Title: A coherent state description of black holes and the late Universe

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Series: Quantum Gravity

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Abstract: Inspired by the corpuscular model of gravity, I will discuss a simplified quantum description of neutral and electrically charged spherically symmetric black holes given by coherent states of toy gravitons. From such states I will derive effective geometries devoid of curvature singularities (replaced by integrable ones) and Cauchy horizons. Lastly, I will present a similar analysis for the Schwarzschild-de Sitter geometry, used as a model for the late Universe, showing that the "reaction" of the de Sitter background to the presence of localised matter sources induces a modified Newtonian dynamics at galactic scales and different values measured for the present Hubble parameter.

Zoom Link: https://pitp.zoom.us/j/93087108672?pwd=ekQrLysrcjRwVWFSZ2J5bndteXBtdz09

A coherent state description of black holes and the late Universe

Andrea Giusti

A "corpuscular" source of inspiration

$$\left(c=1\,,\ G_{\rm N}=\frac{\ell_{\rm P}}{m_{\rm P}}\,,\ \hbar=m_{\rm P}\ell_{\rm P}\right)$$

Corpuscular gravity

Black hole $\simeq$ self-sustained loosely bound state of $N \gg 1$ soft off-shell gravitons

Scaling Laws

$$N \simeq \frac{M^2}{m_{\rm P}^2}\,,\quad \langle k\rangle \simeq \frac{\hbar}{R_{\rm H}}\,,\quad \alpha \simeq \frac{\langle k\rangle^2}{m_{\rm P}^2} \simeq \frac{1}{N}$$

Eff. non-dim. self-coupling

Collective coupling: $\alpha N \simeq 1$ ⟶ system stuck at the critical point

Quantum depletion

graviton-graviton

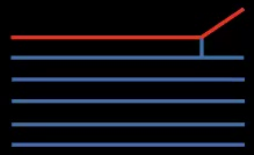
Hawking radiation = graviton depletion

$$\dot{N} \simeq -\frac{1}{\sqrt{N}\ell_{\rm P}}$$

$$\downarrow$$

$$\dot{M} \simeq -\frac{T_{\rm H}^2}{\hbar}\,,\ T_{\rm H} \simeq \langle k\rangle$$

Thermality ~ softness + combinatorics



"matter"-graviton

1/N quantum hair

$$\dot{N}_{\rm m} \simeq \frac{N_{\rm m}}{N}\dot{N}$$

1/N-suppressed

[G. Dvali, C. Gomez, et. al., seminal works]
[A review: A. Giusti, IJGMMP 16 (2019) 1930001]

Assumptions and Objectives

Spacetimes: static and spherically symmetric (+ $g_{tt}\, g_{rr} = -1$)

$$\mathrm{d}s^2 = -(1+2V)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{(1+2V)} + r^2\,\mathrm{d}\Omega^2\,, \quad V = V(r)\,, \quad \mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2$$

Einstein-Hilbert + Matter to linear order: static source + non-relativistic speeds

$$\mathcal{S} = \int \mathrm{d}^4x \left[-\frac{R}{16\pi G_{\mathrm{N}}} + \mathcal{L}_{\mathrm{M}} \right]$$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{S}_{\mathrm{M}}}{\delta g^{\mu\nu}} = \rho(r) \delta^0{}_\mu \delta^0{}_\nu \quad \Rightarrow \quad L \equiv \int \mathrm{d}^3x\, \mathcal{L} \simeq \int \mathrm{d}^3x \left(-\frac{V \triangle V}{8\pi G_{\mathrm{N}}} + \rho\, V \right)$$

+ de Donder gauge

[R. Casadio, A. Giugno, A. Giusti, M. Lenzi, Phys.Rev.D 96 (2017) 044010]

Effective Lagrangian

$$L = \int \mathrm{d}^3x \left(\frac{1}{2}\,\Phi\Box\Phi - J\,\Phi \right) \qquad \Phi(r) \equiv \frac{V(r)}{\sqrt{G_\mathrm{N}}}\,, \quad J(r) \equiv 4\pi\sqrt{G_\mathrm{N}}\,\rho(r)$$

(canonically normalized)

What then? Quantize and construct a coherent state $|g\rangle$ such that

$$\sqrt{G_\mathrm{N}}\,\langle g|\widehat{\Phi}|g\rangle = V(r)$$

i.e., the quantum state that "yields" the classical geometry

(at least in the regions accessible to our experiments)

Remark! Φ is not a fundamental d.o.f., it is just convenient representation of the non-perturbative physics of a compact source. ~ mean-field geometry

[R. Casadio, arXiv:2103.00183 [gr-qc]]

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Quantum coherent state for classical configurations

Quantum vacuum: $|0\rangle \equiv$ "system" devoid of any matter or gravitational excitations

Given the static potential $V = V(r)$ $\Longrightarrow$ Coherent state of the massless scalar field Φ
(mean-field potential)

$$\Box\Phi = (-\partial_t^2 + \triangle)\Phi = 0$$

Normal modes: $u_{\boldsymbol{k}}(t,r) = \mathrm{e}^{-\imath k t}\, j_0(k\, r)\,, \quad r := |\boldsymbol{x}|\,, \quad k := |\boldsymbol{k}|$

$$j_0(x) = \frac{\sin x}{x}$$ Spherical Bessel function

$$\widehat{\Phi}(t,r) = \int_0^\infty \frac{k^2\,\mathrm{d}k}{2\,\pi^2}\left(\frac{\hbar}{2\,k}\right)^{\frac{1}{2}}\left[\widehat{a}_{\boldsymbol{k}}\, u_{\boldsymbol{k}}(t,r) + \widehat{a}^\dagger_{\boldsymbol{k}}\, u^*_{\boldsymbol{k}}(t,r)\right]$$

$$\widehat{\Pi}(t,r) = \imath \int_0^\infty \frac{k^2\,\mathrm{d}k}{2\,\pi^2}\left(\frac{\hbar\, k}{2}\right)^{\frac{1}{2}}\left[\widehat{a}_{\boldsymbol{k}}\, u_{\boldsymbol{k}}(t,r) - \widehat{a}^\dagger_{\boldsymbol{k}}\, u^*_{\boldsymbol{k}}(t,r)\right]$$

$\Longrightarrow$

$$\left[\widehat{\Phi}(t,r), \widehat{\Pi}(t,s)\right] = \frac{\imath\,\hbar}{4\,\pi\, r^2}\,\delta(r-s)$$

iff $$\left[\widehat{a}_{\boldsymbol{k}}, \widehat{a}^\dagger_{\boldsymbol{p}}\right] = \frac{2\,\pi^2}{k^2}\,\delta(k-p)$$

[R. Casadio, arXiv:2103.00183 [gr-qc]]
[R. Casadio, A. Giugno, A. Giusti, M. Lenzi, Phys.Rev.D 96 (2017) 044010]

Vacuum $\widehat{a}_{\boldsymbol{k}}\,|0\rangle = 0$, while the Fock space $\mathcal{F}$ is built in the standard way upon $|0\rangle$

$|\text{Classical}\rangle \quad \Longrightarrow \quad$ Natural choice: coherent state $|g\rangle$

$$\widehat{a}_{\boldsymbol{k}}\,|g\rangle = g(\boldsymbol{k})\,\mathrm{e}^{\imath\,\gamma_{\boldsymbol{k}}(t)}\;|g\rangle$$

such that $\sqrt{G_{\mathrm{N}}}\,\langle g|\widehat{\Phi}|g\rangle = V(r)$

$$\langle g|\,\widehat{\Phi}(t,r)\,|g\rangle = \int_0^\infty \frac{k^2\,\mathrm{d}k}{2\,\pi^2}\left(\frac{2\,\hbar}{k}\right)^{\frac{1}{2}} g(k)\,\cos[\gamma_k(t) - k\,t]\,j_0(k\,r) \quad \Longrightarrow \quad \text{Staticity: } \gamma_k(t) = k\,t$$

$$V(r) = \int_0^\infty \frac{k^2\,\mathrm{d}k}{2\,\pi^2}\,\widetilde{V}(k)\,j_0(k\,r) \quad \Longrightarrow \quad g(k) = \sqrt{\frac{k}{2}\,\frac{\widetilde{V}(k)}{\ell_{\mathrm{P}}}}$$

~ occupation numbers

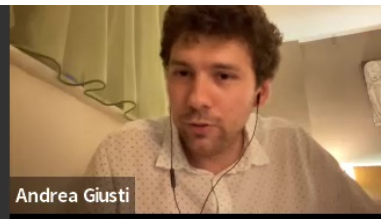
Coherent state: $|g\rangle = e^{-N_{\mathrm{G}}/2}\,\exp\left\{\int_0^\infty \frac{k^2\,\mathrm{d}k}{2\,\pi^2}\,g(k)\,\widehat{a}^\dagger_{\boldsymbol{k}}\right\}|0\rangle$

$$N_{\mathrm{G}} = \int_0^\infty \frac{k^2\,\mathrm{d}k}{2\,\pi^2}\,|g(k)|^2$$

Number of constituents

[R. Casadio, arXiv:2103.00183 [gr-qc]]

[R. Casadio, A. Giugno, A. Giusti, M. Lenzi, Phys.Rev.D 96 (2017) 044010]



Schwarzschild Black Hole

$$\mathrm{d}s^2 = -(1+2V)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{(1+2V)} + r^2\,\mathrm{d}\Omega^2\,, \quad V(r) = -\frac{G_\mathrm{N}\,M}{r}\,, \quad R_\mathrm{H} = 2\,G_\mathrm{N}\,M$$

$$\widetilde{V}(k) = -4\,\pi\,G_\mathrm{N}\,\frac{M}{k^2} \quad \Longrightarrow \quad g(k) = \sqrt{\frac{k}{2}}\,\frac{\widetilde{V}(k)}{\ell_\mathrm{P}} = -\frac{4\,\pi\,M}{\sqrt{2\,k^3}\,m_\mathrm{P}}$$

How many constituents in the coherent state?

$$N_\mathrm{G} = 4\,\frac{M^2}{m_\mathrm{P}^2}\int_0^\infty \frac{\mathrm{d}k}{k}$$

logarithmic divergence both in the IR and UV!

Hence, technically: $\nexists\,|g\rangle \;:\; \langle g|\,\widehat{\Phi}\,|g\rangle = V(r)/\sqrt{G_\mathrm{N}}\,, \forall r > 0$

[R. Casadio, arXiv:2103.00183 [gr-qc]]

This is all well expected!

UV divergence $\Longrightarrow$ Due to the vanishing size of the source!

Regularization: regular quantum matter core of size $R_{\rm s} > 0$

UV cut-off $k_{\rm UV} \sim 1/R_{\rm s}$

IR divergence $\Longrightarrow$ Due to the infinite lifetime of the BH! Schwarzschild is "eternal"

Regularization: account for the necessarily finite lifetime $R_\infty \equiv \tau$

IR cut-off $k_{\rm IR} \sim 1/R_\infty$

Thus:

$$N_{\rm G} = 4\,\frac{M^2}{m_{\rm P}^2}\int_{k_{\rm IR}}^{k_{\rm UV}} \frac{{\rm d}k}{k} = 4\,\frac{M^2}{m_{\rm P}^2}\,\ln\left(\frac{R_\infty}{R_{\rm s}}\right) \simeq \boxed{\frac{M^2}{m_{\rm P}^2}}$$

Corpuscular scaling!

What about $\langle k\rangle$?

$$\langle k\rangle = \frac{1}{N_{\rm G}}\int_{k_{\rm IR}}^{k_{\rm UV}} \frac{k^2\,{\rm d}k}{2\,\pi^2}\,k\,|g(k)|^2 = \frac{4\,M^2}{N_{\rm G}\,m_{\rm P}^2}\left(\frac{1}{R_{\rm s}} - \frac{1}{R_\infty}\right) \simeq \frac{1}{R_{\rm s}} \overset{?}{\neq} \frac{1}{R_{\rm H}}$$

Remark!
Here $[k] = \boldsymbol{length^{-1}}$, the physical momentum is obtained as $\boldsymbol{\hbar k}$

[R. Casadio, arXiv:2103.00183 [gr-qc]]
[R. Casadio, A. Giugno, A. Giusti, M. Lenzi, Phys.Rev.D 96 (2017) 044010]

A more careful look at R_s

Example: Spherical collapse of a ball of dust of (areal) radius $R = R(\tau)$, τ proper time

Radial geodesic in the Schwarzschild spacetime:

$$\dot{R}^2 + 1 - \frac{2\,G_{\rm N}\,M}{R} \simeq \frac{E^2}{M^2} \quad \Longleftrightarrow \quad H \equiv \frac{P^2}{2M} - \frac{G_{\rm N}\,M^2}{R} \simeq \frac{M}{2}\left(\frac{E^2}{M^2} - 1\right), \quad P \equiv M\dot{R}$$

Simple quantization: $R \to \widehat{R}, \quad P \to \widehat{P} = -\imath\,\hbar\,\partial_R\,, \quad \mathscr{H} = L_2\left(\mathbb{R}^3\right)$

$$\widehat{H}\,\Psi = \epsilon\,\Psi \quad \Rightarrow \quad \epsilon_n \simeq -\frac{G_{\rm N}^2\,M^5}{2\,\hbar^2\,n^2}\,, \quad \Psi_n \simeq {\rm e}^{-\frac{x}{n}}\,L^{(1)}_{n-1}\left(\frac{2\,x}{n}\right), \quad x = \frac{M^3\,R}{m_{\rm P}^3\,\ell_{\rm P}}\,, \quad n \geq 1$$

(gen. Laguerre polynomials)

But then: $0 \leq \frac{E_n^2}{M^2} \simeq 1 - \frac{1}{n^2}\left(\frac{M}{m_{\rm P}}\right)^4 \Rightarrow n \geq n_* \simeq \frac{M^2}{m_{\rm P}^2} \Rightarrow R_{n_*} \sim R_{\rm H}$

(Size of the ground state)
$R_{n_*} = R_s$

$\simeq$ up to $O(1)$ multiplicative coefficients

[R. Casadio, Eur.Phys.J.C 82 (2022) 10]

This suggests that $R_{\rm s} \simeq R_{\rm H}$ (strictly speaking $R_{\rm s} \lesssim R_{\rm H}$), and $\boxed{\langle k \rangle \simeq \frac{1}{R_{\rm s}} \simeq \frac{1}{R_{\rm H}}}$

$\Longrightarrow$ Reproduced all the corpuscular scaling laws!

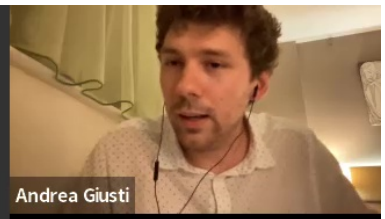
What now?

1) Reconstruct the geometry of the collapsed object
2) Understand how the cut-offs affect it

Requirement

$$V_{\rm q} \equiv \sqrt{G_{\rm N}}\, \langle g | \widehat{\Phi} | g \rangle \simeq V(r)\,, \qquad r \gtrsim R_{\rm H}$$

i.e., consistency with experiments



Quantum Schwarzschild Geometry

Given the characterization of $|g\rangle$ through the cut-offs, one can reverse engineer the corresponding potential:

Quantum potential: $$V_{\rm q}(r) = \int_{k_{\rm IR}}^{k_{\rm UV}} \frac{k^2\,{\rm d}k}{2\,\pi^2}\,\widetilde{V}(k)\,j_0(k\,r) \simeq \frac{2}{\pi}\,V(r)\,{\rm Si}\left(\frac{r}{R_{\rm s}}\right), \qquad V(r) = -\frac{G_{\rm N}\,M}{r}$$

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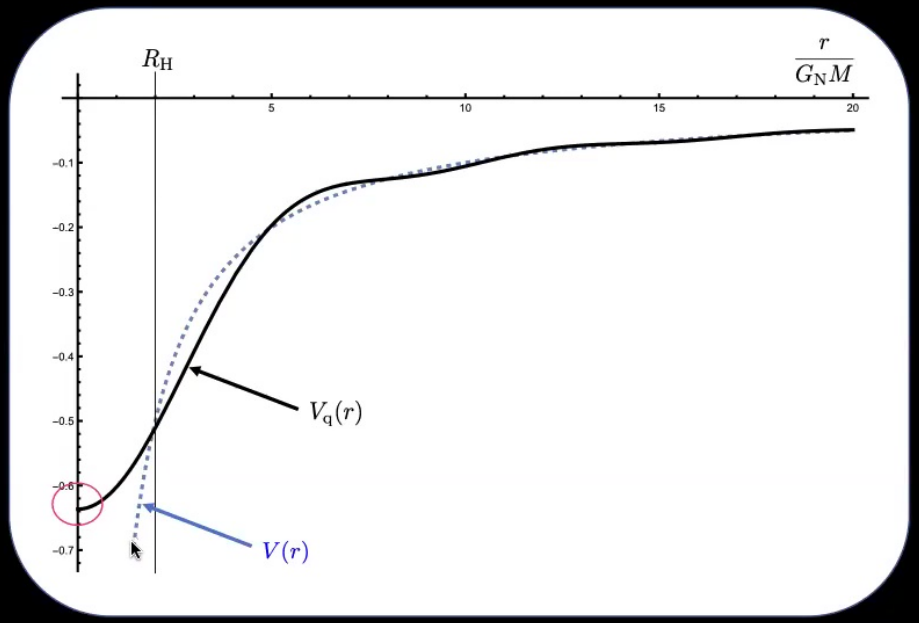
Note that:

$$\lim_{r\to 0^+} V_{\rm q}(r) \simeq -\frac{2}{\pi\, R_{\rm s}}$$

Finite!



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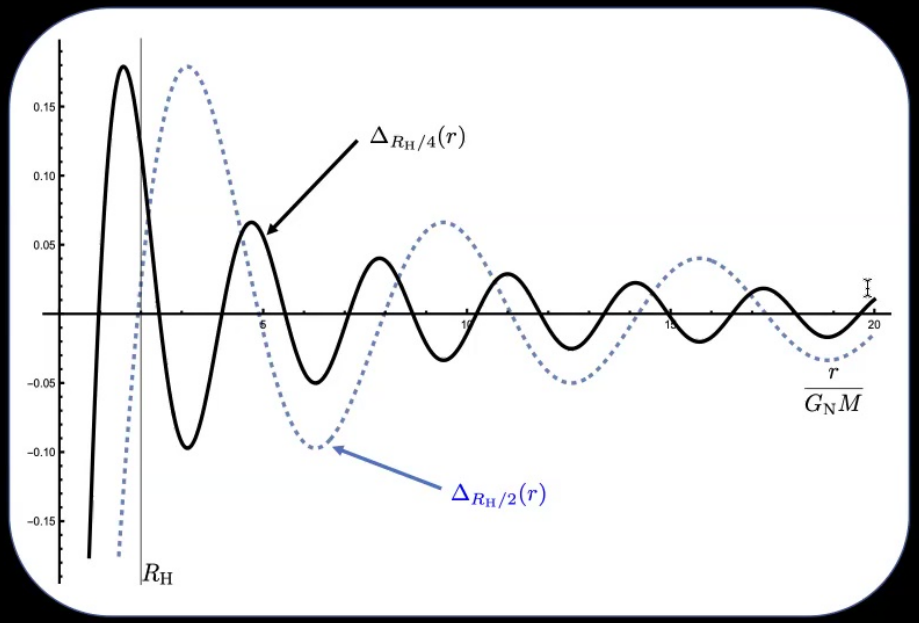
$$V(r) = -\frac{G_{\rm N}\, M}{r}$$

Fluctuations:

$$\Delta_{R_{\rm s}}(r) \equiv \frac{V_{\rm q}(r) - V(r)}{V(r)} \simeq \frac{2}{\pi}\, {\rm Si}\left(\frac{r}{R_{\rm s}}\right) - 1$$

$$\lim_{R_{\rm s}\to 0} \Delta_{R_{\rm s}}(r) = \lim_{r\to\infty} \Delta_{R_{\rm s}}(r) = 0$$



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Quantum-corrected metric

$$\mathrm{d}s^2 = -(1+2V_{\rm q})\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{(1+2V_{\rm q})} + r^2\,\mathrm{d}\Omega^2$$

Horizon: root of $g^{rr} = 1 + 2V_{\rm q}(R_{\rm H}) = 0$

(close to the Schwarzschild one)

Kretschmann scalar: $\mathcal{R}_{\alpha\beta\gamma\delta}\,\mathcal{R}^{\alpha\beta\gamma\delta} \sim \frac{1}{r^4}\,,\ \text{as } r \to 0$ (Schwarzschild: $\sim 1/r^6$)

Tidal forces: relative acceleration of radial geodesics $= -\mathcal{R}^1_{010} \simeq \frac{G_{\rm N}^2 M^2}{R_{\rm s}^4}$ as $r \to 0$

No "spaghettification"

Integrable singularity

Nothing weird happens at $r = 0$

[R. Casadio, arXiv:2103.00183 [gr-qc]]

Reissner-Nordström Black Hole

$$\mathrm{d}s^2 = -(1+2V)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{(1+2V)} + r^2\,\mathrm{d}\Omega^2\,, \quad V(r) = -\frac{G_\mathrm{N}\,M}{r} + \frac{G_\mathrm{N}\,Q^2}{2\,r^2} \equiv V_M + V_Q$$

Horizons: $R_\pm = G_\mathrm{N}\,M \pm \sqrt{G_\mathrm{N}^2\,M^2 - G_\mathrm{N}\,Q^2}$

- $G_\mathrm{N}\,M^2 > Q^2$ two horizons $R_- < R_+$
- $G_\mathrm{N}\,M^2 = Q^2$ extremal $R_- = R_+$
- $G_\mathrm{N}\,M^2 < Q^2$ naked singularity

R_- is a Cauchy horizon $\rightsquigarrow$ mass inflation & loss of predictability

Can our "coherent state approach" alleviate/solve some of these problems?

Building the coherent state for the Reissner-Nordström potential

Classical potential: $$V(r) = -\frac{G_{\rm N}\, M}{r} + \frac{G_{\rm N}\, Q^2}{2\, r^2} \equiv V_M + V_Q$$

$$\widetilde{V}_M = -4\,\pi\, G_{\rm N}\, \frac{M}{k^2}\,, \quad \widetilde{V}_Q = \pi^2\, G_{\rm N}\, \frac{Q^2}{k} \quad \Longrightarrow \quad g(k) = \sqrt{\frac{k}{2}}\, \frac{\widetilde{V}(k)}{\ell_{\rm P}} = g_M(k) + g_Q(k) = -\frac{4\,\pi\, M}{\sqrt{2\, k^3}\, m_{\rm P}} + \frac{\pi^2\, Q^2}{\sqrt{2\, k}\, m_{\rm P}}$$

$$|g\rangle = e^{-N_{\rm G}/2}\, \exp\left\{ \int_0^\infty \frac{k^2\, {\rm d}k}{2\,\pi^2}\, g(k)\, \widehat{a}^\dagger_{\boldsymbol{k}} \right\} |0\rangle$$

$$N_{\rm G} = \int_0^\infty \frac{k^2\, {\rm d}k}{2\,\pi^2} |g(k)|^2 = N_M + N_Q + N_{MQ}$$

Introducing the cut-offs:

$$N_M = 4\, \frac{M^2}{m_{\rm P}^2}\, \ln\left(\frac{R_\infty}{R_{\rm s}}\right), \qquad N_Q = \frac{\pi^2\, Q^4}{8\, m_{\rm P}^2} \left(\frac{1}{R_{\rm s}^2} - \frac{1}{R_\infty^2}\right), \quad N_{MQ} = -\frac{2\,\pi\, M\, Q^2}{m_{\rm P}^2} \left(\frac{1}{R_{\rm s}} - \frac{1}{R_\infty}\right)$$

[R. Casadio, A. Giusti, J. Ovalle, arXiv:2203.03252 [gr-qc]]

What about $\langle k \rangle \equiv \lambda_{\rm G}^{-1}$?

$$\langle k \rangle = \frac{1}{N_{\rm G}} \int_{k_{\rm IR}}^{k_{\rm UV}} \frac{k^2\,{\rm d}k}{2\,\pi^2}\, k\, |g(k)|^2 \equiv \langle k \rangle_M + \langle k \rangle_Q + \langle k \rangle_{MQ}$$

Assuming $R_{\rm s} \ll R_\infty$, then to leading order one finds

$$N_{\rm G} \simeq \frac{4M^2}{m_{\rm P}^2}\, \ln\left(\frac{R_\infty}{R_{\rm s}}\right) + \frac{\pi^2\, Q^4}{8\, m_{\rm P}^2 R_{\rm s}^2} - \frac{2\,\pi\, M\, Q^2}{m_{\rm P}^2\, R_{\rm s}} \qquad \langle k \rangle \simeq \frac{4\, M^2}{m_{\rm P}^2\, R_{\rm s}} + \frac{\pi^2\, Q^4}{12\, m_{\rm P}^2\, R_{\rm s}^3} - \frac{\pi\, M\, Q^2}{m_{\rm P}^2\, R_{\rm s}^2}$$

Taking $2\, G_{\rm N}\, M \gg G_{\rm N}\, Q^2$, then

$$\lambda_{\rm G} = \frac{1}{\langle k \rangle} \simeq R_{\rm s} \left(1 + \frac{\pi\, Q^2}{4\, M\, R_{\rm s}}\right) \ln\left(\frac{R_\infty}{R_{\rm s}}\right)$$

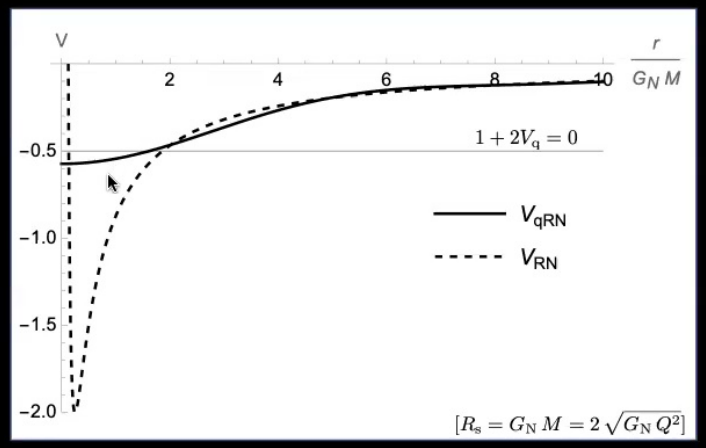
"Typical" wavelength of the gravitons building the geometry

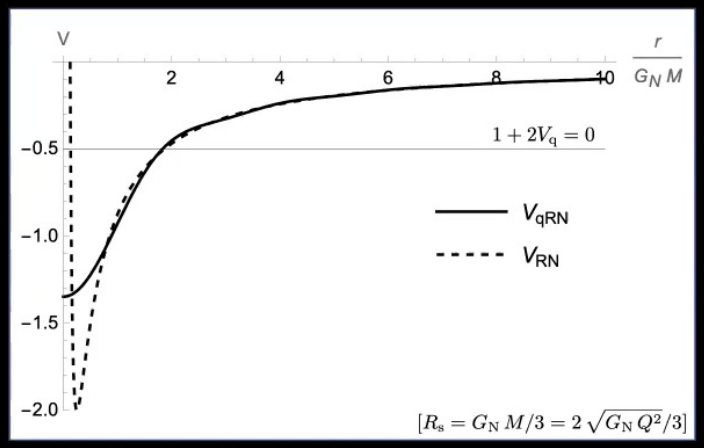
[R. Casadio, A. Giusti, J. Ovalle, arXiv:2203.03252 [gr-qc]]

Quantum corrected Reissner-Nordström spacetime

Quantum potential: $$V_{\rm q}(r) = \int_{k_{\rm IR}}^{k_{\rm UV}} \frac{k^2\,{\rm d}k}{2\,\pi^2}\,\widetilde{V}(k)\,j_0(k\,r) \simeq -\frac{2\,G_{\rm N}\,M}{\pi\,r}\,{\rm Si}\!\left(\frac{r}{R_{\rm s}}\right) + \frac{G_{\rm N}\,Q^2}{2\,r^2}\left[1-\cos\!\left(\frac{r}{R_{\rm s}}\right)\right]$$

Quantum-corrected metric $$\mathrm{d}s^2 = -(1+2V_{\rm q})\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{(1+2V_{\rm q})} + r^2\,\mathrm{d}\Omega^2$$





Curvature singularities?

Kretschmann scalar: $\mathcal{R}_{\alpha\beta\gamma\delta}\,\mathcal{R}^{\alpha\beta\gamma\delta} \sim \frac{1}{r^4}$, as $r \to 0$ (RN: $\sim Q^4/r^8$)

Tidal forces: remain finite as $r \to 0$ No "spaghettification"

Integrable singularity

Nothing weird happens at $r = 0$

Effective stress-energy tensor

$$T^{\mu}_{\ \nu} \equiv \frac{G^{\mu}_{\ \nu}}{8\,\pi\,G_{\rm N}} = \mathrm{diag}\left(-\rho^{\rm eff}, p_r^{\rm eff}, p_t^{\rm eff}, p_t^{\rm eff}\right)$$

$$\rho^{\rm eff} = \frac{Q^2}{8\,\pi\,r^4}\left[1-\cos\left(\frac{r}{R_{\rm s}}\right)\right] + \frac{4\,M\,R_{\rm s} - \pi\,Q^2}{8\,\pi^2\,r^3\,R_{\rm s}}\,\sin\left(\frac{r}{R_{\rm s}}\right) = -p_r^{\rm eff}$$

$$p_t^{\rm eff} = \frac{Q^2}{8\,\pi\,r^4}\left[1-\cos\left(\frac{r}{R_{\rm s}}\right)\right] + \frac{\left(\pi\,Q^2 - 4\,M\,R_{\rm s}\right)\,r\,\cos\left(\frac{r}{R_{\rm s}}\right) + 2\,R_{\rm s}\left(2\,M\,R_{\rm s} - \pi\,Q^2\right)\sin\left(\frac{r}{R_{\rm s}}\right)}{16\,\pi^2\,r^3\,R_{\rm s}^2}$$

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Curvature singularities?

Kretschmann scalar: $\mathcal{R}_{\alpha\beta\gamma\delta}\,\mathcal{R}^{\alpha\beta\gamma\delta} \sim \frac{1}{r^4}$, as $r \to 0$ (RN: $\sim Q^4/r^8$)

Tidal forces: remain finite as $r \to 0$ No "spaghettification"

Integrable singularity

Nothing weird happens at $r = 0$

Effective stress-energy tensor

$$T^{\mu}_{\ \nu} \equiv \frac{G^{\mu}_{\ \nu}}{8\,\pi\,G_{\rm N}} = \mathrm{diag}\left(-\rho^{\rm eff}, p_r^{\rm eff}, p_t^{\rm eff}, p_t^{\rm eff}\right)$$

$$\rho^{\rm eff} = -p_r^{\rm eff} \overset{r\to 0}{\sim} \frac{8\,M\,R_{\rm s} - \pi\,Q^2}{16\,\pi^2\,r^2\,R_{\rm s}^2} \quad \Longrightarrow \quad \int_0^{\varepsilon} \rho^{\rm eff}(r)\,r^2\,{\rm d}r = -\int_0^{\varepsilon} p_r^{\rm eff}(r)\,r^2\,{\rm d}r = \mathcal{O}(\varepsilon)$$

$$p_t^{\rm eff} \overset{r\to 0}{\sim} \frac{16\,M\,R_{\rm s} - 3\,\pi\,Q^2}{192\,\pi^2\,R_{\rm s}^4} \quad \Longrightarrow \quad \int_0^{\varepsilon} p_t^{\rm eff}(r)\,r^2\,{\rm d}r = \mathcal{O}(\varepsilon^2)$$

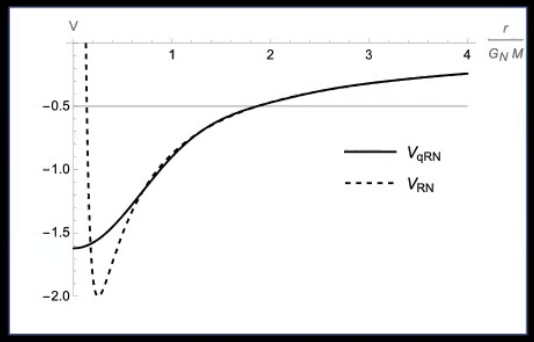
Defining properties of "integrable" singularities

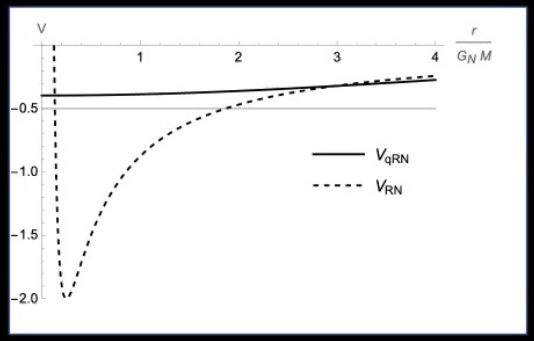
[R. Casadio, A. Giusti, J. Ovalle, arXiv:2203.03252 [gr-qc]]

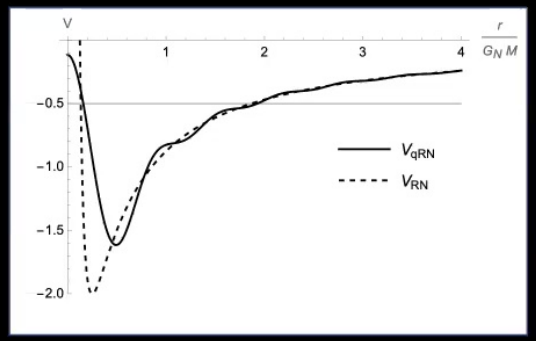
Cauchy and Event horizons

("classical" horizons)

No analytical solutions of $1 + 2V_{\rm q}(R_{\rm H}) = 0$ → Qualitative analysis: no Cauchy horizon for $R_- \lesssim R_{\rm s} \lesssim R_+$







[R. Casadio, A. Giusti, J. Ovalle, arXiv:2203.03252 [gr-qc]]

A quantum state for the late Universe

A simple toy model for the late universe: Schwarzschild-de Sitter geometry

$$\mathrm{d}s^2 = -(1+2V)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{(1+2V)} + r^2\,\mathrm{d}\Omega^2\,, \quad V(r) = -\frac{G_\mathrm{N}\,M}{r} - \frac{\Lambda\,r^2}{6}$$

Λ cosmological constant; $M = 4\pi \int_0^R \rho(r)\,r^2\,\mathrm{d}r$ Mass of a localized matter source of size R

Denoting by R_H the gravitational radius of the source and by L the cosmological horizon one finds that

$$\sqrt{3/\Lambda} \gg 2\,G_\mathrm{N}\,M \quad \Longrightarrow \quad R_\mathrm{H} \approx 2\,G_\mathrm{N}\,M\,, \quad L \approx \sqrt{3/\Lambda}$$

[A. Giusti, S. Buffa, L. Heisenberg, R. Casadio, Phys. Lett. B 826 (2022) 136900]

Characterization of the coherent state for SdS

Construct $|g\rangle$ such that:

$$\sqrt{G_{\rm N}}\,\langle g|\widehat{\Phi}|g\rangle \simeq V(r) \quad \text{for} \quad R_{\rm H} \lesssim r \lesssim L$$

Since we assume $R_{\rm s} \lesssim R_{\rm H} \leq R \ll L$ we can approximately take:

UV cut-off: $R_{\rm s} \sim R_{\rm H}$

IR cut-off: $R_\infty \sim L$

Therefore,

$$N = \int_{R_\infty^{-1}}^{R_{\rm s}^{-1}} \frac{k^2\,{\rm d}k}{2\,\pi^2}|g(k)|^2 = N_M + N_\Lambda + N_{M\Lambda}$$

$$N_M \simeq \frac{M^2}{m_{\rm P}^2}\,\ln\left(\frac{m_{\rm P}\,L}{M\,\ell_{\rm P}}\right), \qquad N_\Lambda \simeq \frac{L^2}{\ell_{\rm P}^2}\,\ln\left(\frac{m_{\rm P}\,L}{M\,\ell_{\rm P}}\right), \qquad N_{M\Lambda} = \frac{M\,L}{\hbar}\,\ln\left(\frac{m_{\rm P}\,L}{M\,\ell_{\rm P}}\right)$$

(holographic) (holographic) "interaction" between the two holographic scalings

[A. Giusti, S. Buffa, L. Heisenberg, R. Casadio, Phys. Lett. B 826 (2022) 136900]

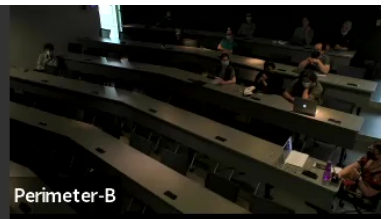
Is it MOND?

Consider the potential $V_{\rm MOND}(r) \simeq \frac{G_{\rm N}\, M}{\ell} \ln\left(\frac{r}{\ell}\right)$, $\ell \simeq \sqrt{G_{\rm N}\, M\, L}$ (in the outer region of a Galaxy)

$$g(k) = \sqrt{\frac{k}{2}}\, \frac{\widetilde{V}_{\rm MOND}(k)}{\ell_{\rm P}} \simeq -\frac{G_{\rm N}\, M}{\ell\, \ell_{\rm P}\, k^{\frac{5}{2}}} \quad \Longrightarrow \quad N_{\rm MOND} = \int_{R_\infty^{-1}}^{R_{\rm s}^{-1}} \frac{k^2\, {\rm d}k}{2\,\pi^2} |g(k)|^2 \simeq \frac{M\, L}{\hbar} \ln\left(\frac{m_{\rm P}\, L}{M\, \ell_{\rm P}}\right) = \boxed{N_{M\Lambda}}$$

We started off with just SdS and we ended up with a quantum state
that contains also something that
resembles a contribution from a MOND-like potential

[A. Giusti, Phys. Rev. D 101 (2020) 124029]
[M. Cadoni, R. Casadio, A. Giusti, W. Mück, M. Tuveri, Phys. Lett. B 776 (2018) 242-248]



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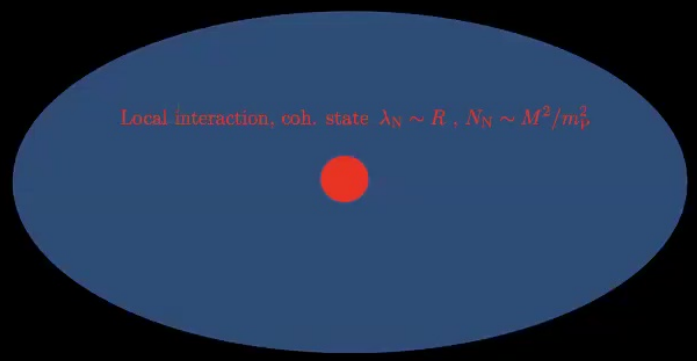
Is it MOND? The corpuscular Dark Force

Consistent with some corpuscular (energy balance) arguments.



dS bound state of gravitons, $\lambda_{dS} \sim L$, $N_\Lambda \sim L^2/\ell_P^2$

Competition between dS scaling
and local (Newtonian) physics
gives rise to
an intermediate-scale force on test particles

$$|a_{DF}(r)| \sim \left(\frac{G_N\, m_B(r)}{L\, r^2}\right)^{\frac{1}{2}} \ , \ \text{at } r \sim \ell$$

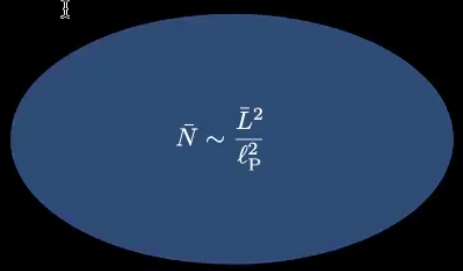
which is the kind of force responsible for the
flattening of Galaxy rotation curves

$N \supset N_{M\Lambda}$ ~ consistency check

[M. Cadoni, R. Casadio, A. Giusti, M. Tuveri, Phys. Rev. D 97 (2018) 044047]

Comments on the H_0 tension

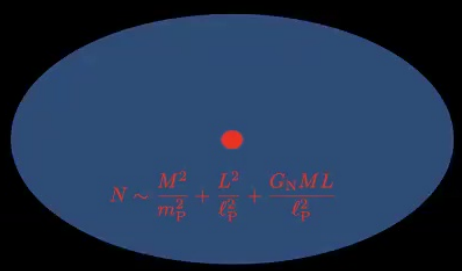
Early Universe ~ quasi-dS



$\Longrightarrow$

+ localized baryonic matter

Late Universe ~ SdS



Assuming the conservation of N → $L \sim \bar{L} - \frac{\ell_P M}{2 m_P}\left(1 + \frac{3 M \ell_P}{4 m_P \bar{L}}\right) < \bar{L}$

Idea: CMB → Carry info about $\bar{L}$, Type Ia Supernovae → Carry info about L

$$H_0^{\rm CMB} \simeq 1/\bar{L}$$
$$H_0^{\rm SNeIa} \simeq 1/L$$

Take $n_g \gg 1$ galaxies of mass M_g
+ 5% baryonic content of the Universe
$(n_g G_N M_g \sim 0.05 L)$

$\Longrightarrow$ $\frac{H_0^{\rm SNeIa} - H_0^{\rm CMB}}{H_0^{\rm CMB}} \simeq 0.05 + \mathcal{O}\left(\frac{1}{n_g}\right)$ [Observed ~ 7-8%]

[A. Giusti, S. Buffa, L. Heisenberg, R. Casadio, Phys. Lett. B 826 (2022) 136900]

Conclusions

- Starting Point: Corpuscular scaling laws
- Coherent state representation for static and spherically symmetric spacetimes
- Coherent state for a quantum field $\Phi \sim$ mean-field potential (not fund. dof)
- Application to: Schwarzschild, Reissner-Nordström, Schwarzschild-de Sitter
- Reproduces Corpuscular scalings
- Resolution of the singularity (integrable) and removal of Cauchy horizons
- In SdS, the coherent state picks up (for free) a contribution of with MOND-like form
- Argument that "relaxes" the H_0 tension