Title: Geometrization of Renormalization Group Histories: (A)dS/CFT correspondence emerging from Asymptotic Safety?

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Abstract: Considering the scale dependent effective spacetimes implied by the functional renormalization group in d-dimensional Quantum Einstein Gravity, we discuss the representation of entire evolution histories by means of a single, (d+1)-dimensional manifold furnished with a fixed (pseudo-) Riemannian structure.

We propose a universal form of the higher dimensional metric and discuss its properties. We show that, under precise conditions, this metric is always Ricci flat; if the evolving spacetimes are maximally symmetric, their (d+1)-dimensional representative has a vanishing Riemann tensor even. The non-degeneracy of the higher dimensional metric is linked to a monotonicity requirement for the running of the cosmological constant, which we test in the case of Asymptotic Safety.

Furthermore, we allow the higher dimensional manifold to be an arbitrary Einstein space, admitting the possibility that the spacetimes to be embedded have a Lorentzian signature, a prime example being a stack of de Sitter spaces. We "derive" the (A)dS/CFT correspondence by applying the gravitational Effective Average Action approach, by solving the corresponding functional RG and the effective Einstein equations, and finally embedding the 4D metrics into the one single 5-dimensional one. It is an intriguing possibility that in this way one might find a specific solution to the general equations which coincides with the 5D kinematic setting which forms the basis of the conjectured (A)dS/CFT correspondence.

Zoom Link: https://pitp.zoom.us/j/92268060878?pwd=M2E1S1RxcHBFbzBiblhpSUJCMWIzUT09

Geometrization of Renormalization Group Histories: (A)dS/CFT Correspondence emerging from Asymptotic Safety ?

Renata Ferrero & Martin Reuter

Based on arXiv:2103.15709 and *work in progress - STAY TUNED!*

Quantum Gravity Seminar, Perimeter Institute - May, 19th 2022

PERIMETER INSTITUTE

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Ingredients

Functional Renormalization Group

Gravitational Effective Average Action Γ_k

Asymptotic Safety

$\mathscr{M}_4$

Scale dependent spacetime metric $g_{\mu\nu}^k$

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Functional Renormalization Group

Gravitational Effective Average Action Γ_k

Asymptotic Safety

Scale dependent spacetime metric $\mathscr{M}_4$ $g^k_{\mu\nu}$

Geometrization

Embedding of the 4D slices into a 5D manifold

$^{d+1}g_{IJ}$ $\mathscr{M}_5$

Embed the family of emerging 4D spacetimes into a single 5D manifold that encodes the complete information about all scales.

Single foliated manifold, the leaves of whose foliation describe the spacetime at a certain value of the RG parameter k: **“Scale-space-time”**

Is it conceivable that there exist general reasons or principles, over and above those inherent in the RG framework, that determine those missing ingredients in a meaningful and physically relevant way?

?

trajectory of 4D geometries + ? = unique 5D geometry

From trajectories to metric of higher dimensions

1 (Functional) RG → Scale dependent effective field equation (Einstein's equation)

$$\{g^k_{\mu\nu}\}_{k\geq 0}$$

RG scale

Family of different Riemannian structures which furnish the same manifold $\mathscr{M}_d$

Trajectory $k \mapsto (\mathscr{M}_d, g^k_{\mu\nu})$

From trajectories to metric of higher dimensions

2 Re-interpret the RG parameter k as an additional coordinate (or its reparametrization $\tau(k)$) together with the x^μ's

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Natural foliation: ADM formalism

determines the signature

Shift

$$ds^2_{d+1} = \varepsilon N(x^I)^2 d\tau^2 + {}^{(d)}g_{\mu\nu}(x^I)\left[dx^\mu + N^\mu(x^I)d\tau\right]\left[dx^\nu + N^\nu(x^I)d\tau\right]$$

Lapse

From trajectories to metric of higher dimensions

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Lapse

The original manifold $\mathscr{M}_d$ is isometrically embedded in $\mathscr{M}_{d+1}$ in a k-dependent way, and so $\mathscr{M}_{d+1}$ comes into being equipped with a natural foliation.

↓

The RG trajectory is described by a single Riemannian structure in $d+1$ dimensions

$$\left(\mathscr{M}_{d+1}, {}^{(d+1)}g_{IJ}\right)$$

From trajectories to metric of higher dimensions

2 Re-interpret the RG parameter k as an additional coordinate (or its reparametrization $\tau(k)$) together with the x^μ's

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Lapse

From trajectories to metric of higher dimensions

Make contact with RG approach

$$^{(d)}g_{\mu\nu}(\tau, x^\rho) = g^k_{\mu\nu}(x^\rho)\Big|_{k=k(\tau)}$$

Assume that $g^k_{\mu\nu}(x^\rho)$ and $k(\tau)$ are known, externally prescribed functions.

From trajectories to metric of higher dimensions

Make contact with RG approach

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What is missing?

N, N^μ, sign(ε)

These are properties of $\mathscr{M}_{d+1}$ which do not follow from the flow equations.

From trajectories to metric of higher dimensions

Make contact with RG approach

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Assume that $g^k_{\mu\nu}(x^\rho)$ and $k(\tau)$ are known, externally prescribed functions.

What is missing?

$\mathscr{D}iff(\mathscr{M}_{d+1})$ already exploited

N, N^μ, sign(ε)

These are properties of $\mathscr{M}_{d+1}$ which do not follow from the flow equations.

The possibility of performing coordinate transformations has been exhausted already in solving the k-dependent effective field equations

Running Einstein metrics

Solutions of rescaling type

k-dependence resides in the conformal factor

Pure Quantum Gravity, Einstein-Hilbert truncation

$$\Gamma_k = \frac{1}{16\pi\, G(k)} \int d^4x\, \sqrt{g}\, \Big(-R(g) + 2\Lambda(k) \Big) + \cdots$$

Effective field equations

$$R_\mu{}^\nu[g^k_{\alpha\beta}] = \frac{2}{d-2}\, \Lambda(k)\, \delta_\mu{}^\nu$$

The only input from the RG equations is the k-dependence of the running cosmological constant $\Lambda(k)$.

$$\Lambda(k) = \sigma\, |\, \Lambda(k)\, |$$

Running Einstein metrics
Solutions of rescaling type

Finding solutions let us fix some convenient reference scale such that

reference metric

Running metric

$$g^k_{\mu\nu}(x^\rho) = Y(k)^{-1}\, g^R_{\mu\nu}(x^\rho) \quad \text{where} \quad g^k_{\mu\nu}\,|_{k=k_R} = g^R_{\mu\nu}$$

$$Y(k) \equiv \frac{|\Lambda(k)|}{|\Lambda_R|} \equiv \frac{H(k)^2}{H_R^2}$$

solve the effective field equations.

Running Einstein metrics

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Running metric $$g^k_{\mu\nu}(x^\rho) = Y(k)^{-1}\, g^R_{\mu\nu}(x^\rho)$$ where $$g^k_{\mu\nu}\,|_{k=k_R} = g^R_{\mu\nu}$$

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solve the effective field equations.

The ADM metric reads: $$^{(d)}g_{\mu\nu}(\tau, x^\rho) = Y(k(\tau))^{-1}\, g^R_{\mu\nu}(x^\rho)$$

from the RG

Focusing on the lapse function

$$N^\mu = 0, N = N(\tau)$$

$$ds^2_{d+1} = \varepsilon\, N(\tau)^2\, d\tau^2 + {}^{(d)}g_{\mu\nu}(\tau, x^\rho)dx^\mu dx^\nu$$

$$\downarrow$$

$$ds^2_{d+1} = \varepsilon\, N(\tau)^2\, d\tau^2 + Y(k(\tau))^{-1}\;\; g^R_{\mu\nu}(x^\rho)dx^\mu dx^\nu$$

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What is a single pseudo-Riemannian manifold $\left(\mathscr{M}_{d+1}, {}^{(d+1)}g_{IJ}\right)$ capable of doing for us that would not already be possible using the original stack of unrelated manifolds $(\mathscr{M}_d, g^k_{\mu\nu})$?

?

Running Einstein metrics

Solutions of rescaling type

Finding solutions let us fix some convenient reference scale such that

reference metric

Running metric $g^k_{\mu\nu}(x^\rho) = Y(k)^{-1}\, g^R_{\mu\nu}(x^\rho)$ where $g^k_{\mu\nu}\big|_{k=k_R} = g^R_{\mu\nu}$

$$Y(k) \equiv \frac{|\Lambda(k)|}{|\Lambda_R|} \equiv \frac{H(k)^2}{H_R^2}$$

solve the effective field equations.

Focusing on the lapse function

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do-Riemannian manifold $(\mathscr{M}_{d+}$
ld not already be possible using
unrelated manifolds $(\mathscr{M}_d, g^k_{\mu\nu})$?

Distinguished higher dimensional geometries

POSTULATE 1

The cosmological constant $\Lambda(k)$ is a strictly increasing function of k.

The higher dimensional metric can be chosen Ricci flat: ${}^{(d+1)}R_{IJ} = 0$

POSTULATE 2

$g^{k}_{\mu\nu}$ is maximally symmetric

The higher dimensional metric can be chosen Riemann flat: ${}^{(d+1)}R^{I}{}_{JKJ} = 0$

Distinguished higher dimensional geometries

Hubble length: $L_H(k) \equiv \dfrac{1}{H(k)}$

Introduce RG time ξ s.t. $\xi = L_H(k(\xi))$ with $\xi(k) = \left[\dfrac{(d-1)(d-2)}{2\,|\,\Lambda(k)\,|}\right]^{1/2}$

Motivation: $Y(k(\xi))^{-1} = H_R^2\; H(k(\xi))^{-2} = H_R^2\; L_H(k(\xi))^2 = H_R^2\,\xi^2$

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N.B. Monotonicity of $\Lambda(k)$ is crucial for the solvability of $\xi = L_H(k(\xi))$

Distinguished higher dimensional geometries

Hubble length: $L_H(k) \equiv \dfrac{1}{H(k)}$

Introduce RG time ξ s.t. $\xi = L_H(k(\xi))$ with $\xi(k) = \left[\dfrac{(d-1)(d-2)}{2|\Lambda(k)|}\right]^{1/2}$

Motivation: $Y(k(\xi))^{-1} = H_R^2\ H(k(\xi))^{-2} = H_R^2\ L_H(k(\xi))^2 = H_R^2\ \xi^2$

N.B. Monotonicity of $\Lambda(k)$ is crucial for the solvability of $\xi = L_H(k(\xi))$

RULE N assumes simplest form as possible: $N(\xi) = 1$

$$ds_{d+1}^2 = \varepsilon\ (d\xi)^2 + \xi^2\ H_R^2\ g_{\mu\nu}^R(x^\rho)dx^\mu dx^\nu$$

Equivalent forms

- η: conformal time

$$ds^2_{d+1} = e^{2H_R\eta}\left[\varepsilon\,(d\eta)^2 + g^R_{\mu\nu}(x^\rho)dx^\mu dx^\nu\right]$$

- IR cutoff

$$ds^2_{d+1} = \left|\frac{\Lambda_R}{\Lambda(k)}\right|\left\{\varepsilon\,\left(\frac{1}{2}\partial_k \ln|\Lambda(k)|\right)^2 \left(L^R_H\,dk\right)^2 + g^R_{\mu\nu}(x^\rho)dx^\mu dx^\nu\right\}$$

- More general conformal form

$${}^{(d+1)}g_{IJ}(x^K)dx^I dx^J = \Omega^2(\gamma)\left[\varepsilon\,(d\gamma)^2 + g^R_{\mu\nu}(x^\rho)dx^\mu dx^\nu\right]$$

Ricci tensor

$$^{(d+1)}g_{IJ}(x^K)dx^I dx^J = \Omega^2(\gamma)\left[\varepsilon\,(d\gamma)^2 + g^R_{\mu\nu}(x^\rho)dx^\mu dx^\nu\right]$$

$$^{(d+1)}R^0{}_0 = -\,\varepsilon\, d\,\Omega^{-2}\left[\frac{\ddot\Omega}{\Omega} - \left(\frac{\dot\Omega}{\Omega}\right)^2\right]$$

$$^{(d+1)}R^0{}_\mu = 0, \qquad {}^{(d+1)}R^\mu{}_0 = 0$$

$$^{(d+1)}R^\mu{}_\nu = \Omega^{-2}\left\{R^\mu{}_\nu - \varepsilon\,\delta^\mu{}_\nu\left[\frac{\ddot\Omega}{\Omega} + (d-2)\left(\frac{\dot\Omega}{\Omega}\right)^2\right]\right\}$$

Ricci flatness

$$^{(d+1)}R_{IJ} = 0 \quad \Longrightarrow \quad \varepsilon = \sigma \quad \textbf{and} \quad \Omega(\eta) = e^{H_R(\eta-\eta_R)} \,,$$

$$ds^2_{d+1} = e^{2H_R(\eta-\eta_R)}\left[\sigma\,(d\eta)^2 + g^R_{\mu\nu}(x^\rho)dx^\mu dx^\nu\right]$$

depends on $\Lambda_0/|\Lambda_0|$

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Strict flatness

$$^{(d+1)}R^I{}_{JKJ} = 0 \quad \Longrightarrow \quad R^{\mu\nu}{}_{\rho\sigma} = \varepsilon\, H_R^2\left[\delta^\mu{}_\rho\delta^\nu{}_\sigma - \delta^\mu{}_\sigma\delta^\nu{}_\rho\right]$$

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The inclusion of the scale variable has “flattened” the curved spacetime. Resulting $\mathscr{M}_{d+1}$:

Spherical slicing of R^{d+1} $$ds^2_{d+1} = (d\xi)^2 + \xi^2\, d\Omega^2_d \qquad (\Lambda_0 > 0)$$

Hyperbolical slicing of $M^{1,d}$ $$ds^2_{d+1} = -\,(d\xi)^2 + \xi^2\, dH^2_d \qquad (\Lambda_0 < 0)$$

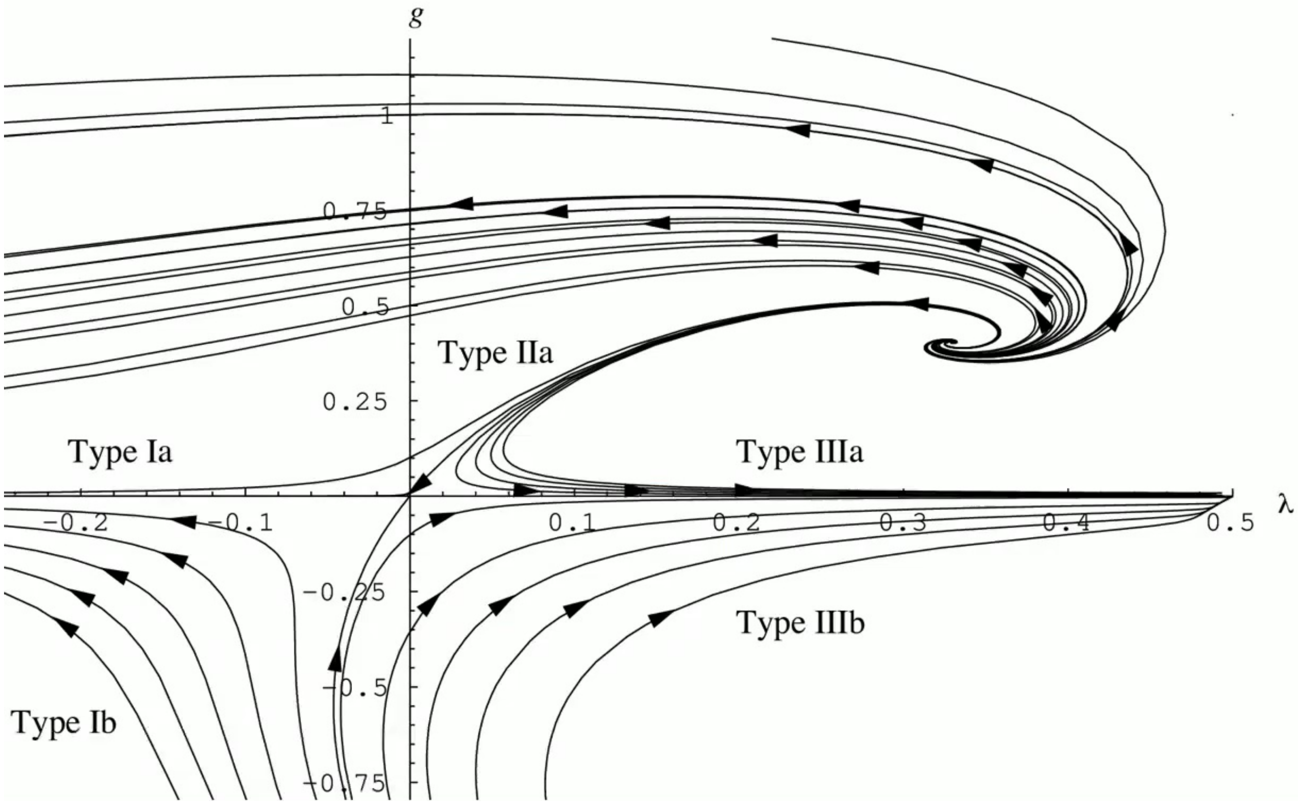
trajectory of 4D geometries + ? = unique 5D geometry

Einstein space + Monotonicity of $\Lambda(k)$ = Ricci flat

Maximally symmetric space + Monotonicity of $\Lambda(k)$ = Riemann flat

Asymptotic Safety

(Einstein-Hilbert truncation)



$\sigma = \mathrm{sign}(\Lambda)$

Type Ia $\sigma = \begin{cases} -1 & \text{for } 0 \leq k < \ell^{-1} \\ +1 & \text{for } k > \ell^{-1} \end{cases}$

Type IIa $\sigma = +1$ for all $k > 0$

Type IIIa $\sigma = +1$ for all $k > 0$

Reuter, Saueressig

Caricature trajectory

$$\Lambda(k) = |\Lambda_0| \cdot \begin{cases} \ell^4 k^4 \pm 1 & \text{for} \quad 0 \leq k \lesssim \hat{k} \\ L^2 k^2 & \text{for} \quad k \gtrsim \hat{k} \end{cases}$$

CHECK:
Monotonicity
of $\Lambda(k)$

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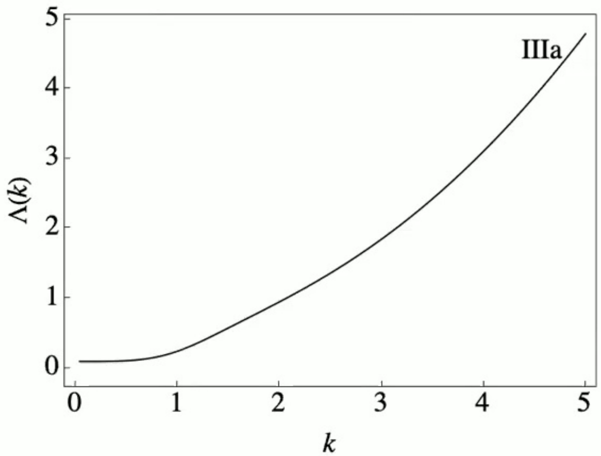
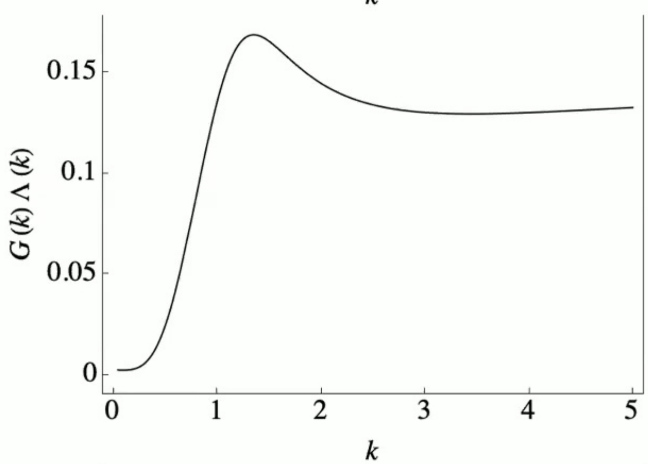
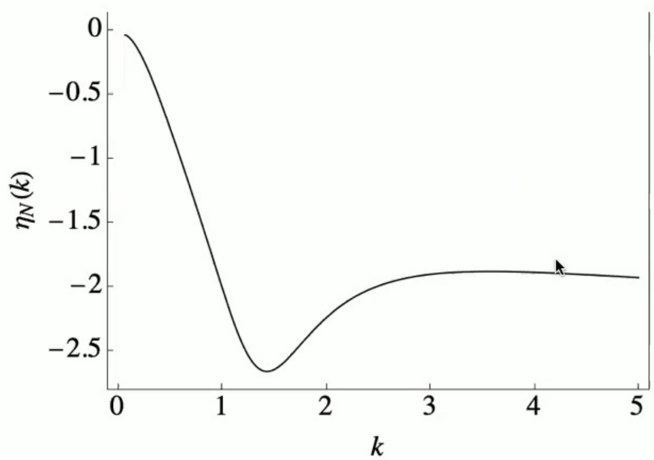


Meaning of the monotonicity? Mode counting?





Resulting $\varepsilon(k)$:

Signature

Type Ia	$\varepsilon(k) = \begin{cases} -1 & \text{for} \quad 0 \leq k < \ell^{-1} \\ +1 & \text{for} \quad k > \ell^{-1} \end{cases}$	$\mathscr{M}_4$ with $(++++)$ $\mathscr{M}_4$ with $(++++)$	$\mathscr{M}_5$ with $(-++++)$ $\mathscr{M}_5$ with $(+++++)$
Type IIa	$\varepsilon(k) = +1$ **for all** $k \geq 0$	$\mathscr{M}_4$ with $(++++)$	$\mathscr{M}_5$ with $(+++++)$
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Change in the topology?

Resulting $\varepsilon(k)$:

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This completes our demonstration that the asymptotically safe trajectories in 4D do indeed comply with the Postulate 1 and are thus eligible for a geometrization based upon the proposed Rule.

Embedding in Einstein manifolds

The signature matters!

Local vs. Global embedding

$$R^{\mu}{}_{\nu}\left[g^{R}_{\alpha\beta}\right] = \frac{2}{(d-2)}\,\Lambda_R\,\delta^{\mu}{}_{\nu} \qquad \Lambda_R > 0$$

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$$^{(d+1)}R^{I}{}_{J} = C\,\delta^{I}{}_{J} \quad \Longrightarrow \quad ^{(\mathbf{d+1})}\boldsymbol{\Lambda} = \frac{1}{2}(d-1)\,C = \frac{1}{2}\,d\,(d-1)\,\left[-\,\varepsilon\,\alpha_1^2 + \alpha_2^2\right] H_R^2$$

Integration constants

$$R^{\mu\nu}{}_{\rho\sigma} = H_R^2\left[\delta^{\mu}_{\rho}\,\delta^{\nu}_{\sigma} - \delta^{\mu}_{\sigma}\,\delta^{\nu}_{\rho}\right] \quad \Longrightarrow \quad ^{(d+1)}R^{IJ}{}_{KL} = C\,d^{-1}\left[\delta^{I}_{K}\,\delta^{J}_{L} - \delta^{I}_{L}\,\delta^{J}_{K}\right]$$

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For every choice of $\{\varepsilon, \alpha_1, \alpha_2\}$ and of the d-dimensional Einstein metric $g^{R}_{\mu\nu}$,
the d+1-dimensional metric ${}^{(d+1)}g_{IJ}$ is maximally symmetric
if, and only if,
$g^{R}_{\mu\nu}$ is maximally symmetric.

The candidates

Which principles and criteria can constrain or, in the ideal case, determine exactly a manifold $\mathscr{M}_{d+1}$ that geometrizes a given trajectory of Lorentzian spacetimes $\left(\mathscr{M}_d, g^k_{\mu\nu}\right)$?

?

Symmetry

The higher-dimensional $\mathscr{M}_{d+1}$ should display the maximum amount of symmetry that is consistent with the symmetry properties of the lower-dimensional metrics $g^k_{\mu\nu}$.

Starting manifold: dS$_d$

If we are given a stack of de Sitter spaces $\left(\mathscr{M}_d = \mathrm{dS}_d, Y(k)^{-1} g^R_{\mu\nu}\right)$,
in which manifolds $\left(\mathscr{M}_{d+1}, {}^{(d+1)}g_{IJ}\right)$ can they possibly be embedded if we demand that the higher-dimensional scale-space-time, too, is Lorentzian and maximally symmetric?

The candidates

$$^{(d+1)}g_{IJ}(x^K)dx^I dx^J = \Omega^2(\gamma)\left[\varepsilon\,(d\gamma)^2 + g^R_{\mu\nu}(x^\rho)dx^\mu dx^\nu\right]$$

AdS$_{d+1}$ and dS$_{d+1}$ arise

$$\varepsilon = 1$$

The scale coordinate has to be spatial.

AdS_{d+1}

$$^{(\mathbf{d+1})}\boldsymbol{\Lambda}_{(1,0)} = -\frac{1}{2}\,d\,(d-1)\,H_R^2 = -\left(\frac{d}{d-2}\right)\,\Lambda_R$$

$$\Omega_{(1,0)}(\gamma) = \frac{1}{\sinh(H_R\,\gamma)}$$

$$^{(d+1)}g^{AdS}_{IJ}(x^K)dx^I dx^J = \frac{1}{\sinh^2(H_R\,\gamma)}\left[(d\gamma)^2 \;+\; d\Sigma_d^2\right], \qquad \gamma \in (-\infty, 0)$$

AdS$_{d+1}$

$$\gamma \mapsto \xi(\gamma) = -\,H_R^{-1}\;\ln\;\tanh\left(-\frac{1}{2}\,H_R\,\gamma\right)$$

dS-slicing

$$^{(d+1)}g^{AdS}_{IJ}(x^K)dx^I dx^J = (d\xi)^2 \;+ \sinh^2(H_R\,\xi)\;d\Sigma_d^2, \qquad \xi \in (0, \infty)$$

$$ {}^{(d+1)}g^{dS}_{IJ}(x^K)dx^I dx^J = \frac{1}{\cosh^2(H_R\,\gamma)}\left[(d\gamma)^2 \;+\; d\Sigma_d^2\right], \qquad \gamma \in (-\infty, \infty) $$

$\mathbf{dS}_{d+1}$

$$ \xi(\,\cdot\,):\;(-\infty,\,\infty)\;\to\;(0,\pi H_R^{-1}), \qquad \gamma \mapsto \xi(\gamma) = 2H_R^{-1}\;\arctan\left(e^{H_R\gamma}\right) $$

dS-slicing

$$ {}^{(d+1)}g^{dS}_{IJ}(x^K)dx^I dx^J = (d\xi)^2 \;+ \sin^2(H_R\,\xi)\;d\Sigma_d^2, \qquad H_R\,\xi \in (0,\pi) $$

Relating foliation and RG scale

$$ {}^{(d+1)}g^{AdS/dS}_{IJ} dx^I dx^J \Big|_{d\xi=0} \;=\; F(H_R\,\xi)^2\, d\Sigma_d^2 \;=\; Y(k)^{-1}\, d\Sigma_d^2 $$

Global coordinates of dS

$$ F(x) = \begin{cases} \sinh(x) & \text{for AdS}_{d+1} \\ \sin(x) & \text{for dS}_{d+1} \end{cases} $$

$$ d\Sigma_d^2 = \frac{1}{H_R^2}\left[-\,dt^2 + \cosh^2(t)\; d\Omega_{d-1}^2 \right] $$

The (d+1)-dimensional spacetime is foliated by leaves with $\xi =$ const, which we would like to interpret as surfaces of equal RG scale k.

$$ F\left(H_R\,\xi\right) \;=\; Y(k)^{-1/2}\,, \qquad k \in \mathbb{R}^+ $$

Relating foliation and RG scale

$$^{(d+1)}g^{AdS/dS}_{IJ}dx^I dx^J \Big|_{d\xi=0} = F(H_R\,\xi)^2\, d\Sigma_d^2 = Y(k)^{-1}\, d\Sigma_d^2$$

Global coordinates of dS

$$F(x) = \begin{cases} \sinh(x) & \text{for AdS}_{d+1} \\ \sin(x) & \text{for dS}_{d+1} \end{cases}$$

$$d\Sigma_d^2 = \frac{1}{H_R^2}\left[-\,dt^2 + \cosh^2(t)\; d\Omega_{d-1}^2\right]$$

The (d+1)-dimensional spacetime is foliated by leaves with $\xi =$ const, which we would like to interpret as surfaces of equal RG scale k.

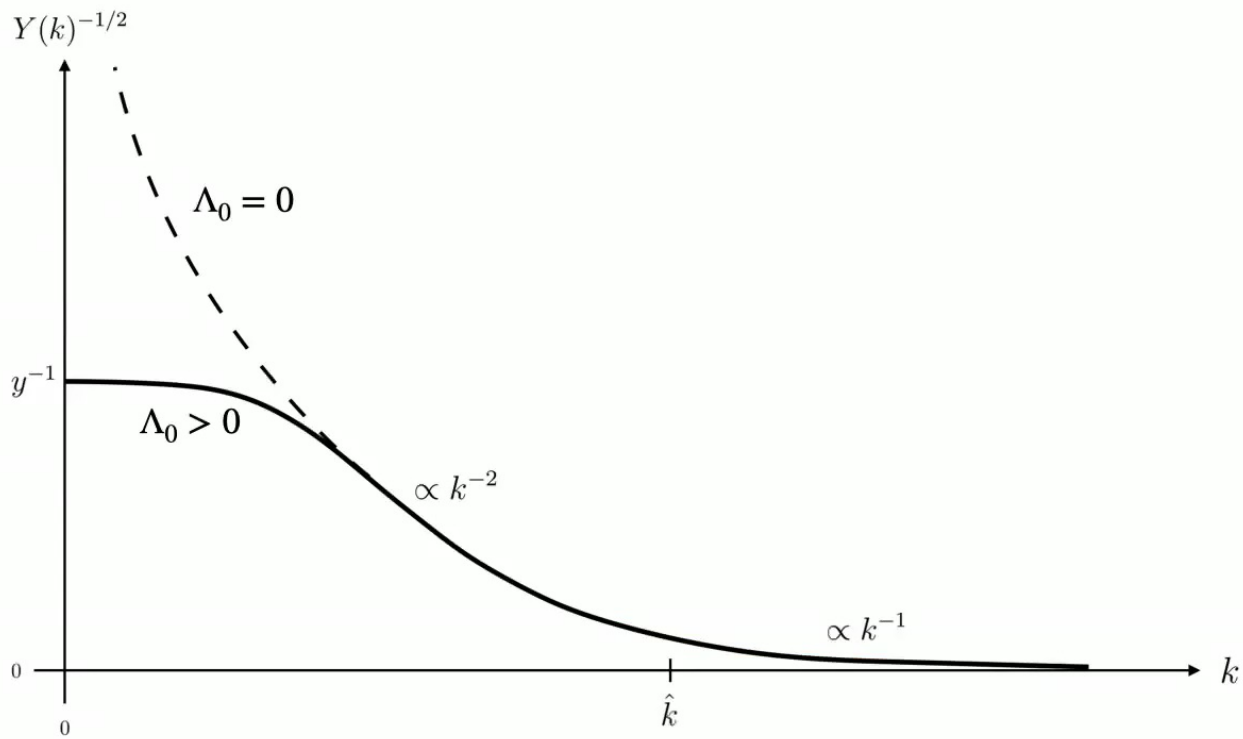
$$F\left(H_R\,\xi\right) = Y(k)^{-1/2}\,, \qquad k \in \mathbb{R}^+$$

This gives rise to a regular coordinate transformation.

$$L_R\, F\left(\frac{\xi}{L_H^R}\right) = L_H(k)$$

It is a "deformed" form of the Ricci flat case.

Input from the RG



The AdS$_5$ candidate

$$\sinh(H_R \xi) \;=\; Y(k)^{-1/2}$$

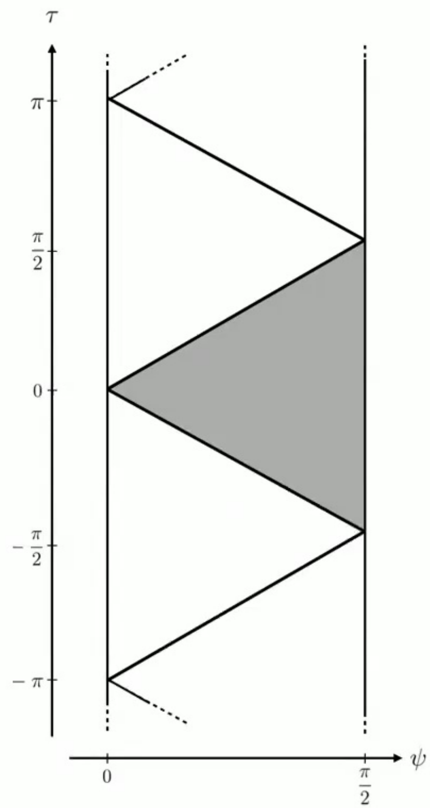
$$k \to \xi(k) \;=\; H_R^{-1}\,\text{arsinh}\Big(\,Y(k)^{-1/2}\Big)$$

This is precisely as it must be if the $k - \xi$ relationship is to qualify as an orientation reversing diffeomorphism.

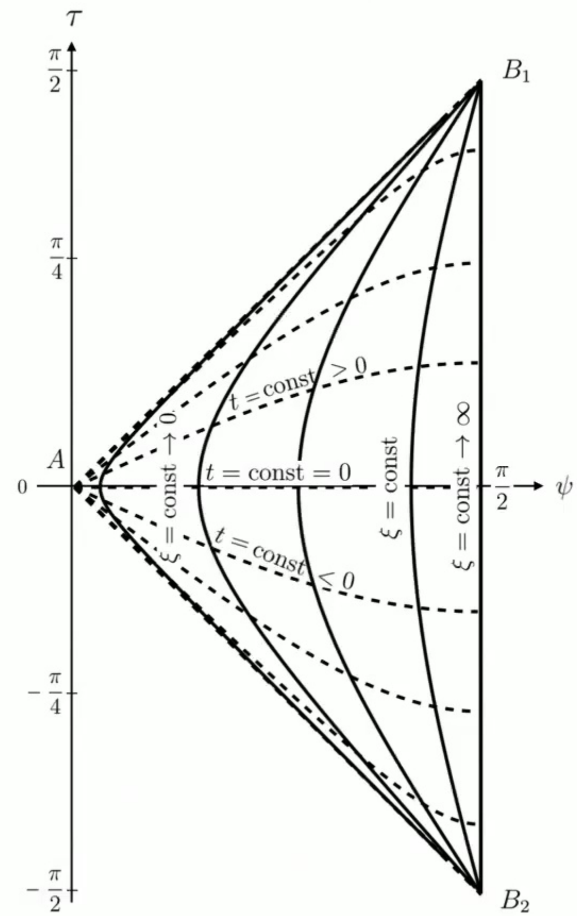
Domain: $k(\,\cdot\,):\ \Big(0, \xi_{max}(y)\Big) \ \to\ \mathbb{R}^+, \quad \xi \ \mapsto\ k(\xi)$ for $\Lambda_0 > 0$

$k(\,\cdot\,):\ \mathbb{R}^+ \ \to\ \mathbb{R}^+, \quad \xi \ \mapsto\ k(\xi)$ for $\Lambda_0 = 0$

Global structure and AdS connection



$$ds^2_{d+1} = \frac{1}{\cos^2(\psi)}\left[-d\tau^2 + d\psi^2 + \sin^2(\psi)\, d\Omega^2_{d+1}\right]$$



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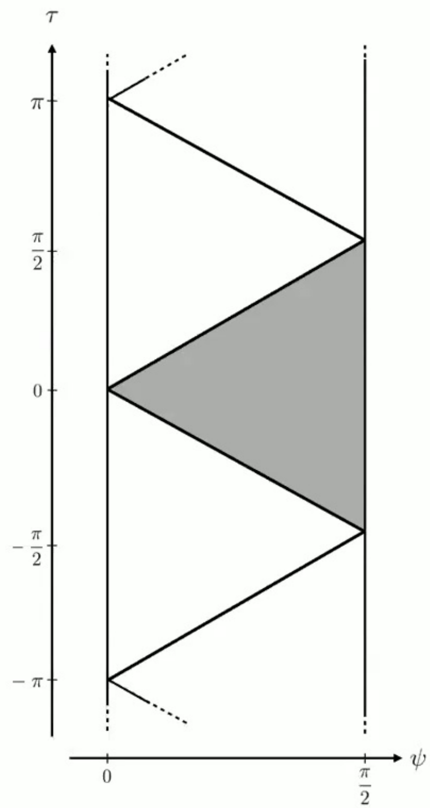
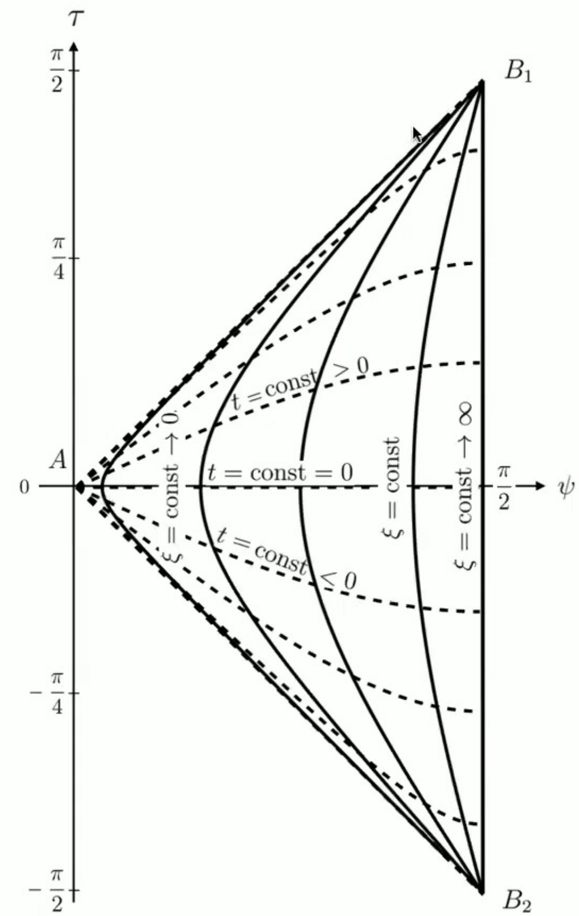
Global structure and AdS connection

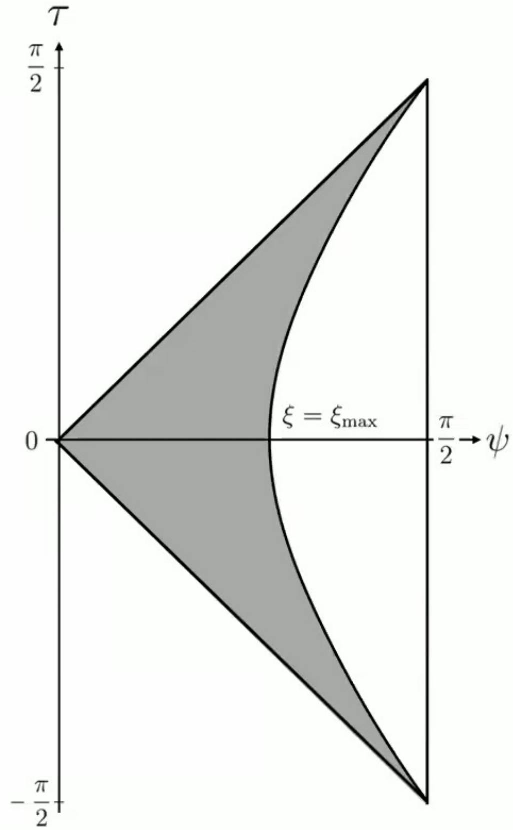


$$ds^2_{d+1} = \frac{1}{\cos^2(\psi)} \left[-\, d\tau^2 \;+\; d\psi^2 \;+\; \sin^2(\psi)\, d\Omega^2_{d+1} \right]$$



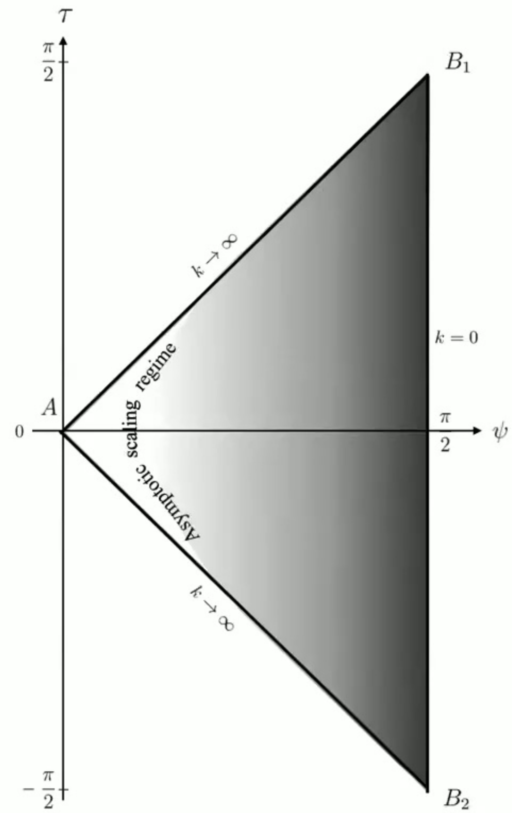
$\mathbf{AdS}_{d+1}$

IIIa



$$\left\{ \left(\mathsf{dS}_d,\, g^k_{\mu\nu}\right),\, k \in \mathbb{R}^+ \right\} \;=\; \left\{ \left(\mathsf{dS}_d,\, g^{k(\xi)}_{\mu\nu}\right),\, \xi \in \left(0,\, \xi_{max}(y)\right) \right\}$$

IIa



$$\left\{ \left(\mathsf{dS}_d, g^k_{\mu\nu}\right), k \in \mathbb{R}^+ \right\}$$

AdS/CFT interpretation

$k \to \infty \Longleftrightarrow \xi \to 0$ Lightline coundaries $AB_{1,2}$ $\Gamma_{k\to\infty} \sim S$

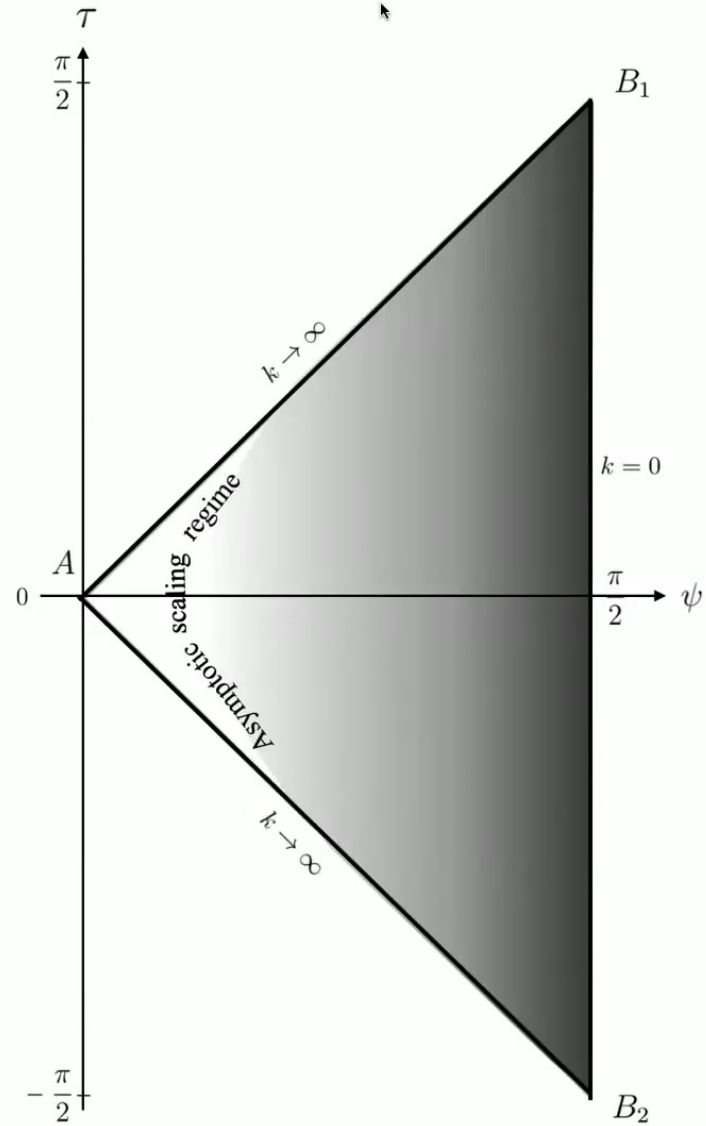
$k \to 0$ Timelike boundary $B_1 B_2$ $\Gamma_{k\to 0} \sim \Gamma$

AdS/CFT

A theory of gravity which lives on the bulk of AdS_5
and is holographically related to a CFT on the boundary.

The analogy is perfect, provided that

the trajectory $\left\{\Gamma_k^{\text{IIa}}, k \in \mathbb{R}^+\right\}$ is conformal
and the limit $\lim_{k\to 0}\Gamma_k \equiv \Gamma$ is the action of a 4D CFT.



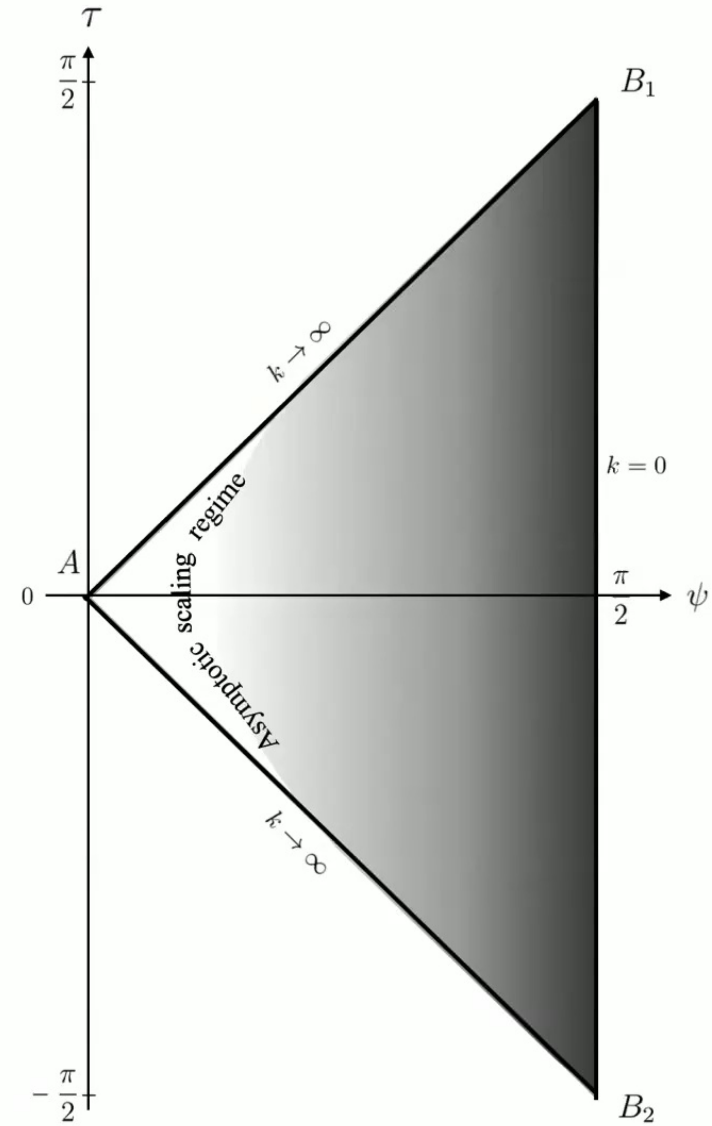
Holography

Within the framework of the gravitational Effective Average Action there exists a natural and perfectly general notion of holography:

Holographic Principle

All actions Γ_k, including the bare one, $S \sim \Gamma_{k\to\infty}$, can be reconstructed from the ordinary effective action $\Gamma_{k=0} = \Gamma$.

The FRG equation defines a meaningful initial value problem also when the direction of the k-evolution is changed from “downward” to “upward”, and the initial condition $\Gamma_{k=0} = \Gamma$ is imposed in the IR rather than UV.



The dS$_5$ candidate

$$\sin(H_R \xi) \;=\; Y(k)^{-1/2}$$

$$k \to \xi(k) \;=\; H_R^{-1}\,\text{arcsin}\left(Y(k)^{-1/2} \right)$$

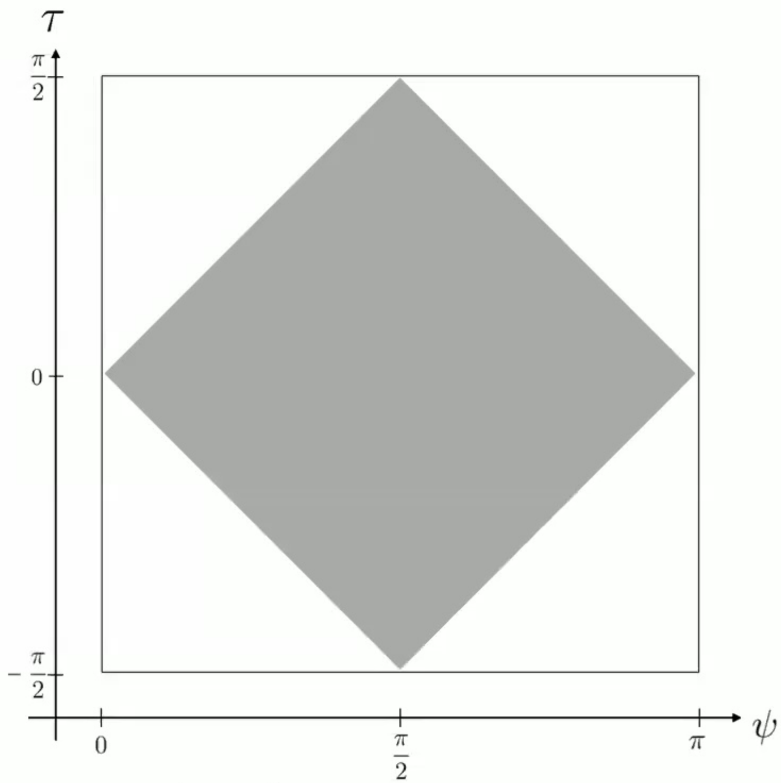
Existence for $y = Y(0)^{1/2} > 1 \quad \Longleftrightarrow \quad L_H^R > L_H(0) \quad \Longleftrightarrow \quad \Lambda_0 > \Lambda_R$

If this constraint is satisfied, then we have the diffeomorphic map

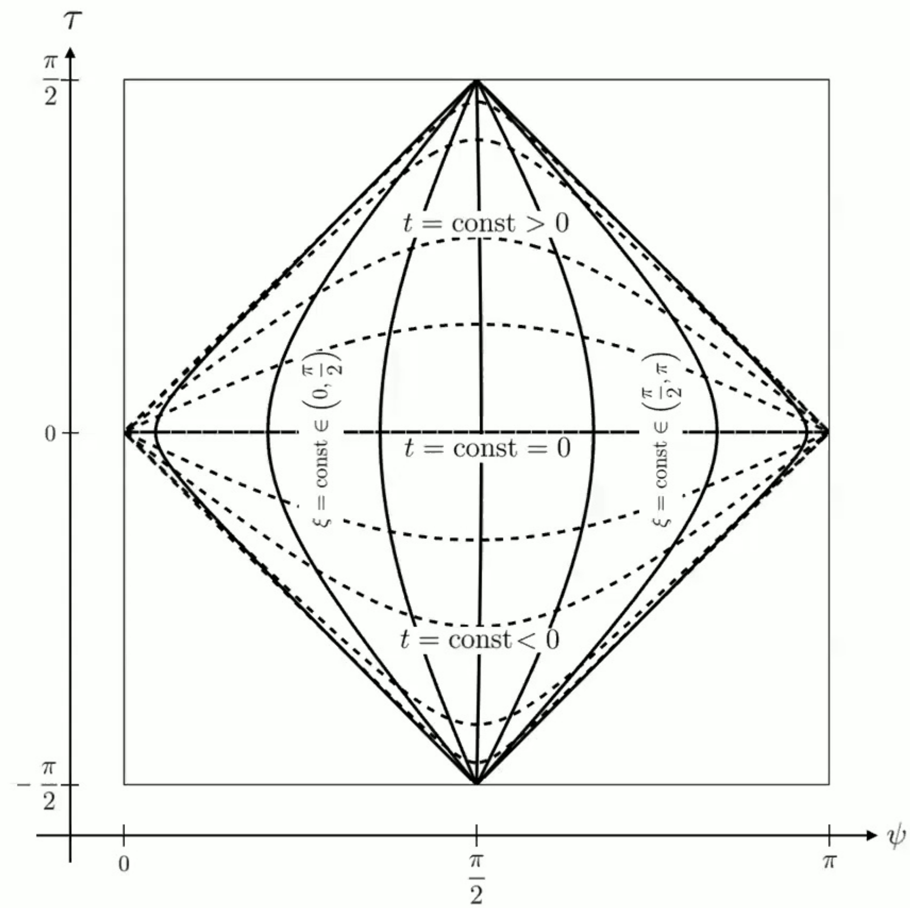
$$\xi(\,\cdot\,):\ \mathbb{R}^+ \to \big(0, \xi_{max}\big), \quad k \mapsto \xi(k)$$

$$k(\,\cdot\,):\ \Big(0,\, \xi_{max}(y)\Big) \;\to\; \mathbb{R}^+, \quad \xi \;\mapsto\; k(\xi)$$

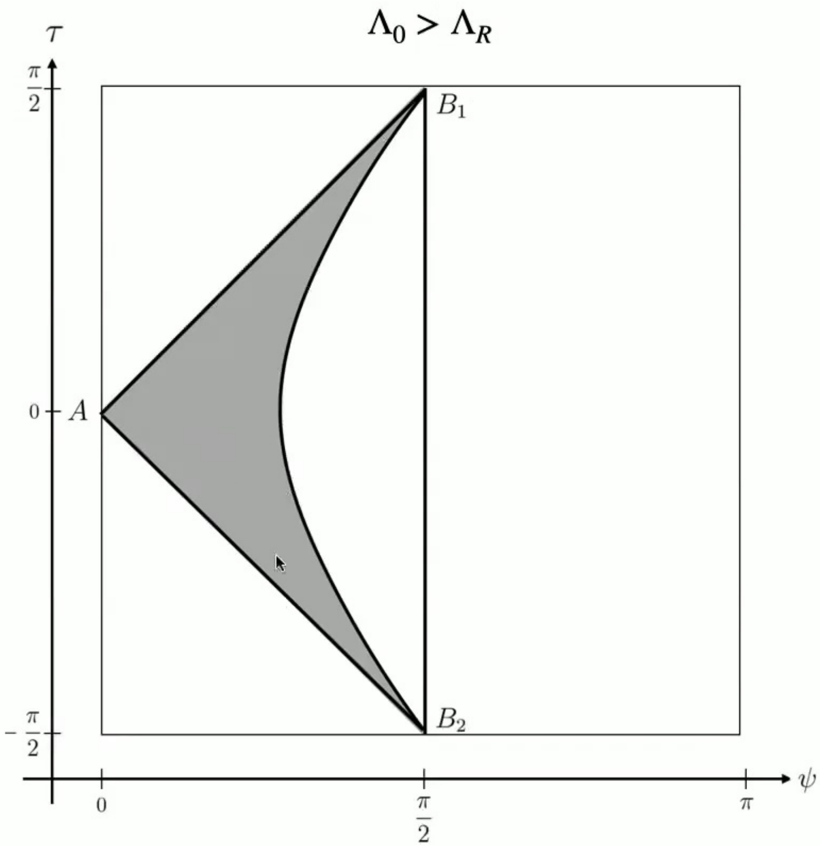
Global structure and dS connection

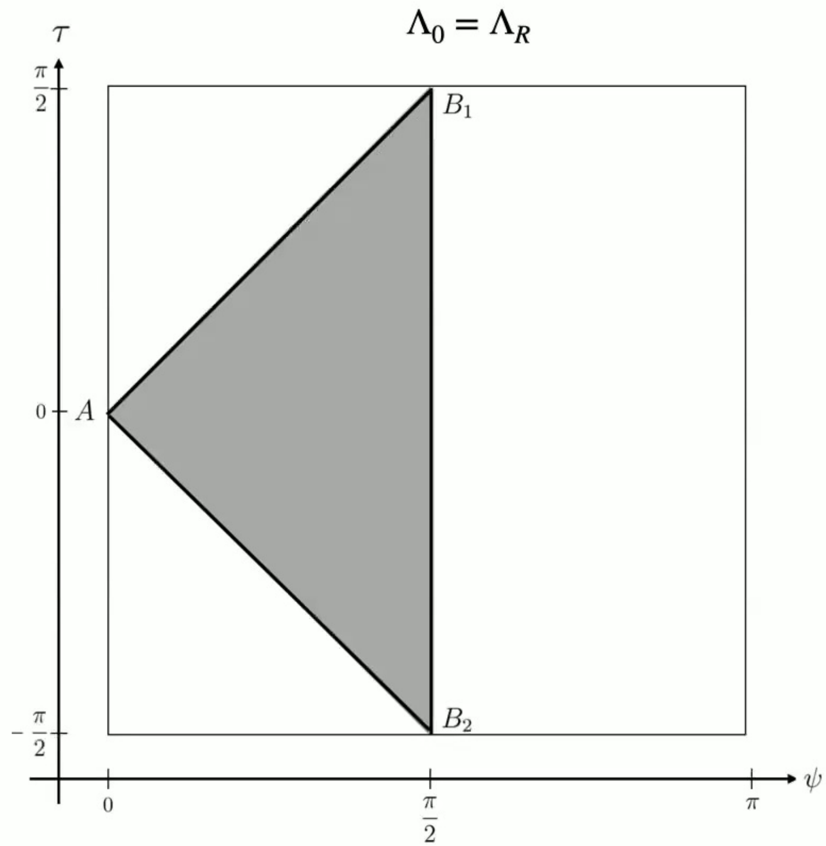


$$ds^2_{d+1} = \frac{1}{\cos^2(\tau)}\left[-d\tau^2 + d\psi^2 + \sin^2(\psi)\, d\Omega^2_{d+1}\right]$$



dS_{d+1}


$\Lambda_0 > \Lambda_R$
τ
$\frac{\pi}{2}$
0
$-\frac{\pi}{2}$
A
B_1
B_2
0
$\frac{\pi}{2}$
π
ψ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τ
$\frac{\pi}{2}$
0
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A
B_1
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0
$\frac{\pi}{2}$
π
ψ

dS/CFT correspondence

$k \to 0$ inner points $B_1 B_2$

At B_1 and B_2 the actions Γ_k with $k = k(\xi)$ and $t \to \pm\infty$, $\xi \to 0$, all seem to "meet".

Critical phenomenon at the UV fixed point

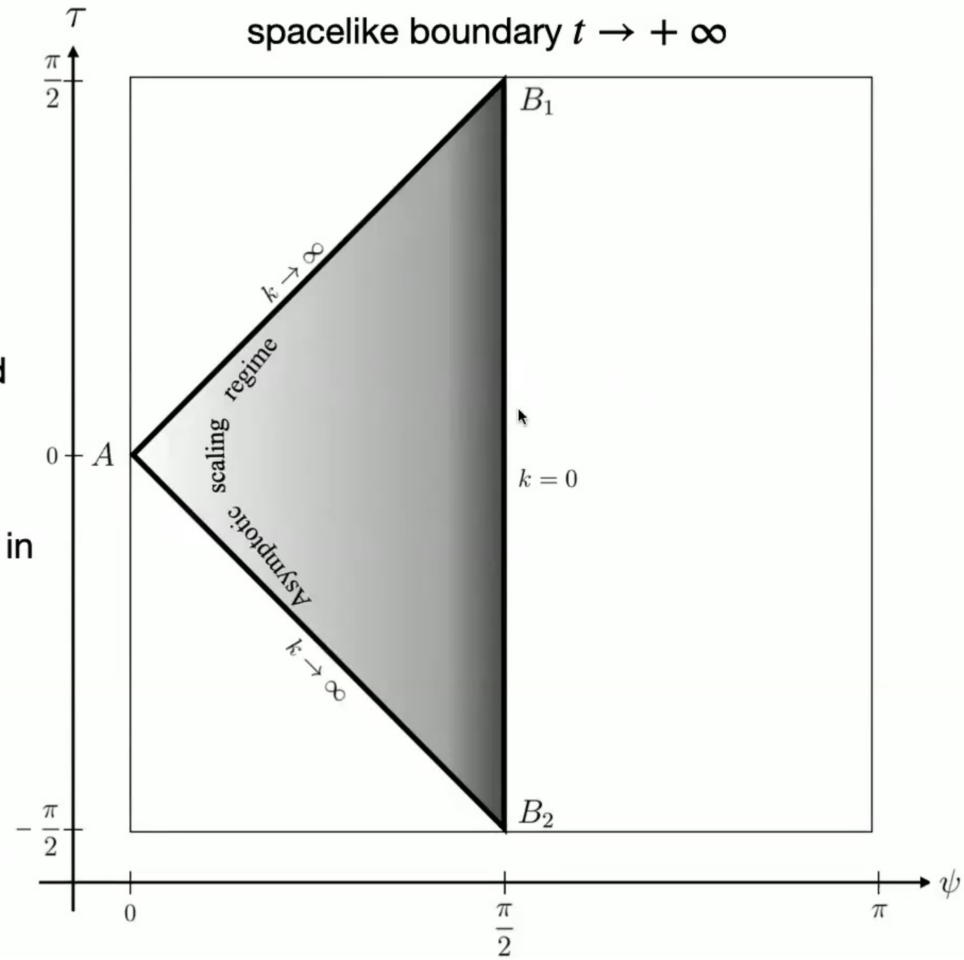
All Effective Action functionals Γ_k are of equal importance, and that therefore fluctuations on all scales are equally important.

The 3-spheres at $t \to \pm\infty$ host a 3D CFT which obtains from the Effective Average Action by evaluating it in the early/late time regime.

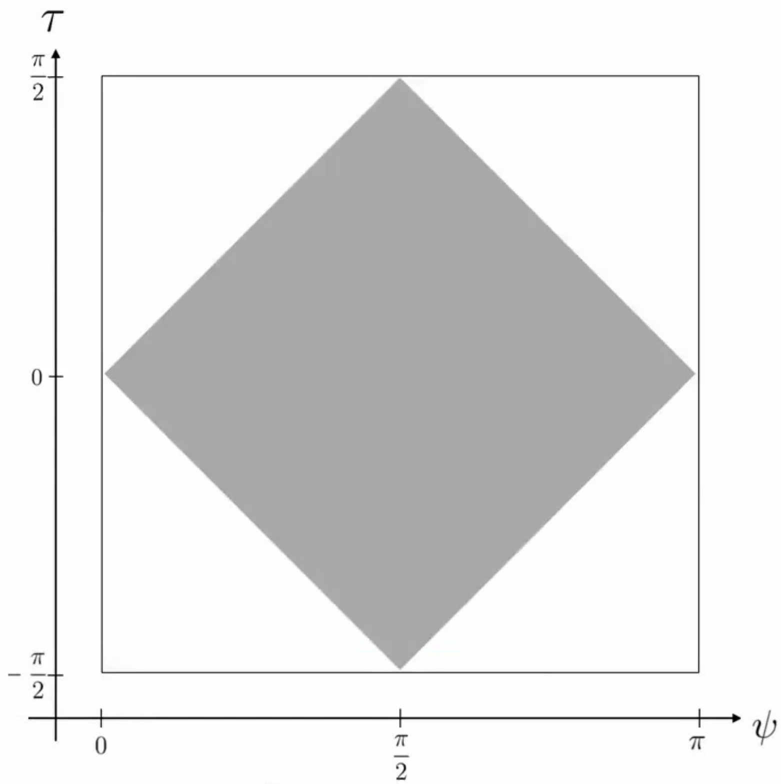
dS/CFT correspondence

Strömінger 2001

S^3



Global structure and dS connection



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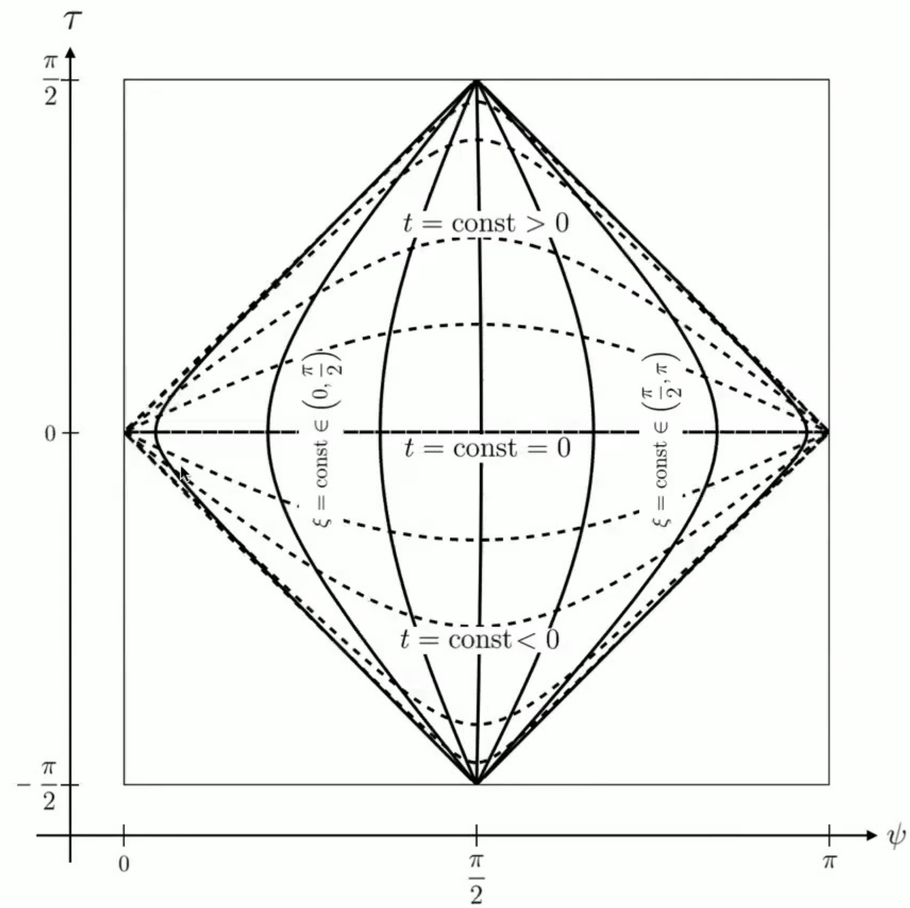

dS/CFT correspondence

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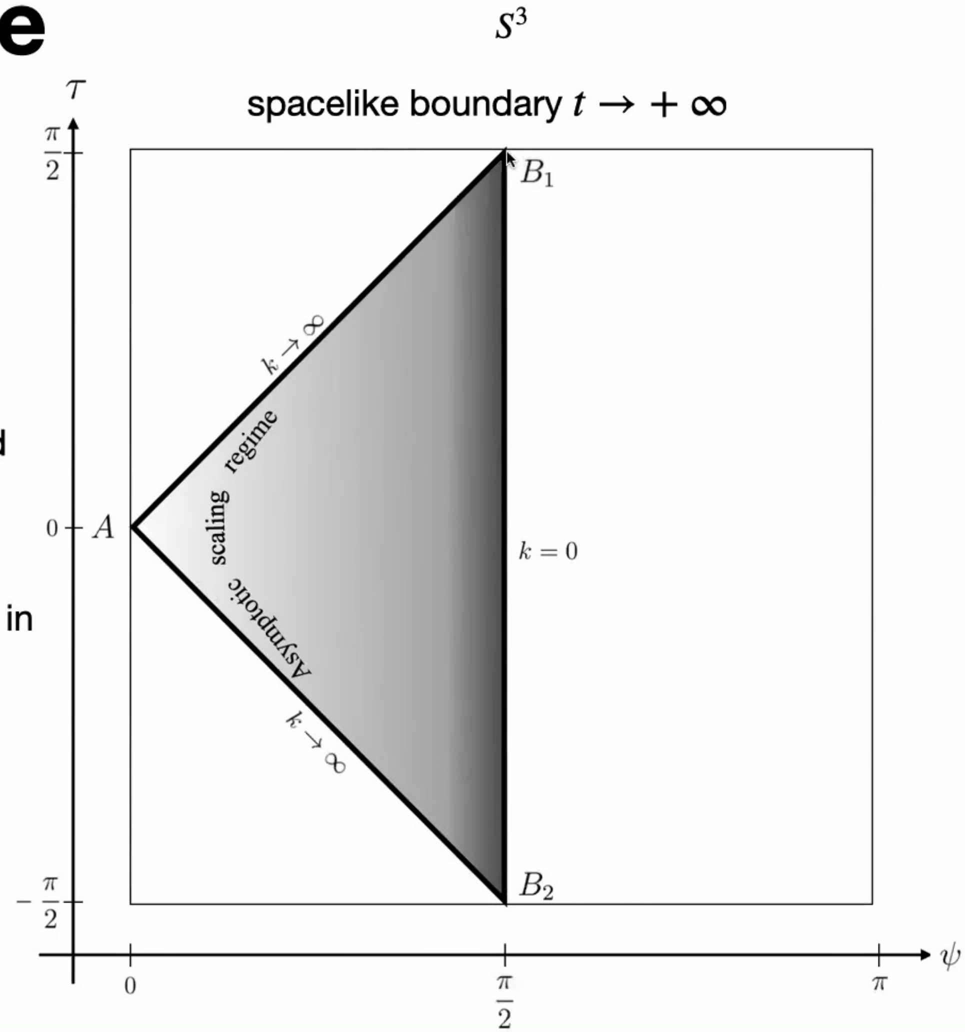
Strominger 2001



Conclusions & Outlook

Various forms of (A)dS/CFT correspondences as specific solutions in nonperturbatively renormalized Quantum Einstein Gravity

Conclusions & Outlook

Various forms of (A)dS/CFT correspondences
as specific solutions in nonperturbatively renormalized Quantum Einstein Gravity

Technical open questions:

CFT at the fixed point

“Downwards holography”: Reconstruction of the effective action.

Generalizations and future work:

Different truncations and matter models → Asymptotical (A)dS embeddings

The other way around? Holographic RG to proof Asymptotic Safety?

Thank you for your attention.

Renata Ferrero & Martin Reuter

PERIMETER INSTITUTE

Based on arXiv:2103.15709 and *work in progress - STAY TUNED!*

Quantum Gravity Seminar, Perimeter Institute - May, 19th 2022

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