Title: Quantum Reference Frames for Superpositions of Spacetimes

Speakers: Anne-Catherine de la Hamette

Series: Quantum Foundations

Date: May 31, 2022 - 3:30 PM

URL: https://pirsa.org/22050065

Abstract: The current theories of quantum physics and general relativity on their own do not allow us to study situations in which spacetime is in a quantum superposition. In this talk, I propose a general strategy to determine the dynamics of objects on an indefinite spacetime metric, using an extended notion of quantum reference frame transformations. First, we study the situation of the gravitational source mass being in a spatial superposition state and, using a generalized principle of covariance, show how to transform to a frame in which the standard theories of GR and QFT allow to determine the dynamics. In the second part, we consider superpositions of conformally equivalent metrics inhabited by a massive quantized Klein-Gordon field. By requiring invariance of the KG equation under quantum conformal transformations, we find that the superposition is transferred to the quantum field in the form of an effective, spacetime dependent mass term. Overall, the proposed strategy allows to construct the respective explicit quantum frame change operators, and to study physical phenomena such as time dilation and cosmological particle production in different quantum frames.

Zoom Link: https://pitp.zoom.us/j/96903859307?pwd=aEtLUy9tME5GL25nTjBVNXVmb2N3Zz09

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Quantum Reference Frames for Superpositions of Spacetimes

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in collaboration with V. Kabel, E. Castro-Ruiz, and Č. Brukner
arXiv:2112.11473 + work in preparation

Perimeter Institute, Waterloo, ON, May 31st, 2022

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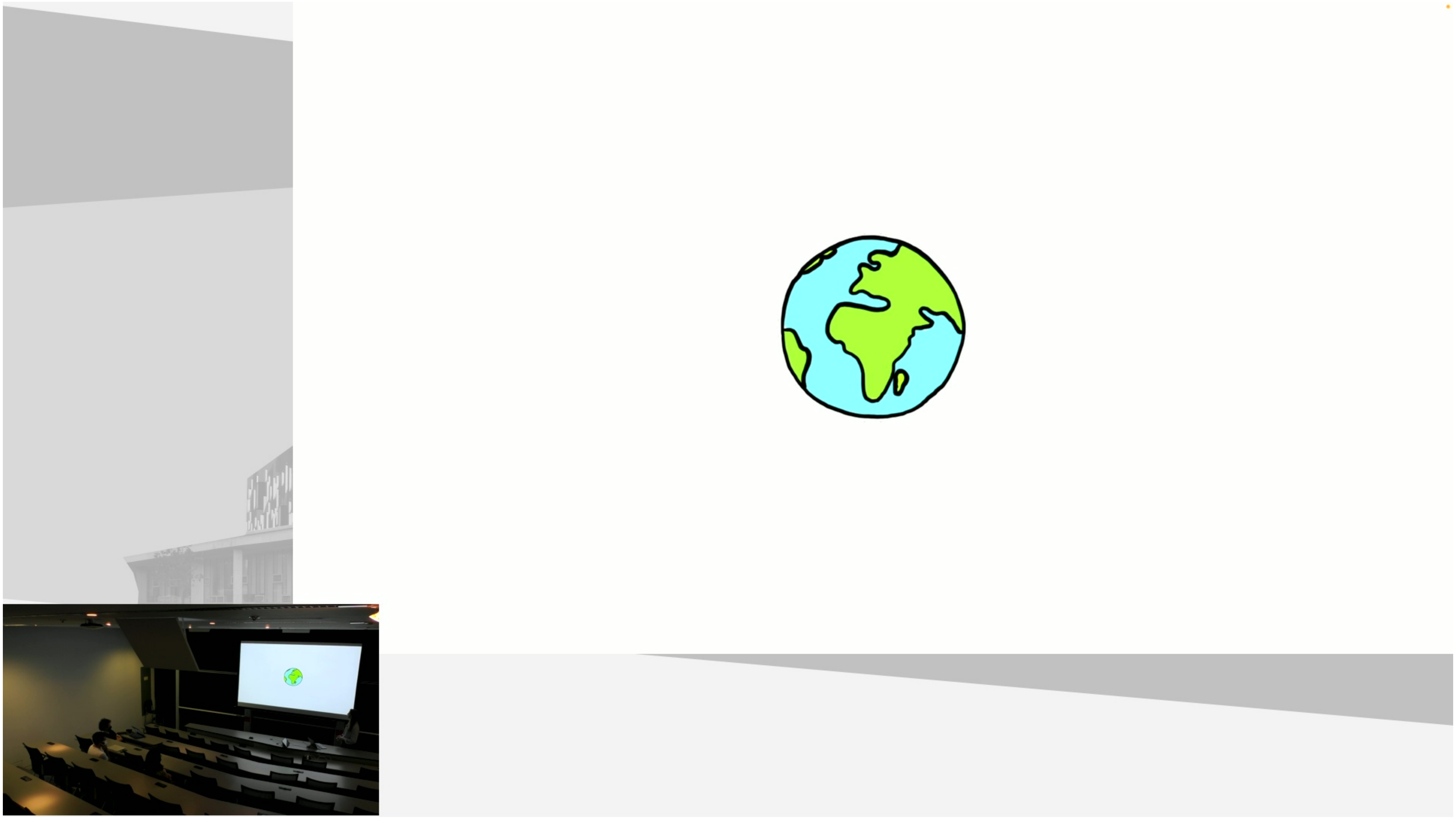
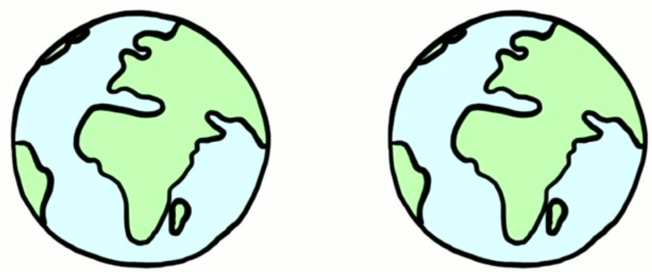
- Falling through masses in superposition: quantum reference frames for indefinite metrics, arXiv:2112.11473
- Spacetimes in superposition: Extended symmetries of the Klein-Gordon field, *to appear*

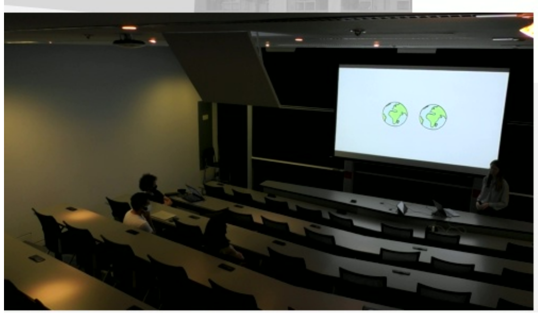
Viktoria Kabel

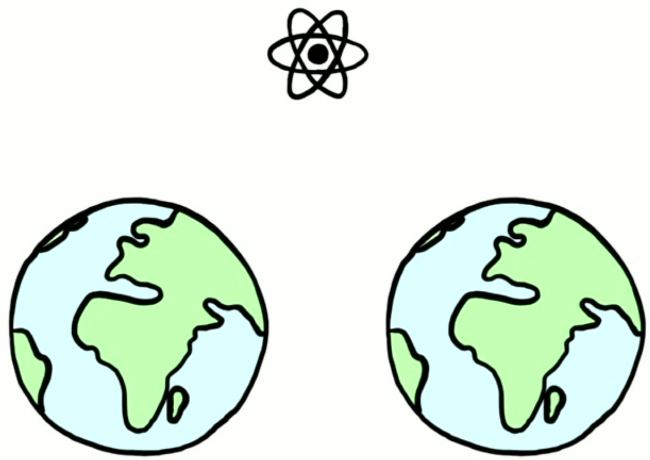
Esteban Castro-Ruiz

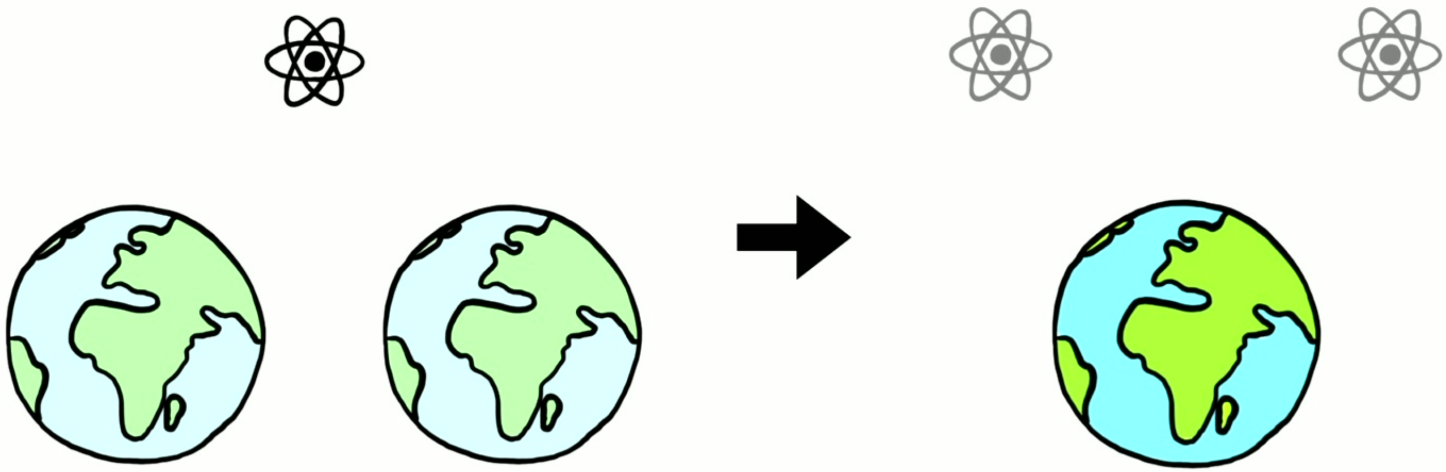
Časlav Brukner

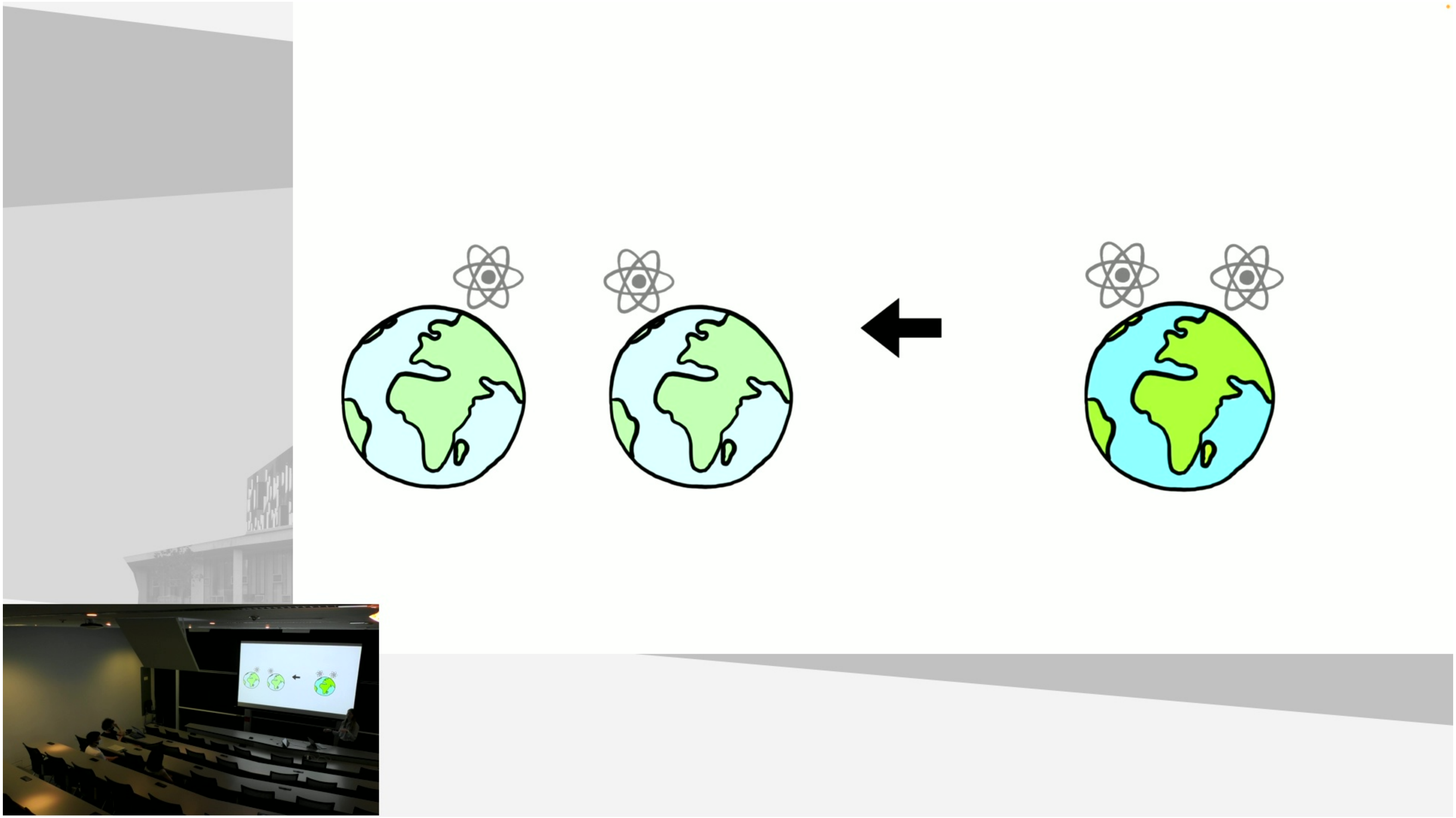












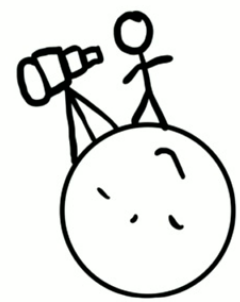
Overview

Falling through masses in superposition

1. Quantum reference frames
2. Generalized principle of covariance
3. Motion of a test particle and time dilation
4. Generalizations

Extended symmetries of KG field in superposition of spacetimes

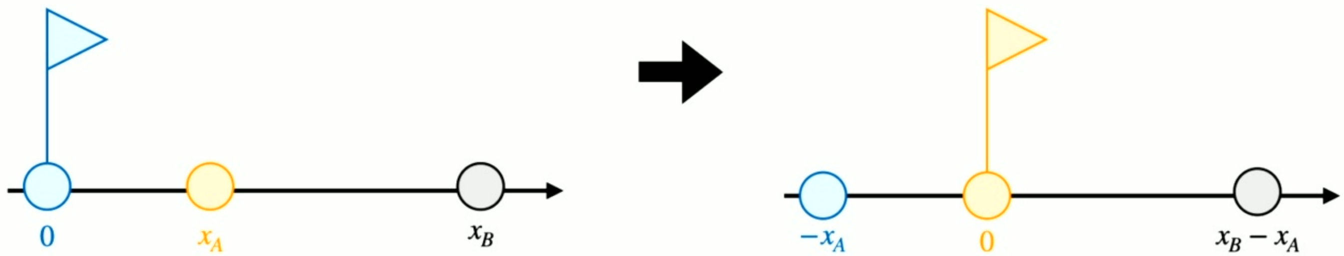
1. Symmetries of the KG field in an expanding universe
2. Quantum conformal transformations
3. Cosmological particle production



Superpositions of spacetimes: extended symmetry principles

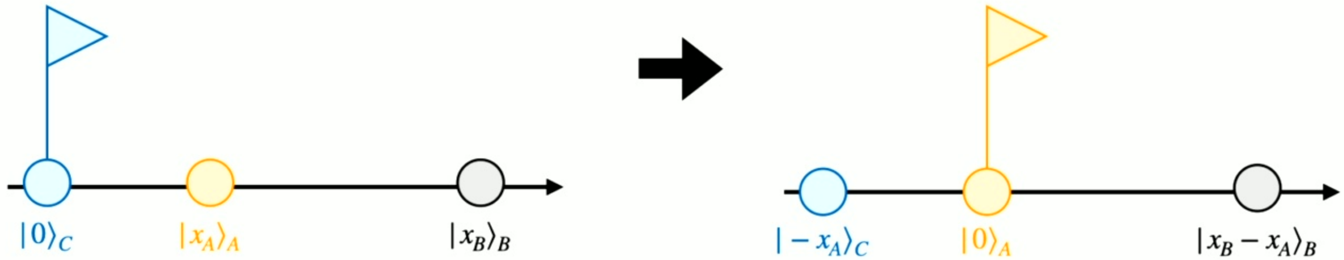
Quantum Reference Frames

Formalism



Quantum Reference Frames

Formalism



$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C |x_A\rangle_A |x_B\rangle_B$$

$$\in \mathscr{H}^{(C)}_{ABC} = \mathscr{H}^{(C)}_A \otimes \mathscr{H}^{(C)}_B \otimes \mathscr{H}^{(C)}_C$$

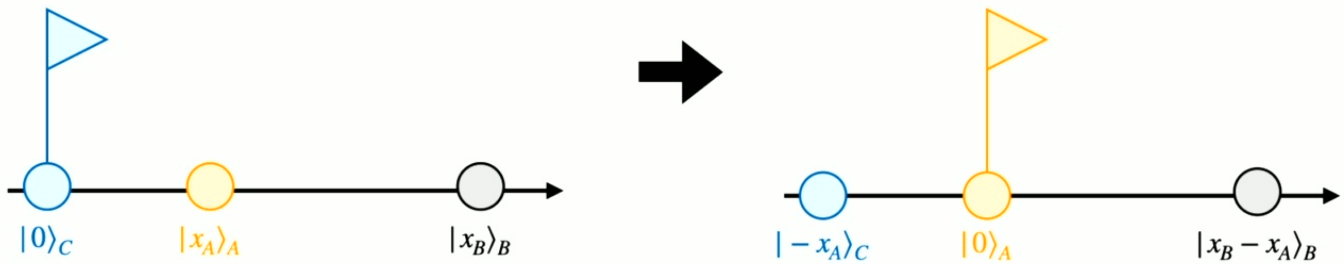
$$|\psi\rangle^{(A)}_{ABC} = |0\rangle_A |-x_A\rangle_C |x_B - x_A\rangle_B$$

$$\in \mathscr{H}^{(A)}_{ABC} = \mathscr{H}^{(A)}_A \otimes \mathscr{H}^{(A)}_B \otimes \mathscr{H}^{(A)}_C$$

F. Giacomini, E. Castro-Ruiz, and Č.Brukner, Nature Comm. 10, 494 (2019).
A.-C. de la Hamette, T. Galley, Quantum 4, 367 (2020).

Quantum Reference Frames

Formalism



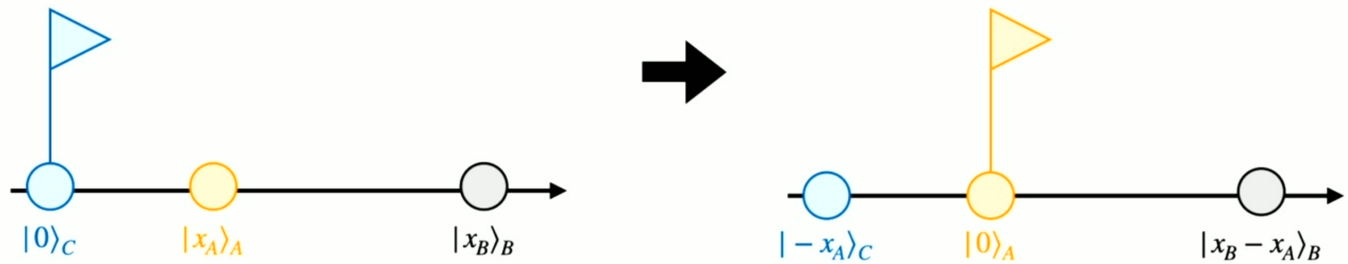
$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C |x_A\rangle_A |x_B\rangle_B$$

$$|\psi\rangle^{(A)}_{ABC} = |0\rangle_A |-x_A\rangle_C \boxed{|x_B - x_A\rangle_B}$$

$$= \mathscr{P}_{CA} |0\rangle_C |x_A\rangle_A \boxed{e^{\frac{i}{\hbar}x_A \hat{p}_B} |x_B\rangle_B}$$

Quantum Reference Frames

Formalism



$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C |x_A\rangle_A |x_B\rangle_B$$

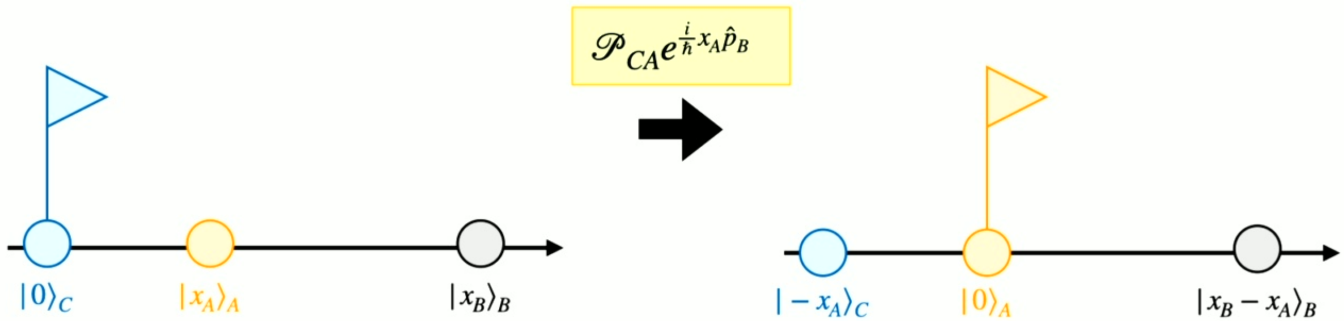
$$|\psi\rangle^{(A)}_{ABC} = |0\rangle_A |-x_A\rangle_C |x_B - x_A\rangle_B$$

$$= \mathscr{P}_{CA} |0\rangle_C |x_A\rangle_A e^{\frac{i}{\hbar} x_A \hat{p}_B} |x_B\rangle_B$$

$$\mathscr{P}_{CA} = SWAP_{CA} \circ \int |-x_A\rangle\langle x_A|_A \, dx_A$$

Quantum Reference Frames

Formalism



$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C |x_A\rangle_A |x_B\rangle_B$$

$$|\psi\rangle^{(A)}_{ABC} = |0\rangle_A |-x_A\rangle_C |x_B - x_A\rangle_B$$

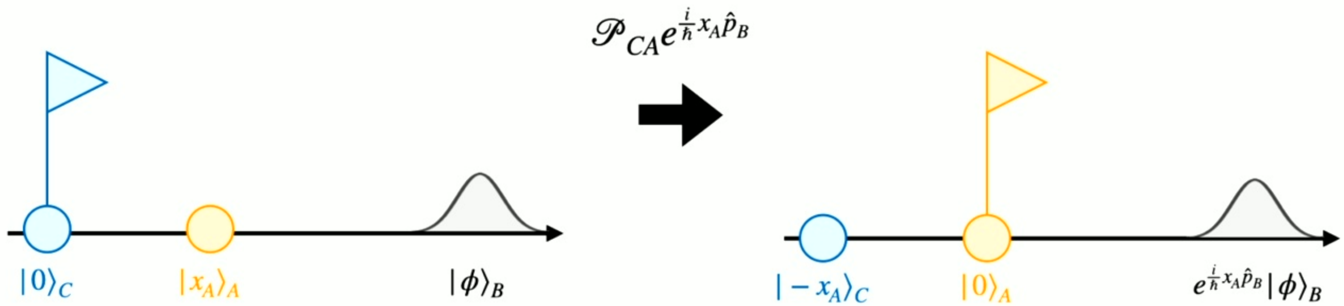
$$= \mathscr{P}_{CA} |0\rangle_C |x_A\rangle_A e^{\frac{i}{\hbar} x_A \hat{p}_B} |x_B\rangle_B$$

$$= \mathscr{P}_{CA} e^{\frac{i}{\hbar} x_A \hat{p}_B} |\psi\rangle^{(C)}_{ABC}$$

$$\mathscr{P}_{CA} = SWAP_{CA} \circ \int |-x_A\rangle\langle x_A|_A \, dx_A$$

Quantum Reference Frames

Formalism

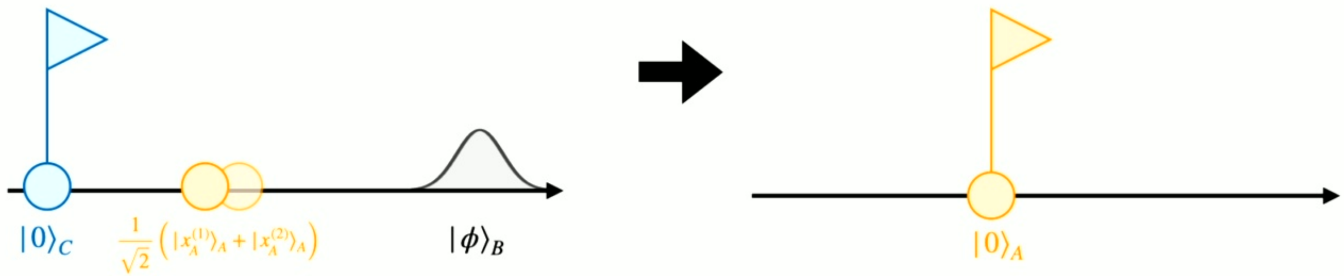


$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C |x_A\rangle_A |\phi\rangle_B$

$|\psi\rangle^{(A)}_{ABC} = \mathscr{P}_{CA} e^{\frac{i}{\hbar} x_A \hat{p}_B} |0\rangle_C |x_A\rangle_A |\phi\rangle_B$

Quantum Reference Frames

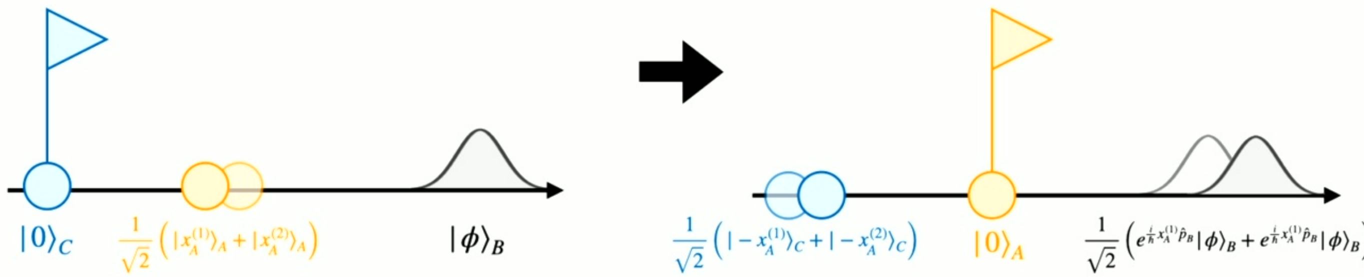
Formalism



$$|\psi\rangle_{ABC}^{(C)} = |0\rangle_C \frac{1}{\sqrt{2}}\left(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A\right)|\phi\rangle_B$$

Quantum Reference Frames

Formalism

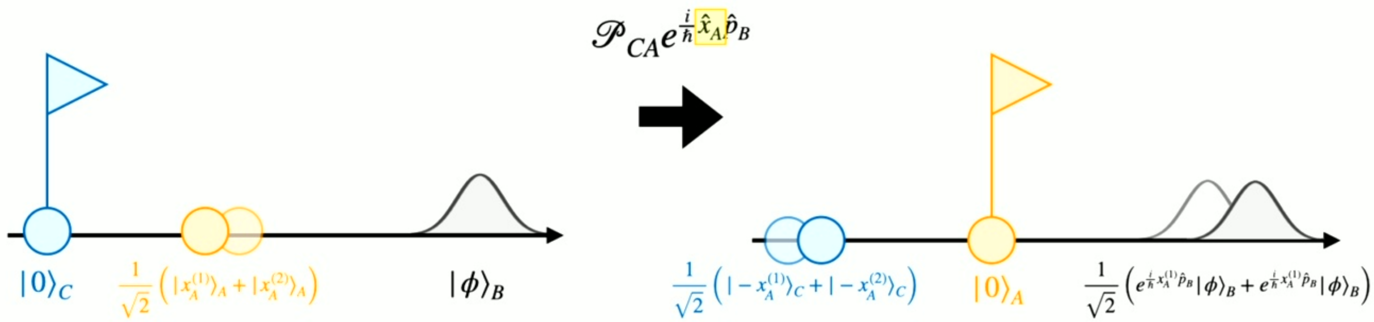


$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C \frac{1}{\sqrt{2}}\left(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A\right)|\phi\rangle_B$$

$$|\psi\rangle^{(A)}_{ABC} = |0\rangle_A \frac{1}{\sqrt{2}}\left(|-x_A^{(1)}\rangle_C e^{\frac{i}{\hbar}x_A^{(1)}\hat{p}_B}|\phi\rangle_B + |-x_A^{(2)}\rangle_C e^{\frac{i}{\hbar}x_A^{(1)}\hat{p}_B}|\phi\rangle_B\right)$$

Quantum Reference Frames

Formalism

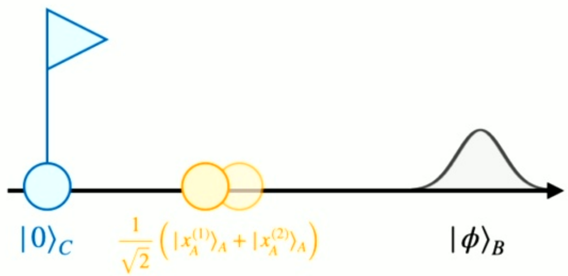


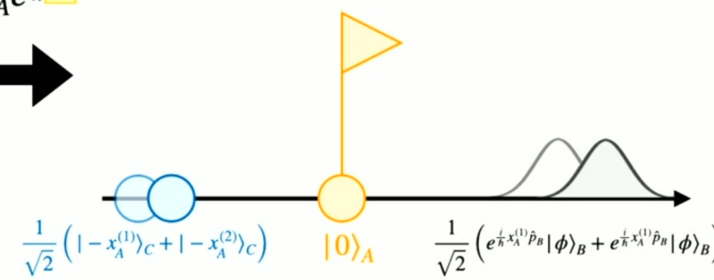
$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C \frac{1}{\sqrt{2}}\left(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A\right)|\phi\rangle_B$$

$$|\psi\rangle^{(A)}_{ABC} = \mathcal{P}_{CA} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}|0\rangle_C \frac{1}{\sqrt{2}}\left(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A\right)|\phi\rangle_B$$

Quantum Reference Frames

Formalism





$$|\psi\rangle_{ABC}^{(C)} = |0\rangle_C|\frac{1}{\sqrt{2}}\left(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A\right)|\phi\rangle_B$$

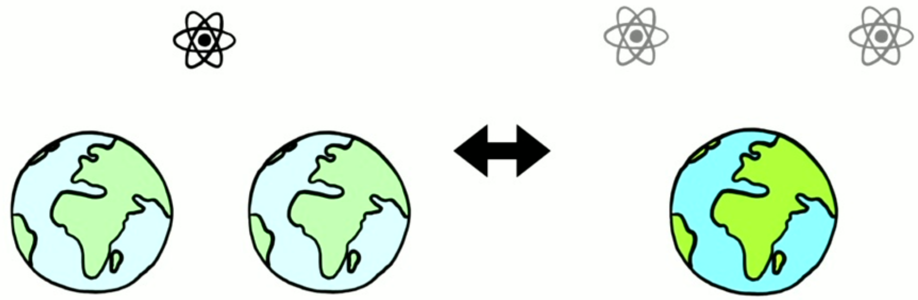
$$|\psi\rangle_{ABC}^{(A)} = \mathscr{P}_{CA}e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}|0\rangle_C\frac{1}{\sqrt{2}}\left(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A\right)|\phi\rangle_B$$

Quantum controlled translation

$$\mathscr{P}_{CA}e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}$$

Quantum Reference Frames

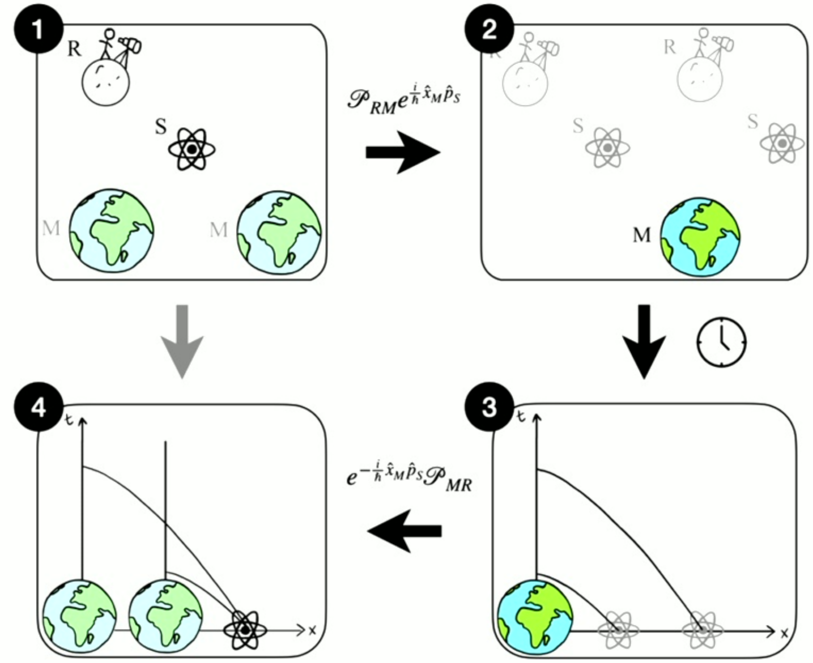
Generalized Principle of Covariance

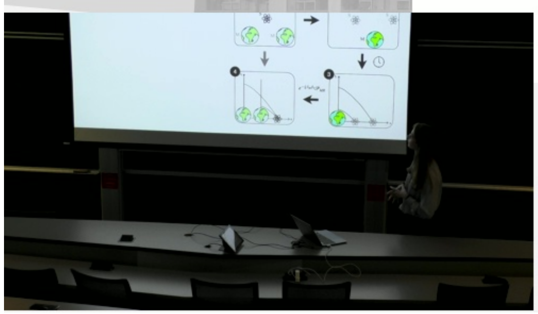


Covariance of dynamical laws under quantum coordinate transformations:
Physical laws retain their form under *quantum* coordinate transformations.

Applications

Motion of a Test Particle



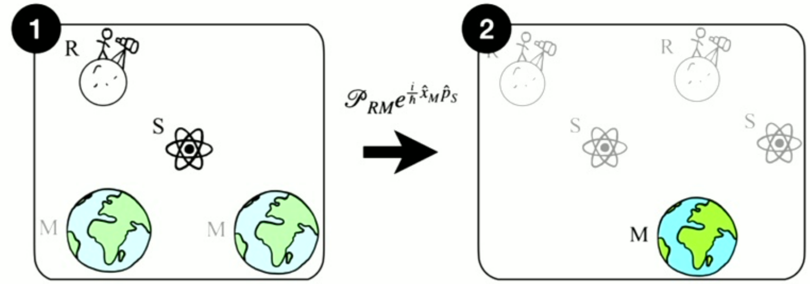


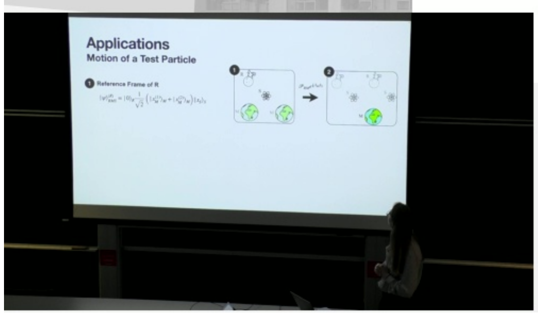
Applications

Motion of a Test Particle

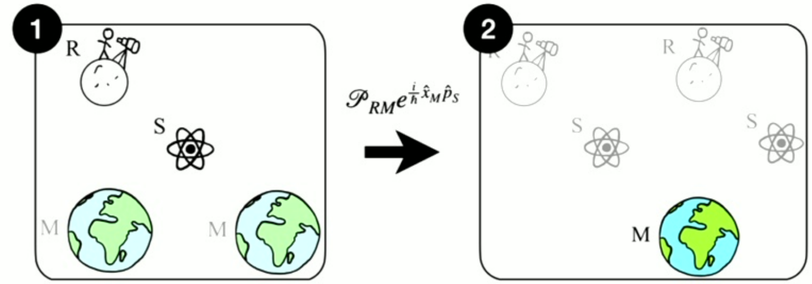
1 Reference Frame of R

$$|\psi\rangle^{(R)}_{RMS} = |0\rangle_R \frac{1}{\sqrt{2}} \left(|x^{(1)}_M\rangle_M + |x^{(2)}_M\rangle_M \right) |x_S\rangle_S$$





Applications
Motion of a Test Particle



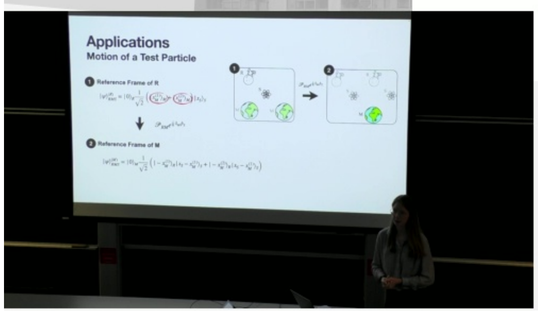
1 Reference Frame of R

$$|\psi\rangle^{(R)}_{RMS} = |0\rangle_R \frac{1}{\sqrt{2}}\left(|x_M^{(1)}\rangle_M + |x_M^{(2)}\rangle_M\right)|x_S\rangle_S$$

$$\Downarrow \quad \mathscr{P}_{RM}e^{\frac{i}{\hbar}\hat{x}_M\hat{p}_S}$$

2 Reference Frame of M

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\left(|-x_M^{(1)}\rangle_R|x_S - x_M^{(1)}\rangle_S + |-x_M^{(2)}\rangle_R|x_S - x_M^{(1)}\rangle_S\right)$$



Applications

Motion of a Test Particle

2 Reference Frame of M

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}} \left(|-x^{(1)}_M\rangle_R |x_S - x^{(1)}_M\rangle_S + |-x^{(2)}_M\rangle_R |x_S - x^{(1)}_M\rangle_S \right)$$

3

$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}} \left(e^{-\frac{i}{\hbar}\Phi^{(1)}} |-x^{(1)}_M\rangle_R |\tilde{x}^{(1)}_S\rangle_S + e^{-\frac{i}{\hbar}\Phi^{(2)}} |-x^{(2)}_M\rangle_R |\tilde{x}^{(2)}_S\rangle_S \right)$$

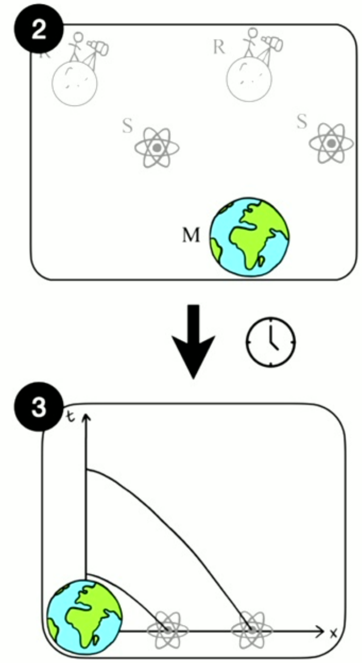
geodesic motion

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

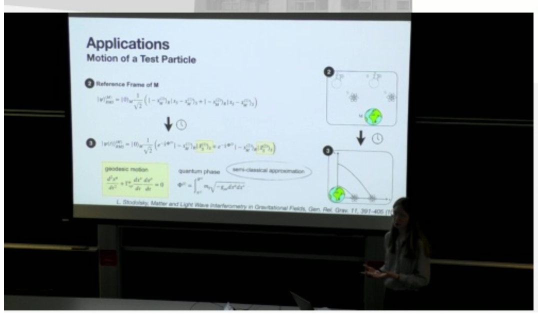
quantum phase

semi-classical approximation

$$\Phi^{(i)} = \int_{A^{(i)}}^{B^{(i)}} m_S \sqrt{-g_{\mu\nu}dx^\mu dx^\nu}$$



L. Stodolsky, Matter and Light Wave Interferometry in Gravitational Fields, Gen. Rel. Grav. 11, 391-405 (1979).

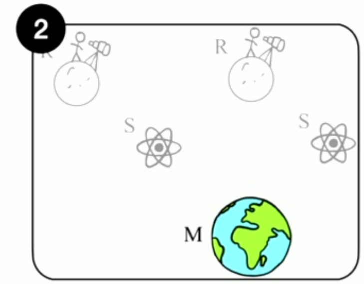


Applications

Motion of a Test Particle

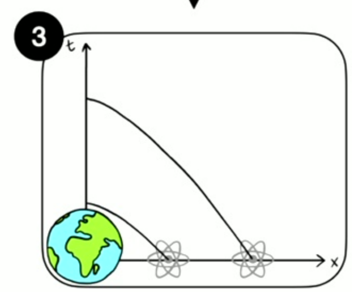
2 Reference Frame of M

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}} \left(|-x_M^{(1)}\rangle_R |x_S - x_M^{(1)}\rangle_S + |-x_M^{(2)}\rangle_R |x_S - x_M^{(1)}\rangle_S \right)$$



3

$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}} \left(e^{-\frac{i}{\hbar}\Phi^{(1)}} |-x_M^{(1)}\rangle_R |\tilde{x}_S^{(1)}\rangle_S + e^{-\frac{i}{\hbar}\Phi^{(2)}} |-x_M^{(2)}\rangle_R |\tilde{x}_S^{(2)}\rangle_S \right)$$



geodesic motion

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

quantum phase

semi-classical approximation

$$\Phi^{(i)} = \int_{A^{(i)}}^{B^{(i)}} m_S \sqrt{-g_{\mu\nu}dx^\mu dx^\nu}$$

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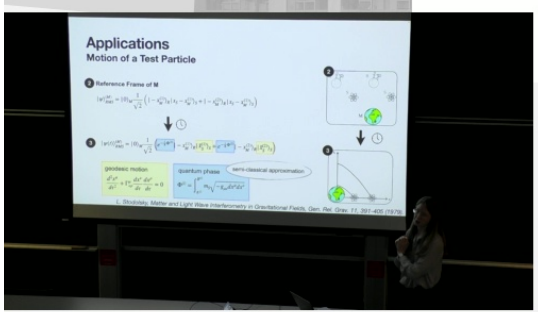


Applications

Motion of a Test Particle

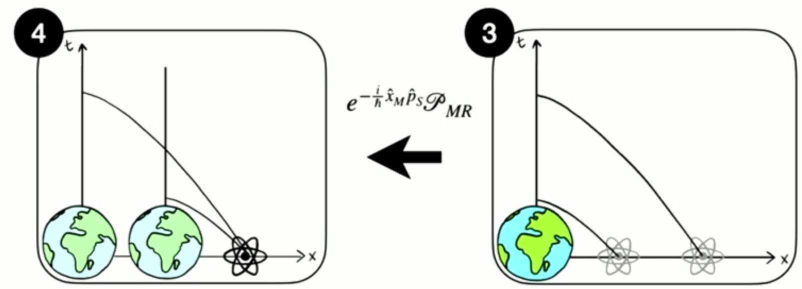
3 Reference Frame of M

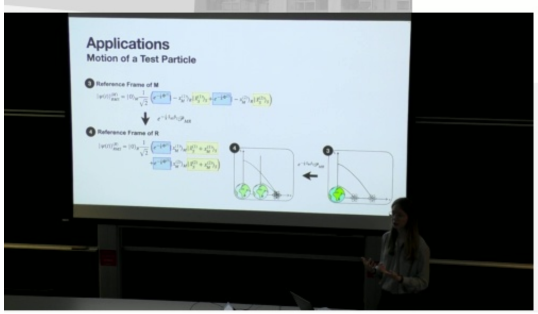
$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}} \left(e^{-\frac{i}{\hbar}\Phi^{(1)}} |-x_M^{(1)}\rangle_R |\tilde{x}_S^{(1)}\rangle_S + e^{-\frac{i}{\hbar}\Phi^{(2)}} |-x_M^{(2)}\rangle_R |\tilde{x}_S^{(2)}\rangle_S \right)$$

$$\Big\downarrow \quad e^{-\frac{i}{\hbar}\hat{x}_M \hat{p}_S}\mathcal{P}_{MR}$$

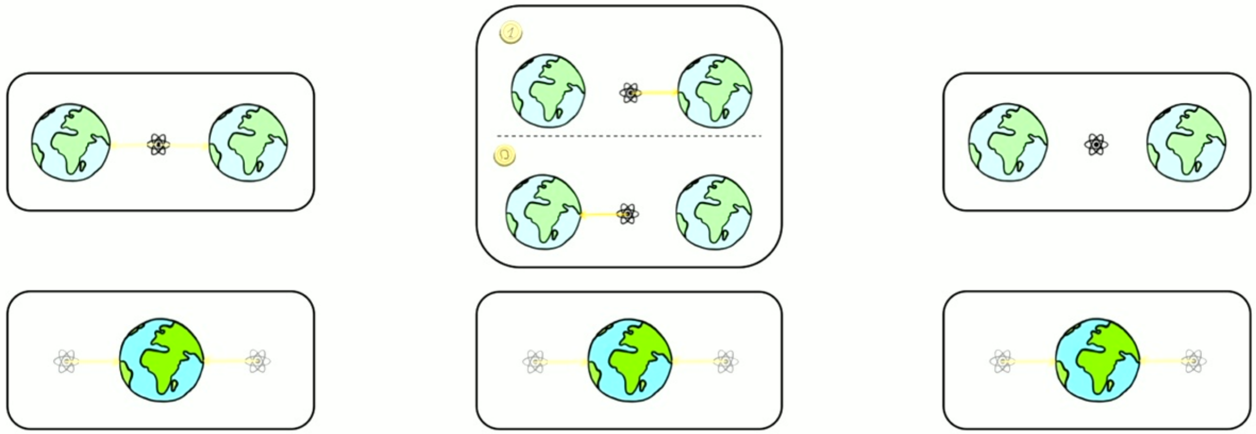
4 Reference Frame of R

$$|\psi(t)\rangle^{(R)}_{RMS} = |0\rangle_R \frac{1}{\sqrt{2}} \Big(e^{-\frac{i}{\hbar}\Phi^{(1)}} |x_M^{(1)}\rangle_M |\tilde{x}_S^{(1)} + x_M^{(1)}\rangle_S$$
$$+ e^{-\frac{i}{\hbar}\Phi^{(2)}} |x_M^{(2)}\rangle_M |\tilde{x}_S^{(2)} + x_M^{(2)}\rangle_S \Big)$$





Comparison with other Approaches

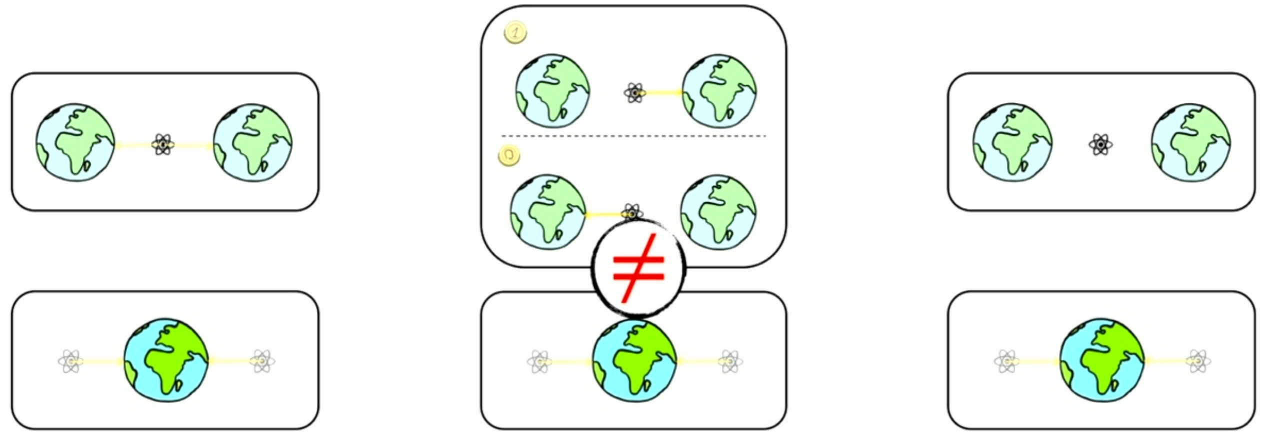


Generalized Covariance

Collapse Models

Semi-Classical Gravity

Comparison with other Approaches



Generalized Covariance

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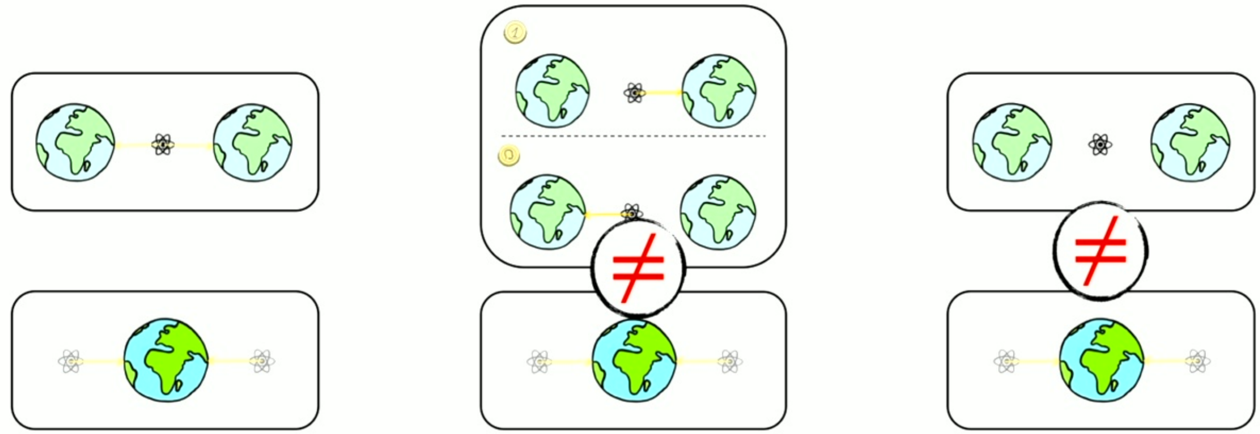
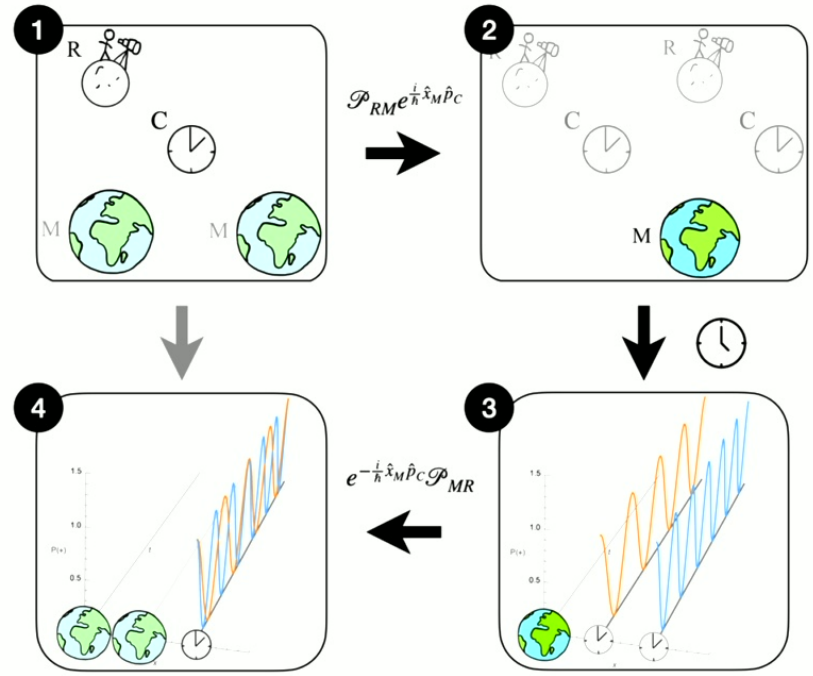


Generalized Covariance | Collapse Models | Semi-Classical Gravity

Application
Time Dilation



Application

Time Dilation

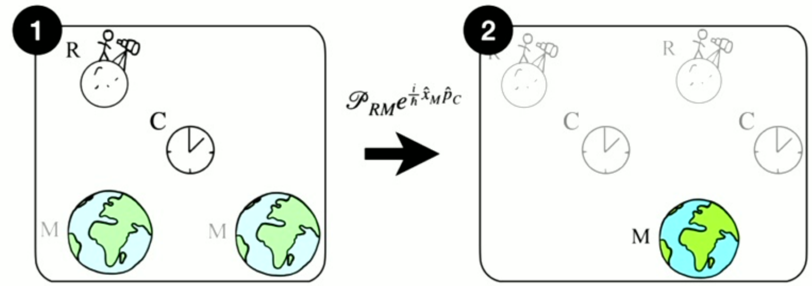
1 Reference Frame of R

$$|\psi\rangle^{(R)}_{RMC} = |0\rangle_R \frac{1}{\sqrt{2}}\left(|x_M^{(1)}\rangle_M + |x_M^{(2)}\rangle_M\right)|x_C\rangle_{C_{ext}} |s(\tau_0)\rangle_{C_{int}}$$

$$\Big\downarrow \quad \mathscr{P}_{RM} e^{\frac{i}{\hbar}\hat{x}_M \hat{p}_C}$$

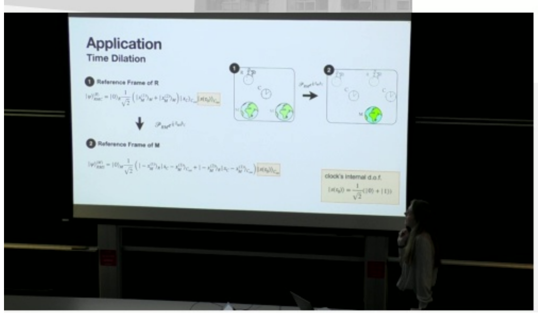
2 Reference Frame of M

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\left(|-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} + |-x_M^{(2)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}}\right)|s(\tau_0)\rangle_{C_{int}}$$



clock's internal d.o.f.

$$|s(\tau_0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

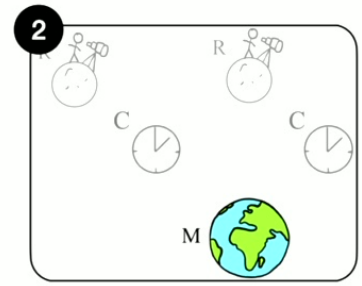


Application

Time Dilation

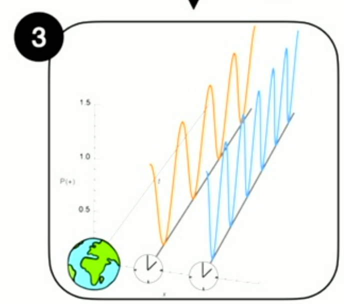
❷ Reference Frame of M

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\left(|-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} + |-x_M^{(2)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}}\right)|s(\tau_0)\rangle_{C_{int}}$$



❸

$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\Big(e^{-\frac{i}{\hbar}\Phi^{(1)}}|-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} |s(\tau_0 + \tau^{(1)})\rangle_{C_{int}} + e^{-\frac{i}{\hbar}\Phi^{(2)}}|-x_M^{(2)}\rangle_R |x_C - x_M^{(2)}\rangle_{C_{ext}} |s(\tau_0 + \tau^{(2)})\rangle_{C_{int}}\Big)$$



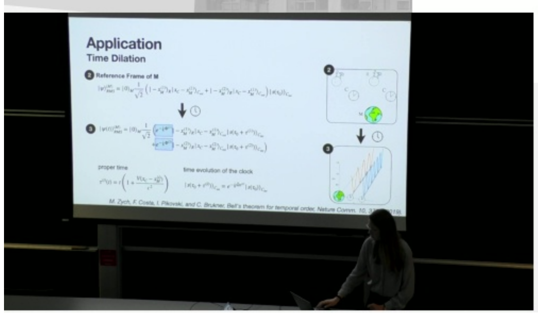
proper time

$$\tau^{(i)}(t) = t\left(1 + \frac{V(x_C - x_M^{(i)})}{c^2}\right)$$

time evolution of the clock

$$|s(\tau_0 + \tau^{(i)})\rangle_{C_{int}} = e^{-\frac{i}{\hbar}\hat{\Omega}\tau^{(i)}}|s(\tau_0)\rangle_{C_{int}}$$

M. Zych, F. Costa, I. Pikovski, and C. Brukner, Bell's theorem for temporal order, Nature Comm. 10, 3772 (2019).



Application

Time Dilation

2 Reference Frame of M

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\left(|-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} + |-x_M^{(2)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}}\right)|s(\tau_0)\rangle_{C_{int}}$$

3

$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\Big(e^{-\frac{i}{\hbar}\Phi^{(1)}}|-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} |s(\tau_0+\tau^{(1)})\rangle_{C_{int}}$$
$$+ e^{-\frac{i}{\hbar}\Phi^{(2)}}|-x_M^{(2)}\rangle_R |x_C - x_M^{(2)}\rangle_{C_{ext}} |s(\tau_0+\tau^{(2)})\rangle_{C_{int}}\Big)$$

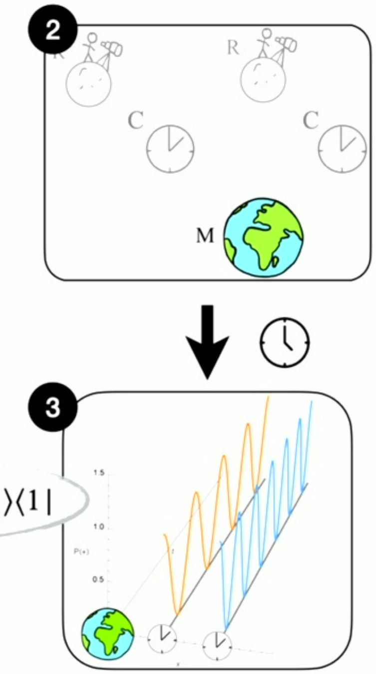
$$\hat{\Omega} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

proper time

$$\tau^{(i)}(t) = t\left(1 + \frac{V(x_C - x_M^{(i)})}{c^2}\right)$$

time evolution of the clock

$$|s(\tau_0+\tau^{(i)})\rangle_{C_{int}} = e^{-\frac{i}{\hbar}\hat{\Omega}\tau^{(i)}}|s(\tau_0)\rangle_{C_{int}}$$



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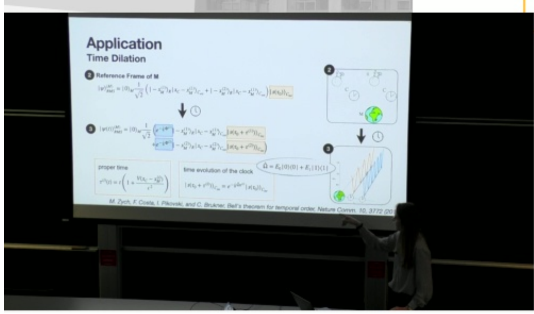


Application

Time Dilation

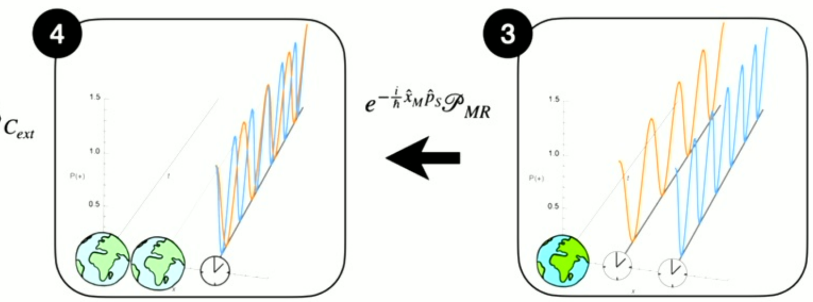
3 Reference Frame of M

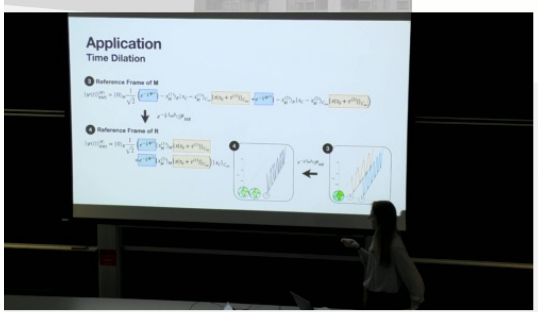
$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}} \left(e^{-\frac{i}{\hbar}\Phi^{(1)}} |-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} |s(\tau_0 + \tau^{(1)})\rangle_{C_{int}} + e^{-\frac{i}{\hbar}\Phi^{(2)}} |-x_M^{(2)}\rangle_R |x_C - x_M^{(2)}\rangle_{C_{ext}} |s(\tau_0 + \tau^{(2)})\rangle_{C_{int}} \right)$$

$$\downarrow \quad e^{-\frac{i}{\hbar}\hat{x}_M \hat{p}_S} \mathcal{P}_{MR}$$

4 Reference Frame of R

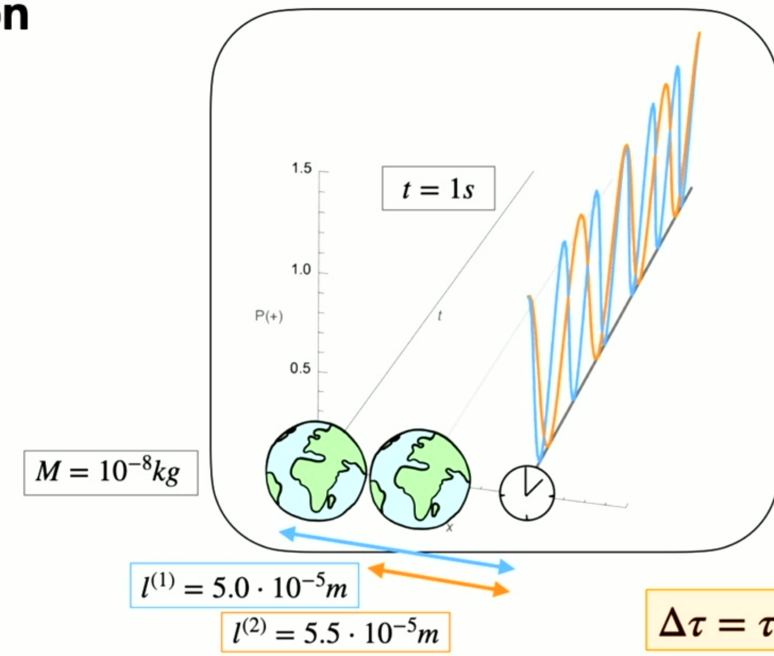
$$|\psi(t)\rangle^{(R)}_{RMS} = |0\rangle_R \frac{1}{\sqrt{2}} \left(e^{-\frac{i}{\hbar}\Phi^{(1)}} |x_M^{(1)}\rangle_M |s(\tau_0 + \tau^{(1)})\rangle_{C_{int}} + e^{-\frac{i}{\hbar}\Phi^{(2)}} |x_M^{(2)}\rangle_M |s(\tau_0 + \tau^{(1)})\rangle_{C_{int}} \right) |x_C\rangle_{C_{ext}}$$





Application

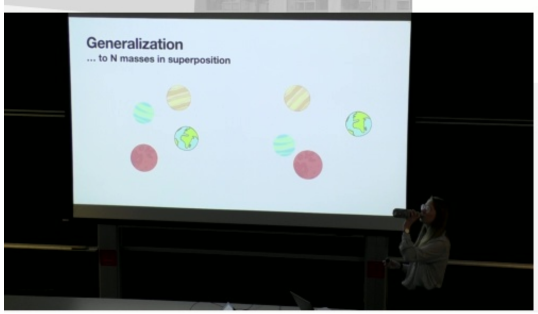
Time Dilation



$$\Delta\tau = \tau^{(2)} - \tau^{(1)} \approx 10^{-32}s$$

Generalization

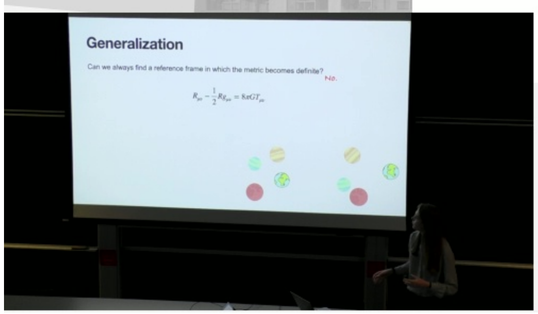
… to N masses in superposition



Generalization

Can we always find a reference frame in which the metric becomes definite? No.

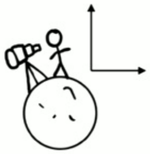
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$



Generalization

Can we always find a reference frame in which the metric becomes definite? No.

$$R_{\mu\nu} \ldots g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



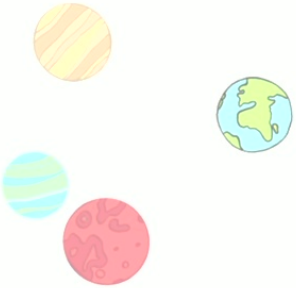
Generalization

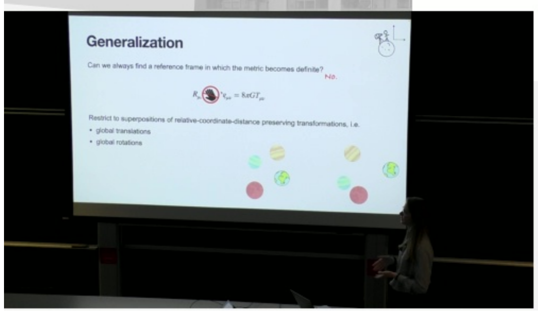
Can we always find a reference frame in which the metric becomes definite? No.

R_{μ}  $g_{\mu\nu} = 8\pi G T_{\mu\nu}$

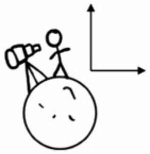
Restrict to superpositions of relative-coordinate-distance preserving transformations, i.e.

- global translations
- global rotations





Generalization



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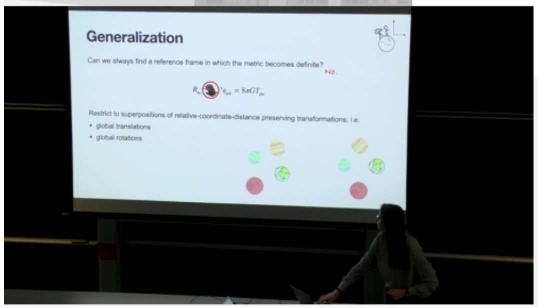
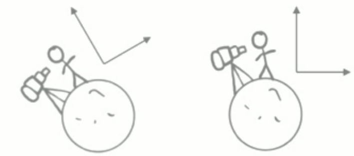


Generalization



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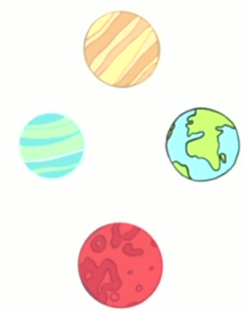
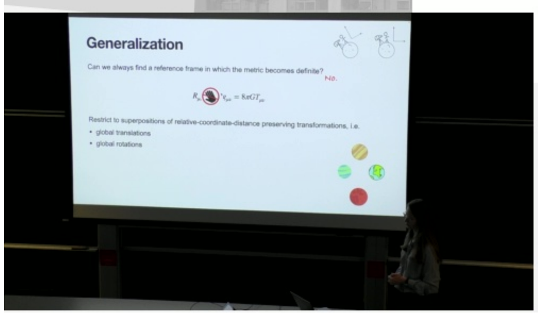
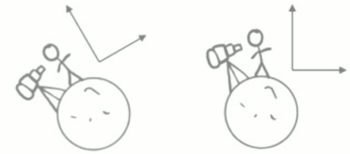




Generalization



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Restrict to superpositions of relative-coordinate-distance preserving transformations, i.e.

- global translations
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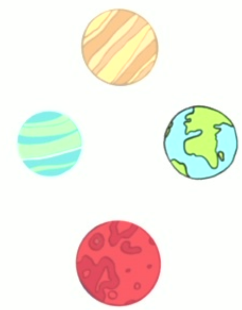
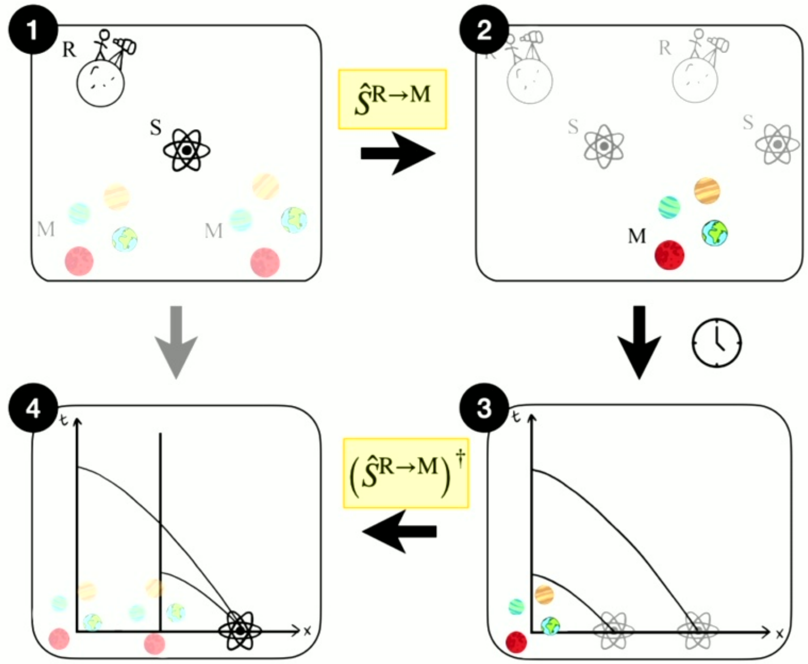
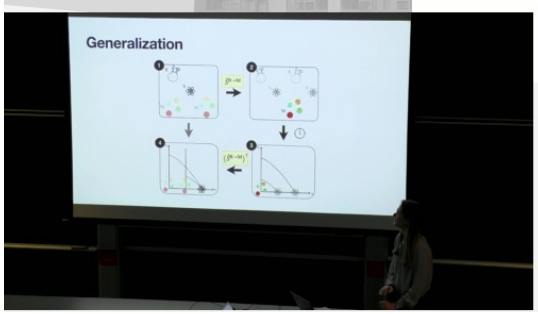
Does not limit us to trivial situations as the presence of other objects breaks the symmetry.



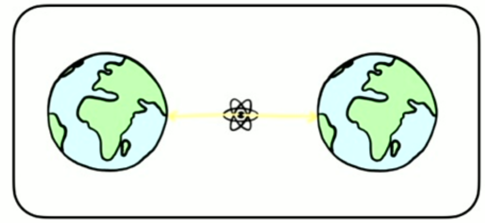
Generalization

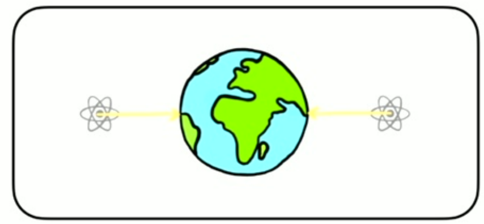


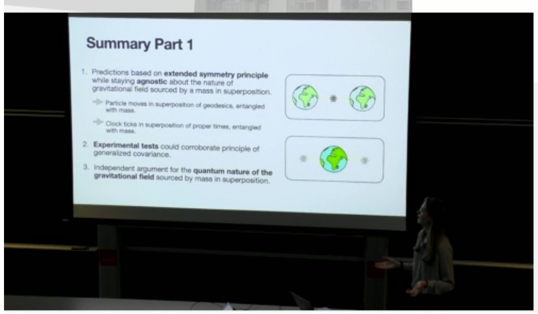


Summary Part 1

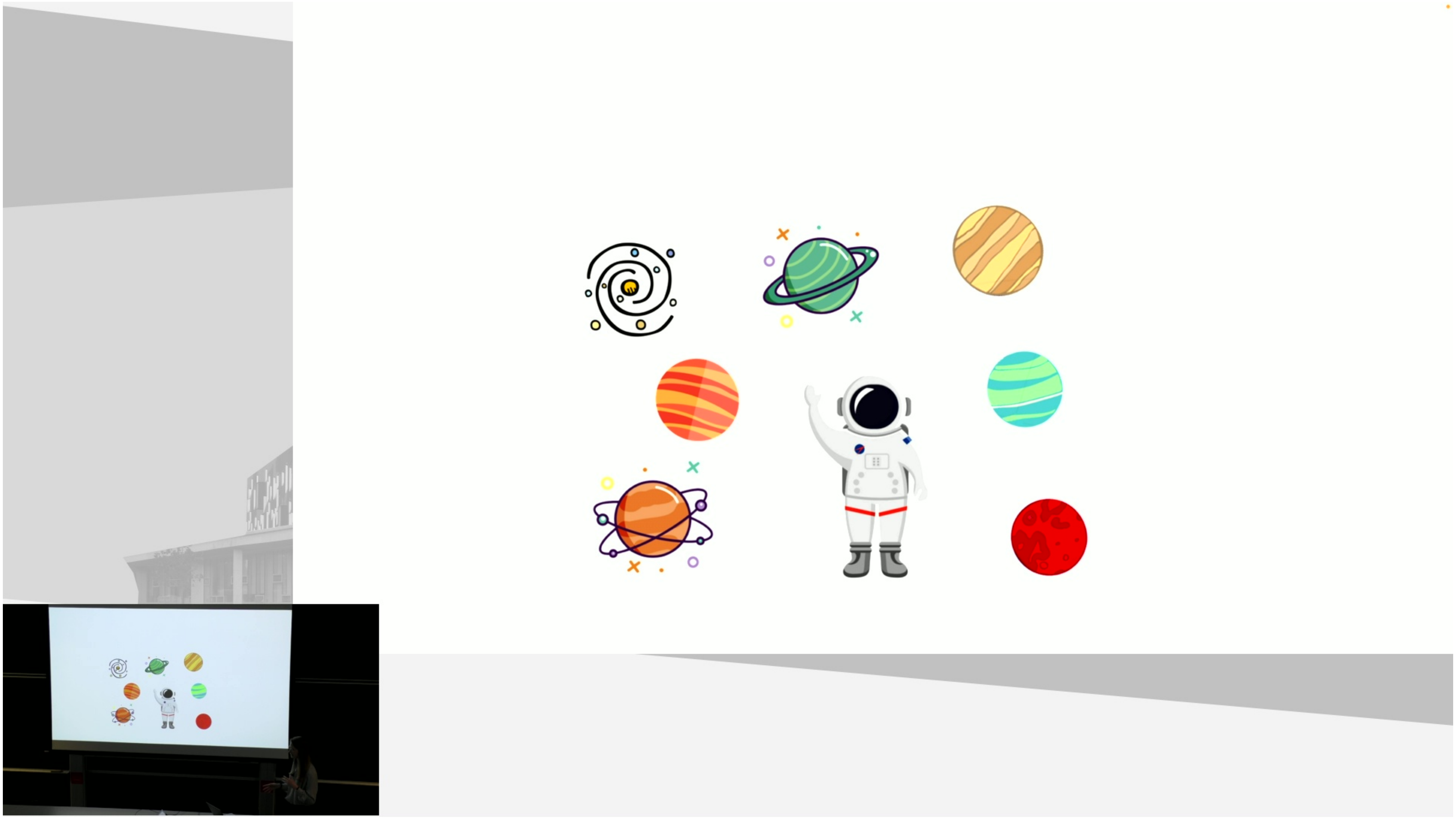
1. Predictions based on **extended symmetry principle** while staying **agnostic** about the nature of gravitational field sourced by a mass in superposition.
   - Particle moves in superposition of geodesics, entangled with mass.
   - Clock ticks in superposition of proper times, entangled with mass.
2. **Experimental tests** could corroborate principle of generalized covariance.
3. Independent argument for the **quantum nature of the gravitational field** sourced by mass in superposition.

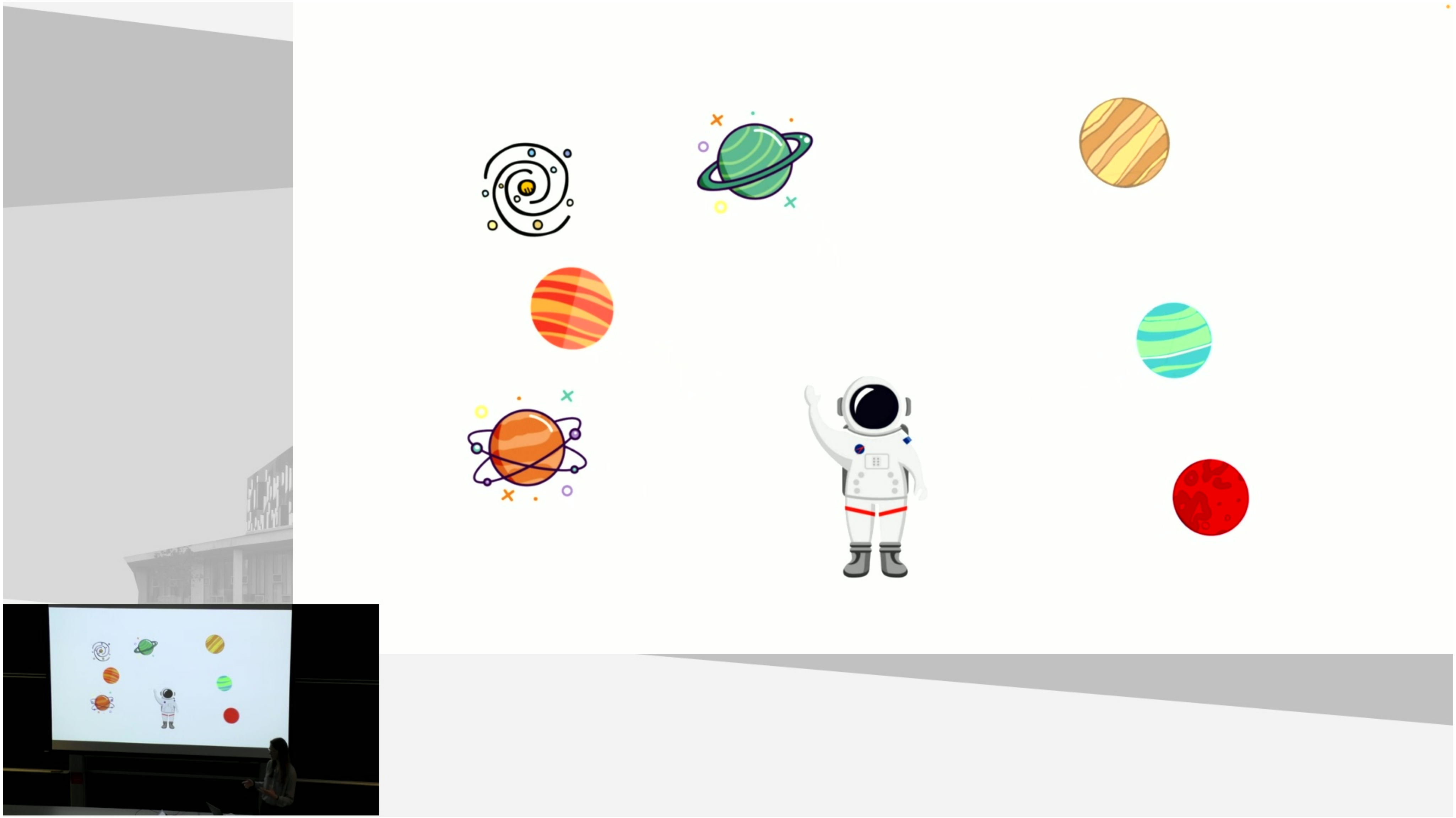


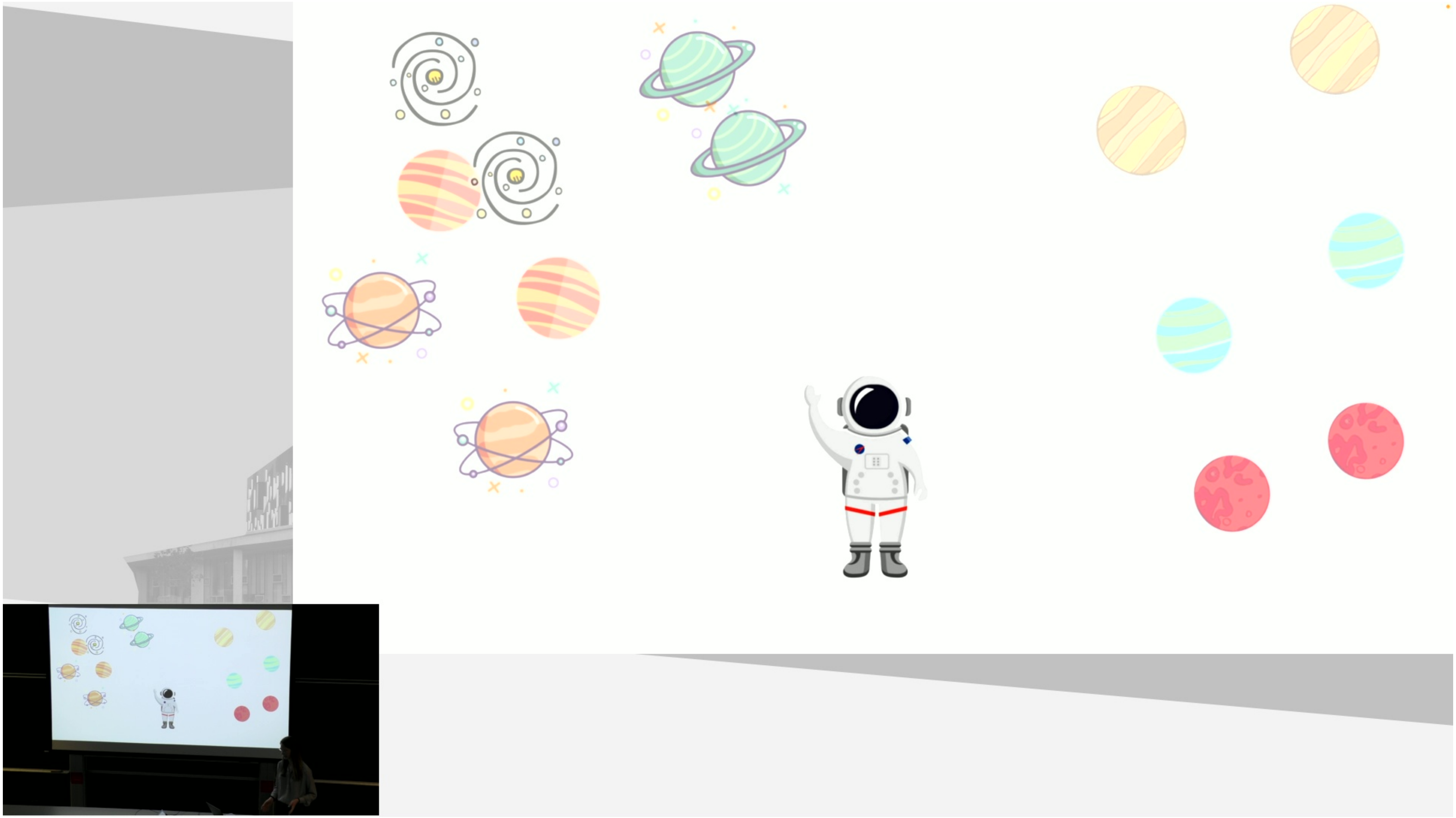




Extended symmetries of KG field in superposition of spacetimes







Motivation

Build intuition and techniques to deal with “true” **quantum metric**, including **fuzziness of spacetime**

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Build intuition and techniques to deal with “true” **quantum metric**, including **fuzziness of spacetime**

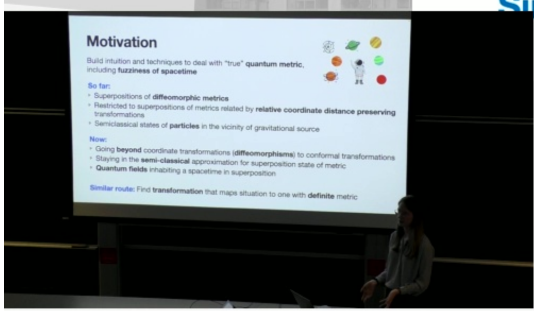
So far:

- Superpositions of **diffeomorphic metrics**
- Restricted to superpositions of metrics related by **relative coordinate distance preserving** transformations
- Semiclassical states of **particles** in the vicinity of gravitational source

Now:

- Going **beyond** coordinate transformations (**diffeomorphisms**) to conformal transformations
- Staying in the **semi-classical** approximation for superposition state of metric
- **Quantum fields** inhabiting a spacetime in superposition

Similar route: Find **transformation** that maps situation to one with **definite** metric



Symmetries of the KG field…
… in an expanding universe

Take a spatially flat ($k = 0$) FLRW spacetime:

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

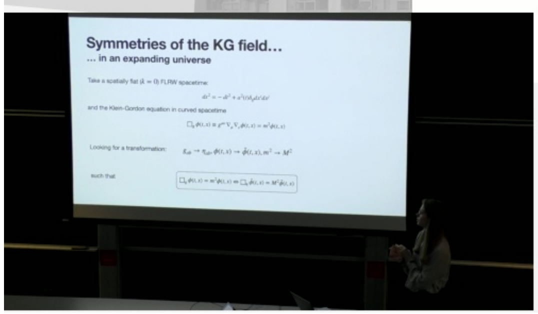
and the Klein-Gordon equation in curved spacetime

$$\Box_g \phi(t,x) \equiv g^{\mu\nu}\nabla_\mu \nabla_\nu \phi(t,x) = m^2\phi(t,x)$$

Looking for a transformation: $g_{ab} \to \eta_{ab}, \phi(t,x) \to \tilde{\phi}(t,x), m^2 \to M^2$

such that

$$\boxed{\Box_g \phi(t,x) = m^2\phi(t,x) \Leftrightarrow \Box_\eta \tilde{\phi}(t,x) = M^2\tilde{\phi}(t,x)}$$



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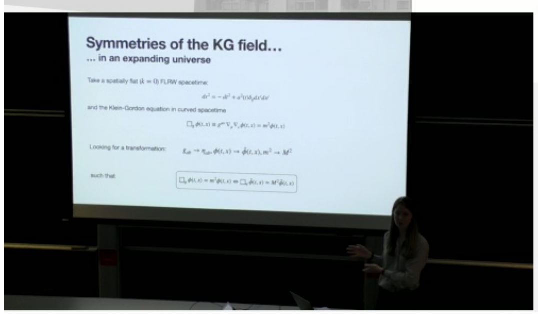


Symmetries of the KG field…

… in an expanding universe

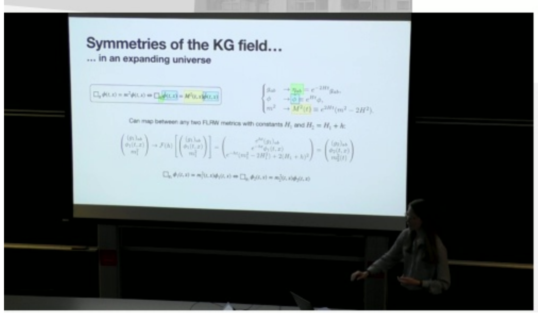
$$\Box_g \phi(t,x) = m^2\phi(t,x) \Leftrightarrow \Box_\eta \tilde{\phi}(t,x) = M^2(t,x)\tilde{\phi}(t,x)$$

$$\begin{cases} g_{ab} & \to \eta_{ab} = e^{-2Ht} g_{ab}, \\ \phi & \to \tilde{\phi} \equiv e^{Ht}\phi, \\ m^2 & \to M^2(t) \equiv e^{2Ht}(m^2 - 2H^2). \end{cases}$$

Can map between any two FLRW metrics with constants H_1 and $H_2 = H_1 + h$:

$$\begin{pmatrix} (g_1)_{ab} \\ \phi_1(t,x) \\ m_1^2 \end{pmatrix} \to \mathcal{F}(h)\left[\begin{pmatrix} (g_1)_{ab} \\ \phi_1(t,x) \\ m_1^2 \end{pmatrix}\right] = \begin{pmatrix} e^{ht}(g_1)_{ab} \\ e^{-ht}\phi_1(t,x) \\ e^{-ht}(m_1^2 - 2H_1^2) + 2(H_1+h)^2 \end{pmatrix} = \begin{pmatrix} (g_2)_{ab} \\ \phi_2(t,x) \\ m_2^2(t) \end{pmatrix}$$

$$\Box_{g_1}\phi_1(t,x) = m_1^2(t,x)\phi_1(t,x) \Leftrightarrow \Box_{g_2}\phi_2(t,x) = m_2^2(t,x)\phi_2(t,x)$$



Symmetries of the KG field…

… in an expanding universe

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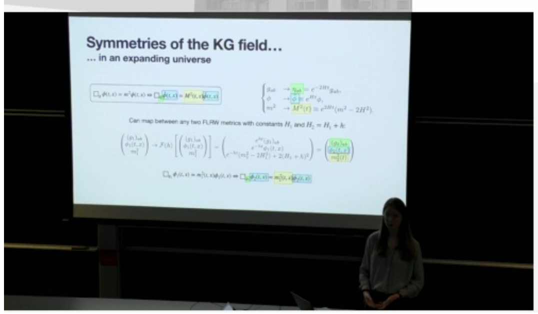

Symmetries of the KG field…

… in an expanding universe

$$\Box_g \phi(t,x) = m^2\phi(t,x) \Leftrightarrow \Box_\eta \tilde{\phi}(t,x) = M^2(t,x)\tilde{\phi}(t,x)$$

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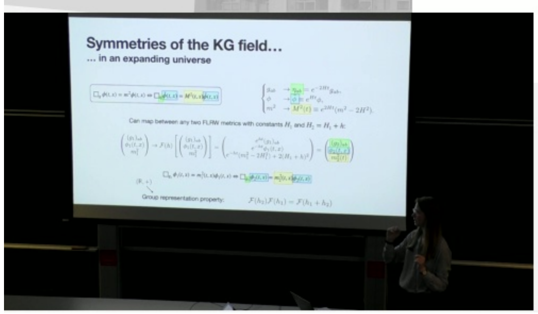
Can map between any two FLRW metrics with constants H_1 and $H_2 = H_1 + h$:

$$\begin{pmatrix} (g_1)_{ab} \\ \phi_1(t,x) \\ m_1^2 \end{pmatrix} \to \mathcal{F}(h)\left[\begin{pmatrix} (g_1)_{ab} \\ \phi_1(t,x) \\ m_1^2 \end{pmatrix}\right] = \begin{pmatrix} e^{ht}(g_1)_{ab} \\ e^{-ht}\phi_1(t,x) \\ e^{-ht}(m_1^2 - 2H_1^2) + 2(H_1+h)^2 \end{pmatrix} = \begin{pmatrix} (g_2)_{ab} \\ \phi_2(t,x) \\ m_2^2(t) \end{pmatrix}$$

$$\Box_{g_1} \phi_1(t,x) = m_1^2(t,x)\phi_1(t,x) \Leftrightarrow \Box_{g_2} \phi_2(t,x) = m_2^2(t,x)\phi_2(t,x)$$

$(\mathbb{R}, +)$

Group representation property: $\mathcal{F}(h_2)\mathcal{F}(h_1) = \mathcal{F}(h_1 + h_2)$



Symmetries of the KG field…

… for conformally related spacetimes

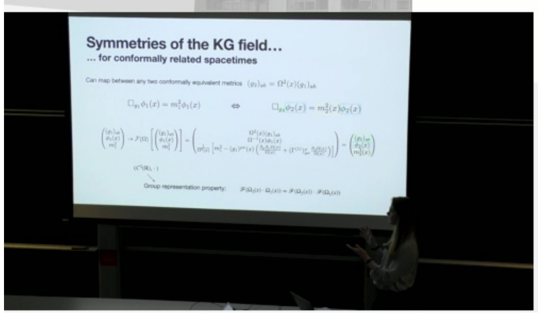
Can map between any two conformally equivalent metrics $(g_2)_{ab} = \Omega^2(x)(g_1)_{ab}$

$$\Box_{g_1}\phi_1(x) = m_1^2\phi_1(x) \qquad \Leftrightarrow \qquad \Box_{g_2}\phi_2(x) = m_2^2(x)\phi_2(x)$$

$$\begin{pmatrix}(g_1)_{ab}\\ \phi_1(x)\\ m_1^2\end{pmatrix} \to \mathcal{F}(\Omega)\left[\begin{pmatrix}(g_1)_{ab}\\ \phi_1(x)\\ m_1^2\end{pmatrix}\right] = \begin{pmatrix}\Omega^2(x)(g_1)_{ab}\\ \Omega^{-1}(x)\phi_1(x)\\ \frac{1}{\Omega^2(x)}\left[m_1^2 - (g_1)^{\mu\nu}(x)\left(\frac{\partial_\mu\partial_\nu\Omega(x)}{\Omega(x)} + (\Gamma^{(1)})^\rho_{\mu\nu}\frac{\partial_\rho\Omega(x)}{\Omega(x)}\right)\right]\end{pmatrix} = \begin{pmatrix}(g_2)_{ab}\\ \phi_2(x)\\ m_2^2(x)\end{pmatrix}$$

$(C^2(\mathbb{R}), \cdot\,)$

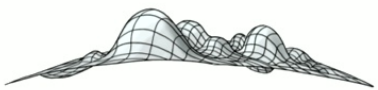
Group representation property: $\mathscr{F}(\Omega_2(x)\cdot\Omega_1(x)) = \mathscr{F}(\Omega_2(x))\cdot\mathscr{F}(\Omega_1(x))$



Symmetries of the quantized KG field…

… in a conformally flat universe

Curved spacetime



inner product: $(\phi_1, \phi_2)_g$

$$\phi(x) = \int d^3k \left(\overbrace{(u_k, \phi)_g}^{a_k} u_k + \overbrace{(u_k^*, \phi)_g}^{a_k^*} u_k^* \right)$$

$$\mathcal{Q} \Big\downarrow \begin{matrix} a_k \to \hat{a}_k \\ a_k^* \to \hat{a}_k^\dagger \end{matrix}$$

$$\hat{\phi}(x) = \int d^3k \hat{a}_k u_k + \hat{a}_k^\dagger u_k^*$$

vacuum state: $\hat{a}_k |0\rangle_g = 0$

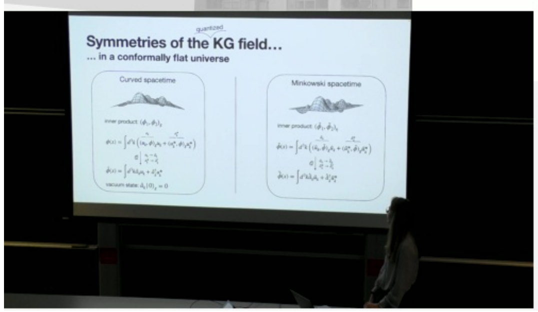
Minkowski spacetime

inner product: $(\tilde{\phi}_1, \tilde{\phi}_2)_\eta$

$$\tilde{\phi}(x) = \int d^3k \left(\overbrace{(\tilde{u}_k, \tilde{\phi})_g}^{\tilde{a}_k} \tilde{u}_k + \overbrace{(\tilde{u}_k^*, \tilde{\phi})_g}^{\tilde{a}_k^*} \tilde{u}_k^* \right)$$

$$\mathcal{Q} \Big\downarrow \begin{matrix} \tilde{a}_k \to \hat{\tilde{a}}_k \\ \tilde{a}_k^* \to \hat{\tilde{a}}_k^\dagger \end{matrix}$$

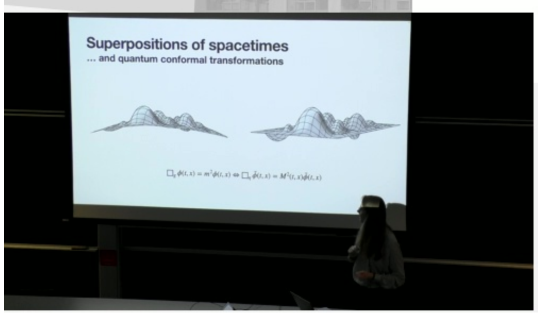
$$\hat{\tilde{\phi}}(x) = \int d^3k \hat{\tilde{a}}_k \tilde{u}_k + \hat{\tilde{a}}_k^\dagger \tilde{u}_k^*$$



Superpositions of spacetimes

… and quantum conformal transformations

$$\Box_g \phi(t,x) = m^2\phi(t,x) \Leftrightarrow \Box_\eta \tilde{\phi}(t,x) = M^2(t,x)\tilde{\phi}(t,x)$$



Superpositions of spacetimes

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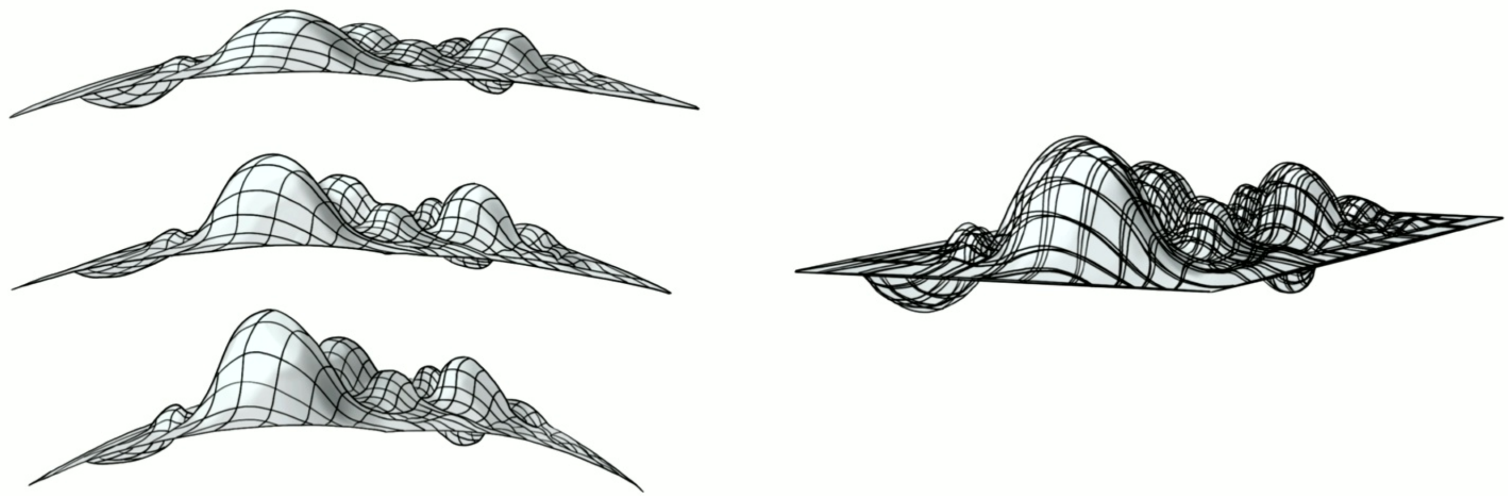
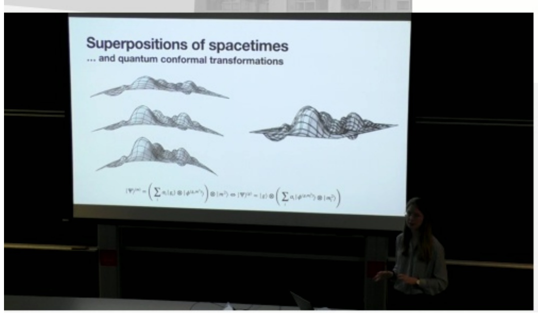


$$|\Psi\rangle^{(m)} = \left(\sum_i \alpha_i |g_i\rangle \otimes |\phi^{(g_i,m^2)}\rangle \right) \otimes |m^2\rangle \Leftrightarrow |\Psi\rangle^{(g)} = |g\rangle \otimes \left(\sum_i \alpha_i |\phi^{(g,m_i^2)}\rangle \otimes |m_i^2\rangle \right)$$



Superpositions of spacetimes

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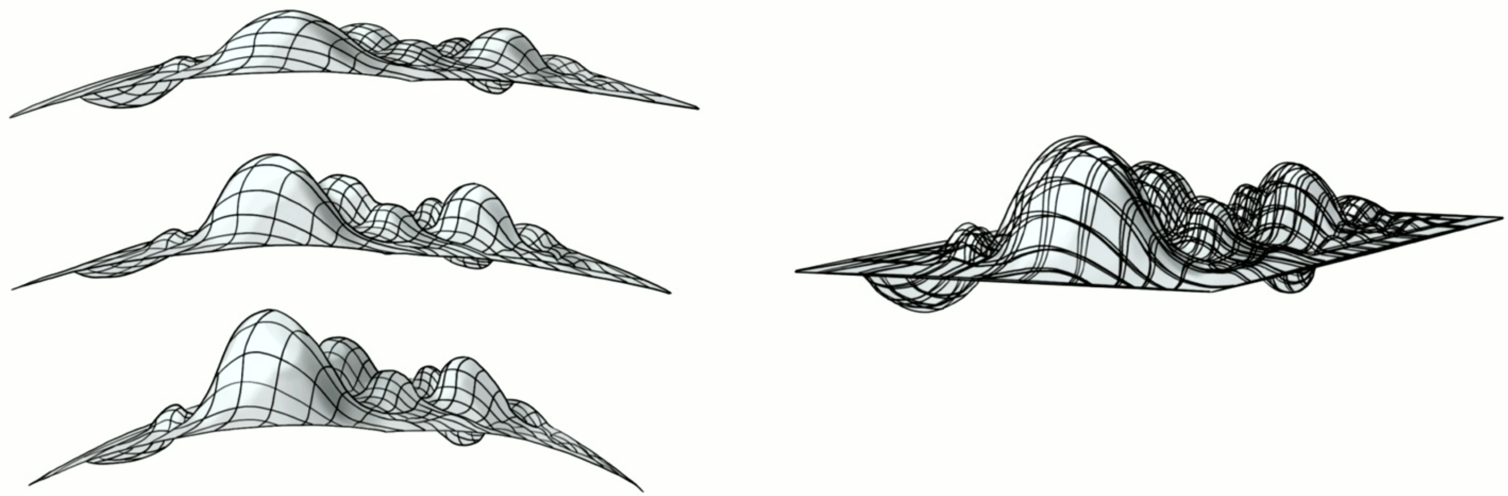

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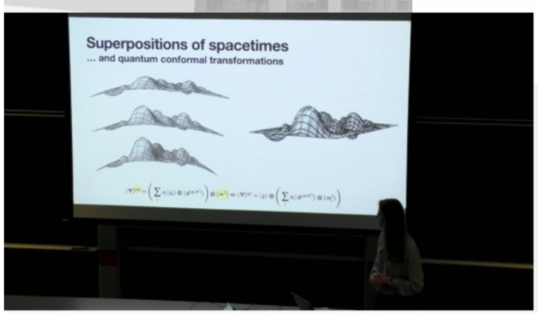

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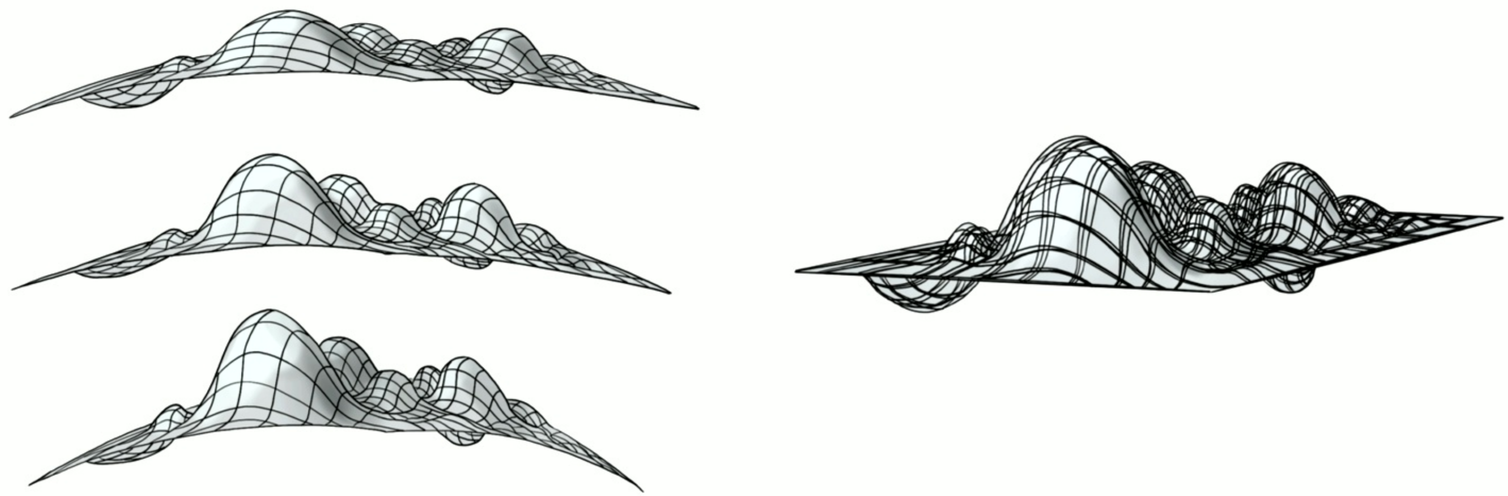

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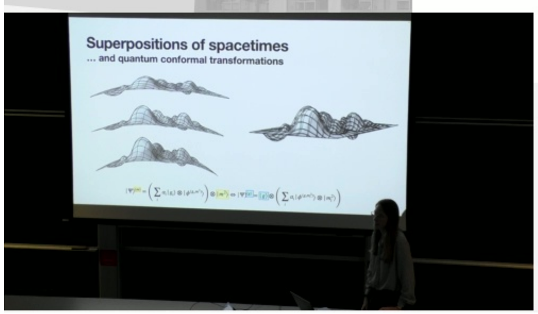

Superpositions of spacetimes

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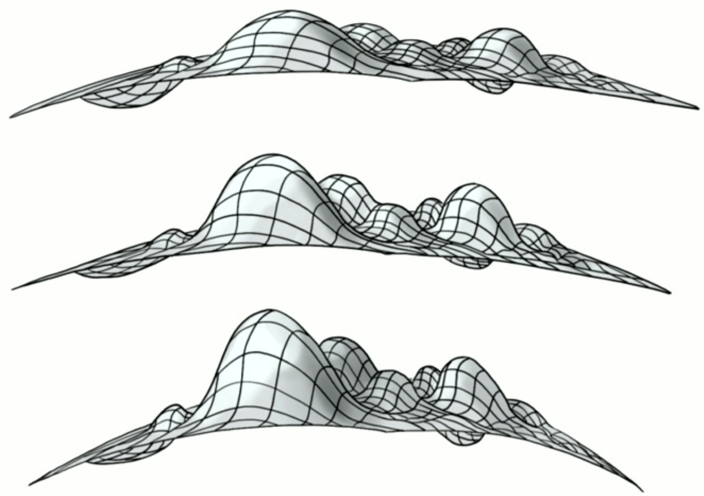
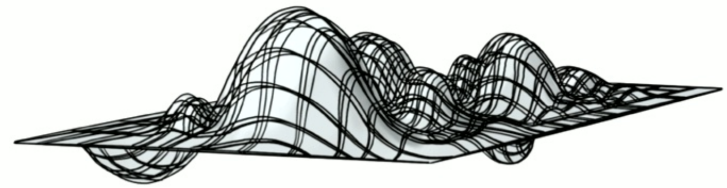
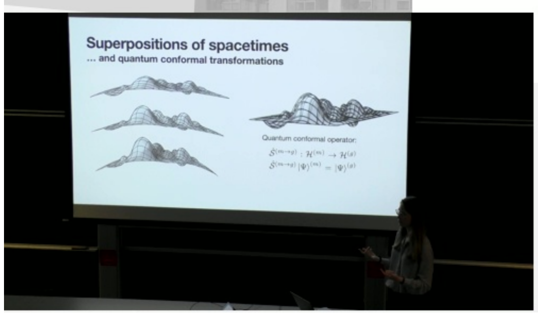




Quantum conformal operator:

$$\hat{\mathcal{S}}^{(m\to g)} : \mathcal{H}^{(m)} \to \mathcal{H}^{(g)}$$

$$\hat{\mathcal{S}}^{(m\to g)} \left|\Psi\right\rangle^{(m)} = \left|\Psi\right\rangle^{(g)}$$



Superpositions of spacetimes

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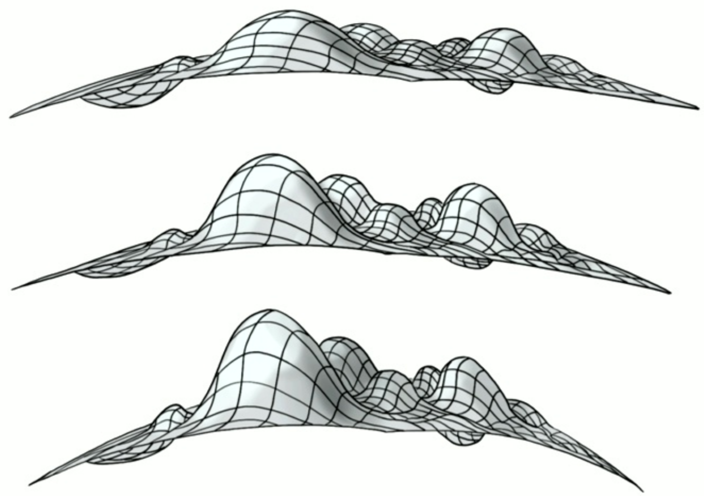

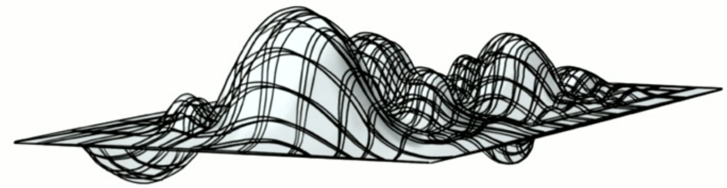

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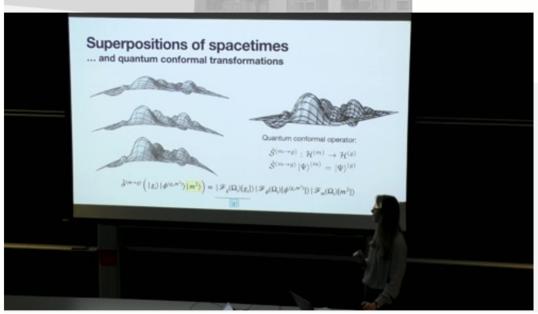
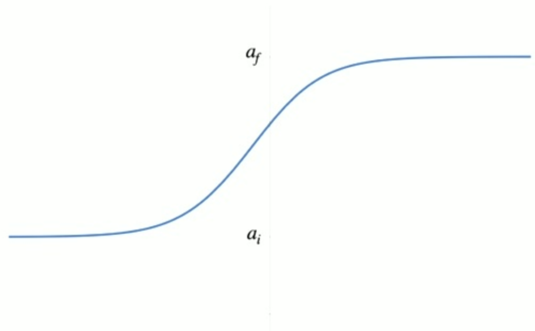
$$\hat{\mathcal{S}}^{(m\to g)}\left(|g_i\rangle|\phi^{(g_i,m^2)}\rangle|m^2\rangle\right) = \underbrace{|\mathscr{F}_g(\Omega_i)[g_i]\rangle}_{|g\rangle}|\mathscr{F}_\phi(\Omega_i)[\phi^{(g_i,m^2)}]\rangle|\mathscr{F}_m(\Omega_i)[m^2]\rangle$$



Cosmological particle production

… in modified Minkowski?

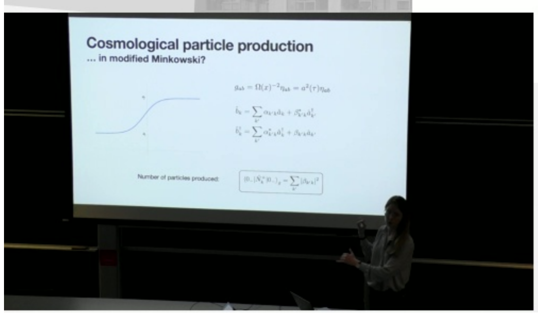


$$g_{ab} = \Omega(x)^{-2}\eta_{ab} = a^2(\tau)\eta_{ab}$$

$$\hat{b}_k = \sum_{k'} \alpha_{k'k}\hat{a}_k + \beta^*_{k'k}\hat{a}^\dagger_{k'}$$

$$\hat{b}^\dagger_k = \sum_{k'} \alpha^*_{k'k}\hat{a}^\dagger_k + \beta_{k'k}\hat{a}_{k'}$$

Number of particles produced: $$\langle 0_- | \hat{N}^+_k | 0_- \rangle_g = \sum_{k'} |\beta_{k'k}|^2$$



Cosmological particle production

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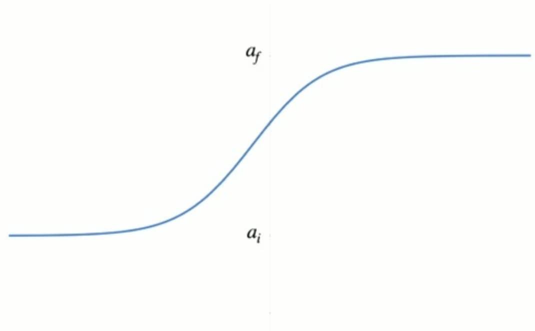
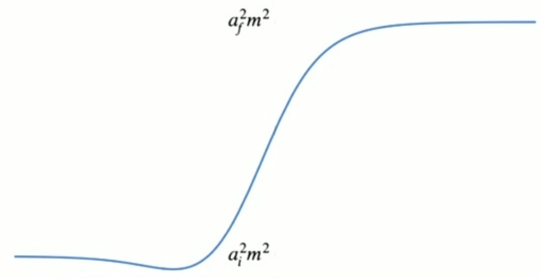
$$g_{ab} = \Omega(x)^{-2}\eta_{ab} = a^2(\tau)\eta_{ab}$$

$$M^2(\tau) = a^2(\tau)m^2 - \frac{\ddot{a}(\tau)}{a(\tau)}$$

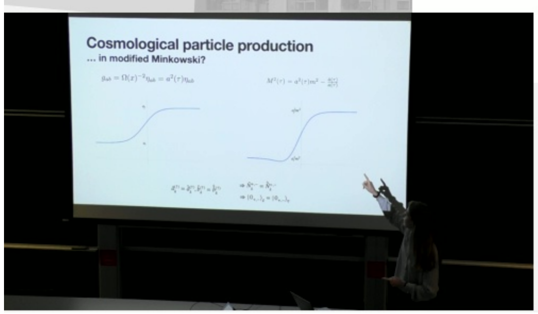




$$\hat{a}_k^{(\dagger)} = \hat{\tilde{a}}_k^{(\dagger)}, \hat{b}_k^{(\dagger)} = \hat{\tilde{b}}_k^{(\dagger)}$$

$$\Rightarrow \hat{N}_k^{+,-} = \hat{\tilde{N}}_k^{+,-}$$

$$\Rightarrow |0_{+,-}\rangle_g = |0_{+,-}\rangle_\eta$$



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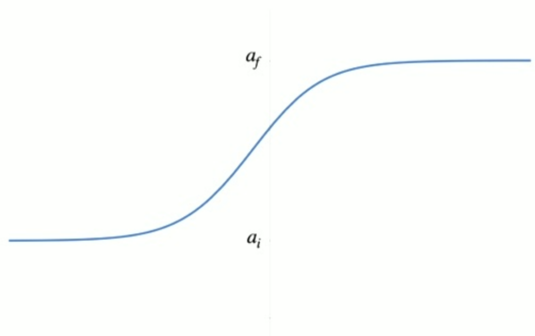
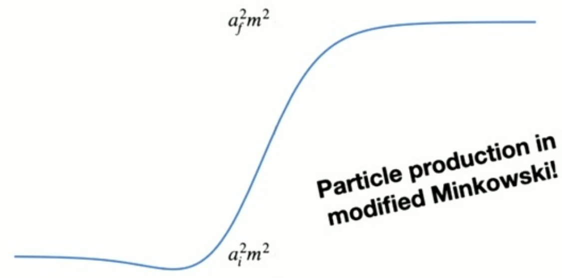
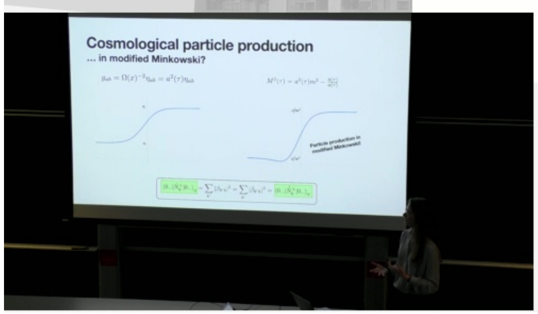




$$\langle 0_- | \hat{N}_k^+ | 0_- \rangle_g = \sum_{k'} |\beta_{k'k}|^2 = \sum_{k'} |\tilde{\beta}_{k'k}|^2 = \langle 0_- | \hat{\tilde{N}}_k^+ | 0_- \rangle_\eta$$

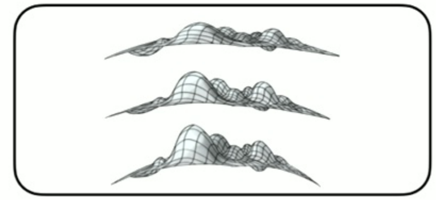


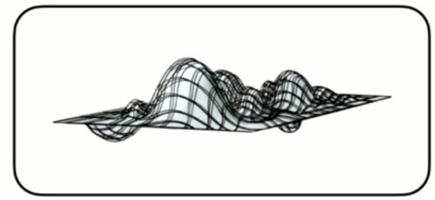
Summary Part 2

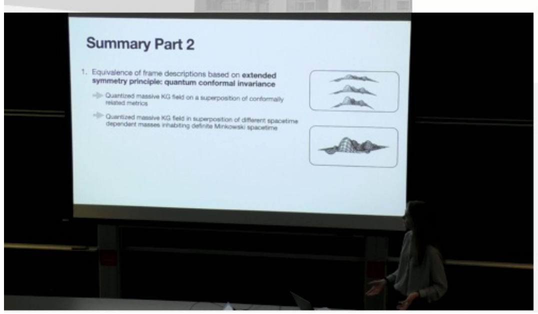
1. Equivalence of frame descriptions based on **extended symmetry principle: quantum conformal invariance**

   - Quantized massive KG field on a superposition of conformally related metrics

   - Quantized massive KG field in superposition of different spacetime dependent masses inhabiting definite Minkowski spacetime

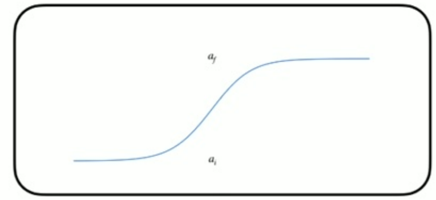


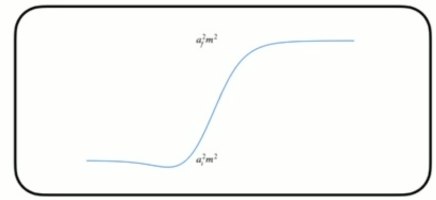


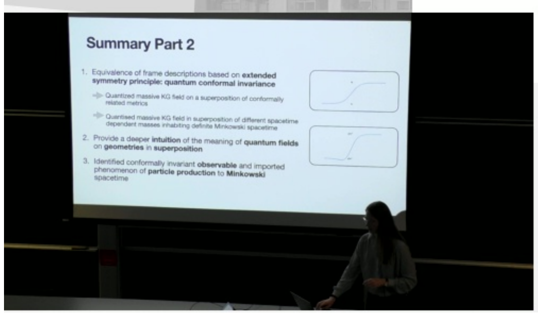


Summary Part 2

1. Equivalence of frame descriptions based on **extended symmetry principle: quantum conformal invariance**
   - Quantized massive KG field on a superposition of conformally related metrics
   - Quantised massive KG field in superposition of different spacetime dependent masses inhabiting definite Minkowski spacetime
2. Provide a deeper **intuition** of the meaning of **quantum fields** on **geometries** in **superposition**
3. Identified conformally invariant **observable** and imported phenomenon of **particle production** to **Minkowski** spacetime







Extended symmetry principles

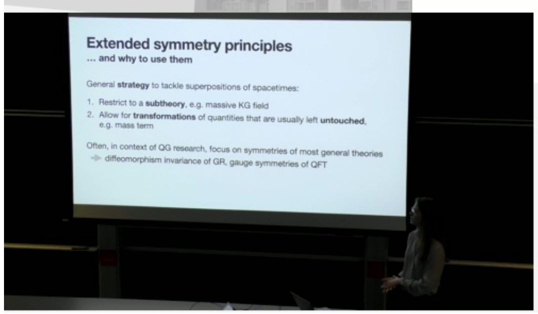
… and why to use them

General **strategy** to tackle superpositions of spacetimes:

1. Restrict to a **subtheory**, e.g. massive KG field
2. Allow for **transformations** of quantities that are usually left **untouched**, e.g. mass term

Often, in context of QG research, focus on symmetries of most general theories

➡ diffeomorphism invariance of GR, gauge symmetries of QFT



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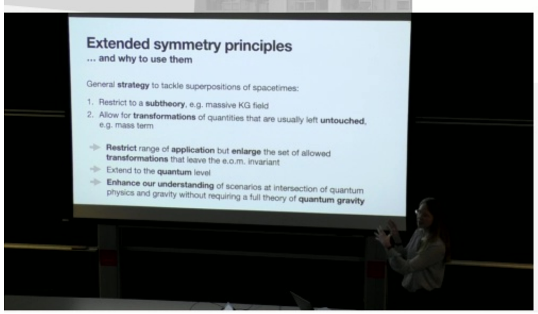
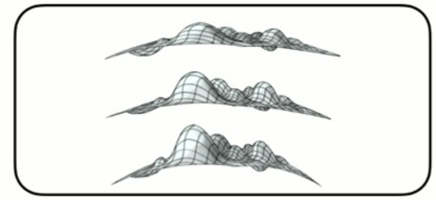
- ➡ **Restrict** range of **application** but **enlarge** the set of allowed **transformations** that leave the e.o.m. invariant
- ➡ Extend to the **quantum** level
- ➡ **Enhance our understanding** of scenarios at intersection of quantum physics and gravity without requiring a full theory of **quantum gravity**

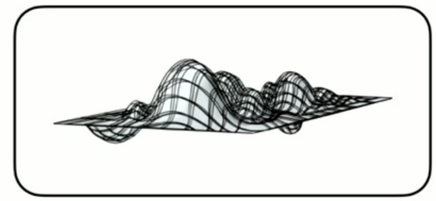


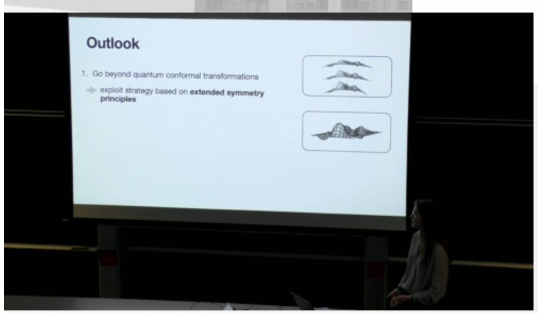
Outlook

1. Go beyond quantum conformal transformations

   ➡ exploit strategy based on **extended symmetry principles**







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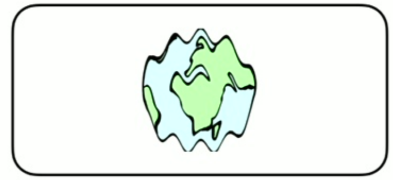
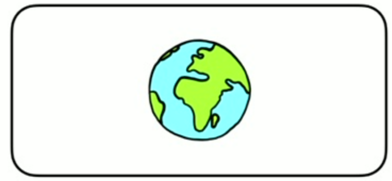


2. Take into account **quantum fuzziness** of spacetime

   ➡ Go **beyond semi-classical approximation** (for massive objects and states of metric)



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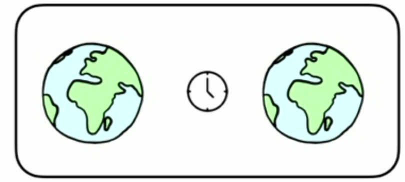

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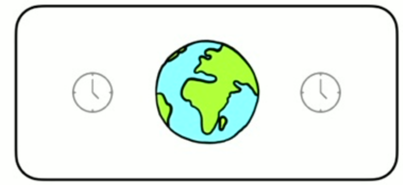
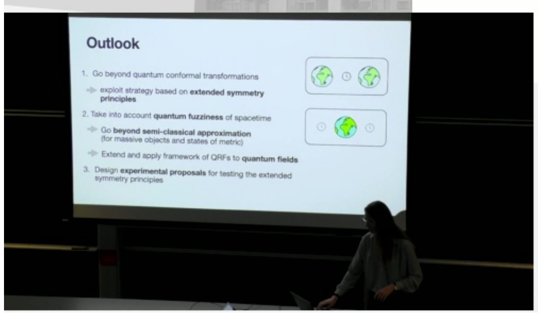
   ➔ Extend and apply framework of QRFs to **quantum fields**



3. Design **experimental proposals** for testing the extended symmetry principles



Outlook

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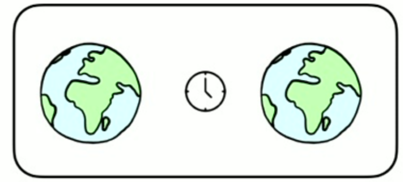

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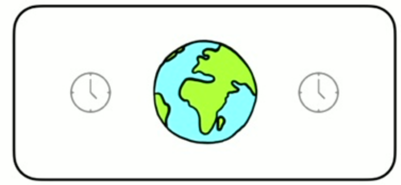

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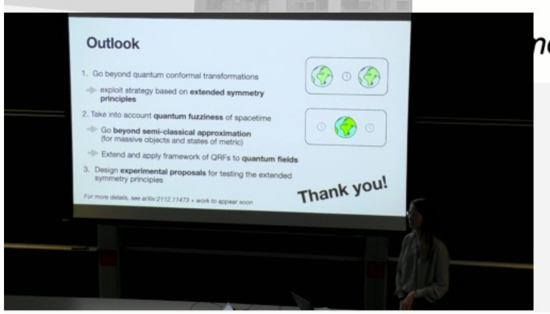
Thank you!



ore details, see arXiv:2112.11473 + work to appear soon