Title: Monodromy and derived equivalences

Speakers: Andrei Okounkov

Collection: Global Categorical Symmetries

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Abstract: This will be an introductory discussion of our joint work with Roman Bezrukavnikov. Given a symplectic resolution X, one may study its Gromov-Witten theory and the monodromy group of the curve-counting functions in the K\"ahler variables. There is also a large group of derived autoequivalences of X coming from its quantization in large prime characteristic, as studied by Bezrukavnikov and collaborators. Conjecturally, the action of the latter group on K(X) is identified with the former group, and we prove this for many X.

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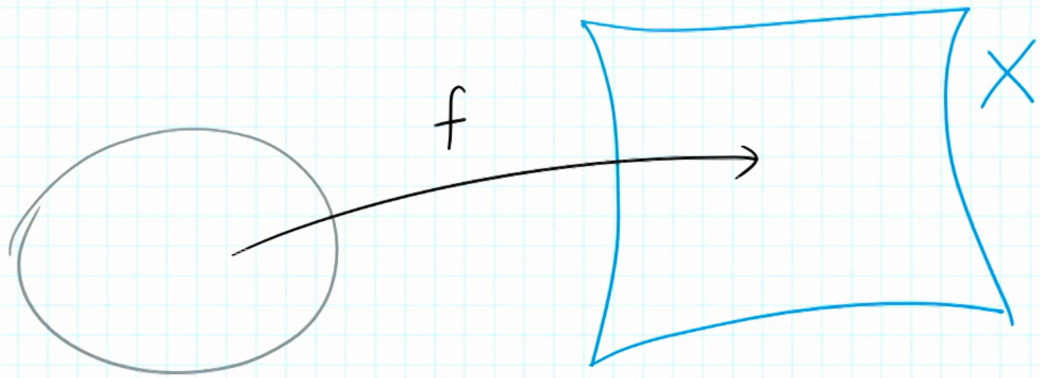
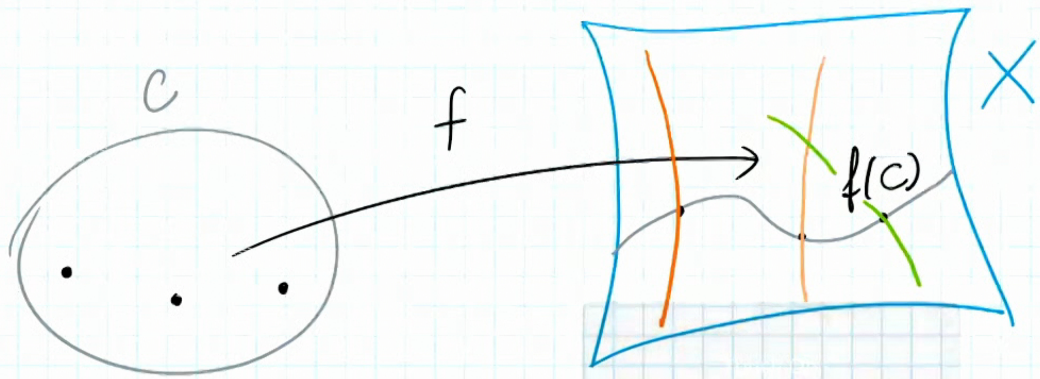
Monodromy and derived equivalences

joint with Roman Bezrukavnikov





$$\langle \alpha, \beta, \gamma \rangle_d = \#\ \text{maps } f \text{ of degree } d$$

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$$\langle \alpha, \beta, \gamma \rangle_d = \# \text{ maps } f \text{ of degree } d$$

Kähler torus

$$H^2(X,\mathbb{C}) / 2\pi i H^2(X,\mathbb{Z})$$

$$\mathrel{\rotatebox{90}{$\in$}}$$

$$(\alpha \star \beta, \gamma) = \sum_{d \in H_2(X,\mathbb{Z})} z^d \langle \alpha, \beta, \gamma \rangle$$

formal series in z

flat connection

Dubrovin

Givental

$$\text{base} = H^2(X,\mathbb{C}) \mathrel{\rotatebox{90}{$\in$}} \lambda$$

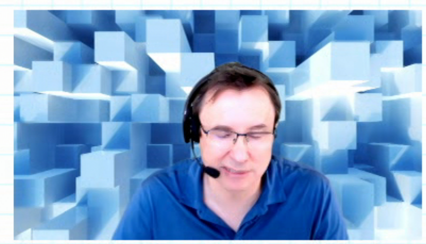
linear

not with constant coeff.

$$H^*(X) \mathrel{\rotatebox{90}{$\in$}} \Psi(z)$$

$$\frac{d}{d\lambda} \Psi(z) = \lambda \star \Psi(z)$$

$$\frac{d}{d\lambda} z^d = \langle \lambda, d \rangle z^d$$



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$$(\alpha \star \beta, \gamma) = \sum_{d \in H_2(X,\mathbb{Z})} z \, \langle \alpha, \beta, \gamma \rangle$$

formal series in z

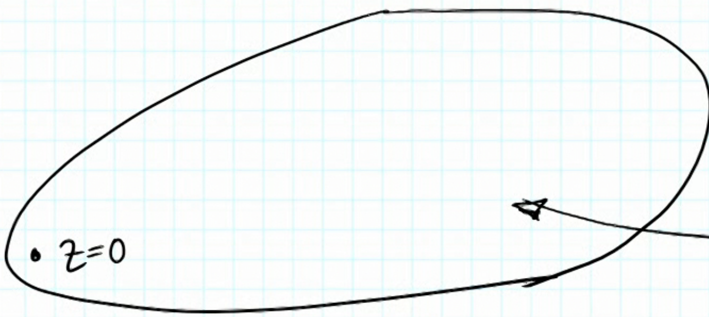
flat connection
Dubrovin
Givental

$$\text{base} = H^2(X,\mathbb{C}) \ni \lambda$$

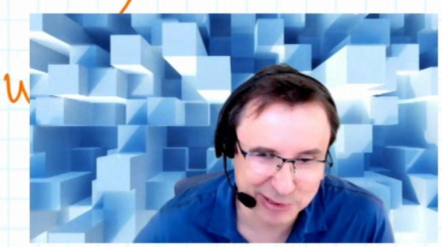
$$\frac{d}{d\lambda} \Psi(z) = \lambda \star \Psi(z), \qquad \Psi \in H^*(X)$$

linear
not with constant coeff.

$$\frac{d}{d\lambda} z^d = \langle \lambda, d \rangle \, z^d$$



toric variety



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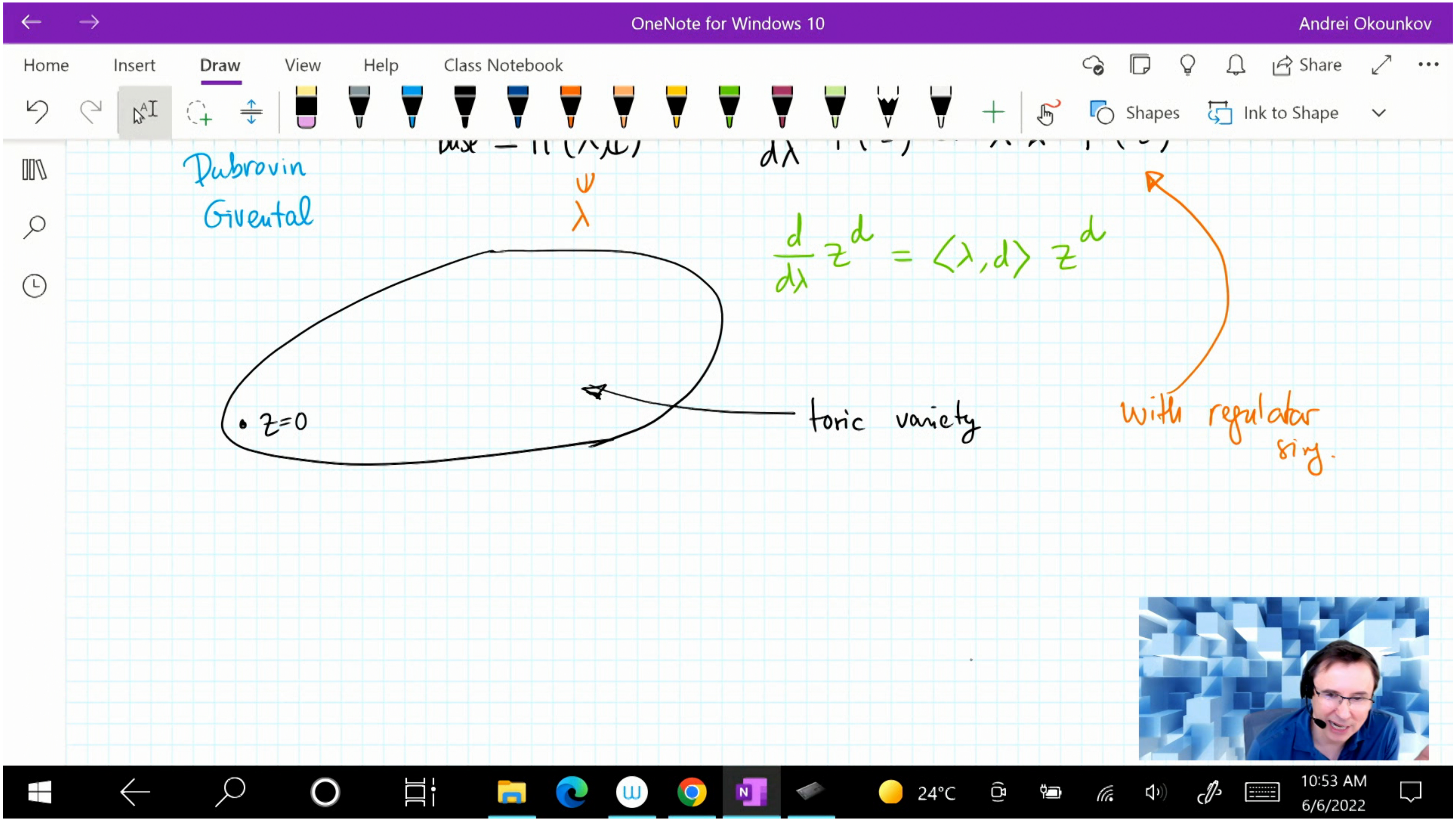
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λ
$\frac{d}{d\lambda} z^d = \langle \lambda, d \rangle z^d$
• z=0
toric variety
with regulator sing.
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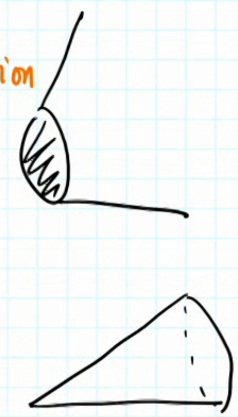
• $t=0$ Toric variety sing.

X = equivariant symplectic resolution

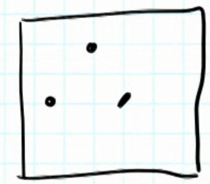


$n=2$

$\mathrm{Hilb}(\mathbb{C}^2, n)$

$\downarrow$

$S^n \mathbb{C}^2$



$\downarrow$

remember mult. Only.

$\subset$ Nakajima quiver varieties

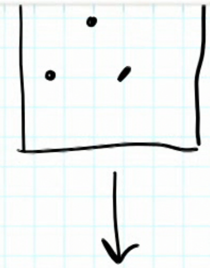
X = equivariant symplectic resolution

$T^*\mathbb{P}^1$

$n=2$

$T^* G/P$

$$\mathrm{Hilb}(\mathbb{C}^2, n) \longrightarrow S^n\mathbb{C}^2$$



remember mult. only

$\subset$ Nakajima quiver varieties

Categorification of the <u>monodromy</u> of |

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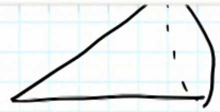

$S^n \mathbb{C}^2$ remember mult. Only.

$T^* G/P$

$\subset$ Nakajima quiver varieties

Categorification of the <u>monodromy</u> of Dubrovin connection

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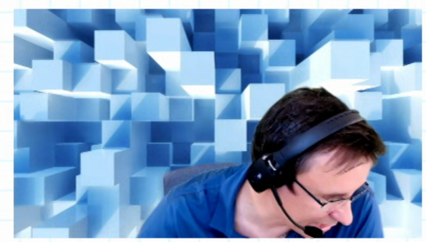
$$X = T^*M$$

$\rightsquigarrow$ sheaf $\hat{\mathcal{O}}_X$ deforming $\mathcal{O}_X$

differential operators on M

or sections of line bundles

$$\frac{d}{dx}$$



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or

sections of line bundles

Pic(X) ⊗ ground field

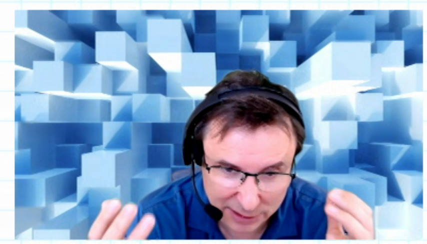
$$\frac{1}{g(x)^{\lambda}} \frac{d}{dx} g(x)^{\lambda} = \frac{d}{dx} + \frac{\lambda}{g(x)}$$

huge center $\left(\frac{d}{dx}\right)^p$, or x^p, ...

$Fr_* \hat{\mathcal{O}}_X$ = remarkable sheaf of Azumaya algebras of dim $p^{\dim X}$

$$\hat{X}_\lambda = \Gamma(\quad)$$

noncommutative algebra deforming functions on 



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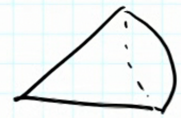
$*$ X

Azumaya algebras of dim $p^{\dim X}$

$$\hat{X}_\lambda = \Gamma(\quad)$$

noncommutative algebra deforming functions on



splits $\simeq \mathrm{End}(\mathcal{E})$ over a sufficiently large set in X

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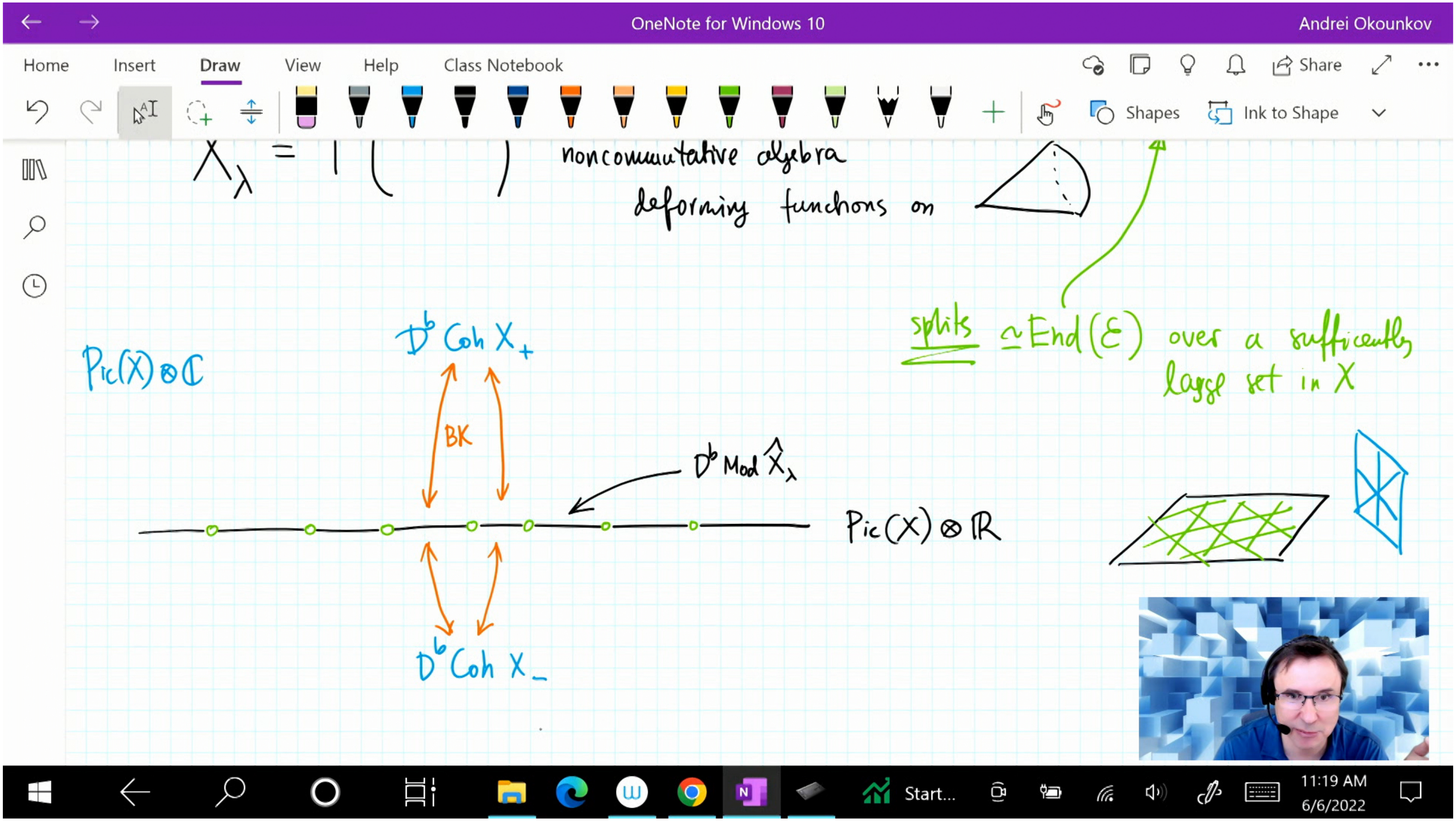
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noncommutative algebra
deforming functions on
splits ≃ End(E) over a sufficiently large set in X
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D^b Coh X_+
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D^b Mod X̂_λ
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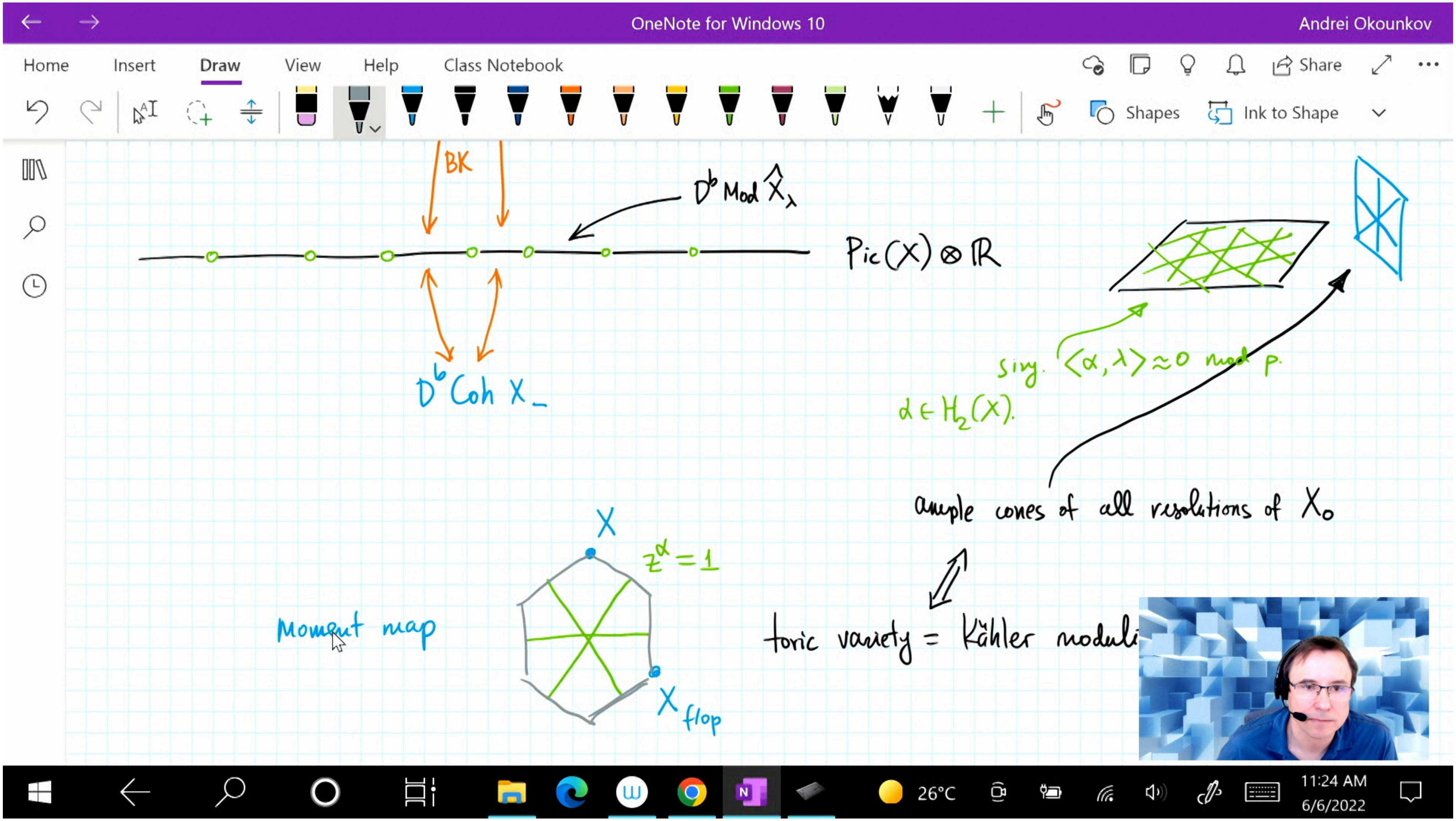
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D^b Mod X̂_λ
Pic(X) ⊗ ℝ
sing. ⟨α, λ⟩ ≈ 0 mod p.
α ∈ H_2(X).
D^b Coh X_−
ample cones of all resolutions of X_0
X
z^α = 1
Moment map
toric variety = Kähler moduli
X flop
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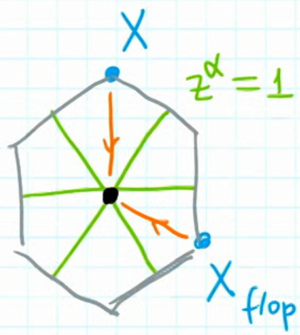
$D^b \mathrm{Coh}\, X_-$

$\alpha \in H_2(X)$.

ample cones of all resolutions of X_0

toric variety = Kähler moduli space

Moment map



Important: how to identify $K(X) \longrightarrow H^\bullet(X)$

Γ-class, Iritani, + ...

Important how to identify $K(X) \longrightarrow H^{\cdot}(X)$

Γ-class, Iritani, + . . .

<u>Thm</u> (202..., BO) This is indeed true for

$X = T^* G/P$ and others with isolated X^A

$= T^* \text{Vector space} \;////\; \prod GL(V_i)$

torus preserving ω_X

role

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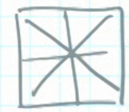
$X = T^* G/P$ and others with isolated X^A

role

$= T^*\,\text{Vector space} \;////\; \prod GL(V_i)$



relate the conjecture for X and X^A

in some direction

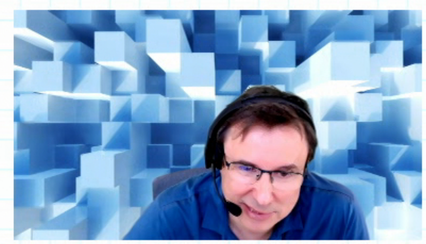
let $a \to \infty$ in enumerative counts

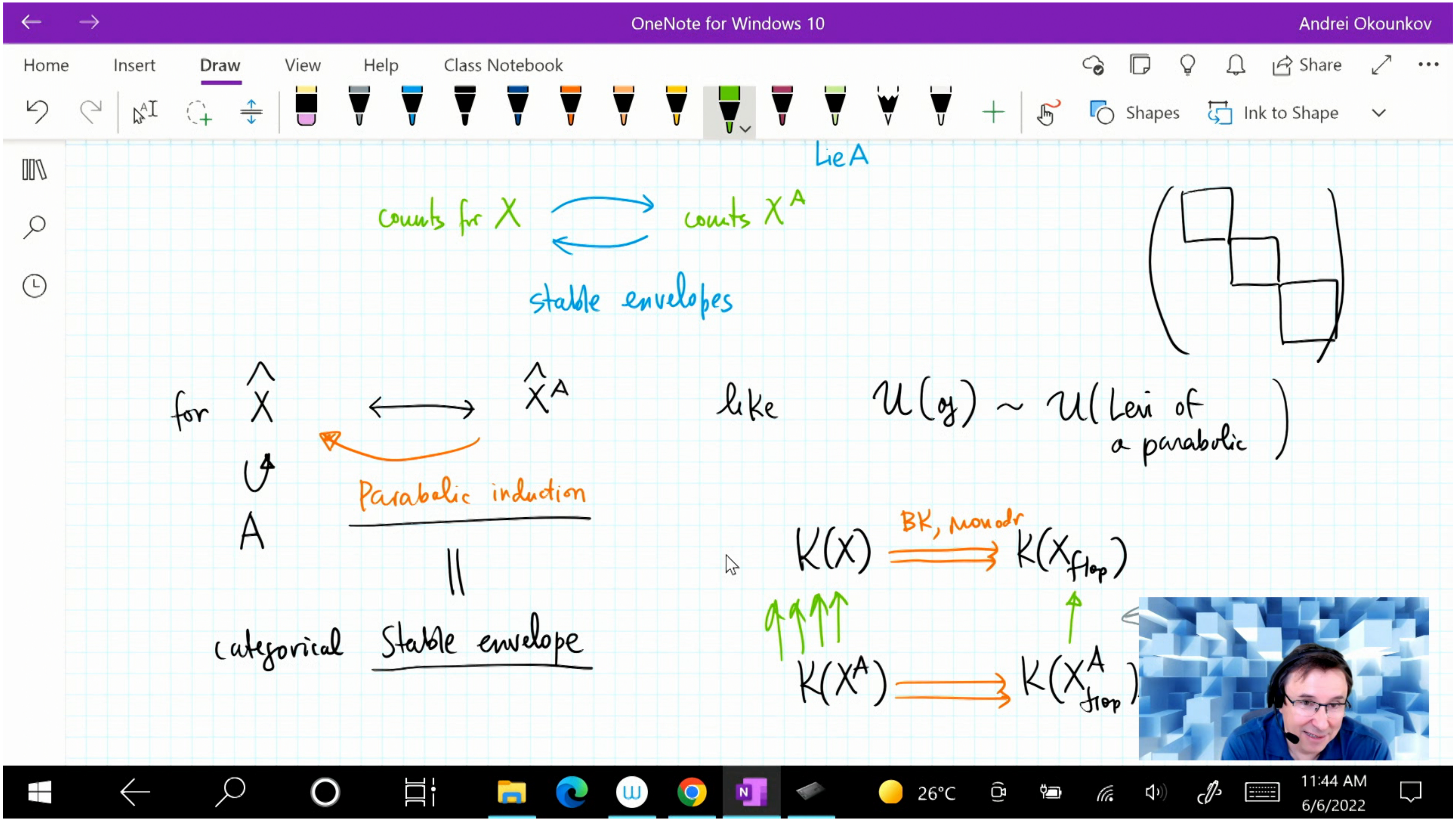
$\cap$

Lie A

$$\text{counts for } X \rightleftarrows \text{counts } X^A$$

stable envelopes

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Lie A
counts for X
counts X^A
stable envelopes
for X
A
X^A
Parabolic induction
categorical Stable envelope
like U(g) ~ U(Levi of a parabolic)
K(X)
BK, monodr
K(X_flop)
K(X^A)
K(X^A_flop)