Title: Atomic clock interferometers: a test for a quantum generalization of Einsteinâ€™s Equivalence Principle and a quantum sensing analysis

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Series: Quantum Foundations

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Abstract: It is unknown how the Einstein Equivalence Principle (EEP) should be modified to account for quantum features. A possibility introduced in arXiv:2012.13754 is that the EEP holds in a generalized form for particles having an arbitrary quantum state. The core of this proposal is the ability to transform to a Quantum Reference Frame (QRF) associated to an arbitrary quantum state of a physical system, in which the metric is locally inertial. I will show that this extended EEP, initially formulated in terms of the local expression of the metric field in a QRF, can be verified in an interferometric setup via tests on the proper time of entangled clocks (arXiv:2112.03303). Moreover, the same setup can be analyzed with quantum sensing techniques (arXiv:2204.03006): I will talk about how gravitational time dilation may be used as a resource in quantum information theory, showing that it may enhance the precision in estimating the gravitational acceleration for long interferometric times.

Zoom Link: https://pitp.zoom.us/j/98626566246?pwd=b09Hdm5QbzhHLytjNkhrekpqVlRtdz09

Atomic clock interferometers: a test for a quantum generalisation of Einstein's equivalence principle and a quantum sensing analysis

Based on

- C. Cepollaro, F. Giacomini, Quantum generalisation of Einstein's Equivalence Principle can be verified with entangled clocks as quantum reference frames, arXiv:2112.03303
- C. Cepollaro, F. Giacomini, MGA Paris, Gravitational time dilation as a resource in quantum sensing, arXiv:2204.03006

Perimeter Institute for Theoretical Physics, 03/06/2022

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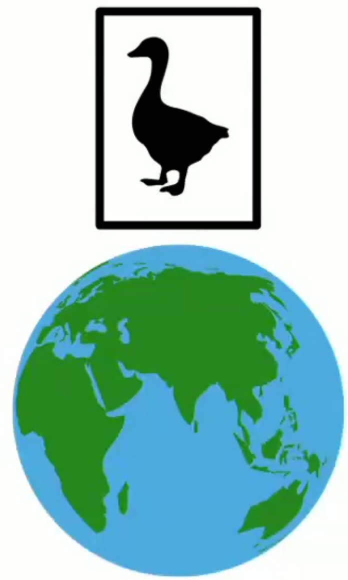
A test for the Einstein's Equivalence Principle for Quantum Reference Frames

Motivation

- Different proposals of a quantum extension of the equivalence principle have been suggested:
  - Y. Aharonov and G. Carmi, Quantum aspects of the equivalence principle, Found. Phys. 3, 493 (1973).
  - R. Penrose, On gravity's role in quantum state reduction, Gen. Rel. Grav. 28, 581 (1996).
  - C. Lämmerzahl, On the equivalence principle in quantum theory, Gen. Relativ. Gravit. 28, 1043 (1996).
  - M. Zych and C. Brukner, Quantum formulation of the Einstein equivalence principle, Nature Physics 14, 1027 (2018).
  - L. Hardy, Implementation of the Quantum Equivalence Principle, in Progress and Visions in Quantum Theory in View of Gravity: Bridging foundations of physics and mathematics (2019).
  - **F. Giacomini and C. Brukner, Einstein's equivalence principle for superpositions of gravitational fields and quantum reference frames, arXiv:2012.13754 [quant-ph] (2021).**
  - C. Marletto and V. Vedral, Sagnac interferometer and the quantum nature of gravity, Journal of Physics Communications 5, 051001 (2021).
- It is fundamental to test these ideas.

The Einstein's Equivalence Principle (EEP)

- In any and every locally inertial frame, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.



C. W. Misner, K. S. Thorne, J. A. Wheeler, Gravitation (W. H. Freeman, 1973).

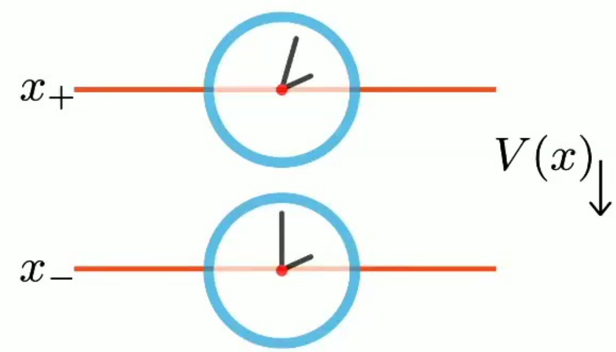
The three aspects of the principle

- Weak Equivalence Principle (WEP): The local effects of motion in a curved spacetime (gravitation) are indistinguishable from those of an accelerated observer in flat spacetime.
- Local Lorentz Invariance (LLI): The outcome of any local nongravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.
- Local Position Invariance (LPI): The outcome of any local nongravitational experiment is independent of where and when in the universe it is performed.

C.M. Will, The Confrontation between General Relativity and Experiment, Living Rev. Relativity 17 (2014).

Classical tests of the principle

- WEP tests: Comparison between accelerations of two bodies of different composition in an external gravitational field.
- LLI tests: Tests of special relativity (e.g., Michelson-Morley experiment).
- LPI tests: Gravitational redshift experiment.
    - What happens when clocks are in a quantum superposition of different heights?

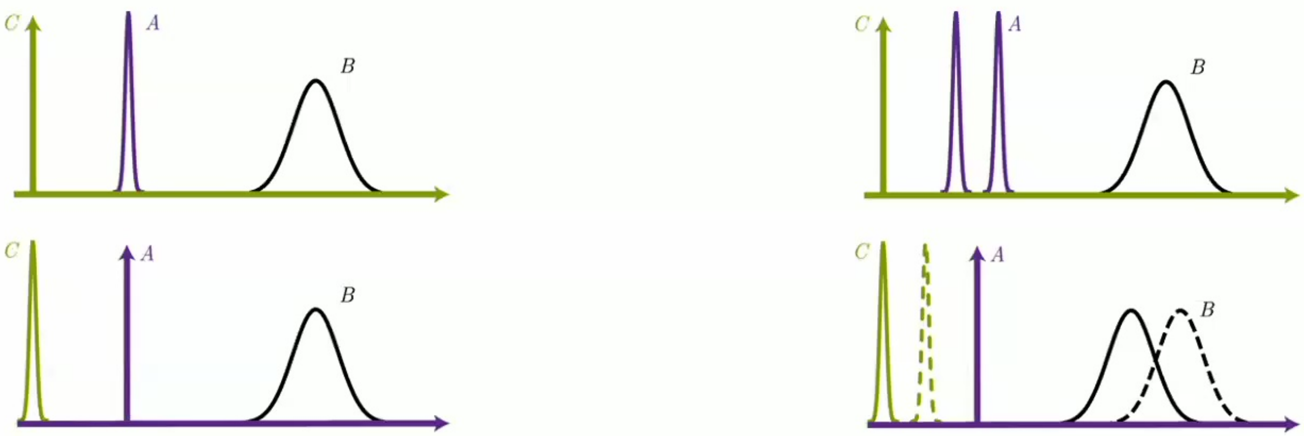


$$\frac{\Delta \nu}{\nu} = \frac{\Delta V}{c^2}$$

C.M. Will, The Confrontation between General Relativity and Experiment, Living Rev. Relativity 17 (2014).

Quantum Reference Frames (QRFs)

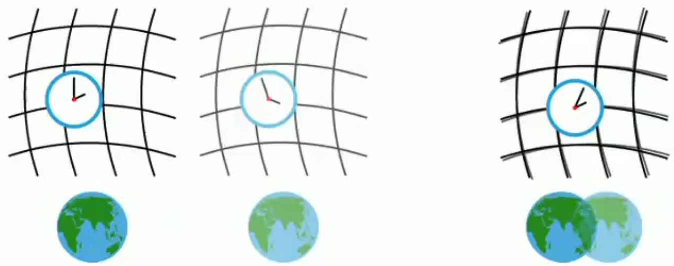
- Quantum Reference Frames are reference frames associated to quantum particles that can be delocalized.



F. Giacomini, E. Castro-Ruiz, Č. Brukner, Quantum mechanics and the covariance of physical laws in quantum reference frames, Nat. Commun. 10, 494 (2019).

Quantum locally inertial frame

- A mass in spatial superposition generates a superposition of gravitational fields. A clock that evolves in this scenario will get entangled with the gravitational field.

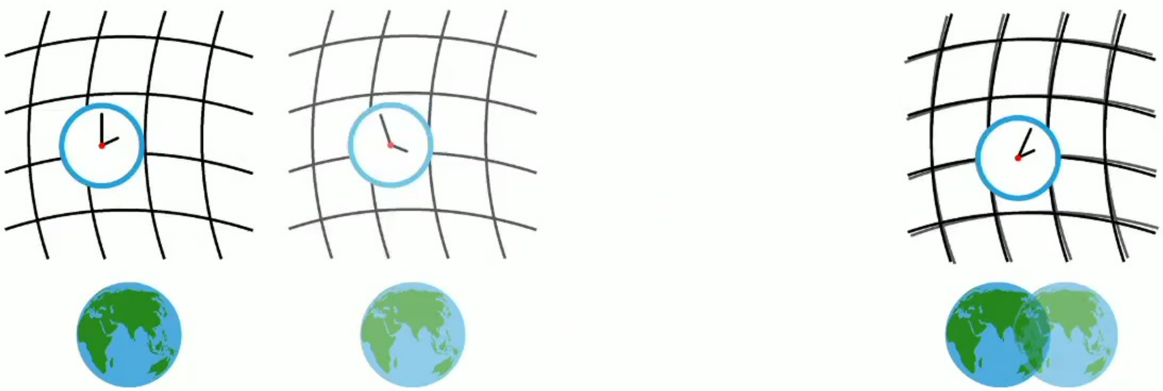


- The **Quantum locally inertial frame** (QLIF) of the clock is a frame associated to a quantum particle where the metric is locally flat and well defined. This can be reached through a QRF transformation.

F. Giacomini, Č. Brukner, Einstein's Equivalence principle for superpositions of gravitational fields and quantum reference frames, arXiv:2012.137544.

The EEP for QRFs

- In any and every **Quantum** Locally Inertial Frame (QLIF), anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.



F. Giacomini, Č. Brukner, Einstein's Equivalence principle for superpositions of gravitational fields and quantum reference frames, arXiv:2012.137544.

The three aspects of the principle revisited

- **(Q-WEP)** The local effects of (quantum) motion in a superposition of uniform gravitational fields are indistinguishable from those of an observer in flat spacetime that undergoes a quantum superposition of accelerations. [1]
- **(Q-LLI)** The outcome of any local nongravitational experiment is independent of the velocity of the freely falling quantum reference frame in which it is performed.
- **(Q-LPI)** The outcome of any local nongravitational experiment is independent of the position of the quantum reference frame in which it is performed.

1. F. Giacomini, E. Castro-Ruiz, Č. Brukner, Quantum mechanics and the covariance of physical laws in quantum reference frames, Nat. Commun. 10, 494 (2019).

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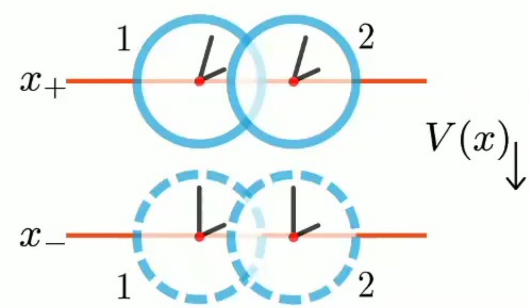
Quantum Universality of Gravitational Redshift (Q-UGR)

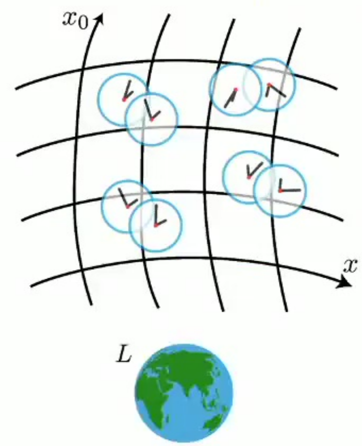
- In order to test Q-LPI, one has to test Q-UGR.
- For example: given two clocks in superposition such that in each branch they are at the same height, the time of the second clock intuitively should not be dilated according to the first clock.
- What is the time of the second clock according to the first one?



Spacetime Quantum Reference Frames (SQRFs)

- SQRFs are reference frames associated to non-interacting quantum particles that move in a gravitational field generated by a mass L, given by a Newtonian metric.
- They have both external and internal degrees of freedom, i.e. they are atomic clocks.
- External degrees of freedom are used to fix the reference frames.
- Internal degrees of freedom are used as clocks, as in the Page-Wootters mechanism.

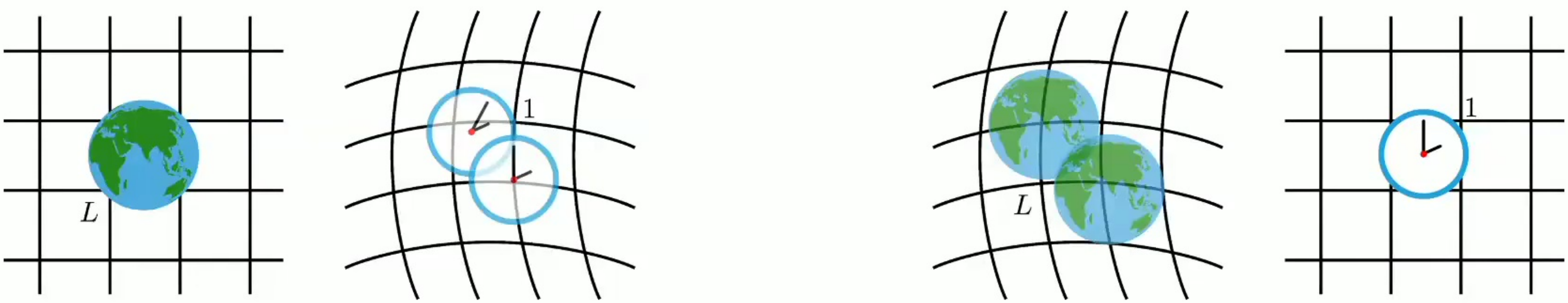
$$g_{00} = 1 + 2\frac{V(\mathbf{x} - \mathbf{x_L})}{c^2},$$
$$g_{01} = g_{10} = 0,$$
$$g_{11} = -1.$$



F. Giacomini, Spacetime Quantum Reference Frames and superpositions of proper times, Quantum 5, 508 (2021).

Spacetime Quantum Reference Frames (SQRFs)

- The formalism allows to describe the QLIF of a particle, and to change from one QLIF to another.

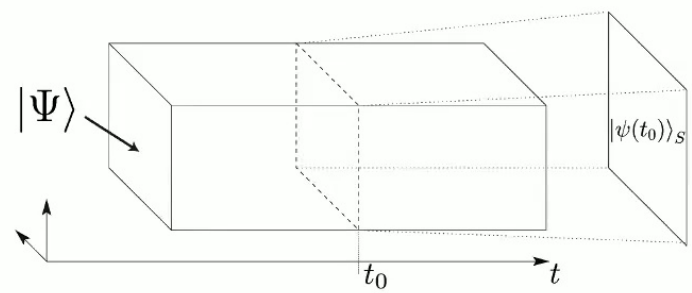


Spacetime Quantum Reference Frames (SQRFs)

- The state from the point of view of a given particle is a history state, as in the Page-Wootters mechanism.

$$|\psi\rangle^{(L)} = \int d\tau_L e^{-\frac{i}{\hbar}\hat{H}^{(L)}\tau_L} |\psi_0^{(L)}\rangle |\tau_L\rangle$$

$$|\psi\rangle^{(L)}(t) = \langle t|\psi\rangle^{(L)}$$

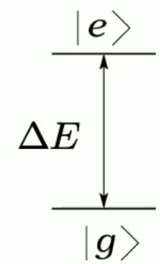


V. Giovannetti, S. Lloyd, L. Maccone, Quantum time, Phys. Rev. D 92, 045033 (2015).

The Hamiltonian in the laboratory frame

$$\hat{H}^{(L)} = mc^2 + \frac{\hat{\mathbf{p}}^2}{2m} + mV(\hat{\mathbf{x}}) + \hat{H}_I\left(1 + \frac{V(\hat{\mathbf{x}})}{c^2} - \frac{\hat{\mathbf{p}}^2}{2m^2c^2}\right)$$

$$\omega' = \omega\left(1 + \frac{V(x)}{c^2} - \frac{p^2}{2m^2c^2}\right) = \omega\frac{d\tau}{dt} \qquad \omega = \frac{\Delta E}{\hbar}$$



- The same Hamiltonian can be found starting from the Newtonian Hamiltonian and performing the substitution[1,2]:

$$m \rightarrow \hat{M} = m + \frac{\hat{H}_I}{c^2}$$

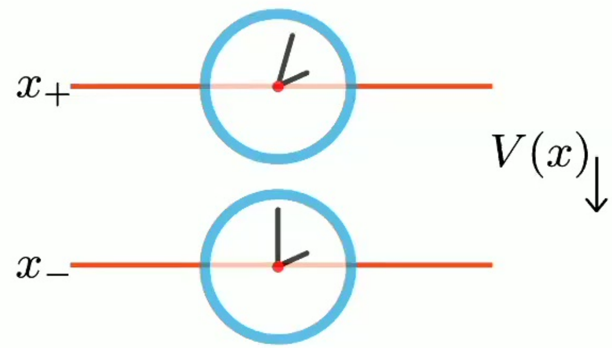
1. Zych, M., Costa, F., Pikovski, I, Č. Brukner, Quantum interferometric visibility as a witness of general relativistic proper time. Nat Commun 2, 505 (2011).
2. M. Zych, Č. Brukner, Quantum formulation of the Einstein Equivalence Principle, Nature Phys 14, 1027–1031 (2018).

UGR and SQRFs

- Using SQRFs we can analyze classical UGR:

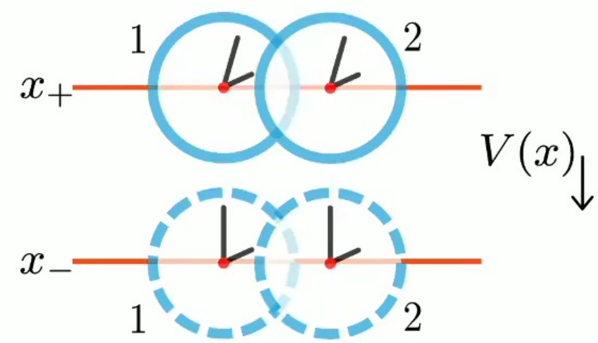
$$|\psi_0\rangle^{(L)}_{AB} = |x_+\rangle_{\mathbf{A}}\,|x_-\rangle_{\mathbf{B}}\,|\tau_{in}=0\rangle_{C_A}\,|\tau_{in}=0\rangle_{C_B}$$

$$|\psi(t_A)\rangle^{(A)}_{BL} = |\Psi(t_A, x_+, x_-)\rangle_{\mathbf{B}L}\left|\left(1+\frac{V(\mathbf{x}_-)-V(\mathbf{x}_+)}{c^2}\right)t_A\right\rangle_{C_B}$$



Examples of Q-UGR

$$|\psi_0\rangle^{(L)}_{AB} = \frac{|x_+\rangle_{\mathbf{A}}\,|x_+\rangle_{\mathbf{B}} + |x_-\rangle_{\mathbf{A}}\,|x_-\rangle_{\mathbf{B}}}{\sqrt{2}}\,|\tau_{in}\rangle_{C_A}\,|\tau_{in}\rangle_{C_B}$$

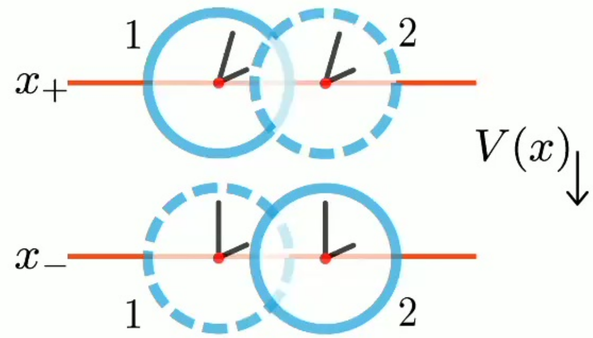


$$|\psi^{(A)}(\tau_A)\rangle_{BL} = \frac{1}{\sqrt{2}}\left(|\Psi(\tau_A, x_+, x_+)\rangle_{\mathbf{B}L} + |\Psi(\tau_A, x_-, x_-)\rangle_{\mathbf{B}L}\right)|\tau_A\rangle_{C_B}$$

- When the two particles are in quantum superposition such that in each branch they are at the same height, there is no time dilation.

Examples of Q-UGR

$$|\psi_0\rangle^{(L)}_{AB} = \frac{|x_+\rangle_{\mathbf{A}} |x_-\rangle_{\mathbf{B}} + |x_-\rangle_{\mathbf{A}} |x_+\rangle_{\mathbf{B}}}{\sqrt{2}} |\tau_{in}\rangle_{C_A} |\tau_{in}\rangle_{C_B}$$



$$|\psi^{(A)}(\tau_A)\rangle_{BL} = \frac{1}{\sqrt{2}} \left(|\Psi(\tau_A, x_+, x_-)\rangle_{\mathbf{B}L} |\tau_A + \Delta\tau\rangle_{C_B} + |\Psi(\tau_A, x_-, x_+)\rangle_{\mathbf{B}L} |\tau_A - \Delta\tau\rangle_{C_B} \right)$$

- In the QRF of the clock there is a superposition of gravitational time dilations, which is due to the relative delocalisation of the clocks: there is a redshift factor for each possible position of particle B as seen from A.

Model for violations

- Let's assume that there are three different types of masses: $\hat{M} = m + \frac{\hat{H}_I}{c^2}$
- Inertial mass, coupled with the momentum. $\hat{M}^{(i)}(\hat{\mathbf{p}})$
- Gravitational mass, coupled with the gravitational field. $\hat{M}^{(g)}(\hat{\mathbf{x}})$
- Rest mass, uncoupled. $\hat{M}^{(r)}$

$$\hat{M}^{(i)}(\hat{\mathbf{p}}) \neq \hat{M}^{(g)}(\hat{\mathbf{x}}) \quad \text{(Q-WEP)} \quad \hat{\eta}(\hat{\mathbf{x}}, \hat{\mathbf{p}}) = \hat{\mathbb{1}} - \hat{M}^{(g)}(\hat{\mathbf{x}})\, \hat{M}^{(i)-1}(\hat{\mathbf{p}})$$

$$\hat{M}^{(r)} \neq \hat{M}^{(i)}(\hat{\mathbf{p}}) \quad \text{(Q-LLI)} \quad \hat{\beta}(\hat{\mathbf{p}}) = \hat{\mathbb{1}} - \hat{H}^{(i)}(\hat{\mathbf{p}})\, \hat{H}^{(r)-1}$$

$$\hat{M}^{(r)} \neq \hat{M}^{(g)}(\hat{\mathbf{x}}) \quad \text{(Q-LPI)} \quad \hat{\alpha}(\hat{\mathbf{x}}) = \hat{\mathbb{1}} - \hat{H}^{(g)}(\hat{\mathbf{x}})\, \hat{H}^{(r)-1}$$

M. Zych, Č. Brukner, Quantum formulation of the Einstein Equivalence Principle, Nature Phys 14, 1027–1031 (2018).

Non-existence of QLIFs when violations are introduced

- When Q-LPI is violated, one finds:

$$|\psi\rangle^{(A)} \sim \int d\tau_A dq_L dt_A \, |\chi(\tau_A, \mathbf{q}_L, t_A)\rangle_{BL} \, |\varphi(\tau_A, \mathbf{q}_L, t_A)\rangle_{C_A}$$

- It is not an history state anymore: the state of the clock in its reference frame depends explicitly on the position of the clock.
- This result means that, in the QLIF of system A, the clock behaves differently according to where it is placed in the gravitational field.
- This is a violation of Q-LPI as defined previously.

Violations in the laboratory QLIF

- It is still possible to describe the point of view of the laboratory. This allows us to make predictions about experiments, even if the EEP for QRFs is violated. The new Hamiltonian, e.g. when only Q-LPI is violated, is

$$\hat{H}^{LPI} = \frac{\mathbf{p}^2}{2m} + mV(\mathbf{x}) + \hat{H}_I^{(r)} + \hat{H}_I^{(g)}(\mathbf{x})\frac{V(\mathbf{x})}{c^2} - \hat{H}_I^{(r)}\frac{\mathbf{p}^2}{2m^2c^2}.$$

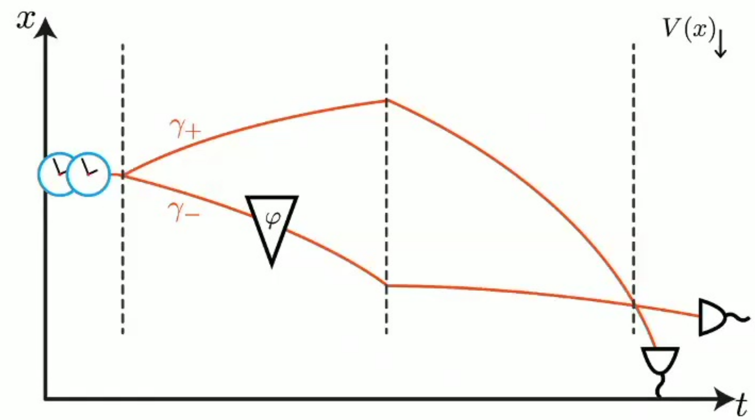
- How can we measure the difference with the standard Hamiltonian?

Atomic clock interferometers

- A natural choice are atomic clock interferometers.

$$|\psi_0\rangle^{(L)}_{AB} = \frac{|x_+\rangle_{\mathbf{A}} |x_+\rangle_{\mathbf{B}} + e^{2i\varphi} |x_-\rangle_{\mathbf{A}} |x_-\rangle_{\mathbf{B}}}{\sqrt{2}} |\tau_{in}\rangle_{C_A} |\tau_{in}\rangle_{C_B}$$

$$|D_\pm\rangle_{\mathbf{AB}} = \frac{|x_+\rangle_{\mathbf{A}} |x_+\rangle_{\mathbf{B}} \pm |x_-\rangle_{\mathbf{A}} |x_-\rangle_{\mathbf{B}}}{\sqrt{2}}$$



The probabilities in the presence of violations

- The measurement probabilities depend on the violation parameters for Q-LPI and Q-LLI:

$$P_{\pm}^{(Q-LPI)} = f_1(\alpha_{ij}(x))$$

$$P_{\pm}^{(Q-LLI)} = f_2(\beta_{ij}(p))$$

- Regarding Q-WEP, it is sufficient to test for the classical parameter η, since:

$$\hat{\eta} = \hat{\eta}(\hat{\alpha}(x), \hat{\beta}(p), \eta)$$

- And one finds: $$P_{\pm}^{(Q-WEP)} = f_3(\eta)$$

Test EEP for QRFs: Take-home messages

- We introduced a model for violations of the Einstein's Equivalence Principle for Quantum Reference Frames.
- We showed that entangled atomic clocks in an atomic clock interferometer can be used to test the Einstein's Equivalence Principle for Quantum Reference Frames.
- We found a link between different proposals of quantum extensions of the equivalence principle.

Quantum sensing

Motivation

- Is it possible, even in principle, to use joint effects of quantum theory and general relativity to enhance the precision of measurements?

Classical estimation theory

- Given a parameter of interest λ and a sample space $\{x\}$, namely a set of all possible outcomes of an experiment, it holds the Cramér-Rao bound:

$$\Delta^2(\lambda) \geq \frac{1}{MF(\lambda)}$$

$\Delta^2(\lambda)$ is the variance, M is the number of measurements.

- The Fisher Information (FI) $F(\lambda)$ is defined as

$$F(\lambda) = \sum_{x \in \chi} \frac{(\partial_\lambda P_\lambda(x))^2}{P_\lambda(x)}$$

$P_\lambda(x)$ is the probability of obtaining a certain outcome x for a fixed value of the parameter of interest λ.

- Given a measurement probability, the FI measures how much information it contains about a parameter of interest.

Quantum estimation theory

- The Quantum Fisher Information (QFI) can be defined analogously with the quantum Cramér-Rao bound:

$$\Delta^2(\lambda) \geq \frac{1}{MG(\lambda)} \qquad F(\lambda) \leq G(\lambda)$$

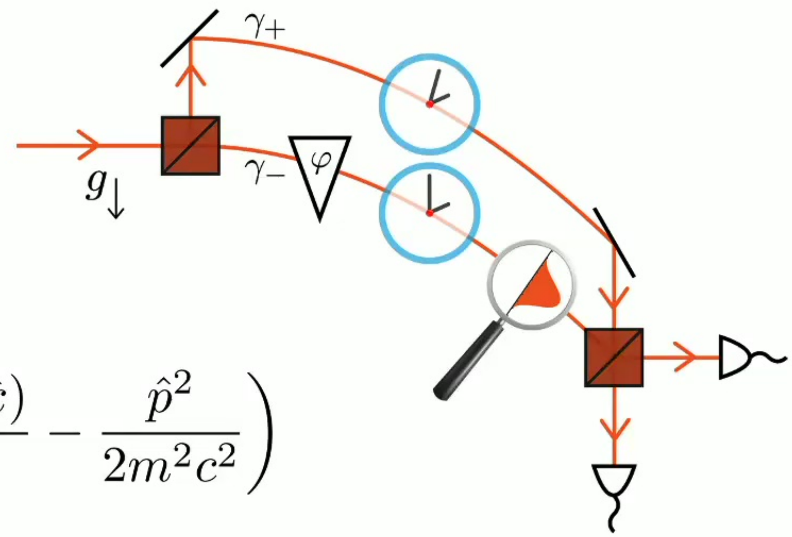
- The QFI of a pure state is $G(\lambda) = 4\left(\langle \partial_\lambda \psi | \partial_\lambda \psi \rangle - |\langle \partial_\lambda \psi | \psi \rangle|^2\right)$

- The QFI measures how much information about a parameter of interest is contained in a quantum state.

- Maximizing the QFI means finding the best quantum state to estimate a parameter, maximizing the FI means find the best measurement given a quantum state.

An atomic clock in a freely falling interferometer

- Let's consider a freely falling interferometer with atomic clocks.
- The external degrees of freedom are Gaussian wave packets.
- The internal degree of freedom is a two-level clock.



$$\hat{H} = mc^2 + \frac{\hat{p}^2}{2m} + mV_F(\hat{x}) + \hat{H}_{int}\left(1 + \frac{V_F(\hat{x})}{c^2} - \frac{\hat{p}^2}{2m^2c^2}\right)$$

$$V_F(\hat{x}) = g(\hat{x} - x_0) + V_N(x_0)$$

$$\hat{H}_{int} = E_0 \left|0\right\rangle \left\langle 0\right| + E_1 \left|1\right\rangle \left\langle 1\right|$$

$$\left|\psi_0\right\rangle = \frac{\left|\psi_+\right\rangle + e^{i\varphi}\left|\psi_-\right\rangle}{\sqrt{2}} \left|\tau_{in}\right\rangle$$

$$\frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}}$$

Quantum sensing analysis: QFI

- We find a QFI that scales as Δt^6
- Previous results without time dilation show a Δt^4 scaling[1,2].
- This shows that gravitational time dilation may be used as a resource in quantum estimation theory.

$$G^{(GTD)}(g) \overset{\Delta t \to \infty}{\propto} \Delta t^6$$

$$G^{(\text{NO GTD})}(g) \propto \Delta t^4$$

1. L. Seveso, V. Peri, M.G.A. Paris, Quantum limits to mass sensing in a gravitational field, J. Phys. A: Math. Theor. 50 235301 (2017)
2. M. Kritsotakis, S.S. Szigeti, J.A. Dunningham, S. A. Haine, Optimal matter-wave gravimetry, Phys. Rev. A 98, 023629 (2018)

Quantum sensing analysis: FI

- The FI scales as: $F(g) \propto \Delta t^2$
- This means that part of the information is lost when the internal degrees of freedom are ignored.
- In order to exploit all the information, both internal and external degrees of freedom sould be measured.

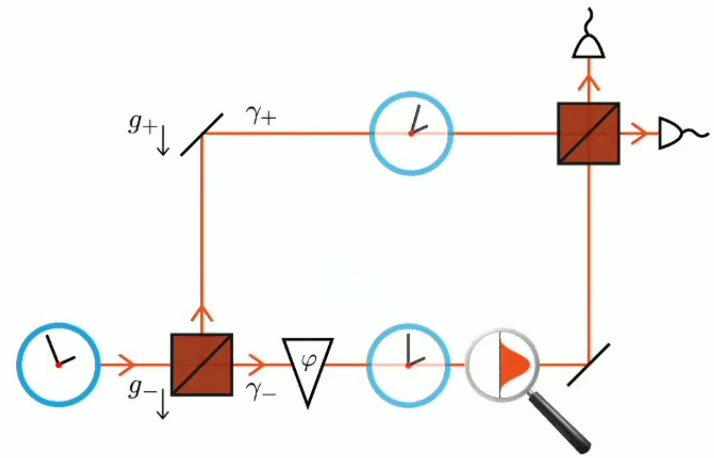
The Mach-Zehnder interferometer

- We insert a potential to balance the gravitational attraction in the two hoizontal trajectories.

$$\hat{H} = mc^2 + \frac{\hat{p}^2}{2m} + \hat{H}_{int}\left(1 + \frac{V_{MZ}(\hat{x})}{c^2} - \frac{\hat{p}^2}{2m^2c^2}\right)$$

- We approximate the potential to be piece-wise linear

$$V_{MZ}(\hat{x}) = \begin{cases} g_+(\hat{x} - x_{+0}) + V_N(x_{+0}) & \text{if } x > x_0 \\ g_-(\hat{x} - x_{-0}) + V_N(x_{-0}) & \text{if } x < x_0 \end{cases}$$



Quantum sensing analysis: QFI and FI

- The setup is sensitive to both $\bar{g}$ and Δg, as an effect of the gravitational time dilation.
- Moreover, the standard interferometric measurement is optimal to extract that information.
- Without gravitational time dilation, they would be zero: there is an enhancement.

$$G^{MZ}(\Delta g) \propto \Delta t^2$$

$$G^{MZ}(\bar{g}) \propto \Delta t^2$$

$$F^{MZ}(\Delta g) \propto \Delta t^2$$

$$F^{MZ}(\bar{g}) \propto \Delta t^2$$

Quantum sensing: take-home messages

- We showed that gravitational time dilation may be used as a resource in quantum sensing of gravimeters, because it enhances the information for long interferometric times.
- To exploit all the advantage, one must measure both internal and external degrees of freedom.

Thank you!

An atomic clock in a freely falling interferometer

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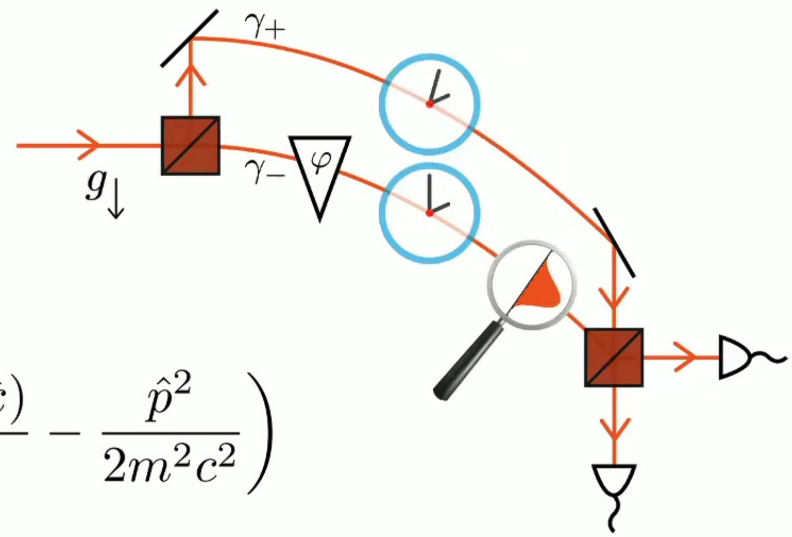
- The internal degree of freedom is a two-level clock.



$$\hat{H} = mc^2 + \frac{\hat{p}^2}{2m} + mV_F(\hat{x}) + \hat{H}_{int}\left(1 + \frac{V_F(\hat{x})}{c^2} - \frac{\hat{p}^2}{2m^2c^2}\right)$$

$$V_F(\hat{x}) = g(\hat{x} - x_0) + V_N(x_0)$$

$$\hat{H}_{int} = E_0 \left|0\right\rangle \left\langle 0\right| + E_1 \left|1\right\rangle \left\langle 1\right|$$

$$\left|\psi_0\right\rangle = \frac{\left|\psi_+\right\rangle + e^{i\varphi}\left|\psi_-\right\rangle}{\sqrt{2}} \left|\tau_{in}\right\rangle$$

$$\left|\tau_{in}\right\rangle = \frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}}$$