Title: Quantum nonlocality without entanglement via indefinite causal order

Speakers: Ravi Kunjwal

Series: Quantum Foundations

Date: June 17, 2022 - 2:00 PM

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Abstract: I will discuss a recent result on an intimate link between two a priori distinct phenomena: quantum nonlocality without entanglement and classically-achievable indefinite causal order. The first phenomenon refers to a multipartite scenario where the parties are unable to perfectly discriminate orthogonal product states drawn from an ensemble of quantum states by using local operations and classical communication (LOCC). The second (hypothetical) phenomenon refers to a multipartite scenario where the parties can communicate classically but the local operations of each party are in the future of the other parties, i.e., they cannot be ordered causally. Specifically, I will show how three separated parties with access to a classical process exhibiting indefinite causal order---the AF/BW process---can perfectly discriminate the states in an ensemble---the SHIFT ensemble---that exhibits quantum nonlocality without entanglement. Time permitting, I will discuss the generalization of this result beyond the tripartite case and comment on its connection with separable operations that are outside LOCC.

Based on joint work with Ämin Baumeler, arXiv:2202.00440.

Zoom Link: https://pitp.zoom.us/j/93727212623?pwd=cjVRL3cvMmhicDRic3lXRFBkNi9xZz09

Unentangled measurements

LOCC: $\{|0+\rangle, |0-\rangle, |1+\rangle, |1,-\rangle\}$
(direct product)

(LOCC = ‘local operations and classical communication’)

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$\{|00\rangle, |01\rangle, |1+\rangle, |1,-\rangle\}$
(adaptive)

Non-LOCC: $\{|000\rangle, |+01\rangle, |1+0\rangle, |01+\rangle,$
$|111\rangle, |-01\rangle, |1-0\rangle, |01-\rangle\}$

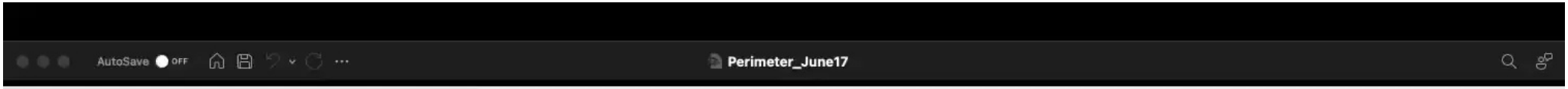
(non-adaptive, no party has a fixed mmt basis)

(LOCC = 'local operations and classical communication')

SHIFT basis

$$\{|000\rangle, |+01\rangle, |1+0\rangle, |01+\rangle,$$
$$|111\rangle, |-01\rangle, |1-0\rangle, |01-\rangle\}$$

- Impossible to perfectly discriminate these locally preparable states via LOCC.
- This, however, assumes *causal* classical communication, i.e., at least one of the three parties is in the global past of the other two parties. This is the 'starting party' of the LOCC protocol that conditions, via CC, the measurements of the other parties.
- What about classical communication that fails to respect causal order?

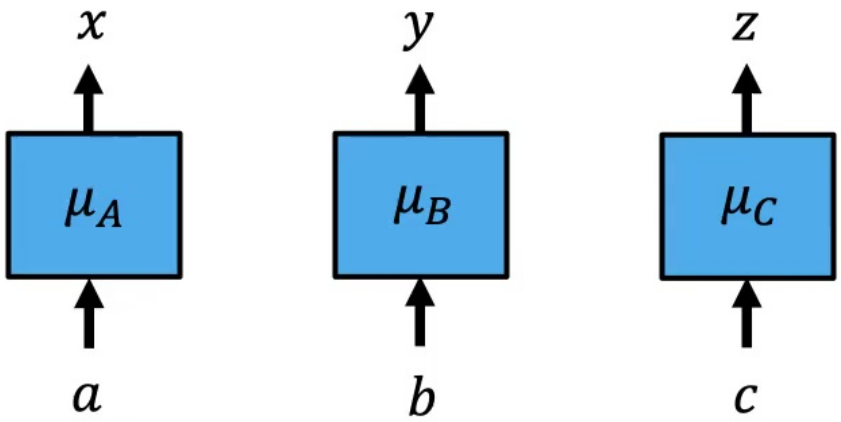


Classical communication without causal order

Process function

- *n* local labs, each with a single classical I/P and classical O/P, free local interventions that map I/P to O/P; within each lab, I/P causally precedes O/P

Process function

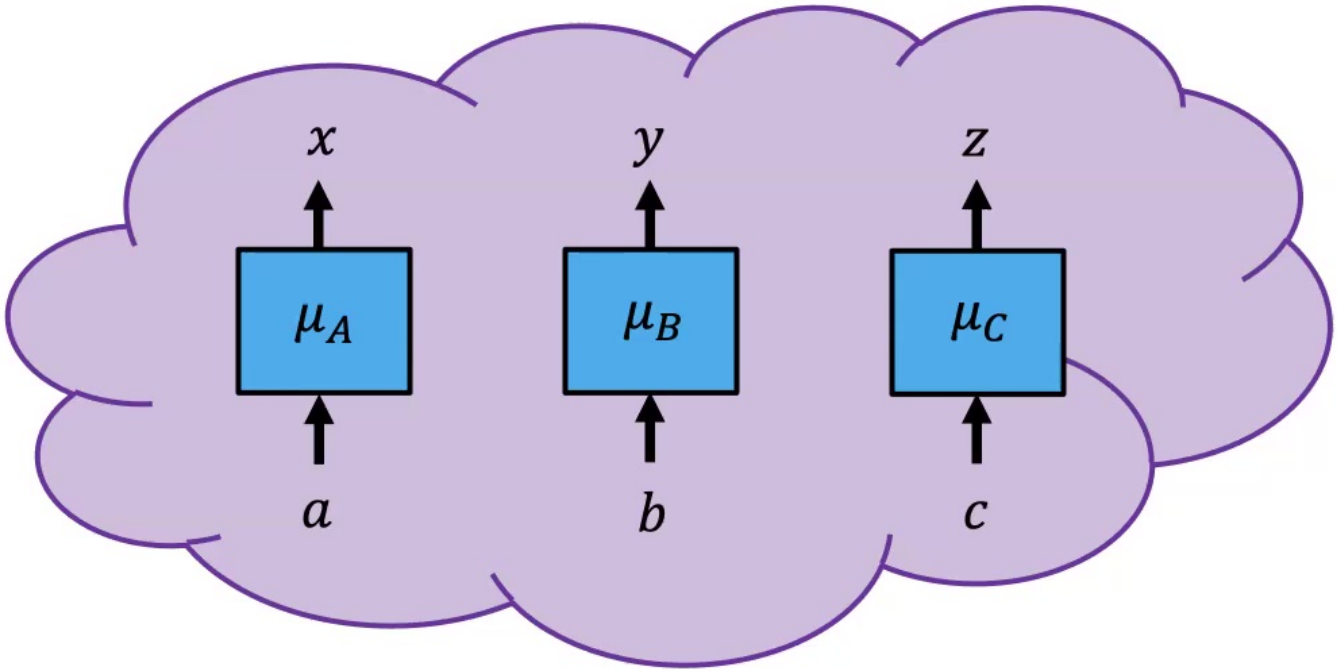


Process function

- *n* local labs, each with a single classical I/P and classical O/P, free local interventions that map I/P to O/P; within each lab, I/P causally precedes O/P

- The labs are embedded in an environment that they can't control or intervene upon; this environment dictates the causal order (definite/indefinite) and the resulting correlations between the labs

Process function



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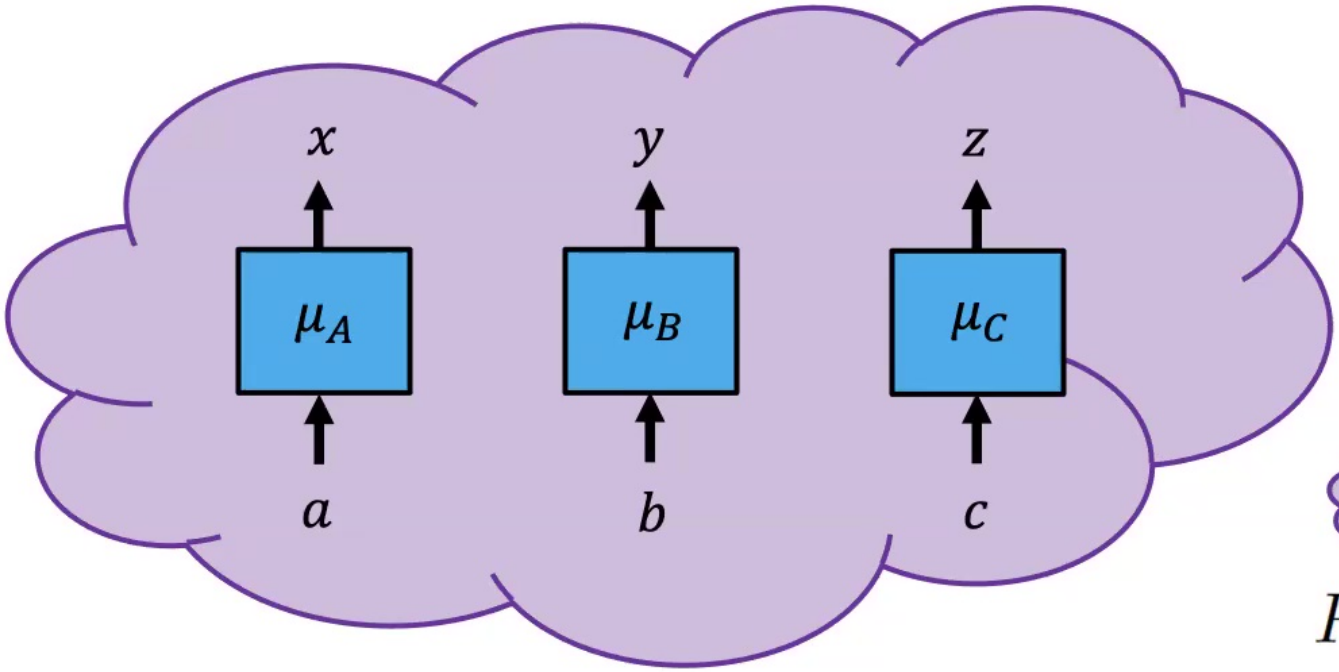

$$P_{a,b,c|x,y,z}$$

Process function

This deterministic classical channel should satisfy logical consistency:

Process function

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for any set of local interventions, there exists a unique set of I/Ps ("fixed point") to the parties that is returned back by the classical channel

deterministic classical channel + logical consistency
= process function

Ä. Baumeler, S. Wolf, arXiv:1511.05444
Ä. Baumeler, E. Tselentis, arXiv:2004.12921

Process function: 1, 2, and 3-party

- A process function is *causal* iff the associated classical channel is a constant function (non-signalling) or admits causal signalling (=> at least one party is in the global past)

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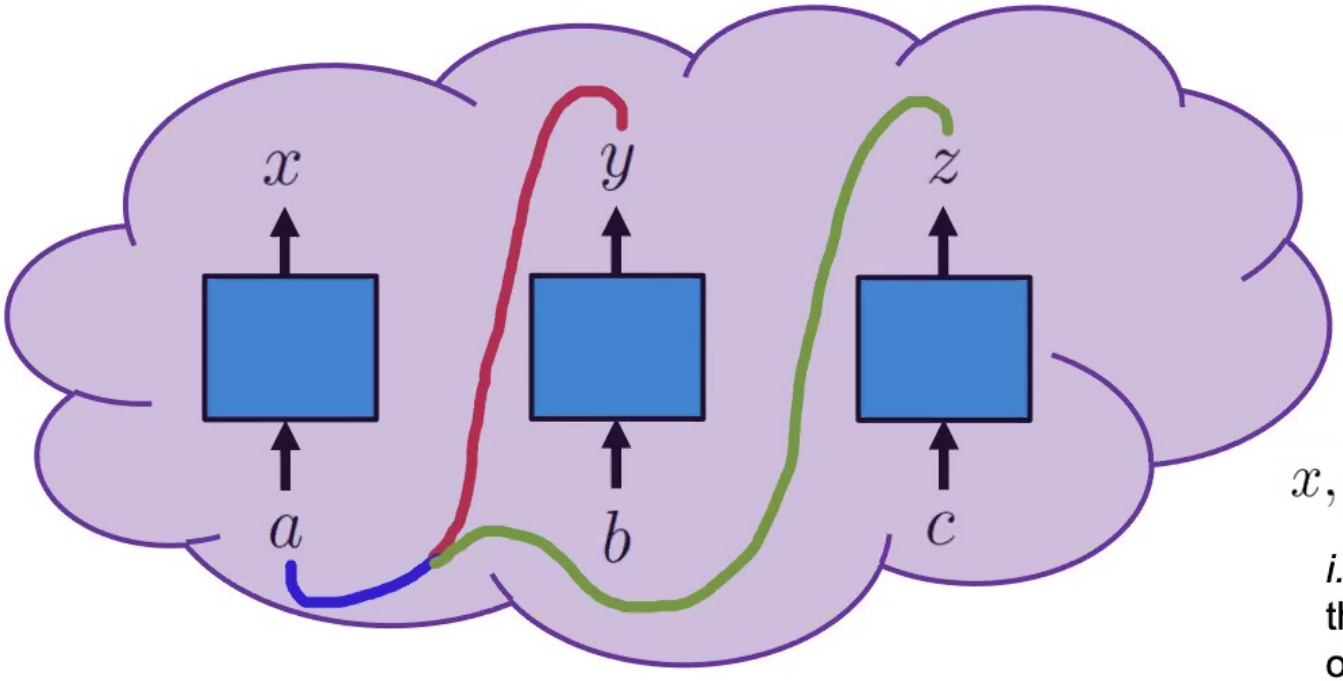
- A 3-party process function need not be causal: process functions can be constant, causal signalling, or non-causal signalling, *e.g.*, AF/BW process

AF/BW process

Tripartite process function that wires the outputs of three labs to their inputs in a noncausal way



$$P_{a,b,c|x,y,z}$$

$$a = (y \oplus 1)z$$
$$b = (z \oplus 1)x$$
$$c = (x \oplus 1)y$$

$$x, y, z, a, b, c \in \{0, 1\}$$

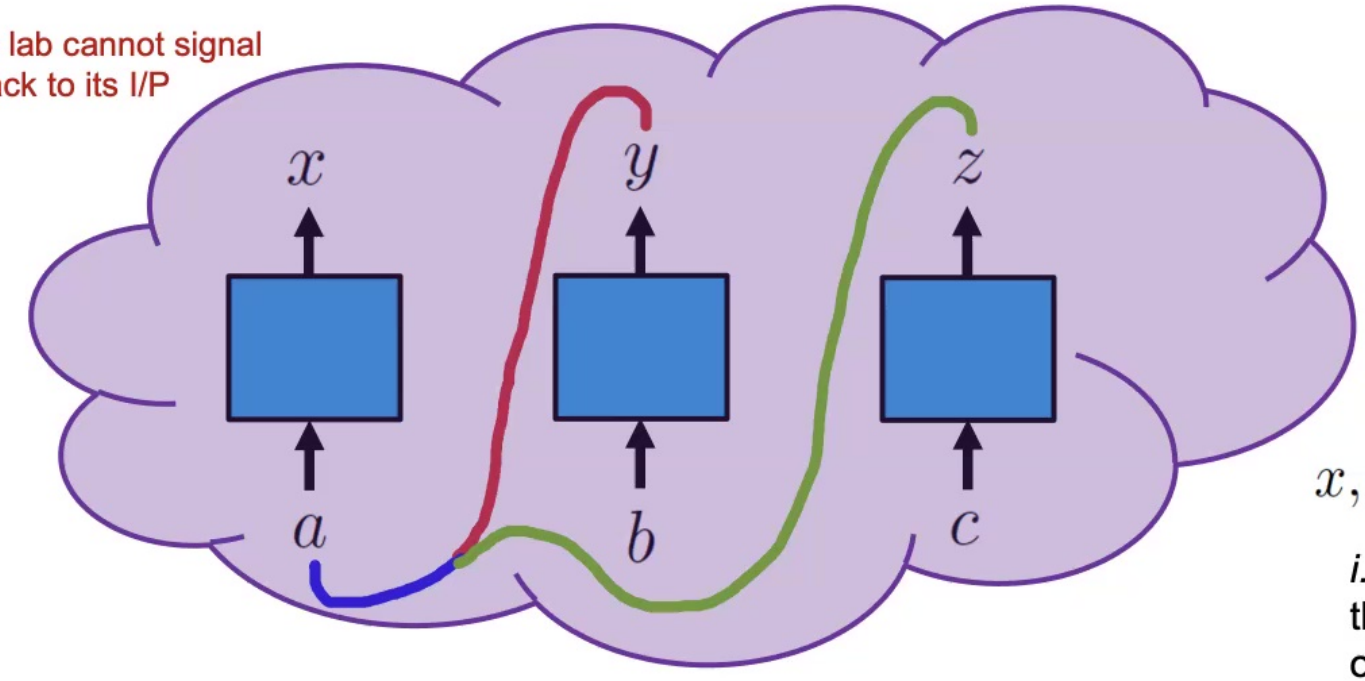
i.e., each party acts in the causal future of the other two

AF/BW process

Tripartite process function that wires the outputs of three labs to their inputs in a noncausal way

Quantum stuff can happen inside the labs!

O/P of a lab cannot signal back to its I/P



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AF/BW process: I/O table

abc	xyz
000	000, 111
001	010, 011
010	100, 110
100	001, 101

("Shift left", "Shift right and flip")

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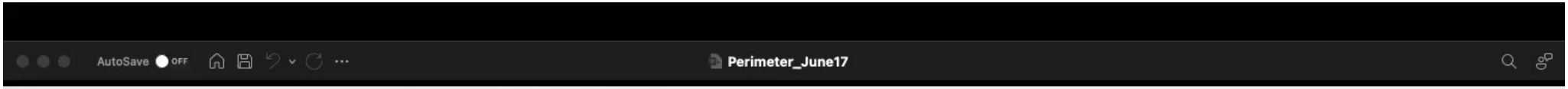

Results

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abc	xyz
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Local implementation of SHIFT measurement

Three parties communicating through the AF/BW process can measure quantum systems in the SHIFT basis using local operations

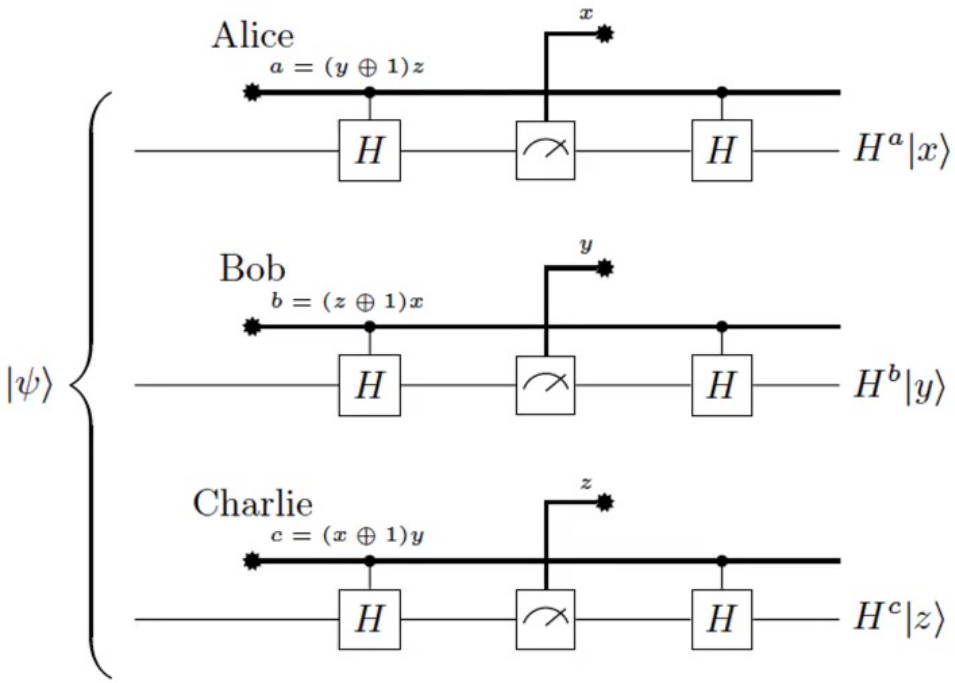

Figure 1. Schematic of protocol to implement the SHIFT-basis measurement on an arbitrary quantum state $|\psi\rangle$ with local operations and classical "acausal" communication. Thick wires represent classical bits, normal wires qubits, and (✸) represents the interface to the AF/BW process.

$$H^0 = I, H^1 = H$$

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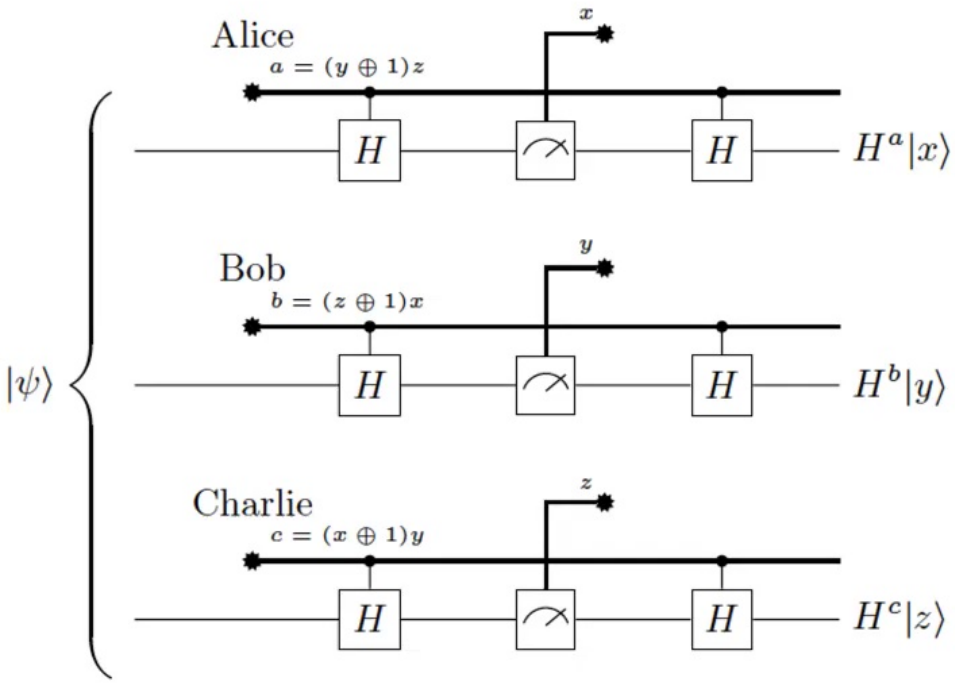

Final state:

$$\sum_{x,y,z} \left|\langle xyz|H^{(a,b,c)}|\psi\rangle\right|^2 H^{(a,b,c)}|xyz\rangle\langle xyz|H^{(a,b,c)}$$

$$H^{(abc)} = H^a \otimes H^b \otimes H^c$$

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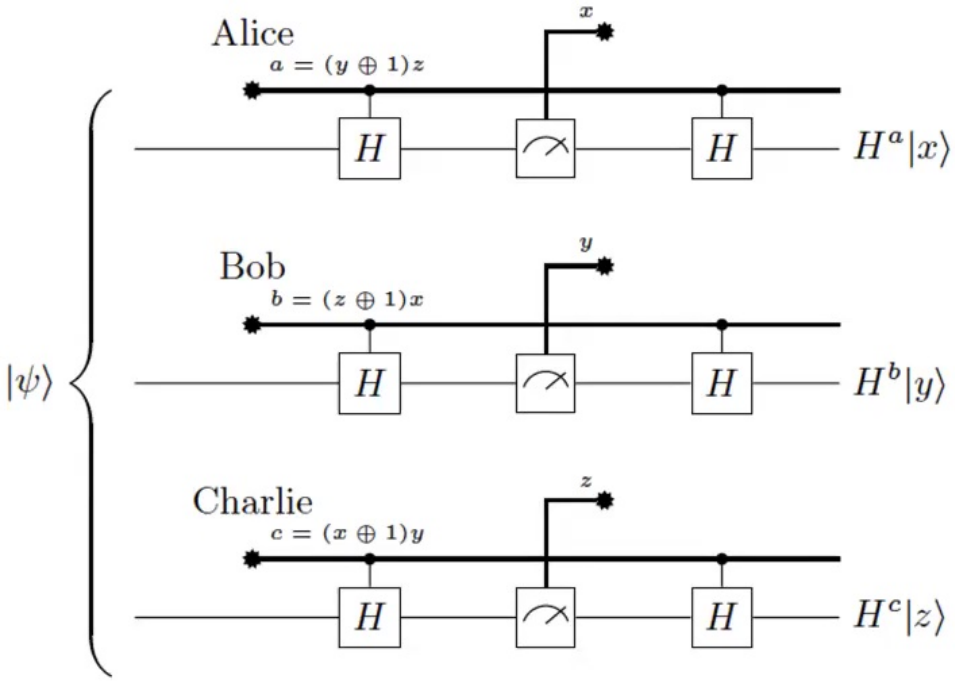

Perfect discrimination protocol:

Bits a,b,c received from AF/BW specify the local basis:
0 for Z basis and 1 for X basis

Bits x,y,z received from the local mmt specify the local basis state: 0 for |0⟩, |+⟩ and 1 for |1⟩,|-⟩

abc	xyz
000	000, 111
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$\Rightarrow$ $\{|000\rangle, |01+\rangle, |1+0\rangle, |+01\rangle, |111\rangle, |01-\rangle, |1-0\rangle, |-01\rangle\}$

SHIFT measurement simulates the AF/BW channel

Turning the SHIFT measurement device into the AF/BW channel

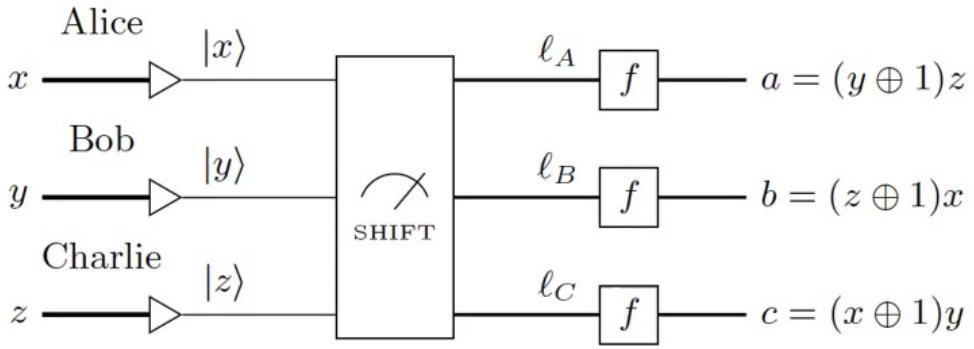

Figure 2: Schematic of protocol to simulate the AF/BW channel from a SHIFT-basis measurement.

Protocol:

- Each party locally prepares a qubit in the Z basis depending on the bit received

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Turning the SHIFT measurement device into the AF/BW channel

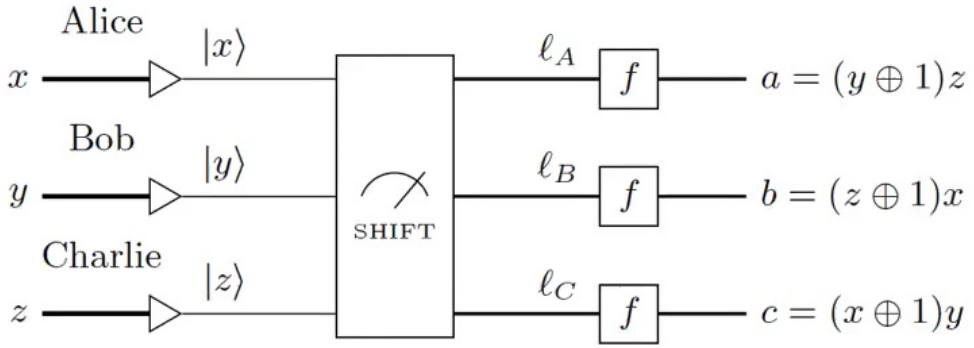

Figure 2: Schematic of protocol to simulate the AF/BW channel from a SHIFT-basis measurement.

Protocol:

- Each party locally prepares a qubit in the Z basis depending on the bit received
- No party has any info about bits received by the other parties.
- Apply SHIFT mmt w/ local outcomes

$$\ell_A, \ell_B, \ell_C \in \{0, 1, +, -\}$$

$$\{|000\rangle, |01+\rangle, |1+0\rangle, |+01\rangle, |111\rangle, |01-\rangle, |1-0\rangle, |-01\rangle\}$$

Quantum NL w/o E from Generalized AF/BW

Maximal violation of causal order = each party can receive a bit from at least one other party

Theorem.— If ω^n is a Boolean n-party classical process that maximally violates causal order, then

$$\mathcal{S}_{\omega^n} := \left\{ H^{(\omega^n(\underline{x}))} |\underline{x}\rangle \mid \underline{x} \in \{0,1\}^n \right\} \qquad (13)$$

is a basis of orthonormal states that exhibits quantum nonlocality without entanglement.

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Examples

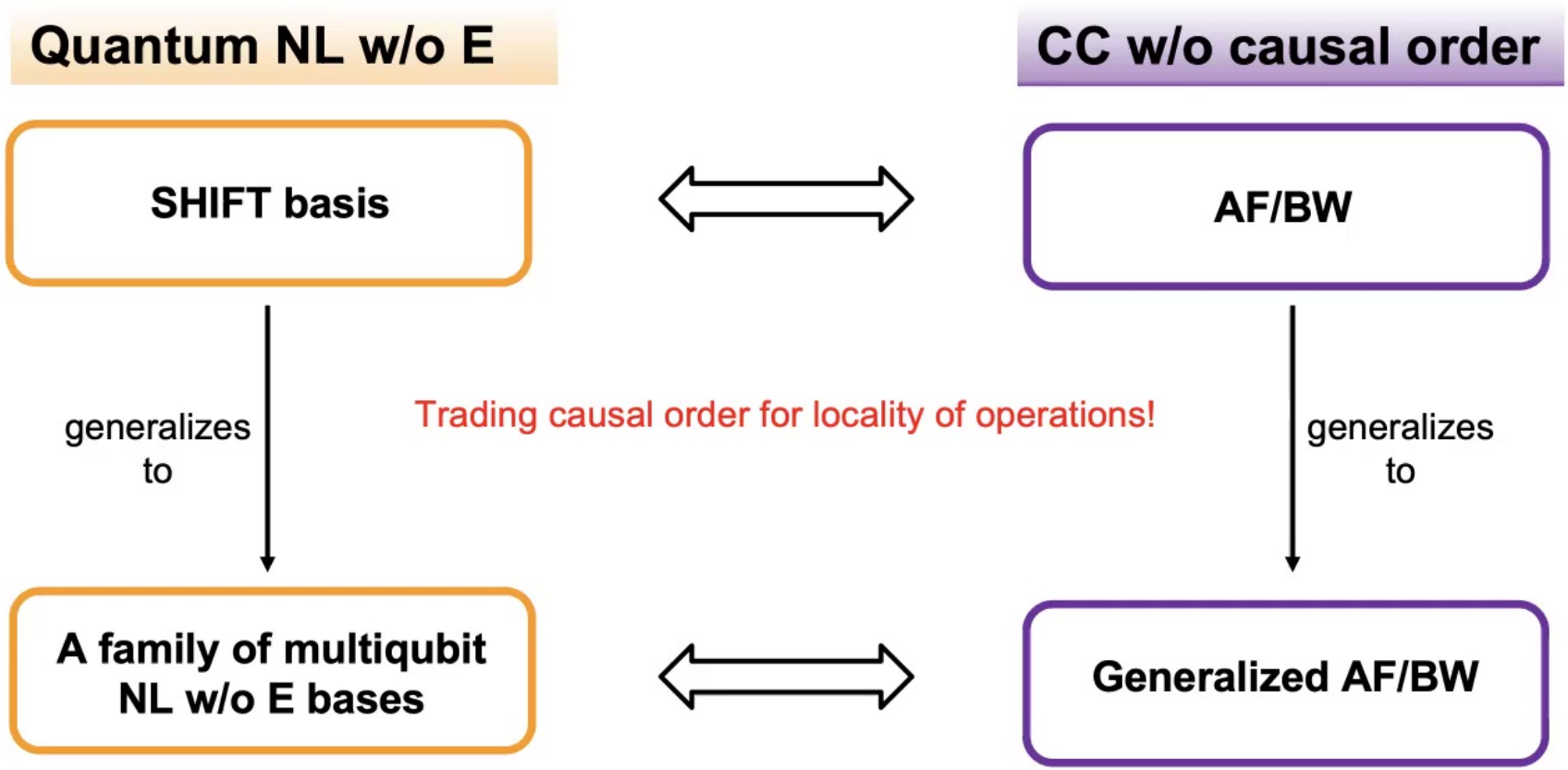
Based on Baumeler *et al.*, arXiv:2104.06234

$$\{|0000\rangle, |0{+}01\rangle, |{+}01{+}\rangle, |001{-}\rangle, |01{+}0\rangle, |{+}{-}01\rangle, |01{-}0\rangle, |0111\rangle, |1{+}0{+}\rangle, |1{+}{+}{-}\rangle, |{-}01{+}\rangle, |1{+}{-}{-}\rangle, |1{-}00\rangle, |{-}{-}01\rangle, |111{+}\rangle, |1{-}1{-}\rangle\}.$$

Based on Araújo *et al.*, PRA 96, 052315 (2017)

$$\{|0000\rangle, |0101\rangle, |0111\rangle, |1010\rangle, |1011\rangle, |1101\rangle, |1110\rangle, |1111\rangle, |001{+}\rangle, |001{-}\rangle, |01{+}0\rangle, |01{-}0\rangle, |1{+}00\rangle, |1{-}00\rangle, |+001\rangle, |{-}001\rangle\}.$$

Takeaway


Quantum NL w/o E
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SHIFT basis
AF/BW
generalizes to
Trading causal order for locality of operations!
generalizes to
A family of multiqubit NL w/o E bases
Generalized AF/BW

Thanks!

And check out our paper for more!
arXiv: 2202.00440