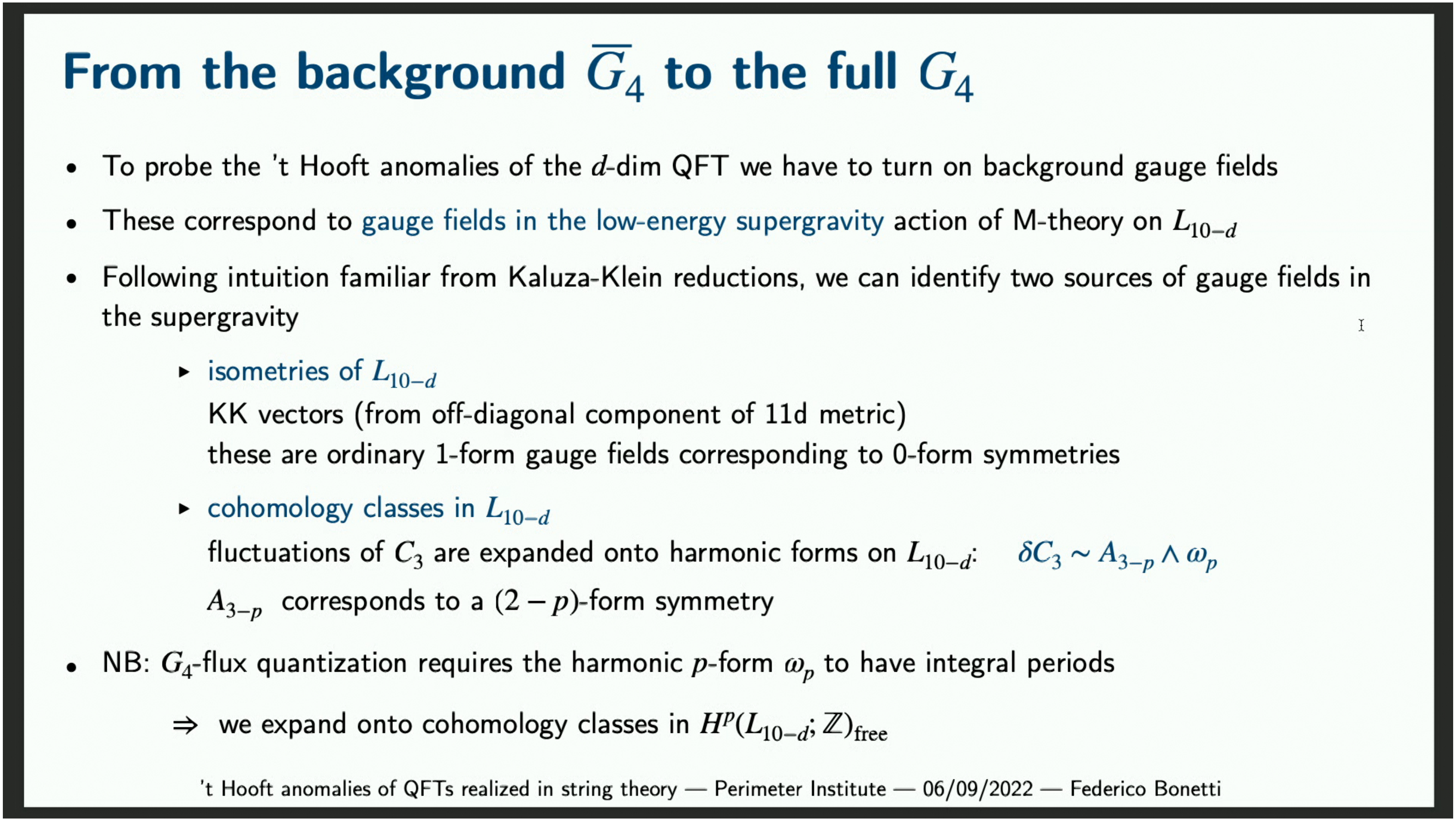
$$L_4 = S^4 \quad , \quad \overline{G}_4 = \mathrm{vol}_{S^4}$$

- M5-branes wrapping a smooth Riemann surface Σ_g

$$L_6 = S^4 \text{ bundle over } \Sigma_g \quad , \quad \overline{G}_4 = \mathrm{vol}_{S^4}$$

- The S^4 is fibered over the Riemann surface to implement a suitable topological twist to preserve 4d $\mathcal{N} = 2$ or $\mathcal{N} = 1$ supersymmetry

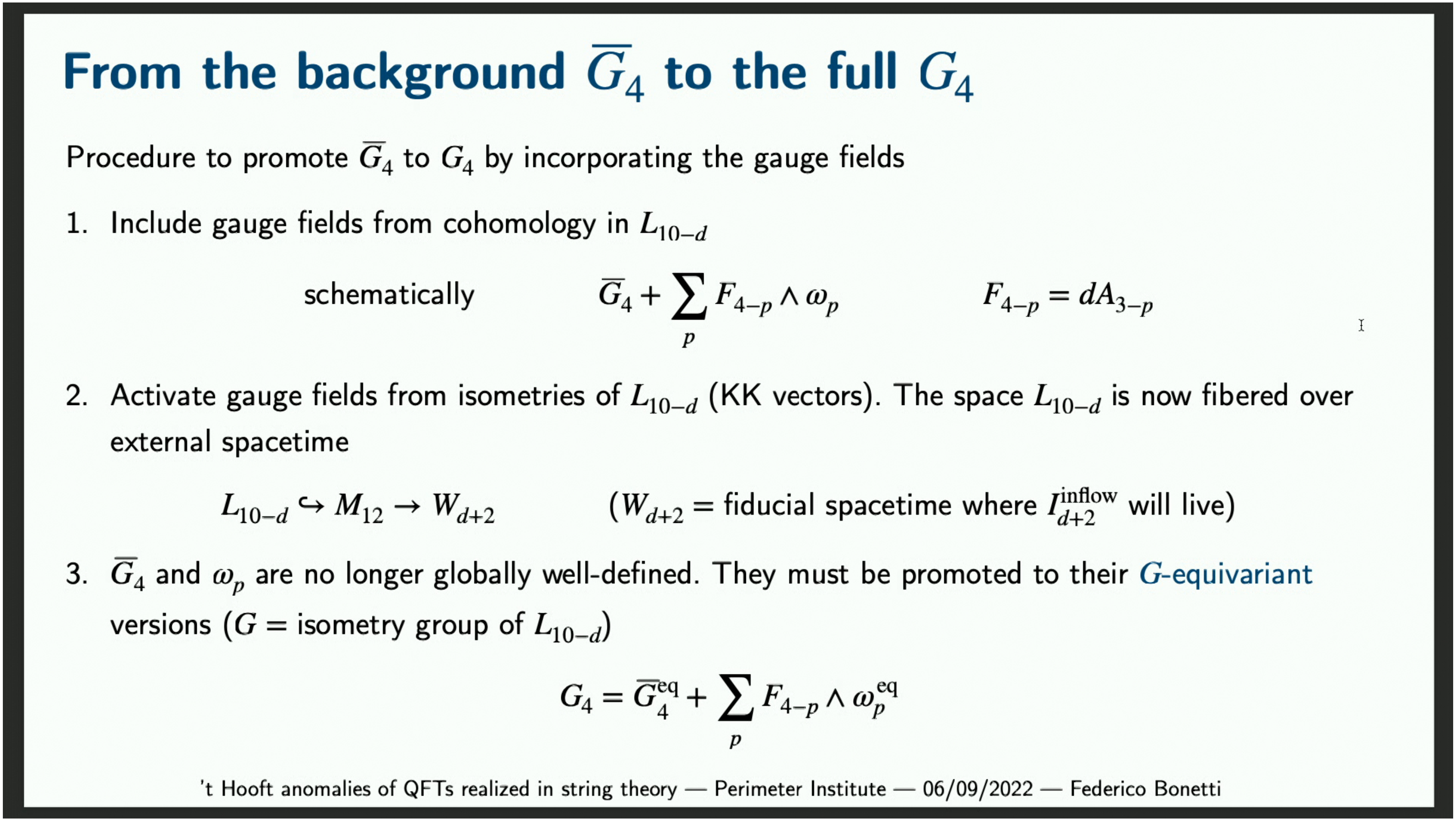
[Gaiotto 09; Gaiotto, Moore, Neitzke 09; Maruyoshi, Taki, Terashima, Yagi 09; Benini, Tachikawa, Wecht 09; Bah, Wecht 11; Bah, Beem, Bobev, Wecht 12 ...]



From the background $\overline{G}_4$ to the full G_4

- To probe the 't Hooft anomalies of the d-dim QFT we have to turn on background gauge fields
- These correspond to gauge fields in the low-energy supergravity action of M-theory on L_{10-d}
- Following intuition familiar from Kaluza-Klein reductions, we can identify two sources of gauge fields in the supergravity
  - isometries of L_{10-d}
    KK vectors (from off-diagonal component of 11d metric)
    these are ordinary 1-form gauge fields corresponding to 0-form symmetries
  - cohomology classes in L_{10-d}
    fluctuations of C_3 are expanded onto harmonic forms on L_{10-d}: $\delta C_3 \sim A_{3-p} \wedge \omega_p$
    A_{3-p} corresponds to a $(2-p)$-form symmetry
- NB: G_4-flux quantization requires the harmonic p-form ω_p to have integral periods

  $\Rightarrow$ we expand onto cohomology classes in $H^p(L_{10-d};\mathbb{Z})_{\text{free}}$



From the background $\overline{G}_4$ to the full G_4

Procedure to promote $\overline{G}_4$ to G_4 by incorporating the gauge fields

1. Include gauge fields from cohomology in L_{10-d}

   schematically $$\overline{G}_4 + \sum_p F_{4-p} \wedge \omega_p \qquad F_{4-p} = dA_{3-p}$$

2. Activate gauge fields from isometries of L_{10-d} (KK vectors). The space L_{10-d} is now fibered over external spacetime

   $$L_{10-d} \hookrightarrow M_{12} \to W_{d+2} \qquad (W_{d+2} = \text{fiducial spacetime where } I^{\text{inflow}}_{d+2} \text{ will live})$$

3. $\overline{G}_4$ and ω_p are no longer globally well-defined. They must be promoted to their G-equivariant versions ($G =$ isometry group of L_{10-d})

   $$G_4 = \overline{G}^{\text{eq}}_4 + \sum_p F_{4-p} \wedge \omega^{\text{eq}}_p$$

Aside: a toy example of equivariant completion

- Let us consider a round S^2

$$ds^2(S^2) = d\theta^2 + \sin^2\theta\, d\phi^2 \quad , \qquad \omega_2 = \frac{1}{2}\sin\theta\, d\theta\, \frac{d\phi}{2\pi}$$

- Let us fiber S^2 over a base space by turning on a connection for the $U(1)_\phi$ isometry

$$\frac{d\phi}{2\pi} \to \frac{d\phi}{2\pi} + A \quad , \qquad \omega_2 \to \frac{1}{2}\sin\theta\, d\theta \left(\frac{d\phi}{2\pi} + A\right) \qquad \text{is no longer closed!}$$

- We restore closure by adding a term proportional to $F = dA$

$$\omega_2^{\text{eq}} = \frac{1}{2}\sin\theta\, d\theta\left(\frac{d\phi}{2\pi} + A\right) - \frac{1}{2}\cos\theta\, F$$

- This procedure is isomorphic to the Cartan model for equivariant cohomology

Final steps in the computation of I_{d+2}^{inflow}

- The full G_4 is plugged in the formula

$$I_{d+2}^{\text{inflow}} = \int_{L_{10-d}} I_{12} \qquad\qquad I_{12} = -\frac{1}{6} G_4 \wedge G_4 \wedge G_4 - G_4 \wedge X_8$$

- Here X_8 is constructed with the tangent bundle of $L_{10-d} \hookrightarrow M_{12} \to W_{d+2}$ (includes KK vectors)
- The integral denotes fiber integration

An important caveat

- The inflow anomaly polynomial balances against the total 't Hooft anomalies of the worldvolume QFT. In many cases of interest, the latter in the IR consists of an interacting SCFT, plus decoupled sectors

- Example: for unwrapped N M5-branes in flat space

$$-I_8^{\text{inflow}} = I_8^{\text{worldvol}} = I_8^{\text{SCFT}} + I_8^{\text{free}}$$

I_8^{SCFT} : the interacting 6d (2,0) SCFT of type A_{N-1}

I_8^{free} : a free 6d (2,0) tensor multiplet

- Typically, the decoupled part of the worldvolume theory is associated to the center-of-mass modes of the M5-brane stack

Physical applications

Applications of the systematic methods described above include:

- study of punctures in the reduction of 6d SCFTs to 4d
- connections to holography
- geometric origin of subtle decoupling phenomena in RG flows
- ...

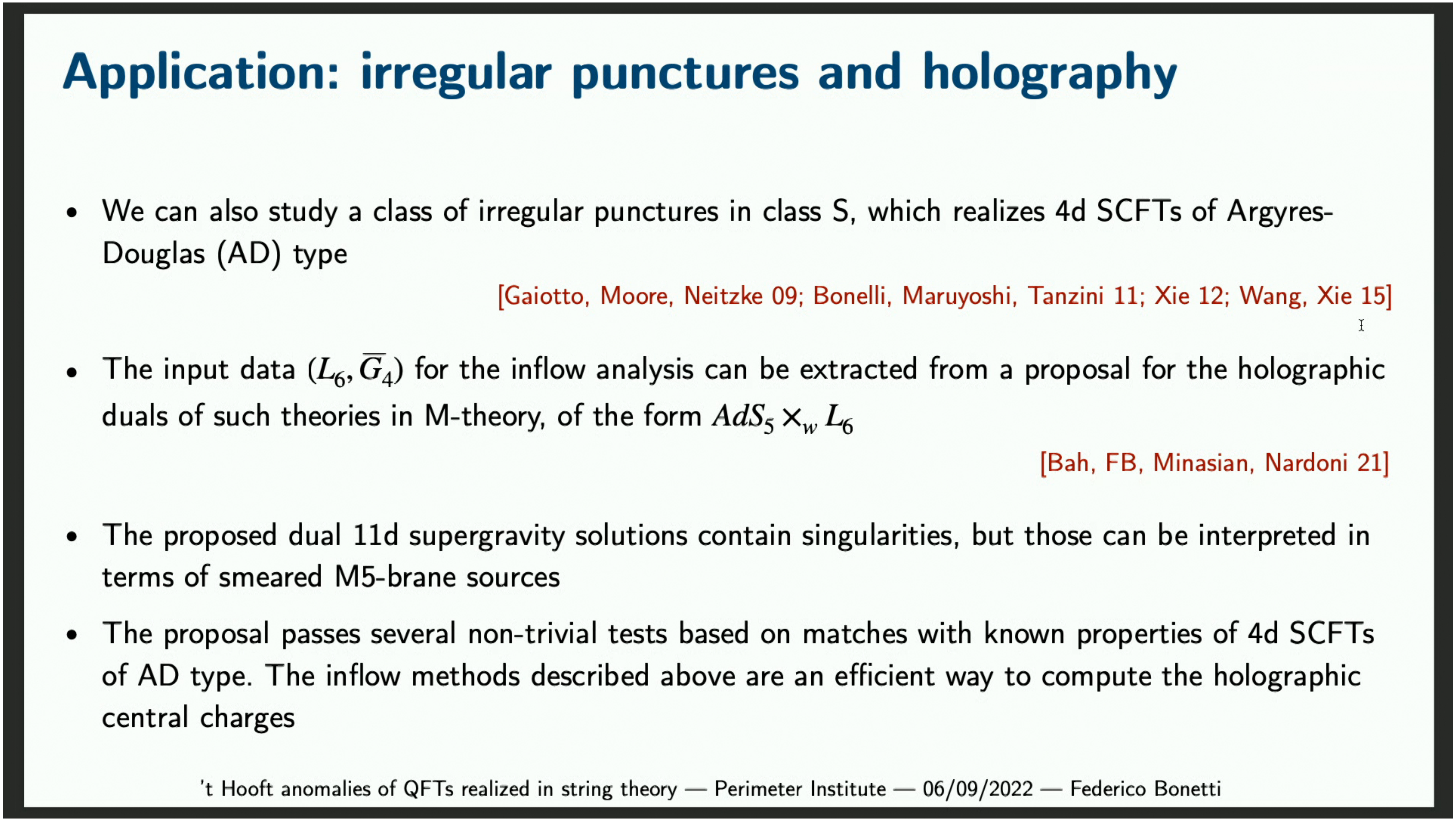
Application: punctured Riemann surfaces

[Bah, FB, Minasian, Nardoni 19]

- As mentioned above, if we consider M5-branes wrapped on a smooth Riemann surface: $S^4 \hookrightarrow L_6 \to \Sigma_g$
- In the presence of punctures, the space L_6 is no longer an S^4 bundle, but acquires a richer topology
- For regular punctures in class S, we can describe L_6 by removing small disks on the Riemann surface (and the S^4 fibers on top of them) and gluing in local puncture geometries

$$L_6 \cong L_6^{\text{bulk}} \cup \bigcup_{i=1}^{n} X_6^i$$

- The puncture geometries admit non-trivial 4-cycles: the pattern of associated flux quanta reproduces the partition of N that labels the regular puncture
- The inflow methods described above applied to this case give a first-principle derivation of the contribution of regular punctures to 't Hooft anomalies of the 4d SCFT
- NB: the contribution of punctures to the 't Hooft anomalies cannot be derived by integrating the anomaly polynomial of the 6d SCFT. It is crucial to go back to the 11d construction in M-theory



Application: irregular punctures and holography

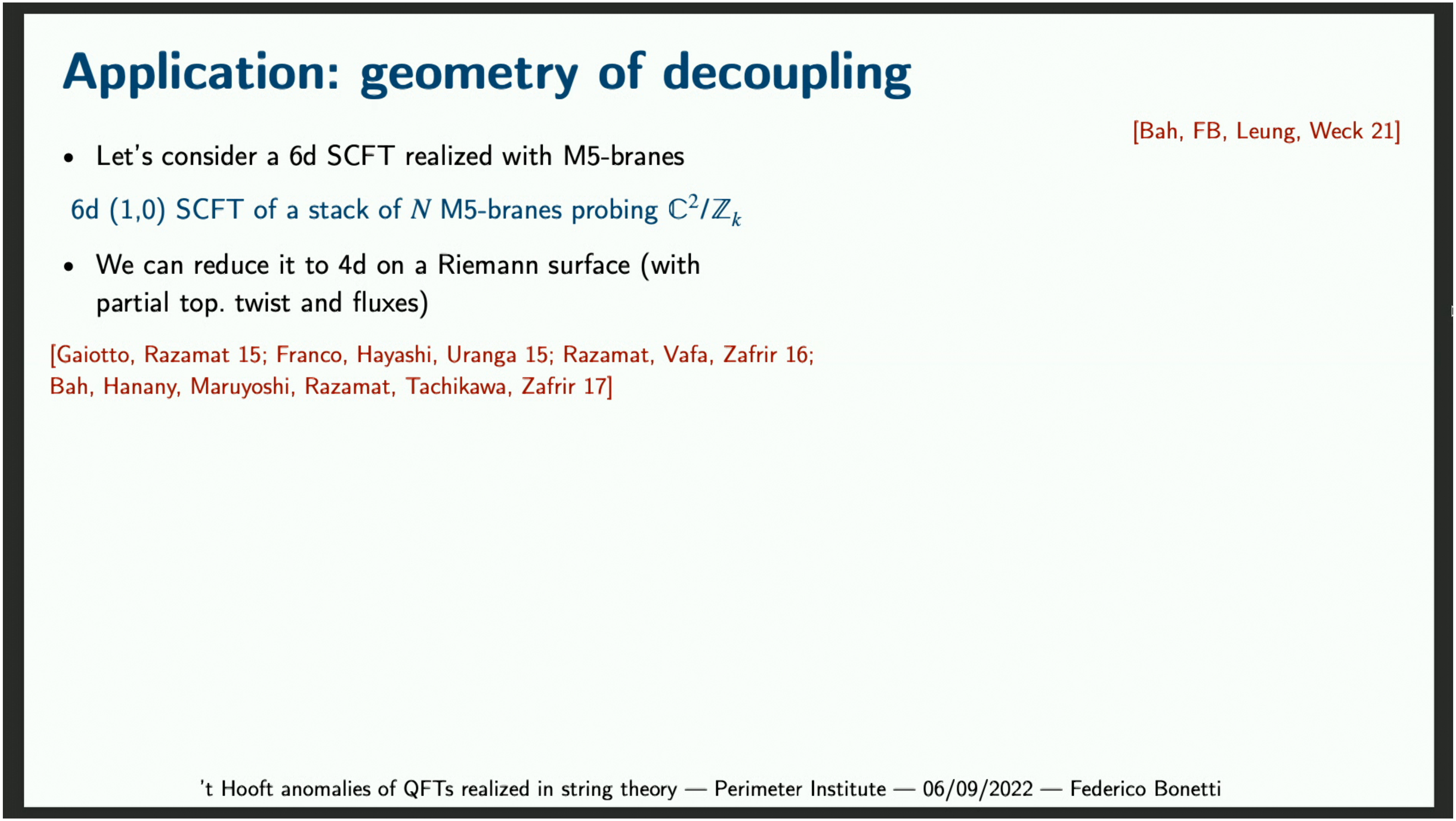
- We can also study a class of irregular punctures in class S, which realizes 4d SCFTs of Argyres-Douglas (AD) type

[Gaiotto, Moore, Neitzke 09; Bonelli, Maruyoshi, Tanzini 11; Xie 12; Wang, Xie 15]

- The input data $(L_6, \overline{G}_4)$ for the inflow analysis can be extracted from a proposal for the holographic duals of such theories in M-theory, of the form $AdS_5 \times_w L_6$

[Bah, FB, Minasian, Nardoni 21]

- The proposed dual 11d supergravity solutions contain singularities, but those can be interpreted in terms of smeared M5-brane sources
- The proposal passes several non-trivial tests based on matches with known properties of 4d SCFTs of AD type. The inflow methods described above are an efficient way to compute the holographic central charges



Application: geometry of decoupling

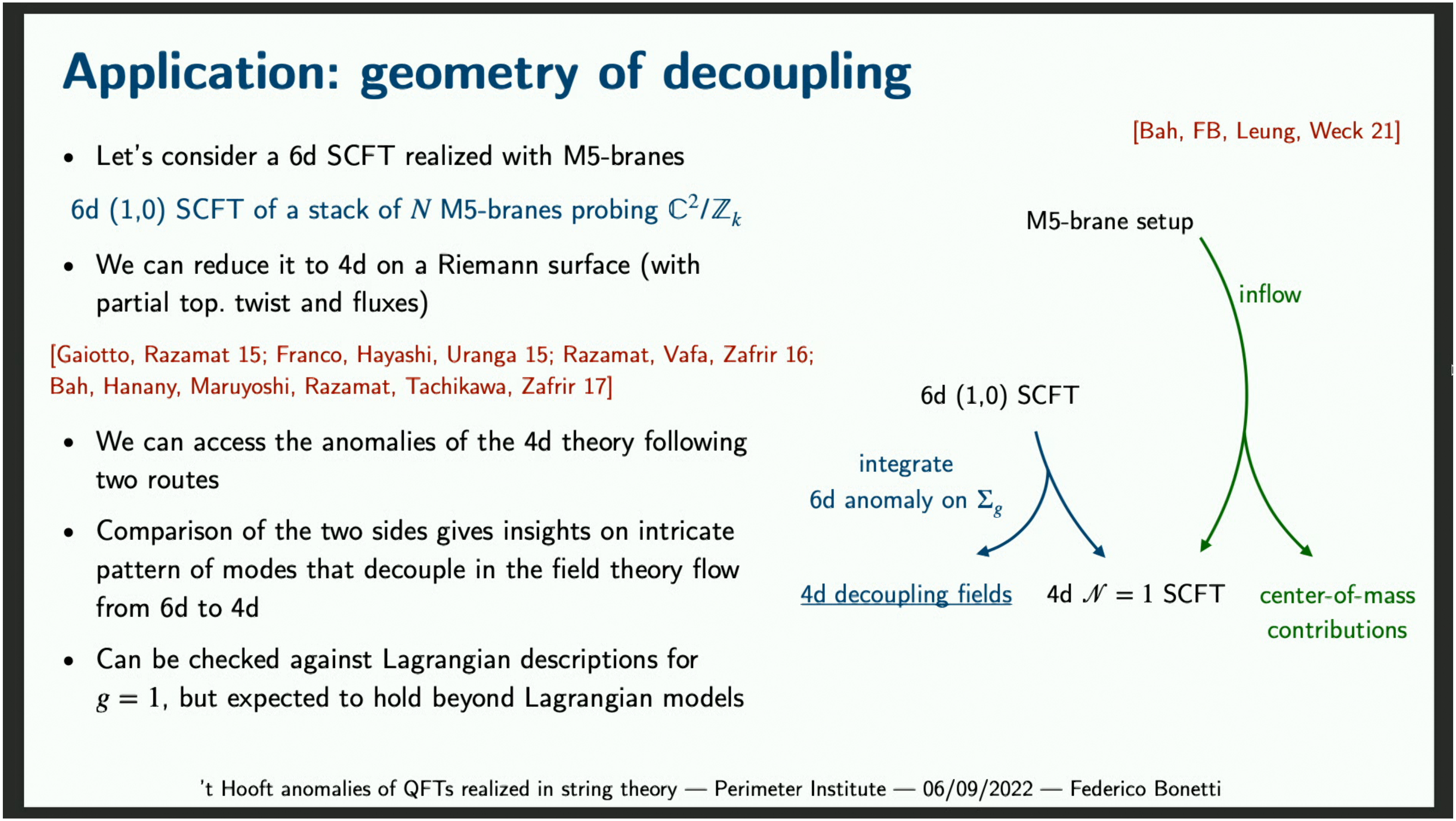
[Bah, FB, Leung, Weck 21]

- Let's consider a 6d SCFT realized with M5-branes

6d (1,0) SCFT of a stack of N M5-branes probing $\mathbb{C}^2/\mathbb{Z}_k$

- We can reduce it to 4d on a Riemann surface (with partial top. twist and fluxes)

[Gaiotto, Razamat 15; Franco, Hayashi, Uranga 15; Razamat, Vafa, Zafrir 16; Bah, Hanany, Maruyoshi, Razamat, Tachikawa, Zafrir 17]



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- We can access the anomalies of the 4d theory following two routes
- Comparison of the two sides gives insights on intricate pattern of modes that decouple in the field theory flow from 6d to 4d
- Can be checked against Lagrangian descriptions for $g = 1$, but expected to hold beyond Lagrangian models

M5-brane setup

inflow

6d (1,0) SCFT

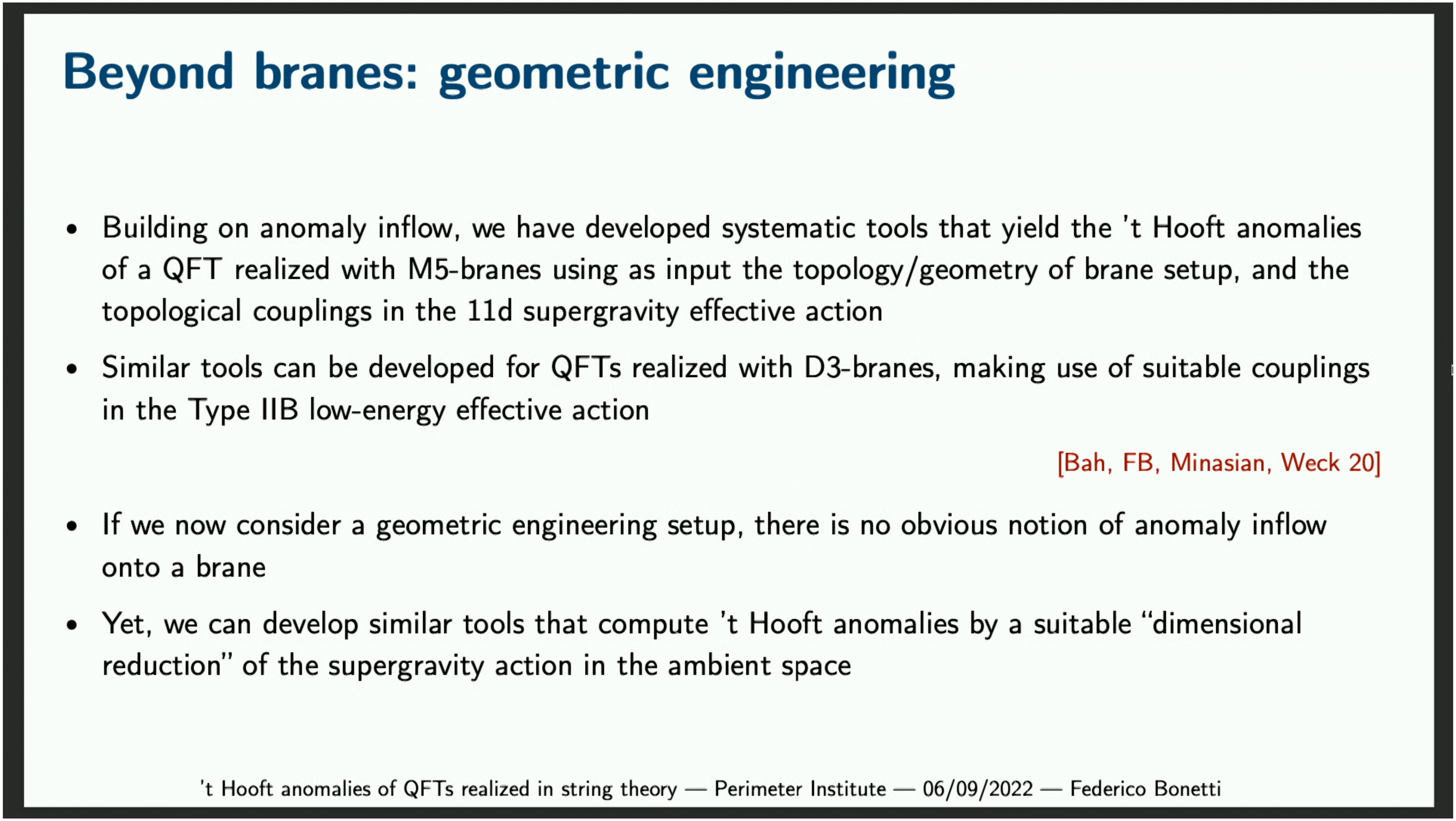
integrate
6d anomaly on Σ_g

4d decoupling fields

4d $\mathcal{N} = 1$ SCFT

center-of-mass
contributions

Geometric engineering: M-theory on Calabi-Yau threefold singularities



Beyond branes: geometric engineering

- Building on anomaly inflow, we have developed systematic tools that yield the 't Hooft anomalies of a QFT realized with M5-branes using as input the topology/geometry of brane setup, and the topological couplings in the 11d supergravity effective action
- Similar tools can be developed for QFTs realized with D3-branes, making use of suitable couplings in the Type IIB low-energy effective action

[Bah, FB, Minasian, Weck 20]

- If we now consider a geometric engineering setup, there is no obvious notion of anomaly inflow onto a brane
- Yet, we can develop similar tools that compute 't Hooft anomalies by a suitable "dimensional reduction" of the supergravity action in the ambient space

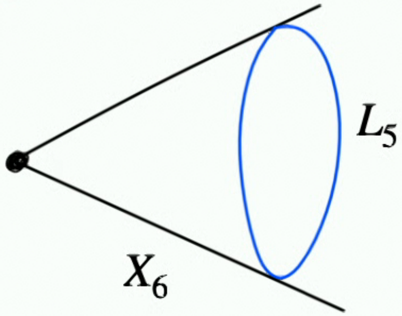
Case study: 5d SCFTs from M-theory

- A vast class of interacting 5d SCFTs can be realized by putting M-theory on a canonical Calabi-Yau threefold singularity

[Seiberg 96; Morrison, Seiberg 96; Intriligator, Morrison, Seiberg 97;
Aharony, Hanany 97; Leung, Vafa 97]

- The non-compact geometry X_6 is a real cone over a 5d Sasaki-Einstein manifold L_5

- For isolated singularities, L_5 is a smooth space

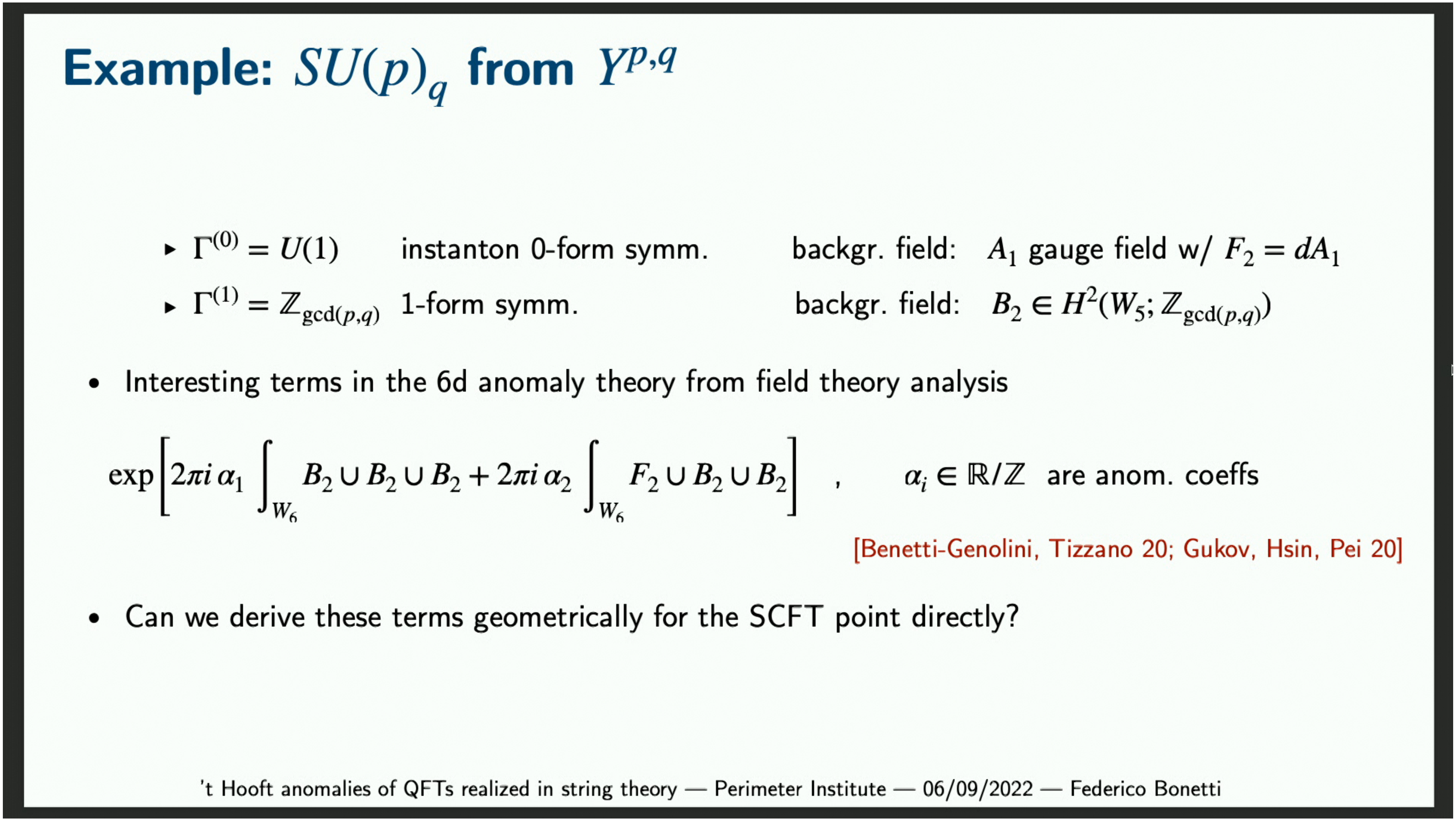


Example: $SU(p)_q$ from $Y^{p,q}$

- If we choose $L_5 = Y^{p,q}$, we realize a 5d SCFT that admits a mass deformation to

  5d $SU(p)_q$ SYM

- Global symmetries manifest from the non-Abelian gauge theory description:
  - $\Gamma^{(0)} = U(1)$ instanton 0-form symm. backgr. field: A_1 gauge field w/ $F_2 = dA_1$
  - $\Gamma^{(1)} = \mathbb{Z}_{\mathrm{gcd}(p,q)}$ 1-form symm. backgr. field: $B_2 \in H^2(W_5; \mathbb{Z}_{\mathrm{gcd}(p,q)})$



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- $\Gamma^{(0)} = U(1)$ instanton 0-form symm. backgr. field: A_1 gauge field w/ $F_2 = dA_1$
- $\Gamma^{(1)} = \mathbb{Z}_{\gcd(p,q)}$ 1-form symm. backgr. field: $B_2 \in H^2(W_5; \mathbb{Z}_{\gcd(p,q)})$

- Interesting terms in the 6d anomaly theory from field theory analysis

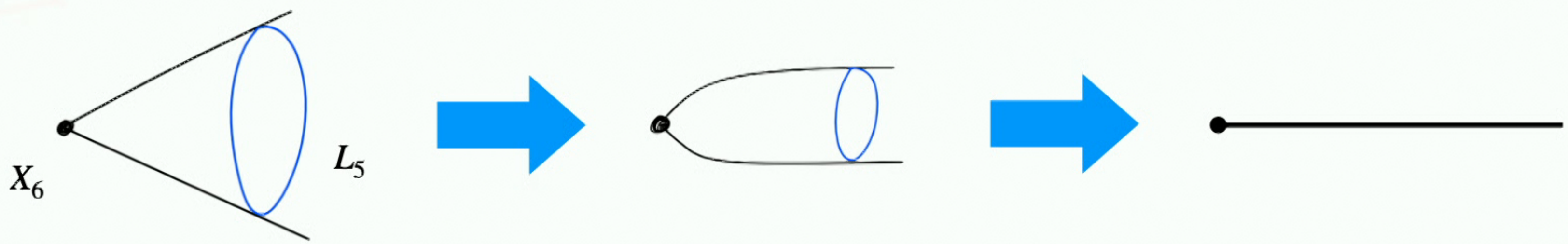
$$\exp\left[2\pi i\,\alpha_1 \int_{W_6} B_2 \cup B_2 \cup B_2 + 2\pi i\,\alpha_2 \int_{W_6} F_2 \cup B_2 \cup B_2\right] \quad , \qquad \alpha_i \in \mathbb{R}/\mathbb{Z} \ \text{ are anom. coeffs}$$

[Benetti-Genolini, Tizzano 20; Gukov, Hsin, Pei 20]

- Can we derive these terms geometrically for the SCFT point directly?

Philosophy: “dimensional reduction” on the link

[Apruzzi, FB, García Etxebarria, Hosseini, Schäfer-Nameki 21]



- X_6 is a singularity linked by the compact space L_5
- Since we are interested in topological couplings, we can deform the cone to a cigar. In the limit of shrinking cigar, we obtain a TFT in 6d
- This philosophy can be applied to extract both BF terms and 't Hooft anomalies

cf. Iñaki's talk and Saghar's gong show and poster

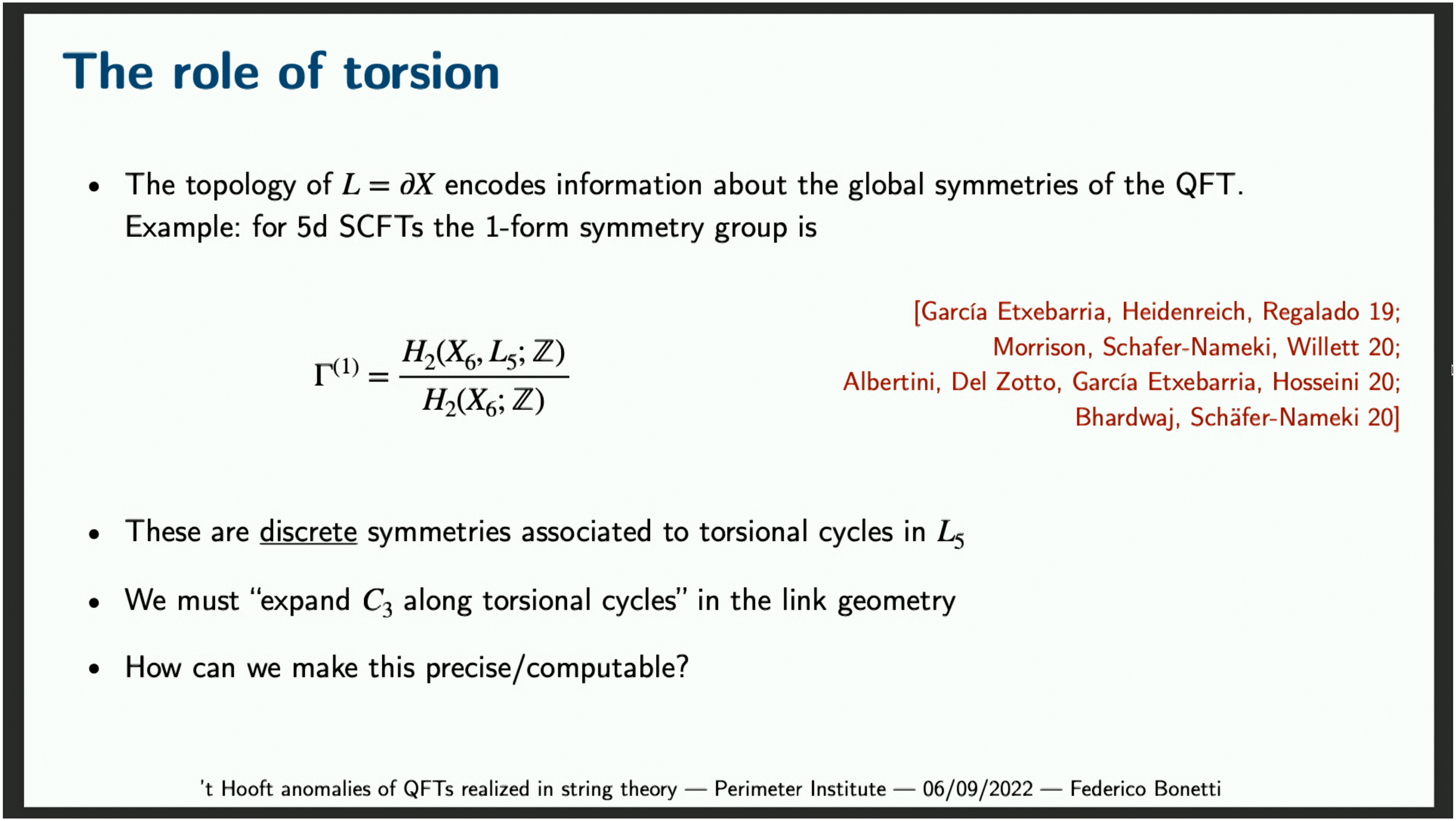
How to obtain anomaly terms

Prescription:

- Perform a dimensional reduction on the compact link geometry L_{d-1} of the topological couplings of 11d supergravity, plus X_8 correction

$$S_{\mathrm{M}}^{\mathrm{top}} = \int_{M_{11}} \left[-\frac{1}{6} C_3 \wedge G_4 \wedge G_4 - C_3 \wedge X_8 \right]$$

- The reduction is to be done à la Kaluza-Klein:
    1. C_3 is expanded schematically as (external background field).(internal form)
    2. we integrate on L_{d-1}



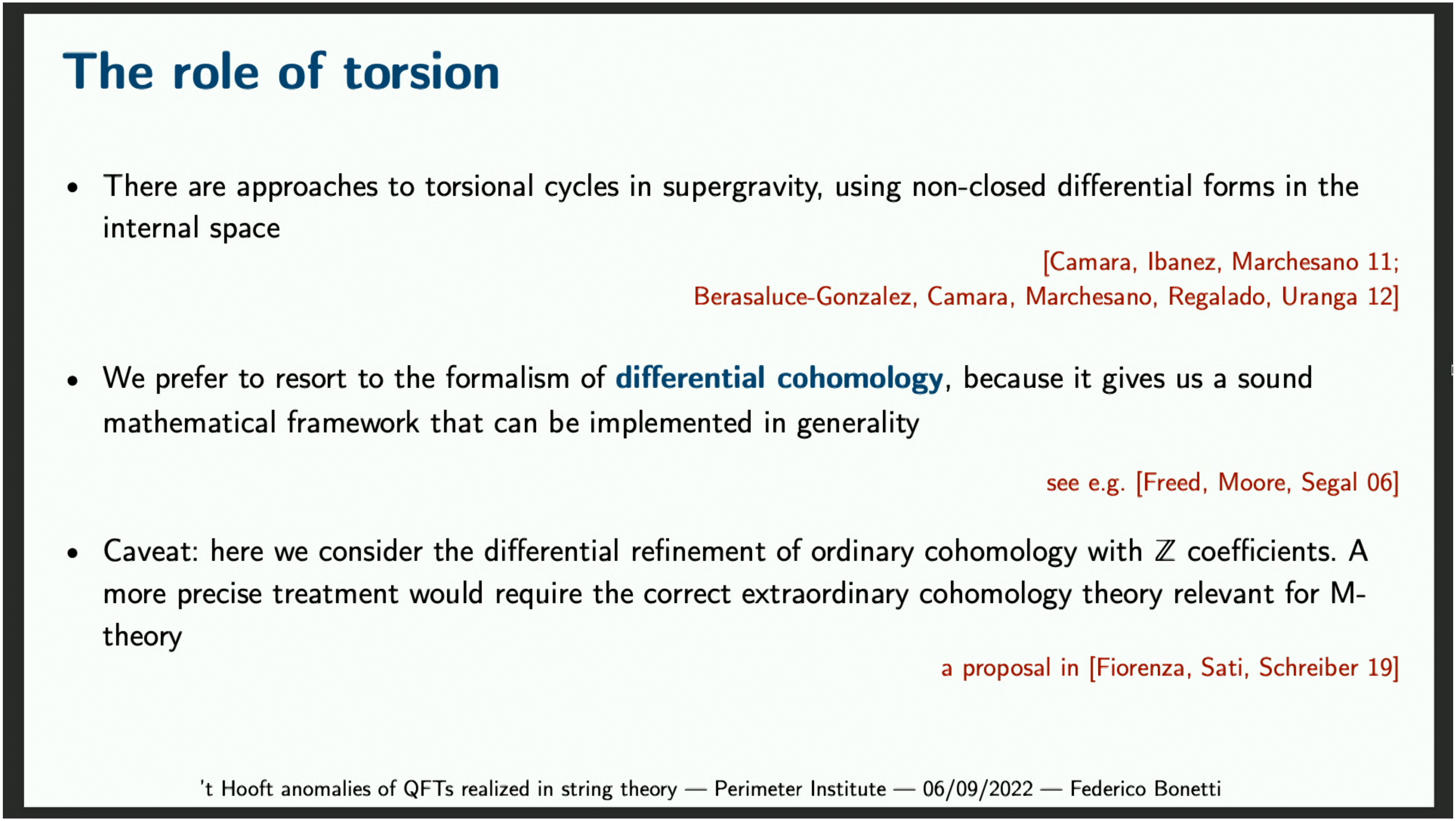
The role of torsion

- The topology of $L = \partial X$ encodes information about the global symmetries of the QFT. Example: for 5d SCFTs the 1-form symmetry group is

$$\Gamma^{(1)} = \frac{H_2(X_6, L_5; \mathbb{Z})}{H_2(X_6; \mathbb{Z})}$$

[García Etxebarria, Heidenreich, Regalado 19;
Morrison, Schafer-Nameki, Willett 20;
Albertini, Del Zotto, García Etxebarria, Hosseini 20;
Bhardwaj, Schäfer-Nameki 20]

- These are discrete symmetries associated to torsional cycles in L_5
- We must "expand C_3 along torsional cycles" in the link geometry
- How can we make this precise/computable?



The role of torsion

- There are approaches to torsional cycles in supergravity, using non-closed differential forms in the internal space

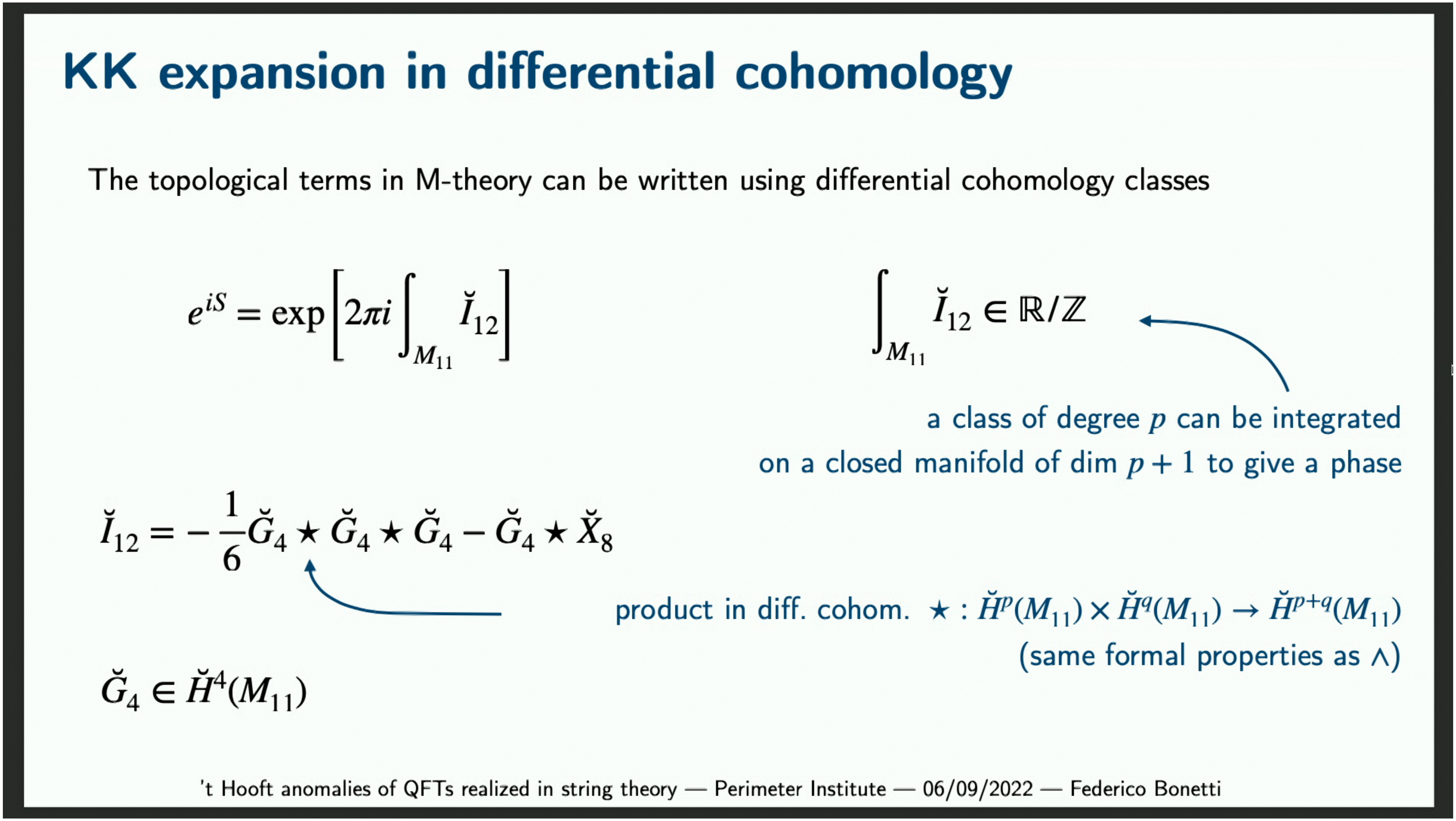
[Camara, Ibanez, Marchesano 11;
Berasaluce-Gonzalez, Camara, Marchesano, Regalado, Uranga 12]

- We prefer to resort to the formalism of **differential cohomology**, because it gives us a sound mathematical framework that can be implemented in generality

see e.g. [Freed, Moore, Segal 06]

- Caveat: here we consider the differential refinement of ordinary cohomology with $\mathbb{Z}$ coefficients. A more precise treatment would require the correct extraordinary cohomology theory relevant for M-theory

a proposal in [Fiorenza, Sati, Schreiber 19]



KK expansion in differential cohomology

The topological terms in M-theory can be written using differential cohomology classes

$$e^{iS} = \exp\left[2\pi i \int_{M_{11}} \breve{I}_{12}\right]$$

$$\int_{M_{11}} \breve{I}_{12} \in \mathbb{R}/\mathbb{Z}$$

a class of degree p can be integrated
on a closed manifold of dim $p+1$ to give a phase

$$\breve{I}_{12} = -\frac{1}{6}\breve{G}_4 \star \breve{G}_4 \star \breve{G}_4 - \breve{G}_4 \star \breve{X}_8$$

product in diff. cohom. $\star : \breve{H}^p(M_{11}) \times \breve{H}^q(M_{11}) \to \breve{H}^{p+q}(M_{11})$
(same formal properties as $\wedge$)

$$\breve{G}_4 \in \breve{H}^4(M_{11})$$

KK expansion in differential cohomology

- 11d spacetime is a product

$$M_{11} = W_6 \times L_5$$

external 6d spacetime
where the SymTFT lives

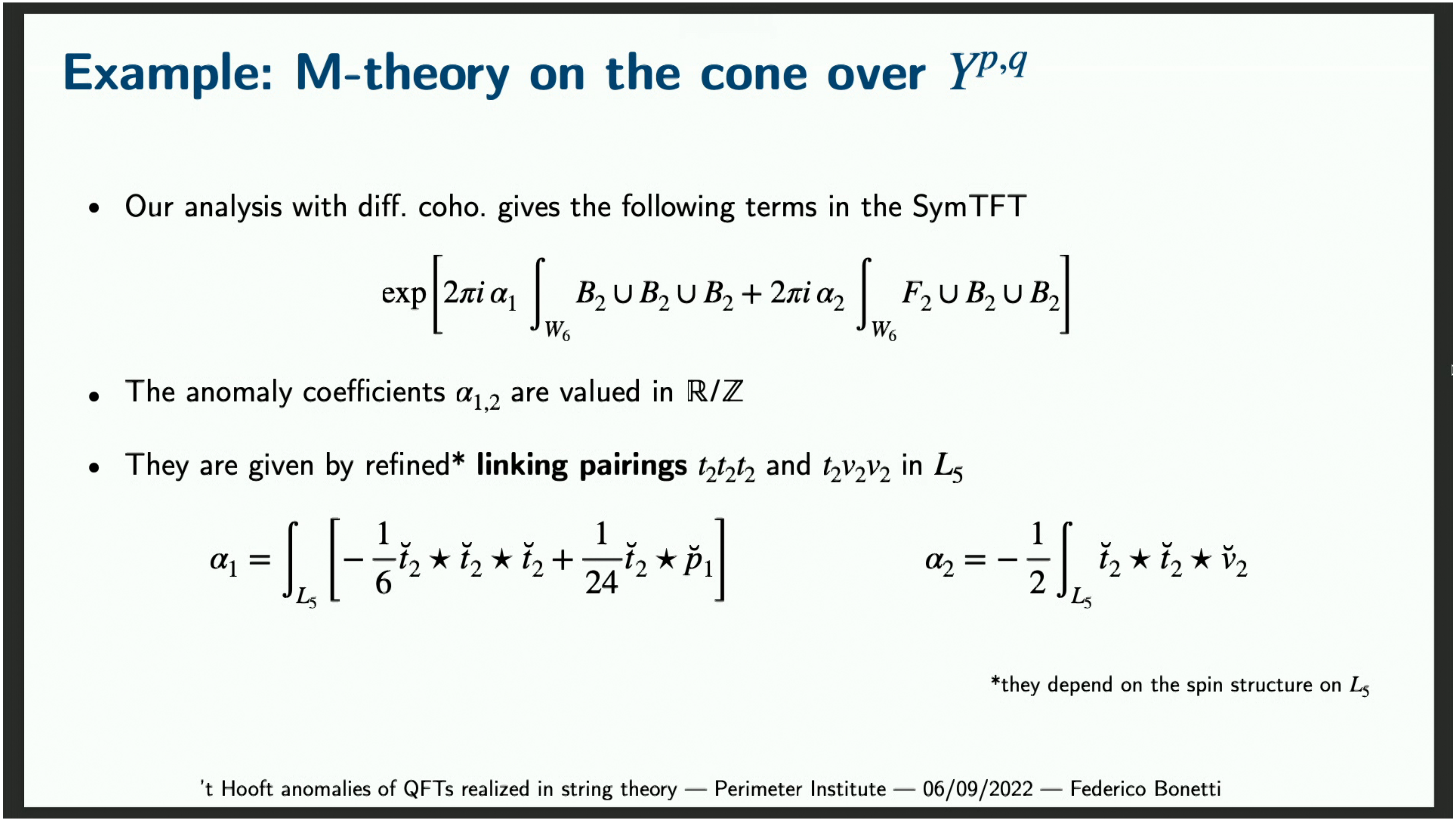
compact internal space
linking singularity

- We can write a KK-like ansatz for $\breve{G}_4$

$$\breve{G}_4 \supset \breve{x}_{4-p} \star \breve{\omega}_p$$

external class $\breve{H}^{4-p}(W_6)$

internal class $\breve{H}^{p}(L_5)$



Example: M-theory on the cone over $Y^{p,q}$

- Our analysis with diff. coho. gives the following terms in the SymTFT

$$\exp\left[2\pi i\,\alpha_1\int_{W_6} B_2\cup B_2\cup B_2+2\pi i\,\alpha_2\int_{W_6} F_2\cup B_2\cup B_2\right]$$

- The anomaly coefficients $\alpha_{1,2}$ are valued in $\mathbb{R}/\mathbb{Z}$
- They are given by refined* **linking pairings** $t_2t_2t_2$ and $t_2v_2v_2$ in L_5

$$\alpha_1=\int_{L_5}\left[-\frac{1}{6}\breve{t}_2\star\breve{t}_2\star\breve{t}_2+\frac{1}{24}\breve{t}_2\star\breve{p}_1\right]\qquad\qquad \alpha_2=-\frac{1}{2}\int_{L_5}\breve{t}_2\star\breve{t}_2\star\breve{v}_2$$

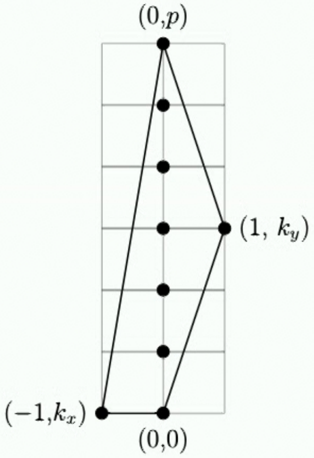
*they depend on the spin structure on L_5

How can we compute the refined linking pairing?

- The refined linking pairings are defined <u>purely in terms of the link</u> $L_5 = Y^{p,q}$
- However, we can

  map the problem to intersection theory of non-torsional cycles in X_6^{res}

- X_6^{res} = a resolution of the singular Calabi-Yau cone X_6
- This example is toric: combinatorial methods



$q = p - (k_x + k)_y$

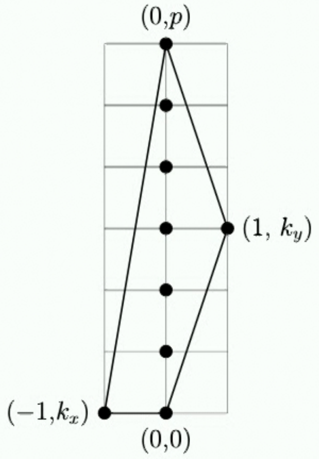
torsional class $t_2 \in H^2(L_5; \mathbb{Z})$ ↔ compact divisor Z

non-tors. class $v_2 \in H^2(L_5; \mathbb{Z})$ ↔ non-compact divisor D

based on [Gordon, Litherland 78]

How can we compute the refined linking pairing?

$$\alpha_1 \int_{W_6} B_2 B_2 B_2 + \alpha_2 \int_{W_6} B_2 B_2 F_2$$



$q = p - (k_x + k)_y$

Results for the anomaly coefficients:

$$\alpha_1 = -\frac{1}{6}\frac{Z\cdot Z\cdot Z}{\gcd(p,q)^3} + \frac{1}{24}\frac{Z\cdot p_1}{\gcd(p,q)} = \frac{qp(p-1)(p-2)}{6\gcd(p,q)^3} \mod 1$$

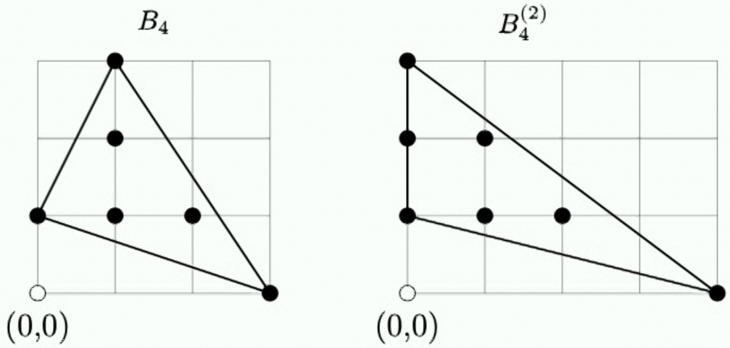
$$\alpha_2 = -\frac{1}{2}\frac{Z\cdot Z\cdot D}{\gcd(p,q)^2} = \frac{p(p-1)}{2\gcd(p,q)^2} \mod 1$$

Agrees with field theory analysis

[Benetti-Genolini, Tizzano 20; Gukov, Hsin, Pei 20]

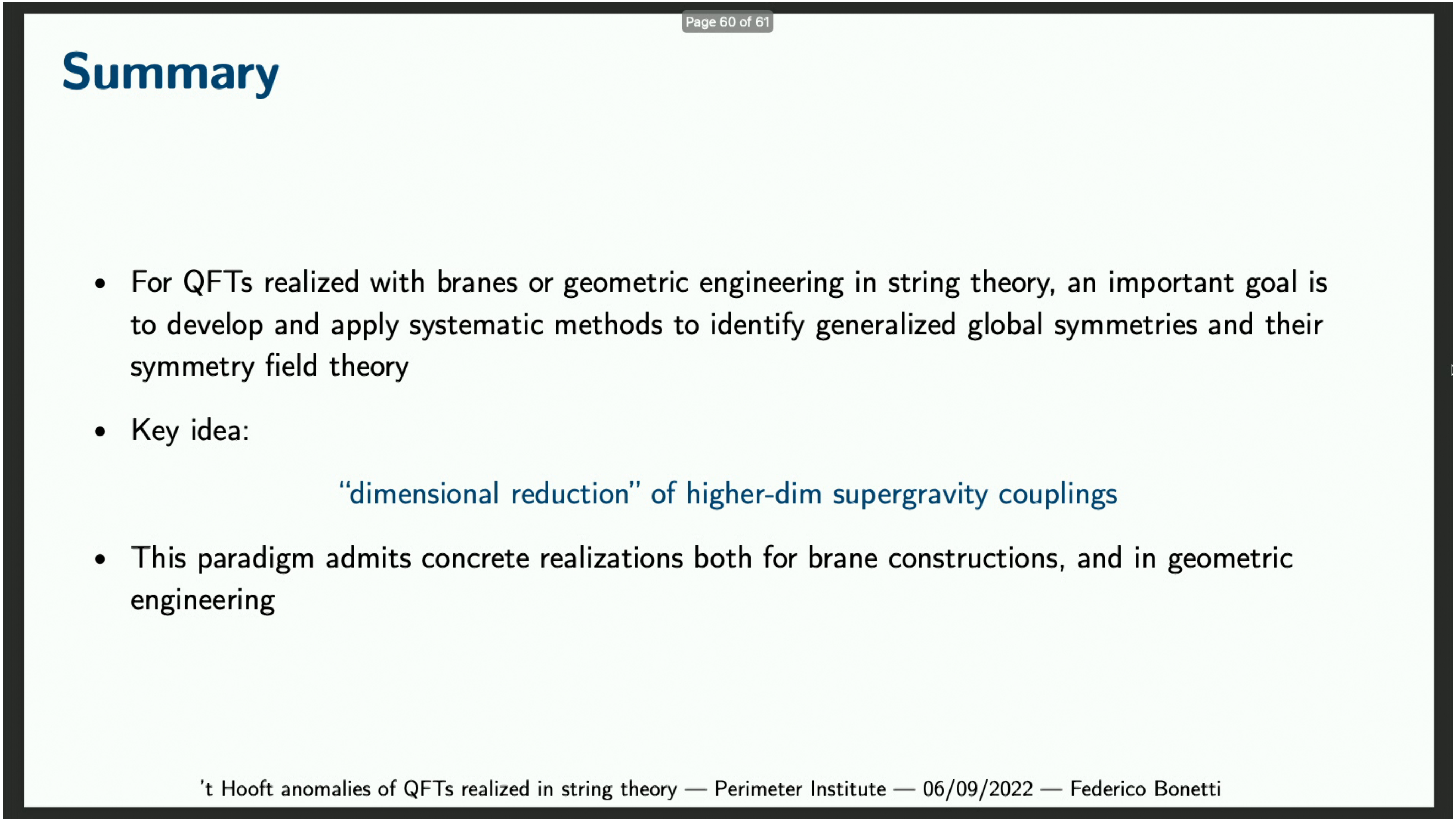
Some remarks on the diff. cohom. approach

- This method applies to 5d SCFTs that do not admit a mass deformation to a non-Abelian gauge theory, for example the B_N, $B_N^{(i)}$ models



[Morrison, Schafer-Nameki, Willett 20; Eckhard, Schäfer-Nameki, Wang 20]

- This approach applies both to toric and non-toric X_6
- See also [Cvetič, Dierigl, Lin, Zhang 21]



Summary

- For QFTs realized with branes or geometric engineering in string theory, an important goal is to develop and apply systematic methods to identify generalized global symmetries and their symmetry field theory
- Key idea:

“dimensional reduction” of higher-dim supergravity couplings

- This paradigm admits concrete realizations both for brane constructions, and in geometric engineering

Outlook

- Explore generalizations to other geometric engineering setups
- Study the effect of torsion in cohomology in brane constructions
- Identify and apply the appropriate mathematical framework to combine isometries of the internal space and torsion (equivariant differential cohomology)
- Develop/apply methods to uncover more refined structures of generalized symmetries

Thank you!